Asset bubbles, economic growth, and a self-fulfilling financial crisis: a dynamic general equilibrium model of infinitely lived heterogeneous agents

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Asset Bubbles, Economic Growth, and a Self-fulfilling Financial Crisis: A Dynamic General Equilibrium Model of Infinitely Lived Heterogeneous Agents*

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Abstract

We develop a dynamic general equilibrium growth model with infinitely lived heterogeneous agents to describe a self-fulfilling financial crisis accompanied by an asset bubble burst as a rational expectations equilibrium. Because of financial market imperfections, asset bubbles appear under mild parameter conditions even though we assume infinitely lived agents. Although these bubbles have both a crowd-in liquidity effect and a crowd-out effect on investment, the former effect always dominates the latter. Thus, a self-fulfilling financial crisis accompanied by an asset bubble burst results in an economic recession. This phenomenon is consistent with empirical observations on financial crises in the existing literature. In addition, we present an effective government policy to avoid self-fulfilling financial crises.

Keywords: Crowd-in effect of bubbles; Financial market imperfections; Sunspots; Self-fulfilling financial crisis; Economic growth.

JEL Classification Numbers: E32; E44; O41

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1 Introduction

Over the past 20 years, numerous countries have suffered from financial crises followed by serious economic recessions. Among them, the Latin American debt crisis in the early 1980s, the Japanese asset price bubble burst in 1990, the Asian financial crisis in 1997, and the US subprime loan crisis in the late 2000s were the most severe.[1] They occurred even though major fundamental economic measures such as growth and inflation rates were fairly sound just before the crises.

Figure 1 presents the time series of the per capita real gross domestic product (GDP), stock prices, and land prices in the United States (panel A) and Japan (panel B). The deviations from the trends of these variables are plotted from 1995 to 2009 in the United States and from 1985 to 1994 in Japan, and they coincide with asset bubbles.[2] An asset bubble is defined as the difference between the fundamental and market values of an asset. In general, it is difficult to identify the fundamental value of an asset correctly. However, in most cases the bubble component exhibits an explosive movement. Thus, we assume here that the trend component of an asset price reflects the movement of its fundamental value and deviations from the trends in the stock and land prices represent their bubble components. As Figure 1 shows, the United States experienced two economic booms during this period. The first was the so-called dot-com bubble that burst in 2001. The figure shows that the burst of the stock price bubble was accompanied by an economic recession. Land prices were relatively stable during this event. The second is a real estate lending boom that resulted in the so-called subprime loan crisis from 2007 to 2008. In this crisis, the burst of the land price bubble was followed by a serious economic downturn and a sharp decline in stock prices. On the other hand, Japan experienced one economic boom during the period shown, which was led by a stock price bubble that burst in 1990. This burst was also followed by a severe economic depression.

These historical observations lead to the following questions. Do asset bubbles promote economic growth? Why do asset bubbles burst? Why does a bubble burst result in an economic recession even though fundamental variables such as the technology level and individuals' tastes do not seem to change just before a bubble burst?[3] To address these ques-

[1] Laeven and Valencia (2008) and Reinhart and Rogoff (2009) provide very useful reviews and datasets on financial crises.

[2] The deviations from the trends are computed by use of the Hodrick-Prescott filter. See Appendix for details of the data sources.

[3] To the best of our knowledge, there is little evidence that exogenous negative technological shocks triggered the bubble burst in past financial crises.

tions, we develop a dynamic general equilibrium model with infinitely lived heterogeneous agents. This research aims to describe a financial crisis accompanied by a bubble burst, which results in a severe recession, as a rational expectations equilibrium of our dynamic general equilibrium model. The financial crisis in our model is self-fulfilling in the sense that rational expectations drive an economy into a financial crisis without any changes in fundamentals such as the productivity or the individuals' tastes. In addition, we present an effective policy to avoid self-fulfilling financial crises.

Traditional growth models dealing with asset bubbles have suggested that asset bubbles crowd investment out and hinder production (e.g., Tirole, 1985; Weil, 1987; Bertocchi and Yong, 1995; Kunieda, 2008; Matsuoka and Shibata, 2012). This effect occurs because if individuals save in the form of an intrinsically useless asset instead of capital, the supply of capital in the next period decreases. Along this line, Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) have developed endogenous growth models with overlapping generations and showed that the growth rate is lower when asset bubbles appear than when no asset bubbles are present, because of the crowd-out effect on investment. We may call these growth models with asset bubbles the first generation models because only the crowd-out effect of asset bubbles on investment emerges in these models. The weak point of the first generation models is that their results are not consistent with the empirical observations that asset bubbles seem to stimulate economic growth and the bubble burst is likely to result in economic downturns, as shown in Figure 1.

In contrast, many researchers such as Kocherlakota (2009), Hirano and Yanagawa (2010), Kiyotaki and Moore (2011), Farhi and Tirole (2011), Martin and Ventura (2011), and Miao and Wang (2011) have recently developed new dynamic general equilibrium models, in which asset bubbles promote investment and economic growth, and investigated business cycles and/or financial crises that cause economic recessions.[4] The growth models analyzing this new trend may be called the second generation models because they focus on the crowd-in liquidity effect of asset bubbles on investment. Our research belongs to this new literature.

We explicitly introduce financial market imperfections into the model. The assumption of an imperfect financial market has twofold importance. The first is related to the existence of asset bubbles. A bubble on an asset is more likely to appear in an economy with an imperfect financial market than one with a perfect financial market. Theorem 3.3 in Santos

[4]Olivier (2000) also develops a dynamic general equilibrium model in which asset bubbles are growth-enhancing; however, the model does not study financial crises.

and Woodford (1997) implies that the necessary condition for an asset bubble to appear is that for any state price of the asset, the present value of the asset diverges to infinity. From our model's perspective, their theorem means that if the equilibrium interest rate is less than the growth rate of the economy, there is a possibility that an asset bubble will develop. In a financially constrained economy, the equilibrium interest rate is generically less than that in a financially unconstrained economy. This is because the demand for borrowing is smaller in a financially constrained economy than in a financially unconstrained economy. Then, Stantos and Woodford's necessary condition for the existence of an asset bubble is more likely to be satisfied in a financially constrained economy. Once asset bubbles emerge, the equilibrium interest rate increases because the market supply of capital decreases owing to the existence of asset bubbles.

The second role of financial market imperfections relates to the liquidity constraints facing investors. As a result of an agency problem in a financially constrained economy, individuals' savings cannot be necessarily invested in good projects yielding high returns, that is, investment opportunities of high-return projects are not realized in such an economy. In this situation, if an intrinsically useless asset, such as a paper asset, takes a positive value, that is, if a bubble on an intrinsically useless asset exists, then the equilibrium interest rate increases. Thus, the intrinsically useless asset, whose rate of return equals the interest rate, becomes a beneficial vehicle to store the output value from today to tomorrow.

Holding the intrinsically useless asset, liquidity constrained investors face two conflicting effects created by asset bubbles. The first is a crowd-out effect on investment. The savings of individuals in an economy are not only invested in investment projects, which produce output, but also used for holding the intrinsically useless asset. Therefore, the positively valued intrinsically useless asset crowds investment out. Holding the intrinsically useless asset crowds the low-return investment projects out because of the increased interest rate. If a positively valued intrinsically useless asset, which yields a higher interest rate than the low-return projects, is carried over to the next period and sold in the financial market to obtain production resources to invest in high-return projects, the equilibrium growth rates may increase. This is the second effect, that is, the crowd-in liquidity effect of asset bubbles. When the financial market is imperfect, the intrinsically useless asset is a beneficial store of value because it yields a higher interest rate than savings yield in an economy without asset bubbles.

In the economy of our model, the productivity of investment projects producing general goods varies among agents, implying that the agents in the economy are heterogeneous in productivity when they engage in production. In equilibrium, we obtain two steady states under certain conditions. One is a steady state in which the intrinsically useless asset has no value, which we call a *bubbleless steady state*. The other is a steady state in which the intrinsically useless asset has a positive value, which we call a *bubbly steady state*. Although both the crowd-in liquidity effect and the crowd-out effect operate in a bubbly equilibrium, the crowd-in liquidity effect always dominates the crowd-out effect in equilibrium. Thus, the growth rate in the bubbly steady state is higher than that in the bubbleless steady state. Therefore, if an asset bubble bursts, the economy will go into a recession.

The aforementioned second generation growth models consider both the crowd-out and crowd-in liquidity effects of asset bubbles in financially constrained economies. These studies can be classified into two groups depending on the modeling strategy. The first group comprising Farhi and Tirole (2011) and Martin and Ventura (2011) employs the overlapping generations framework of Samuelson (1958) and Tirole (1985) and analyzes the crowd-out and crowd-in liquidity effects of bubbles. The second group constructs infinitely lived agent models with heterogeneity across agents in order to analyze the two effects. This group includes Kocherlakota (2009), Hirano and Yanagawa (2010), Kiyotaki and Moore (2011), and Miao and Wang (2011). Among them, Miao and Wang (2011) mainly consider bubbles on a productive asset, whereas the others consider them on an intrinsically useless asset. Along the latter studies, we analyze macroeconomic implications of bubbles on an intrinsically useless asset by assuming infinitely lived agents who are heterogeneous in productivity for general goods production.

It should be noted here that all models other than ours assume a binary distribution with respect to the productivity difference. However, our model assumes that the productivity of agents is continuously distributed among agents. As a result of this continuity in the productivity distribution, we are able to investigate the global dynamics of the equilibrium interest rates.[5] Clarifying the global dynamics in the economy, we are able to derive a two-state stationary sunspot equilibrium in which a financial crisis is described as a rational expectations equilibrium without any changes in fundamental variables. Although Kocherlakota (2009), Wang and Wen (2010), Farhi and Tirole (2011), Martin and Ventura (2011), and Miao and Wang (2011) also derive a two-state stationary sunspot equilibrium, they assume from the beginning of their investigations that one state is bubbleless. In our model, however, the bub-

[5] Wang and Wen (2010) also assume the continuous productivity distribution; however, they do not analytically investigate the dynamics of the economy. Although Miao and Wang (2011) investigate the dynamics of the economy, they study only the local dynamics around the steady states. Moreover, they develop an infinite horizon model with incomplete financial markets.

bleless state endogenously appears in the sunspot equilibrium.[6] Moreover, only our model in the second generation discusses the government policy to avoid self-fulfilling financial crises, although Kocherlakota (2009) and Miao and Wang (2011) discuss the government policy to restore an economy, which experienced a bubble burst, to a healthy state.[7]

The remainder of this paper proceeds as follows. In section 2, we present a model in which agents are ex-ante homogeneous but ex-post heterogeneous because of the productivity shocks regarding general goods production, and they derive a dynamical system with respect to the equilibrium interest rate. In section 3, we discuss both the dynamics and steady states of the economy. Here we derive a condition under which an intrinsically useless asset has a positive value. In section 4, we describe a financial crisis as a rational expectations equilibrium. In section 5, we discuss how to avoid self-fulfilling financial crises, and in section 6 we present concluding remarks and discuss a relationship between the suboptimality of the market equilibrium and the double infinity of commodities and agents studied by Shell (1971).

2 Model

The economy consists of one unit measure of infinitely lived agents and an infinitely lived financial intermediary. Time is discrete and goes from 0 to ∞. As will be seen later, only highly productive agents produce general goods. General goods created at time t are used interchangeably as physical capital, financial capital, and consumption goods at time t. However, they are perishable in one period. This implies that the physical capital, which is used as input for general goods production, depreciates entirely in one period. Financial capital is used as a resource for borrowing and lending in the financial market.

While the infinitely lived agents maximize their lifetime utility, the infinitely lived financial intermediary only accommodates borrowers with loans and accepts deposits from savers to balance its balance sheet.

