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Abstract

In this paper, we have studies the effects of macroeconomic policies on foreign asset accumulation in a wealth effect model used by Bardhan (1967), Kurz (1968), Calvo (1980) and Blanchard (1983). Our results differ dramatically from the ones in Obstfeld (1981). In particular, we have shown that government spending always reduces foreign asset accumulation (or increases foreign borrowing). While Obstfeld's model turned the conventional Mundell-Fleming model on its head, our wealth effect approach has restored its validity.

JEL Classification Numbers: E58, E63, F52.

0.1 I. Introduction

This paper examines the effects of macroeconomic policies on foreign asset accumulation in a small open economy. It obtains policy implications that are very different from many existing studies such as Turnovsky (1985, 1987) and, in particular, Obstfeld (1981). In an often-cited paper, Obstfeld (1981) presents three interesting results regarding the effects of government policies on foreign asset holdings: (1) foreign exchange intervention is found to have no real effects when official foreign reserves earn interest that is distributed to the public; (2) inflation leads to higher long-run consumption and foreign claims; (3) an increase in government consumption induces a surplus on current account in the short run and larger foreign asset accumulation in the long run. The intertemporal optimization framework used by Obstfeld in this study and some related studies Obstfeld (1982, 1990) has also influenced the open economy macroeconomics in the past decade.

In this paper, we are going to show that the policy implications of Obstfeld's model hinge on the special assumption of Uzawa's (1968) time preference and they are totally reversed and substantially changed in a dynamic optimization model with the wealth effect. The Becker's endogenous time preference developed in our paper is adapted from the models of Bardhan (1967), Kurz (1968), Calvo (1980), Blanchard (1983), Barro (2003), Brueckner (2000), and Solow (2003) and defines the representative agent's utility function on foreign asset in addition to consumption and real balances. The main results derived from our model stand in striking contrast to the ones in Obstfeld paper: (1) foreign exchange intervention leads to more foreign asset holdings and more consumption in the long run; (2) if the utility function is separable in consumption and real balances as in Obstfeld (1981), inflation has no effect on the real variables in both short run and long run; if the utility function is nonseparable, inflation results in more foreign asset accumulation when the cross derivative of consumption and real balances is positive; (3) government spending always reduces foreign asset accumulation and crowds out private consumption.

Our paper is organized as follows. Section II sets up a simple wealth effect model with money and discusses the stability and some policy implications of the model. Section III makes detailed comparative study on the effects of macroeconomic policies. We conclude our paper in Section IV.

0.2 II. The Model

We consider a small economy in a competitive world market. The economy is populated with many identical people. We follow Sidrauski (1967), define a representative agent's instantaneous utility as

$$u(c,m) = u(c) + v(m)$$

where c is consumption, m is real balance holdings. Suppose the agent derive the positive utility from consumption goods and money holding, but with positive, but diminshing marginal utility of consumption goods and money holding.

The representative agent's discounted utility over an infinite horizon can be written as

$$\max \int_0^\infty u(c,m)e^{-\Delta_t}dt$$

where

$$\Delta_t = \int_0^t \rho(s(v))dv \tag{(1)}$$

is the Becker's endogenous time preference, s is consumer's expenditure on decreasing the time preference, and $\rho(.) : R \to [0,1]$ satisfies

$$\rho^{'} < 0, \rho^{''} > 0 \tag{(2)}$$

Condition (2) state that with the increasing of spending on s, the time preference will decrease, but the marginal value of it is increasing.

The agent's total wealth defined as

$$a = b + m \tag{(3)}$$

and his budget constraint is

$$\frac{da}{dt} = y + rb + \chi - c - s - \pi m \tag{(4)}$$

where y is output, x is the government transfer, π is the expected inflation rate and r is the returns on the foreign bonds, which is given in the world capital market. The representative agent choose his consumption paths of a and s, holding of foreign bonds and money to maximize his discounted utility, i.e.

$$\max \int_0^\infty u(c,m)e^{-\Delta_t}dt$$

subject to the budget constraint (3) and (4), with Δ_t given by equation (1) and initial bonds holding $b(0)$ is given.

