A more general theory of commodity bundling

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Abstract

This paper extends the standard model of bundling as a price discrimination device to allow products to be substitutes and for products to be supplied by separate sellers. Whether integrated or separate, firms have an incentive to introduce a bundling discount when demand for the bundle is elastic relative to demand for stand-alone products. Product substitutability typically gives an integrated firm a greater incentive to offer a bundle discount (relative to the model with additive preferences), while substitutability is often the sole reason why separate sellers wish to offer inter-firm discounts. When separate sellers coordinate on an inter-firm discount, they can use the discount to overturn product substitutability and relax competition.

1 Introduction

Bundling—the practice whereby consumers are offered a discount if they buy several distinct products—is used widely by firms, and is the focus of a rich economic literature. However, most of the existing literature discusses the phenomenon under relatively restrictive assumptions, namely a consumer's valuation for a bundle of several products is the sum of her valuations for consuming the items in isolation, and bundle discounts are only offered for products sold by the same firm. The two assumptions are related, in that when valuations are additive it is less likely that a firm would wish to reduce its price to a customer who also buys a product from another seller. This paper analyzes the incentive to engage in bundling when these assumptions are relaxed.

There are very many situations in which modelling products as substitutes is relevant. For instance, when visiting a city a tourist may gain some extra utility from visiting art gallery A if she has already visited art gallery B, but the incremental utility is likely to be

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smaller than if she were only to visit A. Joint purchase discounts (or premia) on products offered by separate sellers are rarer, though some examples include:

— A tourist may be able to buy a "city pass", so that she can visit all participating tourist attractions at a discount on the sum of individual entry fees. These could be organized either as a joint venture by the attractions themselves, or implemented by an intermediary which puts together its own bundles given wholesale fees negotiated with attractions.

— Bundling is prevalent in markets for transport services, as is the case with alliances between airlines or when neighboring ski-lifts offer a combined ticket.

— Products supplied by separately-owned firms are often marketed together with discounts for joint purchase. Thus, supermarkets and gasoline stations may cooperate to offer a discount when both services are consumed. Airlines and car rental firms may link up for marketing purposes, and sometimes credit cards offer discounts proportional to spend towards designated flights or hotels.

— Pharmaceuticals are sometimes used as part of a "cocktail" with one or more drugs supplied by other firms. Drugs companies can set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail.

— Marketing data may reveal useful information about a potential customer's purchase history which affects a firm's price to the customer. For instance, information that the customer has chosen to buy firm 1's product may induce firm 2 to discount its price, and an inter-firm discount for the joint purchase of the two products is implemented.

— At a wholesale level, a manufacturer may offer a retailer a discount if the retailer does not stock a rival manufacturer's product. (Such contracts are sometimes termed "loyalty contracts".) This is a situation with a bundle premium instead of a discount.

The plan of the paper is as follows. In section 2, I present a general framework for consumer demand for two products in the presence of product substitutability and bundle discounts. Section 3 covers the case where an integrated firm supplies both products. I revisit the approach to bundling presented in Long (1984), which is used as a major ingredient for the analysis in section 3. Long's result is that the firm has an incentive to bundle when demand for the bundle is more elastic than demand for stand-alone products. Relative to the situation with additive preferences, the integrated firm typically has a greater incentive to offer a bundle discount when products are substitutable. Because the purchase of one product can decrease a consumer's incremental utility from a second, the firm has a direct incentive to reduce the price for a second item, in addition to the rent-extraction motive for bundling familiar from the existing literature. In examples we see that the size of the discount can be above or below the corresponding discount with additive preferences.

In section 4 I turn to the situation where products are supplied by separate sellers. With additive preferences, a firm has a unilateral incentive to offer a bundle discount when product valuations are negatively correlated. When there is full market coverage, a firm has an incentive to offer a joint-purchase discount under plausible conditions on consumer valuations. When products are substitutes, whether a firm has a unilateral incentive to introduce a discount depends on the way that preferences are modelled. When there is a constant disutility of joint consumption, separate sellers typically wish to offer a joint-purchase discount: the fact that a customer has purchased the rival product implies that her incremental valuation for the firm's own item has fallen, and this usually implies that the firm would like to reduce its price to this customer. Alternatively, if a proportion of buyers only want a single item (for instance, a tourist in a city might only have time to visit a single museum) while other consumers have additive preferences, a seller would like, if feasible, to charge a premium when a customer also buys the rival product. In examples, when this form of price discrimination is feasible, one price increases and the other decreases relative to the situation with uniform pricing, and price discrimination results in higher equilibrium profit and higher welfare, but a worse outcome for consumers.

Finally, section 5 investigates partial coordination between separate sellers, which is currently the relevant case for several of the industries mentioned above. Specifically, I suppose that firms first agree on a bundle discount which they fund jointly, and subsequently choose prices without coordination. When valuations are additive, it is shown that such a scheme will usually raise each firm's profit, and, at least in the example considered, its operation will also boost total welfare. However, when sellers offer substitute products, the negotiated discount overturns the innate substitutability of products, inducing firms to raise prices. The resulting "tariff-mediated" product complementarity can induce collusion which harms consumers and overall welfare.

This paper is not the first to investigate these issues. The incentive for an integrated seller to offer a discount for the purchase of multiple items is discussed by Adams and Yellen (1976), Long (1984) and McAfee, McMillan, and Whinston (1989), among many others. The latter two papers showed that it is optimal to introduce a bundle discount whenever the distribution of valuations is statistically independent and valuations are additive, so that a degree of joint pricing is optimal even with entirely unrelated products. Except for Long, these papers assume that valuations are additive.[1] Long (1984) presents what could be

[1] Venkatesh and Kamakura (2003) analyze an integrated firm's incentive to engage in bundling when products are either complements or substitutes. The analysis is carried out using a specific uniform

termed an "economic" model of bundling. Rather than following a diagrammatic exposition concentrating on the details of joint distributions of two-dimensional consumer valuations, he uses standard demand theory—which applies equally to non-additive preferences—to derive conditions under which a bundle discount is optimal.

Schmalensee (1982) and Lewbel (1985) study the incentive for a single-product monopolist to offer a discount if its customers also purchase a competitively-supplied product. Schmalensee supposes that two items are for sale to a population of consumers, and item 1 is available at marginal cost due to competitive pressure while item 2 is supplied by a monopolist. Valuations are additive, but are not independent in the statistical sense. If there is negative correlation in the values for the two items, the fact that a consumer buys item 1 is "bad news" for the monopolist, who then has an incentive to set a lower price to its customers who also buy 1. Lewbel performs a similar exercise but allows the two items to be partial substitutes. In this case, the fact that a consumer buys item 1 is also bad news for the monopolist, and gives an incentive to offer a discount for joint consumption.

Bundling arrangements between separate firms are analyzed by Gans and King (2006), who investigate a model with two kinds of products (gasoline and food, say), and each product is supplied by two differentiated firms. When all four products are supplied by separate firms which set their prices independently, there is no interaction between the two kinds of product. However, two firms (one offering each of the two kinds of product) can enter into an alliance and agree to offer consumers a discount if they buy both products from the alliance. (In their model, the joint pricing mechanism is similar to that used in section 5 below: firms decide on their bundle discount, which they agree to fund equally, and then set prices non-cooperatively.) Gans and King observe that when a bundle discount is offered for joint purchase of otherwise independent products, those products are converted into complements. In their model, in which consumer tastes are uniformly distributed, a pair of firms does have an incentive to enter into such an alliance, but when both pairs do this their equilibrium profits are unchanged from the situation when all four firms set independent prices, although welfare and consumer surplus fall.[2]

Calzolari and Denicolo (2011) propose a model where consumers buy two products and each product is supplied by a single firm. Each firm potentially offers a nonlinear

example, and a consumer's valuation for the bundle is some constant proportion (greater or less than one, depending on whether complements or substitutes are present) of the sum of her stand-alone valuations. The focus of their analysis is on whether pure bundling is superior to linear pricing.

[2] Brito and Vasconcelos (2010) modify this model so that rival suppliers of the same products are vertically rather than horizontally differentiated. They find that when two pairs of firms form an alliance all prices rise relative to the situation when all four products are marketed independently. This result resembles the analysis in section 5 below, where an agreed bundle discount induces collusion in the market.

tariff which depends on a buyer's consumption of its own product and her consumption of the other firm's product. They find that the use of these tariffs can harm consumers compared to the situation in which firms base their tariff only on their own supply. Their model differs in two ways from the one presented in section 4 of this paper. First, in their model consumers have elastic (linear) demands, rather than unit demands, for the two products. Thus, they must consider general nonlinear tariffs, while the firms in my model merely choose a pair of prices. Second, in my model consumers differ in richer way, and a consumer might like product 1 but not product 2, and can vary in the degree of substitutability between products. In Calzolari and Denicolo (2011), consumers differ by only a scalar parameter (the demand intercept for both products), and so all consumers view the two products when consumed alone as perfect substitutes.

Finally, Lucarelli, Nicholson, and Song (2010) discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set two different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS set different prices for similar chemicals depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails combining drugs from different firms. Although in this particular market firms do not price drugs differently depending whether the drug is used in a cocktail, they estimate the impact when one firm engages in this form of price discrimination. They find that a firm will typically (but not always) reduce the price for stand-alone use and raise the price for bundled use.