[6] Wang and Wen (2010) show that even rational agents are willing to invest in bubbles despite a positive probability that they will burst. They also show that by calibration exercises, changes in the perceived systemic risk can generate asset price collapse.

[7] Furthermore, Kocherlakota (2009) and Hirano and Yanagawa (2010) assume that the future value of an intrinsically useless asset can be pledged to derive the bubbly equilibrium. By contrast, we assume that only a part of the net worth currently held by the agents can be pledged.

2.1 Maximization problem

Each agent is endowed with a production function at time t such that:

$$y_t = A\Phi_{t-1}k_{t-1},$$

where y_t is general goods, k_{t-1} is physical capital, and Φ_{t-1} is productivity at time t. The productivity Φ_{t-1} is a function of a stochastic event ω_{t-1}, where $\{\omega_{t-1} \in \Omega \mid \Phi_{t-1}(\omega_{t-1}) \leq \Phi\}$ is an element of a σ-algebra $\mathcal{F}$ of a probability space $(\Omega, \mathcal{F}, P)$. In other words, $\Phi_{t-1}(\omega_{t-1})$ is a random variable received at time $t-1$. Following Angeletos (2007), we assume that the idiosyncratic productivity shocks $\Phi_0(\omega_0), \Phi_1(\omega_1), ...$ are independent and identically distributed across both time and agents (the i.i.d. assumption), implying that the distributions with respect to $\Phi_0, \Phi_1, ...$ are all the same and the stochastic events $\omega_0, \omega_1, ...$ are also independent across both time and agents. Specifically, we assume that Φ has support over $[a, b]$ or $[a, \infty)$, where $a \geq 0$ and $b < \infty$, and its cumulative distribution function is given by $G(\Phi)$, where $G(\Phi)$ is continuous, differentiable and strictly increasing on the support.

Let the histories of stochastic events and the idiosyncratic productivity shocks until time $t-1$ be respectively denoted by $\omega^{t-1} = \{\omega_0, \omega_1, ...\omega_{t-1}\}$ and $\Phi^{t-1} = \{\Phi_0, \Phi_1, ...\Phi_{t-1}\}$. Then, there exists a probability space $(\Omega^t, \mathcal{F}^t, P^t)$, which is a Cartesian product of t copies of $(\Omega, \mathcal{F}, P)$ such that $\Phi^{t-1}(\omega^{t-1})$ is a vector function of the history ω^{t-1} on $(\Omega^t, \mathcal{F}^t, P^t)$.

Because agents receive idiosyncratic productivity shocks, they are ex-post heterogeneous although ex-ante homogeneous. Note that production takes one gestation period. In other words, an idiosyncratic productivity shock Φ_{t-1} with respect to production at time t is realized at time $t-1$. It is assumed that low productivity cannot be insured before the realization of an idiosyncratic productivity shock because no insurance market exists for the idiosyncratic productivity shocks.

An agent maximizes the expected lifetime utility of

$$U_0 = E\left[\sum_{t=0}^{\infty} \beta^t \ln c_t(\omega^t) \middle| \Phi^0\right],$$

where $c_t(\omega^t)$ is consumption at time t and $E[.|\Phi^t]$ is the expected value of a variable given an information set Φ^t at time t. The flow budget constraint of the agent is given by:

$$k_t(\omega^t) + b_t(\omega^t) = A\Phi_{t-1}k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1}) - c_t(\omega^t) \quad \text{for } t \geq 1, \tag{1}$$

where b_t is a deposit if positive and a debt if negative, and r_t is the gross interest rate. At $t = 0$, the flow budget constraint is given by $k_0 + b_0 = w_0 - c_0$, where w_0 is the initial endowment that the agent holds at birth, which is common to all agents. In what follows, ω^t is omitted to save representations, unless the omission is confusing.

If an agent borrows from the financial intermediary, he/she faces a credit constraint. Following Aghion et al. (1999), Aghion and Banerjee (2005), Aghion et al. (2005), and Antrás and Caballero (2009), we assume that the credit constraint is given by

$$b_t \geq -\theta a_t,$$

where $\theta \in [0, \infty)$ is the measure of the extent of the credit constraint and $a_t := A\Phi_{t-1}k_{t-1} + r_t b_{t-1} - c_t$ is his/her saving, or the net worth remaining after he/she consumes at time t. Henceforth, we call a_t the net worth.

As a result of the credit constraint, an agent can borrow financial capital from the financial intermediary only up to θ times his/her net worth. Two types of microfoundations for the credit constraint are provided in the appendix. Because $a_t = k_t + b_t$ from Eq.(1), the credit constraint is rewritten as

$$b_t \geq -\mu k_t, \tag{2}$$

where $\mu := \theta/(1+\theta) \in [0, 1)$ is also the measure of the extent of the credit constraint. Lastly, the non-negativity constraint of physical capital is given by:

$$k_t \geq 0. \tag{3}$$

Each agent maximizes his/her expected lifetime utility U_0 subject to Eqs.(1)-(3).

2.2 Optimal behaviors of agents

While defining $\phi_t := r_{t+1}/A$, we note that it is optimal for agents with $\Phi_t > \phi_t$ to invest in a project, borrow financial capital up to the limit of the credit constraint, and engage in general goods production. In other words, the flow budget constraints of agents with $\Phi_t > \phi_t$ at time t and time $t+1$ are, respectively, given by

$$(1-\mu)k_t = A\Phi_{t-1}k_{t-1} + r_t b_{t-1} - c_t,$$

and

$$k_{t+1} + b_{t+1} = (A\Phi_t - r_{t+1}\mu)k_t - c_{t+1}.$$

Therefore, the Euler equation is obtained as follows:

$$\frac{1}{c_t} = \beta E\Big[\frac{A\Phi_t - r_{t+1}\mu}{1-\mu}\frac{1}{c_{t+1}}|\Phi^t\Big]. \tag{4}$$

Likewise, it is optimal for agents with $\Phi_t < \phi_t$ to deposit a part of their net worth with the financial intermediary without engaging in general goods production. As such, the flow budget constraints of agents with $\Phi_t < \phi_t$ at time t and time $t+1$ are, respectively, given by

$$b_t = A\Phi_{t-1}k_{t-1} + r_t b_{t-1} - c_t,$$

and

$$k_{t+1} + b_{t+1} = r_{t+1}b_t - c_{t+1}.$$

The Euler equation is obtained as follows:

$$\frac{1}{c_t} = \beta E\Big[r_{t+1}\frac{1}{c_{t+1}}\big|\Phi^t\Big]. \tag{5}$$

We note that ϕ_t is a cutoff that divides agents into investors and depositors and has a one-to-one relationship with the interest rate r_{t+1}.

By defining $\tilde{R}_{t+1} := \max\{r_{t+1}, \frac{A\Phi_t - r_{t+1}\mu}{1-\mu}\}$ and from Eqs.(4) and (5), the Euler equations and the flow budget constraints of all agents for any time t can be rewritten in an intensive form such that

$$\frac{1}{c_t} = \beta E\left[\tilde{R}_{t+1}\frac{1}{c_{t+1}}\,|\Phi^t\right] \tag{6}$$

and

$$a_{t+1} = \tilde{R}_{t+1}a_t - c_{t+1}. \tag{7}$$

Because the lifetime utility function is log-linear, it follows from Eqs.(6) and (7), and the transversality condition that the equilibrium dynamic equation of net worth a_t of an agent is given by:[8]

$$a_{t+1} = \beta\tilde{R}_{t+1}a_t. \tag{8}$$

2.3 Financial Intermediary

Following Grandmont (1983) and Rochon and Polemarchakis (2006), we assume that the financial sector is competitive and thus the representative financial intermediary cannot gain profits from its business. As assumed in the previous section, the financial intermediary imposes credit constraints on agents. Besides, it accepts deposits from agents and lends financial capital to investors. The financial intermediary purchases an intrinsically useless asset with the excess total saving.[9]

[8]Eq.(8) is obtained as follows. From Eq.(7), we have $E[a_{t+1}/c_{t+1}|\Phi^t]{=}a_tE[\tilde{R}_{t+1}/c_{t+1}|\Phi^t]{-}1$. Substituting Eq.(6) into this equation, we have $a_t/c_t{=}\beta E[a_{t+1}/c_{t+1}|\Phi^t] + \beta$. From this equation and the law of iterated expectations, we obtain $a_t/c_t{=}\beta^\tau E[a_{t+\tau}/c_{t+\tau}|\Phi^t]{+}\beta + \beta^2 + ... + \beta^\tau$. From the transversality condition, we have $\lim_{\tau\to\infty}\beta^\tau E[a_{t+\tau}/c_{t+\tau}|\Phi^t] = 0$. Therefore, $a_t/c_t = \beta/(1-\beta)$ for all $t \geq 0$ and thus $a_{t+1} = \beta\tilde{R}_{t+1}a_t$.

[9]In the current model, we assume that the financial intermediary purchases an intrinsically useless asset. This assumption is only for convenience and does not change our results.

The balance sheet of the financial intermediary is given by:

$$L_t + B_t = D_t,$$

where L_t and D_t are aggregate loans and deposits, respectively. We assume that the nominal supply of the intrinsically useless asset is constant and is given by M. Accordingly, it follows that $p_t M = B_t$ where p_t is the price of the intrinsically useless asset measured by the general goods at time t. The intrinsically useless asset is freely disposable. Therefore, B_t is non-negative. Because an asset bubble is defined as the difference between the fundamental and market values of an asset, if B_t is strictly greater than zero, we say that a bubble exists on the intrinsically useless asset. Note that for this asset to have a positive value, it must hold that $p_t/p_{t-1} \geq r_t$. Otherwise, the financial intermediary does not buy the asset. Moreover, because there is no opportunity for the financial intermediary to gain profits, it follows that $p_t/p_{t-1} = r_t$ in equilibrium if the asset has a positive value. Therefore, we have a dynamic equation with respect to B_t as follows:

$$B_t = r_t B_{t-1}. \tag{9}$$

Aggregating Eq.(1) across all agents, we find that Eq.(9) holds if and only if the goods market clears.

2.4 Aggregation

We assume that the law of large numbers can be applied to the population in the economy.

Because $a_t = \beta \tilde{R}_t a_{t-1}$ from Eq.(8), the net worth a_t of an agent at time t is given by:

$$a_t = \beta(A\Phi_{t-1}k_{t-1} + r_t b_{t-1}).$$

Note that the distributions of $\Phi_{t-1}k_{t-1}$ and $r_t b_{t-1}$ across agents are independent of the realization of Φ_t due to the i.i.d. assumption regarding $\{\Phi_t\}_{t=0}^{\infty}$. Applying the law of large numbers to the population in the economy, we obtain the aggregate net worth $\tilde{a}_t(\Phi_t)$ across the agents whose productivity realization is Φ_t as follows:

$$\begin{aligned} \tilde{a}_t(\Phi_t) := \int_{\Omega^{t-1}} a_t(\omega^t) dP^{t-1}(\omega^{t-1}) &= \beta \int_{\Omega^{t-1}} (A\Phi_{t-1}k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1})) dP^{t-1}(\omega^{t-1}). \\ &= \beta(Y_t + r_t B_{t-1}), \end{aligned} \tag{10}$$

where $Y_t := \int_{\Omega^{t-1}} A\Phi_{t-1}k_{t-1}(\omega^{t-1})dP^{t-1}(\omega^{t-1})$ is the total output and $B_{t-1} = D_{t-1} - L_{t-1} = \int_{\Omega^{t-1}} b_{t-1}(\omega^{t-1})dP^{t-1}(\omega^{t-1})$ is the intrinsically useless asset held by the financial intermediary. Likewise, because the credit constraint (2) is binding for the agents with $\Phi_t > \phi_t$, the aggregate borrowing, $-\tilde{b}_t(\Phi_t)$, across agents whose productivity realization is $\Phi_t > \phi_t$ is given by:

$$-\tilde{b}_t(\Phi_t) = \frac{\mu}{1-\mu}\tilde{a}_t(\Phi_t) = \frac{\mu\beta}{1-\mu}(Y_t + r_t B_{t-1}) \tag{11}$$

and the aggregate investment, $\tilde{k}_t(\Phi_t)$, across agents whose productivity realization is $\Phi_t > \phi_t$ is given by

$$\tilde{k}_t(\Phi_t) = \tilde{a}_t(\Phi_t) - \tilde{b}_t(\Phi_t) = \frac{\beta}{1-\mu}(Y_t + r_t B_{t-1}). \tag{12}$$

.

