Child labour and inequality

D'Alessandro, Simone and Fioroni, Tamara

University of Pisa, University of Verona

22 April 2011

Child Labour and Inequality

Simone D'Alessandro* Tamara Fioroni†

March 26, 2012

Abstract

This paper focuses on the evolution of child labour, fertility and human capital in an economy characterized by two type of individuals, low and high skilled workers. This heterogeneity allows for an endogenous analysis of inequality generated by child labour. More specifically, according to empirical evidence, we offer an explanation for the emergence of a vicious cycle between child labour and inequality. The basic intuition behind this result is the interdependence between child labour and fertility choices: unskilled parents tend to have a high number of children and to send them to work whereas skilled parents tend to have a low fertility rate and a high investment in education. The fertility differential between high and low skilled increases the fraction of unskilled workers in the labour market which in turns reduces unskilled wage. The fact that children can offers only unskilled labor reinforces such process creating a vicious cycle between child labour and inequality.

JEL classification: J13; J24; J82; K31.

Keywords: Child Labor, Fertility, Human capital, Inequality.

*Dipartimento di Economia, Università di Pisa, Via Ridolfi 10, Pisa, Italy. Tel.: +39 050 2216333; Fax: +39 050 598040. *E-mail address:* s.dale@ec.unipi.it.

†Dipartmento di Scienze Economiche, Università di Verona, Vicolo Campofiore, 2. *E-mail address:* tamara.fioroni@univr.it.

1 Introduction

This paper presents a model where the interaction between child labour, fertility and human capital offers an explanation for a vicious cycle between child labour and inequality. The basic intuition behind this result is the interdependence between child labour and fertility choices: unskilled parents tend to have a high number of children and send them to work whereas skilled parents have a low fertility rate with a high investment in education. The fertility differential between high and low skilled increases the fraction of unskilled workers in the labour market which in turns reduces unskilled wages. The fact that children can offers only unskilled labor reinforces such process creating a vicious cycle between child labour and inequality.

As shown in Figure 1, we find empirical evidence of a positive relationship between inequality and child labour. In this figure, we use the data on children not attending school (i.e.number of out-of-school children as a percentage of all primary school-age children) as a proxy of child labour given the shortage of data on child labour. Even if this measure presents the shortcoming that a child not attending school is not necessarily working, on the other hand it is more easier to monitor children not attending school with respect to children that are working. In addition, the rate of children out of school should give also a measure of children working within the household or engaged in unofficial which are not included in the number of children economically active (see Cigno and Rosati, 2002). Note also that while still positive, the relationship between child labour and inequality has begun to flatten in recent years. A possible explanation of this result could be the increasing attention by national and international organizations to child labour.

Our work is related to a large body of the literature which has developed theoretical and empirical models to study the causes of child labour persistence (see for example Basu and Van, 1998; Basu, 1999, 2000; Baland and Robinson, 2000; Dessy, 2000; Dessy and Pallage, 2001, 2005; Ranjan, 1999, 2001). The benchmark framework is based on two main axioms: the