2 A Framework for Consumer Demand

Consider a market with two products, labeled 1 and 2, where a consumer buys either zero or one unit of each product (and maybe one unit of each). A consumer is willing to pay v_i for product $i = 1, 2$ on its own, and to pay v_b for the bundle of both products. (A consumer obtains payoff zero if she consumers neither product.) Thus a consumer's preferences are described by the vector (v_1, v_2, v_b), which varies across the population of consumers according to some known distribution.[3] A consumer views the two products

[3] In the analysis which follows, we assume that the stand-alone valuations (v_1, v_2) have a continuous marginal density with support on a compact rectangle in $\mathbb{R}^2_+$. Given (v_1, v_2), the distribution of v_b is sometimes deterministic (as in Example 1 below), sometimes discrete (as in Example 2), and sometimes continuous (as in Example 3). All we need to assume about the distribution of (v_1, v_2, v_b) is that it is

as *partial substitutes* whenever $v_b \le v_1 + v_2$. Whenever there is free disposal, so that a consumer can discard an item without cost, we require that $v_b \ge \max\{v_1, v_2\}$ for all consumers.

Only deterministic selling procedures are considered in this paper.[4] Consumers face three prices: p_1 is the price for consuming product 1 on its own; p_2 is the price for product 2 on its own, and $p_1 + p_2 - \delta$ is the price for consuming the bundle of both products. Thus, δ is the discount for buying both products, which is zero if there is linear pricing or negative if consumers are charged a premium for joint consumption. A consumer chooses the option from the four discrete choices which leaves her with the highest surplus, so she will buy both items whenever $v_b - (p_1 + p_2 - \delta) \ge \max\{v_1 - p_1, v_2 - p_2, 0\}$, she will buy product $i = 1, 2$ on its own whenever $v_i - p_i \ge \max\{v_b - (p_1 + p_2 - \delta), v_j - p_j, 0\}$, and otherwise she buys nothing.

As functions of the three tariff parameters (p_1, p_2, δ), denote by Q_1 the proportion of potential consumers who buy only product 1, Q_2 the proportion who buy only product 2, and Q_b the proportion who choose the bundle. It will also be useful to discuss demand when no discount is offered, so let $q_i(p_1, p_2) \equiv Q_i(p_1, p_2, 0)$ and $q_b(p_1, p_2) \equiv Q_b(p_1, p_2, 0)$ be the corresponding demand functions when $\delta = 0$. Indeed, we will see that a firm's incentive to introduce a bundle discount is determined entirely by the properties of the "no-discount" demands q_i and q_b. This is important insofar as these demand functions are easier to estimate from market data than the more hypothetical demands Q_i and Q_b.[5]

Several properties of these demand functions follow immediately from the discrete choice nature of the consumer's problem, and are not contingent on whether the products are partial substitutes. To illustrate, note that total demand for each product is an increasing function of the bundle discount, i.e.,

$$Q_i + Q_b \text{ increases with } \delta \ . \tag{1}$$

sufficiently well behaved that the demand functions shortly defined are differentiable.

[4]Unlike the single-product case, when a monopolist sells two or more products it can often increase its profits if it is able to use stochastic schemes (e.g., where for a specified price the consumer gets product 1 or product 2 but she is not sure which one). See Pavlov (2011) for a recent contribution to this topic, which studies cases with extreme substitutes (all consumers buy a single item) and with additive preferences.

[5]The model of consumer preferences presented here is related to the small empirical literature which estimates discrete consumer choice when multiple goods are chosen simultaneously. For instance, see Gentzkow (2007) who estimates the degree of complementarity between print and online newspapers. In his illustrative model in section 1.A, he supposes that the value of the bundle is the sum of the values of the two individual products plus a constant term (which could be positive or negative), which is similar to Example 1 discussed later in this paper.

To see this, observe that a consumer buys product 1, say, if and only if

$$\max\{v_b - (p_1 + p_2 - \delta), v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ .$$

(The left-hand side above is the consumer's maximum surplus if she buys product 1—either in the bundle or on its own—while the right-hand side is the consumer's maximum surplus if she does not buy the product.) Clearly, the set of such consumers is increasing (in the set-theoretic sense) in δ. In the case of separate supply, analyzed in section 4, this implies that when a firm unilaterally introduces a bundle discount, its rival's profits will rise.

We necessarily have Slutsky symmetry of cross-price effects, so that

$$\frac{\partial Q_2}{\partial p_1} + \frac{\partial Q_2}{\partial \delta} \equiv \frac{\partial Q_1}{\partial p_2} + \frac{\partial Q_1}{\partial \delta} \ ; \ \frac{\partial Q_b}{\partial p_i} + \frac{\partial Q_b}{\partial \delta} \equiv -\frac{\partial Q_i}{\partial \delta} \ . \tag{2}$$

For instance, the left-hand side of (2) says that the effect on demand for good 2 on its own of a price rise of good 1 on its own (which is achieved by increasing p_1 and δ by the same amount so that the bundle price does not change) is the same as the effect on demand for good 1 on its own of price rise for good 2 on its own. Setting $\delta = 0$ in the right-hand expression in (2) implies that the impact of a small bundle discount on the total demand for a product is equal to the impact of a corresponding price cut on bundle demand, i.e.,

$$\left.\frac{\partial (Q_i + Q_b)}{\partial \delta}\right|_{\delta=0} = -\frac{\partial q_b}{\partial p_i} \ . \tag{3}$$

This identity plays a key role when we analyze the profitability of introducing a discount.

One price effect which does depend on the innate substitutability of products is the following:

Claim 1 *Suppose that $v_b \leq v_1 + v_2$ for all consumers. Then when linear prices are used, demand for product i, $q_i + q_b$, weakly increases with p_j.*

(All omitted proofs are contained in the appendix.) Importantly, when a bundle discount is offered, this result can be reversed: even if products are intrinsically substitutes then when $\delta > 0$ the demand for a product can decrease with the stand-alone price of the other product. The observation that a bundle discount can overturn the innate substitutability of products is a recurring theme in the following analysis.

A second property of demand which depends on product substitutability is that any consumer who chooses to buy the bundle at linear prices (p_1, p_2) has $v_j \geq p_j$ for each $j = 1, 2$. To see this, note that if a consumer with preferences (v_1, v_2, v_b) buys the bundle at prices (p_1, p_2), then $v_1 + v_2 - p_1 - p_2 \geq v_b - p_1 - p_2 \geq v_i - p_i$, where the first inequality follows from substitutability and the second is due to the superiority of the bundle to product i on its own. Thus, $\min\{v_1 - p_1, v_2 - p_2\} \geq 0$. This implies that with linear pricing there is no "margin" between buying the bundle and buying nothing, and any consumer who optimally buys the bundle would instead buy a single item (if they change at all) rather than exit altogether when faced with a small price rise. A second implication is that the set of consumers who buy *something* with linear prices (p_1, p_2) consists of those consumers with preferences satisfying $\max\{v_1 - p_1, v_2 - p_2\} \geq 0$. (Clearly, if $v_i \geq p_i$ then the consumer will buy something, since product i on its own yields positive surplus. Those consumers who buy the bundle lie inside this set since they satisfy $\min\{v_1 - p_1, v_2 - p_2\} \geq 0$.) In particular, the fraction of participating consumers, which is $q_1(p_1, p_2) + q_2(p_1, p_2) + q_b(p_1, p_2)$, depends only on the (marginal) distribution of the stand-alone valuations (v_1, v_2).

3 Integrated Supply

3.1 Long's analysis revisited

Suppose that the market structure is such that an integrated monopolist supplies both products. Here, and in section 4 with separate supply, suppose that the constant marginal cost of supplying product i is equal to c_i. To avoid tedious caveats involving corner solutions in the following analysis, suppose that over the relevant range of linear prices there is some two-item demand, so that $q_b > 0$.

In this section I recapitulate the analysis in Long (1984), as the integrated-firm analysis throughout section 3 rests on this. The firm's profit with bundling tariff (p_1, p_2, δ) is

$$\pi = (p_1 - c_1)(Q_1 + Q_b) + (p_2 - c_2)(Q_2 + Q_b) - \delta Q_b \ . \tag{4}$$

Consider the incentive to offer a bundle discount. Starting from linear prices (p_1, p_2), by differentiating (4) we see that the impact on profit of introducing a small discount $\delta > 0$ is

$$\begin{aligned} \left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} &= \left.\left\{(p_1 - c_1)\frac{\partial}{\partial \delta}(Q_1 + Q_b) + (p_2 - c_2)\frac{\partial}{\partial \delta}(Q_2 + Q_b) - Q_b\right\}\right|_{\delta=0} \\ &= -(p_1 - c_1)\frac{\partial q_b}{\partial p_1} - (p_2 - c_2)\frac{\partial q_b}{\partial p_2} - q_b \ , \end{aligned} \tag{5}$$

where the second equality follows from expression (3).

Although Long also considers the asymmetric case, his analysis is greatly simplified when products are symmetric, and for the remainder of section 3 assume that $c_1 = c_2 = c$ and the same density of consumers have taste vector (v_1, v_2, v_b) as have the permuted taste vector (v_2, v_1, v_b). Since the environment is symmetric, for convenience we consider only tariffs which are symmetric in the two products. If the firm offers price p for either product and no bundle discount, write $x_s(p)$ and $x_b(p)$ respectively for the proportion of consumers who buy a single item and who buy the bundle. (Thus, $x_s(p) \equiv q_1(p,p) + q_2(p,p)$ and $x_b(p) \equiv q_b(p,p)$.) From expression (5), a small discount is profitable with stand-alone price p in this symmetric setting if and only if

$$x_b(p) + (p-c)x_b'(p) < 0 \ . \tag{6}$$

Consider whether this is satisfied at the most profitable linear price, p^*. Since p^* maximizes $(p-c)(x_s(p) + 2x_b(p))$, the first-order condition for p^* is

$$x_s(p^*) + 2x_b(p^*) + (p^* - c)(x_s'(p^*) + 2x_b'(p^*)) = 0 \ .$$

Taking this together with expression (6), we see that it is profitable to introduce a bundle discount if

$$\frac{-x_b'(p^*)}{x_b(p^*)} > \frac{-x_s'(p^*)}{x_s(p^*)}$$

so that bundle demand x_b is more elastic than single-item demand x_s at optimal price p^*. This discussion is summarized in this result:[6]

Proposition 1 *Suppose an integrated monopolist supplies two symmetric products. The firm has an incentive to introduce a discount for buying the bundle whenever the demand for a single item is less elastic than the demand for the bundle, so that*

$$\frac{x_b(p)}{x_s(p)} \text{ strictly decreases with } p \ . \tag{7}$$

Condition (7) is intuitive: if the firm initially charges the same price for buying a single item as for buying a second item, and if demand for the latter is more elastic than demand for the former, then the firm would like to reduce its price for buying a second item (and to increase its price for the first item).

