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11 August 2011

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FIRST DRAFT: AUGUST 11, 2011
THIS DRAFT: MARCH 23, 2012
LATEST DRAFT: https://sites.google.com/site/marioghossoub

Abstract. This paper suggests a behavioral, preference-based definition of loss aversion for decision under risk. This definition is based on the initial intuition of Markowitz [30] and Kahneman and Tversky [19] that most individuals dislike symmetric bets, and that the aversion to such bets increases with the size of the stake. A natural interpretation of this intuition leads to defining loss aversion as a particular kind of risk aversion. The notions of *weak loss aversion* and *strong loss aversion* are introduced, by analogy to the notions of weak and strong risk aversion. I then show how the proposed definitions naturally extend those of Kahneman and Tversky [19], Schmidt and Zank [48], and Zank [54]. The implications of these definitions under Cumulative Prospect Theory (PT) and Expected-Utility Theory (EUT) are examined. In particular, I show that in EUT loss aversion is not equivalent to the utility function having an S shape: loss aversion in EUT holds for a class of utility functions that includes S-shaped functions, but is strictly larger than the collection of these functions. This class also includes utility functions that are of the Friedman-Savage [14] type over both gains and losses, and utility functions such as the one postulated by Markowitz [30]. Finally, I discuss possible ways in which one can define an index of loss aversion for preferences that satisfy certain conditions. These conditions are satisfied by preferences having a PT-representation or an EUT-representation. Under PT, the proposed index is shown to coincide with Köbberling and Wakker's [22] index of loss aversion only when the probability weights for gains and losses are equal. In Appendix B, I consider some extensions of the study done in this paper, one of which is an extension to situations of decision under uncertainty with *probabilistically sophisticated* preferences, in the sense of Machina and Schmeidler [27].

Date: March 23, 2012.
Key words and phrases. Loss Aversion, Risk Aversion, Mean-Preserving Increase in Risk, Prospect Theory, Probability Weights, S-Shaped Utility.
JEL Classification: D03, D81.

The main idea behind this work originated from a conversation with Fabio Maccheroni. I thank him for that. I also thank Max Amarante, Peter Wakker, Horst Zank, and especially Marc Henry for their comments on an earlier draft of this paper. Any errors or omissions are entirely mine. Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. Parts of the first draft of this paper were written while I was visiting the department of Decision Sciences of Università Bocconi, which I thank for its hospitality.

> *"A salient characteristic of attitudes to changes in welfare is that losses loom larger than gains. The aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount [...] Indeed, most people find symmetric [50:50] bets [...] distinctively unattractive. Moreover, the aversiveness of symmetric fair bets generally increases with the size of the stake."*
>
> (Kahneman and Tversky [19])

1. Introduction

One of the pillars of what today is called *behavioral economics* is the notion of *loss aversion*, that is, the idea that individuals do not react symmetrically to losses and gains, as measured from a given reference point. In fact, the disutility experienced as a result of a given loss is felt more onerously by most people than the utility that would result form a gain of the same magnitude. There has been numerous empirical justifications of this asymmetry in evaluating economic prospects, starting from the pioneering work of Kahneman and Tversky [19, 51].

The usefulness of loss aversion, as a behavioral phenomenon, is by now well-understood. Many of the "anomalies" of Expected-Utility Theory (EUT) – such as the equity premium puzzle [6, 33], the endowment effect [21], and the status quo bias [47] – have been explained by loss aversion[1]. Nevertheless, loss aversion has always been considered as an intrinsic property of Cumulative Prospect Theory (PT) [19, 51], and was hitherto almost never systematically examined outside of PT, that is, in other models of decision making that might account for reference-dependent behavior. Indeed, much of the popularity of PT stems precisely from its providing a theoretical framework for the behavioral notion of loss aversion, and the way this was done was in terms of the curvature of the utility function (value function) in PT.

Be that as it may, a proper quantification of this behavioral notion remains problematic today. Indeed, to this day, and over 30 years after the ground-breaking work of Kahneman and Tversky, there is no uniquely agreed-upon quantification of loss aversion. The only consensus seemed to be that loss aversion manifests itself *solely* in the curvature of the value function u in PT, but the exact way in which this happens was debated and still is debatable. In fact, all of the definitions of loss aversion used in the literature[2] fall in one of the categories of Table 1.

Table 1. Definitions of Loss Aversion in the Literature

Reference	*Definition given*
Kahneman and Tversky [19, 51]	$u(x) \leqslant -u(-x)\,,\ \forall\ x>0$
Wakker and Tversky [53]	$u(x)-u(y) \leqslant u(-y)-u(-x)\,,\ \forall\ x>y\geqslant 0$
Bowman, Minehart, and Rabin [9]	$u'(y) \leqslant u'(z)\,,\ \forall\ z<0<y$
Neilson [38]	$u(y)/u \leqslant u(z)/z,\ \forall\ z<0<y$

[1] Camerer [10] surveys these and other findings.

[2] See, e.g. Barberis and Huang [4], Barberis et al. [5], Benartzi and Thaler [6], Bowman et al. [9], Kahneman and Tversky [19], Köbberling and Wakker [22], Köszegi and Rabin [23, 24, 25], Neilson [38], Thaler [50], Tversky and Kahneman [51], or Wakker and Tversky [53].

The way in which Kahneman and Tversky initially defined loss aversion is in terms of preferences: most individuals prefer the status quo to any symmetric gamble, or – equivalently – "*losses loom larger than gains*". Two questions then arise naturally here:

(1) Since Kahneman and Tversky's definition of loss aversion – which is arguably the most intuitive definition of loss aversion – is preference-based, why is it that loss aversion has nearly always been considered within PT? It seems justifiable to examine loss aversion outside of PT, in a purely preference-based fashion, i.e. behaviorally.

(2) Assuming that the analysis of loss aversion is restricted to PT, why is loss aversion seen as *only* a property of the DM's *utilitarian risk attitude*, as measured by the curvature of his S-shaped value (utility) function? It is tempting to equate loss aversion with the curvature of the utility function; but this is only a heritage of EUT, where risk attitudes are entirely captured by the curvature of the vNM utility function, and can be measured by the Arrow-Pratt [3, 43] indices of risk aversion. In PT, however, one of the constituents of choice behavior is the intrinsic probability weighting process, and it seems unreasonable that this aspect of choice behavior be left out of the picture when studying the notion of loss aversion. If loss aversion is a property of choice behavior, it should also account for the DM's *probabilistic risk attitude*, as a characteristic of the probability weights, and of the difference between how the DM weights probabilities of gains and probabilities of losses.

If one is not willing to give up on the idea that loss aversion is only a property of the utility function's curvature, one should note that it would still be possible to examine loss aversion outside of PT. Indeed, the notion of reference-dependence, or gain-loss dependent choice behavior, is by no means an exclusive property of PT[3]. It has been noted and argued for since the work of Markowitz [30] who wrote:

> "*Generally people avoid symmetric bets. This suggests that the [utility] curve falls faster to the left of the origin than it rises to the right of the origin. (I.e.,* $U(X) > |U(-X)|, X > 0$*).*"

Even though the term *loss aversion* was not explicitly used by Markowitz [30], the idea behind the predominant view that loss aversion is a property of the utility that manifests itself in the fact that the utility of a given gain is lower than the absolute value of the utility of a loss of the same magnitude was noted by Markowitz [30], about 60 years ago; so was the idea that *people dislike symmetric bets*, which was the definition of loss aversion given by Kahneman and Tversky [19]. Also, the idea of a utility function which is concave on a part of its domain and convex on another part is also not a property of PT *per se*, and it has been advocated by Friedman and Savage [14] in 1948 (although not in the S shape used in PT).

If, on the other hand, one accepts the idea that loss aversion should be viewed as not only a result of the utility function's curvature, but also as a consequence of the difference in which probabilities of gains and probabilities of losses are distorted, then it would still be possible to examine loss aversion outside of PT. As a matter of fact, the idea that individuals are predisposed to distort probabilities differently depending on the amount and/or the sign of outcomes (i.e. whether an outcome is a gain or a loss) can be traced back at least to the work of Mosteller and Nogee [34], Preston and Baratta [44], and Griffith [15]; and, as Zank [54] notes, if such an asymmetry in the weighting of probabilities was not a byproduct of loss aversion, then it would have been a mere coincidence and totally fortuitous. Consequently, any definition of loss aversion formulated strictly

[3] For recent work on reference-dependent behavior outside of PT, I refer to Apesteguia and Ballester [2], Masatlioglu and Ok [31, 32], Ok et al. [39], or Ortoleva [40], for instance.

in terms of the utility function automatically disregards the effect of the probability weighting on the general risk attitude, and hence on the observed phenomenon of loss aversion itself.

Loss Aversion via Preferences, and this Paper's Contributions. The original definition of loss aversion given by Kahneman and Tversky [19] is aversion to *symmetric 50:50 bets*. Although this can naturally be seen as a *preference-based* definition, few theoretical investigations of loss aversion in PT were carried out in terms of preferences. Moreover, few have dealt with the probability weighting process as an inherent constituent of loss aversion, and the ones that were carried out were done in a context where the objects of choice are lotteries, that is, discrete probability distributions (see, e.g. Schmidt and Zank [48] and Zank [54]). Accordingly, the definitions proposed are very specific to that particular case.

Recently, Blavatskyy [8] explored the notion of loss aversion outside of PT, and in a general framework where outcomes are not necessarily monetary, but with a finite state space and where the elements of choice are lotteries. Blavatskyy's [8] definition of loss aversion is behavioral, based on the properties of a preference over a set of lotteries. However, his definition is essentially comparative, and an "absolute" notion of loss aversion is defined as "more loss averse than a loss neutral" preference. The major complication, as the author remarks, is that it is not immediately clear how to define loss neutrality in that context.

It is the object of this paper to examine loss aversion in a purely preference-based fashion, as in Schmidt and Zank [48] and Zank [54], but in a model-free environment, i.e. in terms of preferences that do not necessarily have a PT-representation, and for objects of choice that are more general than lotteries. Some of the results of this paper can be seen as an extension of previous analyses of loss aversion carried out in terms of preferences.

The gist of this paper is a particular stance on what loss aversion is, and is arguably a very natural interpretation of the Markovitz-Kahneman-Tversky view of loss aversion, albeit in a purely behavioral, model-free manner. I take a diametrically opposite view of loss aversion to that of Köbberling and Wakker [22] who wrote:

> "*To a considerable extent, risk aversion as it is commonly observed is caused by loss aversion.*"

I argue that loss aversion is, in fact, a special case of risk aversion, when the latter is defined in terms of preferences, i.e. as aversion to mean-preserving increases in risk (strong risk aversion) and preferring the expected value of a prospect to the prospect itself (weak risk aversion). Roughly speaking, this paper defines loss aversion as nothing more than risk aversion, when restricted to a special collection of objects of choice: those that are symmetric in a sense that will be made precise below. The definition proposed here will be shown to be an extension of those of Kahneman and Tversky [19, 51], Schmidt and Zank [48], and Zank [54].

Specifically, for a preference over a collection of given acts (considered as random variables on some objectively given probability space), I define two kinds of loss aversion: *weak loss aversion* and *strong loss aversion*. The former is defined as preferring the expected value of any symmetric act to the act itself, where the symmetry of an act is defined in terms of its distribution function for the given objective probability measure (Def. 2.2 below). Hence, weak loss aversion is simply defined as weak risk aversion on the collection of all symmetric acts, and it is an extension of the intuitive definition of loss aversion given by Kahneman and Tversky as preferring the status quo 0 to symmetric 50:50 bets, since these bets have zero expectation. The latter kind of loss aversion is defined roughly as strong risk aversion (that is aversion to mean-preserving increases in risk) when restricted to the collection of all symmetric acts. Since any two symmetric acts will have equal means (zero), a preference displays strong loss aversion if – roughly – it preserves second-order stochastic dominance on the collection of symmetric acts. This proposed definition of strong loss aversion will be shown to extend the idea of Kahneman and Tversky that, when one is dealing with bets (binary lotteries), the aversion to symmetric bets increases with the size of the stake.

