Cournot and Bertrand competition with asymmetric costs in a mixed duopoly revisited

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Abstract

We investigate a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities when the public firm is less efficient than the private firm. Thus, if the Singh and Vives assumption of positive primary outputs holds, (i) Bertrand competition or quantity-price competition can occur depending on the degree of public firm's inefficiency when the goods are substitutes. (ii) regardless of its inefficiency, there can be always sustained Bertrand competition when the goods are complements. (iii) the ranking of a private firm's profit is not reversed. However if we relax the parameter restriction imposed implicitly by Singh and Vives (i.e., we adopt Zanchettin (2006) assumption) to allow for a wider range of cost asymmetry, there can be always sustained multiple subgame Nash perfect equilibria in the contract stage by each critical value of the public firm's inefficiency. In particular, Cournot and Bertrand competition coexist if its inefficiency is sufficiently small or large.

JEL: D43, H44, L13.
Keywords: Inefficiency, Cournot-Bertrand Competition, Mixed Duopoly.

1 Introduction

There are several studies of mixed oligopolies, in which public firms maximize their social welfare, whereas the private firms compete with public firms in order to maximize their own profits (see De Fraja and Delbono (1990), De Fraja (2009) and Bös (1991) for general reviews of mixed oligopolies). Studies of mixed oligopolies have become richer and more diverse over the past decade (e.g., Matsumura, 1998; Matsumura and Matsushima, 2004; Heywood and Ye, 2008; Barcena-Ruiz and Casado-Izaga, 2011), and the occurrence of mixed oligopolies in industry has also become a common feature across different economic systems. However, relatively few theoretical analyses of the choice of strategic variables for prices or quantities have emerged in recent publications on mixed oligopolies.

In the real world, the public and private firms frequently interact many levels. In the present research, we study these interactions between public and private firms by allowing them to choose strategically set their own levels of price or quantity competition. Thus, the present study is modeled on a non-cooperative game in which the choice of strategic variables is set in a mixed duopoly. Several authors have analyzed the strategic variables in non-cooperative games. For a purely private duopoly, Singh and Vives (1984) were the first to show that Bertrand competition is more efficient than Cournot competition when goods are differentiated (see also Cheng, 1985; Okuguchi, 1987). They found that Cournot equilibrium profits are greater than Bertrand equilibrium profits when goods are substitutes and vice versa when goods are complements. Moreover, they established that when private firms play the downstream duopoly game, their dominant strategy in a purely private duopoly is to choose quantity contracts if goods

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are substitutes, but to select prices if they are complements[1]. These studies of the choice of strategic variables for prices or quantities, have thus suggested important implications for the determination of market outcomes.

However, one issue that remains to be analyzed is whether the above results are robust to changes in the type of industry competition. To the best of the author's knowledge, no previous studies have considered the case of both private and public firms choosing to set quantities or prices in a mixed duopoly when the public firm is less efficient than the private firm. The present study aims to fill this gap in the literature. In this study, we investigate whether the standard results of the ranking of Cournot competition and Bertrand competition outcomes under a differentiated mixed duopoly hold. More specifically, we illustrate how the choice of strategic variables for setting quantities or prices affects social welfare in a mixed duopoly.

To the author's knowledge, only two previous studies have compared Bertrand and Cournot outcomes in a mixed oligopoly: Ghosh and Mitra (2010), Matsumura and Ogawa (2012) and Choi (2012). Given that the goods are substitutes, Ghosh and Mitra (2010) compared Cournot with Bertrand competition in a mixed oligopoly where the rankings of profit and social welfare are determined without public and private firms choosing strategic variables. Moreover, in a companion paper, considering the nature of goods, Choi (2009) introduced a case in which both private and public firms choose to set prices or quantities when trade unions are included in the mixed duopoly. Matsumura and Ogawa (2012) suggest that choosing Bertrand competition contract is a dominant strategy for both public and private firms when the efficiency gap between the public and private firms is ignored[2]. These studies assumed that the public firm is equally efficient as a private firm. However, the present study crucially differs from Matsumura and Ogawa (2012), Choi (2012) and Ghosh and Mitra (2010) as our focus is primarily on industry competition when private firms are more efficient than the public firm in the absence of trade unions[3].