Consider the familiar knife-edge case where a consumer's valuation for the bundle is the sum of her stand-alone valuations, i.e., $v_b \equiv v_1 + v_2$. With additive valuations, if the firm offers the linear price p for buying either item the consumer's decision is simple: she should buy product i whenever $v_i \geq p$. Define

$$\Psi(p) \equiv \Pr\{v_2 \geq p \mid v_1 \geq p\} \ , \tag{8}$$

[6]Long stated the result in the alternative, but equivalent, form whereby bundling was profitable if the ratio of total demand $x_s + 2x_b$ to the number of customers $x_s + x_b$ decreased when the price p increased. When products are not symmetric, Long shows using a similar analysis that the firm has an incentive to introduce a discount for buying the bundle whenever single-item demand is less elastic than bundle demand, in the sense that $q_b/(q_1 + q_2)$ strictly decreases with an equi-proportional amplification of price-cost mark-ups.

so that

$$\Psi(p) = \frac{x_b(p)}{x_b(p) + \frac{1}{2}x_s(p)}$$

and

$$\frac{x_b(p)}{x_s(p)} = \frac{1}{2} \cdot \frac{\Psi(p)}{1 - \Psi(p)} \ .$$

Proposition 1 implies, therefore, that the firm has an incentive to introduce a bundle discount if

$$\Psi(p) \text{ strictly decreases with } p \tag{9}$$

(at the most profitable linear price p^*). Condition (9) holds, roughly speaking, if v_1 and v_2 are not "too" positively correlated. In particular, a degree of bundling is profitable even if valuations are additive and statistically independent. As we explore in the next section, the more fundamental condition (7) is also useful for situations outside this additive case.

3.2 Bundling with substitute products

Using Long's condition (7), in this section I analyze in more detail the firm's incentive to bundle when preferences are not additive. One advantage of assuming symmetry in the two products is that what is in general the three-dimensional nature of preferences reduces to just two dimensions, since only the highest stand-alone valuation matters out of (v_1, v_2). With this in mind, given preferences (v_1, v_2, v_b), define

$$V_1 \equiv \max\{v_1, v_2\} \ ; \ V_2 \equiv v_b - V_1 \ , \tag{10}$$

so that V_1 is a consumer's maximum utility if she buys only one item and V_2 is her incremental utility from the second item. Note that $v_b = V_1 + V_2$, so that valuations are additive after this change of variables. Given the linear price p for each item, the type-(V_1, V_2) consumer will buy one item if $V_1 \geq p$ and $V_2 < p$, and she will buy both items if $V_2 \geq p$ and $V_1 + V_2 \geq 2p$, and this pattern of demand is depicted on Figure 1A. In general, a consumer might buy both items even if she does not obtain positive surplus from buying only one, so there is a "margin" between buying the bundle and buying nothing. However, if products are substitutes this margin disappears: when $v_b \leq v_1 + v_2$ then $V_2 \leq \min\{v_1, v_2\} \leq V_1$, and the support of (V_1, V_2) lies under the 45^0 line as shown on Figure 1B.

From now on, assume that the products are substitutes. Similarly to (8), define

$$\Phi(p) \equiv \Pr\{V_2 \geq p \mid V_1 \geq p\} = \frac{\Pr\{V_2 \geq p\}}{\Pr\{V_1 \geq p\}} \ . \tag{11}$$

By examining Figure 1B we see that $x_b = (x_b + x_s)\Phi$, or

$$\frac{x_b(p)}{x_s(p)} = \frac{\Phi(p)}{1 - \Phi(p)} \ .$$

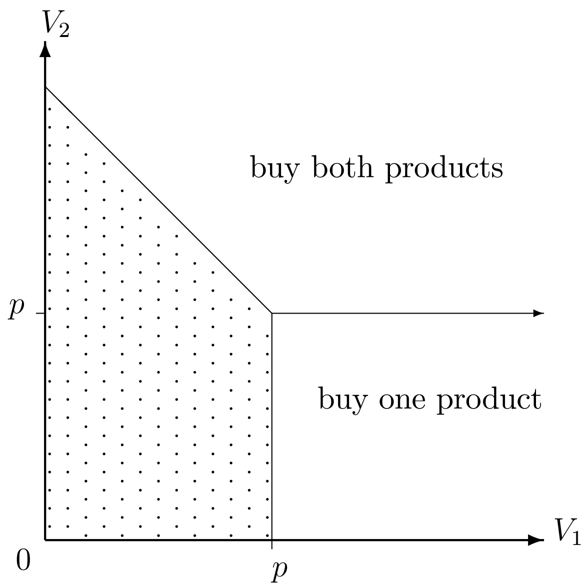

Figure 1A: General case

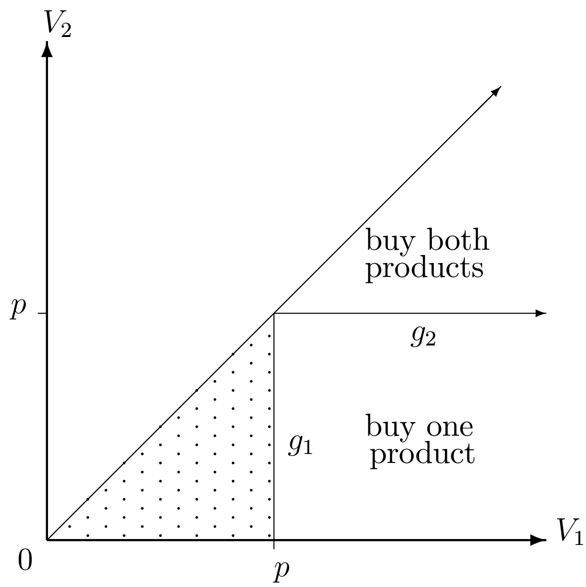

Figure 1B: Substitute products

Therefore, when Φ is strictly decreasing Proposition 1 implies that the monopolist has an incentive to introduce at least a small bundle discount. In fact, we can obtain the following non-local result, which is our main result for integrated supply:

Proposition 2 *Suppose products are substitutes and Φ in (11) is strictly decreasing. Then the most profitable bundling tariff for a monopolist involves a positive bundle discount.*

The fundamental condition which makes bundling profitable for an integrated seller is (7), and this condition applies regardless of whether products are substitutes or not. However, the more transparent condition that Φ in (11) be decreasing only applies when products are substitutes. Otherwise, the pattern of demand looks like Figure 1A above, and Φ does not capture all the relevant demand information.

For $i = 1, 2$, write $G_i(p) = \Pr\{V_i \le p\}$ for the marginal c.d.f. for valuation V_i and $g_i(p) = G_i'(p)$ for the corresponding marginal density. (The densities g_1 and g_2 are the "measures" of the lines marked on Figure 1B.) The condition that Φ is decreasing is equivalent to the hazard rates satisfying $g_1/(1-G_1) < g_2/(1-G_2)$. As is well known, a sufficient condition for this to hold is that the likelihood ratio

$$\frac{g_1(p)}{g_2(p)} \text{ decreases with } p \ . \tag{12}$$

Whenever (12) holds, then, the firm has an incentive to introduce a bundle discount.

Proposition 2 applies equally to an alternative framework where the monopolist supplies a single product, and where consumers consider buying one or two units of this product.

Here, the parameter V_1 represents a consumer's value of one unit and V_2 is her incremental value for the second. Thus when consumers have diminishing marginal utility ($V_2 \leq V_1$) and Φ in (11) is decreasing, the single-product firm will offer a nonlinear tariff which involves a quantity discount.[7] (However, this alternative interpretation of the model is not natural in the separate sellers context of section 4, since we would have to assume that for some reason a supplier could only sell a single unit of the product to a consumer.)

A natural question is whether products being substitutes makes it more likely that the integrated firm wishes to introduce a bundle discount, relative to the same market but with additive valuations. Consider a market where the stand-alone valuations, v_1 and v_2, have a given (symmetric) distribution. We know from Proposition 1 that the firm has an incentive to offer a bundle discount whenever x_b/x_s is decreasing in the linear price p, which is equivalent to the condition that x_b/n decreases with p, where $n \equiv x_s + x_b$ is the fraction of consumers who buy something from the firm. Consider two scenarios: in scenario (a), each consumer's valuation for the bundle is additive, so that $v_b \equiv v_1 + v_2$, while in scenario (b) we have $v_b \leq v_1 + v_2$. Write the fraction of consumers who buy both items at linear price p in scenario (a) as $x_b(p)$ and the corresponding fraction in scenario (b) as $\hat{x}_b(p)$. As discussed in section 2, n is exactly the same function in the two scenarios. Thus, if $\hat{x}_b/x_b$ (weakly) decreases with price, then whenever bundling is profitable under scenario (a) it is sure to be profitable under scenario (b) as well. It is plausible, though not inevitable, that demand $\hat{x}_b$ is more elastic than demand x_b. Since $V_2 \leq \min\{v_1, v_2\}$, it follows that $\hat{x}_b \leq x_b$. Thus, for $\hat{x}_b$ to be more elastic we require that the slope $-\hat{x}_b'$ not be "too much" smaller than $-x_b'$.[8]

Intuitively, when products are substitutes there is an extra motive to offer a bundle discount, relative to the additive case, which is to try to serve customers with a second item even though the incremental utility of the second item is lowered by the purchase of the first item. Once a customer has purchased one item, this is bad news for her willingness-to-pay for the other item, and this often gives the firm a motive to reduce price for the

[7] See Maskin and Riley (1984) for an early contribution to the theory of quantity discounts, where—in contrast to the current paper—consumers differ by only a scalar parameter.

