In a democracy, Bayrou would have won. Application of the Borda Fixed Point method to the 2007 French presidential elections

Colignatus, Thomas

Thomas Cool Consultancy Econometrics

10 May 2007

In a democracy, Bayrou would have won

Application of the Borda Fixed Point method to the 2007 French presidential elections

Thomas Colignatus
http://www.dataweb.nl/~cool
June 27 2007

■ Summary

Democratic nations are advised to have parliaments select the chief executive by the Borda Fixed Point method. The current practice of having direct popular elections using systems that have originated in history is inoptimal and actually quite undemocratic since winners are selected who don't reflect the national sentiment. The paper gives the example of the 2007 French presidential elections. Under the currently and historically grown system of run-off plurality Sarkozy got elected while the more democratic method of Borda Fixed Point would have generated Bayrou. The example uses reasonable assumptions on underlying micro preferences. The parliamentary vote that followed the presidential vote and that annihilated Bayrou increased the paradox but not the principles involved.

Introduction

What would be a proper democratic voting system ? Given the widely differing opinions of voters and the long lists of candidates, it is generally advisable to use indirect representative democracy and proportional representation. In parliamentary elections, parties advocate their preferences, the popular vote determines party sizes, whereupon the elected party professionals can arrive at the final choice of the executive by using both bargaining and more complex voting schemes, in particular the Borda Fixed Point method. In this manner the information overload is reduced, both for voters who don't have to think about long lists of candidates and for voting mechanisms that don't have to calculate with millions of different preferences. Representative democracy also allows for bargaining that allows for optimal compromises.

France in the Spring of 2007 would seem to provide a good testing ground for this theory since there were both presidential and parliamentary elections. The French first had the direct presidential vote on April 22 and May 6 and only secondly the parliamentary vote on June 10 and 17. This order turns the parliamentary vote into a rally for support of the just elected president. Also, the elections were run-off systems that do not generate proportionality. By consequence, France showed two voting paradoxes. The first paradox is that Sarkozy got elected president while it should have been Bayrou, the second paradox is that the parliamentary vote, that followed the presidential one, was in support of Sarkozy and almost annihilated Bayrou, and thus did not expose the first paradox. Thus what happened was that a president got elected by a wrong system and that subsequently popular opinions were somewhat adjusted towards that result, by a sufficient amount such that the subsequently wrong system seemed to generate that support.

These are fundamental issues. One might hold that the election of the executive is a different one than the election of the legislative. Or one might hold that voters would be rational under any voting system so that they would adjust their strategies accordingly so that the final result would always be the same. The literature on voting theory is huge, starting, incidently with the French theorists Borda and Condorcet.

This paper has a limited objective. It will take the first paragraph as the summary conclusion of the literature on "voting theory for democracy". It will subsequently concentrate on the French presidential election as an instance of the "direct single seat election" where a body of voters has to select a single item from a list of candidates. It will use the French data and compare the current French system with the Borda Fixed Point method. **Appendix A** contains some other direct single seat election methods. Subsequently, the paper will conclude and close the main point of this paper. Which main point is that, in a democracy, we would have parliaments that use the Borda Fixed Point to select the executive. This limited objective of this paper is fully supported by Colignatus (2007b) "Voting theory for democracy", 2nd edtion, and it uses the voting routines in "The Economics Pack" which routines only exist for direct single seat elections.

For completeness, **Appendix B** collects some notes on the French parliamentary elections. The Economics Pack currently contains no routines for multiple seat elections. It is not likely that these will be created in the near future. These would be superfluous for a national proportional representation or for a district single representative election or for some combinations of those. These would be rather complex for other cases and likely depend upon local ideosyncracies.

Direct single seat elections

In a direct single seat election, there are (1) a list of items to select one off, (2) voters, e.g. partitioned in parties with weights, (3) the preferences of the voters. Voters can cheat on their preferences and a first way to limit the scope for cheating is to allow for only ordinal preferences. This paper develops a voting example using the 2007 presidential elections in France. It is an example only, and the discussion below will emphasize the limitations of the example.

The selection of a winner in a national election depends not only upon the person's qualities but also upon the voting method. In the 2007 French presidential elections Sarkozy was selected by the current voting method but it should not come as a surprise that under another system Bayrou would have won. What might come as a surprise however is that the current system is not really democratic and that there is a democratic system that would have favoured Bayrou.

The current French presidential election system is called run-off plurality. The two candidates with the most votes of the first round go on to the second round. The run-off plurality scheme has the property of destroying voter preferences, and a ruthless cut-off is used to achieve the simplicity of a final binary choice. An alternative is the Borda Fixed Point method. That method (i) uses all preferences, (ii) allows for some intensity of preference indicated by rank order, (iii)

includes the condition that the winner should also win from its main contender, (iv) finds this main contender under the counterfactual that the winner would not partake. By the current system, Sarkozy got elected. By the Borda Fixed Point method Bayrou would have been elected.

What is relevant for the present discussion are not the candidates and their qualities but the properties of these methods. Run-off plurality is less democratic than Borda Fixed Point. In fact, the system used in France is quite undemocratic since the voters get a president that they would not prefer under optimal voting conditions.