On the other hand, the aggregate deposit, $\tilde{b}_t(\Phi_t)$, across agents whose productivity realization is $\Phi_t < \phi_t$, which is equal to $\tilde{a}_t(\Phi_t)$, is given by

$$\tilde{b}_t(\Phi_t) = \beta(Y_t + r_t B_{t-1}). \tag{13}$$

Note that the right-hand sides of Eqs.(10)-(13) are independent of the realization of Φ_t because of the law of large numbers and the i.i.d. assumption regarding $\{\Phi_t\}_{t=0}^{\infty}$.

From the balance sheet of the representative financial intermediary, we have

$$\begin{aligned} B_t = D_t - L_t &= \int_E \tilde{b}_t(\Phi_t)dP(\omega_t) + \int_{\Omega/E} \tilde{b}_t(\Phi_t)dP(\omega_t) \\ &= \beta(Y_t + r_t B_{t-1})\frac{G(\phi_t) - \mu}{1-\mu}, \end{aligned} \tag{14}$$

where $E = \{\omega_t \in \Omega \mid \Phi_t(\omega_t) \leq \phi_t\}$. Multiplying $A\Phi_t$ on both sides of Eq.(12) and aggregating this equation across all producers, we obtain the total output Y_{t+1} as follows:

$$\begin{aligned} &\int_{\Omega/E} A\Phi_t \tilde{k}_t(\Phi_t)dP(\omega_t) = \int_{\Omega/E} \frac{\beta A \Phi_t}{1-\mu}(Y_t + r_t B_{t-1})dP(\omega_t) \\ \iff \quad & Y_{t+1} = \frac{\beta A F(\phi_t)}{1-\mu}(Y_t + r_t B_{t-1}) \end{aligned} \tag{15}$$

where $F(\phi_t) := \int_{\phi_t}^{\infty} \Phi_t dG(\Phi_t)$.

2.5 Dynamical system

The dynamical system of this economy consists of Eqs.(9), (14), and (15). It is straightforward to derive the dynamic equations for the cutoff ϕ_t and the intrinsically useless asset, respectively, as follows:

$$\frac{G(\phi_t) - \mu}{1 - \mu - \beta(G(\phi_t) - \mu)} = \frac{\phi_{t-1}(G(\phi_{t-1}) - \mu)}{\beta F(\phi_{t-1})} \tag{16}$$

and

$$B_t = A\phi_{t-1}B_{t-1}. \tag{17}$$

From Eq.(14), we note that when $G(\phi_t) = \mu$, the intrinsically useless asset has no value. In this case, the total deposit equalizes with the total loan on the balance sheet of the financial intermediary. The growth rate of the aggregate output is given by

$$\Gamma_{t+1} := \frac{Y_{t+1}}{Y_t} = \frac{A\beta F(\phi_t)}{1-\mu-\beta(G(\phi_t)-\mu)}. \tag{18}$$

In a competitive equilibrium, the economy is recursively expressed by sequences $\{\phi_t, B_t, Y_t\}$ such that for all $t \geq 1$, these three sequences satisfy the difference equations (16), (17) and (18).

3 Steady states and dynamic behavior

3.1 Steady states

If we know the dynamic behavior of ϕ_t, we know the dynamic behavior of B_t and the equilibrium growth rates of the output. Therefore, we intensively analyze Eq.(16). Note from Eq.(14) that the intrinsically useless asset has a positive value if $G(\phi_t) > \mu$ provided that Y_t is strictly greater than zero. Therefore, our discussion about Eq.(16) is focused on the case in which $G(\phi_t) \geq \mu$, and thus, we restrict the domain of the dynamical system of Eq.(16) to $[G^{-1}(\mu), \infty)$ because the intrinsically useless asset is freely disposable.

First, to investigate the existence of the steady states in Eq.(16), we define ϕ^* and ϕ^{**}, which, respectively, satisfy

$$G(\phi^*) = \mu$$
$$1-\mu-\beta(G(\phi^{**})-\mu) = \beta F(\phi^{**})/\phi^{**}.$$

Lemma 1 *Both ϕ^* and ϕ^{**} are uniquely determined.*

Proof: See the appendix.

Although ϕ^* and ϕ^{**} can be rewritten as the functions of the parameters of μ and β and the parameters of the distribution of Φ such that $\phi^*(\mu;\Theta)$ and $\phi^{**}(\mu,\beta;\Theta)$, where Θ is the parameter set of the distribution of Φ, we write just ϕ^* and ϕ^{**} to save notations.

As seen in Figure 2, the value of ϕ^{**} is determined by the intersection of the x axis and the function $H(x) := \beta F(x)/x - [1-\mu-\beta(G(x)-\mu)]$, which is decreasing with respect to x, as demonstrated in the proof of Lemma 1 in the appendix. The dynamical system of Eq.(16) has two steady-state equilibria, ϕ^* and ϕ^{**}, if and only if ϕ^{**} is strictly greater than ϕ^*. This is because the domain of the dynamical system of Eq.(16) is $[\phi^*, \infty)$ because of the free disposability of the intrinsically useless asset. As shown in Figure 2, ϕ^{**} is strictly greater than ϕ^* if and only if $H(\phi^*) > 0$, or equivalently $(1 - G(\phi^*))\phi^* < \beta F(\phi^*)$. In this case, the intrinsically useless asset has a positive value in the steady state ϕ^{**}. Meanwhile, the dynamical system of Eq.(16) has only one steady-state equilibrium if and only if $(1 - G(\phi^*))\phi^* \geq \beta F(\phi^*)$. Proposition 1 summarizes these results.

Proposition 1 *Consider the dynamical system of Eq.(16). (i) There exists only one steady state, ϕ^*, such that $G(\phi^*) = \mu$ if and only if $(1 - G(\phi^*))\phi^* \geq \beta F(\phi^*)$. Moreover, the intrinsically useless asset has no value in this steady state. (ii) There exist two steady states ϕ^* and ϕ^{**}, where $\phi^{**} > \phi^*$, such that $G(\phi^*) = \mu$ and $1-\mu-\beta(G(\phi^{**})-\mu) = \beta F(\phi^{**})/\phi^{**}$ if and only if $(1 - G(\phi^*))\phi^* < \beta F(\phi^*)$. Moreover, the intrinsically useless asset has a positive value in the steady state of ϕ^{**}, implying that asset bubbles appear in this steady state.*

Proof. The discussion just before proposition 1 demonstrates the claims of this proposition. □

In what follows, we call the steady state ϕ^{**} a bubbly steady state if it exists and the steady state ϕ^* a bubbleless steady state.

3.1.1 Examples

First, suppose that Φ has a Pareto distribution such that:

$$G(\Phi) = \begin{cases} 1 - \left(\frac{a}{\Phi}\right)^x & \text{if } \Phi \geq a \\ 0 & \text{if } 0 \leq \Phi < a, \end{cases}$$

where $x > 1$. In this case, $F(\Phi)/\Phi = x(1-G(\Phi))/(x-1)$. From Proposition 1, if $(1-\beta)x < 1$, then the bubbly and bubbleless steady states exist in the economy. There are several findings with the Pareto distribution. First, we find that the existence condition for both steady states is independent of the extent of credit constraints, $\mu \in [0, 1)$. Second, assuming that the mean of the Pareto distribution is constant and x is greater than 2, a decrease in x leads to a mean preserving spread of the Pareto distribution.[10] Therefore, if the Pareto distribution exhibits a mean preserving spread associated with a decrease in x, the bubbly steady state is more likely to appear. Intuitively, if the number of less productive agents increases as x decreases, the supply of deposits increases and the intrinsically useless asset is more likely to be held by the financial intermediary. The asset plays a role of insurance that secures a higher return to depositors in this case than when the financial intermediary does not hold it. Third, if the subjective discount factor is too small, the bubbly steady state does not appear. This is because if the agents in the economy consider their future consumption unimportant, they

[10] If $x \leq 2$, we cannot discuss a mean preserving spread of the Pareto distribution because if $x \leq 2$, the variance of the Pareto distribution does not exist.

need not store their output today for their future consumption. Therefore,the supply of deposits decreases and the financial intermediary is unlikely to hold the intrinsically useless asset.

Now, suppose that Φ has a uniform distribution such that:

$$G(\Phi) = \begin{cases} 0 & \text{if } 0 \leq \Phi < a \\ \frac{\Phi - a}{2(m-a)} & \text{if } a \leq \Phi \leq 2m - a \\ 1 & \text{if } \Phi \geq 2m - a, \end{cases}$$

where m is the mean of the distribution and the support of the distribution is $[a, 2m - a]$. In this case, $\phi^* = 2(m - a)\mu + a$ and $F(\phi^*) = (1 - \mu)[(m - a)\mu + m]$. Therefore, from Proposition 1, both the bubbly and bubbleless steady states exist if and only if $0 < \mu < \beta/(2 - \beta) - [(1 - \beta)a]/[(2 - \beta)(m - a)]$. Differing from the case of the Pareto distribution, this condition is dependent on the extent of credit constraints μ. This existence condition for asset bubbles can be discussed from the perspective of Theorem 3.3 in Santos and Woodford (1997). From Eq.(18), the equilibrium growth rate in the bubbleless steady state is computed as $\Gamma^* := A\beta[(m - a)\mu + m]$ and because $r_{t+1} = A\phi_t$, the equilibrium interest rate in the bubbleless steady state is computed as $r^* := A[2\mu(m - a) + a]$. Therefore, if μ is so small that $\mu < \beta/(2 - \beta) - [(1 - \beta)a]/[(2 - \beta)(m - a)]$, then the equilibrium interest rate in the steady state is smaller than the equilibrium growth rate. This is because the severer credit constraints reduce the demand for financial capital, and thus the equilibrium interest rate decreases. This consequence is consistent with the necessary condition for a bubble on the intrinsically useless asset to appear, which can be deduced from Theorem 3.3 in Santos and Woodford (1997).

As in the case of the Pareto distribution, if the uniform distribution exhibits a mean preserving spread induced by a decrease in a, the bubbly steady state is more likely to appear. Lastly, the effect of a decrease in the subjective discount factor β is also the same as in the case of the Pareto distribution.

We particularly find from these two examples that the appearance of a bubble on the intrinsically useless asset depends on the classes of distributions as well as the fundamental variables such as the agents' productivity, extent of credit constraints, and subjective discount factor. For instance, even though two different productivity distributions have the same mean and variance, asset bubbles can appear in one economy but not in the other if the two economies are endowed with the same fundamental variables but with the different productivity distributions.

3.2 Dynamics

Now, to investigate the dynamic behavior of the economy, we draw phase diagrams. We define functions $\Psi(\phi_t) := \frac{G(\phi_t)-\mu}{1-\mu-\beta(G(\phi_t)-\mu)}$, which is the left-hand side of Eq.(16), and $\Lambda(\phi_{t-1}) := \frac{\phi_{t-1}(G(\phi_{t-1})-\mu)}{\beta F(\phi_{t-1})}$, which is the right-hand side. $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are respectively approximated around the bubbleless steady state such that:

$$\Psi(\phi_t) \approx \frac{G'(\phi^*)}{1-G(\phi^*)}(\phi_t - \phi^*)$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{\phi^* G'(\phi^*)}{\beta F(\phi^*)}(\phi_{t-1} - \phi^*).$$

From these approximations, we obtain the local dynamical system with respect to ϕ_t around the bubbleless steady state as follows:

$$\phi_t - \phi^* = \frac{\phi^*(1-G(\phi^*))}{\beta F(\phi^*)}(\phi_{t-1} - \phi^*).$$

From this equation and Proposition 1, we note that if only the bubbleless steady state exsits in the economy, this bubbleless state is locally unstable, whereas if both bubbly and bubbleless steady states exist in the economy, the bubbleless steady state is locally stable.