[8] An example where the substitutability of products makes the firm *less* likely to engage in bundling is as follows. Suppose that $v_b = v_1 + v_2$ if $\min\{v_1, v_2\} \geq k$ and $v_b = \max\{v_1, v_2\}$ otherwise, where k is a positive constant. Thus, preferences are additive when both stand-alone valuations are high, while if one valuation does not meet the threshold k the incremental value for the second item is zero. With these preferences, whenever the linear price satisfies $p < k$ those consumers with $\min\{v_1, v_2\} \geq k$ will buy both items, and this set does not depend on p. Therefore, bundle demand $\hat{x}_b$ is completely inelastic for $p < k$, while in the corresponding example without substitution (i.e., setting $k = 0$), bundle demand is elastic. Whenever k is large enough that the equilibrium linear price is below k, the firm strictly lowers its profits if it introduces a bundle discount: it reduces its revenue from those who buy the bundle without any compensating boost to overall demand.

second item. With additive preferences, the only motive in this model to use a bundle discount is to extract information rents from consumers, and this motive vanishes if the firm knows consumer preferences. With sub-additive preferences, the firm may wish to offer a bundling tariff even when it knows the customer's tastes. While with integrated supply sub-additive preferences merely give one additional reason to bundle, with separate sellers such preferences will often be the *sole* reason to offer a bundle discount, as discussed in section 4.

3.3 Special cases

In this section I describe three special cases to illustrate this analysis of bundling incentives, as well as some equilibrium bundling tariffs.

Example 1: Constant disutility of joint consumption.

Consider the situation in which for all consumers

$$v_b = v_1 + v_2 - z \tag{13}$$

for some constant $z \geq 0$. Here, to ensure free disposal we assume that the minimum possible realization of v_i is greater than z. With a linear price p_i for buying product i, the pattern of demand is as shown on Figure 2. The next result provides a sufficient condition for bundling to be profitable in this setting.

Claim 2 *Suppose that bundle valuations are given by (13). Suppose that each valuation v_i has marginal c.d.f. F and marginal density f, and the hazard rate $f(\cdot)/(1-F(\cdot))$ is strictly increasing. Then a monopolist has an incentive to offer a bundle discount when condition (9) holds.*

To illustrate, suppose that (v_1, v_2) is uniformly distributed on the unit square $[1,2]^2$, and that $z = \frac{1}{4}$ and $c = 1$. Then an integrated monopolist which uses linear prices will choose $p \approx 1.521$, generating profit of around 0.407. At this price, around 73% of potential consumers buy something, although only 5% buy both products. The most profitable bundling tariff can be calculated to be

$$p \approx 1.594 \ ; \ \delta \approx 0.380 \ , \tag{14}$$

which generates profit of about 0.446, and about 66% of potential consumers buy something but now 28% buy both items. This bundle discount is large enough to outweigh the innate substitutability of the products (i.e., $\delta > z$), and faced with this bundling tariff consumers now view the two products as complements rather than substitutes. (The resulting pattern of demand looks as depicted in Figure 5.) Nevertheless, the discount in (14) is smaller than it is in the corresponding example with additive valuations (i.e., when $z = 0$).[9]

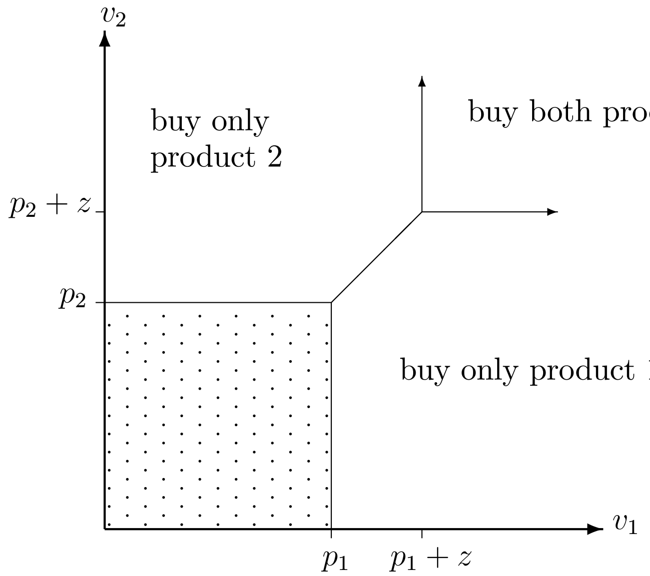

Figure 2: Pattern of demand with constant disutility of joint purchase

Example 2: Time-constrained consumers.

A natural reason why products might be substitutes is that some buyers are only able to consume a restricted set of products, perhaps due to time constraints.[10] To that end, suppose that an exogenous fraction λ of consumers have valuation v_i for stand-alone product $i = 1, 2$ and valuation $v_b = v_1 + v_2$ for the bundle, while the remaining consumers can only buy a single item (and have valuation v_i if they buy item i). (See Figure 3 for an illustration.) For simplicity, suppose that the distribution for (v_1, v_2) is the same for the two groups of consumers. Let $\Psi(\cdot)$ be as defined in (8). It is straightforward to show $\Phi(p) = \lambda\Psi(p)/(2 - \Psi(p))$, so that Φ is decreasing if and only if Ψ is. Proposition 2 therefore implies that when some consumers are time-constrained, an integrated firm has an incentive to offer a bundle discount if and only if (9) holds, i.e., under the same condition as when consumers have additive preferences. The reason is that when the firm offers a bundle discount this only affects the λ unconstrained consumers, and the sign of the impact on profit is just as if all consumers had additive preferences.

[9]When $c = 1$, (v_1, v_2) is uniformly distributed on $[1, 2]^2$ and $v_b \equiv v_1 + v_2$, one can check that $p = \frac{5}{3}$ and $\delta = \frac{\sqrt{2}}{3} \approx 0.47$.

[10]In the context of competitive *intra*-firm bundling, Thanassoulis (2007) also analyzes the situation where an exogenous fraction of consumers wish to buy a single product.

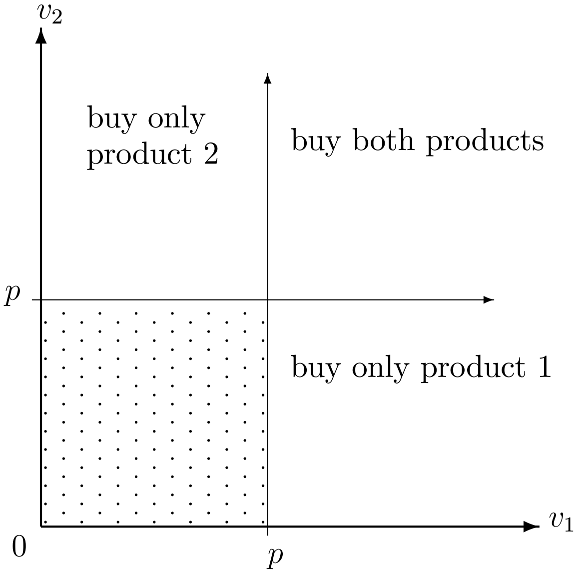

Figure 3A: Unconstrained consumers

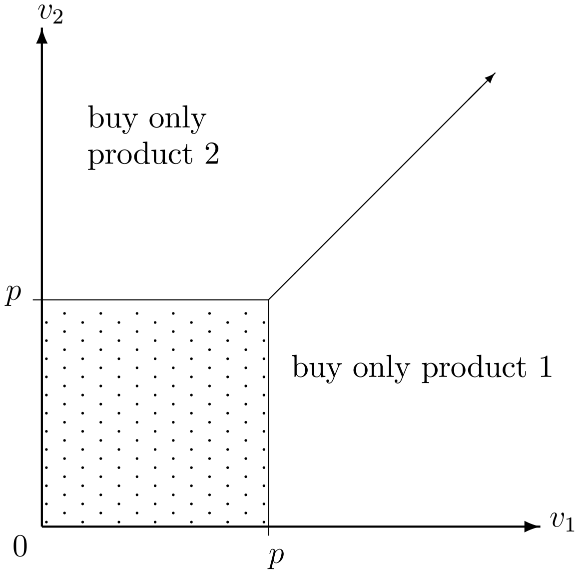

Figure 3B: Time-constrained consumers

Example 3: Stand-alone values (v_1, v_2) *are uniformly distributed on the unit square* $[0, 1]^2$*, and given* (v_1, v_2) *the bundle value* v_b *is uniformly distributed on* $[\max\{v_1, v_2\}, v_1 + v_2]$.

