Empirical analysis of the forecast error impact of classical and bayesian beta adjustment techniques

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Abstract

The paper presents a comparative study of conventional beta adjustment techniques and suggests an improved Bayesian model for beta forecasting. The seminal papers of Blume (1971) and Levy (1971) suggested that for both single security and portfolio there was a tendency for relatively high and low beta coefficients to over predict and under predict, respectively, the corresponding betas for the subsequent time period. We utilize this proven fact to give a Bayesian adjustment technique under a bilinear loss function where the problem of overestimation and underestimation of future betas is rectified to an extent so as to give us improved beta forecasts. The accuracy and efficiency of our methodology with respect to existing procedures is shown by computing the mean square forecast error.

1. Introduction

Systematic risk, as measured by beta, is one of the most fundamental and essential aspect of investment analysis. The method of estimation for the risk measure as predictors of future betas has been under study for the past three decades. The numerous research articles on the subject focus on the refinement of the measure in order to have an optimum prediction. Harrington (1983) highlights the usage of betas to show relative importance of good beta forecasts.

The purpose of the paper is to test empirically the classical and Bayesian beta adjustment techniques for individual securities and portfolios. Different estimation and prediction period lengths are used in the study to evaluate the impact of beta adjustment techniques and portfolio size on the beta forecast error and its components. Here the 'estimation period' is the time period used to compute ex post betas for the estimation of ex ante betas and the 'prediction period' is the time period used to compute realized or predicted betas for comparison with the estimated betas.

Pioneer works of Blume (1971) and Levy (1971) suggest a regular pattern in the betas termed as the regression tendency towards the grand mean of all betas that is one. Further they observed that for both single security and portfolio betas the relatively high and low beta coefficients tend to over predict and under predict, respectively, the corresponding betas for the subsequent time periods.

Vasicek (1973) proposed a Bayesian technique which gave a different dimension to beta estimation. This approach utilizes prior information in the form of prior distribution together with sample information to provide with an optimal predictive posterior estimate, with respect to minimization due to misestimation, under quadratic loss function (Squared error loss function : SELF).

Klemkosky and Martin (1975) investigated various beta adjustment procedures and suggested a combination of Bayesian predictor and a reasonable portfolio size to make an effective risk measure to predict future betas. Eubank and Zumwalt (1979) carried out the investigation for

beta risk classes (quintiles) and varied the length of the sample periods for beta estimation and prediction and hence based inferences on the same.
Bera and Kannan (1986) tested the data and observed possible deviation from normality and concluded that adjustment techniques proposed by Blume and Vasicek may not always be appropriate.

In Bayesian estimation choice of loss function is vital since the objective is to choose an estimator which minimizes the expected loss. In order to develop our model we make use of the fact that relatively high and low beta coefficients tends to get over predicted and under predicted, respectively, for corresponding betas in future time periods. We propose a Bayesian model under bilinear loss function which takes into account the over and under estimation problem. Such a function while remaining convex increase more slowly than SELF and do not overpenalize large but unlikely errors, conventionally used SELF is known to penalize larger deviations too heavily. Further in comparison to Vasicek's technique we use a proper conjugate form, since in latter the process of reparametrization leads to improper definition of prior parameters which may not give appropriate results in case of small sample.

The alternate beta adjustment techniques are discussed in second section sample and testing methodology is explained in the third section, and empirical results are provided in the fourth section.

3. Adjustment Techniques

In literature the fundamental base model to estimate systematic risk has been the single index model:

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\varepsilon}_{it} \qquad (1)$$

where $\tilde{R}_{it}$ is the return on security or portfolio i in period t, $\tilde{R}_{mt}$ is the corresponding market return in period t, α_i and β_i are the regression parameters : $\beta_i = \text{cov}\left(\tilde{R}_i, \tilde{R}_m\right) / \text{var}\left(\tilde{R}_m\right)$ is the slope coefficient, and $\tilde{\varepsilon}_{it}$ is the random error term with $E\left(\tilde{\varepsilon}_{it}\right) = 0$ and $E(\tilde{\varepsilon}_{it}{}^2) = \sigma^2$.

From previous studies, in an attempt to improve the accuracy of the forecasting ability of betas the following alternative beta adjustment techniques were developed and we describe them as follows.
The various adjustment procedures can be expressed as:

$$\widehat{\beta}_{i,t+1} = w_{1t} + w_{2t}\widehat{\beta}_{i,t} \qquad (2)$$

where, for alternate methods of evaluating w_{1t} and w_{2t} we obtain the varying techniques for beta prediction.