On the other hand, $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are, respectively, approximated around the bubbly steady state such that

$$\Psi(\phi_t) \approx \frac{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**})-\mu)}{\beta(F(\phi^{**}))^2}(\phi_t - \phi^{**})$$

and

$$\Lambda(\phi_{t-1}) \approx \frac{[(G(\phi^{**})-\mu) + \phi^{**}G'(\phi^{**})]F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**})-\mu)}{\beta F(\phi^{**})^2}(\phi_{t-1} - \phi^{**}).$$

Therefore, the local dynamical system with respect to ϕ_t around the bubbly steady state is given by

$$\phi_t - \phi^{**} = \Big[\frac{(G(\phi^{**})-\mu)F(\phi^{**})}{\phi^{**}G'(\phi^{**})F(\phi^{**}) + (\phi^{**})^2 G'(\phi^{**})(G(\phi^{**})-\mu)} + 1\Big](\phi_{t-1} - \phi^{**}).$$

We note from this equation that if the bubbly steady state exists, it is locally unstable because the coefficient of $\phi_{t-1} - \phi^{**}$ in the right-hand side is greater than one.

The stability of the two steady states can also be investigated using phase diagrams. Both $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ are increasing functions. We can easily show that $\lim_{\phi_t \to \infty} \Psi(\phi_t) = 1/(1-\beta)$ and $\lim_{\phi_{t-1} \to \infty} \Lambda(\phi_{t-1}) = \infty$. Therefore, configurations of $\Psi(\phi_t)$ and $\Lambda(\phi_{t-1})$ and the dynamic behaviors of the economy are described in Figure 3.

From Eqs.(9) and (14), we have $p_t M = \beta[G(\phi_t) - G(\phi^*)]Y_t/[1 - \mu - \beta(G(\phi_t) - G(\phi^*))]$. Because the price of the intrinsically useless asset is non-predetermined, ϕ_t is also non-predetermined. As noted from Panel A in Figure 3, if only the bubbleless steady state exists, this steady state is unstable. Because ϕ_t is not a predetermined variable, only the bubbleless steady state is a perfect foresight equilibrium and there is no transitional dynamics in this case. The economy does not experience endogenous fluctuations caused by the agents' expectations.

On the other hand, if both the bubbly and bubbleless steady states exist, as discussed above, the bubbleless steady state is locally stable and the bubbly steady state is unstable. In this case, as seen in Panel B in Figure 3, equilibrium is locally determinate in the neighborhood of the bubbly steady state. In other words, if one focuses the analysis on the small neighborhood of ϕ^{**}, the bubbly steady state is only the perfect foresight equilibrium. This implies that even though an exogenous shock associated with fundamental variables, such as productivity and the agents' tastes, occurs, the bubbly steady state equilibrium is always achievable without any excess volatility driven by the agents' expectations. However, it is also true that if we consider the global dynamic behavior of the economy, there exist uncountably many equilibrium trajectories, originating from the left neighborhood of the bubbly steady state, which monotonically converge to the bubbleless steady state. In other words, if we consider the global dynamics of the economy, equilibrium is indeterminate and thus endogenous fluctuations caused by the agents' expectations may appear. In section 4, we investigate the possibility of a sunspot equilibrium.

3.3 Growth rate comparison

From Eq.(18), the growth rate Γ^* in the bubbleless steady state and the growth rate Γ^{**} in the bubbly steady state are computed as

$$\Gamma^* = \frac{\beta A F(\phi^*)}{1 - G(\phi^*)} \tag{19}$$

and

$$\Gamma^{**} = A\phi^{**}, \tag{20}$$

respectively.

Proposition 2 *Suppose that the bubbly steady state exists. The growth rate in the bubbly steady state is strictly greater than that in the bubbleless steady state. Moreover, the growth rate in the bubbly steady state is the highest for any $\phi_t \geq \phi^*$.*

Proof: See the appendix.

The consequences in Proposition 2 contrast with those in the first generation growth models with asset bubbles. In the first generation literature (e.g., Tirole, 1985; Grossman and Yanagawa, 1992; Saint-Paul, 1992; Futagami and Shibata, 2000), a positively valued intrinsically useless asset always reduces the growth rate in an economy because asset bubbles crowd investment out. However, in our model, there is a crowd-in liquidity effect of asset bubbles on investment as well as a crowd-out effect. From Eqs.(9) and (14), we obtain the bubble to output ratio (the B_t/Y_t ratio) as follows:

$$\frac{B_t}{Y_t} = \frac{\beta(G(\phi_t) - G(\phi^*))}{1 - \mu - \beta(G(\phi_t) - G(\phi^*))}.$$

We find from this equation that the further is the distance between ϕ_t and ϕ^*, the greater is the B_t/Y_t ratio. Proposition 2 implies that if ϕ_t is greater than ϕ^* but less than ϕ^{**}, the crowd-in liquidity effect of asset bubbles dominates the crowd-out effect, whereas if ϕ_t is greater than ϕ^{**}, the crowd-out effect bubbles dominates the crowd-in liquidity effect. However, there is no equilibrium such that $\phi_t > \phi^{**}$ as observed in Panel B in Figure 3. Therefore, if the bubbly steady state exists, the crowd-in liquidity effect always dominates the crowd-out effect in equilibrium. In other words, the greater is the B_t/Y_t ratio, the higher is the growth rate in equilibrium if the bubbly steady state exists.

In the financial market, in response to the crowd-out effect of asset bubbles, the interest rate increases. Although the increase in the equilibrium interest rate reduces the number of investors, the existence of asset bubbles crowds the less productive agents out, and they become depositors. Moreover, these depositors acquire higher returns when asset bubbles exist than when they do not exist. Some depositors who are less productive agents today will become productive investors tomorrow. This implies that agents who have turned into highly productive investors have enough liquidity, which can be used for their investment projects because of the higher returns to deposits. This is the crowd-in liquidity effect of asset bubbles. On the other hand, if asset bubbles do not exists, the less productive agents will obtain only the lower returns to deposits. In this case, these agents who will turn into highly productive agents do not have enough liquidity to invest in projects. The crowd-in liquidity effect of asset bubbles is similarly discussed in the second generation models with asset bubbles (e.g., Hirano and Yanagawa, 2010; Farhi and Tirole, 2011; Martin and Ventura, 2011; and Miao and Wang, 2011).

We conclude this section with a remark on the constrained dynamic efficiency.[11] The bubbleless steady state is constrained dynamically inefficient if both the bubbly and bubbleless steady states exist. Because $\beta c_t = (1-\beta)a_t$ for each agent, per capita consumption is obtained as $\tilde{c}_t := (1-\beta)(Y_t + r_t B_{t-1}) = (1-\beta)(Y_t + B_t)$ for $t \geq 1$, where the second equality holds because of Eq.(9) and $\tilde{c}_0 = (1-\beta)w_0$. We note that the growth rates of Y_t and $\tilde{c}_t$ are the same in the bubbleless steady state and those of Y_t, B_t, and $\tilde{c}_t$ are the same in the bubbly steady state. From Proposition 2, the growth rate in the bubbly steady state is the highest for any $\phi_t \geq \phi^*$. Obviously, this means that allocative inefficiency in the bubbleless economy is corrected by the existence of asset bubbles. In Tirole (1985) the existence of asset bubbles also corrects allocative inefficiency in the bubbleless steady-state equilibrium if the bubbleless steady-state equilibrium is dynamically inefficient. However, the mechanism of the correction of inefficiency in our model differs from that in Tirole's model. While in Tirole's model only the crowd-out effect is important for correcting the allocative inefficiency, in our model, both the crowd-out and the crowd-in liquidity effects have importance. In our model, in response to the crowd-out effect, unproductive agents do not invest in projects and thus physical capital is not inefficiently used. In response to the crowd-in liquidity effect, the agents who changed into the highly productive today from the unproductive yesterday can utilize many production resources.

Unlike the model of Tirole and Farhi (2011), constrained dynamic efficiency and constrained Pareto efficiency are not the same concept in our model because of the ex-post heterogeneity of the agents.[12] Therefore, the existence of asset bubbles might not be able to lead to Pareto improvements due to the increased interest rates.

4 Sunspots and financial crisis

To investigate sunspot equilibria, we focus our analysis on the case in which the bubbly steady state appears. As discussed in the previous section, equilibrium is globally indeterminate in this case. In this section, we shall prove the existence of a two-state stationary sunspot equilibrium, which leads the economy to a financial crisis without any changes in fundamental variables such as productivity or the agents' tastes.

[11] See Kunieda (2008) and Tirole and Farhi (2011) for the definition of constrained dynamic efficiency.
[12] Here, we consider ex-post Pareto efficiency.

4.1 Sunspot variables and the equilibrium dynamics

We assume that there is a sunspot variable z_t that follows a two-state Markov process whose support is $\{0,1\}$ such that:

$$Pr(z_t = 1|z_{t-1} = 1) = \pi^a$$
$$Pr(z_t = 0|z_{t-1} = 0) = \pi^b,$$

where π^a and $\pi^b \in (0,1]$ are the transition probabilities of the Markov chain and we assume that $z_0 = 1$. Let the history of sunspot events up to time t be $z^t = \{z_0, z_1, ..., z_t\}$. The sunspots events are independent of the idiosyncratic productivity shocks and they are common among agents.

We assume that agents have rational expectations for the interest rate $r_t(z_t)$ given the sunspot history z^{t-1}. Because the stochastic process of $\{z_t\}_{t=0}^{\infty}$ is a Markov process, the rationally expected value of $r_t(z_t)$ is given by $E[r_t(z_t)|z_{t-1}]$. This implies that agents with $A\Phi_{t-1} > E[r_t(z_t)|z_{t-1}]$ invest in a project at time $t-1$, borrowing financial capital from the financial intermediary, whereas agents with $A\Phi_{t-1} < E[r_t(z_t)|z_{t-1}]$ deposit their financial capital with the financial intermediary at time $t-1$. In other words, the cutoff for the realization of productivity, which divides agents into investors and lenders, is denoted by $\phi_{t-1} = E[r_t(z_t)|z_{t-1}]/A$. Note that although ϕ_{t-1} is a stochastic variable before the realization of z_{t-1}, it becomes a deterministic variable when z_{t-1} is fulfilled.