(Recall that with free disposal we require that v_b be at least $\max\{v_1, v_2\}$, and we require $v_b \le v_1 + v_2$ if products are substitutes.) The support of (V_1, V_2) on Figure 1B in this example is $0 \le V_2 \le V_1 \le 1$, and calculations reveal that the joint density for (V_1, V_2) on this support is $2 \log \frac{V_1}{V_2}$. The marginal densities for V_1 and V_2 are respectively $g_1(p) = 2p$ and $g_2(p) = 2(p - \log p - 1)$. It follows that $x_b(p) = 1 - (p^2 - 2p \log p)$ and $x_s(p) = -2p \log p$. If $c = 0$, the most profitable linear price p maximizes $p(x_s(p) + 2x_b(p))$, which entails $p \approx 0.540$ and profit 0.406. About 70% of potential consumers buy something with this tariff, although just 4% of consumers buy the bundle.

One can check that x_b/x_s strictly decreases with p, and Proposition 1 implies that the firm will wish to offer a bundle discount. One can modify Figure 1B to allow the firm to offer a discount $\delta > 0$, and integrate the density for (V_1, V_2) over the regions corresponding to single-item and bundle demand, to obtain explicit (but tedious) expressions for single-item and bundle demands in terms of the tariff parameters (p, δ). Using these expressions, one can calculate the optimal bundling tariff to be $p \approx 0.648$ and $\delta \approx 0.588$, which yields profit 0.463. Notice that the bundle discount is now deeper compared to the corresponding example with additive values.[11] With this bundling tariff, where the incremental price for the second item is rather small, about 51% of potential consumers buy the bundle and only 15% buy a single item.

[11] When $c = 0$, (v_1, v_2) is uniformly distributed on $[0, 1]^2$ and $v_b \equiv v_1 + v_2$, one can check that $\delta \approx 0.47$.

4 Separate Sellers

4.1 General analysis

I turn now to the situation where the two products are supplied by separate sellers. In contrast to the integrated seller case, here there is no significant advantage in assuming that products are symmetric, and we no longer make that assumption. Suppose that the sellers set their tariffs simultaneously and non-cooperatively. (The next section discusses a setting in which firms coordinate on their inter-firm bundle discount.) When firms offer linear prices—i.e., prices which are not contingent on whether the consumer also purchases the other product—firm i chooses its price p_i^* given its rival's price to maximize $(p_i - c_i)(q_i + q_b)$, so that

$$q_i \left[1 - (p_i^* - c_i)\frac{-\partial q_i/\partial p_i}{q_i}\right] + q_b \left[1 - (p_i^* - c_i)\frac{-\partial q_b/\partial p_i}{q_b}\right] = 0 \ . \tag{15}$$

In some circumstances, a firm can condition its price on whether a consumer also buys the other firm's product. For instance, a museum could ask a visitor to show her entry ticket to the other museum to claim a discount. Suppose now that firm i offers a discount $\delta > 0$ from its price p_i^* to those consumers who purchase product j as well. (Those consumers who only buy product i continue to pay p_i^*.) Then firm i's profit is

$$\pi_i = (p_i^* - c_i)(Q_i + Q_b) - \delta Q_b \ , \tag{16}$$

and the impact on profit of a small joint purchase discount is governed by the sign of $\frac{d\pi_i}{d\delta}\big|_{\delta=0}$, which from (3) is equal to

$$-q_b - (p_i^* - c_i)\frac{\partial q_b}{\partial p_i} \ . \tag{17}$$

When demand for the single item is less elastic than bundle demand, so that $\frac{-\partial q_i/\partial p_i}{q_i} < \frac{-\partial q_b/\partial p_i}{q_b}$, the second term $[\cdot]$ in (15) is strictly negative, i.e., (17) is strictly positive. In this case, offering a discount for joint purchase will raise the firm's profit.

Thus, discounts for joint purchase can arise even when products are supplied by separate firms and when a firm chooses and funds the discount unilaterally. The reason is straightforward: since the own-price elasticity of bundle demand is higher than that of demand for its stand-alone product, a firm wants to offer a lower price to those consumers who also buy the other product. As expression (1) shows, the introduction of a discount will also benefit the rival firm.

We summarise this discussion as:

Proposition 3 *Suppose that demand for the bundle is more elastic than demand for firm i's stand-alone product, in the sense that*

$$\frac{q_b(p_1, p_2)}{q_i(p_1, p_2)} \text{ strictly decreases with } p_i \ . \tag{18}$$

Starting from the situation where firms set equilibrium linear prices p_1^* and p_2^*, firm i has an incentive to offer a discount to those consumers who buy product j. If expression (18) is reversed, so that q_b/q_i increases with p_i, then firm i would like if feasible to charge its customers a premium if they buy product j.

The crucial difference between condition (18) and the corresponding condition (7) with integrated supply is that with a single seller both prices are increased, whereas with separate sellers only one price rises. With substitute products and linear pricing, a firm competes on three fronts. If it raises its price: (i) some consumers will switch from buying the bundle to buying the rival product alone; (ii) some will switch from buying its product alone to buying the rival product alone, and (iii) some consumers will switch from buying its product alone to buying nothing. (As discussed in section 2, with substitutes a possible fourth margin between buying the bundle and buying nothing is absent.) Broadly speaking, condition (18) requires that margins (ii) and (iii) together are less significant, relative to the size of associated demand, than margin (i).

When products are asymmetric, at the equilibrium linear prices it is possible that one firm has an incentive to offer a discount when a customer also buys the other firm's product, but the other firm does not.[12] However, it may well be that both firms choose to offer such a discount. If firm $i = 1, 2$ offers the price p_i when a consumer only buys its product and the price $p_i - \delta_i$ when she also buys the other product, a consumer who buys the bundle pays the price $p_1 + p_2 - \delta_1 - \delta_2$. The issue then arises as to how the combined discount $\delta = \delta_1 + \delta_2$ is implemented. For instance, a consumer might have to buy the two items sequentially, and firms cannot simultaneously require proof of purchase from the other seller when they offer their discount. However, there are at least two natural ways to implement this inter-firm bundling scheme. First, the bundle discount could be implemented via an electronic sales platform which allows consumers to buy products from several sellers simultaneously. Sellers choose their prices contingent on which other products (if any) a consumers buys, a website displays the total prices for the various combinations, and firms receive their stipulated revenue from the chosen combination. With such a mechanism there is no need

[12] One simple way this can happen is when one firm sells to *all* consumers at the equilibrium linear prices, while the other does not. Clearly, the latter firm has nothing to gain from making its price contingent on whether its customers buy the other product, while the former may have such an incentive.

for firms to coordinate their tariffs. Second, there may be "product aggregators" present in the market who put together their own bundles from products sourced from separate firms and retail these bundles to final consumers. In the two-product case discussed in this paper, aggregators bundle the two products together and each firm chooses a wholesale price for its product contingent on being part of the bundle. If the aggregator market is competitive, the price of the bundle will simply be the sum of the two wholesale prices. Again, there is no need for firms to coordinate their prices.

A major difference between this inter-firm bundling discount and the discount offered by an integrated supplier is that with separate sellers the discount is chosen non-cooperatively. A bundle is, by definition, made up of two "complementary" components, namely, firm 1's product and firm 2's product, and the total price for the bundle is the sum of each firm's component price $p_i - \delta_i$. When a firm considers the size of its own discount δ_i, it ignores the benefit this discount confers on its rival. Thus, as usual with separate supply of complementary components, double marginalization will result and the overall discount $\delta = \delta_1 + \delta_2$ will be too small (for given stand-alone prices).

4.2 Special cases

In this section, I analyze in more depth various special cases where separate sellers have an incentive to introduce a joint-purchase discount. Consider first the situation where consumer valuations are additive, so that margin (ii) discussed in section 4 is absent and firms do not compete with each other:

Proposition 4 *Suppose that valuations are additive, i.e.,* $v_b = v_1 + v_2$*. Starting from the situation where firms set equilibrium linear prices, firm* i *has an incentive to offer a discount to those consumers who buy the other product whenever* $\Pr\{v_j \le p \mid v_i\}$ *strictly increases with* v_i*.*

Whenever the valuations are negatively correlated in the strong sense that $\Pr\{v_j \le p \mid v_i\}$ decreases with v_i, then, a firm has an incentive to offer a discount for joint purchase. Somewhat counter-intuitively, those firms which offer products which appeal to very different kinds of consumer (boxing and ballet, say) may wish to offer discounts to consumers who buy the other product.

In the oligopoly context, it is sometimes natural to consider situations with full coverage, so that all consumers buy something for the relevant range of linear prices.[13] (This is

[13] This is not a useful special case to consider in the context of integrated supply. For instance, Armstrong (1996) shows how a monopolist will typically wish to exclude some consumers when consumers have multi-dimensional private information (as they do here).

relevant when the minimum possible realizations of v_1 and v_2 are sufficiently high.) When the outside option of zero is not relevant for any consumer's choice, all that matters for demand is the distribution of *incremental* utilities, and given the triple (v_1, v_2, v_b) define new variables

$$\hat{v}_1 \equiv v_b - v_2 \ ; \ \hat{v}_2 \equiv v_b - v_1 \tag{19}$$

for the incremental valuation for product i given the consumer already has product j.

