Risk-taking, fiscal policies, asset pricing, and stochastic growth with the spirit of capitalism

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Abstract

In this paper, we study risk-taking, fiscal policies, and asset pricing in a stochastic model of growth with non-expected utility function and the spirit of capitalism. With specific assumptions on the production technology, preferences, and stochastic shocks, we derive the explicit solutions to the growth rates of consumption and savings and equilibrium returns on all assets. Finally, we give the effects of fiscal policies, the spirit of capitalism, and stochastic shocks on growth, asset pricing, and welfare.

Keywords: the spirit of capitalism, fiscal policies, asset pricing, stochastic growth, non-expected utility.

JEL Classification: E0, G1, H0, O0.
1 Introduction

In neoclassical growth models, the representative agent chooses a consumption path to maximize his discounted utility, which is defined only on consumption. This motive for wealth accumulation is often taken to be solely driven by one’s desire to increase consumption rewards. It is an important motive, however, not the only one. Because man is a social animal, he also accumulates wealth to gain prestige, social status, and power in the society; see Frank (1985), Cole, Mailath, and Postlewaite (1992, 1995), Fershtman and Weiss (1993), Zou (1994, 1995), Bakshi and Chen (1996), and Fershtman, Murphy and Weiss (1996). Earlier contributions include Duesenberry (1948), Kurz (1968), and Spence (1974). In these wealth-is-status models, the representative agent accumulates wealth not only for consumption but also for wealth-induced status. Mathematically, in light of the new perspective, the utility function can be defined on both consumption, $c$, and wealth, $W$: $u(c_t, W_t)$. In fact, these models is in line with the spirit of capitalism in the sense of Weber (1958) and Keynes (1971): capitalists accumulate wealth for the sake of wealth.

Many authors have used the wealth-is-status and the spirit-of-capitalism models to try to explain growth, savings, and asset pricing. Cole, Mailath, and Postlewaite (1992) have demonstrated how the presence of social status leads to multiple equilibria in long-run growth. Zou (1994, 1995) has studied the spirit of capitalism and long-run growth and shows that a strong capitalist spirit can lead to unbounded growth of consumption and capital even under the neoclassical assumption of production technology. Bakshi and Chen (1996) have explored empirically the relationship between the spirit of capitalism and stock market pricing and offered an attempt towards the resolution of the equity premium puzzle in Mehra and Prescott (1985). They have shown that when investors care about status they will be more conservative in risk taking and more frugal in consumption spending. Furthermore stock prices tend to be more volatile with the presence of the spirit of capitalism.

As for the discussing of risk, fiscal policies, and growth, Eaton (1981), Turnovsky (1993, 1995), Grinols and Turnovsky (1993, 1994), and Obstfeld (1994) have introduced stochastic tax and stochastic government expenditure into the continuous-time growth and asset-pricing models. Under specific assumptions on the production technology, preferences, and stochastic shocks, they have derived explicit solutions to the growth rates of consumption and savings and equilibrium returns on assets. And for the continuous-time stochastic growth model with the role of social status and the spirit of capitalism in capital accumulation, asset pricing, and growth, Gong and Zou (2002) study the fiscal policies and asset pricing in a stochastic growth model with the spirit of capitalism.
Turn to the literature on non-expected utility function, Epstein and Zin (1989, 1991) and Weil (1990) have used it to analysis the asset pricing theory, Obstfeld (1994) developed a continuous-time model in which international risk-sharing can yield substantial welfare gains through its effect on expected consumption growth. Yang (1998) used the stochastic asset pricing model with non-expected utility function and the social status to explain the equity premium puzzle.

Among the enormous literatures, there was seldom paper to discuss the risk, fiscal policies, growth, and social status with the spirit of capitalism and non-expected utility function. This paper integrates these three trends of growth and asset-pricing literature, non-expected utility function, and considers social status, fiscal policies, and asset pricing in a stochastic model of growth. With specific assumptions on the production technology, preferences, and stochastic shocks, it gives the explicit solutions to the growth rates of consumption and savings and equilibrium returns on all assets. Furthermore, it demonstrates how fiscal policies, social status, the spirit of capitalism, and stochastic shocks affect economic growth and asset pricing.

The paper is organized as follows: in section 2, we present a modified growth and asset-pricing framework as in Turnovsky (1995), Bakshi and Chen (1996), and Gong and Zou (2002). In section 3, we derive the optimal conditions for macroeconomic equilibrium. In section 4, using a specific utility function, we present explicit solutions to the consumption-wealth ratio, the mean growth rate of the economy, and the expected real return on bonds and capital. In section 5, we discuss the effects of stochastic shocks and fiscal policies on the economy. In section 6, we discuss the effects of the concern for social status or the spirit of capitalism on asset pricing and growth. We conclude the paper in section 7.

2 The Model

Along with Eaton (1981) and Turnovsky (1995), we assume output $Y$ and government expenditure $G$ to be proportional to the mean-level output, i.e.

$$dY = \alpha K dt + \alpha K dy,$$

$$dG = g\alpha K dt + \alpha K dz,$$

where $\alpha$ and $g$ are positive constants.

Equation (1) asserts that the accumulated flow of output over the period $(t, t + dt)$, given by the right-hand side of this equation, consists of two components. The deterministic component is described as the first term on the right hand, which is the firm’s production technology and has been specified as a linear production function. The second part is the stochastic component, which can be viewed as the shock to the production and assumed
to be temporally independent, normally distributed, and

\[ E(dy) = 0, \quad \text{Var}(dy) = \sigma_y^2 dt. \]

In equation (2), the deterministic part of government expenditure is expressed in terms of a fraction of mean output, and government expenditure has the stochastic shock \( dz \). It is further assumed that \( dz \) is temporally independent, normally distributed, and

\[ E(dz) = 0, \quad \text{Var}(dz) = \sigma_z^2 dt. \]

Following Fischer (1975) and Turnovsky (1995), it is assumed that there are two assets in the economy: government bonds, \( B \) and the capital stock, \( K \). It is postulated, as in Turnovsky (1995) that the stochastic real rate of return on bonds, \( dR_B \), over a period \( dt \), is given by

\[ dR_B = r_B dt + du_B, \]

where \( r_B \) and \( du_B \) will be determined endogenously in the macroeconomic equilibrium.