1. Unadjusted or historical betas are obtained by substituting $w_{1t} = 0$ and $w_{2t} = 1$ in equation (2).

2. Blume's (1971) procedure utilizes simple linear regression in attempting to adjust beta forecasts as follows:

   $$\widehat{\beta}_{i,t} = a_o + b_o \widehat{\beta}_{i,t-1}$$

   where the OLS estimates of a_o and b_o are used in forecasting the beta for third time period. Hence we have $w_{1t} = \widehat{a}_o$ and $w_{2t} = \widehat{b}_o$ in (3).
   Alternately above can be expressed in another form as

$$\hat{\beta}_{i,t+1} = \bar{\beta}_t + \hat{b}_o\left(\hat{\beta}_{i,t} - \bar{\beta}_{t-1}\right) \quad (3)$$

Blume observed that regression coefficients change over time and hence should be careful in using historical rates of regression. He showed that using non-overlapping periods the regression-adjusted betas resulted in smaller beta forecast than unadjusted beta values.

3. Merrill Lynch, Pierce, Fenner and Smith, Inc. (MLPFS) technique is based on the theory that on an average the beta estimates tend to drift towards one. For $\bar{\beta}_t = \bar{\beta}_{t-1} = 1$ in (3) we have the MPLPFS predictive estimate as

$$\hat{\beta}_{i,t+1} = 1 + \hat{b}_o\left(\hat{\beta}_{i,t} - 1\right)$$

The estimates by Blume and MLPFS techniques can be expected to be close whenever the average value of the beta is close to unity. Klemokosky and Martin (1975) found the forecast errors to be nearly identical for the two techniques.

4. Vasicek's (1973) technique is the Bayesian estimation procedure where the sizeable prior information of beta coefficients are used in adjusting beta forecasts. We substitute in (3)

$$w_{1t} = \bar{\beta} d_{ti} \text{ and } w_{2t} = 1 - d_{ti} \; ; \; d_{ti} = \frac{s_{\beta i}^2}{s_\beta^2 - s_{\beta i}^2}$$

where $\bar{\beta}$ and s_β^2 are the prior cross-sectional location and scale parameters, respectively, estimated using the previous periods; $s_{\beta i}^2$ is the estimated variance of $\beta_{i,t}$. The coefficients w_{1t} and w_{2t} are different for each security or portfolio. The optimal estimate is obtained under quadratic loss function which gives the posterior expected mean as the optimal estimate with respect to loss minimization.

Bayesian Adjustment under SELF and Bilinear Loss Functions

In addition to above discussed beta adjustment techniques we propose the following Bayesian model.

We redefine the market model in (1) and derive our results under two cases when variance $\sigma^2 (=1/\tau)$ is known and unknown.

$$\tilde{y} = \alpha_t + \beta_t \tilde{x} + \tilde{\varepsilon}_t \; ; t = 1,2,...,T \quad (4)$$

where $\tilde{y}$ be the return on asset or portfolio and $\tilde{x}$ be return on the market. Further assuming normal distribution of the error disturbances in model that is $\tilde{\varepsilon}_t \sim N\left(0,\tau\right)$.

Case I: Precision τ known

We define the likelihood function under (4) as follows

$$L(\tilde{y} \mid \tilde{x},\alpha,\beta,\tau) = \left(\frac{\tau}{2\pi}\right)^{\frac{T}{2}} \exp\left(-\frac{\tau}{2}\sum_{t=1}^{T}(y_t - \alpha - \beta x_t)^2\right)$$

The prior density is taken to be bivariate normal density and is defined as

$\pi(\alpha,\beta) = BIVN(\alpha_o,\beta_o,\tau r)$; precision matrix $r = \begin{pmatrix} r_1 & r_{12} \\ r_{21} & r_2 \end{pmatrix}$.