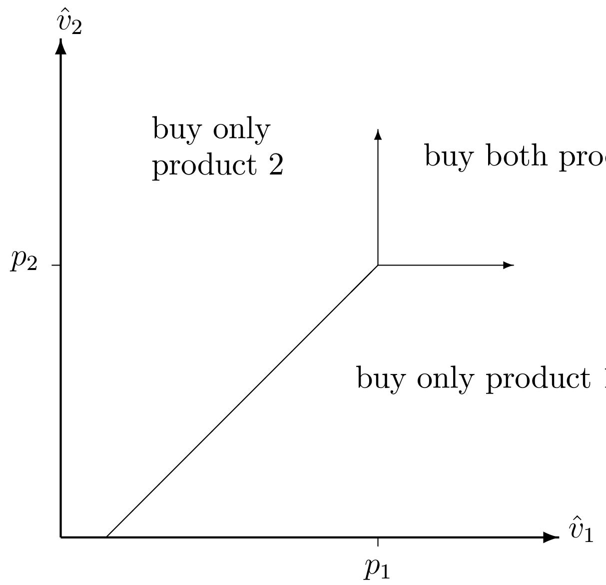

Figure 4: Pattern of demand with full consumer coverage

As depicted on Figure 4, a consumer will buy both items with linear prices (p_1, p_2) provided that $\hat{v}_1 \geq p_1$ and $\hat{v}_2 \geq p_2$, and otherwise she will buy product 1 instead of product 2 when $\hat{v}_1 - p_1 \geq \hat{v}_2 - p_2$. In particular, margin (iii) discussed in section 4.1 is no longer present, and this may boost the incentive to offer a bundle discount. Write $G_i(\hat{v}_i \mid \hat{v}_j)$ for the c.d.f. for $\hat{v}_i$ conditional on $\hat{v}_j$, and write $g_i(\hat{v}_i \mid \hat{v}_j)$ for the associated conditional density. Consider this assumption on the hazard rate:

$$\frac{g_i(\hat{v}_i \mid \hat{v}_j)}{1 - G_i(\hat{v}_i \mid \hat{v}_j)} \text{ strictly increases with } \hat{v}_i \text{ and weakly increases with } \hat{v}_j \ . \tag{20}$$

It is somewhat reasonable to suppose that this hazard rate increases with $\hat{v}_i$. That the hazard rate weakly increases with $\hat{v}_j$ is perhaps less economically natural, but includes independence of $\hat{v}_1$ and $\hat{v}_2$ as a particular case.

Proposition 5 *Suppose at the relevant linear prices there is full consumer coverage. If the incremental valuations in (19) satisfy condition (20), then firm i has an incentive to offer a discount to those consumers who buy the other product.*

When the market is covered, this result suggests that the incentive to introduce a discount contingent on buying another firm's product is present for many pairs of suppliers.

We next consider the impact of inter-firm bundling in two of the examples with non-additive valuations introduced in section 3.3.

Example 1. Here, the pattern of consumer demand was illustrated in Figure 2. Write $H_i(v_i \mid v_j)$ for the c.d.f. for v_i conditional on v_j and $h_i(v_i \mid v_j)$ for the associated conditional density. The next result describes when a firm has a unilateral incentive to offer a bundle discount. (The proof of the claim is similar to that for Proposition 5, and omitted.)

Claim 3 *Suppose that bundle valuations satisfy (13) and the stand-alone valuations satisfy*

$$\frac{h_i(v_i \mid v_j)}{1 - H_i(v_i \mid v_j)} \text{ strictly increases with } v_i \text{ and weakly increases with } v_j \ . \tag{21}$$

Then a seller has an incentive to offer a discount to consumers who buy the rival's product.

It is economically intuitive that products being substitutes of the form (13) will give the firm an incentive to offer a discount when its customers purchase the rival product. If the potential customer purchases the other product, this is bad news for the firm as the customer's incremental value for its product has been shifted downwards by z, and this provides an incentive to offer a lower price.

Consider the same specific example as presented in section 3—that is, (v_1, v_2) uniform on $[1, 2]^2$, $z = \frac{1}{4}$ and $c = 1$—applied to the case with separate sellers. The equilibrium linear price is $p \approx 1.446$ and industry profit is about 0.399. Around 9% of consumers buy both items with this linear price, and 80% buy something. The equilibrium non-cooperative bundling tariff is

$$p_1 = p_2 = 1.476 \ ; \ \delta_1 = \delta_2 = 0.05 \ . \tag{22}$$

Here, the combined bundle discount, $\delta = \delta_1 + \delta_2$, is about one quarter the size of the discount with integrated supply in (14), reflecting the earlier discussion that separate firms will non-cooperatively choose too small a discount. Now, around 14% of consumers buy both items, and industry profit rises to 0.421. Intuitively, when firms offer a bundle discount, this reduces the effective degree of substitution between products, which in turn relaxes competition between firms. As reported in Table 2 below, relative to the outcome with linear pricing, here consumers in aggregate are harmed, but total welfare rises, when firms unilaterally offer a discount. Note that the equilibrium linear price lies between the two discriminatory prices when firms engage in this form of price discrimination.[14]

[14] This is not surprising in the light of the analysis in Corts (1998), who shows that when the two firms wish to set their lower price to the same group of customers (the "weak" market, which in this example is

Example 2. Consider next the situation in which some consumers are time constrained, so that a fraction λ of consumers have additive preferences and the remaining consumers wish to buy either product 1 or product 2. Suppose the distribution of stand-alone valuations (v_1, v_2) is the same for the two kinds of consumer. Then we can obtain the following result:

Claim 4 *Suppose that some consumers are time-constrained, and that stand-alone valuations v_1 and v_2 are independently distributed, where v_i has distribution function $F_i(\cdot)$ and density $f_i(\cdot)$, and where for each i the hazard rate $f_i(\cdot)/(1 - F_i(\cdot))$ is strictly increasing. When the two products are supplied by separate sellers, a seller has no incentive to offer a discount to those consumers who buy the rival product. They would, if feasible, like to charge their customers a higher price when they buy the rival product.*

In this setting, the observation that a consumer wishes to buy both items implies she belongs to the "non-competitive" group of consumers, and a firm would like to exploit its monopoly position over those consumers if feasible. Of course, in many situations, a consumer can hide her purchase from a rival firm, in which case a firm cannot feasibly levy a premium when a customer buys another supplier's product. Comparing Examples 1 and 2 shows that the precise manner in which products are substitutes is important for a firm's incentive to offer a bundling discount unilaterally.

The fundamental condition governing when a firm unilaterally wishes to introduce a joint-purchase discount is (18). All the special cases considered in this section have the same underlying logic, which is to find conditions under which single-item demand is more, or less, elastic than bundle demand. With additive preferences, Proposition 4, shows that negative correlation between the two valuations implies demand for a firm's product on its own is less elastic than demand for the bundle. This is due to the fact that the size of bundle demand is then small relative to stand-alone demand. With full coverage, Proposition 5 (as well as the closely related Claim 3) shows how single-item demand is less elastic than bundle demand, but for a different reason: the margin (i) between buying the bundle and buying only product 2 is more competitive than the margin (ii) between buying only product 1 or only product 2. When some consumers are time-constrained (Claim 4), margin (ii) is now less competitive than margin (i), and a firm wishes to raise its price to those consumers

the set of consumers who buy both products), then the equilibrium non-discriminatory price lies between the two discriminatory prices. However, we cannot apply Corts' result directly, since his argument relies on there being no cross-price effects across the two consumer groups, which is not the case here.

wishing to buy the bundle.[15]

5 Partial Coordination Between Sellers

The analysis to this point has considered the two extreme cases where there is no tariff coordination between separate sellers (section 4), and where there is complete tariff co-ordination (section 3). The problem with complete coordination is that any competition between rivals is eliminated. As discussed in section 4, though, the welfare problem with a policy of permitting no coordination between sellers is that the resulting bundle discount may be inefficiently small (or non-existent). It would be desirable to obtain the efficiency gains which may accrue to bundling without permitting the firms to collude over their regular prices.

One way this might be achieved is if firms first negotiate an inter-firm bundle discount, the funding of which they agree to share, and then compete by choosing their stand-alone prices non-cooperatively. Specifically, suppose the two firms are symmetric and consider the following joint pricing scheme: firms first coordinate on bundle discount δ, and if firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying both products is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold.

Consider first the case where valuations are additive, so that competition concerns are absent. Firm i's profit under this scheme is

$$(p_i - c)(Q_i + Q_b) - \tfrac{1}{2}\delta Q_b \ , \tag{23}$$

where each firm's price is a function of the agreed discount δ as determined by the second-stage non-cooperative choice of prices. The impact of introducing a small $\delta > 0$ on firm i's equilibrium profit is equal to

$$\frac{d}{d\delta}\left\{(p_i - c)(Q_i + Q_b) - \tfrac{1}{2}\delta Q_b\right\}\Big|_{\delta=0} = -\tfrac{1}{2}Q_b\Big|_{\delta=0} + (p^* - c)\,\frac{\partial}{\partial\delta}(Q_i + Q_b)\Bigg|_{\delta=0}$$

$$+\left(\frac{dp_i}{d\delta}\Bigg|_{\delta=0}\right)\left(\frac{\partial}{\partial p_i}[(p_i - c)(Q_i + Q_b)]\Bigg|_{\delta=0, p_i=p_j=p^*}\right) \tag{24}$$

$$+\left(\frac{dp_j}{d\delta}\Bigg|_{\delta=0}\right)\left(\frac{\partial}{\partial p_j}[(p_i - c)(Q_i + Q_b)]\Bigg|_{\delta=0, p_i=p_j=p^*}\right) \tag{25}$$

$$= -\tfrac{1}{2}\left[x_b(p^*) + (p^* - c)x_b'(p^*)\right] \ . \tag{26}$$

[15] In technical terms, the difference between the two cases is that in Figure 4, margin (ii) lies to the left of margin (i), while in Figure 3 the reverse is true.

