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Abstract

In this paper, we empirically examine a heterogenous bounded rationality version of a hybrid New-Keynesian model. The model is estimated via the simulated method of moments using Euro Area data from 1975Q1 to 2009Q4. It is generally assumed that agents' beliefs display waves of optimism and pessimism - so called animal spirits - on future movements in the output and inflation gap. Our main empirical findings show that a bounded rationality model with cognitive limitation provides fits for auto- and cross-covariances of the data which are slightly better than or equal to a model where rational expectations are assumed. This implies that the bounded rationality model provides some structural insights on the expectation formation process at the macro-level for the Euro Area. First, over the whole time interval the agents had expected moderate deviations of the future output gap from its steady state value with low uncertainty. Second, we find strong evidence for an autoregressive expectation formation process regarding the inflation gap. Both observations explain a high degree of persistence in the output gap and the inflation gap.

Keywords: Animal Spirits; Bounded Rationality; Euro Area; New-Keynesian Model; Simulated Method of Moments.

JEL classification: C53, D83, E12, E32.

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1 Introduction

The rational expectations are a flexible and natural way of modeling market behaviors in the dynamic stochastic general equilibrium (DSGE) models, which are widely used by macroeconomists. Since the DSGE approach disposes a convenient analytical tractability under the assumption of rational expectations, this modeling framework serves as a efficient toolbox when analyzing monetary and fiscal policy measures. As Selten (2001) states, however, "modern mainstream economic theory is largely based on a unrealistic picture of human decision theory" since evidence from experimental studies supports information processing with limited cognitive ability of agents rather than perfect information (see Hommes (2011) among others). It is also known in a DSGE modelling framework that a substantial degree of inertia in the reaction of agents to new information (Colander et. al. (2009, p. 8)) should not be overlooked. Indeed, many research has been done on alternative forms of information processing mechanisms in macroeconomics; see e.g. the literature on learning (Evans and Honkaphohja (2001)), rational inattention (Sims (2003)), sticky information (Mankiw and Reis (2002)) or bounded rationality in general (Sargant (1994) and Kahneman (2003)).[1]

The behavioral research program clarifies that the realization of decisions that take place in the economy can be seen as a complex and interactive process between different types of agents. Keynes (1936) already attributed significant irrationality to human nature and discuss the impacts of waves of optimism and pessimism - so called *animal spirits* - on economic outcome. As described by Akerlof and Shiller (2009) and others, the emotional states are reflected in economic behaviors - see also Franke (2012) for his extensive discussions about market behaviors and how expectation formation should be treated in macroeconomic models. In this paper, we attempt to empirically examine the hypothesis that the behavioral heterogeneity will have a macroscopic impact on the economy.

The point of view taken in this paper is that the behavioral model can provide a conceptual framework for a cognitive ability as well as a substantial degree of inertia in the DSGE models. De Grauwe (2011) emphasizes that if agents are known to be either optimists or pessimists, their ability (or better: limitation) and their expectation formation processes affect economic conditions, i.e. movements in employment, the output gap and inflation, more appropriately than standard rational expectation models. Indeed, it is shown in the expectation formation process under bounded rationality that animal spirits can be explicitly modeled by applying discrete choice theory to the group behaviors. Note here that agents compare their expected future values of the output gap and the inflation gap with their realized counterparts. Then the optimistic and pessimistic behaviors are now seen as a by-product of the switching mechanisms

[1] The problems of information transmission mechanisms in rational expectations models are already found in early publications from Shiller (1978), i.e. the applicability of agents to predict future economic outcome due to their perfect knowledge of the whole structure of the model. Camerer (1998) also offers an informative overview of the discussion on this topic in economics.

according to the performance measure from their expectations.[2]

To the best of our knowledge, however, an empirical evaluation of this specific kind of model is missing in the literature so far. Therefore, the purpose of this paper is to measure the effects of psychological behaviors on the economy under consideration of animal spirits in the small-scale DSGE model. To fill the existing gap between the use of the models and their empirical evaluations, we use the moment-based estimation that is applicable to the small-scale DSGE models with rational expectations and bounded rationality (see Franke et. al. (2011) among others). First, we study the model framework of De Grauwe and investigate empirically the role of bounded rationality on the economic behavior in the Euro Area from 1975Q1 to 2009Q4. Accordingly, an important aspect of this paper is to test the bounded rationality hypothesis in order to offer reliable parameter values that can be used for calibration in more realistic-grounded future work, e.g. studying monetary and fiscal policy analysis in a DSGE model without the assumption of rational expectations. Then we discuss differences between two polar cases of expectation formation processes: while the underlying model structure equals a standard three-equations New-Keynesian model (NKM), we allow both for rational expectations and for endogenously-formed expectations according to De Grauwe's specifications.

In our empirical applications, we show that the NKM with rational expecations or bounded rationality can generate auto- and cross-covariances of the output gap, the inflation gap and the interest gap that are matched to the data. A quadratic object function is used in the estimation to measure the distance between the model-generated and empirical moments. As the usual procedure of the method of moments, the global minimum of the objective function provides consistent parameter estimates of the model. Then we evaluate the goodness-of-fit of the data to the model from the value of the quadratic object function, i.e. the lower this value the better the fit of the model-generated moments to their empirical counterparts.

Because of non-linearities under a discrete choice framework, however, the moment conditions are not readily available in closed forms, but have to be approximated by simulation. In this case, the simulated method of moments offers an appropriate approach for estimating bounded rationality models. As a result, we found that the bounded rationality model describes the data at least as good as the model with rational expectations since the estimated values for the quadratic object function in both specifications are small while the corresponding auto- and cross-covariances profiles do not differ across both models. However, the parameters in the models (especially the behavioral parameters in the function which describes the divergence in beliefs) are ill-determined with wide confidence intervals. We attempt to fix this problem by verifying the reliability of the parameter estimates and their finite sample properties of the

[2] In particular, observed movements in the output gap and the inflation gap show a high degree of inertia. However, this empirical fact is not well captured by purely forward-looking NKM (see the discussion on the inflation persistence problem by Chari et. al. (2000)). Within his behaviorial model, De Grauwe replicates such a degree of persistence even without any backward-looking terms in the structural equations of a DSGE model which account for price indexation and habit formation.

moment-based estimation via a Monte Carlo study.

The remainder of the paper is structured as follows. Section 2 introduces a small-scale NKM and discusses two model specifications, i.e. one with rational expectations and one under consideration of the animal spirits. The estimation methodology is presented in section 3. Section 4 then estimates two versions of the model by the moment-based estimation and discusses their empirical results. Afterwards, the properties of the moment-based procedure for estimation are examined through a Monte Carlo study. Finally, section 5 concludes. Appendices collect the detailed solution of the NKM and present both the Delta method for computing the confidence bands and the Monte Carlo study with a large simulation size.