Turning to the second asset, capital, and using the production technology in equation (1), the stochastic real rate of return on capital is

\[ dR_K = \frac{dY}{K} = \alpha dt + \alpha dy = r_K dt + du_K. \]

Thus wealth \( W_t \) is the sum of the holdings of \( B_t \) and \( K_t \), i.e.,

\[ W_t = B_t + K_t. \]

Let \( n_B \) and \( n_K \) denote the fractions of wealth invested in bonds, nominal bonds, and capital, respectively, i.e.,

\[ n_B = \frac{B_t}{W_t}, n_K = \frac{K_t}{W_t}, \]

and \( n_B + n_K = 1 \).

We may assume that, without any loss of generality, taxes are levied on capital income and consumption, namely,

\[ dT = (\tau r_K K + \tau_c c) dt + \tau' K du_K = (\tau \alpha K + \tau_c c_t) dt + \tau' \alpha K dy, \]

where \( \tau \) and \( \tau' \) are the tax rates on the deterministic component of capital income and the stochastic capital income, respectively, and \( \tau_c \) is the tax rate on consumption.
Suppose the utility depends upon consumption and wealth, savings behavior is determined by the consumer’s impatience, risk aversion, and willing to substitute consumption over time, and the utility function satisfy

\[ f((1 - R)u(c_t, W_t)) = (1 - R)u(c_t, W_t)\Delta t + e^{-\beta \Delta t} f((1 - R)E_t u(c_{t+\Delta t}, W_{t+\Delta t})), \]  

where the function \( f(x) \) is given by

\[ f(x) = \frac{1 - R}{1 - \gamma} x^{\frac{1 - \gamma}{1 - \gamma}}, \]  

The recursive form in equation (6) is similar to the function form presented by Obstfeld (1994). It states that current utility depends the current consumption level \( c_t \), the current wealth level \( W_t \), and the expected future utility \( E_t u(c_{t+\Delta t}, W_{t+\Delta t}) \). \( E_t \) is the mathematical expectation conditional on the information before time \( t \). \( u(c_t, W_t) \) is the intra-period utility function, which depends upon current consumption level and the current wealth level. \( R > 0 \) is the coefficient of relative risk aversion with respect to timeless gambles over \( x_t \). \( \beta > 0 \) is the rate of time preference.

If we select the intra-period utility function \( u(c_t, W_t) \) as

\[ u(c_t, W_t) = c_t^{1-\gamma} W_t^{-\lambda}, \]  

where \( 1/\gamma > 0 \) is the elasticity of intertemporal substitution defined over riskless paths of \( x_t \), \( |\lambda| \) measures the investor’s concern with his social status or measures his spirit of capitalism. The larger the parameter \( |\lambda| \), the stronger the agent’s spirit of capitalism or concern for social status. \( \gamma \) and \( \lambda \) satisfy that: \( \gamma > 0 \), and \( \lambda \geq 0 \) when \( \gamma \geq 1 \), and \(-1 < \lambda < 0 \), otherwise.

The effective coefficient of relative risk aversion can be defined as

\[ ERRA = 1 - \frac{1 - \gamma - \lambda}{1 - \gamma} (1 - R) \]  

and we require \( ERRA > 0 \).

If there were no spirit of capitalism, then \( \lambda = 0 \), and we have the version of generalized isoelastic preferences introduced by Svensson (1989), if we further specialize the model without the a spirit of capitalism by imposing time-separability \( (R = \gamma) \), then \( ERRA = \gamma \), and maximizing (6) is reduced to

\[ \max E_0 \int_0^\infty u(c_t) e^{-\beta t} dt \]  

which was discussed by Turnovsky and Eaton, et al.
Next, if we were maintain time separability, but allow a spirit of capitalism, then \( ERRA = \gamma + \lambda \), and maximizing (6) is equivalent to

\[
\max E_0 \int_0^\infty u(c_t, W_t) e^{-\beta t} dt
\]

which discussed by Gong and Zou (2002).

Now the representative agent chooses the consumption-wealth ratio, \( c/W \), the portfolio shares, \( n_B \) and \( n_K \) to maximize his expected utility subject to the budget constraint, i.e.,

\[
\max U(c_t, W_t)
\]

subject to

\[
dW_t = (n_B W_t r_B + n_K W_t (1 - \tau) r_K - (1 + \tau_c) c_t) dt + W_t dw,
\]

\( n_B + n_K = 1 \),

with the given initial stocks of nominal bonds \( B(0) \) and capital \( K(0) \).

Where we denote \( U(c_t, W_t) \) the utility function satisfies equation (6), \( w_t \) is stochastic shock of wealth, and it is given as

\[
dw = n_B du_B + n_K (1 - \tau') du_K.
\]

3 Macroeconomic equilibrium

As in Turnovsky (1995) and Gong and Zou (2002), the economic system in equilibrium determines the rates of consumption and savings, the value of returns on all assets, and the economic growth rate.

The exogenous variables include the preference parameters, technology parameters, and government fiscal policies including government expenditure \( g \), tax rates \( \tau, \tau' \), and \( \tau_c \). The exogenous stochastic processes consist of government expenditure, \( dz \), and productivity shocks, \( dy \), which are taken to be mutually uncorrelated. The remaining stochastic disturbances—real rates of returns on bonds, \( du_B \), and total wealth, \( dw \), are both endogenous and will be determined by the economic system. The remaining endogenous variables include the following: the consumption-wealth ratio, \( c/W \), the mean growth rate of the economy, the expected real returns on two assets, \( r_B \), and \( r_K \), respectively, and the corresponding portfolio shares \( n_B \) and \( n_K \).

To solve the agent’s optimization problem, we introduce the value function

\[
V(W(t), t) = \max U(c_t, W_t)
\]

subject to equations (9) and (10).
If we define \( V(W, t) = e^{-\beta t} X(W) \), we know that the value function for this problem obeys the recursion

\[
f((1 - R)X(W_t)) = \lim_{\Delta t \to 0} \max_{n_B, n_K} \{(1 - R)u(c_t, W_t)\Delta t + e^{-\beta \Delta t} f((1 - R)E_t X(W_{t+\Delta t}))\}
\]

subject to the equations (9) and (10).