The maximization problem of an agent is almost the same as in section 2 except that we replace notations such as $k_t(\omega^t)$ and $E(.|\Phi^t)$ with $k_t(\omega^t, z^t)$ and $E(.|\Phi^t, z^t)$, respectively. Hence, we will not insert the maximization problem here. Given the sunspot history z^{t-1} and from the solutions of the maximization problem and the market clearing conditions, the dynamic behavior of the economy is expressed by almost the same equations as Eqs.(9), (14), and (15) because of the law of large numbers and the i.i.d. assumption regarding Φ_t, except that $r_t(z_t)$ is a random variable.[13] We insert the new equations regarding the aggregate variables below:

$$B_t(z^t) = r_t(z_t)B_{t-1}(z^{t-1}), \tag{21}$$

[13]Now that the rational expectations for the future interest rate $A\phi_{t-1} = E[r_t(z_t)|z_{t-1}]$ differs from the fulfilled interest rate $r_t(z_t)$, some investors may default. We avoid such a situation by imposing the following assumption. Because the net return of an investor at time t is given by $A\Phi_{t-1} - r_t(z_t)\mu$, if $A\phi_{t-1} \geq r_t(z_t)\mu$, no investors default. Because the minimum value that ϕ_{t-1} can take in equilibrium is ϕ^*, the sufficient condition for no investors to default is $\phi^* \geq \mu r_t(z_t)/A$. We assume this condition in what follows. In our two-state stationary sunspot equilibrium, it holds that $\sup r_t(z_t) = A\phi^{**}$. Thus, we may assume that $\phi^* \geq \mu\phi^{**}$. When μ is relatively small, this condition is satisfied. For instance, in the case of the Pareto distribution discussed in section 3.1, if $\mu^x \leq (1-\beta)(x-1)/\beta$, this assumption holds.

$$B_t(z^t) = \beta(Y_t(z^{t-1}) + r_t(z_t)B_{t-1}(z^{t-1}))\frac{G(\phi_t) - \mu}{1-\mu}, \tag{22}$$

$$Y_{t+1}(z^t) = \frac{\beta AF(\phi_t)}{1-\mu}(Y_t(z^{t-1})) + r_t(z_t)B_{t-1}(z^{t-1}))). \tag{23}$$

4.2 Stationary sunspot equilibrium

From Eqs.(21), (22), and (23), we obtain:

$$\frac{G(\phi_t) - \mu}{1 - \mu - \beta(G(\phi_t) - \mu)} = \frac{r_t(z_t)(G(\phi_{t-1}) - \mu)}{A\beta F(\phi_{t-1})}, \tag{24}$$

and

$$\Gamma_{t+1} = Y_{t+1}(z^t)/Y_t(z^{t-1}) = \frac{A\beta\phi_t F(\phi_t)}{1 - \mu - \beta(G(\phi_t) - \mu)}, \tag{25}$$

Taking an expectation for both sides of Eq.(24) given the sunspot event z_{t-1}, we obtain:

$$E\Big[\frac{G(\phi_t) - \mu}{1 - \mu - \beta(G(\phi_t) - \mu)}\Big| z_{t-1}\Big] = \frac{\phi_{t-1}(G(\phi_{t-1}) - \mu)}{\beta F(\phi_{t-1})}. \tag{26}$$

In what follows, we consider a stationary sunspot equilibrium. In a two-state stationary sunspot equilibrium, the interest rates take only two values: $r^H = r(z_t = 1)$ and $r^L = r(z_t = 0)$, where $r^H > r^L$. For $t \geq 1$, we define the two states of ϕ_t in the stationary sunspot equilibrium as follows:

$$\phi^a \;:\; = E[r(z_t)/A | z_{t-1} = 1] = [\pi^a r^H + (1-\pi^a)r^L]/A \tag{27}$$

$$\phi^b \;:\; = E[r(z_t)/A | z_{t-1} = 0] = [(1-\pi^b)r^H + \pi^b r^L]/A, \tag{28}$$

where we assume that $\pi^a > 1 - \pi^b$ and thus $\phi^a > \phi^b$ so that the economy is initially in the high growth state.[14] We also assume that $\pi^a < 1$.[15]

Using ϕ^a and ϕ^b, Eq.(26) is rewritten as the following two equations associated with a two-state stationary sunspot equilibrium depending upon the realization of z_{t-1}:

$$\pi^a \frac{G(\phi^a) - \mu}{1 - \mu - \beta(G(\phi^a) - \mu)} + (1-\pi^a)\frac{G(\phi^b) - \mu}{1 - \mu - \beta(G(\phi^b) - \mu)} = \frac{\phi^a(G(\phi^a) - \mu)}{\beta F(\phi^a)}. \tag{29}$$

$$(1-\pi^b)\frac{G(\phi^a) - \mu}{1 - \mu - \beta(G(\phi^a) - \mu)} + \pi^b \frac{G(\phi^b) - \mu}{1 - \mu - \beta(G(\phi^b) - \mu)} = \frac{\phi^b(G(\phi^b) - \mu)}{\beta F(\phi^b)}. \tag{30}$$

Given the Markov transition probabilities, π^a and π^b, the two-state stationary sunspot equilibrium of the economy consists of four-tuple $\{\phi^a, \phi^b, r^H, r^L\}$ such that $\{\phi^a, \phi^b, r^H, r^L\}$ satisfies Eqs. (27)-(30). In the stationary sunspot equilibrium, given the sunspot history

[14]If $\phi^a < \phi^b$, we will obtain only an equilibrium such that the economy stays in the steady state $\phi^a = \phi^*$ forever because $z_0 = 1$ although we do not provide the analysis for this case.

[15]We will note from Eqs.(29) and (30) that if $\pi^a = 1$, the economy stays in the steady state ϕ^{**} forever because $z_0 = 1$.

z^{t-1}, the realization of z_t in each time recursively determines the output $Y_{t+1}(z^t)$ following Eq.(24) and the intrinsically useless asset $B_t(z^t)$ following Eq.(21).

Proposition 3 characterizes the two-state stationary sunspot equilibrium in our economy.

Proposition 3 *Suppose that* $(1 - G(\phi^*))\phi^* < \beta F(\phi^*)$, *implying that the bubbly steady state equilibrium exists without extrinsic uncertainty. Suppose also that* $1 - \pi^b < \pi^a < 1$. *Then, there exists only a two-state stationary sunspot equilibrium* $\{\phi^a, \phi^b, r^H, r^L\}$ *such that* $\phi^b = \phi^*$ *and* $\phi^a \in (\phi^*, \phi^{**})$ *with* $\pi^b = 1$, *where* $\{r^H, r^L\}$ *are functions of* $\{\phi^a, \phi^b\}$ *satisfying Eqs.(27) and (28).*

Proof: See the appendix.

For the two-state stationary sunspot equilibrium to exist, it must hold that $\pi_b = 1$ and $\phi_b = \phi^*$. The proof of Proposition 3 is illustrated in Figure 4. In particular, to obtain the transition probability, π^b, that is less than one and the state ϕ^b that is not equal to ϕ^*, $\Lambda(\phi^b)$ must be an internally dividing point between $\Psi(\phi^a)$ and $\Psi(\phi^b)$. However, this is impossible because $\Psi(\phi^a) > \Psi(\phi^b) > \Lambda(\phi^b)$. The only possible case to hold in Eq.(30) is that $\pi_b = 1$ and $\phi_b = \phi^*$.

In the two-state stationary sunspot equilibrium in the economy, one state is the bubbleless steady-state equilibrium of ϕ^*, where the growth rate Γ^* is the minimum in $\phi_t \in [\phi^*, \phi^{**}]$. Once the economy falls from the high-growth state of ϕ^a to the low-growth state of $\phi^b = \phi^*$, it is never restored to the high-growth state. The self-fulfilling financial crisis in our model accompanies the bubble burst. In the extant second generation literature on asset bubbles, such as Kocherlakota (2009), Farhi and Tirole (2011), Martin and Ventura (2011), and Miao and Wang (2011), this type of financial crisis equilibrium is investigated. However, they assume from the beginning of their investigations that one state is bubbleless. Differing from them, the bubbleless state in our stationary sunspot equilibrium endogenously appears.

We note that as π^a is close to one, the probability of the occurrence of a self-fulfilling financial crisis approaches zero and ϕ^a is close to ϕ^{**}, which leads the economy to the highest growth as shown in Proposition 2. When the US subprime loan crisis occurred in late 2000, it was said to be the century's greatest crisis. From our model's perspective, we can consider that π^a was very close but not equal to one before the subprime loan crisis.

5 Avoiding self-fulfilling financial crises

In this section, borrowing the idea proposed by Tirole (1985), we present a government policy to avoid self-fulfilling financial crises. If the bubbleless steady-state equilibrium is eliminated by a government policy and the bubbly steady-state equilibrium becomes a unique equilibrium, self-fulfilling financial crises never occur.[16] Tirole (1985) demonstrates that if the intrinsically useless asset is backed by the government, the bubbly steady-state equilibrium becomes a unique equilibrium.

We assume that the government imposes a tax on the agents' net income in Eq.(1) as follows:

$$k_t(\omega^t) + b_t(\omega^t) = [A\Phi_{t-1}k_{t-1}(\omega^{t-1}) + r_t b_{t-1}(\omega^{t-1})](1-\tau_t) - c_t(\omega^t) \quad \text{for } t \geq 1, \tag{31}$$

where $\tau_t \in (0,1)$ is the tax rate at time t, which is common across agents. From Eq.(31), the aggregate tax is given by $\tau_t(Y_t + r_t B_{t-1})$. We assume that the government purchases $\epsilon Y_t/p_t$ of the intrinsically useless asset at time t and thus the dynamic equation of B_t becomes:

$$B_t = r_t B_{t-1} - \epsilon Y_t. \tag{32}$$

Note that ϵ is independent of time and assumed to be infinitesimal. All computations to derive the equilibrium dynamics with respect to ϕ_t are almost the same as in subsection 2.4. Because the agents have to pay tax, we have $a_t = \beta \tilde{R}_t(1-\tau_t)a_{t-1}$, and thus,

$$a_t = \beta(A\Phi_{t-1}k_{t-1} + r_t b_{t-1})(1-\tau_t).$$

Therefore, we obtain

$$B_t = \beta\frac{G(\phi_t)-\mu}{1-\mu}(Y_t + r_t B_{t-1})(1-\tau_t), \tag{33}$$

and

$$Y_{t+1} = \frac{\beta A F(\phi_t)}{1-\mu}(Y_t + r_t B_{t-1})(1-\tau_t). \tag{34}$$

Assuming a balanced government budget, we have $\tau_t(Y_t + r_t B_{t-1}) = \epsilon Y_t$. Applying Eq.(32) to the government's budget constraint, we get

$$1-\tau_t = \frac{Y_t + B_t}{Y_t + B_t + \epsilon Y_t}.$$

Thus, Eqs.(33) and (34), respectively, become

$$B_t = \beta\frac{G(\phi_t)-\mu}{1-\mu}(Y_t + B_t), \tag{35}$$

and

$$Y_{t+1} = \frac{\beta A F(\phi_t)}{1-\mu}(Y_t + B_t). \tag{36}$$

[16]The idea that the government policy eliminates a bad steady-state equilibrium is employed in other areas in macroeconomics. For instance, Benhabib et al. (2002) propose a fiscal policy that eliminates the low-inflation steady state to avoid liquidity traps.

From Eqs.(32), (35) and (36) and using $r_t = A\phi_{t-1}$, we obtain:

$$\frac{(1-\epsilon)(G(\phi_t)-\mu)+\epsilon(1-\mu)/\beta}{1-\mu-\beta(G(\phi_t)-\mu)} = \frac{\phi_{t-1}(G(\phi_{t-1})-\mu)}{\beta F(\phi_{t-1})}. \tag{37}$$

We note that the right-hand side of Eq.(37) is the same as that of Eq.(16), whereas the left-hand side is different because ϵ is strictly greater than zero. Figure 5 presents a phase diagram of the dynamic behavior of ϕ_t. We find from Figure 5 that the equilibrium is globally determinate, implying that only the bubbly steady state ϕ^{***} is a unique perfect foresight equilibrium in this economy. In this case, self-fulfilling financial crises never occur. Moreover, the economy experiences an almost highest growth rate because ϵ is infinitesimal.

6 Concluding Remarks

We have described a self-fulfilling financial crisis accompanied by a bubble burst as a rational expectations equilibrium. In our model, the crowd-in liquidity effect of asset bubbles on investments dominates the crowd-out effect in equilibrium. Therefore, a self-fulfilling financial crisis always results in a severe economic recession. This result is consistent with empirical evidence observed in the history of financial crises.