2 The Model: Rational Expectations and Bounded Rationality

The New-Keyesian three-equations model reads as follows:

$$y_t = \frac{1}{1+\chi}\tilde{E}_t^j y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \tau(\hat{r}_t - \tilde{E}_t^j \hat{\pi}_{t+1}) + \varepsilon_{y,t} \quad (1)$$

$$\hat{\pi}_t = \frac{\nu}{1+\alpha\nu}\tilde{E}_t^j \hat{\pi}_{t+1} + \frac{\alpha}{1+\alpha\nu}\hat{\pi}_{t-1} + \kappa y_t + \varepsilon_{\hat{\pi},t} \quad (2)$$

$$\hat{r}_t = \phi_{\hat{r}}(\phi_{\hat{\pi}}\hat{\pi}_t + \phi_y y_t) + (1-\phi_{\hat{r}})\hat{r}_{t-1} + \varepsilon_{\hat{r},t} \quad (3)$$

where the superscript $j = \{\text{RE,BR}\}$ specifies the rational expectation (RE) model and the bounded rationality (BR) model which is described below. It goes without saying that all variables are given in quarterly magnitudes. Equation (1) describes a hybrid dynamic IS curve and results from the standard utility maximization approach of a representative household. Here the current output gap depends negatively on the real interest rate which indicates consumption smoothing. The composite parameter $\tau \geq 0$ stands for the inverse intertemporal elasticity of substitution. Equation (2) stands for the hybrid New-Keynesian Phillips Curve where the output gap (y_t) is the driving force of inflation due to monopolistic competition and the Calvo price-setting scheme. The slope of the Phillips Curve is given by the parameter $\kappa \geq 0$. The parameter ν denotes the discount factor ($0 < \nu < 1$). According to the Taylor rule with interest rate smoothing (3), the nominal interest gap is a predetermined variable while the monetary authority reacts directly to movements in the output ($\phi_y \geq 0$) and inflation ($\phi_{\hat{\pi}} \geq 0$) gap. We account for intrinsic persistence in this stylized version of the well-known Smets and Wouters (2003, 2005 and 2007) model due to the assumption of backward-looking behavior indicated by the parameters for habit formation χ, price indexation α and interest rate smoothing $\phi_{\hat{r}}$ respectively ($0 \leq \chi \leq 1$, $0 \leq \alpha \leq 1$, $\phi_{\hat{r}} \geq 0$). The exogenous driving forces in the model variables are assumed to be idiosyncratic shocks $\varepsilon_{z,t}$ which are drawn from multivariate normal distributions around mean zero with variances σ_z^2 with variables $z = \{y, \hat{\pi}, \hat{r}\}$.

Note here that we consider the gaps instead of the levels and account explicitly for a time-varying trend in inflation and in the natural rate of interest.

The corresponding gaps are simply given by taking the difference of the actual value for inflation and the interest rate from their trends (i.e. time-varying steady state values) respectively where the latter is computed by applying the Hodrick-Prescott filter with a standard value of the corresponding smoothing parameter of 1600. Accordingly the set of equations models the dynamics in the output gap y_t, the inflation gap $\hat{\pi}_t$ and the nominal interest rate gap $\hat{r}_t$.

It has already been shown by several studies that assuming a constant trend, like a zero-inflation steady state, leads to misleading results. For example, Ascari and Ropele (2009) observe that the dynamic properties (i.e. mainly the stability of the system) depends on the variation in trend inflation. Cogley and Sbordone (2008) also provide evidence for the explanation of inflation persistence by considering a time-varying trend in inflation. In the same vein, one could discard the assumption of a constant natural rate of interest as being empirically unrealistic. In this paper, we follow the empirical approaches proposed by Cogley et. al. (2010), Castelnuovo (2010), Franke et. al. (2011) among others, who also consider gap specifications for inflation (and the nominal interest rate). Furthermore, inflation and money growth are likely to be non-stationary in the Euro Area data. If that is the case, the estimation methodology such as the method of moments approach presented here (or the Generalized Method of Moments in general) will lead to biased estimates.[3] Taken this into account it is reasonable to consider the gaps rather than the levels.

To make the description of the expectation formation processes more explicit, first we consider two polar cases in the theoretical model framework of the NKM. (1) Under rational expectations, the forward-looking terms which are the expectations of the output gap and inflation gap at time $t+1$ in equations (1) and (2) are just given by

$$\tilde{E}_t^{RE} y_{t+1} = E_t y_{t+1} \quad (4)$$

$$\tilde{E}_t^{RE} \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1} \quad (5)$$

where E_t denotes the expectations operator. (2) As regards the other specification, we depart from rational expectations by considering a behaviorial model of De Grauwe (2011). It is generally assumed that agents will be either *optimists* or *pessimists* (in the following indicated by the superscripts O and P, respectively) who form expectations based on their beliefs regarding movements in the future output gap:

$$E_t^O y_{t+1} = d_t \quad (6)$$

$$E_t^P y_{t+1} = -d_t \quad (7)$$

where

$$d_t = \frac{1}{2} \cdot [\beta + \delta\sigma(y_t)] \quad (8)$$

"can be interpreted as the divergence in beliefs among agents about the output gap" (De Grauwe (2011, p. 427)). In contrast to the RE model, both types of

[3] See also Russel and Banerjee (2008) as well as Aussenmacher-Wesche and Gerlach (2008) among others for methodological issues related to non-stationary inflation in the US and the Euro Area.

agents are uncertain about the future dynamics of the output gap and therefore predict a fixed value of y_{t+1} denoted by $\beta \geq 0$. This parameter can be interpreted as the *predicted subjective mean value* of y_t. However, this kind of subjective forecast is generally biased and therefore depends on the volatility in the output gap (given by the unconditional standard deviation $\sigma(y_t) \geq 0$). In this respect, the parameter $\delta \geq 0$ measures the *degree of divergence* in the movement of economic activity. Note that due to the symmetry in the divergence in beliefs, optimists expect that the output gap will differ positively from the steady state value (which for consistency is set to zero) while pessimists will expect a negative deviation by the same amount. The value of δ remains the same across both types of agents.

The expression for the market forecast regarding the output gap in the bounded rationality model is given by

$$\tilde{E}_t^{BR} y_{t+1} = \alpha_{y,t}^O \cdot E_t^O y_{t+1} \ + \ \alpha_{y,t}^P \cdot E_t^P y_{t+1} = (\alpha_{y,t}^O \ - \ \alpha_{y,t}^P) \cdot d_t \tag{9}$$

where $\alpha_y^O + \alpha_y^P = 1$. The probabilities that agents choose a specific forecasting rule, i.e. (6) or (7), are denoted as $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$ respectively. In particular, α_y^O can also be interpreted as the probability being an optimist and vice versa. In the following, we show explicitly how these probabilities are computed. Indeed, the selection of the forecasting rules (6) or (7) depends on the forecast performances of optimists and pessimists given by the mean squared forecasting error of which values can be updated in every period as

$$U_t^k \ = \ \rho U_{t-1}^k \ - \ (1-\rho)(E_{t-1}^k y_t - y_t)^2 \tag{10}$$

where $k = O, \ P$ and the parameter ρ denotes the measure of the memory of agents ($0 \leq \rho \leq 1$). Here $\rho = 0$ means that agents have no memory of past observations while $\rho = 1$ means that they have infinite memory instead. By applying discrete choice theory under consideration of the forecast performances, agents revise their expectations with which different performance measures will be utilized for $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$:[4]

$$\alpha_{y,t}^O \ = \ \frac{\exp(\gamma U_t^O)}{\exp(\gamma U_t^O) - \exp(\gamma U_t^P)} \tag{11}$$

$$\alpha_{y,t}^P \ = \ \frac{\exp(\gamma U_t^P)}{\exp(\gamma U_t^O) - \exp(\gamma U_t^P)} = 1 - \alpha_{y,t}^O \tag{12}$$

where the parameter $\gamma \geq 0$ denotes the intensity of choice: If $\gamma = 0$, the self-selecting mechanism is purely stochastic ($\alpha_{y,t}^O = \alpha_{y,t}^P = 1/2$) and if $\gamma = \infty$, it is fully deterministic ($\alpha_{y,t}^O = \alpha_{y,t}^P = 0$, De Grauwe (2011), p. 429). In other words, if $\gamma = 0$ agents are indifferent in being optimist or pessimist while if $\gamma = \infty$ their expectation formation process is independent of their emotional state. Finally, given the equations (10) to (12), we can rationalize equation (9) by using simple substitution.