In this paper, in order to strategically set prices or quantities when the public firm is less efficient than the private firm, we discuss two cases that are distinguished: (i) we adopt the assumption of positive primary outputs in the Zachenttin (2006) model, allowing for a wider range of cost asymmetry between the public and private firms (i.e., we relax Singh and Vives assumption). (ii) At the same time, we also investigate the case that the Singh and Vives assumption of positive primary outputs holds. In particular, Zanchettin (2006) found that Singh and Vives's (1984) result that firms always make larger profits under quantity competition than under price competition fails to hold[4].

First, using a canonical two-stage game model if and only if the Singh and Vives assumption of positive primary outputs holds, we demonstrate that given the private firm's profit maximization and depending on the degree of public firm's inefficiency when the goods are substitutes, private and public firms can choose to strategically set Bertrand competition or quantity-price competition. Thus, there can be sustained a unique subgame perfect Nash equilibrium in the contract stage. However, when the goods are complement, choosing Bertrand competition con-

[1]Dastidar (1997), Qiu (1997), Lambertini (1997), Häckner (2000), Zanchettin (2006), and Amir and Jin (2001), among others, have analyzed counterexamples based on the framework of Singh and Vives (1984) by allowing for a wider range of cost and demand asymmetries.

[2]Matsumura and Ogawa (2012) introduced the marginal cost of both public and private firms, denoted as m_0 and m_1, however, they ignored the nonnegativity conditions of the public firm's output in solving the problems.

[3]From the empirical studies, it is found that the public firm tends to be less efficient than the private firm. See e.g., Megginson and Netter (2001), La Porta and Lopez-de-Silane (1999), Warzynski (2003), and Nishiyama and Smetter (2007).

[4]Wang (2008) showed that while profit ranking between price and quantity competition can be partially reversed, the traditional result of Singh and Vives (1984) that firms always choose a quantity in a two-stage game continues to hold in the enlarged parameter space.

tract is a dominant strategy for the public and private firms regardless of the degree of public firm's inefficiency. More specifically, if the degree of inefficiency is sufficiently small or large when the the goods are substitutes, the total output under Bertrand competition is larger than that under quantity-price competition. Thus, welfare improvement under Bertrand competition is possible because the negative effect on social welfare is dominated by total output effect. On contrary, if the range of critical value falls in the middle range when the the goods are substitutes, it turns out that depending upon the degree of inefficiency, social welfare under Bertrand competition can be lower or higher than that under quantity-price competition. This also can be understood by comparing total output effect and the negative effect on social welfare. On the other hand, the *private* firm will always opt for Bertrand competition, regardless of whether the goods are substitutes or complements in the first stage(SPNE))[5].

Second, if we relax the parameter restriction imposed implicitly by Singh and Vives (i.e., we adopt Zanchettin (2006) assumption) to allow for a wider range of cost asymmetry, some results are greatly changed. That is, (i) Cournot and Bertrand competition coexist if the degree of the inefficiency of the public firm is sufficiently small or large (ii) our results relies on the different effects exerted by the three forms of competition on prices and market shares with multiple subgame perfect Nash equilibria. This is because the private firm always has an incentive to use limit-pricing policy except for the Cournot competition by adopting Zanchettin (2006) assumption.

Consequently, the main finding in the present study differs from Singh and Vives's (1984) conclusion in which the dominant strategy for private firms in a purely private duopoly is to choose the quantity contract if goods are substitutes and vice versa[6]. Because we endogenously investigate the type of contract in a mixed duopoly, this study differs from the current body of knowledge on this topic in two important aspects. First, previous studies of mixed oligopolies have considered an exogenous type of contract rather than an endogenous one when the public firm is less efficient than the private firm. Second, while previous studies have focused on the opposite results with regard to the Cournot-Bertrand profit differential in a purely private duopoly market, this study not only investigates the case when both private and public firms choose to set prices or quantities, but also determines how social welfare is affected by the type of contract structure.

The organization of the remainder of this paper is as follows. In Section 2, we describe the model. Section 3 presents fixed motives on the type of contract. Section 4 determines firms' endogenous choices of strategic variables. Concluding remarks appear in Section 5.