Here, the two terms (24)–(25) reflect the indirect effect of the discount on the firm's profit via its impact on the two prices, p_i and p_j, both of which vanish, and the final expression (26) follows from (3). Expression (24) vanishes because p^* is the optimal price for firm i when firms choose linear prices (i.e., p^* maximizes $(p_i - c)(q_i + q_b)$). Expression (25) vanishes because changing the other firm's price has no impact on a firm's demand when there is no bundling discount and valuations are additive (i.e., $q_i + q_b$ does not depend on p_j when valuations are additive). Thus, the first-order impact of δ on industry profit is that, for a *fixed* stand-alone price p^*, the discount boosts overall demand but reduces revenue from each bundle sold. Following the discussion in section 3.1, in the additive case expression (26) is positive if and only if (9) holds. To summarize:

Proposition 6 *Suppose that products are symmetric and valuations are additive. Consider the coordinated bundling scheme whereby firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying the bundle is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold. If condition (9) holds, for small discount $\delta > 0$ this scheme increases each firm's profit relative to the situation where the products are sold independently ($\delta = 0$).*

This result suggests that a coordinated bundling scheme of this form could be profitable for many pairs of suppliers, even if they supply unrelated products. Proposition 6 could be seen as a "separate seller" analogue of the result for integrated monopoly derived by Long (1984) and McAfee *et al.* (1989), who showed with additive preferences that when condition (9) was satisfied it was profitable for a monopolist to introduce a bundle discount.

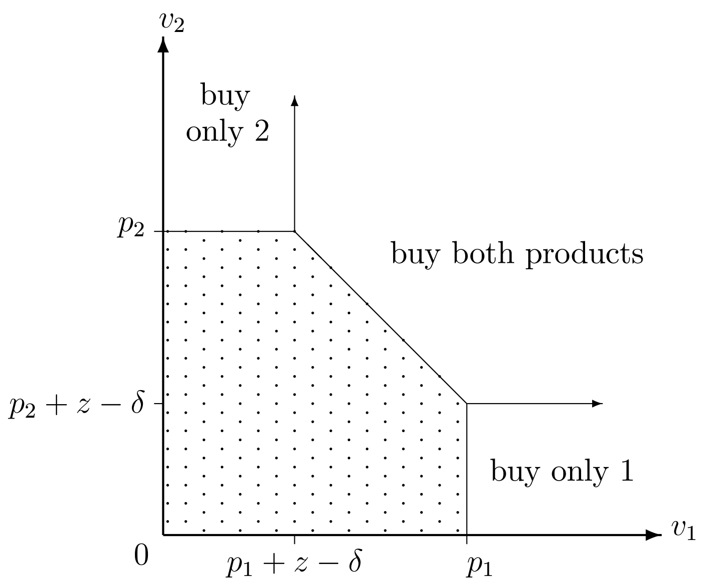

Figure 5: Pattern of demand in Example 1 with bundling discount $\delta > z$

To illustrate, consider the specific case where (v_1, v_2) is uniformly distributed on $[1, 2]^2$ and the marginal cost of each product is $c = 1$. This is a special case of Example 1 above, with z set equal to zero. Figure 5 depicts the pattern of consumer demand in Example 1 for general z when the bundle discount δ is larger than the substitution parameter z, as will turn out to be the case in equilibrium. (The case with $\delta < z$ looks like Figure 2.) Since valuations are additive when $z = 0$, without coordination on the inter-firm discount (so $\delta = 0$) firms set price $p^* = \frac{3}{2}$. The resulting payoffs in the market are reported in the first row of Table 1. If firms first coordinate on δ and then choose price non-cooperatively, one can check that the most profitable discount is $\delta \approx 0.384$, which implements the higher price $p \approx 1.669$.[16] The corresponding payoffs are reported in the second row of this table. In this example, then, allowing the firms to coordinate on an inter-firm discount boosts profit, harms consumers in aggregate, and (slightly) increases overall welfare. Because valuations are additive and statistically independent, firms would not wish unilaterally to introduce a joint-purchase discount in this market.

	industry profit	consumer surplus	welfare
linear pricing ($\delta = 0$)	0.500	0.250	0.750
coordinated discount ($\delta = 0.384$)	0.544	0.210	0.754

Table 1: Market outcomes with and without coordination on discount ($z = 0$)

While the operation of the joint-pricing scheme appears relatively benign when values are additive, this can be reversed when firms offer substitutable products. Consumers benefit, and total welfare rises, when firms are forced to set low prices due to products being substitutes. However, an agreed inter-firm discount can reduce the effective substitutability of products and relax competition between suppliers. To illustrate this effect, modify the preceding example so that $z = \frac{1}{4}$. The impact of partial coordination in this case is reported in Table 2. As derived in section 4.2, with linear pricing firms choose price $p \approx 1.446$ and the resulting payoffs are given in the first row. When firms coordinate on the bundle discount and then choose stand-alone prices non-cooperatively, their most profitable choice is $\delta \approx 0.39$, which implements price $p \approx 1.588$, and payoffs are given in the second row.[17] In contrast to Table 1, now total welfare falls when firms coordinate on the discount, reflecting the high prices which are then induced. For comparison, the third row reports payoffs when firms choose their joint-purchase discount non-cooperatively, when

[16] Recall from footnote 9 that the bundling tariff chosen by an integrated supplier in this example involves a deeper discount but essentially the same stand-alone price.

[17] This tariff is similar to that offered when the firms fully coordinate their retail tariffs, when the tariff is (14). As such, the payoffs when firms fully coordinate their tariffs is similar to the figures given in the second row of Table 2.

the equilibrium tariff is (22) above. For firms and consumers, the resulting outcome is intermediate between the outcomes with linear pricing and with a coordinated discount; however it generates the highest welfare level of the three regimes. At least in this example, a modest bundle discount enhances welfare, but when firms coordinate on the discount, they choose too deep a discount from a welfare perspective.

	industry profit	consumer surplus	welfare
linear pricing ($\delta = 0$)	0.399	0.261	0.660
coordinated discount ($\delta = 0.390$)	0.449	0.202	0.651
non-cooperative discount ($\delta = 0.1$)	0.421	0.244	0.665

Table 2: Market outcomes with and without coordination on discount ($z = \frac{1}{4}$)

Thus, the apparently pro-consumer policy of coordinating to offer a discount for joint purchase may act as a device to sustain collusion. This suggests that negotiated inter-firm discounting schemes operated by firms supplying substitutable products should be viewed with some suspicion by antitrust authorities, although non-cooperative discounting schemes as analyzed in section 4 may actually be welfare-enhancing.

6 Conclusions

This paper has extended the standard model of bundling to allow products to be partial substitutes and for products to be supplied by separate sellers. With monopoly supply, building on Long (1984), we typically found that the firm has an incentive to offer a bundle discount in at least as many cases as with the traditional model with additive valuations. Sub-additive preferences give the firm an additional reason to offer a bundle discount, which is to better target a low price for a second item at those customers who are inclined (with linear prices) to buy a single item. We observed that the impact of substitutability could amplify or diminish the size of the most profitable bundle discount.

When products were supplied by separate firms, we found that a firm often has a unilateral incentive to offer a joint-purchase discount when their customers buy rival products. In such cases, inter-firm bundle discounts are achieved without any need for coordination between suppliers. The two principal situations in which a firm might wish to do this are (i) when product valuations are negatively correlated in the population of consumers, and (ii) when products are substitutes in such a way that bundle demand was more elastic than single-item demand. While product substitutability makes bundle demand *smaller* than it would otherwise be, it need not make such demand more elastic. Plausible kinds of substitution lead firms to offer either a joint-purchase discount or a joint-purchase premium. In an example (Example 1) we saw that when firms price discriminate in this manner, relative to the uniform pricing regime equilibrium profits are higher and welfare rises. One reason why profits rise is that when firms offer an inter-firm bundle discount, this mitigates the innate substitutability of their products and competition is relaxed.

Historically, this form of price discrimination was not often observed. In many cases, in order to condition price on a purchase from a rival supplier, a firm would need a "paper trail" such as a receipt from the rival. One problem with this system is that customers are then encouraged to visit the rival firm first, and because of transaction and travel costs, this might mean that fewer customers would actually come to the firm. A second problem is that it is hard for two firms to offer such discounts, since a customer might have to visit the firms sequentially. However, as discussed in section 4, these two related problems can nowadays often be overcome with modest methods of selling, and we may see greater use of this kind of contingent pricing in future.

A more traditional way to implement inter-firm bundling is for firms to coordinate aspects of their pricing strategy. In this paper I examined one particular kind of coordination, which is where firms agree on a joint purchase discount, and subsequently choose their prices non-cooperatively. Because a bundle discount mitigates the innate substitutability of rival products, separate sellers can use this mechanism to lessen rivalry in the market. Thus, firms often have an incentive to explore joint pricing schemes of this form, and regulators have a corresponding incentive to be wary.

In future work it would be useful to extend the analysis in this paper in at least three directions. First, how do the results change if the products are complements rather than substitutes? Second, what happens if the products in question are intermediate products? It may be that the framework studied here could sometimes be extended to situations where rival manufacturers potentially supply products to a retailer, which then supplies one or both products to final consumers. If products are partial substitutes, might a manufacturer have an incentive to charge a lower price if the retailer also chooses to supply the rival product? This would then be the opposite pricing pattern to the "loyalty pricing" schemes which worry antitrust authorities. Finally, it would be interesting to explore whether a "large" firm has an incentive to exclude smaller firms from its internal bundling policies, with the aim of driving these rivals out of the market. In a famous antitrust case concerning ski-lifts in the Aspen resort, described in Easterbrook (1986), one small ski-lift operator successfully sued a larger operator for not permitting it to participate in its multi-mountain ski-pass scheme.