Moreover, I examine the implications of this proposed approach in PT and show how loss aversion is a consequence of both *tastes*, as measured by the utility function (value function), and *beliefs*, as measured by the probability weighting functions. Furthermore, I show that when the probability weights are equal, a sufficient (although not necessary) condition for loss aversion to hold is that the marginal utility of a given monetary loss is strictly greater than that of a monetary gain of the same amount, which is more or less the definition of loss aversion given by Köszegi and Rabin [23, 24, 25] and Wakker and Tversky [53], for instance.

As I byproduct of my analysis, the definition of loss aversion given in this paper is applicable to situations where the objects of choice are not necessarily lotteries, but can be more general (continuous) distributions. In practice, this is more relevant since in most applications of PT to finance and insurance, for instance, one deals with an underlying (financial or actuarial) risk which has a continuous distribution on the real line or on an interval thereof (see, e.g. Barberis and Huang [4], Bernard and Ghossoub [7], Carlier and Dana [11], He and Zhou [17], or Jin and Zhou [18]). In such circumstances, a proper definition of loss aversion does not exist as yet, to the best of my knowledge.

I also examine loss aversion in EUT, and I show that in that case loss aversion is not equivalent to the utility function having an S shape. I show that loss aversion in EUT holds for a class of utility functions that includes S-shaped functions, but which is strictly larger than the collection of these functions, for it also includes utility functions that are concave-convex of the Friedman-Savage [14] type over both gains and losses, and utility functions such as the one postulated by Markowitz [30], for instance.

Finally, under some gain-loss separability and continuity assumptions on the functional representing the DM's preferences, I propose an index of loss aversion. These assumptions are verified, *inter alia*, by functionals representing PT-preferences or EU-preferences. I then show that under PT, Köbberling and Wakker's [22] index of loss aversion coincides with my proposed index only when the probability weighting functions are identical. In other words, Köbberling and Wakker [22]'s index of loss aversion (and any other index of loss aversion defined solely in terms of the value function) overlooks the effect of the difference between the probability weights on loss aversion.

Outline. Section 2 introduces some notation and preliminary definitions. In section 3, I distinguish two notions of loss aversion: *weak loss aversion* and *strong loss aversion*, by analogy to the notions of *weak risk aversion* and *strong risk aversion*, and I propose a preference-based definition of each of these notions. I define *weak loss aversion* as aversion to symmetric acts, and, just as strong risk aversion is usually defined as aversion to *mean-preserving increases in risk* (e.g. Rothschild and Stiglitz [46]), I define *strong loss aversion* as aversion to a special kind of mean-preserving increase in risk – or, equivalently, as strong risk aversion when restricted to a particular class of symmetric acts. Section 4 examines the implications of these definitions in PT, and gives necessary and sufficient conditions for each to hold. Section 5 considers the specific case of EUT. In particular, I show that in EUT loss aversion is not equivalent to the utility function having an S shape. In section 6, I propose an index of weak loss aversion as well as an index of strong loss aversion for preferences that are *gain-loss separable* and *adequately continuous*, as defined later on.

Finally, section 7 concludes. Appendix A contains most of the proofs and some related analysis. Appendix B suggests two possible extensions of the work done in this paper: (i) the first is an extension to non-monetary outcomes, where the set of consequences is an arbitrary linearly ordered space; and (ii) the second is an extension to situations of decision under uncertainty, where the DM's preference is probabilistically sophisticated in the sense of Machina and Schmeidler [27].

2. Preliminaries

2.1. **Setup and Some Definitions.** Situations of decision under risk can be formulated as situations where (S, Σ, P) is an *objectively* given probability space, and a DM has preference $\succ$ over elements of $B(\Sigma)$, the space of all bounded, real-valued, and Σ-measurable functions on S. Henceforth, the *objective* probability measure P on (S, Σ) will be fixed and taken as given.

Let $B^+(\Sigma)$ denote the cone of nonnegative elements of $B(\Sigma)$, and let $B_s(\Sigma)$ denote the linear space of all simple, real-valued, and Σ-measurable functions on S. That is, $B_s(\Sigma)$ is the collection of finite linear combinations of indicator (characteristic) functions of sets in Σ. Let $B_s^+(\Sigma)$ denote the cone of nonnegative elements of $B_s(\Sigma)$, and, for each $C \in \Sigma$, let $\mathbf{1}_C$ denote the indicator function of C.

For each $n \in \mathbb{N}$, let $B_{s,n}(\Sigma)$ denote the subset of $B_s(\Sigma)$ consisting of those simple functions taking on n distinct values. Using the probability measure P, each collection $B_{s,n}(\Sigma)$ will be identified with the collection $\mathcal{L}_n$ of all lotteries on $\mathbb{R}$ assigning positive probability to only n distinct values. Elements of $\mathcal{L}_n$ take the following form:

$$(\alpha_1, p_1; \ldots; \alpha_n, p_n) \tag{2.1}$$

for some $\{\alpha_1, \alpha_2, \ldots, \alpha_n\} \subset \mathbb{R}$, with $\alpha_i \neq \alpha_j$ for $i \neq j$, and some $\{p_1, p_2, \ldots, p_n\} \subset [0, 1]$ such that $\sum_{i=1}^n p_i = 1$.

The set S is interpreted as the set of all states of the world, and elements of $B(\Sigma)$ are interpreted as the *acts* over which a decision maker (DM) has a (strict) preference $\succ$. Weak preference $\succcurlyeq$ and indifference $\sim$ are defined from strict preference $\succ$ in the usual manner. Finally, I will also assume that $f \sim g$ for any $f, g \in B(\Sigma)$ that are identically distributed under P.

If one denotes by 0 the constant act $g \in B(\Sigma)$ yielding 0 in each state of the world, and if one interprets 0 as the *status quo*, then one can think of elements of $B(\Sigma)$ as deviations from the *status quo*. The DM can then be seen as having a preference $\succ$ over deviations from the *satus quo*.

Finally, each $a \in \mathbb{R}$ will be identified with the constant act $h \in B(\Sigma)$ yielding a in each state of the world. Any notation of the form $a \succ g$ (resp. $g \succ a$), with $a \in \mathbb{R}$ and $g \in B(\Sigma)$, will mean $h \succ g$ (resp. $g \succ h$), where $h \in B(\Sigma)$ is the constant act yielding a in each state of the world. The same applies to weak preference $\succcurlyeq$ and to indifference $\sim$.

An element f of $B(\Sigma)$ is said to have *no mass points* if for any $x \in \mathbb{R}$, $P \circ f^{-1}(\{x\}) = 0$. In particular, f is said to have no mass point at zero when $P \circ f^{-1}(\{0\}) = P(\{s \in S : f(s) = 0\}) = 0$.

Definition 2.1. For any $f \in B(\Sigma)$ let:

(1) $G_f(t) := P(\{s \in S : f(s) \geqslant t\})$; and,
(2) $F_f(t) := P(\{s \in S : f(s) \leqslant t\})$.

Note that if $f \in B(\Sigma)$ has no mass points, then $G_f(t) = P(\{s \in S : f(s) > t\})$ and $F_f(t) = P(\{s \in S : f(s) < t\})$. In particular, if f has no mass points, then G_f is simply the *decumulative distribution function* (or *survival function*) of f for the probability measure P.

2.2. **Comparative Risk.** The literature on comparative risk ("increasing risk") from which some of the definitions appearing in Definition 2.2 are drawn is vast. I refer to Hadar and Russell [16], Müller [36], or Rothschild and Stiglitz [46], for instance.

Definition 2.2. An act $h \in B(\Sigma)$ is called *symmetric around the status quo*, or simply *symmetric*, if for all $t > 0$, one has:

$$G_f(t) = F_f(-t) \tag{2.2}$$

For any $f, g \in B(\Sigma)$ it is said that:

(1) g *dominates* f in the sense of *first-order stochastic dominance*, and one writes $g \succcurlyeq_{fsd} f$, when $F_g(t) \leqslant F_f(t)$, for all $t \in \mathbb{R}$. If, moreover, the inequality is strict for some $t_0 \in \mathbb{R}$, then g is said to *dominate* f in the sense of *strict first-order stochastic dominance*, and one writes $g \succ_{fsd} f$;

(2) g dominates f in the sense of *second-order stochastic dominance*, and one writes $g \succcurlyeq_{ssd} f$, when $\int_{-\infty}^{x} F_g(t)\ dt \leqslant \int_{-\infty}^{x} F_f(t)\ dt$, for all $x \in \mathbb{R}$. If, moreover, the inequality is strict for some $x_0 \in \mathbb{R}$, then g is said to *dominate* f in the sense of *strict second-order stochastic dominance*, and one writes $g \succ_{ssd} f$;

(3) g is a *Mean-Preserving Increase in Risk* of f if f and g have the same mean, and $f \succcurlyeq_{ssd} g$. If, moreover, $f \succ_{ssd} g$ then g will be called a *Strict Mean-Preserving Increase in Risk* of f;

(4) g is a *Mean-Preserving Spread* of f if f and g have the same mean, and there are some $t_1, t_2 \in \mathbb{R}$ with $t_1 \leqslant t_2$, such that:
   (a) $F_g - F_f$ is nondecreasing on $(-\infty, t_1)$;
   (b) $F_g - F_f$ is nonincreasing on (t_1, t_2); and,
   (c) $F_g - F_f$ is nondecreasing on $(t_2, +\infty)$.

(5) g is obtained from f by a *Single Crossing* if there exists some $t_0 \in \mathbb{R}$ such that:

$$F_g(t) \geqslant F_f(t) \ \text{ for all } t < t_0, \text{ and } F_g(t) \leqslant F_f(t) \ \text{ for all } t \geqslant t_0 \tag{2.3}$$

(6) g has a *thicker right tail* than f if $G_g(t) \geqslant G_f(t)$, for all $t > 0$;

(7) g has a *thicker left tail* than f if $F_g(-t) \geqslant F_f(-t)$, for all $t > 0$;

(8) g and f are *adapted* if $G_g(0) = G_f(0)$.

Clearly, if an act g dominates an act f in the sense of first-order stochastic dominance, then, in particular, g has a thicker right tail than f. The converse, however, needs not be true. Moreover, any two symmetric acts which have no mass point at zero are adapted. However, as soon as one has a mass point at zero this might not hold. In particular, any two symmetric acts which have no mass points are adapted. Also, any two symmetric acts have equal means, namely zero. Finally, (2.3) is usually referred to as the "*Single Crossing Condition*", and if g is a *Mean-Preserving Spread* of f then g is obtained from f by a *Single Crossing*, but the converse is not true (see Müller [35, 36]).

In what follows, $BS(\Sigma)$ will denote the collection of all elements of $B(\Sigma)$ that are symmetric:

$$BS(\Sigma) := \Big\{ f \in B(\Sigma) : G_f(t) = F_f(-t)\,,\ \forall t > 0 \Big\} \tag{2.4}$$

2.3. **Weak and Strong Risk Aversion.** The following definitions are the standard way in which risk aversion is defined in terms of preferences, rather than utility functions. A preference displays weak risk aversion if the expected value of a lottery is preferred to the lottery itself. More generally, weak risk aversion is defined as follows.

Definition 2.3. The preference $\succ$ over $B(\Sigma)$ is said to be *weakly risk averse* (resp. *weakly risk neutral*) if for any $f \in B(\Sigma)$ the following holds:

$$\int f\ dP \succ f \quad \left(resp.\ \int f\ dP \sim f \right) \tag{2.5}$$

A preference is said to display strong risk aversion if for two acts f and g that have the same mean, and f dominates g in the sense of second-order stochastic dominance, f is preferred to g. In other words,

Definition 2.4. The preference $\succ$ over $B(\Sigma)$ is said to be *strongly risk averse* (resp. *strongly risk neutral*) if for any $f, g \in B(\Sigma)$ such that g is a *Mean-Preserving Increase in Risk* of f, the following holds:

$$f \succ g \quad (resp.\ f \sim g) \tag{2.6}$$

3. Towards a Definition of Loss Aversion

In this section I give a preference-based definition of loss aversion in terms of the DM's preference $\succ$ over elements of $B(\Sigma)$. My definition is an extension of the original behavioral definition stated in Kahneman and Tversky [19] (p. 279), who wrote:

> *"A salient characteristic of attitudes to changes in welfare is that losses loom larger than gains. The aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount [...] Indeed, most people find symmetric [50:50] bets [...] distinctively unattractive. Moreover, the aversiveness of symmetric fair bets generally increases with the size of the stake."*

It seems then that there are two characteristics of loss aversion: (i) aversion to symmetric 50:50 bets; and (ii) the aversion to such bets increases with the size of the stake. I will refer to the first constituent of loss aversion as *weak loss aversion*, and I will refer to the second constituent of loss aversion as *strong loss aversion*.