The posterior density is given by

$$\pi(\alpha,\beta \mid \tilde{y},\tilde{x},\tau) = \frac{L(\tilde{y}\mid \tilde{x},\alpha,\beta,\tau)\pi(\alpha,\beta)}{\iint_{\Theta} L(\tilde{y}\mid \tilde{x},\alpha,\beta,\tau)\pi(\alpha,\beta)d\alpha d\beta}$$

which evaluates to be

$$\pi(\alpha,\beta \mid \tilde{y},\tilde{x},\tau) = \frac{\tau\,|r+x'x|^{1/2}}{2\pi}\exp\left[-\frac{1}{2}\left\{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}-\begin{pmatrix}\alpha^*\\ \beta^*\end{pmatrix}\right\}'\tau\left(r+x'x\right)\left\{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}-\begin{pmatrix}\alpha^*\\ \beta^*\end{pmatrix}\right\}\right]$$

which is $BIVN(\alpha_o,\beta_o,\tau_1)$; precision matrix $\tau_1 = \tau\left(r+x'x\right) = \tau\begin{pmatrix} T+r_1 & \sum x_t + r_{12} \\ \sum x_t + r_{12} & \sum x_t^2 + r_2 \end{pmatrix}$

where

$$\alpha^* = \frac{\left(\sum x_t^2 + r_2\right)\left(\sum y_t + r_1\alpha_o + r_{12}\beta_o\right) - \left(\sum x_t + r_{12}\right)\left(\sum x_t y_t + r_{12}\alpha_o + r_2\beta_o\right)}{|r+x'x|}$$

$$\beta^* = \frac{(T+r_1)\left(\sum x_t y_t + r_{12}\alpha_o + r_2\beta_o\right) - \left(\sum x_t + r_{12}\right)\left(\sum y_t + r_1\alpha_o + r_{12}\beta_o\right)}{|r+x'x|}$$

Since our purpose is to evaluate β we consider α as the nuisance parameter and integrate it out to have the marginal density function as

$$\pi(\beta \mid \tilde{y},\tilde{x},\tau) = \int_{\Theta} \pi(\alpha,\beta \mid \tilde{y},\tilde{x},\tau)d\alpha = \left(\frac{\tau\,|r+x'x|}{2\pi(T+r_1)}\right)^{1/2}\exp\left[-\frac{\tau\,|r+x'x|}{2(T+r_1)}\left(\beta-\beta^*\right)^2\right]$$

which is normal density with mean β^* and precision $\tau' = \left(\tau\,|r+x'x|\right)/\left(T+r_1\right)$

The Bayes point estimate under squared error loss function (SELF) $L(\beta,\widehat{\beta}) = (\beta-\widehat{\beta})^2$ is the posterior mean. Hence we have the beta estimate to be

$$\widehat{\beta}_{i,t+1} = \beta^*$$

The Bilinear loss function is defined as

$$L(\beta,\widehat{\beta}) = \begin{cases} k_1(\beta-\widehat{\beta}) & if\ \beta \ge \widehat{\beta} \\ k_2(\widehat{\beta}-\beta) & if\ \beta \le \widehat{\beta} \end{cases}$$

where k_1 and k_2 are positive constants chosen so as to reflect the relative importance of underestimation and overestimation, respectively.

The Bayes point estimate under bilinear loss is the quantile function evaluated as

$$\widehat{\beta}_{i,t+1} = F^{-1}(d) = \begin{cases} \beta^* + \Phi^{-1}(\mathrm{u})/\sqrt{\tau'} & if\ \ d \ge \beta^*\ ,\ 0 < u \le 0.5 \\ \beta^* - \Phi^{-1}(1\text{-}\mathrm{u})/\sqrt{\tau'} & if\ \ d \le \beta^*\ ,\ 0.5 < u \le 1 \end{cases} ;\ u = k_1/\left(k_1+k_2\right)$$

If $k_1 = k_2$ we have absolute error function which gives median as the optimal estimate.

Case II: Precision τ unknown

The likelihood function under (4) is

$$L(\tilde{y}\mid \tilde{x},\alpha,\beta,\tau) = \left(\frac{\tau}{2\pi}\right)^{\frac{T}{2}}\exp\left(-\frac{\tau}{2}\sum_{t=1}^{T}(y_t-\alpha-\beta x_t)^2\right)$$

The prior density is taken to be bivariate normal density for (α_o, β_o) and non-informative prior for τ which is given as

$$\pi(\alpha,\beta,\tau) \cong BIVN(\alpha_o,\beta_o,\tau r).\frac{1}{\tau} \text{ ; precision matrix } r = \begin{pmatrix} r_1 & r_{12} \\ r_{21} & r_2 \end{pmatrix}.$$