Now, we get the following proposition:

Proposition 1. The first-order conditions for the optimization problem can be written as follows:

\[
\frac{\partial u(c_t, W)}{\partial c} = (1 - R)f'((1 - R)X(W))(1 + \tau_c)X_W,
\]

\[
(r_B X'(W)W - \eta)dt + \text{cov}(dw, du_B)X''(W)W^2 = 0,
\]

\[
((1 - \tau)r_K X'(W)W - \eta)dt + \text{cov}(dw, (1 - \tau')du_K)X''(W)W^2 = 0,
\]

\[
n_B + n_K = 1,
\]

where \( \eta \) is the Lagrangian multiplier associated with the portfolio selection constraint (10). Furthermore, the optimal solutions of the problem must satisfy the Bellman equation

\[
(1 - R)u(c_t, W_t) - \beta f((1 - R)X(W_t))
\]

\[
+ (1 - R)f'((1 - R)X(W_t))\{(\rho - (1 + \tau_c)\frac{\partial}{\partial W_t})X'_t(W_t)\}
\]

\[
+ \frac{1}{2}\sigma^2_w W_t^2 X''(W_t)\} = 0,
\]

where \( \rho = n_B r_B + n_K (1 - \tau)r_K \), and it is the expected net-of-tax return on total asset holdings.

See Appendix for the proof of proposition 1.

Condition (13) asserts that in the equilibrium the marginal utility of consumption must equal the marginal utility of wealth; conditions (14) and (15) are the asset pricing relationships; condition (10) is the portfolio selection constraint; and equation (16) is the Bellman equation, from which we will solve the value function \( X(W) \).

In order to determine the full equilibrium system, we follow Turnovsky (1995) in discussing government behavior. Equations (2) and (5) describe government expenditure policy and tax policies, both of which are proportional to current output. In the absence of lump-sum taxation, government budget constraint can be described as:

\[
\text{dB} = BdR_B + dG - dT.
\]
From equations (2) and (5), equation (17) can be written in the form
\[ n_B \frac{dB}{B} = (\tau_B n_B + \tau_c \frac{c}{W} + \alpha(g-\tau)n_K)dt + n_B du_B + \alpha n_K dz - \tau' \alpha n_K dy. \] (18)

The equilibrium product market requires
\[ dK = dY - cdt - dG, \] (19)

where \( G \) follows the stochastic process of equation (2). Now we have

**Proposition 2** The equilibrium system of the economy can be summarized as equations (10), (13)-(15), the Bellman equation (16), and
\[ \frac{dK}{K} = [\alpha(1-g) - \frac{c}{n_K W}] dt + \alpha(dy - dz) \equiv \phi dt + \alpha(dy - dz). \] (20)

with the transversality condition (TVC) plus the initial conditions.

Furthermore, the stochastic component of real rate of return on bonds, \( du_B \), and total wealth, \( dw \), are determined by:
\[ dw = \alpha(dy - dz), \] (21)
\[ du_B = \frac{\alpha}{n_B}[(1 - n_K(1 - \tau'))dy - dz]. \] (22)

**Proof:** Equation (20) can be derive directly from equations (1), (2), and (19). Because of the intertemporal constancy of portfolio shares, we have that all the real assets grow at a common stochastic rate, i.e.,
\[ \frac{dW}{W} = \frac{dK}{K} = \frac{dB}{B}. \] (23)

Combing with equations (9), (18), (20), and (23), we get
\[ dw = n_B du_B + n_K(1 - \tau')\alpha dy = \alpha(dy - dz) = \frac{1}{n_B}[n_B du_B + \alpha n_K(dz - \tau'dy)]. \]

From the equations above, and noticing the fact \( n_B + n_K = 1 \), it is easy to get \( dw \) and \( du_B \). Q.E.D.

Equations (20)-(22) enable us to compute all the necessary covariances and variances in the full equilibrium system. Equation (21) implies that the stochastic shocks of government expenditure and production determine the stochastic rate of return on bonds.
4 An explicit example

Specify the utility function given in equation (8), and we have

**Proposition 3** For the special utility function (8), the first-order optimal conditions for the optimization problem are

\[
\frac{c_t}{W_t} = \frac{\beta\rho(1-\gamma-\lambda)-\frac{1}{2}\sigma_w^2(1-\gamma-\lambda)(\frac{1-\gamma-\lambda}{1+\lambda}(1-R)-1)}{\gamma(1+\tau)(\frac{1-\gamma}{1+\lambda})},
\]  
(24)

\[
(r_B - \eta W \frac{1-\gamma-\lambda}{\delta(1+\lambda)(1-R)}dt = ERRAcov(dw, du_B),
\]  
(25)

\[
((1-\tau)r_K - \eta W \frac{1-\gamma-\lambda}{\delta(1+\lambda)(1-R)}dt = ERRAcov(dw, (1-\tau')du_K),
\]  
(26)

where \(\eta\) is the Lagrangian multiplier associated with constraint (10),

\[
\rho = n_Br_B + n_K(1-\tau)r_K,
\]

\[
dw = n_Bdu_B + n_K(1-\tau')du_K,
\]

\[
\sigma_w^2 = n_B^2\sigma^2_B + n_K^2(1-\tau')^2\sigma^2_K + 2n_Bn_K(1-\tau')\sigma_{BK}.
\]

Equation (24) gives the consumption-wealth ratio. For a logarithmic utility function in consumption, i.e., \(\gamma = 1\), we get \(c/W = \beta\). Therefore, the consumption-wealth ratio is always equal to the time discount rate. If \(\gamma \neq 1\), then the effect of an increase in the expected net-of-tax return on the consumption-wealth ratio will be

\[
\frac{d(c/W)}{d\rho} = \frac{1 - 1/\gamma}{(1+\tau)\gamma},
\]

which is similar to the conclusion of Gong and Zou (2002). Therefore, an increase in the expected net-of-tax return \(\rho\) will raise the consumption-wealth ratio if \(1/\gamma < 1\), and lower it otherwise. This can be explained as follows. When \(1/\gamma < 1\), the elasticity of intertemporal substitution, \(1/\gamma\), is relatively small. The representative agent will increase current consumption more than investment and wealth. On the other hand, when \(1/\gamma > 1\), the elasticity of intertemporal substitution is relatively large, and the agent will increase wealth holding more than consumption.

Similar analysis holds for the effect of the variance of wealth, \(\sigma_w^2\), on \(c/W\):

\[
\frac{d(c/W)}{d\sigma_w^2} = \frac{1}{2} \frac{ERRA}{\gamma(1+\tau_c)\frac{1}{1-\gamma}}.
\]

Therefore, an increase in the variance of wealth reduces the consumption-wealth ratio when \(\gamma > 1\), and increases the ratio when \(\gamma < 1\).
Equations (25) and (26) illustrates the asset pricing relationships. The term of $W^\frac{1-\gamma}{\gamma+1} \frac{(\tau-1)}{1-\gamma} \delta(1-\tau)\delta(1-R)$ in equation (25) implies that the return on bonds is equal to the riskless return plus a risk premium, which is proportional to the covariance between total wealth and bonds. Similarly, in equation (26), for the net return on the risky capital, it is also equal to the riskless return plus a risk premium, which is also proportional to the covariance between total wealth and risky capital. In the absence of risk, these three equations imply that the net returns on the three assets are all equal.

