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Abstract

We develop a Kaleckian model of growth with an endogenous employment rate and investigate the features following Cassetti(2003) which has considered bargaining power of workers and firms and technical progress. We assume that both of the target wage share and the technical progress depend on the rate of change of employment rate, and they become zero in steady state. We also assume that capital accumulation is a decreasing function in employment to consider maturity which defines the present capitalism society.

From the above refinements, we get results different from Cassetti(2003). An increase in the saving rate does not make the growth rate decrease, but the utilization decrease. In addition to that, an increase in the rate of labor productivity exerts a positive impact on growth.

Key words: Income distribution, Bargaining, Growth, Technical progress, maturity

JEL classifications: E12, E22, E25

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Introduction

At the beginning, heterodox schools have developed the conflict theory of inflation and the interaction between economic growth and income distribution separately. But, gradually the researches which integrated them like Marglin (1984), Dutt (1992), Cassetti (2002, 2003), and Sasaki (2011a, 2011b, 2010), have emerged.

This paper is inspired by Cassetti (2003). Cassetti (2003) develops the model with technical progress and discusses if the standard Kaleckian results, the paradox of thrift and the direct relationship between wages and accumulation, are confirmed and the effect of technical progress.

The one special feature is that the target wage share increases directly with the rate of change of employment. This comes from the idea that workers' fear of unemployment depends on the rate of change of employment because it affects the workers' bargaining strength. We adopt the rate of change of employment rate instead of it because it is better to reveal the fear of unemployment. Even if the rate of change of employment is constant, workers' fear is supposed to increase when the rate of change of population or new entry to labor market increases and the rate of change of employment rate decreases.

The second feature is that technical progress depends on the rate of productivity. But, here too, we adopt the rate of change of employment rate instead of it because we pay attention to the aspect technical progress responds to shortages of labor like Dutt (2006) rather than Kaldor-Verdoon Law.

We also assume that capital accumulation is a decreasing function in employment to consider maturity which defines the present capitalism society, following Skott and Zipperer (2010a, 2010b). By doing so we can avoid a criticism to Kaleckian that there is no constant employment rate in steady state.

We are interested in the longer term as employment rate can be ad-

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1 We can take up Okishio (1959) as the former and Rowthorn (1981) and Dutt (1984) as the latter.
3 Lavoie (1992) adopts the same assumption. But it is different from us in that we consider that the rate of change of employment rate converges to zero in the long-run.
justed to a value\(^4\). Thus, it can be thought that our model extends the Cassetti(2003) model in the long run. We investigate how such extensions change the results in Cassetti(2003).

We begin in Section 1 with the structure of the model. The equilibrium values and the comparative statics are then shown in Section 2. Finally, in the conclusion we discuss our results.

1 The structure of the model

We fundamentally follow Cassetti(2003). We assume two social classes, capitalists and workers, goods for both consumption and investment, and demand constraint economy.

Capitalists save a constant fraction of their profit, while workers spend all their income. Thus, we get the Cambridge equation:

\[ g = sr \]  

(1)

where \( g \), \( s \), and \( r \) are the rate of growth, the saving propensity of profit earners, and the profit rate.

We can write the profit rate as

\[ r = \frac{1}{k}mu \]  

(2)

where \( k \), \( m \), and \( u \) are the capital stock to the real full capacity output which is constant, the share of profits in national income, and the actual utilization rate.

The rate of accumulation desired by firms \( g^d \) is as follows.

\[ g^d = \gamma + \epsilon u + \sigma \lambda - \mu e \quad \gamma > 0, \epsilon > 0, \sigma > 0, \mu > 0 \]  

(3)

where \( \gamma \), \( \lambda \), and \( e \) are the autonomous rate of growth of capital, the rate of productivity growth, and the employment rate. In a maturity economy employment rate can be a constraint because the increase strengthens workers vis-a-vis management, and then animal spirits may fall off, as Skott and Zipperer(2010b) pointed out\(^5\). It is meant in the fourth term of right hand in (3)\(^6\).

\(^4\)In the long run we need to consider an equilibrium value of capacity utilization. But the issue is beyond range of this paper. Refer to Skott(2011) for the concise explanation.

\(^5\)The origin of this specification is the argument on “inflation barrier” by Robinson(1962). Flaschel and Skott(2005) also adopt same one.