The posterior density becomes

$$\pi(\alpha,\beta,\tau \mid \tilde{y},\tilde{x}) = \frac{\tau^{\frac{T}{2}-1}}{\Gamma\frac{T}{2}} S^{\frac{T}{2}} e^{-\tau S} . \frac{\tau \left| r + x'x \right|^{1/2}}{2\pi} \exp\left[-\frac{1}{2}\left\{ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} - \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} \right\}' \tau\left(r + x'x\right) \left\{ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} - \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} \right\} \right]$$

which is $Gamma\left(\frac{T}{2}, S\right) BIVN(\alpha_o,\beta_o,\tau_1)$; precision matrix $\tau_1 = \tau\left(r + x'x\right)$

where $S = \frac{1}{2}\left(\sum y_t^2 - \left(\alpha_o \ \beta_0\right) r \begin{pmatrix} \alpha_o \\ \beta_o \end{pmatrix} - \left(\alpha^* \ \beta^*\right)\left(r + x'x\right) \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} \right)$, α^* and β^* as defined above.

In order to evaluate β we consider α and τ both as the nuisance parameter and integrate them out to have the marginal density as

$$\pi(\beta \mid \tilde{y},\tilde{x}) = \iint_{\Theta} \pi(\alpha,\beta,\tau \mid \tilde{y},\tilde{x}) \, d\alpha \, d\tau$$

$$= B\left(\frac{T}{2},\frac{1}{2}\right)^{-1} \left(\frac{T\left| r + x'x \right| / 2(T + r_1)S}{T} \right)^{1/2} \left[1 + \frac{T\left| r + x'x \right| / 2(T + r_1)S}{T} \left(\beta - \beta^*\right)^2 \right]^{-\frac{(T+1)}{2}}$$

which is three-parameter *t* density with T *d.f.*, mean β^* and precision $\tau' = \frac{T\left| r + x'x \right|}{2(T + r_1)S}$.

The Bayes estimate under SELF is $\hat{\beta}_{i,t+1} = \beta^*$

The Bayes point estimate under Bilinear loss function is evaluated as

$$\hat{\beta}_{i,t+1} = F^{-1}(d) = \begin{cases} \beta^* + \left[\left\{ \frac{T}{\tau'} \left(I_{l'}^{-1}\left(\frac{T}{2},\frac{1}{2}\right) \right)^{-1} - 1 \right\} \right]^{\frac{1}{2}} & if \ d \geq \beta^* \text{ , } 0.5 < u \leq 1, \ l' = 2(1-u) \\ \beta^* - \left[\left\{ \frac{T}{\tau'} \left(I_{l'}^{-1}\left(\frac{T}{2},\frac{1}{2}\right) \right)^{-1} - 1 \right\} \right]^{\frac{1}{2}} & if \ d \leq \beta^* \text{ , } 0 < u \leq 0.5, \ l' = 2u \end{cases}$$

where $u = k_1 / \left(k_1 + k_2\right), I_l\left(p,q\right) = \frac{1}{B(p,q)} \int_0^l x^{p-1}(1-x)^{q-1} dx \ ; p,q > 0$ is the incomplete beta function.

In both the above cases the optimum values of k_1, k_2 lies in the interval [0.01, 0.05].

2. Sample and Testing Methodology

The study has been carried out based on daily return data of 60 companies from BSE 500 that were part of the index from 1 January 2000 to 31 December 2010 with the BSE index as the market proxy. These companies are well traded and belong to diverse industry groups. The daily returns have been adjusted for stock splits, bonus and right issues. The required data on stocks and indices was collected from Centre for Monitoring Indian Economy (CMIE) database: PROWESS and Bombay Stock Exchange (BSE) website. For the risk-free rate, 91-

day Treasury bill rates has been taken as proxy and complied from the Reserve Bank of India (RBI) website.

Estimation period lengths of 2 and 3 years are utilized in accordance with prediction period lengths of 1, 2 and 3 years for the beta calculations to analyze the impact of different period lengths on the beta forecast errors. The unadjusted and Bayesian techniques require two consecutive time periods, whereas for the Blume and MLPFS methods three consecutive time periods are required. Table 1 provides a set of comparisons made on the basis of three time periods *t - 1, t* and *t + 1*.