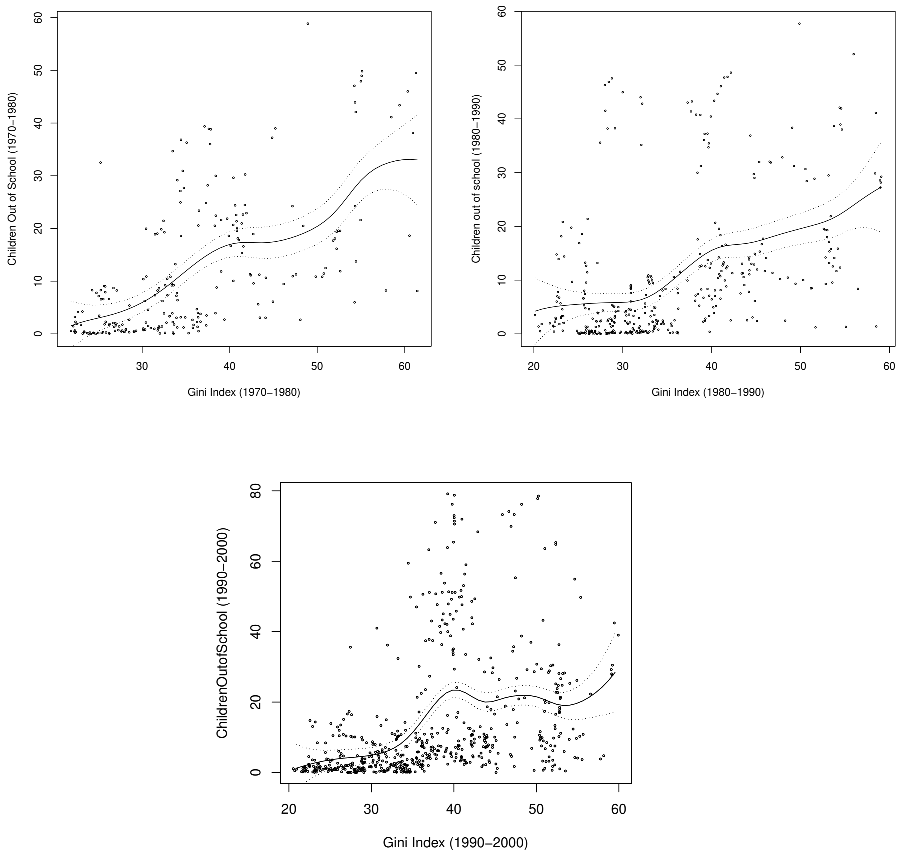

Figure 1: Children out of school and Gini Index (1970-1980, 1980-1990, 1990-2000). Nonparametric kernel smoother. Per capita GDP data are from Penn World Table 7.0. Gini Index data are from World Income Inequality Database. Children out of school data are from World Development Indicators (2010)

luxury axiom and the substitution axiom (Basu and Van, 1998 and Basu, 1999). The luxury axiom implies that parents send children to work if their income is below a certain threshold. The substitution axiom implies that adult labour and child labour are substitutes. These axioms lead to multiple equilibria in the labour market, with one equilibrium where adult wage is low and children work and another where adult wage is high and children do not work.

This framework has been extended by Dessy (2000), Hazan and Berdugo (2002) and Doepke and Zilibotti (2005) which introduce endogenous fertility choices. They analyze the relationship between child labour, fertility and human capital showing the existence of multiple development path. In early stages of development, the economy is in a development trap where child labour is abundant, fertility is high and output per capita is low. Technological progress, allows a take-off from the underdevelopment trap because it gradually increases the wage differential between parental and child labour and hence the return of investment in education.

However these contributions do not consider the presence of inequality, i.e. the economy can follow different paths of growth that are characterized – in equilibrium – by a unique level of human capital. We extend this framework taking into account two groups of individuals with two different levels of human capital. In this respect our work is related to the literature on inequality, differential fertility, and economic growth. In particular, Moav (2005) develops a theory of fertility that offers an explanation for the persistence of poverty within and across countries. The basic idea is that the cost of child quantity increases with parent's human capital since the opportunity cost of time is high. High-income families choose low fertility rates and high investment in education. This implies that high income persist in the dynasty. On the other hand, poor households choose relatively high fertility rates with relatively low investment in their offspring's education. Therefore, their offspring are poor as well. De la Croix and Doepke (2003, 2004) argue that a higher inequality, by increasing fertility differential between rich and poor families, lowers average education and, therefore, growth. The motivation for this result is that a large fertility differential implies more weight on children with little education. From a similar perspective, Galor and Zang (1997) show that due to financial market imperfections countries with smaller average family size and with more equal distribution of income attain a high economic growth. Morand (1999) develops a model in which fertility decisions are motivated by old-age support. He shows that a rich economy with an equal distribution of income evolves on a growth path with increasing levels of human capital. By contrast, a poor economy can take off on the growth path only if there exists a class of relatively high skilled agents.

The interaction between fertility differentials and inequality is also the focus of Dahan and Tsiddon (1998), Kremer and Chen (2002). Dahan and Tsiddon (1998) show that fertility and income distribution follow an inverted U-shaped dynamics during the transition to the steady state. In the first stage of development the high fertility among poor leads to a higher proportion of poor and thus to a greater inequality. When income inequality reaches a certain threshold, wage differential is high enough to lead some poor agents to invest in human capital. Thus the number of skilled agents rises, fertility declines and income becomes more equally distributed. Kremer and Chen (2002) develop a model that generates multiple steady states of inequality, when the initial proportion of skilled workers is high the economy converges to a steady state with low inequality, on the other hand when the initial proportion of skilled workers is too low the economy converges to a steady state with high inequality.

In this paper we show that, by incorporating child labour in this framework, a persistent vicious cycle between inequality and child labour can persist for generations. In particular we develop an overlapping generation model with two types of workers - low and high skilled. According to the existent literature, we assume that child labour is a perfect substitute for unskilled adult but children are relatively less productive. Adults allocate their time endowment between work and child raring. They choose the number of children and their time allocation between schooling and work. Hence, households can have two, possible, sources of income: income by parents and child income. Human capital of children is an increasing, strictly concave function of the time devoted to school which in turns depends on parent's income (see Galor and Weil, 2000; Galor and Tsiddon, 1997; De la Croix and Doepke, 2004; Hazan and Berdugo, 2002). As a result the model shows an intergenerational persistence in education levels. This effect with the differential fertility between low and high skilled parents produces a continuous increase in the inequality and child labour and an average impoverishment within the country. According to the model of Doepke (2004), our paper suggests that child labour regulation is essential in reducing permanently inequality.