3.1. **Weak loss aversion.** The definition of *weak loss aversion* as *aversion to symmetric 50:50 bets* given by Kahneman and Tversky [19] was formulated in terms of bets, i.e. elements of $\mathcal{L}_2$. Zank [54] generalized this definition to elements of $\mathcal{L}_3$. Here, I further generalize this preference-based definition of *weak loss aversion* to general acts, i.e. elements of $B(\Sigma)$. First, however, I recall the aforementioned definitions. Although the authors did not explicitly call this phenomenon *weak loss aversion*, I will use this terminology in the definitions attributed to them.

Definition 3.1 (Kahneman and Tversky [19])**.** A DM with a preference $\succ$ over bets in $\mathcal{L}_2$ is called *weakly loss averse* if, for all $x > 0$, one has:

$$(0,1) \succ (x, 0.5; -x, 0.5) \tag{3.1}$$

where $(0,1)$ denotes the constant simple act yielding the payoff 0 with certainty, that is, the act 0.

This definition is often referred to as *aversion to symmetric 50:50 bets*, and says that a loss averse individual will always prefer the *status quo* (with certainty) to any bet paying some $x > 0$ with probability 0.5 and $-x$ with the same probability. As Zank [54] notes, the requirement that the symmetric acts be 50:50 bets is not essential to the definition of (absolute) loss aversion; only symmetry is. He then proposes the following definition:

Definition 3.2 (Zank [54])**.** A DM with a preference $\succ$ over lotteries in $\mathcal{L}_3$ is called *weakly loss averse* if, for all $x > 0$ and all $p \in (0, 0.5]$, one has:

$$(0,1) \succ (x, p; 0, 1-2p; -x, p) \tag{3.2}$$

where $(0,1)$ denotes the constant simple act yielding the payoff 0 with certainty.

This definition of weak loss aversion simply drops the requirement that acts be 50:50 bets, but keeps the essential symmetry requirement: a loss averse individual will always prefer the *status quo* (with certainty) to any lottery paying some $x > 0$ with probability $0 < p \leqslant 0.5$, $-x$ with the same probability, and 0 with probability $1 - 2p$.

In order to generalize the previous definitions to preferences over elements of $B(\Sigma)$, observe first that the essential requirement is symmetry, and recall that $BS(\Sigma)$ (eq. (2.4)) denotes the collection of all symmetric elements of $B(\Sigma)$.

Definition 3.3. The DM's preference $\succ$ is called *weakly loss averse* if for all $f \in BS(\Sigma) \setminus \{0\}$ one has:

$$0 \succ f \tag{3.3}$$

The preference $\succ$ will be called *weakly loss neutral* if for any act $f \in BS(\Sigma)$ one has $0 \sim f$.

Definition 3.3 is a natural extension of Definition 3.1 and Definition 3.2. Indeed, $f \in \mathcal{L}_2$ is symmetric if and only if it has the form $(x, 0.5; -x, 0.5)$, for some $x > 0$. Similarly, $f \in \mathcal{L}_3$ is symmetric if and only if it has the form $(x, p; 0, 1-2p, 0; -x, p)$, for some $x > 0$ and some $p \in (0, 0.5]$. Recall that 0 denotes the constant act yielding zero in all states of nature, i.e. with certainty.

Clearly, if $\succ$ is weakly risk averse (Definition 2.3), then $\succ$ is weakly loss averse, since symmetric acts have zero expectation. Hence, *weak loss aversion* is just a special case of *weak risk aversion*. The two concepts coincide on the collection of all symmetric nonzero acts.

3.2. **Strong loss aversion.** I referred to the second aspect of the behavioral definition of loss aversion given by Kahneman and Tversky [19] (p. 279), namely that the aversion to symmetric fair bets increases with the stake, as *strong loss aversion*. Schmidt and Zank [48] generalized the definition given by Kahneman and Tversky [19] from bets to elements of $\mathcal{L}_3$. Here, I further generalize this preference-based definition to general acts, i.e. elements of $B(\Sigma)$, after recalingl the aforementioned definitions. Here again, although Kahneman and Tversky [19] and Schmidt and Zank [48] did not explicitly name this phenomenon *strong loss aversion*, I will use this terminology in the definitions attributed to them.

Definition 3.4 (Kahneman and Tversky [19])**.** A DM with a preference $\succ$ over bets in $\mathcal{L}_2$ is called *strongly loss averse* if, for all $x > y > 0$, one has

$$(y, 0.5; -y, 0.5) \succ (x, 0.5; -x, 0.5) \tag{3.4}$$

In Definition 3.4 above, if y were allowed to be equal to 0 then one would recover *weak loss aversion* as a sepecial case of *strong loss aversion*, since the lottery $(0, 0.5; 0, 0.5)$ is simply the *status quo* $0 = (0, 1)$.

Noting that symmetry of the bets is the essential feature of the above definition rather than their binary nature, Schmidt and Zank [48] generalized this definition to lotteries in $\mathcal{L}_3$, as stated in the definition below.

Definition 3.5 (Schmidt and Zank [48])**.** A DM with a preference $\succ$ over lotteries in $\mathcal{L}_3$ is called *strongly loss averse* if, for all $x > y > 0$ and all $p \in (0, 0.5]$, one has:

$$(y, p; 0, 1-2p; -y, p) \succ (x, p; 0, 1-2p; -x, p) \tag{3.5}$$

Letting $X := (x, p; 0, 1-2p; -x, p)$ and $Y := (y, p; 0, 1-2p; -y, p)$, where $x > y > 0$ as in Definition 3.5, then both X and Y are symmetric, and X is a *strict mean-preserving increase in risk* of Y. The essential features of these two lotteries are the following:

(1) The absolute size of the payoff y is smaller than the absolute size of the payoff x;
(2) The two lotteries are symmetric around zero;
(3) The two lotteries have the same mean, namely zero;
(4) $P\{X \geqslant 0\} = P\{Y \geqslant 0\}$;
(5) Letting $\mathcal{D} := \{-x, -y, 0, y, x\}$ be the joint domain of X and Y, the following holds:
  (i) For all $r \in \mathcal{D}$ one has $\sum\limits_{t \in \mathcal{D},\ t \leqslant r} \Big[P\{X \leqslant t\} - P\{Y \leqslant t\}\Big] \geqslant 0$;
  (ii) For $r = -x$ one has $\sum\limits_{t \in \mathcal{D},\ t \leqslant r} \Big[P\{X \leqslant t\} - P\{Y \leqslant t\}\Big] = p > 0$; and,
  (iii) $\sum\limits_{t \in \mathcal{D}} \Big[P\{X \leqslant t\} - P\{Y \leqslant t\}\Big] = 0$

It can also be easily verified that any two symmetric lotteries X and Y in $\mathcal{L}_3$ that satisfy conditions (4) and (5) above are of the form $X = (x, p; 0, 1-2p; -x, p)$ and by $Y = (y, p; 0, 1-2p; -y, p)$, with $x > y > 0$. The same applies to symmetric elements of $\mathcal{L}_2$, i.e. symmetric bets of the form

$(x, 0.5; -x, 0.5)$, with the exception that, in this case, condition (4) above is superfluous. Indeed, fix any symmetric $X, Y \in \mathcal{L}_2$. Then X and Y are of the form $X = (x, 0.5; -x, 0.5)$ and $Y = (y, 0.5; -y, 0.5)$ (and so condition (4) is automatically verified). Then, $0 < y < x$ if and only if condition (5) above holds (with $\mathcal{D} = \{-x, -y, y, x\}$ in this case).

Definition 3.6. For any $f, g \in B(\Sigma)$, it is said that g is a *Symmetric Mean-Preserving Spread in Symmetric Act* of f when:

(1) both f and g are symmetric; and,
(2) g is a *Mean-Preserving Spread* of f.

If, in addition, f and g are adapted, it is then said that g is an *Adapted and Symmetric Mean-Preserving Spread in Symmetric Act* of f.

Since any two symmetric acts have equal means, $g \in B(\Sigma)$ is a *Symmetric Mean-Preserving Spread in Symmetric Act* of $f \in B(\Sigma)$ if and only if both f and g are symmetric and there are some $t_1, t_2 \in \mathbb{R}$ with $t_1 \leqslant t_2$, such that:

(1) $F_g - F_f$ is nondecreasing on $(-\infty, t_1)$;
(2) $F_g - F_f$ is nonincreasing on (t_1, t_2); and,
(3) $F_g - F_f$ is nondecreasing on $(t_2, +\infty)$.

Definition 3.7. For any $f, g \in B(\Sigma)$, g is called a *Symmetric* (resp. *Strict Symmetric*) *Mean-Preserving Increase in Symmetric Risk* of f when:

(1) both f and g are symmetric; and,
(2) g is a *Mean-Preserving* (resp. *Strict Mean-Preserving*) *Increase in Risk* of f.

If, in addition, f and g are adapted, it is then said that g is an *Adapted and Symmetric* (or *Strict Symmetric*) *Mean-Preserving Increase in Symmetric Risk* of f.

Since any two symmetric acts have equal means, $g \in B(\Sigma)$ is a *Symmetric* (resp. *Strict Symmetric*) *Mean-Preserving Increase in Symmetric Risk* of $f \in B(\Sigma)$ if and only if both f and g are symmetric and $f \succcurlyeq_{ssd} g$ (resp. $f \succ_{ssd} g$).

The lotteries $X := (x, p; 0, 1-2p; -x, p)$ and $Y := (y, p; 0, 1-2p; -y, p)$ in Definition 3.5 are adapted, symmetric, and such that X is a Strict Mean-Preserving Increase in Risk of Y; that is, X is an *Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk* of Y. This motivate the following definition of strong loss aversion:

Definition 3.8. The DM's preference $\succ$ is called *strongly loss averse* if for all $f, g \in B(\Sigma)\backslash\{0\}$ such that g is an *Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk* of f, one has:

$$f \succ g \tag{3.6}$$

Equivalently, $\succ$ is *strongly loss averse* if for all $f, g \in BS(\Sigma)\backslash\{0\}$ that are adapted and such that $f \succ_{ssd} g$, one has $f \succ g$.

Clearly, if $\succ$ is strongly risk averse (Definition 2.4), then $\succ$ is strongly loss averse. Hence, *strong loss aversion* is just a special case of *strong risk aversion*. Moreover, a preference displays strong loss aversion if it preserves (strict) second-order stochastic dominance on the collection of all symmetric and adapted acts.

4. Loss Aversion in Cumulative Prospect Theory

4.1. **Cumulative Prospect Theory (PT).** A *PT-DM* is defined as a DM whose choice behavior is described by PT [19, 51]. PT has four major components that distinguish it from EUT, as a paradigm for decision making under risk. First, the carriers of value are *deviations* of wealth from a reference level (the *status quo*), rather than values of wealth. Second, the PT-DM reacts differently towards gains and losses, and his risk attitude is represented by an *S-shaped value function* that is concave on positive outcomes and convex on negative outcomes, exhibiting diminishing sensitivity on both domains. Third, individuals do not value random outcomes using probabilities but base their decisions on *distorted* probabilities. Fourth, the PT-DM exhibits *loss aversion*, i.e. losses "matter" more to him than gains.

Definition 4.1. The value function u is defined as follows:

$$u(x) = \begin{cases} u^+(x) & \text{if } x \geqslant 0 \\ -u^-(-x) & \text{if } x < 0 \end{cases} \tag{4.1}$$

where $u^+ : \mathbb{R}^+ \to \mathbb{R}^+$ and $u^- : \mathbb{R}^+ \to \mathbb{R}^+$are both concave, strictly increasing, continuously differentiable, bounded, and take the value zero at zero. Then the value function u is typically S-shaped.

The third component of the PT-DM's preference representation is the probability weighting. Probabilities (given by the reference probability measure P) are distorted differently if they correspond to probabilities of losses (negative deviations from the *status quo*) or gains (positive deviations from the *status quo*). They are respectively denoted by T^+ and T^- and defined as follows:

Definition 4.2. The probability distortions (or probability weighting functions) are the mappings $T^+ : [0,1] \to [0,1]$ and $T^- : [0,1] \to [0,1]$ such that:

(1) $T^+(0) = T^-(0) = 0$ and $T^+(1) = T^-(1) = 1$;

(2) T^+ and T^- are strictly increasing and differentiable.