Using the fact

$$d\Delta_t = \rho(s(t))dt$$

we can transfer our model into the form

$$\max \int_0^{\infty} \frac{u(c,m)}{\rho(s(v))} e^{-\Delta_t} d\Delta$$

subject to

$$\frac{da}{d\Delta} = \frac{y + rb + \chi - c - s - \pi m}{\rho(s(v))}$$

and equation (3)

Define the Hamiltonian

$$H = \frac{u(c,m)}{\rho(s(v))} + \lambda \frac{y + rb + \chi - c - s - \pi m}{\rho(s(v))} + \mu \frac{a - b - m}{\rho(s(v))}$$

The first-order conditions are summarized as follows

$$u_c = \lambda \tag{(5)}$$

$$u_m = \lambda\pi + \mu \tag{(6)}$$

$$\lambda r = \mu \tag{(7)}$$

$$-\frac{u(c,m)}{\rho(s(v))^2}\rho' - \lambda\frac{1}{\rho} - \lambda\frac{(r-n)a + w + \chi - c - s - (\pi + r)m}{\rho(s(v))^2}\rho' - \mu\frac{a - b - m}{\rho(s(v))^2}\rho' = 0 \tag{(8)}$$

$$\frac{d\lambda}{d\Delta} = \frac{\lambda - \mu}{\rho(s(v))} \tag{(9)}$$

and transversality condition

$$\lim_{\Delta \to \infty} \lambda a e^{-\Delta} = 0$$

From equations (5), (6) and (7), we have

$$u_c = \lambda \tag{(5)}$$

$$u_m = \lambda(\pi + r) \tag{(10)}$$

Equation (5) is the formally condition which states marginal utility of consumption equals marginal value of wealth. Equations shows that the marginal utility of money holding equals real interest rate measured by marginal utility of consumption.

With the aid of $d\Delta_t = \rho(s(t))dt$, Euler equation (9) can be transferred into the form

$$\frac{d\lambda}{dt} = \lambda(\rho(s) - r) \qquad ((11))$$

and the transervesality conditions can be rewritten as

$$\lim_{\Delta\to\infty} \lambda a e^{-\Delta} = 0$$

0.3 Macroeconomic Equilibrium

In order to derive the macroeconomic equilibrium, we must consider the exchange market. Suppose the home price of the goods is p, and the corresponding world price is p^* . From purchasing power parity, we have

$$p = Ep^* \qquad ((12))$$

where E is the exchange rate. With proper normalization, p^* can be set to one.

To fully spell out the dynamics, we need to specify the government sector. Government revenue comes from money creation and interest earnings from the central bank's reserves, i.e., , and R denotes the amount of reserves. Government also consumes goods, g, makes transfer, x, to the representative agent. So its budget is given by

$$g + x = \frac{dM}{dt}/p + rR \qquad ((13))$$

or

$$g + x = \frac{dM}{dt}/M\frac{M}{p} + rR$$

Let the money growth rate be a positive constant σ

$$\frac{dM}{dt}/M = \sigma$$

Then we can write equation (13) as

$$g + x = \sigma m + rR \qquad ((14))$$

By definition, $m = M/p$, we have

$$\frac{dm}{dt} = (\frac{dM}{dt}/M - \frac{dp}{dt}/p)m \qquad ((15))$$

On the perfect foresight path, the expected inflation rate is equal to the actual inflation rate:

$$\frac{dp}{dt}/p = \frac{de}{dt}/e = \pi$$

where e is expected rate of exchange rate depreciation. Therefore

$$\frac{dm}{dt} = (\sigma - \pi)m \qquad ((16))$$

Now, the macroeconomic equilibrium of the economy is summarized by equations (3),(4), (5), (8), (10), (11), (14), (16), and the transversality condition.