2 The Model

The basic structure is a differentiated duopoly model, which is a simplified version of Singh and Vives's (1984) model. Consider two single-product firms that produce differentiated products that are supplied by a public firm (firm 0) and a private firm (firm 1). We assume that the representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}\left(x_i^2 + 2bx_ix_j + x_j^2\right), \quad i \neq j; i, j = 0, 1,$$

[5]By contrast, Choi (2009) found that a public firm always chooses Bertrand competition and a private firm chooses either a Bertrand or a Cournot type of contract regardless of whether the goods are substitutes or complements (i.e., multiple SPNEs) when the public and private firms are equally efficient in the presence of trade unions.

[6]López (2007) and López and Naylor (2004) introduced the union utilities of private firms into a model of the choice of strategic variables for setting prices or quantities. They showed that Singh and Vives's (1984) result holds unless unions are powerful and place considerable weight on the wage argument in their utility functions.

where x_i denotes the output of firm i $(i = 0, 1)$. The parameter $b \in (0, 1)$ is a measure of the degree of substitutability among goods, while a negative $b \in (-1, 0)$ implies that goods are complements. In the main body of analysis, we focus on the imperfect substitutability case of $b \in (0, 1)$. However, we will mention later the imperfect complementarity case of $b \in (-1, 0)$ since it is easy to calculate when goods are complements[7]. Thus, the inverse demand is characterized by

$$p_i = 1 - bx_j - x_i, \tag{1}$$

where p_i is firm i's market price and x_i denotes the output of firm i $(i = 0, 1)$. Hence, we can obtain the direct demands as

$$x_i = \frac{1 - b + bp_j - p_i}{1 - b^2} \tag{2}$$

provided the quantities are positive.

The private firm has constant marginal cost of production, which is normalized to zero. The public firm also has constant marginal cost of production, however, it is assumed to be less efficient than the private firm. Let $\theta > 0$ be the inefficiency of the public firm. For simplicity, following Pal (1998), we assume that the profit of private firm 1 and the profit of the public firm 0 as follows:

$$\pi_1 = p_1 x_1 \quad \text{and} \quad \pi_0 = (p_0 - \theta)x_0,$$

respectively[8]. There is no fixed costs and the marginal cost of the public firm is constant at θ, where we assume $1 > \theta > 0$. Zanchettin (2006) introduced the parameter $a = (1 - 0) - (1 - \theta)$ (thus, $a = \theta$ in this paper) to measure the degree of cost asymmetry between the two firms. As pointed out by Zanchettin (2006), the monopoly outcome in which the private firm becomes a monopoly under either price or quantity competition prevails if $1 - (b/2) \leq \theta$[9]. Thus, following Zanchettin (2006), we set $1 - (b/2) > \theta$.

As is customarily assumed in a mixed oligopoly, the public firm's objective, SW, is to maximize welfare, which is defined as the sum of the consumer surplus and the profits of individual firms. Thus, the public firm aims to maximize social welfare, which is defined as

$$SW = U - \sum_{i=0}^{1} p_i x_i + \sum_{i=0}^{1} \pi_i = x_0 + x_1 - \frac{(x_0^2 + x_1^2)}{2} - cx_1 x_0 - \theta x_0 \tag{3}$$

where $U - \sum_{i=0}^{1} p_i x_i$ is the consumer surplus, and π_i is the profit of both the private and public firms. Note that the last term of (3) is the negative effect on social welfare since we assume that $1 > \theta > 0$.

This study considers that each firm can make two types of binding contracts with consumers, as described by Singh and Vives (1984) and López (2007). Thus, a two-stage game is conducted. The timing of the game is as follows. In the first stage, the private and public firms simultaneously commit to choosing strategic variable, i.e., either price or quantity (which determines the type of contract), to set in the mixed duopoly. In the second stage, each firm chooses its quantity

[7] We exclude independent case since the choice of strategic variables for setting prices or quantities does not change social welfare in a mixed duopoly.

[8] One drawback of the present model is that it does not explain the difference in efficiency between the private and the public firms, however, it assumes the public firm's inefficiency exogenously. See De Fraja (2009, pp. 5-6) for discussion of the public firm's inefficiency.

[9] Compared to Singh and Vives (1984), the space allows for a larger range of cost asymmetries between the two firms.

or price simultaneously, in order to maximize its objective knowledge of the strategic variable of the public and private firms. As in Singh and Vives (1984), we adopt the same assumption. i.e., that there are prohibitively high costs associated with changing the type of contract in the first stage.