Under PT, the DM's preference $\succ$ over elements of $B(\Sigma)$ has a representation in terms of a functional V^{PT} defined below.

Definition 4.3. For a given $f \in B(\Sigma)$, define the functional

$$\begin{aligned} V^{PT} &: B(\Sigma) \to \mathbb{R} \\ f &\mapsto V^{PT}(f) \end{aligned} \tag{4.2}$$

by

$$V^{PT}(f) := V^+\left(f^+\right) - V^-\left(f^-\right) \tag{4.3}$$

where for each $f \in B(\Sigma)$, $f^+ = max(f,0)$ is the nonnegative part of f, $f^- = (-f)^+$ is the nonpositive part of f. Moreover,

$$V^+\left(f^+\right) = \int_0^{+\infty} T^+\left(G_{u^+(f^+)}(t)\right)\, dt \quad \text{and} \quad V^-\left(f^-\right) = \int_0^{+\infty} T^-\left(G_{u^-(f^-)}(t)\right)\, dt,$$

and u^+ and u^- are given in Definition 4.1, and T^+ and T^- are given in Definition 4.2.

4.2. **Weak and Strong loss aversion for PT preferences.**

Proposition 4.4. *Let $\succ$ be a DM's preference over acts, i.e. elements of $B(\Sigma)$. Suppose that $\succ$ has a representation in terms of a PT functional V^{PT}. Then the DM is weakly loss averse if and only if*

$$0 < \ \Omega_{\succ}(f) := \frac{\int_0^{+\infty} T^+\left(G_f(t)\right)\ du^+(t)}{\int_0^{+\infty} T^-\left(G_f(t)\right)\ du^-(t)} < 1,\ \ \forall f \in BS(\Sigma)\backslash\{0\} \tag{4.4}$$

Moreover, the DM is strongly loss averse if and only if for any $f, g \in B(\Sigma)\backslash\{0\}$ such that g is an Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk of f, the following holds:

(1) $\int_0^{+\infty} T^-\left(G_f(t)\right)\ du^-(t) < \int_0^{+\infty} T^-\left(G_g(t)\right)\ du^-(t)$ *whenever* $\int_0^{+\infty} T^+\left(G_f(t)\right)\ du^+(t) = \int_0^{+\infty} T^+\left(G_g(t)\right)\ du^+(t)$*;*

(2) $\int_0^{+\infty} T^+\left(G_g(t)\right)\ du^+(t) < \int_0^{+\infty} T^+\left(G_f(t)\right)\ du^+(t)$ *whenever* $\int_0^{+\infty} T^-\left(G_f(t)\right)\ du^-(t) = \int_0^{+\infty} T^-\left(G_g(t)\right)\ du^-(t)$*; and,*

(3) $\Omega_{\succ}(g, f) := \frac{\int_0^{+\infty}\left[T^+(G_g(t)) - T^+\left(G_f(t)\right)\right]du^+(t)}{\int_0^{+\infty}\left[T^-(G_g(t)) - T^-\left(G_f(t)\right)\right]du^-(t)} < 1$ *otherwise.*

Proposition 4.4, the proof of which is given in Appendix A, shows that both weak and strong loss aversion are a consequence of both *tastes* (as measured by u^+ and u^-) and *beliefs* (as measured by the probability weighting functions T^+ and T^-). In particular,

(i) If u^+ and u^- are identical, having the exact same curvature and shape, then weak loss aversion might persist due to the effect of the probability weights T^+ and T^-;

(ii) If the probability weights T^+ and T^- are identical, then weak loss aversion might persist due to the effect of the functions u^+ and u^-;

(iii) Finally, if $T^+ = T^-$ and $u^+ = u^-$, then $\Omega_{\succ}(f) = 1$, for any $f \in BS(\Sigma)\backslash\{0\}$, and so $\succ$ is a weakly loss neutral preference.

Furthermore, if $\succ$ is a preference over $B(\Sigma)$ having a representation in terms of a PT functional such that $T^+ = T^-$, then a sufficient (although not necessary) condition for weak loss aversion to hold (i.e. for (4.4) to be verified) is that the marginal utility of a given monetary loss is strictly greater than that of a monetary gain of the same amount, that is, for any $t \geqslant 0$,

$$\left(u^-\right)'(t) > \left(u^+\right)'(t) \tag{4.5}$$

This is more or less the definition of loss aversion given by Köszegi and Rabin [23, 24, 25] and Wakker and Tversky [53], for instance[4]. Section 5.2 gives examples of utility functions that satisfy (4.5). Such utility functions include, *inter alia*, those postulated by Markowitz [30].

[4]Note that a definition of loss aversion of the form $u'(x) < u'(-x)$, $\forall\, x > 0$, can be obtained from the definition of Wakker and Tversky [53], appearing in Table 1, by taking limits.

5. Loss Aversion in Expected-Utility Theory (EUT) with a Status Quo

Suppose that a DM has a preference $\succ$ over elements of $B(\Sigma)$, with the zero vector 0 interpreted as the *status quo*, admitting a representation in terms of an EU-functional. That is, I assume that there exist some increasing, bounded and differentiable utility function $u : \mathbb{R} \to \mathbb{R}$ such that $u(0) = 0$ (so that $u \circ 0 = 0$, u is nonnegative over $\mathbb{R}^+$ and nonpositive over $\mathbb{R}^-$), and for any two acts $f, g \in B(\Sigma)$ one has:

$$f \succ g \iff V(f) > V(g) \tag{5.1}$$

where $V(h) := \int u \circ h \, dP$, for all $h \in B(\Sigma)$, so that $V(0) = 0$.

Proposition 5.1. *For each $h \in B(\Sigma)$, $V(h)$ can be written as:*

$$V(h) = \int_0^{+\infty} P\Big(\{h \geqslant t\}\Big) \, du(t) + \int_0^{+\infty} P\Big(\{h \leqslant -t\}\Big) \, du(-t) \tag{5.2}$$

Proposition 5.1, the proof of which is given in Appendix A, simply rewrites $V(h)$ as a PT-functional. This section will show that, even in this setting, loss aversion is not equivalent to the utility function u having an S shape.

5.1. Weak and Strong loss aversion in EUT.

The proof of the following proposition is omitted since it is immediate.

Proposition 5.2. *In this setting, a necessary and sufficient condition for weak loss aversion is given by*

$$\begin{aligned}&\int_0^{+\infty} P\Big(\{f \geqslant t\}\Big) \, u'(t) \, dt \\ &\qquad - \int_0^{+\infty} P\Big(\{f \geqslant t\}\Big) \, u'(-t) \, dt < 0, \quad \forall f \in BS(\Sigma) \setminus \{0\}\end{aligned} \tag{5.3}$$

A necessary and sufficient condition for strong loss aversion is given by:

$$\begin{aligned}&\int_0^{+\infty} \Big[P\Big(\{g \geqslant t\}\Big) - P\Big(\{f \geqslant t\}\Big)\Big] \, u'(t) \, dt \\ &\qquad - \int_0^{+\infty} \Big[P\Big(\{g \geqslant t\}\Big) - P\Big(\{f \geqslant t\}\Big)\Big] \, u'(-t) \, dt \; < \; 0\end{aligned} \tag{5.4}$$

for all $g, f \in BS(\Sigma) \setminus \{0\}$ such that g is an Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk of f.

5.2. Weak and Strong loss aversion without S-Shaped Utilities.

Equations (5.3) and (5.4) imply that a sufficient (although not necessary) condition for both weak and strong loss aversion to hold when preferences have an EU representation is that the utility function $u : \mathbb{R} \to \mathbb{R}$ satisfies the condition

$$u'(-t) > u'(t), \; \forall t \geqslant 0 \tag{5.5}$$

This is verified by all utility functions of the form

$$u(x) = \begin{cases} v(x) & \text{if } x \geqslant 0 \\ -\lambda v(-x) & \text{if } x < 0 \end{cases} \tag{5.6}$$

for some $\lambda > 1$ and some nondecreasing function $v : \mathbb{R}^+ \to \mathbb{R}^+$ with $v(0) = 0$. This class of utility functions includes the usual S-shaped value function of PT but is strictly larger than the collection of these functions, for it contains all functions of the form (5.6) for even a convex function v, for instance (Example 5.3), or a function v of the Friedman-Savage type [14] (Example 5.4), or even a utility function u such as the one postulated by Markowitz [30] (Example 5.5). In sum, both weak and strong loss aversion might hold even for utility functions which are not S-shaped.

Example 5.3 (Reversed S-shaped utility)**.** Consider the utility function u_1 defined by

$$u_1(x) = \begin{cases} v_1(x) & \text{if } x \geqslant 0 \\ -\lambda v_1(-x) & \text{if } x < 0 \end{cases} \tag{5.7}$$

where $\lambda > 1$ and $v_1 : \mathbb{R}^+ \to \mathbb{R}^+$ is *convex* and such that $v_1(0) = 0$. Then u_1 has a *reversed S shape* as illustrated in Figure 1 below, and satisfies equation 5.5. Hence weak loss aversion holds in PT and both weak and strong loss aversion hold in EUT with the utility function u_1.

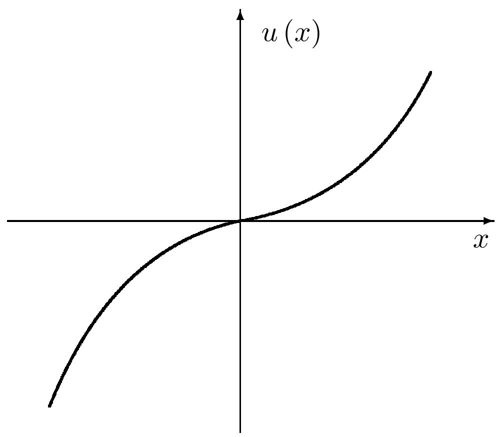

FIGURE 1. An example of a utility function u of the form given by (5.6) and having a reversed S-shape. Here, I have taken $\lambda = 1.25$.

Example 5.4 (Friedman-Savage utility)**.** As an attempt to explain the propensity of economic agents to engage in gambling and insurance purchasing simultaneously, all the while avoiding departures from the expected-utility paradigm, Friedman and Savage [14] hypothesized that the utility function ought to have the shape given in Figure 2.

Consider the utility function u_2 defined by

$$u_2(x) = \begin{cases} v_2(x) & \text{if } x \geqslant 0 \\ -\lambda v_2(-x) & \text{if } x < 0 \end{cases} \tag{5.8}$$

where $\lambda > 1$ and $v_2 : \mathbb{R}^+ \to \mathbb{R}^+$ is of the Friedman-Savage type, normalized so that $v(0) = 0$ as in Figure 2. Then u_2, illustrated in Figure 3 below, satisfies equation 5.5 and hence implies that weak loss aversion holds in PT and both weak and strong loss aversion hold in EUT.

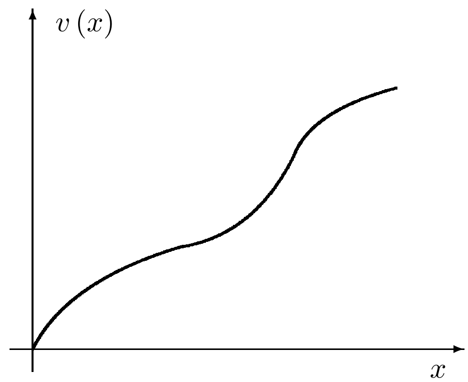

Figure 2. An example of a utility function v of the Friedman-Savage type.

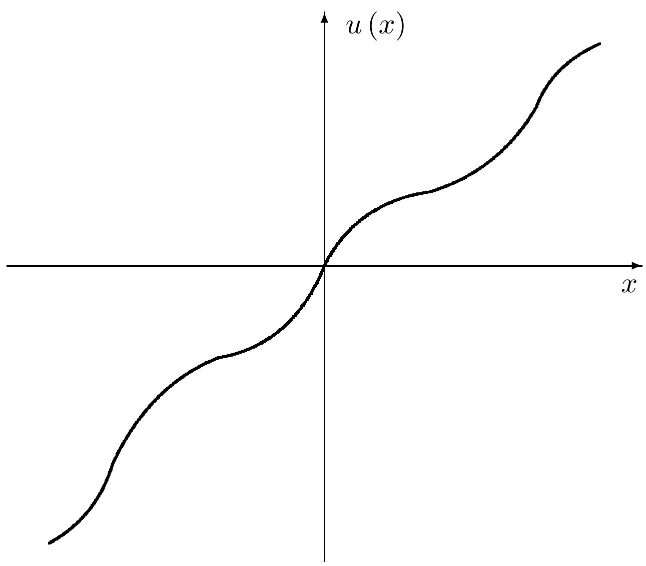

Figure 3. An example of a utility function u of the form given by (5.6), where the function v is of the Friedman-Savage type and with $\lambda = 1.25$.