0.3.1 Short-run equilibrium

From equations (3), (5), (8), (10), we can express λ, s, and π as the functions of c, b, m, r, σ, g, R, and y.

$$\lambda = \lambda(c, b, m, r) \qquad ((17))$$

$$s = s(c, b, m, r, \sigma, g, R, y) \qquad ((18))$$

$$\pi = \pi(c, b, m, r) \qquad ((19))$$

and from the appendix we have

$$\lambda_c = \frac{1}{u_{cc}} < 0, \lambda_b = 0, \lambda_r = 0, \lambda_m = 0 \qquad ((20))$$

$$\pi_c = -\frac{u''(c)(\pi + r)}{u'(c)} > 0, \pi_b = 0, \pi_r = -1, \pi_m = \frac{u_{mm}}{u'(c)} < 0 \qquad ((21))$$

$$\begin{aligned} s_c^* &= \frac{u_c m \rho' \pi_c - u_{cc}\rho}{u\rho''}, s_b^* = \frac{u_c r \rho'}{-u\rho''} > 0, s_r^* = \frac{u_c m \rho' \pi_r - u_c(b+R)\rho'}{u\rho''} > 0, \\ s_m^* &= \frac{u_c m \rho' \pi_m - u_m \rho'}{u\rho''} > 0, s_\sigma^* = \frac{u_c m \rho'}{-u\rho''} > 0, s_y^* = \frac{u_c \rho'}{-u\rho''} > 0, \\ s_R^* &= \frac{u_c r \rho'}{-u\rho''} > 0, s_g^* = \frac{u_c \rho'}{u\rho''} < 0 \end{aligned} \qquad ((22))$$

From equation (20), we know that marginal value of wealth is a decreasing function of consumption level. Equation (21) states that, with the increasing of consumption, the expected inflation rate will increase, but with the increasing of interest rate and money demand, the expected inflation rate will decrease. In order to eliminate the complication, equation (22) presents a steady-state relation.

0.3.2 Dynamics

Substitute equations (17), (18), and (19) into equations (4), (11), and (16), we get the full dynamic system of foreign bonds, consumption and real balances

$$\frac{db}{dt} = y + rb + rR - g - c - s(c, b, m, r, \sigma, g, R, y) \tag{(23)}$$
$$\frac{dc}{dt} = -\frac{u'(c)}{u''(c)}(r - \rho(s(c, b, m, r, \sigma, g, R, y))) \tag{(24)}$$
$$\frac{dm}{dt} = (\sigma - \pi(c, b, m))m \tag{(25)}$$

And the steady state satisfies

$$\begin{aligned} y + rb^* + rR - g - c^* - s(c^*, b^*, m^*, r, \sigma, g, R, y) &= 0 \\ -\frac{u'(c^*)}{u''(c^*)}(r - \rho(s(c^*, b^*, m^*, r, \sigma, g, R, y))) &= 0 \\ (\sigma - \pi(c^*, b^*, m^*))m^* &= 0 \end{aligned}$$

To understand the stability of the system, we linearize equations (23), (24), and (25) around the steady state values

$$\begin{pmatrix} \frac{db}{dt} \\ \frac{dc}{dt} \\ \frac{dm}{dt} \end{pmatrix} = \begin{pmatrix} r - s_b^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & -\pi_c m^* & -\pi_m m^* \end{pmatrix} \begin{pmatrix} b - b^* \\ c - c^* \\ m - m^* \end{pmatrix}$$

The determinant of coefficient matrix of above linear system is given by

$$\begin{aligned} \Delta &= \begin{pmatrix} r - s_b^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & -\pi_c m^* & -\pi_m m^* \end{pmatrix} \\ &= -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & 0 \\ s_b & s_c & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m[r(s_c\pi_m - s_m\pi_c) + s_b\pi_m] \\ &= -\frac{u'(c)}{u''(c)}\rho' mr \frac{u''(\pi + r)u_m\rho' + u_{mm}(u_{cc}\rho + \rho')}{-u\rho'' u'} \\ &< 0 \end{aligned}$$

which is negative. In this case, the dynamic system has one negative and two positive characteristic roots because the product of the three roots is negative and the sum of the three roots is also positive and is given by the trace of the 3x3 matrix, . Therefore, the dynamic system has a unique perfect foresight path near the steady state.