Since $\rho$ is still endogenous in terms of holding shares for various assets, we now use the full equilibrium system to derive explicit solutions to $c=W; n_B; n_K; r_B; r_K$. With proposition 3, and from the optimal conditions (21) and (22) plus equation (18), we have:

$$\sigma_w^2 = \alpha^2(\sigma_y^2 + \sigma_z^2)dt,$$

$$\text{cov}(dw, du_B) = \frac{\alpha^2}{n_B}[(1 - n_K(1 - \tau'))\sigma_y^2 + \sigma_z^2]dt,$$

and

$$\text{cov}(dw, (1 - \tau')du_K) = \alpha^2(1 - \tau')\sigma_y^2 dt.$$

and Proposition 4 The mean return on bonds and the stochastic growth rate of the economy are

$$r_B = \alpha(1 - \tau) - \frac{1-\gamma}{\gamma+1} \frac{(1-R)-1}{n_B} \alpha^2(\tau'\sigma_y^2 + \sigma_z^2),$$

$$\phi = r_Bn_B + (g - \tau)\alpha n_K + \tau c \frac{c}{W} = \rho - (1 + \tau_e)\frac{c}{W}.\quad (27)$$

The first term on the right-hand side of equation (27) is the net (after-tax) return on capital, which is the same as in Turnovsky (1995); the second term on the right-hand side is the stochastic component of the return on bonds.

With proposition 5, we now have our main theorem of this section:

Theorem 1 The explicit solutions of the economy system are

$$c \frac{c}{W} = \frac{\beta}{\gamma(1+\tau_e)(\frac{1-\gamma}{1+\gamma})} - \frac{\alpha(1-\tau) - \frac{1-\gamma}{\gamma+1} \frac{(1-R)-1}{n_B} \alpha^2(2\tau' - 1)\sigma_y^2 + \sigma_z^2)}{\gamma(1+\tau_e)(\frac{1-\gamma}{1+\gamma})},$$

$$\phi = \alpha(1 - \tau) - \alpha^2(\frac{1-\gamma}{1+\gamma} (1 - R) - 1)(\tau'\sigma_y^2 + \sigma_z^2) - (1 + \tau_e)\frac{c}{W},\quad (29)$$

$$n_K = \frac{c/W}{\alpha(\tau - g) + (1+\tau_e)c/W + \alpha^2(\frac{1-\gamma}{1+\gamma} (1-R)-1)(\tau'\sigma_y^2 + \sigma_z^2)},$$

$$n_B = 1 - n_K,$$
and the TVC

$$\lim_{t \to \infty} E(W^{1 - \gamma - \lambda(1 - R)} e^{-\beta t}) = 0. \quad (32)$$

Proof: Notice the conditions

$$n_B + n_K = 1,$$

$$\rho = n_B r_B + n_K r_K (1 - \tau),$$

$$r_B = \alpha (1 - \tau) + \frac{1 - \gamma - \lambda}{1 - \gamma} (1 - R)^{-1} \alpha^2 (\tau' \sigma_y^2 + \sigma_z^2).$$

We obtain

$$\rho = \alpha (1 - \tau) + \alpha^2 \left( \frac{1 - \gamma - \lambda}{1 - \gamma} (1 - R) - 1 \right) (\tau' \sigma_y^2 + \sigma_z^2).$$

Thus, we have equations (29) and (30). With equation (17), we have

$$\phi = \alpha (1 - g) - \frac{c}{n_K W},$$

and equation (31).

Q.E.D.

With equation (31), the portfolio shares of government bonds are determined as a residual from the portfolio-selection constraint $n_K + n_B = 1$.

Please also note that the transversality condition (32) can be shown to be equivalent to $c/W > 0$. In fact, since

$$dW = \phi W dt + W dw,$$

we have

$$W(t) = W(0) e^{(\phi - \frac{1}{2} \sigma_y^2) t + w(t) - w(0)}.$$

The TVC will be met if

$$\lim_{t \to \infty} E(e^{(\phi - \frac{1}{2} \sigma_y^2) \frac{1 - \gamma - \lambda}{1 - \gamma} (1 - R) t} e^{-\beta t}) = 0.$$

Equation (32) can be met for a positive consumption-wealth ratio and $\frac{1 - \gamma - \lambda}{1 - \gamma} (1 - R) < 0$.

5 Comparative dynamics

Now we discuss how stochastic shocks (in production and government spending) and government fiscal policies affect the equilibrium.

Effects of stochastic shocks

Differentiating with respect to $\sigma_z^2$ and $\sigma_y^2$, respectively, in equation (29), we have for $\gamma > 1$, $\tau' < 50%$

$$\frac{\partial c/W}{\partial \sigma_z^2} = \frac{(1 - \gamma - \lambda (1 - R) - 1) \alpha^2}{2(1 + \tau_c)(1 - \gamma)} > 0,$$
\[
\frac{\partial c/W}{\partial \sigma_y^2} = \frac{(1-\gamma\lambda(1-R)-1)(2\tau'-1)\alpha^2}{2(1+\tau_c)\gamma} < 0.
\]

Therefore, when the intertemporal elasticity of substitution is relatively small, a higher variance in government expenditure increases the consumption-wealth ratio, whereas the stochastic shock in production lowers the consumption-wealth ratio.

On the other hand, when \( \gamma < 1 \), we have just the opposite results, namely,

\[
\frac{\partial c/W}{\partial \sigma_z^2} = \frac{(1-\gamma\lambda(1-R)-1)\alpha^2}{2(1+\tau_c)\gamma} < 0,
\]

\[
\frac{\partial c/W}{\partial \sigma_y^2} = \frac{(1-\gamma\lambda(1-R)-1)(2\tau'-1)\alpha^2}{2(1+\tau_c)\gamma} > 0.
\]

From equation (30), the equilibrium growth rate, \( \phi \), varies with the stochastic shocks of government spending as follows. For all values of \( \gamma \),

\[
\frac{\partial \phi}{\partial \sigma^2} = -\frac{1}{2}\alpha^2\left(\frac{1-\gamma\lambda}{1-\gamma}(1-R)-1\right)\left(1 + \frac{1}{\gamma}\right) < 0,
\]

because \( ERRA > 0 \). Therefore, more volatility in government spending always increases the rate of economic growth. This is true because an increase in \( \sigma_z^2 \) raises the risk of bonds. The agent reduces his holding of government bonds and invests more in capital, which in turn leads to more output growth.