We conclude this paper with a remark on the relationship between our model and the double infinity of commodities and agents that was studied by Shell (1971). The suboptimality of our growth model can be reinterpreted from the perspective of the double infinity of commodities and agents. As clarified by Cozzi (2001), even though each agent in an economy is assumed to be infinitely lived, the economy would exhibit the double infinity feature as in the overlapping generations models. To consider this more concretely, suppose that $\mu = 0$, that is, investors cannot borrow at all. If the insurance market for the agents' incomes is complete, consumption in each time is the same across agents even though they are ex-post heterogeneous in their productivity because they are ex-ante homogeneous. In this case, each agent decides once and for all on the schedule of his/her lifetime consumption and savings at time zero as in the manner of the Arrow-Debreu model. Then, equilibrium in this economy is constrained Pareto optimal. However, there is no insurance market in the economy in our model, and each agent stepwise decides on the plan of his/her consumption and savings at each point in time when he/she acquires information about his/her productivity. As a result of this stepwise decision making, an identical infinitely lived agent seems to be a newly born person today who differs from the person yesterday, given a bequest from yesterday's person. In other words, by the combination of the incomplete insurance market and the imperfect financial market, the lifetime of an infinitely lived agent is cut into countably many pieces as if he/she turned into countably many agents. In this sense, there is double infinity of commodities and agents in our economy. In response to the double infinity, asset bubbles could appear under certain parameter conditions and indeterminacy and sunspots are possible. Although asset bubbles have a role in insuring agents' incomes, they can only partially insure the incomes as demonstrated in Cozzi (2001).

Appendix

Data description for Figure 1

Data were drawn from various databases. We gathered the annual data for the United States and Japan over a period from 1980 to 2009. We obtained the data for per capita real GDP in the two countries from the Penn World Table 7.0 database created by Heston et al. (2011), which is titled "PPP Converted GDP Per Capita (Laspeyres), derived from growth rates of c, g, i, at 2005 constant prices." The data for land prices in Japan were collected from the Nationwide Urban Land Price Index database created by the Japan Real Estate Institute. In particular, we used the land price index of six major cities in the dataset. The data for land prices in the United States were assembled from the dataset titled "CSW-based price index: aggregate land data, quarterly, 1975:1-2011:1," which is created by Morris A. Davis and Jonathan Heathcote (2007). The data for the stock price in Japan were drawn from the Nikkei Indexes, which is a database for various Japanese stock price indexes created by Nikkei Inc. In particular, we used the annual data from the Nikkei Stock Average. For stock prices in the United States, we used the S&P Composite Stock Price Index. The data for this index were downloaded from Robert Shiller's web page at: http://www.econ.yale.edu/shiller/data.htm. To obtain the real variables, all the land and stock price indexes were deflated by the consumer price index, which was collected from the database of the World Development Indicators created by the World Bank (2011). We converted all the obtained real variables into the indexes using 2000 as the base year. Applying the Hodrick-Prescott filter to these converted indexes, we obtained the deviations from the trends of each variable.

Microfoundations for the credit constraint (2)

Microfoundation I

Following Aghion et al. (1999), Aghion and Banerjee (2005) and Aghion et al. (2005), we assume that financial market imperfections arise simply from the possibility that borrowers may not pay off their obligations.

The net worth that each investor prepares for his/her investment project by his/her own is a_t. If he/she borrows $-b_t$ from the financial intermediary, his/her total resources to invest are $k_t = a_t - b_t$ at time t. The return on one unit of investment is $A\Phi_t$. If an investor consistently pays off his/her obligations, then he/she will acquire a net income, $A\Phi_t k_t + r_{t+1} b_t$ at time $t+1$. Meanwhile, if the investor does not repay his/her obligations, he/she will incur a cost δk_t to conceal his/her revenue. In this case, the financial intermediary monitors the investor and is able to capture the investor with a probability of p_{t+1}. Thus, his/her expected income is given by $A\Phi_t k_t - \delta k_t + p_{t+1} r_{t+1} b_t$.

Under this lending contract, the incentive compatibility constraint for the investor not to default is given by:

$$A\Phi_t k_t + r_{t+1} b_t \geq [A\Phi_t - \delta] k_t + p_{t+1} r_{t+1} b_t, \tag{A1}$$

or equivalently,

$$b_t \geq -\frac{\delta}{r_{t+1}(1 - p_{t+1})} k_t, \tag{A2}$$

The left-hand side of Eq. (A1) represents the revenue that the investor obtains when he/she invests in a project and consistently pay off his/her obligations. The right-hand side is the gain when he/she defaults.

To achieve the probability p_{t+1} to detect the investor's concealing his/her revenue, the financial intermediary incurs an effort cost, $b_t C(p_{t+1})$, which is increasing and convex with respect to p_{t+1}. As in Aghion and Banerjee (2005), we assume $C(p_{t+1}) = \kappa \log(1 - p_{t+1})$, where κ is strictly greater than δ so that our study is meaningful.[17] The financial intermediary can choose an optimal probability by solving a maximization problem such that

$$\max_{p_{t+1}} \; - p_{t+1} r_{t+1} b_t - \kappa \log(1 - p_{t+1}) b_t.$$

As $-b_t > 0$, this maximization problem is rewritten as:

$$\max_{p_{t+1}} \; p_{t+1} r_{t+1} + \kappa \log(1 - p_{t+1}).$$

From the first-order condition, we have

$$r_{t+1} = \frac{\kappa}{1 - p_{t+1}}. \tag{A3}$$

As the interest rate r_{t+1} increases, the financial intermediary chooses the high probability to detect defaulting investors. From Eqs. (A2) and (A3), we obtain:

$$b_t \geq -\frac{\delta}{\kappa} k_t,$$

[17] If $\delta \geq \kappa$, no investors face binding credit constraints.

or equivalently,

$$b_t \geq -\frac{\delta}{\kappa - \delta} a_t. \tag{A4}$$

As the agent's productivity Φ_t is not observable, the financial intermediary does not impose investor-specific credit constraints. The financial intermediary must know the investors' net worth, a_t. As long as it imposes a credit constraint given by inequality (A4) on all agents, nobody will default in equilibrium. As $\delta < \kappa$, we can let $\theta := \delta/(\kappa - \delta) \in [0, \infty)$, and thus,

$$b_t \geq -\theta a_t,$$

which is a credit constraint in the main text. δ and κ are associated with a default cost and a monitoring cost, respectively. θ represents the extent of the credit constraint.

Microfoundation II

We extend the microfoundation for a credit constraint presented by Antràs and Caballero (2009) in a manner suitable for our model. We consider the participation constraint faced by the financial intermediary and the incentive compatibility constraint of the investors such that they do not back out of their investment projects.

It is assumed that at the end of time t and after investment has occurred, any investor can back out of his/her investment project at no cost, taking some fraction of his investments, $(1-\mu)(a_t - b_t)$, where $0 < \mu < 1$, and not repaying his obligations to the financial intermediary. In this case, the investor will engage in general goods production somewhere in the economy.

If an investor absconds at the end of time t, the financial intermediary can reclaim the remainder of investments, $\mu(w_t - b_t)$. It is assumed that the financial intermediary can relend the remainder of the investments in the financial market. Thus, when making a financial contract with an investor, the financial intermediary faces a participation constraint such that:

$$r_{t+1}\mu(a_t - b_t) \geq -r_{t+1}b_t,$$

or equivalently

$$b_t \geq -\frac{\mu}{1-\mu} a_t.$$

On the other hand, the incentive compatibility constraint for a borrower, such that he/she does not to abscond from engaging in his/her project at the end of time t, is given by:

$$A\Phi_t(a_t - b_t) + r_{t+1}b_t \geq A\Phi_t(1-\mu)(a_t - b_t). \tag{A5}$$

For investors with Φ_t such that $r_{t+1} - \mu A\Phi_t \leq 0$, Eq. (A5) always holds. Therefore, we focus on investors with Φ_t such that $r_{t+1} - \mu A\Phi_t > 0$. Then, Eq. (A5) is rewritten as

$$b_t \geq -\frac{\mu}{(\phi_t/\Phi_t) - \mu} a_t. \tag{A6}$$

As $\phi_t/\Phi_t \leq 1$ in equilibrium, it follows that $-\mu/((\phi_t/\Phi_t) - \mu) \leq -\mu/(1-\mu)$, implying that Eq. (A6) is redundant. In other words,, if the financial intermediary imposes a credit constraint $b_t \geq -\mu a_t/(1-\mu)$, which is the participation constraint of the financial intermediary, investors never default. By letting $\mu/(1-\mu) := \theta$, we obtain the credit constraint $b_t \geq -\theta a_t$, as shown in the main text. As μ, or equivalently θ, increases, it becomes more difficult for the investors to withdraw their investment without repaying their obligations.

Proof of Lemma 1

Obviously, there exists a unique value of ϕ^* because $G(.)$ is a strictly increasing function over the support of Φ. Regarding ϕ^{**}, we note that $H(x) := \beta F(x)/x - [1 - \mu - \beta(G(x) - \mu)]$ is strictly decreasing in $(0, b)$ or $(0, \infty)$ because $H'(x) = -\beta F(x)/x^2 < 0$ in $(0, b)$ or $(0, \infty)$. In addition, $\lim_{x\to 0} H(x) = \infty$ and $\lim_{x\to\infty} H(x) = -1 + \mu(1-\beta) < 0$. Therefore, ϕ^{**}, which is the solution of $H(x) = 0$, is uniquely determined. □

Proof of Proposition 2

Note that because the bubbly steady state exists, it follows that $\phi^{**} > \phi^*$. As shown in Eq.(18), the growth rate $\Gamma_{t+1} = Y_{t+1}/Y_t$ is given by:

$$J(\phi_t) := \frac{A\beta F(\phi_t)}{1 - \mu - \beta(G(\phi_t) - \mu)}.$$

From this, we obtain:

$$J'(\phi_t) := H(\phi_t)\frac{A\beta\phi_t G'(\phi_t)}{[1 - \mu - \beta(G(\phi_t) - \mu)]^2},$$

where $H(\phi_t) = \beta F(\phi_t)/\phi_t - [1 - \mu - \beta(G(\phi_t) - \mu)]$ as defined in the proof of Lemma 1. As demonstrated in the proof of lemma 1, $H(\phi_t)$ is strictly decreasing and $H(\phi^{**}) = 0$. Therefore, $J'(\phi_t)$ is strictly greater than zero if $\phi^* < \phi_t < \phi^{**}$ and it is strictly less than zero if $\phi_t > \phi^{**}$. Therefore, the maximum of $J(\phi_t)$ is $J(\phi^{**})$ and of course $J(\phi^*) < J(\phi^{**})$. □

Proof of Proposition 3

Because $\pi^a > 1 - \pi^b$, it follows that $\phi^a > \phi^b$. Suppose that $\phi^a > \phi^{**}$. Then, $\Lambda(\phi^a) > \Psi(\phi^a) > \Psi(\phi^b)$. However, Eq.(29) does not hold with this inequality. Now, suppose that $\phi^a = \phi^{**}$.