0.4 Comparative Static Solutions

We know that the steady-state value (b^*, c^*, m^*) satisfies

$$y + rb^* + rR - g - c^* - s(c^*, b^*, m^*, r, \sigma, g, R, y) = 0 \tag{(26)}$$

$$-\frac{u'(c^*)}{u''(c^*)}(r - \rho(s(c^*, b^*, m^*, r, \sigma, g, R, y))) = 0 \tag{(27)}$$

$$(\sigma - \pi(c^*, b^*, m^*))m^* = 0 \tag{(28)}$$

And from equations (17), (18), and (19), we can determine the steady-state value λ^*, s^*, and π^*. Next, we analyze the effects of exogenous variables on the steady-state value b^*, c^*, m^*, λ^*, s^*, and π^* .

Total differentiate on equations (26), (27), and (28), we get

$$\begin{pmatrix} r - s_b^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & -\pi_c m^* & -\pi_m m^* \end{pmatrix} \begin{pmatrix} db^* \\ dc^* \\ dm^* \end{pmatrix}$$
$$= \begin{pmatrix} s_\sigma^* \\ -\frac{u'(c)}{u''(c)}\rho' s_\sigma^* \\ -m^* \end{pmatrix} d\sigma + \begin{pmatrix} 1 + s_g^* \\ -\frac{u'(c)}{u''(c)}\rho' s_g^* \\ 0 \end{pmatrix} dg + \begin{pmatrix} -b - R + s_r^* \\ -\frac{u'(c)}{u''(c)}\rho' s_r^* \\ 0 \end{pmatrix} dr \tag{(30)}$$
$$+ \begin{pmatrix} -r + s_R^* \\ -\frac{u'(c)}{u''(c)}\rho' s_R^* \\ 0 \end{pmatrix} dR + \begin{pmatrix} -1 + s_y^* \\ -\frac{u'(c)}{u''(c)}\rho' s_y^* \\ 0 \end{pmatrix} dy$$

We have

Proposition 1: Inflation increases foreign asset accumulation and consumption level, but decreases domestic real balance holdings.

Proof: See the Appendix, we have

$$\frac{db}{d\sigma} = \frac{\Delta_b^\sigma}{\Delta} > 0, \frac{dc}{d\sigma} = \frac{\Delta_c^\sigma}{\Delta} > 0, \frac{dm}{d\sigma} = \frac{\Delta_m^\sigma}{\Delta} < 0$$

Proposition 2: Government spending always increases long-run foreign asset accumulation..

Proof:

$$\frac{db}{dg} = \frac{\Delta_b^g}{\Delta} > 0, \frac{dc}{dg} = \frac{\Delta_c^g}{\Delta} = 0, \frac{dm}{dg} = \frac{\Delta_m^g}{\Delta} = 0$$

Proposition 3: With the increasing of output, long-run foreign asset accumulation will decrease.

$$\frac{db}{dy} = \frac{\Delta_b^y}{\Delta} < 0, \frac{dc}{dy} = \frac{\Delta_c^y}{\Delta} = 0, \frac{dm}{dy} = \frac{\Delta_m^y}{\Delta} = 0$$

Proposition 4: The world interest rate will decrease long-run foreign asset accumulation, but increase domestic consumption and real balance holding.

$$\frac{db}{dr} = \frac{\Delta_b^r}{\Delta} < 0, \frac{dc}{dr} = \frac{\Delta_c^r}{\Delta} > 0, \frac{dm}{dr} = \frac{\Delta_m^r}{\Delta} > 0$$