Example 5.5 (Markowitz utility)**.** Markowitz [30, pp. 152-153] gave an example illustrating how a Friedman-Savage utility function might contradict the commonly observed facts that (i) individuals of moderate wealth will usually not accept actuarially fair gambles involving a possibility of a relatively large loss; and, (ii) individuals of either small or rather large wealth do typically engage in gambling activities (purchase of lottery tickets or participation in stock markets).

He then proposed a utility function that not only avoids such complications but also recognizes the fact that people typically value losses and gains differently. This is a utility function over both positive and negative levels of wealth, having three inflection points: the first one on the negative domain, the second one at the origin (or *status quo*), and the third one on the positive domain. The function is first concave, then convex, then concave, and finally convex, and is such that $|u(-x)| > u(x)$, for all $x > 0$.

The utility function given in Figure 4 is of the Markowitz type, with the additional assumption that there is some $\lambda > 1$ such that $|u(-x)| = \lambda u(x)$, for all $x > 0$. That is, for each $x \neq y > 0$, we have $|u(-x)|/u(x) = |u(-y)|/u(y) = \lambda > 1$. This is then a utility of the form given by (5.6).

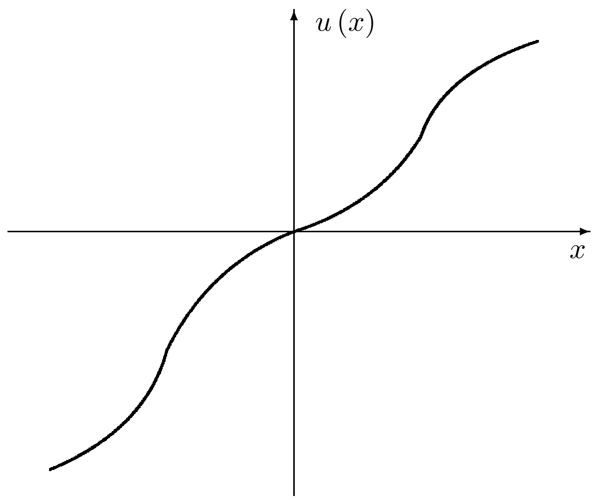

FIGURE 4. An example of a utility function u of the form given by (5.6) and of the type postulated by Markowitz [30]. Here, I have taken $\lambda = 1.25$.

6. Towards an Index of Loss Aversion

In the previous section I defined the notions of *weak loss aversion* and *strong loss aversion* for a DM with preference $\succ$ over $B(\Sigma)$. Now, suppose that the DM's preference $\succ$ admits a representation in terms of a functional $\Psi : B(\Sigma) \to \mathbb{R}$. That is, for all $f, g \in B(\Sigma)$,

$$f \succ g \iff \Psi(f) > \Psi(g) \tag{6.1}$$

In this section I propose an index of both weak and strong loss aversion for $\succ$, under some conditions on the functional Ψ. Recall that $B(\Sigma)$ is a Banach space when equipped with the supnorm $\|.\|_{sup}$ defined by $\|f\|_{sup} := \sup\{|f(s)| : s \in S\} < +\infty$, for each $f \in B(\Sigma)$ (e.g. [13, IV.2.12]).

6.1. **An Index of Weak loss aversion.**

Since the functional Ψ represents the DM's preference $\succ$, it follows that a necessary and sufficient condition for $\succ$ to be weakly loss averse is that

$$\Psi(f) < \Psi(0), \ \forall f \in BS(\Sigma) \backslash \{0\} \tag{6.2}$$

Definition 6.1. The functional Ψ (and, by extension, the binary relation $\succ$) is said to be *gain-loss separable* if there are mappings $\Psi^+ : B^+(\Sigma) \to \mathbb{R}^+$ and $\Psi^- : B^+(\Sigma) \to \mathbb{R}^+$ such that:

1. for each $f \in B(\Sigma)$, $\Psi(f) = \Psi^+(f^+) - \Psi^-(f^-)$, where f^+ and f^- are respectively the positive and negative parts of f;
2. $\Psi^+(0) = \Psi^-(0) = 0$, and for each $h \in B^+(\Sigma) \backslash \{0\}$, $\Psi^+(h) > 0$ and $\Psi^-(h) > 0$.

For instance, preferences having a representation in terms of a PT-functional and those having an EU-representation are gain-loss separable (Proposition A.5).

Proposition 6.2. *If $\succ$ is gain-loss separable then a necessary and sufficient condition for $\succ$ to be weakly loss averse is that*

$$0 < \frac{\Psi^+ (f^+)}{\Psi^- (f^-)} < 1, \ \forall f \in BS (\Sigma) \backslash \{0\} \tag{6.3}$$

In light of Proposition 6.2 (the proof of which is immediate and will be skipped), an index of weak loss aversion can be defined to be inversely proportional to the shortest distance between 0 and the ratio $\Psi^+ (f^+) / \Psi^- (f^-)$, for $f \in BS (\Sigma) \backslash \{0\}$. For such an index to be meaningful, some continuity properties of the functional Ψ must be imposed.

Definition 6.3. If $\succ$ is gain-loss separable, I will say that $\succ$ is *first-kind adequate* when the map Φ defined below is supnorm-continuous, where:

$$\begin{aligned} \Phi : BS (\Sigma) \backslash \{0\} &\to \mathbb{R}^+ \backslash \{0\} \\ f \mapsto \Phi (f) &:= \Psi^+ \left(f^+\right) / \Psi^- \left(f^-\right) \end{aligned} \tag{6.4}$$

For instance, preferences having a representation in terms of a PT-functional and those having an EU-representation are first-kind adequate (Proposition A.6).

Definition 6.4. Let $\mathcal{K}$ denote the collection of all supnorm-compact subsets of $BS (\Sigma) \backslash \{0\}$. If $\succ$ is first-kind adequate and weakly loss averse, then for any $K \in \mathcal{K}$, I define the *Index of Weak loss aversion of $\succ$ over K*, denoted by $ALA^K_{\succ}$, as follows:

$$ALA^K_{\succ} := 1 \Big/ \inf \ \Big\{\Phi (f) : f \in K\Big\} \tag{6.5}$$

Note that if $\succ$ is first-kind adequate and weakly loss averse, then for any $f \in BS (\Sigma) \backslash \{0\}$, $\Phi (f) \in (0, 1)$, and so $ALA^K_{\succ} \in (0, 1)$, for each $K \in \mathcal{K}$.

This definition of an index of weak loss aversion for weakly loss averse preferences induces a natural definition of comparative weak loss aversion as follows:

Definition 6.5. Let $\succ_1, \succ_2$ be two first-kind adequate and weakly loss averse preferences, and fix $K \in \mathcal{K}$. I say that $\succ_1$ is more weakly loss averse than $\succ_2$ over K if $ALA^K_{\succ_1} > ALA^K_{\succ_2}$. Similarly, I say that $\succ_1$ is at least as weakly loss -averse as $\succ_2$ over K if $ALA^K_{\succ_1} \geqslant ALA^K_{\succ_2}$.

If, for each $K \in \mathcal{K}$, $ALA^K_{\succ_1} > ALA^K_{\succ_2}$ (resp. $ALA^K_{\succ_1} \geqslant ALA^K_{\succ_2}$), I say that $\succ_1$ is more weakly loss averse than $\succ_2$ (resp. at least as weakly loss averse as $\succ_2$).

6.2. **An Index of Strong loss aversion.** Defining an index of strong loss aversion is more complicated. I suggest an approach here, although it does not seem fully satisfactory to me.

Since the functional Ψ represents the DM's preference $\succ$, it follows that a necessary and sufficient condition for $\succ$ to be strongly loss averse is that for any $f, g \in BS (\Sigma) \backslash \{0\}$ such that g is an *Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk* of f (that is, for any $f, g \in BS (\Sigma) \backslash \{0\}$ such that f and g are adapted and $f \succ_{ssd} g$), one has:

$$\Psi (f) > \Psi (g) \tag{6.6}$$

Proposition 6.6. *If $\succ$ is gain-loss separable then a necessary and sufficient condition for $\succ$ to be strongly loss averse is that for any $f, g \in BS(\Sigma)\backslash\{0\}$ such that g is an Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk of f, one has:*

(1) $\Psi^{+}(f^{+}) > \Psi^{+}(g^{+})$, *whenever* $\Psi^{-}(f^{-}) = \Psi^{-}(g^{-})$;

(2) $\Psi^{-}(f^{-}) < \Psi^{-}(g^{-})$, *whenever* $\Psi^{+}(f^{+}) = \Psi^{+}(g^{+})$; *and,*

(3) $0 < \Big[\Psi^{+}(g^{+}) - \Psi^{+}(f^{+})\Big] \Big/ \Big[\Psi^{-}(g^{-}) - \Psi^{-}(f^{-})\Big] < 1$, *otherwise.*

Proof. Immediate. □

Define H as the collection of all ordered pairs (g, f) of acts in $BS(\Sigma)\backslash\{0\} \times BS(\Sigma)\backslash\{0\}$ that satisfy the following properties:

(1) g is an Adapted and Strict Symmetric Mean-Preserving Increase in Symmetric Risk of f;

(2) $\Psi^{+}(f^{+}) \neq \Psi^{+}(g^{+})$; and

(3) $\Psi^{-}(f^{-}) \neq \Psi^{-}(g^{-})$.

Let $\mathcal{H}$ denote the collection of all subsets of H that can be written as a Cartesian product of supnorm-compact subsets of $BS(\Sigma)\backslash\{0\}$. That is,

$$\mathcal{H} := \Big\{K_1 \times K_2 \subseteq H : K_1, K_2 \in \mathcal{K}\Big\} \tag{6.7}$$

Definition 6.7. If $\succ$ is gain-loss separable, I will say that $\succ$ is *second-kind adequate* when for each $K_1 \times K_2 \in \mathcal{H}$, the map Υ_{K_1,K_2} defined below is supnorm-continuous in each variable, where:

$$\begin{aligned} &\Upsilon_{K_1,K_2} : K_1 \times K_2 \to \mathbb{R}\backslash\{0\} \\ &(g, f) \mapsto \Upsilon(g, f) := \Big[\Psi^{+}\left(g^{+}\right) - \Psi^{+}\left(f^{+}\right)\Big] \Big/ \Big[\Psi^{-}\left(g^{-}\right) - \Psi^{-}\left(f^{-}\right)\Big] \end{aligned} \tag{6.8}$$

For instance, preferences having a representation in terms of a PT-functional and those having an EU-representation are second-kind adequate (Proposition A.7).

Definition 6.8. If $\succ$ is strongly loss averse and second-kind adequate, then for each $K_1 \times K_2 \in \mathcal{H}$, I define the *Index of Strong loss aversion of $\succ$ over* $K_1 \times K_2$, denoted by $RLA_{\succ}^{K_1 \times K_2}$, as follows:

$$RLA_{\succ}^{K_1 \times K_2} := 1 \Bigg/ \left[\inf_{g \in K_1} \inf_{f \in K_2} \Upsilon(g, f)\right] \tag{6.9}$$

This definition of an index of strong loss aversion for strongly loss averse preferences induces a natural definition of comparative strong loss aversion as follows:

Definition 6.9. Let $\succ_1, \succ_2$ be two second-kind adequate and strongly loss averse preferences, and fix $K_1 \times K_2 \in \mathcal{H}$. I say that $\succ_1$ is more strongly loss averse than $\succ_2$ over $K_1 \times K_2$ if $RLA_{\succ_1}^{K_1 \times K_2} > RLA_{\succ_2}^{K_1 \times K_2}$. Similarly, I say that $\succ_1$ is at least as strongly loss averse as $\succ_2$ over $K_1 \times K_2$ if $RLA_{\succ_1}^{K_1 \times K_2} \geqslant RLA_{\succ_2}^{K_1 \times K_2}$.