[4] See also Westerhoff (2008, p. 199) and Lengnick and Wohltmann (2011, p. 7) among others for an application of discrete choice theory to models in finance and macroeconomics.

The same logic can be applied for the inflation gap expectations. Following the behavioral heterogeneity proposed by De Grauwe (2011, pp. 436), we assume that agents will be either so called *inflation targeters* or *extrapolators*.[5] In the former case, the central bank anchors expectations by announcing a target for the inflation gap $\bar{\hat{\pi}}$. From the view of the inflation targeters, this pre-commitment strategy is judged to be fully credible. Hence the corresponding forecasting rule becomes

$$E_t^{tar}\hat{\pi}_{t+1} = \bar{\hat{\pi}} \tag{13}$$

where we assume $\bar{\hat{\pi}} = 0$.[6] On the other hand, the extrapolators form their expectations in a static way and will expect that the future value of the inflation gap equals its past value, i.e.

$$E_t^{ext}\hat{\pi}_{t+1} = \hat{\pi}_{t-1}. \tag{14}$$

This leads to an expression for the market forecast for the inflation gap similar to (9):

$$\tilde{E}_t^{BR}\hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar}E_t^{tar}\hat{\pi}_{t+1} + \alpha_{\hat{\pi},t}^{ext}E_t^{ext}\hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar}\bar{\hat{\pi}} + \alpha_{\hat{\pi},t}^{ext}\hat{\pi}_{t-1}. \tag{15}$$

The forecast performances of inflation targeters and extrapolators are given by the mean squared forecasting error written as

$$U_t^s = \rho U_{t-1}^s \; - \; (1-\rho)(E_{t-1}^s\hat{\pi}_t - \hat{\pi}_t)^2 \tag{16}$$

where $s = (tar, ext)$ and finally we may write that:

$$\alpha_{\hat{\pi},t}^{tar} = \frac{\exp(\gamma U_t^{tar})}{\exp(\gamma U_t^{tar}) - \exp(\gamma U_t^{ext})} \tag{17}$$

$$\alpha_{\hat{\pi},t}^{ext} = \frac{\exp(\gamma U_t^{ext})}{\exp(\gamma U_t^{tar}) - \exp(\gamma U_t^{ext})} = 1 - \alpha_{\hat{\pi},t}^{tar}. \tag{18}$$

Here $\alpha_{\hat{\pi},t}^{tar}$ denotes the probability to be an inflation targeter, which is the case if the forecast performance using the announced inflation gap target is superior to the extrapolation of the inflation gap expectations and vice versa. Note here that the memory (ρ) as well as the intensive of choice parameter (γ) do not differ across both expectation formation processes regarding the output and inflation gap.

In general, all model specifications are described by the following system in canonical form:

$$AX_t + BX_{t-1} + CX_{t+1} + \varepsilon_t = 0 \tag{19}$$

where

[5] The concept of behavioral heterogeneity has already been developed in financial market models, see e.g. Chiarella and He (2002) as well as Hommes (2006) among others.

[6] In this respect an inflation *gap* target of zero percent implies, that the European Central Bank seek to minimize the deviation of its (realized) target *rate* of inflation from the corresponding time-varying steady state value, where in the optimum this deviation should be zero.

$$X_t = \begin{pmatrix} y_t \\ \hat{\pi}_t \\ \hat{r}_t \end{pmatrix},\ X_{t-1} = \begin{pmatrix} y_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \end{pmatrix},\ X_{t+1} = \begin{pmatrix} \tilde{E}_t^j y_{t+1} \\ \tilde{E}_t^j \hat{\pi}_{t+1} \\ \tilde{E}_t^j \hat{r}_{t+1} \end{pmatrix},\ \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{\hat{\pi},t} \\ \varepsilon_{\hat{r},t} \end{pmatrix}$$

The corresponding system matrices are given by:

$$A = \begin{pmatrix} 1 & 0 & \tau \\ -\lambda & 1 & 0 \\ -\phi_{\hat{r}}\phi_y & -\phi_{\hat{r}}\phi_\pi & 1 \end{pmatrix},\ B = \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 & 0 \\ 0 & -\frac{\alpha}{1+\alpha\nu} & 0 \\ 0 & 0 & -(1-\phi_{\hat{r}}) \end{pmatrix} \quad (20)$$

and

$$C = \begin{pmatrix} -\frac{1}{1+\chi} & -\tau & 0 \\ 0 & -\frac{\nu}{1+\alpha\nu} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (21)$$

Recall that for the rational expectations model we assume

$$\begin{aligned} \tilde{E}_t^{RE} y_{t+1} &= E_t y_{t+1} \\ \tilde{E}_t^{RE} \hat{\pi}_{t+1} &= E_t \hat{\pi}_{t+1} \end{aligned}$$

and for the bounded rationality model we assume

$$\begin{aligned} \tilde{E}_t^{BR} y_{t+1} &= (\alpha_{y,t}^{O} - \alpha_{y,t}^{P}) d_t \\ \tilde{E}_t^{BR} \hat{\pi}_{t+1} &= \alpha_{\hat{\pi},t}^{tar} \bar{\hat{\pi}} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1} \end{aligned}$$

where we also consider equations (10) to (18) with $\bar{\hat{\pi}} = 0$.

Note that in the end, the bounded rationality model turns out to be purely backward-looking (cf. equations (10) and (16)) while the forward- and backward-looking behaviors are contained in the rational expectation model. The solution to the system is computed by backward-induction and the method of undetermined coefficients respectively which are shown in appendix A1.

In order to test the hypothesis of the behavioral heterogeneity in the model of De Grauwe (2011), we present the estimation methodology and the empirical results for the bounded rationality parameters (i.e., β and δ). It will be discussed in the next section.

3 The Method of Moment

The method of moments estimation comprises distributional properties of empirical data X_t, $t = 1, \cdots, T$. The sample covariance matrix at lag k is defined by:

$$m_t(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})' \tag{22}$$

where $\bar{X} = (1/T) \sum_{t=1}^{T} X_t$ is the vector of sample mean. The sample average of discrepancy between the model-generated and the empirical moments is denoted as $g(\theta; X_t)$:

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^{T} (m_t^* - m_t) \tag{23}$$

where m_t^* is the empirical moment function and m_t the model-generated moment function (cf. (22)). Given that the length of the business cycles lies between (roughly) one and eight years in the Euro Area, the estimation should not be based on too long a lag horizon. A reasonable compromise is a length of two years with which we will use auto- and cross-covariances of the interest rate gap, the output gap and the inflation gap; a lag k, where $k = 0, \cdots, 8$. We have a p dimensional vector of moment conditions ($p = 78$) by avoiding double counting at the zero lags in the cross relationships (see Franke et. al. (2011)). Note here that it is our interest to match the auto- and cross-covariances up to a lag length of 8.[7] θ is a $l \times 1$ vector of unknown parameters with a parameter space Θ.

We obtain the parameter estimates from the following quadratic objective function during the minimization process:

$$Q(\theta) = \arg \min_{\theta \in \Theta} \; g(\theta; X_t)' \; W \; g(\theta; X_t) \tag{24}$$

with the weight matrix W estimated consistently in several ways (see Andrews (1991)). A striking feature of the method of moments approach is transparency. In particular, it is easy to check the goodness-of-fit of the model from the moment conditions of interest, i.e. the dynamic properties of the model can be tested by evaluating graphically the match of the estimated and model-generated moments as discussed in the following section.[8] The present study

[7] The Delta method is used to compute the confidence bands in the auto- and cross-covariance moment estimation (see appendix A2 for details).