Proposition 5: The central bank's purchase of foreign claims from the public with domestic currency will lead to less foreign asset accumulation.

$$\frac{db}{dR} = \frac{\Delta_b^R}{\Delta} < 0, \frac{dc}{dR} = \frac{\Delta_c^R}{\Delta} = 0, \frac{dm}{dR} = \frac{\Delta_m^R}{\Delta} = 0$$

0.5 IV. Conclusion

In this paper, we have studies the effects of macroeconomic policies on foreign asset accumulation in a wealth effect model used by Bardhan (1967), Kurz (1968), Calvo (1980), Blanchard (1983), Yin (2008), and Zhang and Xu (2011). Our results differ dramatically from the ones in Obstfeld (1981). In particular, we have shown that government spending always reduces foreign asset accumulation (or increases foreign borrowing). While Obstfeld's model turned the conventional Mundell-Fleming model on its head, our wealth effect approach has restored its validity.

Evaluating the consequences of macroeconomic policies is complicated; and the results are often very sensitive to the optimization framework we have utilized. Our wealth effect model only provides a different perspective to the problems and it should be taken as complementary to many existing models.

0.6 Appendix A Derive of Short-run Equilibrium

From equation (3), (5), (5), and (8), we get

$$u_c = \lambda \tag{(A1)}$$

$$u_m = \lambda(\pi + r) \tag{(A2)}$$

$$-\frac{u(c,m)}{\rho(s(v))^2}\rho' - \lambda\frac{1}{\rho} - \lambda\frac{y + rb + \sigma m + rR - g - c - s - \pi m}{\rho(s(v))^2}\rho' = 0 \tag{(A3)}$$

we get

$$\lambda = \lambda(c, b, m, r)$$
$$s = s(c, b, m, r, \sigma, g, R, y)$$
$$\pi = \pi(c, b, m, r)$$

and, we have

$$\lambda_c = \frac{1}{u_{cc}} > 0, \lambda_b = 0, \lambda_r = 0, \lambda_m = 0 \tag{(A4)}$$

$$\pi_c = -\frac{u''(c)(\pi + r)}{u'(c)} > 0, \pi_b = 0, \pi_r = -1, \pi_m = \frac{u_{mm}}{u'(c)} < 0 \tag{(A5)}$$

$$s_c = \frac{u_c m\rho'\pi_c - u_{cc}\rho - u_{cc}\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho'}{u\rho'' + u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_b = \frac{u_c r\rho'}{-u\rho'' - u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_r = \frac{u_c m\rho'\pi_r - u_c(b + R)\rho'}{u\rho'' + u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_m = \frac{u_c m\rho'\pi_m - u_m\rho' - u_c\{\sigma - \pi\}\rho''}{u\rho'' + u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_\sigma = \frac{u_c m\rho'}{-u\rho'' - u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_y = \frac{u_c\rho'}{-u\rho'' - u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_R = \frac{u_c r\rho'}{-u\rho'' - u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$
$$s_g = \frac{u_c\rho'}{u\rho'' + u_c\{y + rb + \sigma m + rR - g - c - s - \pi m\}\rho''}$$

Substituting the steady-state conditions (27), (28), and (29) into the above equations, we get

$$\begin{aligned} s_c^* &= \frac{u_c m\rho'\pi_c - u_{cc}\rho}{u\rho''}, s_b^* = \frac{u_c r\rho'}{-u\rho''} > 0, s_r^* = \frac{u_c m\rho'\pi_r - u_c(b + R)\rho'}{u\rho''} > 0, \\ s_m^* &= \frac{u_c m\rho'\pi_m - u_m\rho'}{u\rho''} > 0, s_\sigma^* = \frac{u_c m\rho'}{-u\rho''} > 0, s_y^* = \frac{u_c\rho'}{-u\rho''} > 0, \\ s_R^* &= \frac{u_c r\rho'}{-u\rho''} > 0, s_g^* = \frac{u_c\rho'}{u\rho''} < 0 \end{aligned} \tag{(A6)}$$