If, for each $K_1 \times K_2 \in \mathcal{H}$, $RLA_{\succ_1}^{K_1 \times K_2} > RLA_{\succ_2}^{K_1 \times K_2}$ (resp. $RLA_{\succ_1}^{K_1 \times K_2} \geqslant RLA_{\succ_2}^{K_1 \times K_2}$), I say that $\succ_1$ is more strongly loss averse than $\succ_2$ (resp. at least as strongly loss averse as $\succ_2$).

6.3. **An Index of Weak loss aversion for PT Preferences.** Let $\mathcal{K}$ denote the collection of all supnorm-compact subsets of $BS(\Sigma)\backslash\{0\}$, as above. Then by the first-kind adequateness of $\succ$, and as an immediate adaptation of Definition 6.4, one can define an index of weak loss aversion for PT preferences over each $K \in \mathcal{K}$ as follows:

Definition 6.10. If $\succ$ is a PT preference which is weakly loss averse, then for each $K \in \mathcal{K}$, the *Index of Weak loss aversion of* $\succ$ *over* K, denoted by $ALA_{\succ}^{K}$, will be defined as follows:

$$ALA_{\succ}^{K} := 1 \Big/ \inf\left\{\Omega_{\succ}(f) : f \in K\right\} \tag{6.10}$$

where the function $\Omega_{\succ}(.)$ is as defined in equation (4.4).

Example 6.11. Köbberling and Wakker [22]'s index of loss aversion is defined as

$$LA_{kw} := \frac{\lim\limits_{x \to 0^-} (u^-)'(x)}{\lim\limits_{x \to 0^+} (u^+)'(x)} \tag{6.11}$$

In the usual parameterization of PT, the value function u is a piecewise-power value function of the form:

$$u(x) = \begin{cases} u^+(x) & \text{if } x \geqslant 0 \\ -u^-(-x) & \text{if } x < 0 \end{cases} \tag{6.12}$$

where $u^+(x) = x^{\alpha}$, for some $\alpha \in (0,1)$, and $u^-(x) = \lambda x^{\beta}$, for some $\beta \in (0,1)$ and some $\lambda > 1$. Moreover, it is usually assumed that $\alpha = \beta$, and λ is often referred to as the "coefficient of loss aversion". One can easily verify that, in this case, $LA_{kw} = \lambda > 1$.

Moreover, if $\succ$ is a weakly loss averse preference (in the sense of this paper) over $B(\Sigma)$ that admits a representation in terms of a PT functional with the previous parameterization (piecewise-power value function with equal coefficients), then for any symmetric nonzero act f, one has:

$$\begin{aligned} \Omega_{\succ}(f) &= \frac{\int_0^{+\infty} T^+(G_f(t))\ du^+(t)}{\int_0^{+\infty} T^-(G_f(t))\ du^-(t)} = \left(\frac{1}{\lambda}\right)\left(\frac{\int_0^{+\infty} T^+(G_f(t))\ \phi(t)\ dt}{\int_0^{+\infty} T^-(G_f(t))\ \phi(t)\ dt}\right) \\ &= \left(\frac{1}{LA_{kw}}\right)\left(\frac{\int_0^{+\infty} T^+(G_f(t))\ \phi(t)\ dt}{\int_0^{+\infty} T^-(G_f(t))\ \phi(t)\ dt}\right) \in (0,1) \end{aligned} \tag{6.13}$$

where $\phi(t) = t^{\alpha-1} = t^{\beta-1}$. Consequently, for each $K \in \mathcal{K}$, one has:

$$ALA_{\succ}^{K} = \frac{1}{\inf_{f \in K}\ \Omega_{\succ}(f)} = \frac{LA_{kw}}{\inf_{f \in K}\ \frac{\int_0^{+\infty} T^+\left(G_f(t)\right)\ \phi(t)\ dt}{\int_0^{+\infty} T^-\left(G_f(t)\right)\ \phi(t)\ dt}} > 1 \tag{6.14}$$

It is clear from equation (6.14) that when the probability weighting functions T^+ and T^- are identical, then for each $K \in \mathcal{K}$, one has $ALA_{\succ}^{K} = LA_{kw}$. In other words, Köbberling and Wakker [22]'s index of loss aversion (and any other index of loss aversion defined solely in terms of the value function) overlooks the effect of the difference between the probability weights on loss aversion.

6.4. **An Index of Strong loss aversion for PT Preferences.** Let $\mathcal{H}$ be defined as in (6.7). Then by the second-kind adequateness of $\succ$, and as an immediate adaptation of Definition 6.8, one can define an index of strong loss aversion for PT preferences over each $K_1 \times K_2 \in \mathcal{H}$ as follows:

Definition 6.12. If $\succ$ is a PT preferences which is strongly loss averse, then for each $K_1 \times K_2 \in \mathcal{H}$, I define the *Index of Strong loss aversion of* $\succ$ *over* $K_1 \times K_2$, denoted by $RLA_{\succ}^{K_1 \times K_2}$, as follows:

$$RLA_{\succ}^{K_1 \times K_2} := 1 \Big/ \left[\inf_{g \in K_1} \inf_{f \in K_2} \Omega_{\succ}(g, f) \right] \tag{6.15}$$

where the function $\Omega_{\succ}(.,.)$ is defined as in Proposition 4.4.

Example 6.13. If $\succ$ is a strongly loss averse preference over $B(\Sigma)$ that admits a representation in terms of a PT functional with a piecewise-power value function with equal coefficients, then for any pair $(g, f) \in K$, one has:

$$\Omega_{\succ}(g, f) = \left(\frac{1}{LA_{kw}} \right) \left(\frac{\int_0^{+\infty} \left[T^+ (G_g(t)) - T^+ (G_f(t)) \right] \ \phi(t) \ dt}{\int_0^{+\infty} \left[T^- (G_g(t)) - T^- (G_f(t)) \right] \ \phi(t) \ dt} \right) \tag{6.16}$$

where $\phi(t) = t^{\alpha - 1} = t^{\beta - 1}$. Consequently, for each $K_1 \times K_2 \in \mathcal{H}$, one has:

$$RLA_{\succ}^{K_1 \times K_2} = \frac{LA_{kw}}{\inf_{g \in K_1} \ \inf_{f \in K_2} \ \left(\frac{\int_0^{+\infty} \left[T^+(G_g(t)) - T^+\left(G_f(t)\right) \right] \ \phi(t) \ dt}{\int_0^{+\infty} \left[T^-(G_g(t)) - T^-\left(G_f(t)\right) \right] \ \phi(t) \ dt} \right)} > 1 \tag{6.17}$$

Equation (6.17) shows that when the probability weighting functions T^+ and T^- are identical, then for each $K_1 \times K_2 \in \mathcal{H}$, $RLA_{\succ}^{K_1 \times K_2} = LA_{kw}$. In other words, just as I mentioned above, Köbberling and Wakker [22]'s index of loss aversion (and any other index of loss aversion defined solely in terms of the value function) overlooks the effect of the difference between the probability weights on loss aversion.

7. Conclusion and Some Open Questions

Based on the initial intuitive definition of loss aversion advocated by Kahneman and Tversky [19] (and noted earlier by Markowitz [30]), I gave a purely preference-based definition of *weak loss aversion* (aversion to symmetric acts) and *strong loss aversion* (aversion to adapted and strict symmetric mean-preserving increases in symmetric acts). Weak loss aversion is a particular kind of weak risk aversion, and strong loss aversion is particular kind of strong risk aversion.

I then examined the implications of these definitions under Cumulative Prospect Theory (PT), and gave a necessary and sufficient condition for each of weak loss aversion and strong loss aversion to hold. My analysis of loss aversion under PT also generalizes that of Schmidt and Zank [48] and Zank [54], and shows the importance of the probability weighting functions in the determination of loss aversion, both absolute and relative. I also examined both weak and strong loss aversion under Expected-Utility Theory (EUT), and showed that under EUT a sufficient (although not necessary) condition on the utility for both weak and strong aversion to hold is that the marginal utility of a given monetary loss is strictly greater than that of a monetary gain of the same amount, which is the definition of loss aversion usually used in the literature. I showed that, although an S-shaped utility function which is steeper for losses than for gains implies that both weak and strong loss aversion hold under EUT, the class of utility functions for which this is true is strictly larger than the collection of those S-shaped utilities.

Finally, under some gain-loss separability and continuity assumptions on the functional representing the DM's preferences, I proposed an index for both weak and strong loss aversion. These assumptions are verified, *inter alia*, by functionals representing PT-preferences or EUT-preferences. I then show that under PT, Köbberling and Wakker's [22] index of loss aversion coincides with my proposed index only when the probability weighting functions are identical. In other words, Köbberling and Wakker [22]'s index of loss aversion (and any other index of loss aversion defined solely in terms of the value function) overlooks the effect of the difference between the probability weights on loss aversion.

The study carried out in Section 6 naturally suggests some questions that are left for future research. For instance, (i) can an index of loss aversion be defined so as to measure the effects of both *weak* and *strong loss aversion* simultaneously? (ii) Is it possible to define an index of loss aversion over the collection of all elements of choice, rather than over a specific compact subset thereof? (iii) How can one define an index of loss aversion for *gain-loss separable* preferences without imposing the additional *first-* and *second-kind adequateness* conditions?

Appendix A. Proofs and Related Analysis

A.1. **Capacities and the Choquet Integral.**

Definition A.1. A *capacity* on (S, Σ) is a set function $\nu : \Sigma \to [0,1]$ such that

(1) $\nu(\varnothing) = 0$;

(2) $\nu(S) = 1$; and,

(3) ν is monotone: for any $A, B \in \Sigma$, $A \subseteq B \Rightarrow \nu(A) \leqslant \nu(B)$.

An example of a capacity on a measurable space (S, Σ) is a set function $\nu := T \circ P$, where P is a probability measure on (S, Σ) and $T : [0,1] \to [0,1]$ is increasing with $T(0) = 0$ and $T(1) = 1$.

Definition A.2. For a given capacity ν and a given $\psi \in B^+(\Sigma)$, the *Choquet integral* $\oint \psi \, d\nu$ of ψ with respect to ν is defined by

$$\oint \psi \, d\nu := \int_0^{+\infty} \nu(\{s \in S : \psi(s) \geqslant t\}) \, dt \tag{A.1}$$

Remark A.3. For any capacity ν on (S, Σ) and for any $\psi \in B^+(\Sigma)$, the following holds (see, e.g. Marinacci and Montrucchio [29, Proposition 4.8]):

$$\oint \psi \, d\nu := \int_0^{+\infty} \nu(\{s \in S : \psi(s) \geqslant t\}) \, dt = \int_0^{+\infty} \nu(\{s \in S : \psi(s) > t\}) \, dt \tag{A.2}$$

A.2. **More on the PT-functional.** The PT-functional V^{PT} given in equation (4.3) can be rewritten in several forms, as the following proposition shows. First, however, note that for each $f \in B(\Sigma)$ one can rewrite $V^{PT}(f)$ as a difference of two Choquet integrals. Namely,

$$V^{PT}(f) := V^+\left(f^+\right) - V^-\left(f^-\right),$$

where:

(1) $f^+ = max\,(f, 0)$ and $f^- = (-f)^+$;
(2) $V^+\,(f^+) = \oint u \circ f^+ \; dT^+ \circ P$;
(3) $V^-\,(f^-) = \oint u \circ f^- \; dT^- \circ P$.

Proposition A.4. *For a given $f \in B\,(\Sigma)$, for u^+ and u^- as in Definition 4.1, and for T^+ and T^- as in Definition 4.2, the following quantities are all equal:*

$$\int_0^{+\infty} T^+\,(G_f\,(t))\;du^+\,(t) + \int_{-\infty}^{0} T^-\,(F_f\,(t))\;du^-\,(-t) \tag{A.3}$$

and

$$V^{PT}\,(f) := \int_0^{+\infty} T^+\left(G_{u^+(f^+)}\,(t)\right)\;dt - \int_0^{+\infty} T^-\left(G_{u^-(f^-)}\,(t)\right)\;dt \tag{A.4}$$

and

$$\int_0^{+\infty} T^+\,(G_f\,(t))\;du^+\,(t) - \int_0^{+\infty} T^-\,(F_f\,(-t))\;du^-\,(t) \tag{A.5}$$

and

$$\int_0^{+\infty} T^+\left(G_f\left(\left(u^+\right)^{-1}(t)\right)\right)\;dt - \int_0^{+\infty} T^-\left(F_f\left(-\left(u^-\right)^{-1}(t)\right)\right)\;dt \tag{A.6}$$

Proof. See Bernard and Ghossoub [7] (pp. 300-301) for the fact the first three representations are equivalent. Wakker [52] also gives some similar characterizations.