This paper does not discuss historical issues. They may be briefly indicated for perspective. For historical reasons, some countries still use direct presidential elections. Their methods of selection create all kinds of voting paradoxes and frequently lead to choices that do not properly reflect national sentiment. The main voting paradox is that the "popular vote" is said to serve the interests of the voters while instead that direct method goes against the voters' interest. Historically, the current suboptimal situation in national elections can best be explained by a serious deficiency by voting theorists. The explanation is stated in Colignatus (2007b) and can be briefly summarized as follows. Starting with Kenneth Arrow in 1951 voting theorists have emphasized impossibilities and paradoxes in voting instead of designing systems that would work. This has resulted into a literature that is pervasive in its cynicism and anti-democractic inclination. Some authors even advise dictatorship. In a massive "betrayal by the clerks", voting theorists at the academia have entrenched points of view and are not open to the idea of optimal voting methods. As a result, democracy suffers greatly, not only in national elections but also in, say, union elections or company board elections. It helps to have examples that show avenues for improvement.

The data

For voting we need items (candidates), voter preferences, and voter weights.

The following data on France have been retrieved from Wikipedia (2007). This internet encyclopedia can be unstable, both over long periods of time (a shift of interest) and even at freak moments (a hacker), but the retrieved data fit some other reports in the media. The following table summarizes the data in a useful format.

The voting items are in the rows and the voting rounds are in the columns. The main contenders are Sarkozy and Royal, who move on to the second round. The third column gives an estimate how the Bayrou vote of the first round was split over Sarkozy and Royal in the second round, with a remainder of either blank or spoilt vote ("Zero2"). This table expresses an assumption that all votes for Le Pen in the first round went to Sarkozy in the second round, so that some 3.7 million from Bayrou's first round helped Sarkozy in the second round. The table also expresses the assumption that all votes of the smaller leftist groups in the first round went to Royal in the second round, so that some 2.6 million of Bayrou's first round helped Royal in the second round. The number of voters in the 2nd round was also a bit larger, with the difference indicated in the row "New".

DataTable

	Round1	Round2	Bayrou2nd
Sarkozy	11448663	18983408	–
LePen	3834530	–	–
Sarkozy2	–	–	3700215
Royal	9500112	16790611	–
Leftist	4695327	–	–
Royal2	–	–	2595172
Bayrou	6820119	–	–
Other	420645	–	–
Valid	36719396	35774019	–
Zero	534846	1569450	–
Zero2	–	–	524732
New	–	–	89227
Total	37254242	37343469	–

The lower rows give blank or spoilt votes and people abstaining in the first round and joining up in the second round. The available data only give a net number and it may be that even more people joined while compensating for others dropping out. Overall turnout was about 84% so potentially there can be quite some flows here. The sizable increase in blank and spoilt votes in the second round is a bit curious since the whole system is intended such that there are only two options available in that second round and thus it makes little sense to try to insert a third option - though people apparently did.

Assumptions on preferences

Above assumptions already show that we lack proper data on the preferences. The available data are at the aggregate level, give us only a first and second choice for the total. At the micro level anything might be possible. This means that the discussion here is only indicative. But the example remains sufficiently clear for our stated purpose, to show that the current French voting system is inoptimal (even though it is better than the US system). Any example suffices, also a fabricated one. For the 2007 French presidential elections we can make some reasonable assumptions so that the example becomes a bit more realistic.

The following assumptions namely seem reasonable approximations. In this section we express the preferences by lists of names of decreasing preference. (Eventually the voting routines use lists with increasing preferences.)

(1) For voters on Sarkozy in the first round, perhaps a large share would vote for LePen as second choice. How many, is unknown. It is not so meaningful to use Le Pen's score against Chirac some years ago, since that was another candidate, and some years ago. We can insert an arbitrary split, say with 2 million voters. The majority of Sarkozy's voters would put Le Pen at the bottom of the list.

```
x = 2000000;
sA = {11448663 - x,
   {Sarkozy, Bayrou, Other, Zero, Abstain, Royal, LePen, Leftist}};
sB = {x, {Sarkozy, LePen, Bayrou, Other, Zero, Abstain, Royal, Leftist}};
```

(2) The voters for Le Pen would prefer Sarkozy in the second round and Bayrou above others.

```
l = {3834530, {LePen, Sarkozy, Bayrou, Other, Zero, Abstain, Royal, Leftist}};
```

(3) The Royal backing might have some subgroups who put Sarkozy or even LePen in second place. We neglect the possibility of such subgroups however, selecting only one single preference, that puts Bayrou in third place.

```
r = {9500112, {Royal, Leftist, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}};
```

(4) The extreme left might rather abstain than vote for Royal or Bayrou. But overall it seems reasonable that we assign Royal to the second position and Bayrou to the third.

```
left =
  {4695327, {Leftist, Royal, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}};
```

(5) The Bayrou vote is the one that we need to split.

```
bA = {3700215, {Bayrou, Sarkozy, Other, Zero, Abstain, Royal, LePen,
Leftist}};
bB = {2595172, {Bayrou, Royal, Other, Zero, Abstain, Sarkozy, LePen,
Leftist}};
bC = {524732, {Bayrou, Zero, Abstain, Sarkozy, Royal, Other, LePen,
Leftist}};
```

(6) For the other vote on the right we can assume the most interesting case that their second vote is for Sarkozy.

```
o = {420645, {Other, Sarkozy, Bayrou, Zero, Abstain, LePen, Royal, Leftist}};
```