Section 2 describes the model and the optimal individual choices. In Section 3, we investigate the properties of the short run general equilibrium. Section 4 analyses the long run dynamics of the model. Section 5 concludes.

2 The Model

We analyze an overlapping-generation economy which is populated by N_t individuals. Each of them is endowed with a level of human capital, h_t^i. This level is endogenously determined by parent's choice on the children's time allocation between labour and schooling. Adults can supply skilled or unskilled labour, while children can only supply unskilled labour.

2.1 Production

For the sake of the argument we assume that labour is the only production factor. Production occurs according to a neoclassical, constant return to scale, Cobb-Douglas production technology using unskilled and skilled labour as inputs. The output produced at time t, Y_t, is

$$Y_t = \psi(H_t)^{\mu}(L_t)^{1-\mu} = \psi(s_t)^{\mu}L_t; \qquad 0 < \mu < 1, \tag{1}$$

where $s_t \equiv H_t/L_t$ is the ratio of skilled H_t to unskilled labor L_t employed in production in period t, and $\psi > 0$ is the technological level. Producers in period t choose the level of unskilled labour , L_t, and the efficiency units of labour, H_t, so as to maximize profits. Thus the wage of unskilled workers, i.e. w_t^u, and the wage rate per efficiency unit w_t^s are

$$w_t^u = \psi(1-\mu)\,(s_t)^{\mu}\,, \tag{2}$$

and

$$w_t^s = \psi\mu\,(s_t)^{\mu-1}\,. \tag{3}$$

2.2 Preferences

Members of generation t live for two periods: childhood and adulthood. In the childhood, individuals may either work, go to school or both. In the adulthood, agents supply unskilled or skilled labour. Individual's preferences are defined over consumption , i.e. c_t^i, the number of children n_t^i, and the human capital of children h_{t+1}^i.[1] The utility function of an agent i of generation t is given by

$$U_t^i = \alpha \ln c_t^i + (1-\alpha)\ln(n_t^i h_{t+1}^i), \tag{4}$$

where $\alpha \in (0,1)$ is the altruism factor.

We suppose that children born with some basic human capital a, which can be increased by attending school. In particular, human capital of children in period $t+1$ is an increasing, strictly concave function of the time devoted to school

$$h_{t+1}^i = a(b + e_t^i)^{\beta}, \tag{5}$$

where $a, b > 0$ and $\beta \in (0,1)$.

[1] As it is clear from equation (4), we assume that parents are aware of the human capital of their children rather than their income. Although the results of the model are not crucially affected by this choice, we believe that this is a more realistic assumption, see for instance De la Croix and Doepke (2003) and Galor (2005) for discussion on this point.

Parents allocate their income between consumption and child rearing. In particular, raising each born child takes a fraction $z \in (0,1)$ of an adult's income. In addition, parents allocates the time endowment of children between schooling, $e_t^i \in [0,1]$, and labour force participation $(1-e_t^i) \in [0,1]$. We assume that, each child can offer only $\theta \in [0,z)$ units of unskilled labour, that is children are substitutes for unskilled adult workers but relatively less productive.[2] Therefore, each household can have two, possible, sources of income: i) parent income, $I_t^i = \max\{w_t^s h_t^i, w_t^u\}$ and, ii) child income, $(1-e_t^i)\theta w_t^u$. Indeed, while children can work only as unskilled workers, parents will choose to work in the sector that guarantees them the highest income. Hence, the budget constraint is

$$c_t^i \leq (1 - z n_t^i) I_t^i + (1 - e_t^i)\theta w_t^u n_t^i. \tag{6}$$

2.3 Individual choices

Each household chooses c_t^i, n_t^i and e_t^i so as to maximize the utility function (4) subject to the budget constraint (6). Given the wage ratio, the optimal consumption, the optimal schooling and the optimal number of children chosen by member i of generation t are

$$c_t = \alpha I_t^i; \tag{7}$$

$$e_t^i = \begin{cases} 0 & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{r_t^i \beta z - \theta(\beta+b)}{\theta(1-\beta)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ 1 & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{8}$$

and

[2] The assumption $z > \theta$ means that the ratio between the income of child labour when the child just works and the cost of rising a child – i.e. the relative return of child rearing (θ/z) – is less than 1. This further implies that it is not possible to increase income by simply "producing" more children.

$$n_t^i = \begin{cases} \frac{(1-\alpha)r_t^i}{zr_t^i-\theta} & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)r_t^i}{zr_t^i-\theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{9}$$

where $r_t^i \equiv I_t^i/w_t^u$. In particular,

$$r_t^i = \begin{cases} 1 & \text{if } w_t^s h_t^i \leq w_t^u, \\ \frac{w_t^s h_t^i}{w_t^u} & \text{if } w_t^s h_t^i > w_t^u. \end{cases} \tag{10}$$

Since $h_t^i = a(b + e_{t-1}^i)^\beta$, the ratio r_t^i is a function of the level of education chose in period $t-1$.