Equation (A.6) follows immediately either form equation (A.5) by a simple change of variable, or from equation (A.4). In fact, since u^+ and u^- are strictly increasing, so are $\left(u^+\right)^{-1}$ and $\left(u^-\right)^{-1}$. Moreover, $\left(u^+\right)^{-1}(0) = \left(u^-\right)^{-1}(0) = 0$. Therefore, for each $f \in B\,(\Sigma)$,

$$\begin{aligned}\int_0^{+\infty} T^+\left(G_{u^+(f^+)}\,(t)\right)\;dt &= \int_0^{+\infty} T^+\left(G_f\left(\left(u^+\right)^{-1}(t)\right)\right)\;dt \\ &= \int_0^{+\infty} T^+\,(G_f\,(t))\;du^+\,(t)\end{aligned}$$

and,

$$\begin{aligned}\int_0^{+\infty} T^-\left(G_{u^-(f^-)}\,(t)\right)\;dt &= \int_0^{+\infty} T^-\left(F_f\left(-\left(u^-\right)^{-1}(t)\right)\right)\;dt \\ &= \int_0^{+\infty} T^-\,(F_f\,(-t))\;du^-\,(t)\end{aligned}$$

□

Proposition A.5. *Define the mappings $\Psi^+ : B^+\,(\Sigma) \to \mathbb{R}^+$ and $\Psi^- : B^+\,(\Sigma) \to \mathbb{R}^+$ by:*

$$\Psi^+\left(f^+\right) = \int_0^{+\infty} T^+\left(G_{f^+}\left(\left(u^+\right)^{-1}(t)\right)\right)\;dt$$

and,

$$\Psi^-\left(f^-\right) = \int_0^{+\infty} T^-\left(G_{f^-}\left(\left(u^-\right)^{-1}(t)\right)\right)\;dt$$

for each $f \in B(\Sigma)$. *Then,* $\Psi(0) = \Psi^+(0) = \Psi^-(0) = 0$. *Moreover, for each* $h \in B^+(\Sigma)\backslash\{0\}$, $\Psi^+(h) > 0$ *and* $\Psi^-(h) > 0$.

Proof. From equation (A.6) one can write, for each $f \in B(\Sigma)$,

$$V^{PT}(f) = \Psi^+\left(f^+\right) - \Psi^-\left(f^-\right)$$

Now, for any $a \geqslant 0$, writing a for the element of $B^+(\Sigma)$ yielding the constant a for each $s \in S$, one has:

$$\begin{aligned}\Psi^+(a) &:= \int_0^{+\infty} T^+\left(P\left[u^+(a) \geqslant t\right]\right)\, dt = \int_0^{+\infty} T^+(1)\,\mathbf{1}_{u^+(a)\geqslant t}\, dt \\ &= \int_0^{u^+(a)} 1\, dt = u^+(a)\end{aligned}$$

Similarly, $\Psi^-(a) := \int_0^{+\infty} T^-(P[u^-(a) \geqslant t])\, dt = u^-(a)$. But since $u^+(0) = u^-(0) = 0$, it follows that $\Psi(0) = \Psi^+(0) = \Psi^-(0) = 0$.

Finally, for each $h \in B^+(\Sigma)\backslash\{0\}$, $\Psi^+(h) > 0$ and $\Psi^-(h) > 0$. This is an immediate consequence of the fact that the Lebesgue and Riemann integrals of a Riemann-integrable function coincide [1, Theorem 11.32] and the fact that any bounded and continuous a.e. (for Lebesgue measure) function is Riemann-integrable [1, Theorem 11.30]. Indeed, since distribution functions have at most a countable number of discontinuities, since the Lebesgue measure of any countable set is zero, and since both T^+ and T^- are bounded and continuous (being differentiable), it follows that Ψ^+ and Ψ^- coincide with the corresponding Lebesgue integrals. The rest follows from standard properties of Lebesgue integrals [1, Theorem 11.16]. □

Proposition A.6. *Both PT-preferences and EU-preferences are first-kind adequate.*

Proof. It suffices to show that this is true for PT-preferences since EU-preferences are a special case thereof (Proposition 5.1). Now, as a functional on $B(\Sigma)$, the Choquet integral (with respect to some given capacity) is supnorm-continuous being Lipschitz continuous [29, Proposition 4.11]. Moreover, for any $f \in BS(\Sigma)$, $F_f(-t) = G_f(t)$ for each $t > 0$. The rest then follows trivially since, when defined, the ratio of two continuous real-valued functions is continuous [1, Corollary 2.29]. □

Proposition A.7. *Both PT-preferences and EU-preferences are second-kind adequate.*

Proof. Similar to the proof of Proposition A.6. □

A.3. **Proof of Proposition 4.4.** Immediate consequence of (A.5) and of the fact that for any $f \in BS(\Sigma)$, $F_f(-t) = G_f(t)$ for each $t > 0$. □

A.4. **Proof of Proposition 5.1.** Fix an arbitrary $h \in B(\Sigma)$, and let $\eta := u \circ h$. Denote by η^+ (resp. η^-) the positive (resp. negactive) part of η. Then by definition of the Lebesgue integral,

$$V(h) = \int_S u \circ h\, dP = \int_S \eta^+\, dP - \int_S \eta^-\, dP \tag{A.7}$$

Moreover,

$$\int_S \eta^+ \, dP = \int_0^{+\infty} P\Big(\{\eta^+ \geqslant t\}\Big) \, dt \tag{A.8}$$

and

$$\int_S \eta^- \, dP = \int_0^{+\infty} P\Big(\{\eta^- \geqslant t\}\Big) \, dt \tag{A.9}$$

Now, denote by h^+ (resp. h^-) the positive (resp. negative) part of h and define the increasing and differentiable functions $u^+, u^- : \mathbb{R}^+ \to \mathbb{R}^+$ by:

$$u(x) = \begin{cases} u^+(x) & \text{if } x \geqslant 0 \\ -u^-(-x) & \text{if } x \leqslant 0 \end{cases} \tag{A.10}$$

Then $u(0) = u^+(0) = u^-(0) = 0$, and $u(x) \geqslant 0$ if and only if $x \geqslant 0$. Consequently, one can easily verify that[5]:

(1) $\eta^+ = u^+ \circ h^+$; and,
(2) $\eta^- = u^- \circ h^-$.

Therefore,

$$V(h) = \int_0^{+\infty} P\Big(\{u^+ \circ h^+ \geqslant t\}\Big) \, dt - \int_0^{+\infty} P\Big(\{u^- \circ h^- \geqslant t\}\Big) \, dt \tag{A.11}$$

$V(h)$ given by equation (A.11) is a PT functional over (S, Σ, P), when both probability weightings are simply the identity function on $[0,1]$, and when u^+ and u^- are defined from u as was done above. Consequently, by Proposition A.4, one has:

$$\begin{aligned} V(h) &= \int_0^{+\infty} P\Big(\{u^+ \circ h^+ \geqslant t\}\Big) \, dt - \int_0^{+\infty} P\Big(\{u^- \circ h^- \geqslant t\}\Big) \, dt \\ &= \int_0^{+\infty} P\Big(\{h \geqslant t\}\Big) \, du^+(t) + \int_{-\infty}^{0} P\Big(\{h \leqslant t\}\Big) \, du^-(-t) \\ &= \int_0^{+\infty} P\Big(\{h \geqslant t\}\Big) \, du^+(t) - \int_0^{+\infty} P\Big(\{h \leqslant -t\}\Big) \, du^-(t) \end{aligned} \tag{A.12}$$

Furthermore, $du^+(t) = du(t)$ and $du^-(t) = -du(-t)$. Therefore,

$$V(h) = \int_0^{+\infty} P\Big(\{h \geqslant t\}\Big) \, du(t) + \int_0^{+\infty} P\Big(\{h \leqslant -t\}\Big) \, du(-t) \tag{A.13}$$

□

[5] η is positive if and only $\eta = \eta^+$. But η^+ equals the composition of u^+ with some function g_1. Moreover, $\eta = u \circ h$ and $u(x)$ is positive if and only x is positive. Therefore, η is positive if and only if h is positive. Thus, $g_1 = h^+$. Similarly, η is negative if and only if $\eta = -\eta^-$. But η^- equals the composition of u^- with some function g_2. Moreover, $\eta = u \circ h$ and $u(x)$ is negative if and only x is negative. Hence, $\eta = u \circ h$ is negative if and only h is negative, which happens when $h = -h^-$. Therefore, $-\eta^- = -u^-(g_2) = -u^-\left(-(-h^-)\right) = -u^-\left(h^-\right)$, so that $g_2 = h^-$.

Appendix B. Some Extensions

B.1. **Extension to Non-Monetary Outcomes.**

B.1.1. *Setup.* The previous analysis can be extended to preference over acts with non-monetary outcomes. Suppose, for instance, that S is an arbitrary nonempty set interpreted as the set of states of the world, and Σ is a σ-algebra of subsets of S, called events. As previously, we suppose that there exists an *objectively* given probability measure P on (S, Σ). Let X be a vector space of consequences (or prizes) over the field $\mathbb{R}$, and assume that X is linearly ordered by a total vector order $\succeq_X$. In particular, this assumes that the vector order is compatible with the linear structure. That is, for all $x, y, z \in X$, and for all $\alpha \in \mathbb{R}$, $x \succeq_X y$ if and only if $\alpha.x + z \succeq_X \alpha.y + z$. Endow X with the order topology[6], and let 0_X denote the zero vector in X. Denote by C_X^+ the positive cone of X induced by $\succeq_X$, and let $C_X^- := X \backslash C_X^+$. Elements of C_X^+ are interpreted as the *nonnegative* vectors and elements of C_X^- are interpreted as the *nonpositive* vectors.

Let x_0 be an arbitrary but fixed vector in X. One can then embed $\mathbb{R}$ into X by identifying the real number α with the vector $\alpha.x_0 \in X$. By a slight abuse of notation, I will write $\alpha \in X$ in lieu of $\alpha.x_0 \in X$, where $\alpha \in \mathbb{R}$. Hence, I identify $\mathbb{R}$ with the subset $\{\alpha.x_0 : \alpha \in \mathbb{R}\}$ of X. Since $\succeq_X$ is compatible with the linear structure of X, it then follows that for any $\alpha, \beta \in \mathbb{R}$, $\alpha \geqslant \beta$ if and only if $\alpha \succeq_X \beta$. In particular, for all $r \in \mathbb{R}^+$, one has $r \succeq_X 0_X$. I will also say that a vector z in X is $\succeq_X$-bounded if there are some $z^*, z_* \in X$ such that $z^* \succeq_X z \succeq_X z_*$. In particular, $y \in C_X^+$ is $\succeq_X$-bounded if there is some $y^* \in X$ such that $y^* \succeq_X y$.