3 Market Equilibrium under a Mixed Duopoly

3.1 Results: Fixed Contract Motives with Solutions for Substitutes

Before the type of contract is applied to the model to identify the point of equilibrium, we explain four different cases of contract games. In Bertrand competition, firms set prices, whereas in Cournot competition, firms set quantities. In mixed cases, either firm 0 sets the price and firm 1 sets the quantity or, vice versa. Such a game is solved by backward induction (i.e., the solution concept used is the subgame perfect Nash equilibrium (SPNE)).

3.1.1. [Cournot Competition Game]: Assume that firm $i(i = 0, 1)$ faces the demand functions given by $p_i = 1 - cx_j - x_i$. In the second stage, the first order conditions for firms 0 and 1 are

$$\frac{\partial SW}{\partial x_0} = 1 - cx_1 - \theta - x_0 = 0, \quad \frac{\partial \pi_1}{\partial x_1} = 1 - cx_0 - 2x_1 = 0, \tag{4}$$

respectively. Solving the first-order conditions in Eq. (4), we obtain following equilibrium values when the goods are substitutes.

$$x_0^{cc} = \frac{2 - b - 2\theta}{2 - b^2}, \quad x_1^{cc} = \frac{1 - b + b\theta}{2 - b^2}, \quad p_0^{cc} = \theta, \quad p_1^{cc} = \frac{1 - b + b\theta}{2 - b^2},$$
$$\pi_1^{cc} = \frac{(1 - b + b\theta)^2}{(2 - b^2)^2}, \quad SW^{cc} = \frac{7 - 6b - 2b^2 + 2b^3 - \theta(8 - 6b - 2b^2 + 2b^3) + \theta^2(4 - b^2)}{2(2 - b^2)^2}.$$

Note that the public firm is active in Cournot equilibrium (i.e., $x_0^{cc} > 0$) provided that $1 - \frac{b}{2} > \theta$, which implies that Zanchettin's (2006) assumption of positive primary outputs requires $1 - \frac{b}{2} > \theta$. Note also that when the goods are complements, the public firm always remains active in Cournot equilibrium. As it will be investigated below, regardless of the value of θ, the public firm is always active in each competition in the case of complements.

3.1.2. [Firm 0 Sets Quantity, Firm 1 Sets Price (Quantity-Price)]: Let firm 1 optimally choose its price as a best response to any quantity chosen by public firm 0, and let public firm 0 optimally choose its quantity as a best response to any price chosen by private firm 1. Each demand function that each firm i faces is given by

$$x_1 = 1 - bx_0 - p_1 \quad \text{and} \quad p_0 = 1 - b + bp_1 - (1 - b^2)x_0,$$

respectively. Thus, the best reply functions of the public and private firms with respect to x_0 and p_1 are given by:

$$\frac{\partial SW}{\partial x_0} = 1 - \theta - c - x_0 + c^2 x_0 = 0, \quad \frac{\partial \pi_1}{\partial p_1} = 1 - cx_0 - 2p_1 = 0. \tag{5}$$

Solving the first-order conditions in Eq. (5) and substituting the pair (x_0, p_1) into the pair (x_1, p_0) yields

$$p_1^{cb} = \frac{1-b+b\theta}{2(1-b^2)}, \quad x_0^{cb} = \frac{1-b-\theta}{1-b^2}, \quad p_0^{cb} = \frac{b-b^2+2\theta}{2(1-b^2)}, \quad x_1^{cb} = \frac{1-b+b\theta}{2(1-b^2)},$$
$$\pi_1^{cb} = \frac{(1-b+b\theta)^2}{4(1-b^2)^2}, \quad SW^{cb} = \frac{7-6b-9b^2+8b^3-\theta(8-6b-10b^2+8b^3)+\theta^2(4-5b^2)}{8(1-b^2)^2}.$$

Clearly, the public firm remains active in the quantity-price equilibrium if $1-b>\theta$[10]. On the contrary, if $1-\frac{b}{2}>\theta\geq 1-b$, quantity-price competition leads to the limit-pricing equilibrium of the private firm[11].