Agent i, according to her level of human capital h_t^i, chooses to work as unskilled if, and only if, $w_t^s h_t^i < w_t^u$, while she works as skilled if, and only if, $w_t^s h_t^i > w_t^u$. Finally, if $w_t^s h_t^i = w_t^u$ agent i is indifferent to work as skilled or unskilled. Note that, from equation (10), if parents find convenient to work as unskilled, their choices on fertility and education do not depend on income since $r_t^i = 1$ – see equations (8) and (9). [3]

3 General Equilibrium

This last result highlights the emergence of a marked asymmetry between agents who offer skilled and unskilled work. For the sake of the argument, we assume that in the initial period, $t = 0$, population is divided in two groups which are endowed with two different levels of human capital a low level of human capital h_0^u and a high level h_0^s, with $h_0^u < h_0^s$. We show that such a difference may persist across generations. At any period t, if $h_t^u < h_t^s$, low skilled workers choose to work as unskilled if and only if $w_t^s h_t^u < w_t^u$, while if $w_t^s h_t^u > w_t^u$ they would prefer to work as skilled.[4] Since we assume perfect mobility of labour at equilibrium the ratio w_t^u/w_t^s must satisfy $w_t^s h_t^u \leq$

[3] According to the existent literature the model shows a trade-off between quantity and quality of children. See, for istance, Hazan and Berdugo (2002) for an analysis with constant wages.

[4] The superscript u and s refers to low and high skilled workers respectively.

w_t^u; otherwise all the labour force would offer skilled labour, which it is not possible given equation (2). A similar argument applies to high skilled workers; thus $w_t^s h_t^s \geq w_t^u$. Therefore, for any $h_t^u \leq h_t^s$, at equilibrium

$$h_t^u \leq \frac{w_t^u}{w_t^s} \leq h_t^s. \tag{11}$$

From equations (10) and (11), it holds

$$r_t^u = 1, \tag{12}$$

and

$$1 \leq r_t^s \leq \frac{h_t^s}{h_t^u}, \tag{13}$$

for all $t \in \mathbb{N}_0$.

Equation (11) clarifies that this model presents three different regimes, which influence the equilibrium outcome and, more precisely, determine different supplies of labour for low and high skilled workers. Two regimes are corner solutions. If at equilibrium $w_t^s h_t^s = w_t^u$, high skilled workers are indifferent to work as skilled or unskilled. On the other hand, if at equilibrium $w_t^s h_t^u = w_t^u$, a fraction of low skilled workers work as skilled. In the other case, when $h_t^u < \frac{w_t^u}{w_t^s} < h_t^s$, low skilled only work as unskilled and high skilled only as skilled.

It is worth to point out that if in a certain period t, market equilibrium implies $w_t^s h_t^s = w_t^u$, in period $t+1$ there will be no difference between low and high skilled workers, since all the population gets the same adult income and makes the same schooling and fertility decisions. This argument does not apply when $w_t^s h_t^u = w_t^u$, since in that case high skilled workers get a higher income equal to $w_t^s h_t^s$ that is greater than w_t^u if $h_t^s > h_t^u$.

3.1 Internal equilibrium

Let us assume that in period t, $1 < r_t^s < \frac{h_t^s}{h_t^u}$. As we pointed out above, under such condition low skilled workers find convenient to work as unskilled and high skilled as skilled. Thus, at equilibrium – if it exists – the economy is characterized by two classes of income ($w_t^s h_t^s > w_t^u$), which make different fertility and schooling decisions – see equations (8) (9).

The existence of two income classes implies that the aggregate demand is:

$$D_t = c_t^u N_t^u + c_t^s N_t^s, \tag{14}$$

where N_t^u and N_t^s are, respectively, the number of low and high skilled agents. Moreover, from equations (2), (3) and (7), $c_t^u = \alpha\psi(1-\mu)s_t^\mu$ and $c_t^s = \alpha h_t^s \psi \mu s_t^{\mu-1}$.