[8] One might suggests to apply the Bayesian estimation methodology since over the last two decades the Bayesian approach has gained considerable popularity to evaluate DSGE models. Indeed, the Bayesian estimation has the advantage (among others) that the moments and the distributions of the parameters of interest can be estimated directly from the likelihood values for a model. Furthermore, prior information can be flexibly incorporated and the posterior probability is used for statistical inference. However, we need some caution for a Bayesian analysis when prior information is unavailable at least for the behavioral parameters β and δ.

uses the heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimator suggested by Newey and West (1987). The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions:

$$\widehat{\Gamma}_T(j) = \frac{1}{T}\sum_{t=j+1}^{T}(m_t - \bar{m})(m_t - \bar{m})' \tag{25}$$

The lag length can be set to the values of $j = 5$ to construct the estimate of the covariance matrix (Newey and West (1994)):

$$\widehat{\Omega}_{NW} = \widehat{\Gamma}_T(0) + \sum_{j=1}^{5}\Big(\widehat{\Gamma}_T(j) + \widehat{\Gamma}_T(j)'\Big) \tag{26}$$

The weight matrix is set as the inverse of the estimated covariance matrix. However, the high correlation of moments that we consider makes the estimated covariance matrix near singular. Therefore, we use the diagonal matrix entries as the weighting scheme; i.e., $\widehat{W} = \widehat{\Omega}_{ii}^{-1}$. Under certain regularity conditions, one can derive the following asymptotic distribution of the method of moments estimation for the parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, V) \tag{27}$$

where $V = [(DWD')^{-1}]D'W\Omega WD[(DWD')^{-1}]'$. When the weight matrix is chosen optimally ($\widehat{W} = \Omega^{-1}$), V becomes $(DWD')^{-1}$; see Lee and Ingram (1991) among others. D is the gradient vector of moment functions evaluated around the point estimates:

$$\widehat{D}_T = \left.\frac{\partial m(\theta; X_T)}{\partial \theta}\right|_{\theta=\widehat{\theta}_T} \tag{28}$$

However, we ignore the off-diagonal components of the matrix $\widehat{\Omega}_{NW}$ when estimating the weight matrix. The estimated confidence bands, then, become wider since a weighting scheme in the objective function is not optimal. The ill-determined parameters and some related issues in statistical inference will be discussed later via a Monte Carlo study.

Under RE, we can obtain the simple analytical moment conditions of the model. However, for the BR model, the analytical expressions for the moment conditions are not readily available due to the non-linear discrete choice framework. To circumvent this problem, we use the simulated method of moments to identify the behavioral parameters in the BR model. The simulated method of moments is particularly suited to a situation where the model is easily simulated by replacing theoretical moments. Then the model-generated moments in (23) are replaced by their simulated counterparts:

$$m_t = \frac{1}{S\cdot T}\sum_{t=1}^{S\cdot T}\widetilde{m}_t \tag{29}$$

First, we simulate the data from the model and compute the moment conditions ($\widetilde{m}_t$) in order to approximate the analytical moments (m_t). Note here that we denote by S the simulation size and set it to the value of 100. The asymptotic normality of the simulated method of moments holds under certain regularity conditions (Duffie and Singleton (1993)). Finally, we use the J test to evaluate compatibilities of the moment conditions.

$$J \equiv T \cdot Q(\widehat{(\theta)}) \ \rightarrow^d \ \chi^2_{p-l} \tag{30}$$

where the J-statistic is asymptotically χ^2 distributed with $(p-l)$ degrees of freedom (over-identification).

4 Empirical Application to the Euro Area

In this section, we first describe the data used to estimate the model parameters. Then we present our empirical results on the structural and behavioral parameters. Finally, we evaluate the finite sample properties of the moment-based estimator via a Monte Carlo simulation study.

4.1 Data

The data source for the New Keynesian model is the 10th update of the Area-wide Model quarterly database described in Fagan et. al. (2001). The output gap and interest rate gap are computed from real GDP and nominal short-term interest rate respectively using the Hodrick-Prescott filter with a standard value of the corresponding smoothing parameter of 1600. The inflation gap measure is the quarterly log-difference of the Harmonized Index of Consumer Prices (HICP) instead of the GDP deflator.[9] The sample for this data set is available from 1970:Q1. As we use the data over five years in a rolling window analysis to estimate the perceived volatility of the output gap $\sigma(y_t)$, the data applied in this study cover the period 1975:Q1-2009:Q4.

4.2 Results

We first estimate the BR model parameters using the moment-based estimation in the previous section. Afterward we compare it to the benchmark case, namely the RE model and attempt to identify the effects of divergence in beliefs on the inflation and output gap dynamics. As it is common in an overwhelming amount of empirical studies, the discount parameter ν is calibrated at a value of 0.99. We also fix the intensity of choice parameter γ to unity which may allow us to find more accurate estimates of the behavioral parameters in the expectation formation process. By fixing those parameters in the final estimation, we can

[9]We resort to the HICP instead of the conceptually more appropriate implicit GDP-deflator which is common in the literature, since the former is more in line with micro data evidence. For instance, Forsells and Kenny (2004) show that inflation expectations can be approximated by micro-level data like consumer surveys (i.e. in the European Commission survey indicators). Also see Ahrens and Sacht (2011, pp. 10–11) for a more detailed discussion on using the HICP instead of the GDP-deflator in macroeconomic studies.

avoid high-dimensionality of the parameter space and reduce the uncertainty of the estimates.[10] Given these assumptions, we separately obtain the estimates for remaining parameters from the rational and bounded rationality model via the moment-based estimation. They are presented in the following table.

Table 1: Estimates of the RE and BR Model

Label	RE	BR
α	0.765	0.203
	(0.481 - 1.000)	(0.000 - 0.912)
χ	1.000	0.950
	-	(0.000 - 1.000)
τ	0.079	0.387
	(0.000 - 0.222)	(0.000 - 0.927)
κ	0.035	0.219
	(0.011 - 0.058)	(0.075 - 0.362)
ϕ_y	0.497	0.673
	(0.058 - 0.936)	(0.404 - 0.942)
$\phi_{\hat{\pi}}$	1.288	1.073
	(1.000 - 1.944)	(1.000 - 1.775)
$\phi_{\hat{r}}$	0.604	0.673
	(0.411 - 0.797)	(0.523 - 0.824)
σ_y	0.561	0.827
	(0.354 - 0.768)	(0.463 - 1.190)
$\sigma_{\hat{\pi}}$	0.275	0.743
	(0.097 - 0.453)	(0.449 - 1.046)
$\sigma_{\hat{r}}$	0.421	0.244
	(0.140 - 0.701)	(0.000 - 0.624)
β	-	2.221
		(0.000 - 9.747)
δ	-	0.665
		(0.000 - 7.877)
ρ	-	0.003
		(0.000 - 1.000)
J	56.30	40.30

Note: The data cover the period spanning 1975:Q1 - 2009:Q4 (T=140 observations). The parameters ν and γ are fixed to the values of 0.99 and 1 respectively. We set the rolling window of 5 years (20 observations) to compute the perceived volatility of the output gap, i.e. the unconditional standard deviation of y_t denoted by $\sigma(y_t)$. The 95% asymptotic confidence intervals are given in brackets.