Then, it must hold that $\pi^a = 1$, which contradicts the assumption of $\pi^a < 1$. Therefore, $\phi^a < \phi^{**}$. Suppose that $\phi^b > \phi^*$. Then, $\Psi(\phi^a) > \Psi(\phi^b) > \Lambda(\phi^b)$. However, Eq.(30) does not hold with this inequality. Therefore, only the possible case is that $\phi^b = \phi^*$ and $\pi^b = 1$. Conversely, if $\phi^b = \phi^*$ and $\pi^b = 1$, then there exists $\{\phi^a, \pi^a\}$, such that $0 = 1 - \pi^b < \pi^a < 1$ and Eq.(29) is satisfied. $\square$

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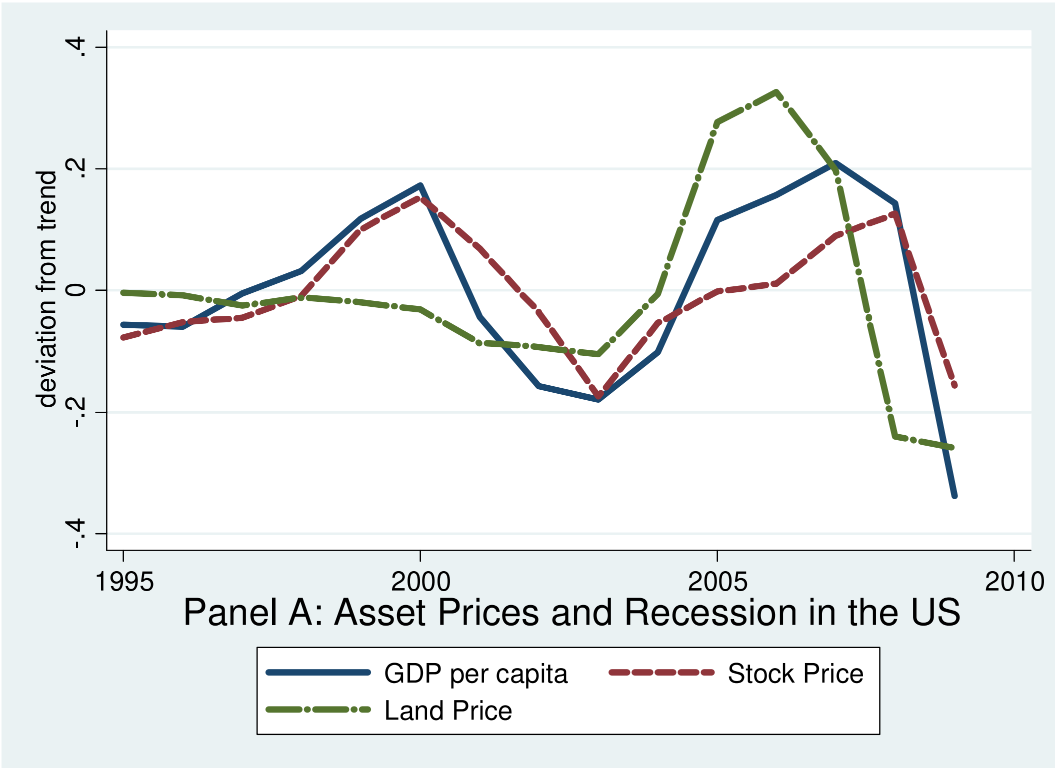


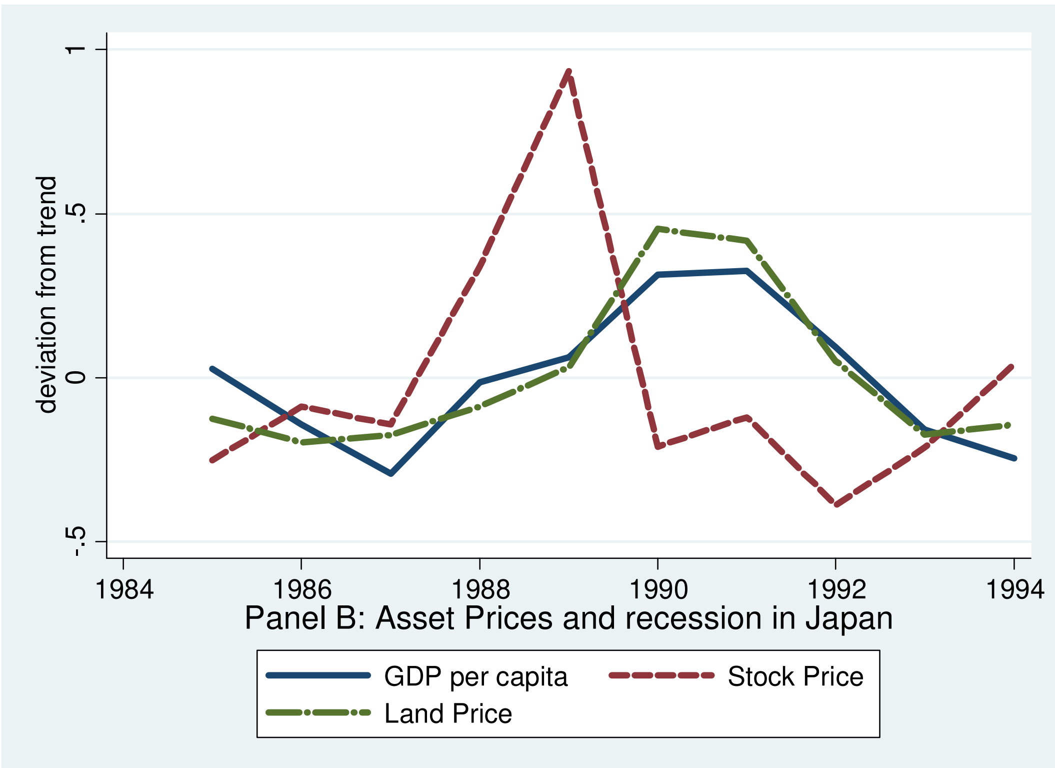


Figure 1: Asset Prices and Recessions

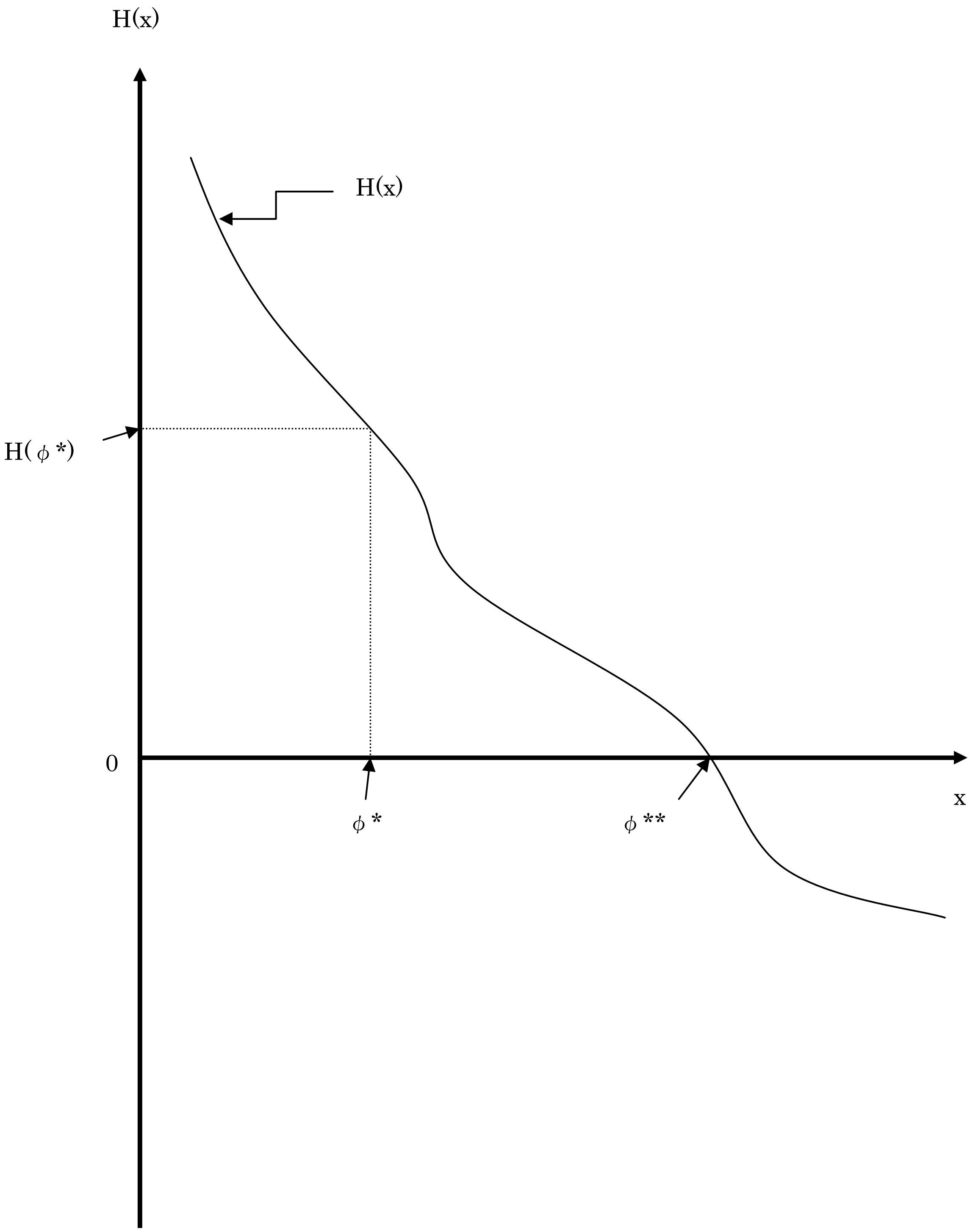
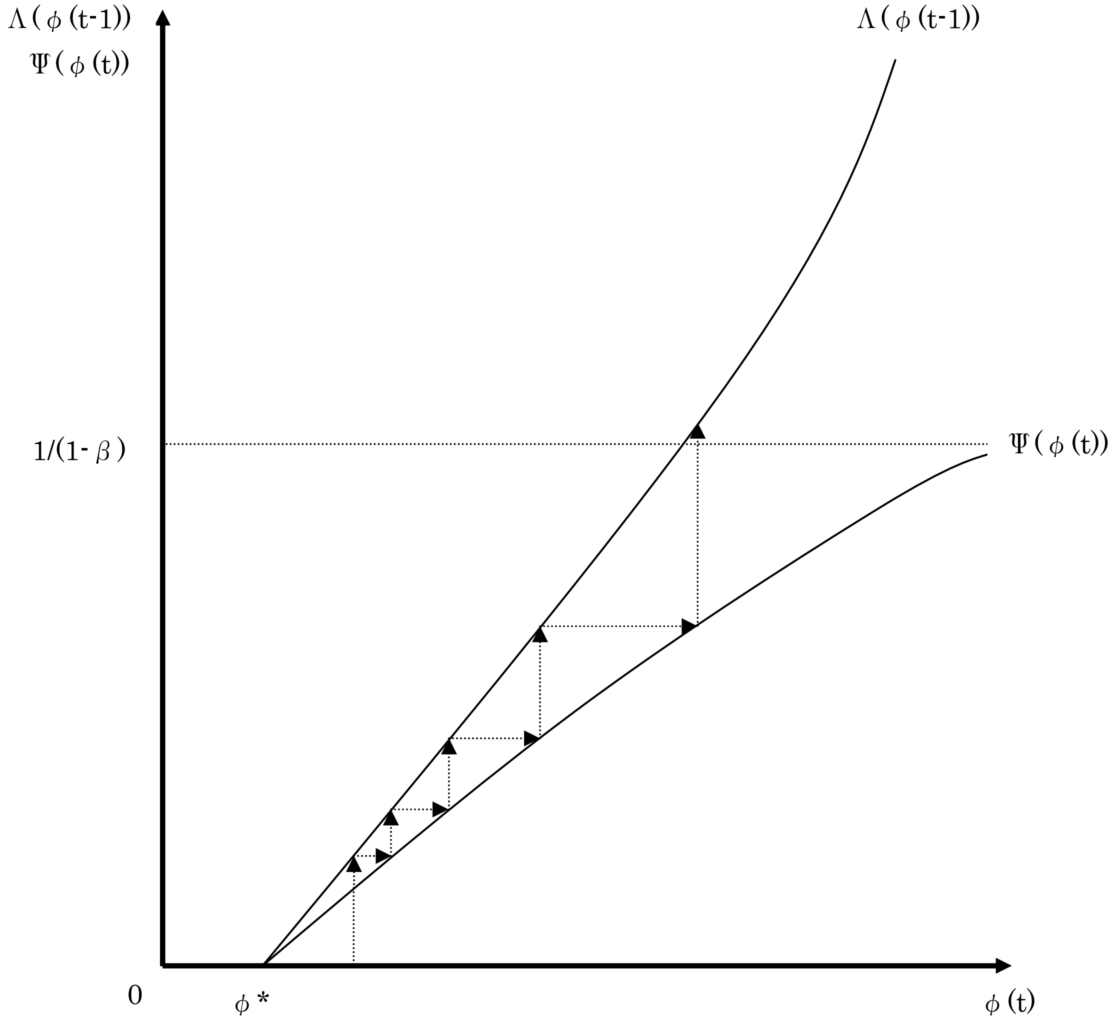
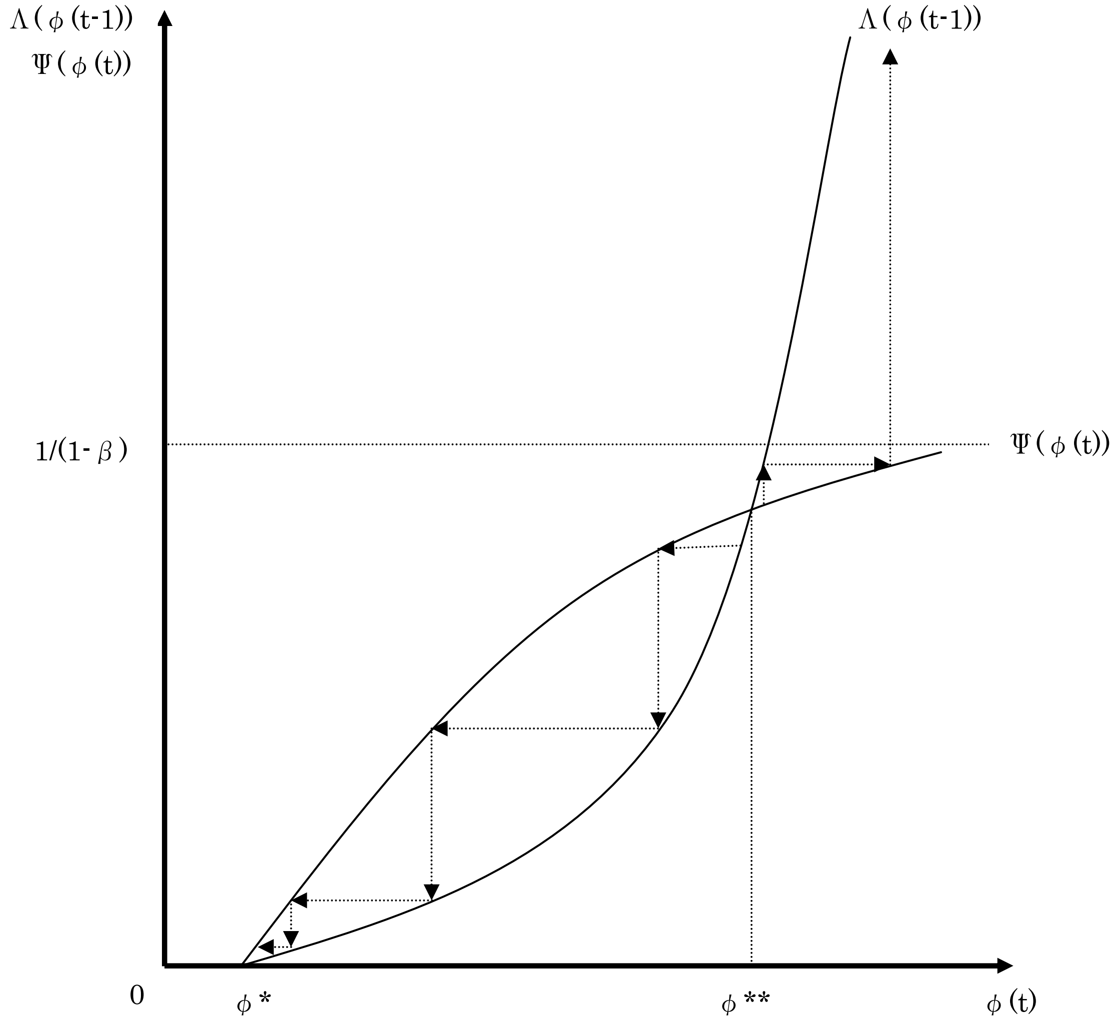