(7) The zero (blank or spoilt) vote of the first round is marginal and gets an arbitray allocation. But they are not motivated to vote for the main contenders, so those drop to the lowest positions.

```
z = {534846, {Zero, Abstain, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}};
```

(8) The people who abstained in the first round but joined in the second round are a curious lot. There can all kinds of psychological considerations here, but the simplest one is that their second vote is a protest, where they move from abstinention to zero. Note though that this is a net number, so the true process might be more complex.

```
a = {89227, {Abstain, Zero, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}};
```

In summary, we have these assumptions on the preferences, reading a high value on the left and a low value on the right.

```
PrefData = {sA, sB, l, r, left, bA, bB, bC, o, z, a}
```

9448663	{Sarkozy, Bayrou, Other, Zero, Abstain, Royal, LePen, Leftist}
2000000	{Sarkozy, LePen, Bayrou, Other, Zero, Abstain, Royal, Leftist}
3834530	{LePen, Sarkozy, Bayrou, Other, Zero, Abstain, Royal, Leftist}
9500112	{Royal, Leftist, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}
4695327	{Leftist, Royal, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}
3700215	{Bayrou, Sarkozy, Other, Zero, Abstain, Royal, LePen, Leftist}
2595172	{Bayrou, Royal, Other, Zero, Abstain, Sarkozy, LePen, Leftist}
524732	{Bayrou, Zero, Abstain, Sarkozy, Royal, Other, LePen, Leftist}
420645	{Other, Sarkozy, Bayrou, Zero, Abstain, LePen, Royal, Leftist}
534846	{Zero, Abstain, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}
89227	{Abstain, Zero, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}

Setting up the voting problem

We already mentioned the items:

```
Items = {Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain};
```

We select the votes from the PrefData and check that their total is indeed the total of the second round.

```
Votes = First /@ PrefData
```

{9448663, 2000000, 3834530, 9500112, 4695327, 3700215, 2595172, 524732, 420645, 534846, 89227}

```
% // Add
```

37343469

For the voting routines we assign scores 1 to 8 to the candidates, giving the highest value to the candidate of the highest preference. The order of the Items determines where a score is put. For example, the voters for Sarkozy will assign value 8 to the first position, Sarkozy, 7 to their second choice Bayrou in the 5th position, and so on.

```
prefs = PrefToList /@ (Pref @@ # &) /@ Reverse /@ Last /@ PrefData
```

$$\begin{pmatrix} 8 & 2 & 3 & 1 & 7 & 6 & 5 & 4 \\ 8 & 7 & 2 & 1 & 6 & 5 & 4 & 3 \\ 7 & 8 & 2 & 1 & 6 & 5 & 4 & 3 \\ 2 & 1 & 8 & 7 & 6 & 5 & 4 & 3 \\ 2 & 1 & 7 & 8 & 6 & 5 & 4 & 3 \\ 7 & 2 & 3 & 1 & 8 & 6 & 5 & 4 \\ 3 & 2 & 7 & 1 & 8 & 6 & 5 & 4 \\ 5 & 2 & 4 & 1 & 8 & 3 & 7 & 6 \\ 7 & 3 & 2 & 1 & 6 & 8 & 5 & 4 \\ 1 & 3 & 2 & 5 & 4 & 6 & 8 & 7 \\ 1 & 3 & 2 & 5 & 4 & 6 & 7 & 8 \end{pmatrix}$$

This gives all elements to set up the voting problem. The votes will be expressed as ratio's summing to 1.

```
SetVotingProblem[Votes, Items, prefs]
```

$$\left\{\text{Number of Voters} \to 11,\ \text{Number of items} \to 8,\ \text{Votes are nonnegative and add up to 1} \to \text{True},\right.$$
$$\text{Preferences fit the numbers of Voters and Items} \to \text{True},\ \text{Type of scale} \to \text{Ordinal},$$
$$\text{Preferences give a proper ordering} \to \text{True},\ \text{Preferences add up to} \to \{36\},$$
$$\left.\text{Items} \to \{\text{Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain}\},\ \text{Votes} \to \left\{\frac{9448663}{37343469}, \frac{2000000}{37343469}, \frac{3834530}{37343469}, \frac{3166704}{12447823}, \frac{1565109}{12447823}, \frac{1233405}{12447823}, \frac{2595172}{37343469}, \frac{524732}{37343469}, \frac{140215}{12447823}, \frac{178282}{12447823}, \frac{89227}{37343469}\right\}\right\}$$

The run-off plurality routine

To properly model run-off plurality we would have to account for shifts in preferences and participation between the rounds. The candidates would keep the same name but they might shift their political positions. The following routine however is simple, assumes that the numbers and preferences are exactly the same for the first and second round, and it does not account for abstination and zero votes other than including them in the items. By consequence we get slightly different numbers than the official figures. De denominators include not just the "valid" votes but also the zero and new ones. The 1.6 million "zero" votes of the second round are allocated to either Sarkozy or Royal, so that in this simulation run both get slightly more votes. (PM. To get the same result in the second round as in the official figures we might substract the 1.6 million zero votes from both rounds, and distribute this proportionally. It is needlessly complicated to do this just for this example.)

```
RunOffPlurality[] // N
```