Several observations are worth mentioning. The parameter estimate of the degree of price indexation α is much higher in the RE (0.765) compared to the BR (0.203) model. It follows that the expressions which are in front of the forward- and backward-looking terms in the Phillips Curve indicate a higher

[10] The system with many parameters is likely to have a likelihood with multiple peaks, some of which are located in uninteresting or implausible regions of the parameter space. Indeed, the global minimum is difficult to find with high-dimensionality of parameter space. Therefore we changed the starting values in the optimization for more robust estimates and had high levels of confidence.

weight on $\tilde{E}_t^j \hat{\pi}_{t+1}$ (i.e. $\frac{\nu}{1+\alpha\nu} > \frac{\alpha}{1+\alpha\nu}$). Then it is pertinent to emphasize that the result is more pronounced for the BR (0.82 > 0.18) compared to the RE model (0.56 > 0.43). For the latter, this means that there is strong evidence for a hybrid structure. The empirical applications of the the BR model show that the dynamics of the inflation gap are primarily driven by the expectations (i.e. the evaluation of the forecast performance) for the inflation gap if cogitative limitation of agents is assumed. This is not necessarily true under rational expectations. In other words, we find strong evidence for an autoregressive expectation formation process since the estimated value for α is high and $\tilde{E}^{BR}\hat{\pi}_{t+1} = \alpha^{ext}_{\hat{\pi}_t,t}\hat{\pi}_{t-1}$ holds. Regarding the dynamic IS equation, the output gap is influenced by the forward- and backward-looking terms at the same proportion, i.e. $\frac{1}{1+\chi} = \frac{\chi}{1+\chi} = 0.5$ since the empirical estimates show that $\chi = 1$ and $\chi = 0.950$ hold for the RE and the BR models respectively. In particular, this degree of habit persistence indicates that past observations strongly matter for the dynamics of the output gap.[11] Finally, the parameter estimate for the degree of interest rate smoothing indicates that there is a moderate degree of persistence in the nominal interest rate gap for the both models since $\phi_{\hat{r},t}$ is slightly lower than observed in the literature (e.g. in Smets and Wouters (2003)).[12]

Furthermore, while the empirical estimates for κ and τ in the RE model indicate a small degree of inherited persistence due to changes in the real interest rate gap and the output gap respectively, this does not hold for the BR model. Here the changes in the output gap have a strong impact ($\kappa = 0.219$) on movements in the inflation gap relative to the RE case ($\kappa = 0.035$). For the output gap, inherited persistence plays fundamental role in shaping the dynamics of this economic indicator which can be seen through the high values of inverse intertemporal elasticity of substitution. For the BR model, this value ($\tau = 0.387$) is much larger than the one for the RE model ($\tau = 0.079$). It implies that the tendency towards consumption smoothing in the BR is so strong when compared to the RE model. To sum up, our results show that in the BR model cross-movements in the output and inflation gap account for persistence in both variables (under consideration of perfect habit formation $\chi = 1$) rather than price indexation alone. This can be seen through the high values of κ and τ compared to α. For the RE model, the opposite holds.

The output and inflation gap shocks, whose magnitudes are estimated to be $\sigma_y = 0.827$ and $\sigma_{\hat{\pi}} = 0.743$ respectively, are larger than those of the RE model.

[11] It is likely that this results from the fact that no autoregressive process in the dynamic IS curve is assumed. However, similar experiments show that estimation results are the best outcome in the case when only impulse shocks are considered, i.e. in the absence of exogenous persistence.

[12] It must be stated that the sample period in Smets and Wouters (2003) captures the period from 1980Q2 to 1999Q4. In their paper, they apply Bayesian estimation on a medium scale model for the Euro Area. Their results are different especially for the parameters τ and $\phi_{\hat{\pi}_t}$ which are estimated to be higher (0.739 and 1.684). In contrast, the estimated values for κ and ϕ_y are relatively small (0.01 and 0.10). However, we apply a moment-based estimation on Euro Area data over a different time interval while considering gap specifications of $\hat{\pi}_t$ and $\hat{r}_t$. Hence a direct comparison of our results with the ones of Smets and Wouters has to be done with some caution.

The results reveal that the volatilities of the output and inflation gap are mitigated with the waves of behavioral heterogeneity. For instance, the waves of optimism and pessimism act as a persistent force in the output gap fluctuations with peaks and troughs. Figure 1 illustrates that the peak of the fluctuation in the simulated output gap (middle-left panel) corresponds to the market optimism (lower-left panel) and vice versa. The qualitative interpretation remains almost the same for the inflation gap dynamics (middle- and lower-right panel respectively) - but the dynamics of extrapolators are highly volatile reflecting the large second moment of the empirical inflation gap (upper-right panel). The goodness-of-fit of the model could not be directly compared by illustrating the simulated time series (middle-panels), but one can see that the series resemble qualitatively the empirical counterparts (empirical moments; upper-panels). Finally, the nominal interest rate shocks $\sigma_{\hat{r}}$ in the RE model are estimated to be roughly twice as large as in the BR model.

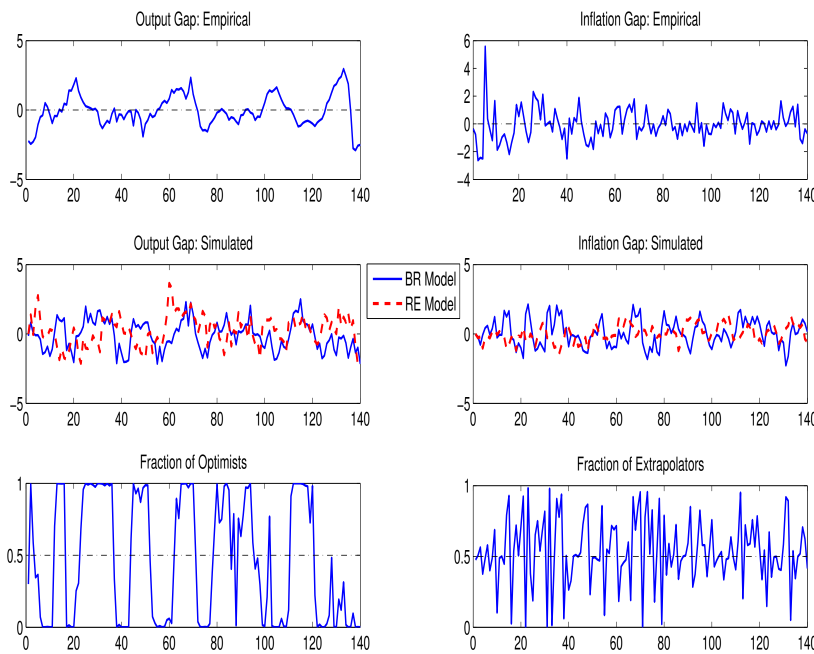

Figure 1: Dynamics in the Output Gap and the Inflation Gap.

Upper and middle panels plot empirical and simulated values for the output gap (left) and the inflation gap (right) while lower panels plot the corresponding fraction of market optimists (left) and extrapolators (right). The simulated time series are computed using the parameter estimates for both models given in Table 1.

The numerical differences in the parameter estimates regarding the inherited persistence parameters as well as the standard deviations in the shocks might be explained by the absence of the self-selection mechanism in the RE model. In fact, this results of the empirical comparison between two models show that the switching mechanisms account for a large degree of persistence and volatility in the BR model.

The remaining parameter estimates confirm the known results from the literature where the monetary policy coefficient on the output gap is low while the opposite holds for the coefficient on the inflation gap. The latter indicates that the Taylor principle holds over the whole sample period. Nevertheless, the results for the BR model indicate a stronger concern in output gap movements relative to the dynamics in the inflation gap. Again, the opposite is true for the RE model. It is worth mentioning that the estimation results indicate a monetary policy coefficient on the output gap ϕ_y of 0.673 which is in line with the observations of De Grauwe (2011, pp. 443-445). He shows that flexible inflation targeting can reduce both output gap and inflation gap variability at a minimum level if ϕ_y lies in the range of 0.6 to 0.8. We can make a valid comparison of these parameter values because the animal spirits have a direct impact on the output gap variability through the expectation channel. In particular, strict inflation targeting does not affect the forecast performance of market optimism and pessimism. This leads to highly volatile movements in the output gap and inflation gap (due to a high value of κ). Thus the model predicts under volatile output gap movements that the pre-commitment to a credible inflation target will have a small impact on the price stabilization. Put it differently, the amplification effects of the policy on the forecast performances of the inflation extrapolators will result in higher inflation variability.