But for the shocks to the productivity, take differentiate on equation (29), we have

\[
\frac{\partial \phi}{\partial \sigma^2} = -\frac{1}{2}\alpha^2\left(\frac{1-\gamma\lambda}{1-\gamma}(1-R)-1\right)\left(\frac{2\tau'-1+\gamma}{\gamma}\right) < 0.
\]

Thus, the mean growth rate of the economy can increase or decrease depending on the values of \( \gamma \) and other parameters. For example, when \( \gamma > 1 \), we have \( \frac{\partial \phi}{\partial \sigma^2} > 0 \); when \( \gamma < 1 \), the effects of the variance of the productivity shocks on the mean growth rate is ambiguous, when \( \tau' < \frac{1-\gamma}{2} \), the effects is positive, while \( \tau' > \frac{1-\gamma}{2} \), the effects is negative. Our results confirm the complicated pictures of the effects of stochastic shocks on output growth in Obstfeld (1994) and Turnovsky (1995).

The dependence of the shares of asset holding on the stochastic shocks can be derived from equation (31):

\[
\frac{\partial n_K}{\partial \sigma_z^2} = n_K\left\{\frac{(1-n_K(1+\tau_c))\partial(c/W)/\partial \sigma_z^2}{c/W} - \alpha^2\left(\frac{1-\gamma\lambda}{1-\gamma}(1-R)-1\right)\frac{n_K}{c/W}\right\},
\]

\[
\frac{\partial n_K}{\partial \sigma_y^2} = n_K\left\{\frac{(1-n_K(1+\tau_c))\partial(c/W)/\partial \sigma_y^2}{c/W} - \frac{n_K}{c/W}\alpha^2\left(\frac{1-\gamma\lambda}{1-\gamma}(1-R)-1\right)\tau'\right\}.
\]

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The first equation above tells us that the stochastic shock in government expenditure will enhance the holding of risky capital for \( \gamma > 1 \). In the second equation the effect of the stochastic shock in production on the holding of risky capital is ambiguous. As for the holding shares of government bonds, \( n_B \) and \( n_D \), we can use the portfolio-selection condition and derive their responses to various shocks and fiscal policies. We leave the exercise to the reader.

We have derived the value function \( X(W) \) in appendix B. Let \( W(0) \) denote the initial stock of wealth. We have the following welfare function:

\[
X(W(0)) = \delta W(0) \frac{1-\gamma}{1-\gamma} (1-R),
\]

where

\[
\delta = (1 + \tau_c) \frac{1}{1-\gamma} \frac{1-R}{1-\gamma} \frac{1}{1-\gamma} \frac{1-R}{1-\gamma} (1-R)^{-1} \left( \frac{C_t}{W_t} \right)^{1-\gamma}.
\]

However, \( W(0) \) is itself endogenously determined by

\[
W(0) = \frac{K_0}{n_K}.
\]

Therefore, with some simple manipulations, welfare is given by:

\[
X(K_0) = n_K^{\frac{1-\gamma}{1-\gamma} (1-R)} (1+\tau_c)^{\frac{1-R}{1-\gamma} (1+\gamma-\lambda)} (1-R)^{-1} \left( \frac{C_t}{W_t} \right)^{\frac{1-R}{1-\gamma}} K_0^{\frac{1-\gamma}{1-\gamma} (1-R)},
\]

where \( c/W \) and \( n_K \) are determined as in Theorem 1. Taking differentiation in equation (33), we get

\[
\frac{dX}{X} = \frac{\gamma+\lambda-1}{1-\gamma} (1-R) \frac{dn_K}{n_K} + \frac{1-R}{1-\gamma} \frac{d(c/W)}{c/W}.
\]

Now we have

\[
\frac{\partial X}{\partial \sigma^x_2} = \frac{\gamma+\lambda-1}{1-\gamma} (1-R) \frac{X \partial n_K/\partial \sigma^2_2}{n_K} + \frac{1-R}{1-\gamma} \frac{X \partial (c/W)/\partial \sigma^2_2}{c/W},
\]

\[
\frac{\partial X}{\partial \sigma^y_2} = \frac{\gamma+\lambda-1}{1-\gamma} (1-R) \frac{X \partial n_K/\partial \sigma^2_2}{n_K} + \frac{1-R}{1-\gamma} \frac{X \partial (c/W)/\partial \sigma^2_2}{c/W}.
\]

These equations imply that the effects on welfare of the stochastic shocks in government expenditure and production are ambiguous.

**Effects of fiscal policies**

Now we turn to how taxes on capital income and consumption impact on the equilibrium.

First, differentiating all endogenous variables with respect to the tax on the deterministic part of capital income, \( \tau \), in equations (29), (30), and (31), we have
\[
\frac{d(c/W)}{d\tau} = \frac{\alpha(1 - \gamma)}{\gamma(1 + \tau_c)},
\]
\[
\frac{d\phi}{d\tau} = -\frac{\alpha}{\gamma} < 0,
\]
\[
\frac{dn_K}{d\tau} = n_K \left\{ \frac{(1 - n_K(1 + \tau_c))\partial(c/W)/\partial\tau - \alpha n_K}{c/W} \right\},
\]
\[
\frac{dX}{d\tau} = X\left\{ \gamma + \frac{\lambda - 1}{1 - \gamma}(1 - R) \frac{\partial n_K/\partial\tau}{n_K} + \frac{1 - R}{1 - 1/\gamma} \frac{\partial(c/W)/\partial\tau}{c/W} \right\}.
\]

If \( \gamma = 1 \), \( c/W \) is independent of the tax rate, because in this case \( c/W = \beta \), which is independent of \( \tau \). When \( 0 < \gamma < 1 \), we notice that a rise in the taxation on the deterministic component of capital income has an ambiguous effect on welfare. But, it is clear that
\[
\frac{d(c/W)}{d\tau} > 0, \quad \frac{d\phi}{d\tau} < 0.
\]
Therefore, a higher tax on the deterministic component of capital income will increase the consumption-wealth ratio and decrease the economic growth rate. This can be explained as follows: a higher tax on capital income will lower the return on capital. As the agent switches away from capital to bonds and consumption, this reduces capital accumulation, lowers the growth rate, and increases the consumption-wealth ratio.