CheckVote::adj : NumberOfItems adjusted to 2

$$\left\{\text{First} \to \left\{\text{Sum} \to \begin{pmatrix} \text{Abstain} & 0.00238936 \\ \text{Bayrou} & 0.182632 \\ \text{Leftist} & 0.125734 \\ \text{LePen} & 0.102683 \\ \text{Other} & 0.0112642 \\ \text{Royal} & 0.254398 \\ \text{Sarkozy} & 0.306577 \\ \text{Zero} & 0.0143223 \end{pmatrix}, \text{Ordering} \to \begin{pmatrix} 0.00238936 & \text{Abstain} \\ 0.0112642 & \text{Other} \\ 0.0143223 & \text{Zero} \\ 0.102683 & \text{LePen} \\ 0.125734 & \text{Leftist} \\ 0.182632 & \text{Bayrou} \\ 0.254398 & \text{Royal} \\ 0.306577 & \text{Sarkozy} \end{pmatrix}, \text{Max} \to \{\text{Sarkozy}, 0.306577\}, \text{Select} \to \{\}\right\},\right.$$

$$\left.\text{Sum} \to \begin{pmatrix} \text{Royal} & 0.466338 \\ \text{Sarkozy} & 0.533662 \end{pmatrix}, \text{Ordering} \to \begin{pmatrix} 0.466338 & \text{Royal} \\ 0.533662 & \text{Sarkozy} \end{pmatrix}, \text{Max} \to \{\text{Sarkozy}, 0.533662\}, \text{Select} \to \text{Sarkozy}\right\}$$

The Borda Fixed Point routine

Using the Borda Fixed Point routine, Bayrou is selected.

```
BordaFP[]
```

Bayrou

It appears that Bayrou would already have been selected by the Borda method itself but his position is also stable in the sense that adding or removing a candidate would not quickly change his winning status.

```
BordaAnalysis[] // N
```

$$\left\{\text{Select} \to \text{Bayrou, BordaFPQ} \to \{\text{True}\},\right.$$
$$\text{WeightTotal} \to \{4.99957, 2.53173, 4.88274, 3.47337, 6.58486, 5.444, 4.53948, 3.54426\},$$
$$\text{Position} \to (\,5.\,),\ \text{Ordering} \to \left.\begin{pmatrix} 2.53173 & \text{LePen} \\ 3.47337 & \text{Leftist} \\ 3.54426 & \text{Abstain} \\ 4.53948 & \text{Zero} \\ 4.88274 & \text{Royal} \\ 4.99957 & \text{Sarkozy} \\ 5.444 & \text{Other} \\ 6.58486 & \text{Bayrou} \end{pmatrix}\right\}$$

In this case we have lumped together all “other” candidates into one person, “who” comes as a surprising second. Perhaps our results have been overly influenced by this ? The following section shows that this is not the case.

Reducing the number of candidates

From the list of preferences we can also select just the three main contenders. In that case the preferences range from 1 to 3 instead of 1 to 8. The main conclusion does not change. The run-off plurality scheme selects Sarkozy while Borda FP selects Bayrou.

```
ReduceVotingProblem[{Sarkozy, Royal, Bayrou}]
```

```
CheckVote::adj : NumberOfItems adjusted to 3
```

$$\left\{\text{Number of Voters} \to 11,\ \text{Number of items} \to 3,\ \text{Votes are nonnegative and add up to 1} \to \text{True},\right.$$
$$\text{Preferences fit the numbers of Voters and Items} \to \text{True},\ \text{Type of scale} \to \text{Ordinal},$$
$$\text{Preferences give a proper ordering} \to \text{True},\ \text{Preferences add up to} \to \{6\},$$
$$\text{Items} \to \{\text{Sarkozy, Royal, Bayrou}\},\ \text{Votes} \to \left\{\frac{9448663}{37343469}, \frac{2000000}{37343469}, \frac{3834530}{37343469}, \frac{3166704}{12447823}, \frac{1565109}{12447823}, \frac{1233405}{12447823}, \frac{2595172}{37343469}, \frac{524732}{37343469}, \frac{140215}{12447823}, \frac{178282}{12447823}, \frac{89227}{37343469}\right\}\Big\}$$

```
Preferences
```

$$\begin{pmatrix} 3 & 1 & 2 \\ 3 & 1 & 2 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

```
RunOffPlurality[] // N
```

CheckVote::adj : NumberOfItems adjusted to 2

$$\left\{\text{First} \to \left\{\text{Sum} \to \begin{pmatrix} \text{Bayrou} & 0.199344 \\ \text{Royal} & 0.380132 \\ \text{Sarkozy} & 0.420524 \end{pmatrix}, \text{Ordering} \to \begin{pmatrix} 0.199344 & \text{Bayrou} \\ 0.380132 & \text{Royal} \\ 0.420524 & \text{Sarkozy} \end{pmatrix}, \text{Max} \to \{\text{Sarkozy}, 0.420524\}, \text{Select} \to \{\}\right\},\right.$$
$$\left.\text{Sum} \to \begin{pmatrix} \text{Royal} & 0.466338 \\ \text{Sarkozy} & 0.533662 \end{pmatrix}, \text{Ordering} \to \begin{pmatrix} 0.466338 & \text{Royal} \\ 0.533662 & \text{Sarkozy} \end{pmatrix}, \text{Max} \to \{\text{Sarkozy}, 0.533662\}, \text{Select} \to \text{Sarkozy}\right\}$$

```
BordaFP[]
```