Table 1

Period Length (Years)			**Estimation Periods**		**Prediction Period**
t - 1	***t***	***t + 1***	***t - 1***	***t***	***t + 1***
2	2	1	1/2000-12/2001	1/2002-12/2003	1/2004-12/2004
2	2	2	1/2000-12/2001	1/2002-12/2003	1/2004-12/2005
2	2	3	1/2000-12/2001	1/2002-12/2003	1/2004-12/2006
3	3	1	1/2002-12/2004	1/2005-12/2007	1/2008-12/2008
3	3	2	1/2002-12/2004	1/2005-12/2007	1/2008-12/2009
3	3	3	1/2002-12/2004	1/2005-12/2007	1/2008-12/2010

In order to make empirical comparisons of the risk measure for portfolios, the betas were computed using market model given in (1) for period *t* and ranked in ascending order of magnitude. The ranked securities were selected sequentially for portfolios containing n = 3, 6 and 10 securities.

The forecasting accuracy of various adjustment techniques were compared using the statistically tractable Mean Square Error (MSE) measure.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\beta}_{i,t+1} - \hat{\beta}_{i,t}\right)^2$$

where $\hat{\beta}_{i,t+1}$ and $\hat{\beta}_{i,t}$ are the prediction and estimation period betas respectively for security or portfolio *i*, and n is the number of security or portfolio betas.

Mincer and Zarnowitz (1969) partitioned MSE into three components as follows:

$$\text{MSE} = \left(\bar{\beta}_{t+1} - \bar{\beta}_t\right)^2 + \left(1 - b_1\right)^2 S_{\beta_t}^2 + \left(1 - R_{\beta_{t+1},\beta_t}^2\right) S_{\beta_{t+1}}^2 \quad (5)$$

where $\bar{\beta}_{t+1}$ and $\bar{\beta}_t$ are means of the realizations and predictions, respectively; b_1 is the slope coefficient of the regression of $\hat{\beta}_{i,t+1}$ on $\hat{\beta}_{i,t}$; $S_{\beta_{t+1}}^2$ and $S_{\beta_t}^2$ are the sample variances of $\hat{\beta}_{i,t+1}$ and $\hat{\beta}_{i,t}$,respectively, and $R_{\beta_{t+1},\beta_t}^2$ is the coefficient of determination for $\hat{\beta}_{i,t+1}$ on $\hat{\beta}_{i,t}$.

The first component is the bias which measures the portion of MSE due to overestimation and underestimation of the mean between subsequent periods. The second term defines the inefficiency component and indicates the tendency of forecast error to be positive for low predicted values and negative for high predicted values. The last component is the random error. The inferences are drawn from detailed analysis of these three components.

4. Empirical Results

In order to carry out the study we make use of two different sets of estimation and prediction periods one set in normal economic condition and another set in economic meltdown. The MSEs for various techniques are given in following Table 2 and 3.