As already noted, the present paper focuses on the estimation of the bounded rationality parameters. First, we come to the conclusion that over the whole sample period, the optimistic agents have expected a fixed divergence of belief of $\beta = 2.221$. Roughly speaking, the optimists have been really optimistic that the future output gap will differ *positively* by slightly above two percent from its steady state value. Due to the symmetric structure of the divergence in beliefs, over the same sample period pessimistic agents instead were moderately pessimistic since from their point of view the future output gap was expected to be 2.221 percent *below* its steady state value. Furthermore, both types of agents felt confident about their expectations due to the fact that the estimate for the variable component in the divergence of pessimistic beliefs is very low ($\delta = 0.665$). It is also shown that there is a low degree of uncertainty connected to the expected value of y_t. In conjuncture with the results for (and assumptions of) the parameters which indicate endogenous and inherited persistence (α, χ, κ and τ), the high subjective mean value of the output gap β - in conjunction with the dynamics induced by the self-selecting mechanisms (see the corresponding fractions in the lower-panels in Figure 1) - explains a high volatility of the output gap. Based on discrete choice theory, this strengthens the optimistic agents' belief about the future output gap to diverge in the data since over (or under)-reactions to underlying shocks across the Euro Area occur. The same observation holds for dynamics of the inflation gap. The proportion of the extrapolators in the economy correspond to the inflation gap movements: the higher the fraction of extrapolators, the more volatile the inflation gap dy-

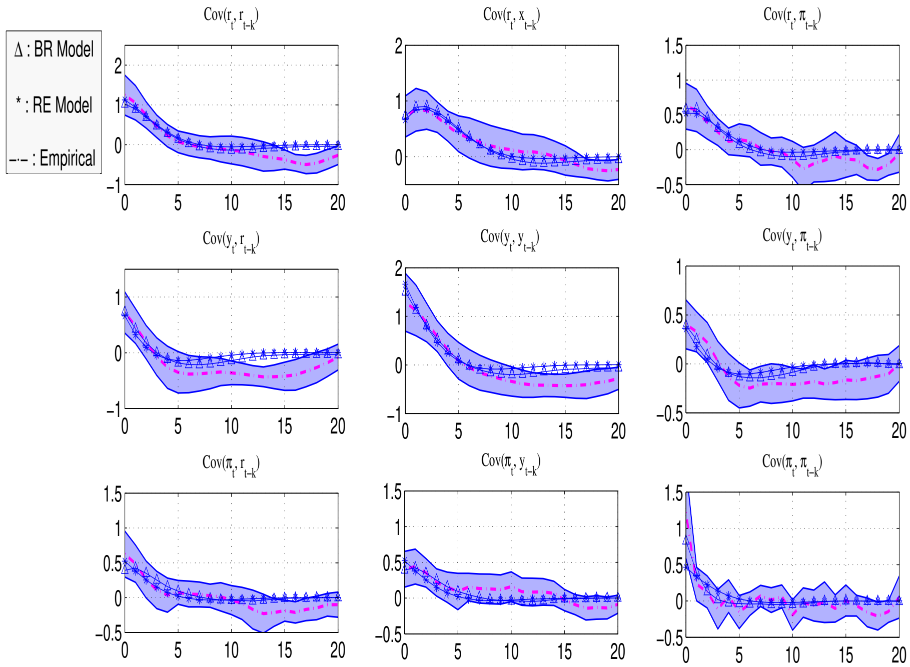

Figure 2: Model Covariance (Cov) Profiles in Euro Area.

The dashed line results from the empirical covariance estimates. The shaded area is the 95% confidence bands around the empirical moments. The triangle (BR) and star (RE) lines indicate the model generated ones. The confidence bands are computed via the Delta method (see appendix A2).

namics. Finally, ρ is estimated to be zero, i.e. past errors are not taken into account (cf. equations (10) and (16)). This leads to the conclusion that strict forgetfulness or cognitive limitation holds, which is a requirement for observing animal spirits (cf. De Grauwe (2011, p. 440)).

Indeed, visual inspection shows a fairly remarkable goodness-of-fit of the data to the models (see Figure 2). The match the both models achieve looks very good over the first few lags and still fairly good over the higher lags until the pertinent lag 8. In any case, all of the moments are now inside the confidence intervals of the empirical moments. This even holds true for some covariances up to lag 20. This is also confirmed by the values of the loss function J for the RE (56.30) and BR (40.30) model given in the last row of Table 1. Furthermore, the picture shows a remarkable fit of the BR model which leads to some confidence in the estimation procedure. We conclude from that, that a bounded rationality model with cognitive limitation provides fits for auto- and cross-covariances of the data which are slightly better than or equal to a model where rational expectations are assumed[13]

In general, compared to the RE model, parameter estimates of the BR model generally have wide confidence intervals because the estimation uncertainty is large due to the non-linearity of the model. Especially, the bounded rationality parameters β and δ (among others) are ill-determined when standard estimation methods are applied to identify the latent processes in the forecasting heuristics (11) and (12) as well as (17) and (18). The wide confidence intervals indicate that the parameter estimates must be taken with caution - even in the case that the matching of the moment conditions is striking, i.e. the value of the loss function is unambiguously small. In order to check the reliability of parameter estimates with wide confidence intervals, we present a Monte Carlo experiment in the next subsection. We show that the estimation uncertainty results presented here can be reduced by increasing the sample size.

4.3 Monte Carlo Study

In this section, we investigate the reliability of statistics obtained in the previous section. First, we examine the small sample properties of the method of moment with a Monte Carlo (MC) study. To analyze the finite sample properties in the macro data, we consider three sampling periods in the data generating process (T=100, 200, 500). The experimental true parameters are drawn from the parameter estimates in the previous section. After 550 observations are simulated, we discard the first 50 observations to trim a transient period. In the RE model, we compute the empirical moment conditions and its Newey-West weight matrix of each artificial time series and estimate the parameters using the method of moment estimator over 1,000 replications. The same procedure is used to estimate the parameters of the BR model. However, this makes the computation expensive for the simulated method of moment estimator. We reduce the computational cost by keeping the simulation size ($S = 10$) and the number of Monte Carlo iterations relatively small, i.e. 200 replications.[14]

Table 2 summarizes the results from the MC experiment for the RE model. We report the mean and the root mean square error (RMSE). The true values of the parameters are stated in the second column. It is observed that the method of moment estimation of the RE model has good finite sample properties; see the RMSE sensitivity to variations in sample size. Note here that we use the optimization tool (Matlab version R2010a) with the *fmincon* solver.[15]

[13] Accordingly to these results, one could also prefer the BR to the RE model since it could be argued that - due to the different values of J - the BR model fits the data slightly better than the RE model does. Nevertheless, significant differences between two models have to be tested by a formal model comparison method since the models do not have any difficulties to fit the empirical moments at the 5% significant interval (see also Jang (forthcoming) among others).

[14] The implementation of the MC study on the model with a large simulation size (i.e. S=100) does not have a large change in parameter estimates; see appendix A3. The theoretical approximation error rates of analytical moments are 10% and 1% for the simulation sizes $S = 10$ and 100 respectively. Since a large simulation size is expensive to compute, we report the MC results from a small simulation size ($S = 10$).