Bayrou

```
BordaAnalysis[] // N
```

$$\left\{\text{Select} \to \text{Bayrou}, \text{BordaFPQ} \to \{\text{True}\},\right.$$
$$\left.\text{WeightTotal} \to \{1.95419, 1.84647, 2.19934\}, \text{Position} \to (3.), \text{Ordering} \to \begin{pmatrix} 1.84647 & \text{Royal} \\ 1.95419 & \text{Sarkozy} \\ 2.19934 & \text{Bayrou} \end{pmatrix}\right\}$$

Strategy

If Bayrou had made it to the second round then he would have beaten Sarkozy. One can imagine different voting strategies coming into play. Royal could have known this already and withdrawn from the race. Vanity gave her a president who may be opposite to her ideals. But perhaps if she had dropped from the race, voters would have started regarding Bayrou as a socialist candidate, so that he might have lost his overall appeal. So, Royal's supporters perhaps should have been wiser than she is and voted for Bayrou already in the first round. Perhaps many Sarkozy voters voted for Royal to make sure that she got more votes than Bayrou.

Whatever all that may be, given above preferences, Bayrou would beat Sarkozy, which is one reason why he is a Borda Fixed Point winner. The Borda Fixed Point method is not immune to strategic voting but less sensitive than current systems.

```
WinnerOfPair[Sarkozy, Bayrou]
```

Bayrou

Conclusion

Above discussion has taken aggregate data from the 2007 French presidential elections, added some arbitrary though not unreasonable assumptions on the underlying preferences, and showed that the winner depends, once the preferences are given, upon the voting method. The situation can be judged on the properties of these methods.

Democratic nations are advised to have parliaments select the chief executive by the Borda Fixed Point method, where those parliaments are proportional representations. The current practice of having direct popular elections using systems that have originated in history is inoptimal and actually quite undemocratic.

The main conclusion of this paper is for students of voting theory, and in particular for those without vested interests in current theoretical errors. Voting theory, both in the academic journals and in public statements, needs to be realigned with Colignatus (2007b) since otherwise democratic nations get wrong advice.

Appendix A: Other voting systems

■ Introduction

The body of the text compares the current French system for electing a president, that happens to be "run-off plurality", with the Borda Fixed Point method. This appendix states the results of some other systems. It is not necessary to evaluate these other systems in full since this is done in Colignatus (2007b) and in the literature in general. It may only be useful to state the current outcomes and perhaps provide a short comment.

Resetting the main data:

```
Items = {Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain};
```

```
SetVotingProblem[Votes, Items, prefs]
```

$$\Big\{\text{Number of Voters} \to 11,\ \text{Number of items} \to 8,\ \text{Votes are nonnegative and add up to 1} \to \text{True},$$
$$\text{Preferences fit the numbers of Voters and Items} \to \text{True},\ \text{Type of scale} \to \text{Ordinal},$$
$$\text{Preferences give a proper ordering} \to \text{True},\ \text{Preferences add up to} \to \{36\},$$
$$\text{Items} \to \{\text{Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain}\},\ \text{Votes} \to \Big\{\frac{9448663}{37343469}, \frac{2000000}{37343469}, \frac{3834530}{37343469},$$
$$\frac{3166704}{12447823}, \frac{1565109}{12447823}, \frac{1233405}{12447823}, \frac{2595172}{37343469}, \frac{524732}{37343469}, \frac{140215}{12447823}, \frac{178282}{12447823}, \frac{89227}{37343469}\Big\}\Big\}$$

```
StatusQuo[] = Abstain
```

Abstain

■ Borda

We already have seen the Borda result above but it can be useful to explicitly mention it just by itself.

```
Borda[]
```

Bayrou

```
BordaAnalysis[] // N
```

{Select → Bayrou, BordaFPQ → {True},
WeightTotal → {4.99957, 2.53173, 4.88274, 3.47337, 6.58486, 5.444, 4.53948, 3.54426},
Position → (5.), Ordering →

2.53173	LePen
3.47337	Leftist
3.54426	Abstain
4.53948	Zero
4.88274	Royal
4.99957	Sarkozy
5.444	Other
6.58486	Bayrou

}

■ Condorcet

Bayrou is also the Condorcet winner. There is no cycle.

```
PairwiseMajority[] // N
```

{VoteMargin → VoteMargin

0.	0.761211	0.0673237	0.206313	−0.158951	0.0447953	0.0392207	0.0392207
−0.761211	0.	−0.631569	0.206313	−0.687521	−0.687521	−0.687521	−0.687521
−0.0673237	0.631569	0.	0.715109	−0.239736	−0.0726441	−0.100747	−0.100747
−0.206313	−0.206313	−0.715109	0.	−0.206313	−0.239736	−0.239736	−0.239736
0.158951	0.687521	0.239736	0.206313	0.	0.944048	0.966577	0.966577
−0.0447953	0.687521	0.0726441	0.239736	−0.944048	0.	0.938474	0.938474
−0.0392207	0.687521	0.100747	0.239736	−0.966577	−0.938474	0.	0.995221
−0.0392207	0.687521	0.100747	0.239736	−0.966577	−0.938474	−0.995221	0.