\(^6\)The maturity may be regarded as an inconsistency with the workers’ target income share of (8) in that it adopts the employment rate level. Although we can add the level effect to (8), we adopt the above specification for a tractability. We also have to mention that such a level effect is needed for an existence of an employment rate in steady state.
We write the endogenous technical progress function in linear form
\[
\lambda = \lambda_0 + \lambda_1 (g - \lambda - n) - \lambda_2 m \quad \lambda_0 > 0, \lambda_1 > 0, \lambda_2 > 0
\] (4)
where \( n \) is the growth rate of labor. \( g - \lambda - n \) is the rate of change of employment rate with a capacity utilization\(^7\). The second term of right hand in (4) means that a faster rate of growth in change of employment results in a faster rate of labour-augmenting technological change. The specification follows Dutt(2006). The third term of right hand in (4) means that an increase in the real wage which amounts to a decrease in the profit share accelerates the productivity pace.

When firms set the price as a mark-up on labor costs, we get
\[
p = \frac{w}{a} \left( \frac{1}{1 - m} \right)
\] (5)
where \( p, w, \) and \( a \) are price, nominal wage, and labor productivity.

Differentiating (5) with respect to time, we obtain
\[
\frac{\dot{p}}{p} = \frac{\dot{w}}{w} + \frac{\dot{m}}{1 - m} - \lambda
\] (6)

Workers set the nominal wage in purpose of limiting the firm’s profits share. When the expectation is adaptive, the rate of wage inflation is
\[
\frac{\dot{w}}{w} = \theta_w (m - m_w) \quad \theta_w > 0
\] (7)
where \( m_w \) is workers’ target income share which depends on the fear of unemployment. Because we can think that the rate of growth of employment rate shows the fear, we get
\[
m_w = v_0 - v_1 (g - \lambda - n) \quad v_0 > 0, v_1 > 0
\] (8)

Firms adopt adaptive expectations and adjust the price to get closer to the target share. The rate of price inflation is
\[
\frac{\dot{p}}{p} = \theta_f (m_f - m)
\] (9)
where \( m_f \) is firms’ target income share which is assumed to be constant \( m_0 \)\(^8\).

\(^7\)We are interested in long run. Thus, here we assume that a product market equilibrates so that there is a capacity utilization.

\(^8\)We may need to consider the expected rate of inflation in (7) and (9). It can be also thought that the endogeneity in \( m_f \) changes our results. These are the suggestions by Peter Skott. We adopt Cassetti(2003)’s assumptions as possible as we can because we weight its extension to the long run.
The dynamic equation for the employment rate with a capacity utilization is

\[ \dot{e} = g_d - \lambda - n \]  

(10)

Next, we intensify the above model.

From (4), we get

\[ \lambda = \frac{\lambda_0 + \lambda_1 (g - n) - \lambda_2 m}{1 + \lambda_1} \]  

(11)

From (3), (11), and \( g = g^d \),

\[ g = \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \epsilon u + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_2} - \mu e \right) \]  

(12)

The specification is still problematic as Skott(2012) pointed out. However, it is in beyond the purpose of this paper.

Next, from (1), (2), and (12), we get

\[ u = \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \epsilon u + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_2} - \mu e \right) \]  

(13)

When we substitute (13) for (12), we get

\[ g = \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \right) \right) + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \]  

(14)

Then, from (11) and (14), we get

\[ \lambda = \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} + \frac{\lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \gamma + \epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \right) \right\} \]  

(15)

Substituting (14) and (15) for (8), we find the relation between \( m_w, m, \) and \( e \)

\[ m_w = v_0 - v_1 \left[ \frac{1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \gamma + \epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \gamma + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \right) \right\} \right. \]  

\[ - \frac{(1 + \lambda_1) \epsilon}{1 + \lambda_1 - \sigma \lambda_1} + \sigma \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \left\} - \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - n \right] \]  

(16)
From (6), (15), and (16), the dynamic equation of profit share \( m \) becomes

\[
\dot{m} = \frac{\theta_f (m_0 - m) - \theta_w (m - v_0 - v_1 \lambda_0 - \lambda_1 n - \lambda_2 m)}{1 - m} - v_1 n + \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} \\
+ \frac{\lambda_1 - \theta_w v_1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \gamma + \epsilon \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \epsilon \lambda_0 - \lambda_1} - \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \right\}
\]

(17)

substituting (14) and (15) for (10), the dynamic equation of employment rate \( e \) is

\[
\dot{e} = \frac{1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \gamma + \epsilon \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \epsilon \lambda_0 - \lambda_1} - \frac{\lambda_0 - \lambda_1 n - \lambda_2 m}{1 + \lambda_1} - \mu e \right\}
\]

(18)

We now have seven main endogenous variables, \( u, g, \lambda, m_w, r, m, \) and \( e \) and seven equations, (2) and (13)∼(18). The relation for determination is as follows. In (17) and (18), we assume arbitrary values of \( m \) and \( e \). Thus, \( u \) in (13), \( g \) in (14), \( \lambda \) in (15), and \( m_w \) in (16) are determined. Then \( r \) is determined in (2). After the determination, \( m \) and \( e \) move in (17) and (18). Thus, the system is completed.