Table 2

Period 1		**Comparative Techniques**						
Estimation	Prediction	Unadj.	Blume	MLPFS	Vasicek	BM1*	BM2*	
1/2002 - 12/2003							τ known	τ unknown
		Individual Securities						
	1/2004-12/2006							
	Mean Square Error	0.08231	0.04594	0.04895	0.07598	0.08038	0.07845	0.07479
	MSE Components:							
	Bias	0.01264	0.00135	0.00436	0.01614	0.01264	0.01096	0.00540
	Inefficiency	0.02753	0.00244	0.00244	0.01810	0.02571	0.02542	0.02742
	Random Error	0.04215	0.04215	0.04215	0.04175	0.04204	0.04207	0.04197
	1/2004-12/2005							
	Mean Square Error	0.09617	0.06751	0.07130	0.09249	0.09462	0.09260	0.08750
	MSE Components:							
	Bias	0.01577	0.00250	0.00628	0.01966	0.01578	0.01389	0.00751
	Inefficiency	0.01544	0.00006	0.00006	0.00854	0.01406	0.01386	0.01532
	Random Error	0.06496	0.06496	0.06496	0.06428	0.06478	0.06484	0.06467
	1/2004-12/2004							
	Mean Square Error	0.12706	0.09577	0.10134	0.12463	0.12546	0.12299	0.11588
	MSE Components:							
	Bias	0.02437	0.00648	0.01205	0.02915	0.02438	0.02202	0.01374
	Inefficiency	0.01339	0.00000	0.00000	0.00700	0.01207	0.01189	0.01324
	Random Error	0.08929	0.08929	0.08929	0.08848	0.08902	0.08908	0.08891
		3-Security Portfolios						
	1/2004-12/2006							
	Mean Square Error	0.05334	0.06208	0.07933	0.05031	0.05160	0.04860	0.04779
	MSE Components:							
	Bias	0.01264	0.00067	0.01792	0.01416	0.01264	0.01010	0.00775
	Inefficiency	0.02752	0.04822	0.04822	0.02324	0.02579	0.02525	0.02696
	Random Error	0.01319	0.01319	0.01319	0.01291	0.01317	0.01324	0.01308
	1/2004-12/2005							
	Mean Square Error	0.05029	0.05088	0.07235	0.04833	0.04900	0.04582	0.04430
	MSE Components:							
	Bias	0.01577	0.00016	0.02163	0.01748	0.01578	0.01293	0.01024
	Inefficiency	0.01539	0.03160	0.03160	0.01218	0.01411	0.01370	0.01501
	Random Error	0.01912	0.01912	0.01912	0.01867	0.01910	0.01919	0.01905
	1/2004-12/2004							
	Mean Square Error	0.06583	0.05720	0.08842	0.06418	0.06460	0.06070	0.05846
	MSE Components:							
	Bias	0.02437	0.00032	0.03154	0.02648	0.02438	0.02080	0.01736
	Inefficiency	0.01361	0.02903	0.02903	0.01051	0.01241	0.01202	0.01329
	Random Error	0.02785	0.02785	0.02785	0.02719	0.02781	0.02788	0.02782

*BM1: Bayesian Model under SELF, BM2: Bayesian Model under Bilinear loss function.

Period 1		Comparative Techniques						
Estimation	Prediction	Unadj.	Blume	MLPFS	Vasicek	BM1	BM2	
1/2002 - 12/2003							τ known	τ unknown
		6-Security Portfolios						
	1/2004-12/2006							
	Mean Square Error	0.04965	0.07125	0.09075	0.04785	0.04783	0.04381	0.04492
	MSE Components:							
	Bias	0.01264	0.00123	0.02073	0.01350	0.01264	0.00935	0.00886
	Inefficiency	0.02947	0.06248	0.06248	0.02693	0.02768	0.02688	0.02860
	Random Error	0.00754	0.00754	0.00754	0.00742	0.00750	0.00758	0.00745
	1/2004-12/2005							
	Mean Square Error	0.04232	0.05318	0.07740	0.04116	0.04091	0.03674	0.03723
	MSE Components:							
	Bias	0.01577	0.00048	0.02470	0.01674	0.01578	0.01207	0.01152
	Inefficiency	0.01637	0.04252	0.04252	0.01445	0.01502	0.01445	0.01569
	Random Error	0.01018	0.01018	0.01018	0.00998	0.01011	0.01022	0.01002
	1/2004-12/2004							
	Mean Square Error	0.05517	0.05587	0.09102	0.05420	0.05379	0.04872	0.04895
	MSE Components:							
	Bias	0.02437	0.00007	0.03523	0.02556	0.02438	0.01971	0.01901
	Inefficiency	0.01453	0.03953	0.03953	0.01268	0.01325	0.01272	0.01388
	Random Error	0.01626	0.01626	0.01626	0.01596	0.01616	0.01629	0.01606
		10-Security Portfolios						
	1/2004-12/2006							
	Mean Square Error	0.03964	0.07013	0.09163	0.03867	0.03804	0.03337	0.03547
	MSE Components:							
	Bias	0.01264	0.00188	0.02338	0.01326	0.01264	0.00884	0.00936
	Inefficiency	0.02547	0.06672	0.06672	0.02390	0.02388	0.02297	0.02462
	Random Error	0.00153	0.00153	0.00153	0.00151	0.00152	0.00156	0.00149
	1/2004-12/2005							
	Mean Square Error	0.03361	0.05229	0.07896	0.03305	0.03235	0.02754	0.02911
	MSE Components:							
	Bias	0.01577	0.00091	0.02758	0.01647	0.01578	0.01149	0.01208
	Inefficiency	0.01454	0.04808	0.04808	0.01334	0.01332	0.01268	0.01385
	Random Error	0.00330	0.00330	0.00330	0.00325	0.00326	0.00337	0.00318
	1/2004-12/2004							
	Mean Square Error	0.04516	0.05337	0.09202	0.04478	0.04392	0.03810	0.03961
	MSE Components:							
	Bias	0.02437	0.00000	0.03865	0.02523	0.02438	0.01897	0.01973
	Inefficiency	0.01339	0.04596	0.04596	0.01222	0.01220	0.01162	0.01268
	Random Error	0.00740	0.00740	0.00740	0.00732	0.00733	0.00751	0.00720