,
1. → {StatusQuo → Abstain, Sum → {6., 1., 2., 0., 7., 5., 4., 3.}, Max → 7., Condorcet winner → Bayrou,
Pref → Pref(Leftist, LePen, Royal, Abstain, Zero, Other, Sarkozy, Bayrou),
Find → Bayrou, LastCycleTest → False, Select → Bayrou},
N → {Sum → {0.999133, −3.93655, 0.76548, −2.05326, 4.16972, 1.88801, 0.0789547, −1.91149},
Pref → Pref(LePen, Leftist, Abstain, Zero, Royal, Sarkozy, Other, Bayrou), Select → Bayrou}, All → Bayrou}

■ Plurality

Sarkozy gets the most votes but not more than 50% (when allowing for the whole field).

```
Plurality[] // N
```

$$\left\{\text{Sum} \to \begin{pmatrix} \text{Abstain} & 0.00238936 \\ \text{Bayrou} & 0.182632 \\ \text{Leftist} & 0.125734 \\ \text{LePen} & 0.102683 \\ \text{Other} & 0.0112642 \\ \text{Royal} & 0.254398 \\ \text{Sarkozy} & 0.306577 \\ \text{Zero} & 0.0143223 \end{pmatrix}, \text{Ordering} \to \begin{pmatrix} 0.00238936 & \text{Abstain} \\ 0.0112642 & \text{Other} \\ 0.0143223 & \text{Zero} \\ 0.102683 & \text{LePen} \\ 0.125734 & \text{Leftist} \\ 0.182632 & \text{Bayrou} \\ 0.254398 & \text{Royal} \\ 0.306577 & \text{Sarkozy} \end{pmatrix}, \text{Max} \to \{\text{Sarkozy}, 0.306577\}, \text{Select} \to \{\}\right\}$$

■ Approval

Under strategic behaviour approval voting reduces to Plurality. Otherwise approval voting requires a base from which approval is measured, which base can vary per individual. Assuming that the base is uniformly the same, e.g. as abstention, is a rather strong assumption. It would be misleading to present a calculation here.

■ Pareto

Pareto voting would hold that a voter could veto any result that would be worse than the status quo. In this case we would choose the status quo as individual abstention so that a veto would be immaterial. The issue only arises materially when there would be proposals on the table that infringe upon individual or minority rights.

Appendix B: Notes on the French parliamentary elections 2007

■ The system

Wikipedia (2007b): “The procedure by which deputies are elected is a mixture of first past the post and run-off systems. A candidate must take an absolute majority (more than 50%) in their constituency to win in the first round, and receive the support of at least 25% of all registered voters. Otherwise, if they get at least 12.5% of the votes of all registered voters in the first round, or are one of the top two candidates remaining, they go through to the second round, where only a simple plurality is needed to win.”

The number of seats is 577. The first round was on June 10 and the second on June 17.

■ Limitations

NRC Handelsblad (2007a): (i) districts, non proportional, run-off plurality, (ii) double mandats, “the current Assemblée has for example 269 mayors and hundres of regional parliamentarians. Only 56 members have no other governmental function. Eleven ministers are candidate in their own district.” (iii) “if a president asks a majority then he gets one”. (iv) Quote: “France is a one-party state”. (v) Premier Jospin created in 2001 the system to have first the presidential election before the parliamentary election, “in order to prevent the risk of a *cohabitation*” (president and premier of different political ideas), (vi) Quote of Christophe Barbier, editor-in-chief of L’Express: “parliament is in a permanent state of disintegration”. (vii) Quote of Guy Carcassone, Univ. Nanterre: “Parliament does not play its intended role as balancing power.” (viii) Quote of Phillipe Manière, Institut Montaigne: “France is an incomplete democracy, a monarchy that never has ended”.

NRC Handelsblad (2007b) (after the first round): (i) “President Sarkozy and premier Fillon are already certain of an all powerful majority.” (ii) “The French “agenda law”: when the elections for parliament are just after those for the president, the party of the president will win”.

Some points in the political analysis: (i) The winning president expresses all kinds of promisses and starts with sweets, saving the sour for after the parliamentary elections, (ii) politicians from the losing sides switch loyalty to get on board still just in time.

■ The results

The results have been taken from French Government (2007) and Wikipedia (2007b). By themselves, the results seem a bit opaque because the much larger abstention and the seeming swings between the rounds. In the first round 110 seats are taken, of which 98 by Sarkozy, so the results of the second round are no longer representative.

The best comparison can be made between the first rounds of both the presidential and parliamentary elections, and the final results in parliamentary seats. Percentages can be expressed in terms of the valid votes, obliterating the no-shows and protest votes. The difference of 2 million in registered voters by the way is not negligible.

DataTableParl

	Pres−1	Parl−1	Seats
Sarkozy	11448663	10289028	313
Rightist	–	1570630	32
LePen	3834530	1116005	0
Royal	9500112	6436136	186
Leftist	4695327	2818640	41
Bayrou	6820119	1981121	3
Other	420645	1811472	2
Valid	36719396	26023032	577
Zero	534846	9201921	–
Total	37254242	35224953	577

DataTableParlPerc

	Pres−1	Parl−1	Seats
Sarkozy	0.311788	0.395382	0.542461
Rightist	0	0.0603554	0.0554593
LePen	0.104428	0.0428853	0.
Royal	0.258722	0.247325	0.322357
Leftist	0.12787	0.108313	0.0710572
Bayrou	0.185736	0.0761295	0.00519931
Other	0.0114557	0.0696103	0.0034662
Valid	1.	1.	1.
Zero	0.0145658	0.353607	0
Total	1.01457	1.35361	1.