When \( \gamma > 1 \), we still find that capital income taxation reduces the holding share of risky capital and lowers the growth rate:
\[
\frac{d\phi}{d\tau} < 0, \quad \frac{dn_K}{d\tau} < 0.
\]
But it reduces the consumption-wealth ratio: \( \frac{d(c/W)}{d\tau} < 0 \).

Second, we look at the effects on the equilibrium of the tax on the stochastic component of capital income:
\[
\frac{\partial(c/W)}{\partial\tau'} = \frac{(1 - \gamma - \lambda)(1 - R)(1 - \gamma)}{(1 + \tau_c)(\tau_c^2)},
\]
\[
\frac{\partial\phi}{\partial\tau'} = -\alpha^2((1 - \gamma - \lambda)(1 - R) - 1)\sigma^2 \frac{1}{\gamma} > 0,
\]
\[
\frac{\partial n_K}{\partial\tau'} = n_K \frac{1 - n_K(1 + \tau_c)}{c/W} \frac{\partial(c/W)}{\partial\tau'} + \alpha^2 \frac{(1 - \gamma - \lambda)(1 - R) - 1)\sigma^2 n_K}{c/W},
\]
\[
\frac{\partial X}{\partial\tau'} = X \frac{\gamma + \frac{\lambda - 1}{1 - \gamma}(1 - R) \frac{\partial n_K/\partial\tau'}{n_K} + \frac{1 - R}{1 - 1/\gamma} \frac{\partial(c/W)/\partial\tau'}{c/W}}{\frac{\partial(c/W)/\partial\tau'}{c/W}}.
\]
These results are very similar to the ones for the tax on the deterministic component of capital income. Still,

$$\frac{\partial (c/W)}{\partial \tau'} < 0, \quad \frac{\partial \phi}{\partial \tau'} > 0,$$

when $0 < \gamma < 1$; and

$$\frac{\partial (c/W)}{\partial \tau'} > 0, \quad \frac{\partial \phi}{\partial \tau'} > 0, \quad \frac{\partial n_k}{\partial \tau'} > 0,$$

when $\gamma > 1$.

Finally, we examine the effects of the consumption tax on the equilibrium. Recall that from the Ramsey-Cass-Koopmans model, the consumption tax does not affect the rate of economic growth and long-run capital accumulation. In the long run, it only crowds out private consumption. Here we have

$$\frac{\partial (c/W)}{\partial \tau_c} = -\frac{c/W}{1 + \tau_c} < 0.$$

That is to say, increasing the consumption tax will reduce the consumption-wealth ratio because a higher consumption tax decreases private consumption directly, and the agent has more money to invest in capital and bonds, which in turn increases wealth. Therefore, the consumption-wealth ratio decreases as a result of a higher consumption tax.

For the growth rate, a rise in the consumption tax results in

$$\frac{\partial \phi}{\partial \tau_c} = -\frac{c}{W} + (1 + \tau_c) \frac{c/W}{1 + \tau_c} = 0.$$

Hence, the consumption tax rate has no effects on the growth rate.

As for welfare, we have

$$\frac{\partial X}{\partial \tau_c} = \frac{\gamma + \lambda - 1}{1 - \gamma} (1 - R) \frac{X \partial n_k / \partial \tau_c}{n_K} + \frac{1 - R}{1 - 1/\gamma} \frac{X \partial (c/W) / \partial \tau_c}{c/W} - \frac{X}{1 + \tau_c}.$$

Therefore, $\frac{\partial X}{\partial \tau_c} < 0$ when $\gamma > 1$. The explanation is simple. Since the elasticity of intertemporal substitution is small, current consumption will not be severely cut as a result of a consumption tax, whereas current investment in assets is reduced. In the long run, the agent will accumulate less assets and earn less income. His consumption and asset holdings are all reduced in the long run. Since welfare is defined on both consumption and wealth accumulation, his long-run welfare is also lower. For $\gamma < 1$, the welfare effect of a consumption tax is ambiguous because the direct effect of a higher consumption tax reduces consumption. But with a larger elasticity of intertemporal substitution the agent may increase his asset holdings, which in turn can lead to more asset accumulation and more income. This rising income can give rise to more long-run consumption. Again since the agent’s welfare is defined on both consumption and asset holdings, his welfare may also rise in this case.
6 Effects of the spirit of capitalism

In this section, we will discuss how the spirit of capitalism or the concern for social status affects asset pricing and economic growth.

First, we give the equilibrium asset-pricing relationships. Following Turnovsky (1995), we define the market portfolio as $Q = n_B W + n_K W$, and the return rate on the market portfolio as

$$r_Q \equiv \rho = r_B n_B + r_K (1 - \tau) n_K.$$ 

Now we have

**Proposition 5** The equilibrium asset-pricing relationships are

$$r_i - \frac{\eta W}{\delta (1 - \gamma - \lambda)} = \beta_i (r_Q - \frac{\eta W}{\delta (1 - \gamma - \lambda)}),$$

where $i = B, K$,

$$\beta_B = \frac{\text{cov}(dw, du_B)}{\text{var}(dw)} = \frac{(1 - n_K (1 - \tau')) \sigma_B^2 + \sigma_z^2}{n_B (\sigma_y^2 + \sigma_z^2)},$$

$$\beta_K = \frac{\text{cov}(dw, du_K)}{\text{var}(dw)} = \frac{(1 - \tau') \sigma_y^2}{\sigma_y^2 + \sigma_z^2}.$$

**Proof:** From equations (26) and (27), we have

$$\frac{\eta W}{\delta (1 - \gamma - \lambda)} = \alpha (1 - \tau) + \frac{(1 - \gamma - \lambda (1 - R) - 1) \alpha^2 (1 - \tau') \sigma_y^2}{n_B (\sigma_y^2 + \sigma_z^2)},$$

and

$$r_Q = \alpha (1 - \tau) - \frac{(1 - \gamma - \lambda (1 - R) - 1) \alpha^2 (1 - \tau') \sigma_y^2 + \sigma_z^2)}{\sigma_y^2 + \sigma_z^2}.$$ 

So, we obtain

$$r_Q - \frac{\eta W}{\delta (1 - \gamma - \lambda)} = \frac{(1 - \gamma - \lambda (1 - R) - 1) \alpha^2 (1 - \tau') \sigma_y^2 + \sigma_z^2)}{\sigma_y^2 + \sigma_z^2},$$

and using proposition 3 we get the conclusion. Q.E.D.