At time t, since all the low skilled adults choose to work as unskilled, the supply of unskilled labour, L_t is given by the labour supplied by low skilled adults, i.e. $(1 - zn_t^u)N_t^u$, plus the labour supplied by the children of low and high skilled parents, i.e. $(1 - e_t^u)n_t^u N_t^u$ and $(1 - e_t^s)n_t^s N_t^s$. At equilibrium this supply must be equal to the total demand of unskilled labour. Thus,

$$L_t = (1 - zn_t^u)N_t^u + \theta[(1 - e_t^u)n_t^u N_t^u + (1 - e_t^s)n_t^s N_t^s]. \tag{15}$$

The supply of skilled labour, must be equal to the demand of skilled labour, that is

$$H_t = (1 - zn_t^s)h_t^s N_t^s. \tag{16}$$

From equations (1) and (14), the equilibrium in the goods market yields

$$L_t = \alpha \frac{(1-\mu)s_t N^u + \mu h^s N^s}{s_t}. \tag{17}$$

The ratio between equations (16) and (17) defines the equilibrium level of s_t

$$s_t^* = \frac{h_t^s}{1-\mu}\left[\frac{1 - zn_t^s}{\alpha} - \mu\right]\frac{1}{x_t}, \tag{18}$$

where $x_t \equiv N_t^u / N_t^s$. Note that s_t^* depends only on the choice of n_t^s. The other variables N_t^s, N_t^u and h_t^s are given at period t because they depend on choices made in period $t-1$. In order to understand the relation between s_t^* and n_t^s it is convenient to rewrite r_t^s. From equations (2), (3) and (18), we obtain,

$$r_t^s = \frac{w_t^s h_t^s}{w_t^u} = \frac{\mu\alpha x_t}{1 - zn_t^s - \mu\alpha}. \tag{19}$$

Therefore we can have different values of s_t^* and r_t^s according to the values of fertility of skilled workers at time t.

From equation (9), the fertility choice of high skilled workers is given by

$$n_t^s = \begin{cases} \frac{(1-\alpha)r_t^s}{zr_t^s-\theta} & \text{if} \quad r_t^s \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)r_t^s}{zr_t^s-\theta(1+b)} & \text{if} \quad \frac{\theta(\beta+b)}{\beta z} \leq r_t^s \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if} \quad r_t^s \geq \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{20}$$

Thus the function n_t^s takes different values according to the value of r_t^s. By solving the system given by equations (19) and (20) we get the equilibrium level of r_t^s and n_t^s. We obtain

$$r_t^{s*} = \begin{cases} \frac{2\theta\alpha\mu x_t}{\theta(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_1(x_t)}} & \text{if} \quad x_t \leq x_2 \\ \frac{2\theta(1+b)\alpha\mu x_t}{\theta(1+b)(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_2(x_t)}} & \text{if} \quad x_2 \leq x_t \leq x_3 \\ \frac{\mu x_t}{1-\mu} & \text{if} \quad x_3 \leq x_t \end{cases} \tag{21}$$

where $\Delta_1(x_t) = [z\alpha\mu x_t - \theta(1-\alpha\mu)]^2 + 4\theta(1-\alpha)z\alpha\mu x_t$ and $\Delta_2(x_t) = [z\alpha\mu x_t - \theta(1+b)(1-\alpha\mu)]^2 + 4\theta(1-\beta)(1-\alpha)z\alpha\mu x_t$; $x_2 = \frac{\theta(b+\beta)[b\alpha(1-\mu)-\beta(1-\alpha)]}{z\alpha\mu\beta b}$, $x_3 = \frac{\theta(1-\mu)(1+b)}{\mu\beta z}$.

Note that the equilibrium value of r_t^{s*} depends only on the ratio between the number of low and high skilled workers. Moreover, in an internal equilibrium it must hold that $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$. Thus it is possible that for some values of x_t does not exist an internal solution. Figure 2 clarifies this result. The function r_t^{s*} is a piecewise function defined in the interval $\underline{x} \leq x_t \leq \bar{x}$ – that implies $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$ – where an internal equilibrium always exists. In the case presented in Figure 2, as long as x_t increases r_t^{s*} becomes equal to $\frac{h_t^s}{h_t^u}$ before reaching the level $\frac{\theta(1+b)}{\beta z}$, that is the level which ensures $e_t^s = 1$. Depending on the values of parameters many different cases are possible, but such an analysis does not give mach insight. In the Appendix we shows that the derivative of r_t^{s*} with respect of x_t is always positive.