2 Equilibrium values and comparative statics

In this section, we induce equilibrium values and do comparative statics.

When \( \dot{m} = \dot{e} = 0 \) in (17) and (18), we get

\[
m^* = \frac{\theta_f m_0 + \theta_w v_0 + \lambda_0}{\theta_f + \theta_w + \lambda_2}
\]

(19)

From (4) and (19),

\[
\lambda^* = \lambda_0 - \lambda_2 \frac{\theta_f m_0 + \theta_w v_0 + \lambda_0}{\theta_f + \theta_w + \lambda_2}
\]

(20)

Using (20) and \( \dot{e} = 0 \),

\[
g^* = \frac{\lambda_0 (\theta_f + \theta_w) - \lambda_2 (\theta_f m_0 + \theta_w v_0)}{\theta_f + \theta_w + \lambda_2} + n
\]

(21)

substituting (21) for (1), we get

\[
r^* = \frac{1}{s} \left( \lambda_0 + n - \lambda_2 \frac{\theta_f + \theta_w + \lambda_2}{\theta_f m_0 + \theta_w v_0 + \lambda_0} \right)
\]

(22)
Using (2), (19), and (22), we get

$$u^* = \frac{k}{s} \left\{ \frac{(\lambda_0 + n)(\theta_f + \theta_w + \lambda_2)}{\theta_f m_0 + \theta_w v_0 + \lambda_0} - \lambda_2 \right\}$$

(23)

Substituting (20) and (23) for (3), we have

$$e^* = \left( \frac{\epsilon k}{sm^*} - 1 \right)(\lambda_0 - \lambda_2 m^* + n) + \sigma(\lambda_0 - \lambda_2 m^*) + \gamma$$

$$\mu$$

(24)

The relation among equilibrium values is as follows. First of all, labor-management negotiation and technical progress determine the profit share $m$. The technical progress also determines the growth rate $g$. Then the growth rate determines the profit rate $r$ and capacity utilization $u$. Finally, $g$ and $u$ determine employment $e$.

The results of comparative statics are as Table 1. We mention results to which we have to pay attention.

**P.1** An increase in the workers’ pay demand which is a fall in $v_0$ or a rise in $\theta_w$ causes an increase in the utilization, growth rate, and technical progress rate and a decrease in the profit share, employment rate, and profit rate.

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Table 1: Comparative Statics

<table>
<thead>
<tr>
<th></th>
<th>$m^*$</th>
<th>$e^*$</th>
<th>$u^*$</th>
<th>$g^*$</th>
<th>$\lambda^*$</th>
<th>$r^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$v_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$m_0$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$s$</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>+</td>
<td>±</td>
<td>±</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

9Refer to Appendix 1 for the stability condition and Appendix 2 for the explanation about calculations.
P.2 An increase in the firms’ price demand which is a rise in $\theta_f$ or $m_0$ causes an increase in the profit share, employment rate, and profit rate and a decrease in the utilization, growth rate, and technical progress rate.

The above results are symmetric. An increase in the profit share means distribution from workers to firms, so it makes the profit rate increase. But the whole demand decreases. Thus, utilization and growth rate decrease. Technical progress rate decreases because of an increase in the profit share. The employment rate decreases corresponding to an increase in the growth rate.

P.3 An increase in the saving rate causes a decrease in the employment rate, utilization rate, and profit rate. It doesn’t affect the profit share. Thus, the growth rate and technical progress don’t change.

It is the reason that the variation of saving rate doesn’t affect labor management negotiation and technical progress in our model.

P.4 Increases in the parameters of investment which are $\gamma$, $\epsilon$, and $\sigma$ cause just a decrease in the employment rate.

The variation of demand in investment is adjusted by only employment rate because real investment is determined by factors of technical progress and profit share.

P.5 Increases in the parameters of technical progress which are a rise in $\lambda_0$ and a fall in $\lambda_2$ cause an increase in the profit share, growth rate, technical progress, and profit rate.

This result also reflects on the feature in our model which factors of technical progress determine the growth rate.

3 Conclusion

We have developed a Kaleckian model of growth with an endogenous employment rate and investigated the features. Although the platform of our model is Cassetti(2003), it has some different characteristics. We are interested in longer term which there is a value of employment rate in steady
state. Thus, we assumed that both of the target wage share and the technical progress depend on the rate of change of employment rate, and they become zero in steady state. We also assumed that capital accumulation is a decreasing function in employment to consider maturity which defines the present capitalism society.