Again $\frac{\eta W}{\delta (1 - \gamma - \lambda)}$ is equal to the return on the riskless, nominal bonds, $r_D$. Equation (44) indicates that the returns on risky assets (bonds and capital) are given by the familiar consumption-based capital asset pricing model with $r_Q$ as the return on the market portfolio.

Furthermore, if we define the return on the market portfolio in the absence of the spirit of capitalism as $\tilde{r}_Q$, then, in our definition of the return of the market portfolio $r_Q$, we set $\lambda = 0$. Hence

$$\tilde{r}_Q = \alpha (1 - \tau) + R \alpha^2 (\tau' \sigma_y^2 + \sigma_z^2).$$
This is just the return on the market portfolio in Turnovsky (1995). At the same time, we have
\[ \frac{\gamma W^{R-1}}{\delta(1-R)} = \alpha(1 - \tau) - R\alpha^2(1 - \tau')\sigma^2_y. \]

Hence, we obtain the asset-pricing relationships as
\[ \tilde{r}_i - \frac{\gamma W^{R-1}}{\delta(1-R)} = \beta_i (\tilde{r}_Q - \frac{\gamma W^{R-1}}{\delta(1-R)}). \]  
(35)

Because \( \tilde{r}_Q < r_Q \), simple calculations yield
\[ r_i - \frac{\gamma W^{1-\gamma}(R-1)}{\delta(1-\gamma)(1-R)} > \tilde{r}_i - \frac{\gamma W^{R-1}}{\delta(1-R)}. \]  
(36)

Equation (45) implies that, with the spirit of capitalism, the gap between the returns on risky assets and the return on the risk-free asset will be enlarged. Like Bakshi and Chen (1996), our findings can be used to partially explain the equity premium puzzle in Mehra and Prescott (1985).

For the growth rate, social welfare, and portfolio selection, we have

**Proposition 6** The effects of the spirit of capitalism on \( c/W, n_K, \phi, \) and \( X \) are as follows

\[ \frac{\partial(c/W)}{\partial \lambda} = \frac{(1-\gamma)(1-R)}{2\gamma(1+\tau_c)} \alpha^2(\sigma^2_y + \sigma^2_z) + \beta(1-\gamma) \frac{(1-\gamma)}{\gamma(1-\gamma-\lambda)^2(1+\tau_c)} \]

\[ + \frac{(1-\gamma)(1-R)}{\gamma(1+\tau_c)} \alpha^2(\tau\sigma^2_y + \sigma^2_z), \]

\[ \frac{\partial \phi}{\partial \lambda} = \alpha^2 \frac{1-R}{1-\gamma} (\tau\sigma^2_y + \sigma^2_z) \frac{(1-\gamma)(1-R)}{2\gamma} \alpha^2(\sigma^2_y + \sigma^2_z) \]

\[ - \frac{\beta(1-\gamma)}{\gamma(1-\gamma-\lambda)^2} - \frac{(1-\gamma)(1-R)}{\gamma} \alpha^2(\tau\sigma^2_y + \sigma^2_z), \]

\[ \frac{\partial n_K}{n_K \partial \lambda} = (1 - (1 + \tau_c)n_K) \frac{\partial(c/W)}{c/W} + \frac{n_K}{c/W} \alpha^2(\tau\sigma^2_y + \sigma^2_z) (1 - R), \]

\[ \frac{\partial X}{\partial \lambda} = \frac{\gamma + \lambda - 1}{1-\gamma} (1-R) \frac{X\partial n_K/\partial \lambda}{n_K} + \frac{1-R}{1-\gamma} \frac{X \partial(c/W)/\partial \lambda}{c/W}. \]

If \( R < 1 \), and \( \gamma > 1 \), we have \( \lambda > 0 \). Then

\[ \frac{\partial \phi}{\partial \lambda} > 0, \frac{\partial n_K}{\partial \lambda} > 0. \]

Similarly, if \( R < 1 \), and \( \gamma < 1 \), we have \( \lambda < 0 \). In this case \( |\lambda| \) measures the spirit of capitalism, and

\[ \frac{\partial \phi}{\partial (-\lambda)} > 0, \frac{\partial n_K}{\partial (-\lambda)} > 0. \]
Therefore, an increase in the spirit of capitalism will always increase the growth rate and the holding share of risky capital. With a strong spirit of capitalism, the agent cares more about his social status and the power of wealth, and will accumulate more wealth and take more risk in investment in order to improve his social status.

If we further impose the condition that \( \tau' < \frac{1}{2}(1 - \frac{\sigma_z^2}{\sigma_y^2}) \), then

\[
\frac{\partial c/W}{\partial \lambda} < 0, \quad \frac{\partial X}{\partial \lambda} > 0,
\]

when \( R < 1, \gamma > 1, \) and \( \lambda > 0 \). Furthermore, with the same condition on the tax rate on the stochastic component of capital income, i.e., \( \tau' < \frac{1}{2}(1 - \frac{\sigma_z^2}{\sigma_y^2}) \),

\[
\frac{\partial c/W}{\partial (-\lambda)} < 0, \quad \frac{\partial X}{\partial (-\lambda)} > 0.
\]

when \( R < 1, \gamma < 1 \) and \( \lambda < 0 \). Given the assumption on the tax rate, a strong spirit of capitalism always reduces the consumption-wealth ratio. Since the agent’s utility is defined on both consumption and wealth accumulation, his long-run welfare rises as a result of higher wealth and possibly even higher consumption.

When \( R > 1 \), the effects of the spirit of capitalism on the consumption-wealth ratio, growth rate, asset holding share, and the welfare are ambiguous.

### 7 Conclusion

In this paper, we have extended the existing frameworks of stochastic growth and asset pricing to stochastic growth framework with non-expected utility function, the spirit of capitalism and concern for social status are discussed also. In this extended model, we have studied how stochastic shocks in production and government spending affects consumption, wealth accumulation, economic growth, and welfare. This paper has further extended the studies by Eaton, Grinols, Obstfeld, Turnovsky, and Gong and Zou, among others, to consider the impact of various taxes on the consumption-wealth ratio, growth, and welfare.