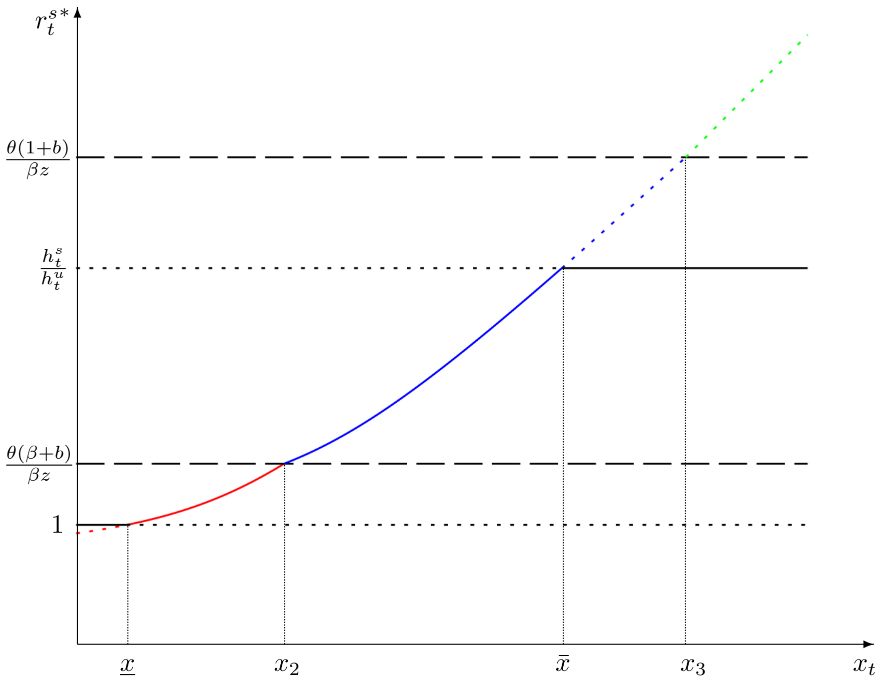

Figure 2: A numerical illustration of r_t^{s*} as a function of x_t. An internal equilibrium exists if and only if $\underline{x} \leq x_t \leq \bar{x}$. Value of parameters: $\alpha = 0.9$; $\mu = 0.3$; $z := 0.3$; $\theta = 0.25$; $\beta = 0.4$; $b = 0.2$; $a = 1$.

Given r_t^{s*}, it is easy to obtain the equilibrium values for all the other variables of the model. Note, that for $x_t \leq x_2$ high (and low) skilled workers do not invest in education, for $x_2 < x_t < x_3$ high skilled workers send children to work and to school, $e_t^s \in (0, 1)$, while for $x_3 \leq x_t$ they send children only to school ($e_t^s = 1$).

3.2 Corner solutions

The previous analysis clarifies that the equilibrium of the system depends on the level of x_t, that is the ratio between low and high skilled in the economy. If this ratio is smaller than $\underline{x}$, the number of high skilled is too high, given the technology available, and therefore the wage of the high skilled is equal to the wage of the unskilled ($w_t^s h_t^s = w_t^u$). This implies that for any $0 \leq x_t \leq \underline{x}$, $r_t^u = r_t^{s*} = 1$ (see Figure 2).

On the other hand, if $x_t > \bar{x}$, high skilled workers are too small with respect to low skilled. Thus the wage for unit of human capital (w_t^s) is high enough to allow low skilled to work as skilled getting the same wage of unskilled labour ($w_t^s h_t^u = w_t^u$). This implies that for any $\bar{x} \leq x_t$, $r_t^{s*} = \frac{h_t^s}{h_t^u}$ (see Figure 2).

4 Long Run

Fertility choices of the two groups affect the relative size of high and low skilled labour. This relation is crucial in determining the wage ratio, and hence the dynamics of human capital.

Since $N_{t+1}^u = n_t^u N_t^u$ and $N_{t+1}^s = n_t^s N_t^s$, the population dynamics is given by

$$N_{t+1} = n_t^u N_t^u + n_t^s N_t^s. \tag{22}$$

At the same time it is possible to determine the dynamics of $x_t \equiv N_t^u / N_t^s$. Indeed we have that

$$x_{t+1} = \frac{n_t^u}{n_t^{s*}} x_t. \tag{23}$$

The fertility of low skilled workers $n_t^u \geq n_t^{s*}$, where

$$n_t^u = \begin{cases} \frac{(1-\alpha)}{z-\theta} & \text{if } 1 \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)}{z-\theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq 1 \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } 1 \geq \frac{\theta(1+b)}{\beta z}, \end{cases} \tag{24}$$

and in any internal equilibrium, from equations (20) and (26)

$$n_t^{s*} = \begin{cases} \frac{\theta(1-\alpha\mu) - z\alpha\mu x_t + \sqrt{\Delta_1(x_t)}}{2\theta z} & x_t \leq x_2 \\ \frac{\theta(1+b)(1-\alpha\mu) - z\alpha\mu x_t + \sqrt{\Delta_2(x_t)}}{2\theta z} & x_2 \leq x_t \leq x_3 \\ \frac{(1-\alpha)}{z} & x_t \geq x_3 \end{cases} \tag{25}$$