$$p_1^L = \frac{b-1+\theta}{b}, \quad x_1^L = \frac{1-\theta}{b}, \quad \pi_1^L = \frac{(1-\theta)(b-1+\theta)}{b^2}, \quad SW^L = \frac{(1-\theta)(2b-1+\theta)}{2b^2}. \tag{6}$$

3.1.3. [Bertrand Competition Game]; Consider that firm i faces the direct demand as in Eq. (2). In the second stage, by maximizing social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm) as follows: The public firm's and private firm's objectives are given by

$$\frac{\partial SW}{\partial p_0} = \theta + cp_1 - p_0 = 0, \quad \frac{\partial \pi_1}{\partial p_1} = 1-c+cp_0-2p_1 = 0, \tag{7}$$

respectively. Straightforward computation yields the equilibrium values under Bertrand competition

$$x_1^{bb} = \frac{1-b+b\theta}{(1-b^2)(2-b^2)}, \quad x_0^{bb} = \frac{1-b-\theta}{1-b^2}, \quad p_1^{bb} = \frac{1-b+b\theta}{2-b^2}, \quad p_0^{bb} = \frac{b(1-b)+2\theta}{2-b^2},$$
$$\pi_1^{bb} = \frac{(1-b+b\theta)^2}{(1-b^2)(2-b^2)^2}, \quad SW^{bb} = \frac{7-6b-15b^2+12b^3+11b^4-8b^5-3b^6+2b^7-\theta\Omega+\theta^2\Psi}{2(1-b^2)^2(2-b^2)^2},$$
$$where \quad \Omega = 8-6b-18b^2+16b^3+14b^4-8b^5-4b^6+2b^7 \quad and \quad \Psi = 4-9b^2+4b^4-2b^6.$$

Similarly, the public firm remains active in the Bertrand equilibrium if $1-b>\theta$. On the contrary, when $1-\frac{b}{2}>\theta\geq 1-b$, Bertrand competition leads to the limit-pricing equilibrium as in previous cases.

3.1.4. [Firm 0 Sets Price, Firm 1 Sets Quantity (Price-Quantity)]: Let firm 0 optimally choose its price as a best response to any quantity chosen by private firm 1, and let private firm 1 optimally choose its quantity as a best response to any price chosen by public firm 0. The first order conditions for firms 0 and 1 are, respectively

$$\frac{\partial SW}{\partial p_0} = \theta - p_0 = 0, \quad \frac{\partial \pi_1}{\partial x_1} = 1-b+bp_0-2(1-b^2)x_1 = 0. \tag{8}$$

[10]The public firm is active in quantity-price equilibrium provided that $1-b>\theta$, if and only if the Singh and Vives assumption of positive primary outputs holds. The market share of the public firm may be zero under quantity-price competition but is still positive under Cournot competition.

[11]When prices are such that $1-p_0-b(1-p_1)\leq 0$ from $x_1 = \frac{1-b+bp_0-p_1}{1-b^2} \leq 0$, the demand of good 1 reduces to $x_1 = 1-p_1$.

Substituting the pair (x_1, p_0) into the pair (x_0, p_1) yields the equilibrium values under the price-quantity, denoted as π_1^{bc}, p_i^{bc}, x_i^{bc}, and SW^{bc} are given by

$$x_1^{bc} = \frac{1-b+b\theta}{2(1-b^2)}, \quad x_0^{bc} = \frac{2-b-b^2-\theta(2-b^2)}{2(1-b^2)}, \quad p_1^{bc} = \frac{1-b+b\theta}{2}, \quad \pi_1^{bc} = \frac{(1-b+b\theta)^2}{4(1-b^2)},$$
$$p_0^{bc} = \theta, \quad SW^{bc} = \frac{7-6b-8b^2+6b^3+b^4-\theta(8-6b-10b^2+6b^3+2b^4)+\theta^2(4-5b^2+b^4)}{8(1-b^2)^2}.$$

Clearly, the public firm remains active in the price-quantity equilibrium if $1-\frac{b}{2-b^2} > \theta$[12]. If $1-\frac{b}{2} > \theta > 1-\frac{b}{2-b^2}(> 1-b)$, then it is straightforward to verify that the limit-pricing solution is given by Eq. (6) as in the previous case.