Some conclusions are:

(1) Sarkozy did not maintain the 19 million votes of the final round of the presidential elections. He even dropped to below the original support of that first round. His final result in parliament is caused by the overall drop in participation and the plurality run-off voting system.

(2) Bayrou who should have been president, i.e. based upon the original preferences revealed by the two rounds of the presidential elections, was annihilated by the same voting system (and of course the irrationality of the French voter not to recognize the right voting system, or the right strategy within the wrong system to get the same result).

(3) Significant minorities are not or inadequately represented in French parliament.

(4) With the current system of electing parlement, it would still be suboptimal to have parliament elect the president. The results are too far from proportionality to give a proper reflection of the national sentiment.

(5) Having voters go to the ballot box four times to achieve a suboptimal and undemocratic result does not report well for a country that says to be proud of its rationality and democracy.

Literature

Thomas Colignatus is the preferred name of Thomas Cool in science.

EWP references are to the Economics Working Papers Archive at the Washington University at St. Louis: http://econwpa.wustl.edu. See also http://www.dataweb.nl/~cool.

To enhance completeness, the following lists the literature in Colignatus (2007b) as well (which in itself need not be complete).

Ancot & Hughes Hallet (1984), "Establishing public preferences for coalition government: an empirical study in economic planning behavior", De Economist, p153-170

Aronson, E. (1972, 1992a), "The social animal", Freeman, sixth edition 1992

Aronson, E. (ed) (1973, 1992b), Readings about The Social Animal", Freeman, sixth edition 1992

Arrow, K, & T. Scitovsky eds. (1969), "Readings in welfare economics", Allen & Unwin

Arrow, K. & F. Hahn (1971), “General competitive analysis”, North Holland

Arrow, K. (1950), “A difficulty in the concept of social welfare”, J. of political economy, pp328-346, reprinted in Arrow & Scitovsky (1969) pp147-168

Arrow, K. (1951, 1963), “Social choice and individual values”, J. Wiley

Barrow, J.D. (1998), “Impossibility. The limits of science and the science of limits”, Vintage

Bernstein (1992), “Capital ideas”, The Free Press

Bernstein (1996), “Against the gods”, Wiley

Blanchard, O. & S. Fischer (1989), “Lectures on macroeconomics”, MIT

Bochenski (1956, 1970), “A history of formal logic”, 2nd edition, Chelsea

Csikszentmihalyi (1997), “Finding flow”, Basic Books

Colignatus (1990c), “Why a social welfare (meta) function does exit: The Arrow Impossibility Theorem for Social Choice resolved, A better analysis suggested,” internal note Central Planning Bureau, The Hague, reprinted in Colignatus (1992b) pp57-104

Colignatus (1992a), “On the reasonableness and the moral desirability of the conditions of Arrow’s Theorem”, published in Colignatus (1992b) pp273-285

Colignatus (1992b), “Definition and Reality in the general theory of political economy; Some background papers 1989-1992”, ISBN 905518-207-9, Magnana Mu Publishing and Research, Rotterdam

Colignatus (1996b), “A constitutional amendment for an Economic Supreme Court”, ewp-get/9604003

Colignatus (1997b) “The solution to Arrow’s difficulty in social welfare”, ewp-get/9707001

Colignatus (1999a), “Proper definitions for risk and uncertainty”, EconWPA ewp-get/9902002

Colignatus (2000), "Definition and Reality in the General Theory of Political Economy", Samuel van Houten Genootschap, ISBN 90-802263-2-7, JEL 2000-1325

Colignatus (2000a), "Aspects of testing (matching, ranking and rating)", ewp-get/0004004

Colignatus (2000b), "The choice on sustainability: information or the meta-SWF approach to a shift of preferences", ewp-othr/0004004

Colignatus (2000c), "The seminal contribution of Roefie Hueting to economic science: Theory and measurement of Sustainable National Income", November 16 2000, see the internet

Colignatus (2001), "Voting Theory For Democracy", first edition, ISBN-90-804774-3-5

Colignatus (2005), "Definition and Reality in the General Theory of Political Economy", 2nd edition, Dutch University Press

Colignatus (2007a), "A Logic of Exceptions", ISBN 978-90-804774-4-5

Colignatus (2007b), “Voting theory for democracy”, 2nd edition, Thomas Cool Econometrics & Consultancy

Cool (1999, 2001), “The Economics Pack, Applications for Mathematica”, Scheveningen, JEL-99-0820, ISBN 90-804774-1-9 (website update 2007)

Cullis, J. & P. Jones (1992), "Public finance & public choice", McGraw-Hill

Dahl & Lindblom (1976), “Politics, economics and welfare”, Chicago

Dahl, R (1956), “A preface to democratic theory”, Chicago

Dasgupta, A.K. & D.W. Pearce (1980), “Cost-benefit analysis”, Macmillan

DeLong, H. (1971), “A profile of mathematical logic”, Addison Wesley

Doel, J. van den, and B.C.J. van Velthoven (1990), “Democratie en welvaartstheorie”, Samson

Dopp (1969), “Formale Logik”, Benziger Verlag, Einsiedeln

Eatwell c.s. (1998), “The New Palgrave”, Macmillan (entries risk, risk aversion, Arrow' Theorem, Social Welfare)

Elo, A.E. (1978), "The Rating of Chess Players, Past and Present", Arco Publishing, Inc., New York.