The direct effect of the spirit of capitalism on the economy has been also explicitly considered in this paper. It is shown that the existence of the spirit of capitalism can better explain the difference between the rates of return on government bonds and risky stock—the Mehra-Prescott risk-premium puzzle. In the spirit-of-capitalism or wealth-is-status model, the gap between the returns on risky assets and the risk-free asset is always larger. Furthermore, a higher spirit of capitalism or a stronger concern for social status can lead to higher output growth, more holding of risky capital, higher welfare, and a lower consumption-wealth ratio.
Consider the optimization problem:

\[
\max U(c_t, W_t)
\]

subject to

\[
\frac{dW_t}{W_t} = (\rho - (1 + \tau_c) \frac{c_t}{W_t}) dt + dw_t,
\]

where the utility function satisfies

\[
f((1 - R)u(c_t, W_t)) = \frac{1 - R}{1 - \gamma} c_t^{1-\gamma} W_t^{-\lambda} \Delta t + e^{-\beta \Delta t} f((1 - R)u(c_{t+\Delta t}, W_{t+\Delta t})),
\]

and

\[
f(x) = \frac{1 - R}{1 - \gamma} x^{1-\gamma},
\]

\[
\rho = n_B r_B + n_K (1 - \tau) r_K,
\]

\[
dw = n_B du_B + n_K (1 - \tau') du_K.
\]

From equation (A5), we have

\[
\sigma_w^2 = n_B \sigma_B^2 + n_K (1 - \tau') \sigma_K^2 + 2n_B n_K (1 - \tau') \sigma_{BK}.
\]

To solve the problem, we define the value function \( V(W, t) \), and

\[
V(W, t) = e^{-\beta t} X(W)
\]

The Lagrangian function associated with the problem is:

\[
\max \{u(c_t, W_t) - \beta f((1 - R)X(W_t))
\]

\[
+ (1 - R)f'((1 - R)X(W_t))((\rho - (1 + \tau_c) \frac{c_t}{W_t})W_t X'(W_t) + \frac{1}{2} \sigma_w^2 W_t^2 X''(W_t))
\]

\[
+ \eta(1 - n_B - n_K) \}\}.
\]

In this case, the corresponding first-order conditions for maximization are:

\[
\frac{\partial u(c, W)}{\partial c} = (1 - R)f'((1 - R)X(W))(1 + \tau_c)X'(W),
\]

\[
(r_B X'(W) W - \eta) dt + \text{cov}(dw, du_B) X''(W) W^2 = 0,
\]

\[
((1 - \tau)r_K X'(W) W - \eta) dt + \text{cov}(dw, (1 - \tau') du_K) X''(W) W^2 = 0,
\]

\[
n_B + n_K = 1.
\]
These equations determine the optimal choices of $c/W$, $n_B$, $n_K$, and $\eta$ as the functions of $X'(W)$ and $X''(W)$. Furthermore, the value function must satisfy the Bellman equation

$$u(c_t, W_t) - \beta f((1 - R)X(W_t)) + (1 - R)f'((1 - R)X(W_t))\{(\rho - (1 + \tau)c_t)\frac{c_t}{W_t}W_tX'(W_t) + \frac{1}{2}\sigma^2 w W_t^2 X''(W_t)\} = 0.$$ 

Now we have completed the proof of proposition 1.

For the specified utility function (8), the form of the value function is postulated as:

$$X(W) = \delta W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R)}, \quad (A12)$$

where $\delta$ is to be determined.

Differentiating with respect to $W$ yields

$$X'(W) = \delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - 1},$$

$$X''(W) = \delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)(\frac{1 - \gamma - \lambda}{1 - \gamma} - 1)W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - 2}.$$ 

Substituting for $c$ in the Bellman equation (A13) leads to

$$\frac{c_t}{W_t} = (1 - R)\frac{1 - \gamma}{1 - \gamma} \delta \frac{1 - \gamma}{1 - \gamma} (1 + \tau c_t)^{-1/\gamma} \left(\frac{1 - \gamma - \lambda}{1 - \gamma}\right)^{-1/\gamma}. \quad (A13)$$

So, we get

$$\delta \frac{1 - \gamma}{1 - \gamma} = \frac{\beta - \rho(1 - \gamma - \lambda) - \frac{1}{2}\sigma^2 (1 - \gamma - \lambda)(\frac{1 - \gamma - \lambda}{1 - \gamma} - (1 - R))}{\gamma(1 - R)\frac{1 - \gamma}{1 - \gamma} (1 + \tau c_t)^{-1/\gamma} \left(\frac{1 - \gamma - \lambda}{1 - \gamma}\right)^{-1/\gamma}}. \quad (A14)$$

Thus

$$\frac{c_t}{W_t} = \frac{\beta - \rho(1 - \gamma - \lambda) - \frac{1}{2}\sigma^2 (1 - \gamma - \lambda)(\frac{1 - \gamma - \lambda}{1 - \gamma} - (1 - R))}{\gamma(1 + \tau c_t)(\frac{1 - \gamma}{1 - \gamma})},$$

$$(r_B\delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - \eta}dt + \text{cov}(dw, du_B)\delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)(\frac{1 - \gamma - \lambda}{1 - \gamma} - (1 - R))W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - 1}) = 0,$$

$$((1 - \tau)\rho R \delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - \eta}dt + \text{cov}(dw, (1 - \tau')du_K)\delta(\frac{1 - \gamma - \lambda}{1 - \gamma})(1 - R)(\frac{1 - \gamma - \lambda}{1 - \gamma} - (1 - R))W^{\frac{1 - \gamma - \lambda}{1 - \gamma}(1 - R) - 1}) = 0.$$ 

Thus, we have obtained all the expressions in proposition 4:

$$\frac{c_t}{W_t} = \frac{\beta - \rho(1 - \gamma - \lambda) - \frac{1}{2}\sigma^2 (1 - \gamma - \lambda)(\frac{1 - \gamma - \lambda}{1 - \gamma} - (1 - R))}{\gamma(1 + \tau c_t)(\frac{1 - \gamma}{1 - \gamma})}, \quad (A15)$$

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\( r_B - \frac{\eta W^{\frac{1-\gamma-\lambda}{\delta(1 - \gamma - \lambda)(1 - R)}}}{\delta(1 - \gamma - \lambda)(1 - R)} dt + \text{cov}(dw, du_B)(\frac{1-\gamma-\lambda}{1-\gamma}(1 - R) - 1) = 0, \quad (A16) \)

\[ ((1 - \tau)r_K - \frac{\eta W^{\frac{1-\gamma-\lambda}{\delta(1 - \gamma - \lambda)(1 - R)}}}{\delta(1 - \gamma - \lambda)(1 - R)} ) dt + \text{cov}(dw, (1 - \tau')du_K)(\frac{1-\gamma-\lambda}{1-\gamma}(1 - R) - 1) = 0. \quad (A17) \]
References