Figure 2: Existence of the bubbly steady state



Panel A: Dynamic behavior without the bubbly steady state

Figure 3



Panel B: Dynamic behavior with the bubbly steady state

Figure 3

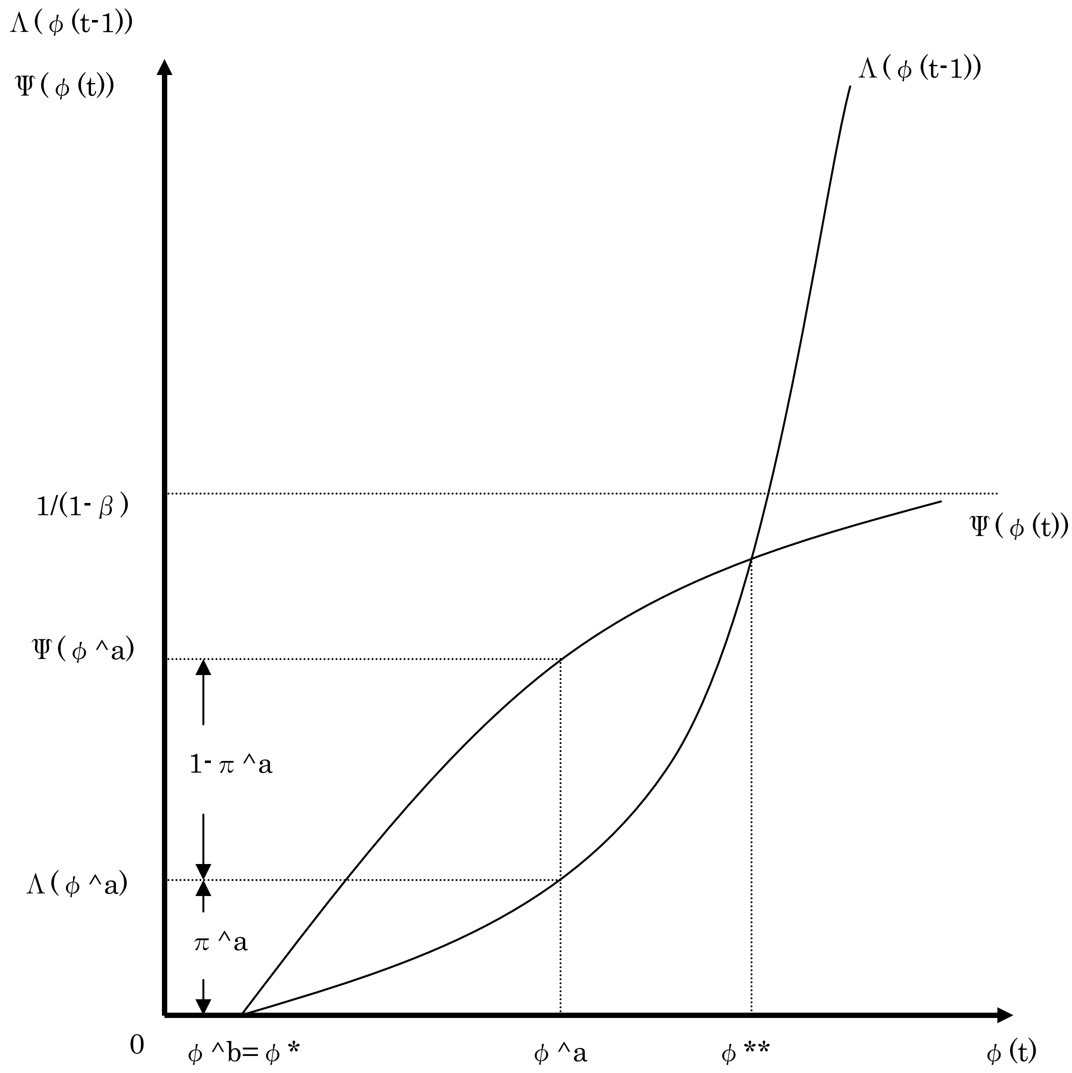
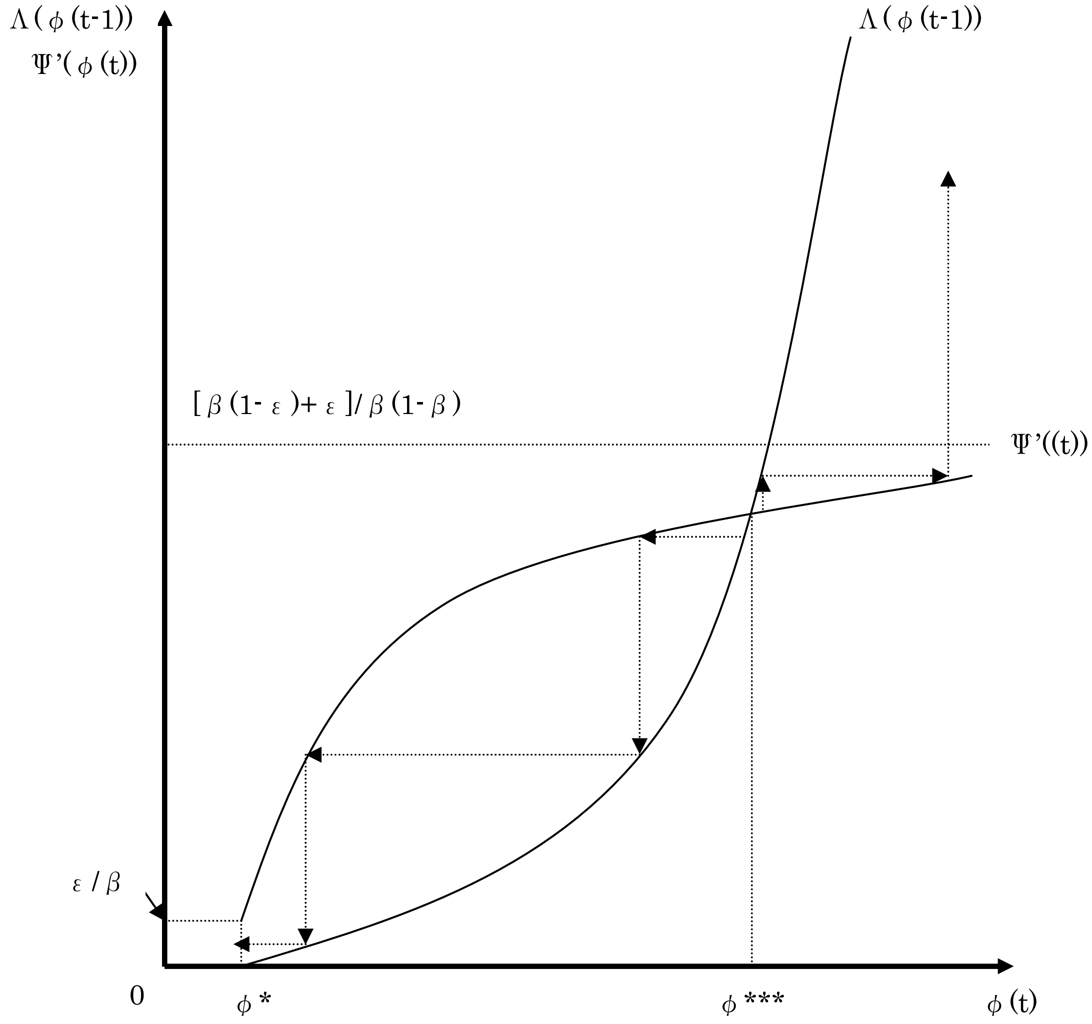


Figure 4: Two-state stationary sunspot equilibrium



Ψ'(ϕ (t)) is the left-hand side of Eq.(37)

Figure 5: Avoiding self-fulfilling financial crises