0.7 Appendix B

From Equation (30), we have

$$
\begin{aligned}
&\begin{pmatrix} r - s_b^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & -\pi_c m^* & -\pi_m m^* \end{pmatrix} \begin{pmatrix} db^* \\ dc^* \\ dm^* \end{pmatrix} \\
&= \begin{pmatrix} s_\sigma^* \\ -\frac{u'(c)}{u''(c)}\rho' s_\sigma^* \\ -m^* \end{pmatrix} d\sigma + \begin{pmatrix} 1 + s_g^* \\ -\frac{u'(c)}{u''(c)}\rho' s_g^* \\ 0 \end{pmatrix} dg + \begin{pmatrix} -b - R + s_r^* \\ -\frac{u'(c)}{u''(c)}\rho' s_r^* \\ 0 \end{pmatrix} dr \qquad ((30)) \\
&+ \begin{pmatrix} -r + s_R^* \\ -\frac{u'(c)}{u''(c)}\rho' s_R^* \\ 0 \end{pmatrix} dR + \begin{pmatrix} -1 + s_y^* \\ -\frac{u'(c)}{u''(c)}\rho' s_y^* \\ 0 \end{pmatrix} dy
\end{aligned}
$$

$$
\begin{aligned}
\Delta_b^\sigma &= \begin{pmatrix} s_\sigma & -1 - s_c & -s_m \\ -\frac{u'(c)}{u''(c)}(\rho' s_\sigma) & \frac{u'(c)}{u''(c)}\rho' s_c & \frac{u'(c)}{u''(c)}\rho' s_m \\ -m & -\pi_c m & -\pi_m m \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} 0 & -1 & 0 \\ -s_\sigma & s_c & s_m \\ 1 & \pi_c & \pi_m \end{pmatrix} \\
&= -\frac{u'(c)}{u''(c)}\rho' m[-s_m - s_\sigma \pi_m] = -\frac{u'(c)}{u''(c)}\rho' m \frac{u_m \rho'}{u\rho''} < 0
\end{aligned}
$$

$$
\begin{aligned}
\Delta_b^g &= \begin{pmatrix} 1 + s_g^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}\rho' s_g & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ 0 & -\pi_c m^* & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} 1 & -1 & 0 \\ -s_g & s_c & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix} \\
&= -\frac{u'(c)}{u''(c)}\rho' m[(s_c \pi_m - s_m \pi_c) - s_g \pi_m] = \Delta/r < 0
\end{aligned}
$$

$$
\begin{aligned}
\Delta_b^r &= \begin{pmatrix} -b - R + s_r^* & -1 - s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(\rho' s_r^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ 0 & -\pi_c m^* & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} -b - R & -1 & 0 \\ -s_r & s_c & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix} \\
&= -\frac{u'(c)}{u''(c)}\rho' m[-(b + R)(s_c \pi_m - s_m \pi_c) - s_r \pi_m] \\
&= -\frac{u'(c)}{u''(c)}\rho' m[-(b + R)(s_c \pi_m - s_m \pi_c) - \frac{u' m \rho' \pi_r \pi_m}{u\rho''} - (b + R)s_b \pi_m / r] \\
&= -\frac{u'(c)}{u''(c)}\rho' m \frac{u' m \rho' \pi_m}{u\rho''} - (b + R)\Delta/r > 0
\end{aligned}
$$

$$\Delta_b^y = \begin{pmatrix} -1+s_y^* & -1-s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(\rho' s_y^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ 0 & -\pi_c m^* & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} -1 & -1 & 0 \\ -s_y & s_c & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[-(s_c\pi_m - s_m\pi_c) - s_y\pi_m] = -\Delta/r > 0$$