We define a long run equilibrium a trajectory in which individual choices do not change from one period to the next. Since choices at any period t are affected by income, in a long run equilibrium wage ratio must be constant which means that there must be a constant proportion of skilled and unskilled labour. However, as long as there is inequality in the economy (the income of high skilled is higher than the income of low skilled), the fertility choices between the two groups are different: the ratio x_t changes over time.

There are three limit cases, where inequality disappears after the first period.

First, if, $1 \geq \frac{\theta(1+b)}{\beta z}$, low skilled workers send their children only to school. The fertility choices of high and low skilled workers are the same and all the population will be characterized by the maximum level of human capital.

Second, if, $1 \leq \frac{\theta(\beta+b)}{\beta z}$ and in period $t = 0$, $x_0 \leq x_2$, high skilled workers send their children to work. Thus next generation will be characterized by the minimum level of human capital: the differences between the two classes disappear.

Third, if in period $t = 0$, $x_0 \leq \underline{x}$ thus high skilled workers get a level of wage equal to w_0^u. Also in this case, their choice of fertility and education are equal to the choice of low skilled workers.

Beside those limit cases, the economy is characterized by an increase in inequality in the transition. Indeed given that at any period the equilibrium only depends on x_t, we can easily characterized the possible trajectories taking into account Figure 2.

For any $\max\{\underline{x}, x_2\} < x_t < \bar{x}$, the fertility of high skilled workers is permanently lower than the fertility of low skilled workers. Thus from equation (23) x_t rises over time. This process leads to a continuous increase in r_t^{s*} which in turn leads to an increase in the human capital of children of skilled parents, i.e. h_{t+1}^s. Thus the presence of differential fertility leads on the one hand to an increases w_t^s and a reduction of w_t^u and on the other hand to a high h_{t+1}^s. This process generates a continuous increase in the inequality and an increase in child labour, since generation by generation the number of high-skilled workers decreases and become richer whereas the number of low skilled workers increases and become poorer.

The increase in h_t^s leads also to an increase in $\bar{x}$, which may allow the dynamics of human capital to reach its maximum level. However, the continuos increase in x_t implies that at a certain period, $t = \tilde{t}$, $x_{\tilde{t}} = \bar{x}$. In that situation low skilled workers are indifferent to work as skilled or as unskilled. A share of low skilled will work as skilled in order to maintain the equality $w_t^s h_t^u = w_t^u$. Thus for any $t > \tilde{t}$ the wage ratio is constant. The economy is in a long run equilibrium where wages do not change, but the presence of differential fertility induces an increase of low skilled workers which send their children to work (at least partially). Thus the high-skilled fraction of population tend to zero, while an higher fraction of low skilled workers tend to work as skilled. In other words, the economy in the long run will be populated only by low skilled workers which continue to send children to work.

5 Final Remarks

This paper is built on the idea that the persistence of child labour is strictly linked to the presence of inequality within the country. For this reason we present a model where the population is divided in two groups endowed with two different level of human capital. We study how this initial heterogeneity affect the distribution of income in the long run. The crucial result of this analysis is that the increase in the return of human capital is not sufficient to induce a transition to a high-skilled economy. The presence of two groups, with different levels of initial human capital, generates a continuous increase in the income of the high skilled workers with respect to those endowed with a low level of human capital. The presence of endogenous fertility induces low income group to make an higher number of children. Thus, child labour will increase. The substitutability between adult and child labour increases the resilience of this result: the economy is trapped in an equilibrium with a high fraction of the population with low income and low human capital. In other words, we find a vicious cycles between child labour and inequality.

This framework can be easily extended to evaluate the issues currently discussed in the literature. For instance, further research is needed to analyze the role of technical progress and international labour standards. With respect to the first issue, preliminary results seem to reject the hypothesis that technical progress can, by itself, induce the low-skilled group to invest in children's education. Another interesting application of the model is to evaluate public policies that through taxation on high-skilled individual may subsidies the low-skilled labour inducing them to invest in education. This policy may generate together a reduction of inequality and the disappearance of child labour.