Ferguson, Th. (1967), “Mathematical statistics”, Academic Press

Freeman, J. (1994), "Simulating neural networks with Mathematica", Addison-Wesley

French Government (2007), “Resultats des elections legislatives”, June 17, http://www.interieur.gouv.fr/sections/a_votre_service/resultats-elections/LG2007/FE.html, retrieved from source

Good, A. & F. Graybill (1950, 1963), "Introduction to the theory of statistics", McGraw-Hill

Hart (1961, 1997), “The concept of law”, Oxford

Hicks (1981), “Collected essays on economic theory, volume I; Wealth and welfare”, Blackwell

Hornby (1985), “Oxford advanced learner’s dictionary of current English”, Oxford

Huang, C. & P. Crooke (1997), "Mathematics and Mathematica for economists", Blackwell

Johansen (1969), “An examination of the relevance of Kenneth Arrow’s general possibility theorem for economic planning”, Economics of planning 9, pp5-41

Jorgenson, D. (1990), “Aggregate consumer behavior and the measurement of social welfare”, Econometrica 58 no 5

pp1007-1040

Luce & Raiffa (1957), “Games & decisions”, J. Wiley

Luenberger, D. (1998), “Investment science”, Oxford

Maeder, R. (1991), "Programming in Mathematica", Addison-Wesley

Mas-colell, A., M. Whinston and J. Green (1995), “Microeconomic theory”, Oxford

Moulin, H. (1988), "Axioms of cooperative decision making", Econometric Society monographs ; 15, CUP

Mueller (1989), “Public Choice II”, Cambridge

Noguchi, Asahi (1993), “General Equilibrium models”, in Varian (ed), “Economic and financial modeling with Mathematica”, Springer Telos 1993 pp 104-123

NRC-Handelsblad (2007a), “In permanente staat van ontbinding”, June 9 2007

NRC-Handelsblad (2007b), “Franse oppositie strijdt voor pluralisme”, June 11 2007

Paulus, J.A. (1990), "Ik denk, dus ik lach", Ooievaar pockets ("I think, therefore I laugh", Columbia University Press 1985)

Pikkemaat, A. (1990), “Arrow’s significance for economic science in the Netherlands”, De Economist 138 No 3 p235-255

Rafiei, D. & A. Mendelzon (2000), "What is this Page Known for? Computing Web Page Reputations", http://www9.org/w9cdrom/index.html

Rijken van Olst, H. (1969), "Inleiding tot de statistiek", Part I-II-III, Van Gorcum

Ruckeer, R. (1988), "Mind tools", Penguin

Saari, D, (internet), "The symmetry and complexity of elections" (S&C)

Saari, D, (internet), "Explaining all three alternative voting outcomes" (E3)

Saari, D, (internet), "Connecting and resolving Sen's and Arrow's theorems" (C&R)

Saari, D. (2001a), Chaotic elections, AMS, 2001, www.ams.org

Saari, D. (2001b), Decisions and Elections. Explaining the unexpected, CUP

Schrijver (1987), “Analyse van macro-economische beleidspreferenties met behulp van optimale besturingstheorie”, Research memorandum 35, CPB

Sen, A. (1970), "Collective choice and social welfare", North Holland

Sen, A. (1986), “Social choice theory,” p1073-1181 in Arrow & Intrilligator eds. (1986), “Handbook of mathematical economics, Volume III”, North Holland

Sen, A. (1999a), “Development as freedom”, Knopf

Sen, A. (1999b), “The possibility of social choice”, AER p349-366 (excluding literature)

Shone, R. (1998), "Economic dynamics", Cambridge (uses Mathematica)

Sudgen, R. (1999), “Book review: On voting. A public choice approach. By Gordon Tullock”, JEL 1715-1716 (December)

Swank, O. (1989). “The objectives of Democrats and Republicans In the United States, 1956—1984”, Rotterdam

Takayama, A. (1974), “Mathematical economics”, The Dryden Press

The Economist (2000),

The New Palgrave, Macmillan 1988 - see Eatwell

Theil, H. (1971), “Principles of econometrics”, John Wiley

Tinbergen, J. (1956), “Economic policy: principles and design”, North-Holland

Tinbergen, J. (1985), “Production, income and welfare: the search for an optimal social order”, Weatsheaf books, Harvester press

Tobin, J. (1990), “On the theory of macroeconomic policy”, Tinbergen Lecture, De economist 138 pp1-14

Varian, H. (ed) (1993), “Economic and financial modeling with Mathematica”, Springer Telos

Varian, H. (ed) (1996), “Computational economics and finance”, Springer Telos

Weingast B.R. and D. Wittman (2006), "The Oxford Handbook of Political Economy", OUP

Wikipedia (2007a), Data on the French presidential elections, http://en.wikipedia.org/wiki/Politics_of_France (Retrieved May 9 2007)

Wikipedia (2007b), Data on the French parliamentary elections, http://en.wikipedia.org/wiki/-French_legislative_election%2C_2007 (Retrieved June 27 2007)

Wilson, J.Q. (1993), “The moral sense”, The Free Press

Wolfram, S. (1992), “Mathematica”, Addison-Wesley

Wolfram, S. (1996), “Mathematica 3.0”, Cambridge