$$\Delta_b^R = \begin{pmatrix} -r+s_R^* & -1-s_c^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(\rho' s_R^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ 0 & -\pi_c m^* & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} -r & -1 & 0 \\ -s_R & s_c & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[-r(s_c\pi_m - s_m\pi_c) - s_R\pi_m] = -\Delta > 0$$

$$\Delta_c^\sigma = \begin{pmatrix} r-s_b & s_\sigma & -s_m \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b) & -\frac{u'(c)}{u''(c)}\rho' s_\sigma & \frac{u'(c)}{u''(c)}\rho' s_m \\ -\pi_b m & -m & -\pi_m m \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & 0 & 0 \\ s_b & -s_\sigma & s_m \\ 0 & \pi_c & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m r(-s_\sigma\pi_m - s_m\pi_c) = -\frac{u'(c)}{u''(c)}\rho' m r\frac{u_m\rho'\pi_c}{u\rho''} < 0$$

$$\Delta_c^g = \begin{pmatrix} r-s_b^* & 1+s_g^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & -\frac{u'(c)}{u''(c)}\rho' s_g^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & 0 & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & 1 & 0 \\ s_b & -s_g & s_m \\ 0 & 0 & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[-rs_g\pi_m - s_b\pi_m] = 0$$

$$\Delta_c^r = \begin{pmatrix} r-s_b^* & -b-R+s_r^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & -\frac{u'(c)}{u''(c)}\rho' s_r^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & 0 & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -b-R & 0 \\ s_b & -s_r & s_m \\ 0 & 0 & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[-rs_r\pi_m + s_b\pi_m(b+R)] = -\frac{u'(c)}{u''(c)}\rho' m\frac{u_c m\rho'}{u\rho''} < 0$$

$$\Delta_c^y = \begin{pmatrix} r-s_b^* & -1+s_y^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & -\frac{u'(c)}{u''(c)}\rho' s_y^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & 0 & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & 0 \\ s_b & -s_y & s_m \\ 0 & 0 & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[-rs_y\pi_m + s_b\pi_m] = 0$$

$$\Delta_c^R = \begin{pmatrix} r - s_b^* & -r + s_R^* & -s_m^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & -\frac{u'(c)}{u''(c)}\rho' s_R^* & \frac{u'(c)}{u''(c)}\rho' s_m^* \\ -\pi_b m^* & 0 & -\pi_m m^* \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -r & 0 \\ s_b & -s_R & s_m \\ 0 & 0 & \pi_m \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[r(-s_R\pi_m + s_b\pi_m)] = 0$$

$$\Delta_m^\sigma = \begin{pmatrix} r - s_b & 1 - s_c & s_\sigma \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b) & \frac{u'(c)}{u''(c)}\rho' s_c & -\frac{u'(c)}{u''(c)}\rho' s_\sigma \\ -\pi_b m & -\pi_c m & -m \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} -r & 1 & 0 \\ -s_b & s_c & s_\sigma \\ 0 & \pi_c & 1 \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[r(s_c - s_\sigma\pi_c) - s_b] = -\frac{u'(c)}{u''(c)}\rho' m[\frac{r(u'' + u'\rho')}{-u\rho''}] > 0$$

$$\Delta_m^g = \begin{pmatrix} r - s_b^* & -1 - s_c^* & 1 - s_g^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & -\frac{u'(c)}{u''(c)}\rho' s_g^* \\ -\pi_b m^* & -\pi_c m^* & 0 \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & 1 \\ s_b & s_c & -s_g \\ 0 & \pi_c & 0 \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[rs_g\pi_c + s_b\pi_c] = 0$$

$$\Delta_m^r = \begin{pmatrix} r - s_b^* & -1 - s_c^* & -b - R + s_r^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & -\frac{u'(c)}{u''(c)}\rho' s_r^* \\ -\pi_b m^* & -\pi_c m^* & 0 \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & b + R \\ s_b & s_c & -s_r \\ 0 & \pi_c & 0 \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[(b + R)s_b\pi_c + rs_r\pi_c] = -\frac{u'(c)}{u''(c)}\rho' m\frac{ru_c m\rho'\pi_r\pi_c}{u\rho''} < 0$$