Table 3

Period 2		**Comparative Techniques**						
Estimation	Prediction	Unadj.	Blume	MLPFS	Vasicek	BM1	BM2	
1/2005 - 12/2007							τ known	τ unknown
		Individual Securities						
	1/2008-12/2010							
	Mean Square Error	0.05316	0.06624	0.07699	0.05125	0.05253	0.05105	0.04896
	MSE Components:							
	Bias	0.00868	0.00631	0.01705	0.00722	0.00825	0.00675	0.00470
	Inefficiency	0.00002	0.01547	0.01547	0.00047	0.00006	0.00005	0.00008
	Random Error	0.04447	0.04447	0.04447	0.04356	0.04423	0.04425	0.04418
	1/2008-12/2009							
	Mean Square Error	0.05441	0.06778	0.07879	0.05252	0.05375	0.05222	0.05007
	MSE Components:							
	Bias	0.00918	0.00673	0.01774	0.00768	0.00873	0.00718	0.00506
	Inefficiency	0.00003	0.01585	0.01585	0.00054	0.00008	0.00007	0.00011
	Random Error	0.04520	0.04520	0.04520	0.04430	0.04494	0.04496	0.04489
	1/2008-12/2008							
	Mean Square Error	0.05134	0.06133	0.07094	0.04929	0.05064	0.04934	0.04756
	MSE Components:							
	Bias	0.00675	0.00468	0.01429	0.00547	0.00636	0.00506	0.00331
	Inefficiency	0.00010	0.01216	0.01216	0.00005	0.00004	0.00005	0.00003
	Random Error	0.04449	0.04449	0.04449	0.04377	0.04424	0.04424	0.04423
		3-Security Portfolios						
	1/2008-12/2010							
	Mean Square Error	0.02441	0.02255	0.02872	0.02354	0.02402	0.02167	0.02136
	MSE Components:							
	Bias	0.00868	0.00502	0.01119	0.00811	0.00825	0.00538	0.00594
	Inefficiency	0.00000	0.00180	0.00180	0.00009	0.00003	0.00001	0.00004
	Random Error	0.01573	0.01573	0.01573	0.01534	0.01574	0.01628	0.01537
	1/2008-12/2009							
	Mean Square Error	0.02466	0.02278	0.02913	0.02379	0.02426	0.02182	0.02153
	MSE Components:							
	Bias	0.00918	0.00540	0.01175	0.00859	0.00873	0.00577	0.00635
	Inefficiency	0.00001	0.00191	0.00191	0.00012	0.00004	0.00002	0.00006
	Random Error	0.01547	0.01547	0.01547	0.01508	0.01548	0.01602	0.01511
	1/2008-12/2008							
	Mean Square Error	0.02069	0.01811	0.02351	0.01970	0.02026	0.01828	0.01790
	MSE Components:							
	Bias	0.00675	0.00358	0.00898	0.00625	0.00636	0.00388	0.00436
	Inefficiency	0.00016	0.00076	0.00076	0.00003	0.00009	0.00013	0.00007
	Random Error	0.01378	0.01378	0.01378	0.01342	0.01380	0.01427	0.01347