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APPENDIX

Proof of Claim 1: A type-(v_1, v_2, v_b) consumer buys product 1 if and only if

$$\max\{v_b - p_1 - p_2, v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ . \tag{27}$$

I claim that the difference between the two sides in (27), that is

$$\max\{v_b - p_1 - p_2, v_1 - p_1\} - \max\{v_2 - p_2, 0\} \ , \tag{28}$$

is weakly increasing in p_2 for all (v_1, v_2, v_b). (This then implies that the set of consumer types who buy product 1 is increasing, in the set-theoretic sense, in p_2, and so the measure of such consumers is increasing in p_2.) The only way in which expression (28) could strictly decrease with p_2 is if $v_b - p_1 - p_2 > v_1 - p_1$ and $v_2 - p_2 < 0$. However, since products are substitutes we have $v_b \leq v_1 + v_2$, which implies that the above pair of inequalities are contradictory. This establishes the result. ■

Proof of Proposition 2: We know already that choosing $\delta > 0$ is more profitable than choosing $\delta = 0$ when expression (7) holds, which in turn is true when (11) is strictly decreasing. Therefore, it remains to rule out the possibility that a tariff with a quantity *premium* is optimal. So suppose to the contrary that the firm makes greatest profit by charging P_1 for the first item and $P_2 > P_1$ for the second. By modifying Figure 1B to allow $P_2 > P_1$, one sees that the firm's profit takes the additively separable form

$$(1 - G(P_1))(P_1 - c) + (1 - G(P_2))\Phi(P_2)(P_2 - c)$$

where we write $G(p) \equiv \Pr\{V_1 \leq p\}$. This profit is therefore greater than when the firm offers either of the linear prices P_1 and P_2. That is to say

$$(1-G(P_1))(P_1-c)+(1-G(P_2))\Phi(P_2)(P_2-c) \geq (1-G(P_1))(P_1-c)+(1-G(P_1))\Phi(P_1)(P_1-c)$$

or

$$(1-G(P_2))\Phi(P_2)(P_2-c) \geq (1-G(P_1))\Phi(P_1)(P_1-c) \ , \tag{29}$$

and

$$(1-G(P_1))(P_1-c)+(1-G(P_2))\Phi(P_2)(P_2-c) \geq (1-G(P_2))(P_2-c)+(1-G(P_2))\Phi(P_2)(P_2-c)$$

or

$$(1-G(P_1))(P_1-c) \geq (1-G(P_2))(P_2-c) \ . \tag{30}$$

Since (11) is strictly decreasing, (29) implies that

$$(1-G(P_2))\Phi(P_2)(P_2-c) > (1-G(P_1))\Phi(P_2)(P_1-c) \ ,$$

which contradicts expression (30). Thus, the most profitable tariff involves $P_2 < P_1$. ■

Proof of Claim 2: From Figure 2 we see that with linear price p for either product we have

$$x_b(p) = (1-F(p+z))\Psi(p+z) \ ; \ x_s(p) = (1-F(p))(2-\Psi(p)) - x_b(p) \ ,$$

and so (11) is given by

$$\Phi(p) = \frac{x_b(p)}{x_s(p)+x_b(p)} = \frac{(1-F(p+z))\Psi(p+z)}{(1-F(p))(2-\Psi(p))} \ .$$

Differentiating shows that Φ is strictly decreasing with p if and only if

$$\frac{\Psi'(p)}{2-\Psi(p)} + \frac{\Psi'(p+z)}{\Psi(p+z)} < \frac{f(p+z)}{1-F(p+z)} - \frac{f(p)}{1-F(p)} \ .$$

Since F is assumed to have an increasing hazard rate, the right-hand side of the above is non-negative, while if condition (9) holds then the left-hand side is strictly negative. Therefore, Φ is strictly decreasing and Proposition 2 implies the result. ■

Proof of Proposition 4: Let $F_i(v_i)$ and $f_i(v_i)$ be respectively the marginal c.d.f. and the marginal density for v_i, and let $H(v_i) \equiv \Pr\{v_j \leq p_j^* \mid v_i\}$, where p_j^* is firm j's equilibrium linear price. Then

$$q_i(p_i, p_j^*) = \int_{p_i}^{\infty} H(v_i) f_i(v_i) dv_i \ ; \ q_b(p_i, p_j^*) = \int_{p_i}^{\infty} (1-H(v_i)) f_i(v_i) dv_i \tag{31}$$

and

$$-\frac{\partial q_i}{\partial p_i} = H(p_i)f_i(p_i) \ ; \ -\frac{\partial q_b}{\partial p_i} = (1 - H(p_i))f_i(p_i) \ .$$

Since H is assumed to be strictly increasing in v_i, it follows from (31) that

$$q_i(p_i, p_j^*) > H(p_i)(1 - F_i(p_i)) \ ; \ q_b(p_i, p_j^*) < (1 - H(p_i))(1 - F_i(p_i))$$

and so

$$-\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} < \frac{f_i(p_i)}{1 - F_i(p_i)} < -\frac{1}{q_b}\frac{\partial q_b}{\partial p_i}$$

and Proposition 3 implies the result. ■

Proof of Proposition 5: Suppose $p_i \geq p_j$ as depicted on Figure 4. (The case where $p_i < p_j$ is handled in a similar manner.) Let $h_j(\hat{v}_j)$ denote the marginal density for $\hat{v}_j$. From the figure we see that

$$q_b = \int_{p_j}^{\infty} (1 - G_i(p_i \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \ ; \ -\frac{\partial q_b}{\partial p_i} = \int_{p_j}^{\infty} g_i(p_i \mid \hat{v}_j)h_j(\hat{v}_j)d\hat{v}_j \ .$$

From assumption (20) we obtain

$$\begin{aligned} -\frac{\partial q_b}{\partial p_i} &= \int_{p_j}^{\infty} g_i(p_i \mid \hat{v}_j)h_j(\hat{v}_j)d\hat{v}_j \\ &= \int_{p_j}^{\infty} \frac{g_i(p_i \mid \hat{v}_j)}{1 - G_i(p_i \mid \hat{v}_j)}(1 - G_i(p_i \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \\ &\geq \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)}q_b \ . \end{aligned}$$

Similarly,

$$q_i = \int_0^{p_j} (1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \ ; \ -\frac{\partial q_i}{\partial p_i} = \int_0^{p_j} g_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \ .$$

From assumption (20) we obtain

$$\begin{aligned} -\frac{\partial q_i}{\partial p_i} &= \int_0^{p_j} g_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)h_j(\hat{v}_j)d\hat{v}_j \\ &= \int_0^{p_j} \frac{g_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)}{1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j)}(1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \\ &< \int_0^{p_j} \frac{g_i(p_i \mid \hat{v}_j)}{1 - G_i(p_i \mid \hat{v}_j)}(1 - G_i(\hat{v}_j + p_i - p_j \mid \hat{v}_j))h_j(\hat{v}_j)d\hat{v}_j \\ &\leq \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)}q_i \ . \end{aligned}$$

It follows that

$$-\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} < \frac{g_i(p_i \mid p_j)}{1 - G_i(p_i \mid p_j)} \leq -\frac{1}{q_b}\frac{\partial q_b}{\partial p_i}$$

and Proposition 3 implies the result. ■

Proof of Claim 4: From Figure 3 we see that

$$q_b = \lambda(1 - F_1(p_1))(1 - F_2(p_2)) \ ; \ -\frac{\partial q_b}{\partial p_1} = \lambda f_1(p_1)(1 - F_2(p_2))$$

so that

$$-\frac{1}{q_b}\frac{\partial q_b}{\partial p_1} = \frac{f_1(p_1)}{1 - F_1(p_1)} \ . \tag{32}$$

The demand for product 1 on its own comes from two sources: the unconstrained and the constrained consumers. Write

$$q_1 = \lambda x + (1 - \lambda)X$$

where x and X are respectively product 1 demand from the unconstrained and constrained consumers. From Figure 3 we see that

$$x = 1 - F_1(p_1) \ ; \ X = F_2(p_2)(1 - F_1(p_1)) + \int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))dv_2 \ .$$

We need to show that (18) is reversed, so that $-\frac{1}{q_1}\frac{\partial q_1}{\partial p_1}$ is greater than (32). But $-\frac{1}{q_1}\frac{\partial q_1}{\partial p_1}$ is a weighted sum of $-\frac{1}{x}\frac{\partial x}{\partial p_1}$ and $-\frac{1}{X}\frac{\partial X}{\partial p_1}$, and $-\frac{1}{x}\frac{\partial x}{\partial p_1}$ is exactly equal to (32). It follows that (18) is reversed if and only if $-\frac{1}{X}\frac{\partial X}{\partial p_1}$ is greater than (32). But

$$\begin{aligned}
-\frac{\partial X}{\partial p_1} &= f_1(p_1)F_2(p_2) + f_2(p_2)(1 - F_1(p_1)) + \int_{p_2}^{\infty} f_2(v_2)f_1(v_2 + p_1 - p_2)dv_2 \\
&= f_1(p_1)F_2(p_2) + f_2(p_2)(1 - F_1(p_1)) \\
&\qquad + \int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))\frac{f_1(v_2 + p_1 - p_2)}{1 - F_1(v_2 + p_1 - p_2)}dv_2 \\
&> f_1(p_1)F_2(p_2) + f_2(p_2)(1 - F_1(p_1)) + \frac{f_1(p_1)}{1 - F_1(p_1)}\int_{p_2}^{\infty} f_2(v_2)(1 - F_1(v_2 + p_1 - p_2))dv_2 \\
&> \frac{f_1(p_1)}{1 - F_1(p_1)}X \ .
\end{aligned}$$

The first inequality follows from the assumption that $f_1/(1 - F_1)$ is an increasing function, while the second inequality can be verified directly. This completes the proof. ■