A Appendix

In this section we show that r_t^s is always an increasing function of x_t. In order to simplify the notation we denote: $A = 2\theta\alpha\mu$, $B = \theta(1-\alpha\mu)$, $C = z\alpha\mu$, $D = 4\theta(1-\alpha)z\alpha\mu$. Given this simplification we can rewrite equation 26 as follows

$$r_t^{s*} = \begin{cases} \frac{Ax_t}{B+Cx_t-\sqrt{(Cx_t-B)^2+Dx_t}} & \text{if } x_1 \leq x_t \leq x_2 \\ \frac{A(1+b)x_t}{B(1+b)+Cx_t-\sqrt{[Cx_t-B(1+b)]^2+D(1-\beta)x_t}} & \text{if } x_2 \leq x_t \leq x_3 \\ \frac{\mu x_t}{1-\mu} & \text{if } x_3 \leq x_t \leq x_4 \end{cases} \tag{26}$$

Thus the derivative of the first line is positive if

$$2B\sqrt{(Cx_t-B)^2+Dx_t} > 2B(Cx_t-B)-Dx_t, \tag{27}$$

where squaring both sides we get that it always holds

$$\alpha(1-\mu) > 0 \tag{28}$$

The derivative of the second line is very similar. It is positve if

$$2B(1+b)\sqrt{(Cx_t-B(1+b))^2+D(1-\beta)x_t} > 2B(1+b)(Cx_t-B(1+b))-D(1-\beta)x_t. \tag{29}$$

Squaring both sides we get that it always holds

$$b(1-\alpha\mu)+\alpha(1-\mu)+\beta(1-\alpha) > 0 \tag{30}$$

References

Baland, J. and J. Robinson (2000). Is child labor inefficient? *Journal of Political Economy 108*(4), 663–679.

Basu, K. (1999). Child labor: Cause, consequence, and cure, with remarks on international labor standards. *Journal of Economic Literature 37*(3), 1083–1119.

Basu, K. (2000). The intriguing relation between adult minimum wage and child labour. *Economic Journal 110*, 50–61.

Basu, K. and P. Van (1998). The economics of child labor. *American Economic Review 88*(3), 412–427.

Cigno, A. and F. Rosati (2002). Does globalization Increase Child Labor? *World Development 30*(9), 1579–1589.

Dahan, M. and D. Tsiddon (1998). Demographic Transition, Income Distribution, and Economic Growth. *Journal of Economic Growth 3*(1), 29–52.

De la Croix, D. and M. Doepke (2003). Inequality and Growth: Why Differential Fertility Matters. *American Economic Review 93*(4), 1091–1113.

De la Croix, D. and M. Doepke (2004). Public versus Private education when differential fertility matters. *Journal of Development Economics 73*(2), 607–629.

Dessy, S. E. (2000). A defense of compulsive measures against child labor. *Journal of Development Economics 62*, 261–275.

Dessy, S. E. and S. Pallage (2001). Child labor and coordination failures. *Journal of Development Economics 65*(2), 469–76.

Dessy, S. E. and S. Pallage (2005). A Theory of the worst form of child labour. *The Economic Journal 115*, 68–87.

Doepke, M. (2004). Accounting for fertility decline during the Transition to Growth. *Journal of Economic Growth 9*, 347–383.

Doepke, M. and F. Zilibotti (2005). The Macroeconomics of Child Labour Regulation. *American Economic Review 95*(5), 1492–1524.

Galor, O. (2005). From Stagnation to Growth: Unified Growth Theory. *In: Aghion P, Durlauf S (eds) The Handbook of Economic Growth North-Holland, Amsterdam 171-293.*

Galor, O. and D. Tsiddon (1997). The Distribution of Human Capital and Economic Growth. *Journal of Economic Growth 2*(1), 93–124.

Galor, O. and D. N. Weil (2000). Population ,Technology, and Growth: from Malthusian Stagnation to the DenographicTransition and Beyond. *American Economic Review 90*(4), 806–828.

Galor, O. and H. Zang (1997). Fertility Income distribution and Economic Growth. *Japan and the World Economy 9*, 197–229.

Hazan, M. and B. Berdugo (2002). Child Labour, Fertility, and Economic Growth. *The Economic Journal 112*(482), 810–828.

Kremer, M. and D. Chen (2002). Income Distribution Dynamics with Endogenous Fertility. *Journal of Economic Growth 7*(3), 227–258.

Moav, O. (2005). Cheap Children and the Persistence of Poverty. *The Economic Journal 115*, 88–110.

Morand, O. F. (1999). Endogenous Fertility, Income Distribution, and Growth. *Journal of Economic Growth 4*, 331–349.

Ranjan, P. (1999). An economic analysis of child labor. *Economics Letters 64*(1), 99–105.

Ranjan, P. (2001). Credit constraints and the phenomenon of child labor. *Journal of Development Economics 64*(1), 81–102.