Period 2		Comparative Techniques						
Estimation	Prediction	Unadj.	Blume	MLPFS	Vasicek	BM1	BM2	
1/2005 - 12/2007							τ known	τ unknown
		6-Security Portfolios						
	1/2008-12/2010							
	Mean Square Error	0.01373	0.00956	0.01254	0.01327	0.01334	0.01027	0.01029
	MSE Components:							
	Bias	0.00868	0.00398	0.00696	0.00832	0.00825	0.00460	0.00552
	Inefficiency	0.00005	0.00058	0.00058	0.00013	0.00010	0.00006	0.00013
	Random Error	0.00501	0.00501	0.00501	0.00481	0.00500	0.00560	0.00463
	1/2008-12/2009							
	Mean Square Error	0.01425	0.00985	0.01294	0.01377	0.01386	0.01067	0.01072
	MSE Components:							
	Bias	0.00918	0.00432	0.00741	0.00881	0.00873	0.00496	0.00592
	Inefficiency	0.00006	0.00052	0.00052	0.00016	0.00012	0.00008	0.00016
	Random Error	0.00501	0.00501	0.00501	0.00481	0.00500	0.00562	0.00463
	1/2008-12/2008							
	Mean Square Error	0.01165	0.00904	0.01157	0.01108	0.01124	0.00866	0.00855
	MSE Components:							
	Bias	0.00675	0.00271	0.00524	0.00643	0.00636	0.00323	0.00400
	Inefficiency	0.00006	0.00149	0.00149	0.00001	0.00002	0.00004	0.00001
	Random Error	0.00484	0.00484	0.00484	0.00464	0.00486	0.00539	0.00453
		10-Security Portfolios						
	1/2008-12/2010							
	Mean Square Error	0.01402	0.01109	0.01312	0.01365	0.01362	0.01006	0.01001
	MSE Components:							
	Bias	0.00868	0.00363	0.00566	0.00841	0.00825	0.00399	0.00507
	Inefficiency	0.00007	0.00219	0.00219	0.00014	0.00013	0.00008	0.00019
	Random Error	0.00527	0.00527	0.00527	0.00509	0.00524	0.00598	0.00475
	1/2008-12/2009							
	Mean Square Error	0.01418	0.01090	0.01301	0.01382	0.01378	0.01005	0.01008
	MSE Components:							
	Bias	0.00918	0.00396	0.00606	0.00890	0.00873	0.00433	0.00544
	Inefficiency	0.00010	0.00204	0.00204	0.00018	0.00017	0.00012	0.00024
	Random Error	0.00491	0.00491	0.00491	0.00474	0.00488	0.00560	0.00440
	1/2008-12/2008							
	Mean Square Error	0.00911	0.00821	0.00991	0.00875	0.00870	0.00556	0.00563
	MSE Components:							
	Bias	0.00675	0.00242	0.00412	0.00651	0.00636	0.00272	0.00362
	Inefficiency	0.00001	0.00343	0.00343	0.00000	0.00000	0.00000	0.00000
	Random Error	0.00236	0.00236	0.00236	0.00224	0.00234	0.00284	0.00201

For individual securities under normal market conditions we observe that MSE decreases with increase in length of prediction period length but in extreme situation we observe that the reverse is true.

In case of portfolios under normal conditions we see that for equal estimation and prediction period the MSE is coming out to be least and under recession the 3 year estimation length and 1 year prediction period gives the least MSE.

The portfolio size definitely cause the MSEs to reduce substantially for unadjusted as well as all the adjusted beta techniques. For both individual securities and portfolios the adjustment procedures appear to be very effective in reducing MSE than unadjusted technique.

Under normal market conditions Blume's procedure has the inefficiency and random components accounted for more than half the total MSE and in comparison to other methods this technique gives the least and negligible bias component. Further in Bayesian techniques the inefficiency component is negligible and bias is very small wherein the random component dominates the total MSE. Hence bias and random component hold a good proportion in total MSE.

The inferences differ when under extreme economic setting. The bias and random component seems to have the maximum proportion of total MSE for all the adjustment techniques. But in Blume and MLPFS techniques inefficiency component also hold in some proportion.

Bayesian techniques are seen to have the least MSE in comparison to conventional procedures. In comparison to Vasicek and Bayesian model under SELF the Bayesian model under bilinear loss gives us the least MSE which shows that overestimation and underestimation when taken into account ensures the efficiency and accuracy of the results.

5. Conclusion

The findings of the study suggest that betas adjustment techniques are required for reducing the forecast error associated with relatively higher or lower betas. Also the techniques are useful in reducing the MSE for shorter estimation and prediction periods. Beta as an investment risk measure should have an optimal estimation period lengths for a specified investment (prediction) period. While comparing, Blume and Bayesian techniques we observe that Bayesian techniques outperform classical methods in most of the cases. Further, we observe that the Blume's technique helps to capture the over and under estimation in the beta measure, this information can be utilized optimally to apply the Bayesian model under bilinear loss function and improve the accuracy of the estimates.

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