Let $\mathcal{B}_X$ denote the Borel σ-algebra on X, that is, the σ-algebra generated by the order topology of X. Let $B(\Sigma, \mathcal{B}_X)$ denote the collection of all $\Sigma/\mathcal{B}_X$-measurable mappings of S into X. Elements of $B(\Sigma, \mathcal{B}_X)$ are interpreted as the acts over which a DM has preference $\succ$. I will say that $f \in B(\Sigma, \mathcal{B}_X)$ is *nonnegative* if $f(S) \subseteq C_X^+$, and I will denote by $B^+(\Sigma, \mathcal{B}_X)$ the collection of all nonnegative elements of $B(\Sigma, \mathcal{B}_X)$. Denoting by $B^-(\Sigma, \mathcal{B}_X)$ the set $B(\Sigma, \mathcal{B}_X) \backslash B^+(\Sigma, \mathcal{B}_X)$, and denoting by $\overrightarrow{0_X}$ the constant act yielding the zero vector 0_X in each state of the world, one can interpret $\overrightarrow{0_X}$ as the *status quo* and elements of $B^+(\Sigma, \mathcal{B}_X)$ (resp. $B^-(\Sigma, \mathcal{B}_X)$) as *gains* (resp. *losses*). Therefore, acts in $B(\Sigma, \mathcal{B}_X)$ can be seen as *deviations from the status quo.*

B.1.2. *Defining Weak loss aversion.* In this situation, each act $f \in B(\Sigma, \mathcal{B}_X)$ will induce a Borel probability measure $P \circ f^{-1}$ on $(X, \mathcal{B}_X)$. One can then define *symmetry* of an act f as follows:

Definition B.1. An act $f \in B(\Sigma, \mathcal{B}_X)$ is called *symmetric around the status quo*, or simply *symmetric*, if for all $a \in C_X^+ \backslash \{0_X\}$, one has:

$$G_f(a) := P\Big(\{s \in S : f(s) \succeq_X a\}\Big) = P\Big(\{s \in S : -a \succeq_X f(s)\}\Big) := F_f(-a) \tag{B.1}$$

Denote by $BS(\Sigma, \mathcal{B}_X)$ the collection of all symmetric acts, that is,

$$BS(\Sigma, \mathcal{B}_X) := \Big\{f \in B(\Sigma, \mathcal{B}_X) : G_f(a) = F_f(-a),\ \forall a \in C_X^+ \backslash \{0_X\}\Big\} \tag{B.2}$$

Then one can then define weak loss aversion as aversion to symmetric acts:

[6] On any totally ordered vector space, the order topology is the natural generalization of the usual topology on $\mathbb{R}$. A subbase for this topology consists of those sets of the form $\{x \in X : x \succeq_X a\}$ or $\{x \in X : a \succeq_X x\}$, for some $a \in X$ [20, p. 58]. The order topology is the finest locally convex topology $\mathcal{T}$ for which every order-bounded set is $\mathcal{T}$-bounded (see also [37] or [41]).

Definition B.2. The DM's preference $\succ$ over acts in $B(\Sigma, \mathcal{B}_X)$ is called *weakly loss averse* if for all $f \in BS(\Sigma, \mathcal{B}_X) \backslash \{\overrightarrow{0_X}\}$ one has:

$$\overrightarrow{0_X} \succ f \tag{B.3}$$

The preference $\succ$ will be called *weakly loss neutral* if for any act $f \in BS(\Sigma, \mathcal{B}_X)$ one has $\overrightarrow{0_X} \sim f$.

B.1.3. *Defining Strong loss aversion.* One can immediately generalize the notion of first-order stochastic dominance as follows:

Definition B.3. For any $f, g \in B(\Sigma, \mathcal{B}_X)$, I will say that g dominates f in the sense of *first-order stochastic dominance*, and I write $g \succcurlyeq_{fsd} f$, when

$$P\Big(\{s \in S : g(s) \succeq_X a\}\Big) \geqslant P\Big(\{s \in S : f(s) \succeq_X a\}\Big), \ \forall a \in X \tag{B.4}$$

By extension, for any $f, g \in B(\Sigma, \mathcal{B}_X)$, I will say that $P \circ g^{-1}$ dominates $P \circ f^{-1}$ in the sense of *first-order stochastic dominance*, and I write $P \circ g^{-1} \succcurlyeq_{fsd} P \circ f^{-1}$, when $g \succcurlyeq_{fsd} f$.

However, in order to define strong loss aversion one needs to generalize the definition of an adapted and symmetric mean-preserving increase in symmetric risk to the case where outcomes are non-monetary. This poses some deep and serious mathematical complications. For instance, to define the notion of *equal means* one requires a proper definition of integration of X-valued functions on S, bearing in mind that X is *not* a Banach space, but merely an ordered linear space, and so the Bochner integral [12], the Pettis integral [49], and other related integrals are not possible candidates. Alternatively, one can use some notion of an integral of a function with values in an ordered space with some appropriately defined linear structure, such as the ones in Kundu and Lahiri [26], Phillips [42], or Roth [45], for instance. This will not be pursued here and will be left for future research.

B.2. **Loss Aversion under Probabilistic Sophistication.** In this paper I studied the notion of loss aversion for decision under risk, or equivalently, when there is a given *objective* probability measure on the state space. A natural question to ask here is whether one can extend this study to a situation of *decision under uncertainty*, i.e. where no objective probability measure is given *a priori*, but where beliefs are instead entirely determined form preferences over the elements of choice (the acts). As a first step towards such a general approach to loss aversion, I examine in this subsection a methodology for defining loss aversion when preferences are *probabilistically sophisticated* [27, 28], as defined below.

B.2.1. *Setup and Definitions.* The setting here is similar to that of Section 2, with the exception that there is no *objective* probability measure on the state space. Namely, S is a collection of *states*, Σ is a σ-algebra of *events* on S, and a DM has preference $\succ$ over elements of $B(\Sigma)$, the linear space of all bounded, real-valued, and Σ-measurable functions on S.

Definition B.4. If μ is any probability measure on (S, Σ), then for any $f, g \in B(\Sigma)$, then g is said to *dominate* f in the sense of *μ-first-order stochastic dominance*, and one writes $g \succcurlyeq^{\mu}_{fsd} f$, when $\mu\Big(\{s \in S : g(s) \leqslant t\}\Big) \leqslant \mu\Big(\{s \in S : f(s) \leqslant t\}\Big)$, for all $t \in \mathbb{R}$.

If, moreover, $\mu\Big(\{s \in S : g(s) \leqslant x\}\Big) < \mu\Big(\{s \in S : f(s) \leqslant x\}\Big)$, for some $x \in \mathbb{R}$, it will be said that g *dominates* f in the sense of *strict μ-first-order stochastic dominance*, and this will be written as $g \succ^{\mu}_{fsd} f$.

By extension, for any $f, g \in B(\Sigma)$ and for any probability measure μ on (S, Σ), it will be said that $\mu \circ g^{-1}$ dominates $\mu \circ f^{-1}$ in the sense of *first-order stochastic dominance* (resp. *strict first-order stochastic dominance*), and written $\mu \circ g^{-1} \succcurlyeq_{fsd} \mu \circ f^{-1}$ (resp. $\mu \circ g^{-1} \succ_{fsd} \mu \circ f^{-1}$), when $g \succcurlyeq^{\mu}_{fsd} f$ (resp. $g \succ^{\mu}_{fsd} f$).

Definition B.5. If If μ is any probability measure on (S, Σ), a functional $\Psi : B(\Sigma) \to \mathbb{R}$ is said to be *monotone with respect to μ-first-order stochastic dominance* (resp. *monotone with respect to strict μ-first-order stochastic dominance*), or *preserves μ-first-order stochastic dominance* (resp. *preserves strict μ-first-order stochastic dominance*), if for any $f, g \in B(\Sigma)$

$$g \succcurlyeq^{\mu}_{fsd} f \ \Big(\text{resp. } g \succ^{\mu}_{fsd} f\Big) \ \Rightarrow \ \Psi(g) \geqslant \Psi(f) \ \Big(\text{resp. } \Psi(g) > \Psi(f)\Big) \tag{B.5}$$

Similarly, it is said that Ψ is *anti-monotone with respect to μ-first-order stochastic dominance* (resp. *anti-monotone with respect to strict μ-first-order stochastic dominance*), if for any $f, g \in B(\Sigma)$

$$g \succcurlyeq^{\mu}_{fsd} f \ \Big(\text{resp. } g \succ^{\mu}_{fsd} f\Big) \ \Rightarrow \ \Psi(g) \leqslant \Psi(f) \ \Big(\text{resp. } \Psi(g) < \Psi(f)\Big) \tag{B.6}$$

Definition B.6. A DM with preference $\succ$ over elements of $B(\Sigma)$ is said to be *probabilistically sophisticated* if there exists a (subjective) probability measure μ on (S, Σ) and a map $\Psi_\mu : B(\Sigma) \to \mathbb{R}$, such that:

(1) for all $f, g \in B(\Sigma)$:

$$f \succ g \iff \Psi_\mu(f) > \Psi_\mu(g) \tag{B.7}$$

(2) $\Psi_\mu : B(\Sigma) \to \mathbb{R}$ is monotone with respect to strict μ-first-order stochastic dominance.

In this case, $\succ$ is said to be *probabilistically sophisticated with respect to μ.*

As an immediate consequence of this definition, and of the fact that the functional Ψ_μ hence obtained preserves strict μ-first-order stochastic dominance, one has the following lemma:

Lemma B.7. *Suppose that the DM has preference $\succ$ over elements of $B(\Sigma)$. Define the equivalence relation $\sim$, representing indifference, from the strict preference $\succ$ in the usual manner. If the DM is probabilistically sophisticated with respect to a probability measure μ on (S, Σ), then for all $f, g \in B(\Sigma)$*

$$\mu \circ f^{-1}\Big((-\infty, t]\Big) = \mu \circ g^{-1}\Big((-\infty, t]\Big), \ \forall t \in \mathbb{R} \ \Rightarrow \ f \sim g \tag{B.8}$$

Now, define a *comparative likelihood* relation over Σ, from the preference $\succ$ over $B(\Sigma)$, as follows:

Definition B.8. Let $A, B \in \Sigma$. The event A will be said to be more likely than B, written as $A \succ^l B$, if for $x > y$ in $\mathbb{R}$,

$$\begin{bmatrix} x & \text{if } s \in A \\ y & \text{if } s \notin A \end{bmatrix} \succ \begin{bmatrix} x & \text{if } s \in B \\ y & \text{if } s \notin B \end{bmatrix} \tag{B.9}$$

The following result is a consequence of Definition B.6:

Proposition B.9. *Suppose that the DM has preference $\succ$ over elements of $B(\Sigma)$, and let $\succ^l$ be defined from $\succ$ as in Definition B.8. If the DM is probabilistically sophisticated with respect to a probability measure μ on (S, Σ), then for all $A, B \in \Sigma$,*

$$A \succ^l B \iff \mu(A) > \mu(B) \tag{B.10}$$

B.2.2. *Defining Weak loss aversion.* Proposition B.9 suggests a definition of *weak loss aversion* in terms of the comparative likelihood relation $\succ^l$ on Σ defined above. First, however, we define *symmetry* in terms of $\succ^l$.

Definition B.10. Let $f \in B(\Sigma)$ be a given act, and for each $t > 0$ define the events $A_{f,t}, B_{f,t} \in \Sigma$ by

$$A_{f,t} := f^{-1}([t, +\infty)) = \{s \in S : f(s) \geqslant t\} \tag{B.11}$$

and

$$B_{f,t} := f^{-1}((-\infty, -t]) = \{s \in S : f(s) \leqslant -t\} \tag{B.12}$$

We then say that the act f is *symmetric* if for any $t > 0$,

$$A_{f,t} \sim^l B_{f,t} \tag{B.13}$$

where the relation $\sim^l$ is defined from the *comparative likelihood* relation $\succ^l$ over Σ in the usual manner.

A natural way to define *weak loss aversion* in this setting is then the following:

Definition B.11. The DM's preference $\succ$ over acts in $B(\Sigma)$ is called *weakly loss averse* if for any act $f \in B(\Sigma)$ which is symmetric in the sense of Definition B.10, one has

$$0 \succ f \tag{B.14}$$

where 0 denotes the constant act $g \in B(\Sigma)$ yielding 0 in each state of the world. The preference $\succ$ will be called *weakly loss neutral* if for any act $f \in B(\Sigma)$ which is symmetric in the sense of Definition B.10, one has $0 \sim f$.

B.2.3. *Defining Strong loss aversion.* Definition 3.8 characterizes *strong loss aversion* as aversion to a specific kind of mean-preserving increase in risk. However, the definition of a mean-preserving increase in risk is in terms of objective distribution functions of the acts involved, that is, when there is a given *objective* probability measure on the state space. While it seems relatively straightforward to define the notions of single crossings, mean-preserving spreads, adaptedness, and symmetry in terms of preferences (i.e. purely behaviorally), it is not clear to me at this point how to define the notion of second-order stochastic dominance without a reference *objective* probability measure. Of course, one way to deal with this complication is to assume that the preference over acts is probabilistically sophisticated with respect to a (subjective) probability measure μ and then define second-order stochastic dominance as *μ-second-order stochastic dominance*, but this will not be a *preference-based* definition *per se*.

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