$$\Delta_m^y = \begin{pmatrix} r - s_b^* & -1 - s_c^* & -1 + s_y^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & -\frac{u'(c)}{u''(c)}\rho' s_y^* \\ -\pi_b m^* & -\pi_c m^* & 0 \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & -1 \\ s_b & s_c & -s_y \\ 0 & \pi_c & 0 \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[\pi_c r s_y + s_b\pi_c] = 0$$

$$\Delta_m^R = \begin{pmatrix} r - s_b^* & -1 - s_c^* & -r + s_R^* \\ -\frac{u'(c)}{u''(c)}(-\rho' s_b^*) & \frac{u'(c)}{u''(c)}\rho' s_c^* & -\frac{u'(c)}{u''(c)}\rho' s_R^* \\ -\pi_b m^* & -\pi_c m^* & 0 \end{pmatrix} = -\frac{u'(c)}{u''(c)}\rho' m \begin{pmatrix} r & -1 & -r \\ s_b & s_c & -s_R \\ 0 & \pi_c & 0 \end{pmatrix}$$

$$= -\frac{u'(c)}{u''(c)}\rho' m[rs_R\pi_c + rs_b\pi_c] = 0$$

0.8 References

Bardhan, P., Optimal Foreign Borrowing, in Essays on the Theory of Optimal Economic Growth, edited by Karl Shell, MIT Press, 1967.

Barro, R., (2003). Determinants of Economic Growth in a Panel of Countries, *Annals of Economics and Finance* 4, 231-274.

Blanchard, Olivier, Debt and Current Account Deficit in Brazil, in Financial Policies and the World Capital Market, edited by Aspe Armella, Dornbusch and Obstfeld, University of Chicago Press, 1983.

Brueckner, J. K. (2000). Fiscal Decentralization in Developing Countries: The Effects of Local Corruption and Tax Evation, *Annals of Economics and Finance* 1: 1-16.

Buiter, Willem, Time Preference and International Lending and Borrowing in an Overlapping-Generations Model,Journal of Political Economy 89, No.4, 1981, pp.769-797.

Calvo, Guillermo, Financial Opening, Crawling Peg and the Real Exchange Rate, mimeo, 1980.

Kurz, Mordecai, Optimal Economic Growth and Wealth Effects, International Economic Review 9, October 1968, pp.348-357.

Obstfeld, Maurice, Macroeconomic Policy, Exchange-Rate Dynamics, and Optimal Asset Accumulation, Journal of Political Economy 89, December 1981, pp.1142-1161.

______, Aggregate Spending and the Terms of Trade: Is There a Laursen-Metzler Effect, Quarterly Journal of Economics 97, 1982, 251-270.

______, Intertemporal Dependence, Impatience, and Dynamics. Journal of Monetary Economics 26, 1990, 45-75.

Solow, R. (2003). Reflections on Growth and Development, *Annals of Economics and Finance* 4, 219-229.

Turnovsky, Stephen, Domestic and Foreign Disturbances in an Optimizing Model of Exchange-Rate Determination, Journal of International Money and Finance 1985, pp.151-171.

____ ,Optimal Monetary Growth with Accommodation Fiscal Policies in a Small Open Economy, Journal of International Money and Finance, 1987, pp.179-193

Uzawa, Hirofumi, Time Preference, the Consumption Function, and Optimal Asset Holdings. In Value, Capital, and Growth, edited by J. N. Wolfe, Chicago: Aldine, 1968.

Yin, H., (2008). Fiscal Disparities and the Equalization Effects of Fiscal Transfers at the Country Level in China, *Annals of Economics and Finance* 9-1: 115-149.

Zhang, P., and Mann, X. (2011). The View from the Country: China's Regional Inequalities of Socio-Economic Development, *Annals of Economics and Finance* 12-1: 183-198