From the above refinements, we got results different from Cassetti(2003). An increase in the saving rate does not make the growth rate decrease, but the utilization decrease. This resulted from our formalization that the growth rate is determined by factors of technical progress and class conflicts in steady state. In Cassetti(2003), the growth rate decreases in the case as a canonical Kaleckian model because of no labor constraint in it. In addition to that, we got the result that an increase in the rate of labor productivity exerts a positive impact on growth. In Cassetti(2003), the effect is ambiguous because it also has the contrary effect, the slowdown in workers’ aspirations caused by the fall in the employment rate of growth. Our model has does not have that effect in steady state. The direct impact of productivity parameters on the productivity growth also dominates the indirect one through an increase in profit rate.

Then, what is the relation between our model and Cassetti(2003)? We think that Cassetti(2003) is short run and the adjustment on employment is not finished, while our model is long run and the adjustment on employment is finished.

We left out the endogenous depreciation rate which reveals “obsolescence effect” in Cassetti(2003) for simplicity. The issue should be taken on later.

Appendix 1

Elements of Jacobian in (17) and (18) are as follows.

\[ a_{11} = -\theta_f - \theta_w - \frac{\lambda_2}{1 + \lambda_1} - \theta_w v_1 \frac{\lambda_2}{1 + \lambda_1} + \frac{\lambda_1 - \theta_w v_1}{1 + \lambda_1 - \sigma \lambda_1} \]

\[ a_{12} = \frac{\lambda_1 - \theta_w v_1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ -\epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \frac{\mu}{k} - \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \mu - \mu \right\} < 0 \]
\[ a_{21} = \frac{1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \epsilon \left( \frac{\lambda_0 - \lambda_1 m - \lambda_2 m}{k} - \frac{\gamma + \sigma \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \right) - \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \left( \frac{\lambda_0}{1 + \lambda_1 - \sigma \lambda_1} - \epsilon \right) \right\} \]

\[ a_{22} = \frac{1}{1 + \lambda_1 - \sigma \lambda_1} \left\{ -\epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} \epsilon - \mu \right\} e < 0 \quad (28) \]

From (25)\(\sim\) (28), we get

\[ \text{det} = \frac{(\theta_f + \theta_w + \lambda_2) e \mu}{1 + \lambda_1 - \sigma \lambda_1} \left\{ \epsilon \frac{1 + \lambda_1}{1 + \lambda_1 - \sigma \lambda_1} + 1 \right\} > 0 \quad (29) \]

Thus, when we assume \( a_{11} < 0 \), the formula is carried by \( \text{trace} < 0 \) and the system is stable in neighborhood of the equilibrium.

**Appendix 2**

The results in Table 1 come from the equilibrium values in (19)\(\sim\)(24), but the following equations which show relation between \( m^* \) and the rest of endogenous equilibrium variables may be useful for the interpretation.

From (4) and \( \hat{e} = 0 \),

\[ \lambda^* = \lambda_0 - \lambda_2 m^* \quad (30) \]

\[ g^* = \lambda_0 - \lambda_2 m^* + n \quad (31) \]

From (1) and (31),

\[ r^* = \frac{1}{s} (\lambda_0 - \lambda_2 m^* + n) \quad (32) \]

From (1), (2), and (31),

\[ u^* = \frac{k}{s} \left( \frac{\lambda_0 + n}{m^*} - \lambda_2 \right) \quad (33) \]

We show some results of calculations which are supposed to be useful.

From (23),

\[ \frac{du^*}{d\lambda_0} = \frac{k}{s} (\theta_f + \theta_w + \lambda_2) \frac{\theta_f m_0 + \theta_w v_0 - n}{(\theta_f m_0 + \theta_w v_0 + \lambda_0)^2} \quad (34) \]

10
From (21),

\[
\frac{dg^*}{d\lambda_2} = -\frac{\theta_fm_0 + \theta_wv_0 + \lambda_0}{\theta_f + \theta_w + \lambda_2} + \lambda_2 \frac{\theta_fm_0 + \theta_wv_0 + \lambda_0}{(\theta_f + \theta_w + \lambda_2)^2}
\]

\[
= -\frac{(\theta_fm_0 + \theta_wv_0 + \lambda_0)(\theta_f + \theta_w + \lambda_2) + \lambda_2(\theta_fm_0 + \theta_wv_0 + \lambda_0)}{(\theta_f + \theta_w + \lambda_2)^2}
\]

\[
= \frac{-(\theta_fm_0 + \theta_wv_0 + \lambda_0)(\theta_f + \theta_w)}{(\theta_f + \theta_w + \lambda_2)^2} < 0 \quad (35)
\]

References