[15] Especially the interior-point algorithm has a number of advantages over other algorithms (i.e., active-set, trust-region-reflective, and sqp). For example, the implementation of the interior-

Table 2: Monte Carlo Study for the RE Model

		T=100		T=200		T=500	
Label	True (θ^0)	Mean	RMSE	Mean	RMSE	Mean	RMSE
α	0.750	0.802	0.174	0.778	0.125	0.763	0.079
χ	1.000	0.943	0.128	0.939	0.127	0.946	0.103
τ	0.085	0.100	0.062	0.088	0.043	0.083	0.029
κ	0.035	0.047	0.026	0.042	0.016	0.039	0.009
ϕ_y	0.500	0.518	0.267	0.487	0.167	0.487	0.107
$\phi_{\hat{\pi}}$	1.250	1.350	0.309	1.322	0.217	1.296	0.146
$\phi_{\hat{r}}$	0.600	0.623	0.111	0.615	0.076	0.611	0.046
σ_y	0.600	0.632	0.127	0.627	0.090	0.623	0.059
$\sigma_{\hat{\pi}}$	0.275	0.248	0.077	0.263	0.049	0.270	0.030
$\sigma_{\hat{r}}$	0.400	0.234	0.240	0.289	0.181	0.345	0.105
J		31.58		24.12		20.10	

Note: ν is set to the value of 0.99. The reported statistics are based on 1,000 replications. RMSE is the root mean square error.

In comparison with the results of the RE model, we found that the simulated method of moments of the BR model has somewhat poor finite sample properties regarding the parameters α, τ, β, and δ; see Table 3. However, the large uncertainty for the parameter estimates can be mitigated by more observations in the data. On the other side, note here that we could consistently recover the true values for the other parameter estimates. This implies that the parameter estimates almost converge to the true ones as the sample size increases. Another finding from the MC study is that the RMSE values for the behavioral parameters (β and δ) in the discrete choice are higher than those for other structural parameters. This indicates that the behavioral parameters are generally ill-determined. Nevertheless, as we increase the sample size (i.e., T=500), the RMSE gets smaller. The large sample allows us to make more accurate inference about the group behaviors in the market expectation formation processes. Put it differently, as market behavior is unobservable in most cases, we need a large sample size to consistently estimate the behavioral parameters. Nevertheless, the estimated results for the behavioral parameters can be seen as confident starting values used for calibration exercises like e.g. (optimal) monetary and fiscal policy analysis.

Finally, the J test is used to evaluate the validity of the models from the artificial data. The null hypothesis that the model is the true one is not rejected according to the over-identification test for both the RE and the BR model. The J test for over-identifying restrictions show that the BR model fits the data slightly better than the RE model on average. Nevertheless, the direct diagnostic comparison between two models must be made with caution because the BR model has more parameters than the RE model does.

point algorithm for large-scale linear programming is considerably simpler than for the other algorithms. Also it can handle nonlinear non-convex optimization problems of non-linear objective functions in the discrete choice.

Table 3: Monte Carlo Study for the BR Model

		T=100		T=200		T=500	
Label	True (θ^0)	Mean	RMSE	Mean	RMSE	Mean	RMSE
α	0.200	0.309	0.308	0.361	0.297	0.271	0.175
χ	1.000	0.679	0.445	0.813	0.292	0.841	0.241
τ	0.385	1.138	1.270	0.613	0.347	0.566	0.234
κ	0.215	0.243	0.091	0.220	0.050	0.227	0.036
ϕ_y	0.675	0.763	0.190	0.697	0.099	0.697	0.076
$\phi_{\hat{\pi}}$	1.100	1.092	0.129	1.063	0.092	1.086	0.077
$\phi_{\hat{r}}$	0.670	0.685	0.056	0.674	0.035	0.682	0.025
σ_y	0.825	0.886	0.257	0.894	0.114	0.875	0.083
$\sigma_{\hat{\pi}}$	0.740	0.613	0.190	0.651	0.109	0.701	0.058
$\sigma_{\hat{r}}$	0.240	0.163	0.137	0.184	0.117	0.167	0.121
β	2.250	2.837	1.876	2.331	0.970	2.369	0.760
δ	0.650	1.418	1.547	1.004	0.918	0.870	0.623
ρ	0.000	0.203	0.271	0.085	0.131	0.089	0.133
J		27.94		21.68		20.58	

Note: ν is set to the value of 0.99. The reported statistics are based on 200 replications. RMSE is the root mean square error.

5 Conclusion

In this paper, we attempt to provide empirical evidence for the behavioral assumptions in the model of De Grauwe (2011). The validity of the model assumption on the cognitive limitation (e.g. because of different individual emotional states) is empirically tested using the historical Euro area data. Although the assumption of rational expectations leads to a convenient analytical tractability of the difference equations models, it may be empirically more convincing to consider several important issues known from the seminal work of Akerlof and Shiller (2009).

Indeed, we hypothesize that historical movements of macro dynamics in Euro Area are influenced by the waves of optimism and pessimism. To examine the effects of expectations on the output and inflation gap, we follow the behavioral approach of De Grauwe (2011) who assumes divergence in beliefs about the future value of both variables. The corresponding decision rules for market optimism and pessimism are given by the forecast performance of the agents from a discrete choice theory. To see this, first we estimate both a NKM with bounded rationality using the moment-based estimation. Afterwards, we contrast a standard hybrid version of the three-equations New-Keynesian model of rational expectations with a version of the same model where we assume bounded rationality in expectation formation processes.

Our main empirical findings show that a bounded rationality model with cognitive limitation provides fits for auto- and cross-covariances of the Euro Area data which are slightly better or equal to a model where rational expectations are assumed - even though we are not judging the performance of both models relative to each other. Therefore our empirical results of the BR model offer some new insights into expectation formation processes for the Euro Area. First, over the whole time interval the agents had expected moderate deviations of the output gap from its steady state value with low uncertainty. Second, we find strong evidence for an autoregressive expectation formation process regarding the inflation gap. Both observations explain a high degree of persistence in the output gap and the inflation gap. Based on discrete choice theory and the self-selection process of the agents, we found that animal spirits strengthen the optimistic's belief about the future output gap to diverge in the historical Euro Area data. Furthermore, we examine the properties of the moment-based procedure for estimation of the rational expectation and the bounded rationality models through a Monte Carlo study.

To the best of our knowledge, such kind of experiments have not been done before in the literature. However, the empirical test of bounded rationality (viz. the assumption of the divergence in beliefs) has to be treated carefully because the behavioral parameters with their non-linear modeling approach are generally ill-determined. Nevertheless, we provide empirical evidence in support of De Grauwe (2011, fn. 4) for understanding the group's over- and under-reaction to the economy. In order to identify the effects of individual expectation formation processes on the economy, in further research, the decision rules i.e. the transition rules from one state of the economy to another could be calculated based on survey data (for example see Lux (2009)). Thus these probabilities are then treated as exogenous and (in contrast in the De Grauwe model) are computed under consideration of the underlying time series using discrete choice theory. Finally and only if the estimation of small-scale models is considered to be satisfactory, one can further continue the model estimation with much more richer models like e.g. the medium-scale version developed by the Smets and Wouters (2005, 2007). We leave these issues to future research.

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Appendix

A1: Solution of the NKM

We solve for the dynamics of the system

$$AX_t + BX_{t-1} + CX_{t+1} + \varepsilon_t = 0. \tag{31}$$

In case of the BR model, the solution is given by

$$X_t = -A^{-1}[BX_{t-1} + CX_{t+1} + \varepsilon_t] \tag{32}$$

where the matrix A is of full rank, i.e. its determinant is not equal to zero, given the parameter estimates in section 4. Under consideration of the heuristics for the forecasts regarding the output and inflation gap expectations, the forward looking term X_{t+1} is substituted by the equivalent expressions for the discrete choice mechanism given in section 2. It follows that the model becomes purely backward-looking and thus (32) can be solved by backward-induction.

In contrast, the RE model is both backward- and forward-looking. Therefore we apply the method of undetermined coefficients in order to solve the model. We claim that the law of motion which describes the analytical solution is given by

$$X_t = \Omega X_{t-1} + \Phi\varepsilon_t \tag{33}$$

where $\Omega \in \mathbb{R}^{3\times3}$ and $\Phi \in \mathbb{R}^{3\times3}$ are the solution matrices. The former is a stable matrix as long as (similar to the matrix A in the BR case) its determinant is not equal to zero, which ensures the invertibility of Ω. This is confirmed given the estimation results in section 4. We substitute (33) into (31) which yields

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega X_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega^2 X_{t-1} + \Omega\Phi\varepsilon_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

Hence the reduced form can be rewritten as

$$(C\Omega^2 + A\Omega + B)X_{t-1} + (A\Phi + C\Omega\Phi + I)\varepsilon_t = 0 \tag{34}$$

with I being the identity matrix. Note that $\varepsilon_t \sim N(0, \sigma_z^2)$ with $z = \{y, \hat{\pi}, \hat{r}\}$ and thus $E_t\varepsilon_{t+1} = 0$. In order to solve equation (34) all the terms in brackets must be zero.[16] Thus the solution matrices can be uniquely determined. We may write that as

$$C\Omega^2 + A\Omega + B = 0 \Rightarrow \Omega = -(C\Omega + A)^{-1}B. \tag{35}$$

[16] Obviously the trivial solution $X_{t-1} = \Gamma_t = \varepsilon_t = 0$ is discarded.

In order to solve the quadratic matrix equation (35) numerically, we employ the brute force iteration procedure mentioned in Binder and Pesaran (1995, p. 155, fn 26). Hence an equivalent recursive relation of (35) is given by

$$\Omega_n = -(C\Omega_{n-1} + A)^{-1}B \tag{36}$$

with an arbitrary number of iteration steps N where $n = \{1, 2, ..., N\}$. We define $\Omega_0 = \xi I$ with $0 \leq \xi \leq 1$. The iteration process (36) proceeds until $||\Omega_n - \Omega_{n-1}|| < \xi$ where ξ is an arbitrarily small number. Given the solution of Ω, the computation of Φ is straightforward:

$$A\Phi + C\Omega_n\Phi + I = 0 \Rightarrow \Phi = -(A + C\Omega_n)^{-1}. \tag{37}$$

Note that the solution under the method of undetermined coefficients equals the one under the method used in Matlab Dynare. This is confirmed when comparing the outcome for (35) and (37) computed by using Matlab Dynare to the results one would get by applying the brute force iteration procedure. Since this procedure is much more convenient to use within our estimation routine, we abstain from using Dynare.

A2: Delta Method and Confidence Interval for Auto- and Cross-covariances

The Delta method is a common technique for providing the first-order approximations to the variance of a transformed parameter; see chapter 5 of Davidson and Mackinnon (2004) among others. In the study, we use the Delta method when computing the standard errors of the estimated auto- and cross-covariances of the data. The covariance is defined as follows:

$$\gamma_{ij}(h) = E[(X_{i,t} - \mu_i)(X_{j,t+h} - \mu_j)'], \quad t = 1, \cdots, T \tag{38}$$

where γ_{ij} is the auto-covariance function when $i = j$. Otherwise γ_{ij} denotes the cross-covariance between $X_{i,t}$ and $X_{j,t+h}$. h is the lag in data and μ_i(or μ_j) is the sample mean of the variable X_i(or X_j). The covariance function in Equation (38) proceeds with a simple multiplication:

$$\begin{aligned} \gamma_{ij}(h) &= E[X_{i,t} \cdot X'_{j,t+h}] - \mu_i \cdot E[X'_{j,t+h}] \\ &= \mu_{ij} - \mu_i \cdot \mu_j \end{aligned} \tag{39}$$

where μ_{ij} denotes $E[X_{i,t} \cdot X'_{j,t+h}]$. Now we see that $\gamma_{ij}(h)$ is a transformed function of the population moments μ_i, μ_j and μ_{ij}. Denote the vector μ as the collection of the moments: $\mu = [\mu_i \quad \mu_j \quad \mu_{ij}]$. We differentiate the covariance function with respect to the vector μ:

$$D = \frac{\partial \gamma_{ij}(h)}{\partial \mu} = \begin{bmatrix} \frac{\partial \gamma_{ij}(h)}{\partial \mu_i} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_j} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_{ij}} \end{bmatrix} = \begin{bmatrix} -\mu_j \\ -\mu_i \\ 1 \end{bmatrix} \tag{40}$$

Therefore the Delta method provides the asymptotic distribution of the estimate $\widehat{\gamma}_{ij}$ by matching the sample moments of the data.

$$\sqrt{T}(\gamma_{ij} - \widehat{\gamma}_{ij}) \;\sim\; N(0, D'SD). \tag{41}$$

For some suitable lag length q, we use a common HAC estimator of Newey and West (1994) when estimating the covariance matrix of sample moments. Specifically, we follow the advice in Davidson and MacKinnon (2004, p.364) and scale q with $T^{1/3}$. Accordingly we may set $q = 5$ for the Euro area data.

$$\widehat{\Sigma}_\mu = \widehat{C}(0) + \sum_{k=1}^{q} \Big(1 - \frac{k}{q+1}\Big)[\widehat{C}(k) + \widehat{C}(k)'] \tag{42}$$

$$\widehat{C}(k) = \frac{1}{T} \sum_{t=k+1}^{T} [f(z_t) - \widehat{\mu}][f(z_{t-h}) - \widehat{\mu}]'$$

where $f(z_t) = [X_i,\ X_j,\ X_i \cdot X_j]$. We use the optimal weight matrix $S = \widehat{\Sigma}_\mu^{-1}$ in estimating the covariance matrix of moments. Let s_γ be $\sqrt{D'SD}$. Then the 95% asymptotic confidence intervals for auto- and cross-covariance estimates become:

$$[\gamma_{ij} - 1.96 \cdot s_\gamma, \quad \gamma_{ij} + 1.96 \cdot s_\gamma] \tag{43}$$

A3: Large-scale Simulation Study for the BR Model

We report the results of a simulation study for the BR model when a large simulation size is used; S=100. At present, it is pertinent to emphasize that the model estimates using a large simulation size have slightly smaller values for the RMSEs than ones from a small simulation size in the section 4.3. However, the simulation studies are computationally expensive as the sample size increases. Note here that we only report the case of $T = 100$.

Table 4: Monte Carlo Study for the BR Model

		T=100	
Label	True (θ^0)	Mean	RMSE
α	0.200	0.226	0.260
χ	1.000	0.709	0.421
τ	0.385	0.939	1.164
κ	0.215	0.234	0.098
ϕ_y	0.675	0.722	0.165
$\phi_{\hat{\pi}}$	1.100	1.113	0.143
$\phi_{\hat{r}}$	0.670	0.678	0.059
σ_y	0.825	0.933	0.268
$\sigma_{\hat{\pi}}$	0.740	0.690	0.116
$\sigma_{\hat{r}}$	0.240	0.165	0.138
β	2.250	2.581	1.569
δ	0.650	1.179	1.122
ρ	0.000	0.212	0.291
J		28.42	

Note: ν is set to the value of 0.99. The reported statistics are based on 200 replications. RMSE is the root mean square error.