4 The Choice of Contract under a Mixed Duopoly

Once the equilibria for the four fixed types of contract and social-welfare levels have been derived as discussed in the preceding section, the type of contract can be determined endogenously by comparing each social-welfare level and private firm's profit. Therefore, we consider the cases of substitutes and complements at the same time. In addition, in order to strategically set prices or quantities when the public firm is less efficient than the private firm, we discuss two cases that are distinguished: (i) $1-\frac{b}{2} > 1-\frac{b}{2-b^2} > 1-b > \theta$, and (ii)$1-\frac{b}{2} > \theta > 1-\frac{b}{2-b^2} > 1-b$.

To employ the two-stage game, let "C" and "B" represent, respectively, Cournot and Bertrand competition with regard to each firm's choice. In this section, the SPNE will be found in the first stage for any given pair of competition types. Recall that we consider positive values of b are associated with substitutes and negative values with complements in the mixed market. Thus, if condition $1-\frac{b}{2} > 1-\frac{b}{2-b^2} > 1-b > \theta$ is satisfied, then the payoff matrix for the contract game can be represented as shown Table 1.

Table 1: Contract Game when $1-\frac{b}{2} > 1-\frac{b}{2-b^2} > 1-b > \theta$

Firm 1

		C	B
Firm 0	C	SW^{cc}, π_1^{cc}	SW^{cb}, π_1^{cb}
	B	SW^{bc}, π_1^{bc}	SW^{bb}, π_1^{bb}

Comparing π_1^{cc} with π_1^{cb}, and π_1^{bc} with π_1^{bb}, we find that, regardless of the nature of goods,

$$\pi_1^{cc} - \pi_1^{cb} = 3b^2 - 4 < 0, \quad \pi_1^{bc} - \pi_1^{bb} = b^2 - 4 < 0, \tag{9}$$

respectively. This is because regardless of what type the public firm chooses under either substitutes or complements, the output and price are higher when the private firm chooses to play Bertrand competition than when it chooses to play Cournot competition.

On the other hand, comparing SW^{bb} with SW^{cb}, we find

$$SW^{bb} - SW^{cb} = 4b^2 - 8b^3 + b^4 + 6b^5 - 3b^6 - \theta(8b^3 + 8b^4 + bc^5 - bc^6) - \theta^2(8b^4 + 3b^6), \tag{10}$$

when the goods are substitutes with $b \in (0,1)$, and when the goods are complements with $b \in (-1,0)$. By directly applying Eq. (9) to a discriminant, we have the roots, θ for $b \in (-1,0)$ and $b \in (1,0)$ because it is a second-order polynomial of θ and the maximum value is attained from $(8b^4 + 3b^6) > 0$ regardless of the nature of goods. Thus, Table 2 provides two roots, one positive and the other negative.

[12]The public firm is active in quantity-price equilibrium provided that $1-b > \theta$, if and only if the Singh and Vives assumption of positive primary outputs holds'

Table 2: Roots for θ with c when $1-\frac{b}{2} > 1-\frac{b}{2-b^2} > 1-b > \theta$

	$SW^{bb} - SW^{cb}$ if $b \in (0,1)$		$SW^{bb} - SW^{cb}$ if $b \in (-1,0)$	
c	θ_1	θ^s	θ_2	θ^c
0.1	-13.90593506	2.879783131	-4.436244115	13.48481199
⋮	⋮	⋮	⋮	⋮
0.21	-7.105525023	1.075968373	-2.479607931	6.367786933
0.22	-6.782865469	0.99229723	-2.395495358	6.075458571
0.3	-4.850474896	0.506258854	-1.905216584	4.44532138
0.5	-3.036602853	0.122317138	-1.24863589	2.677207319
⋮	⋮	⋮	⋮	⋮
0.96	-1.53882347	0.000129455	-0.337494341	1.417210948
0.97	-1.517436943	6.85709E-05	-0.319979814	1.402225712
0.99	-1.47530519	6.69887E-06	-0.285066	1.372916249

Since we assume that $1 > \theta > 0$,

$$SW^{bb} > SW^{cb} \quad \text{if} \quad \theta^s \in (0, 0.22) \quad \text{or} \quad \theta^s \in [0.97, 1) \tag{11}$$

when the goods are substitutes. Otherwise, when the goods are substitutes and the range of critical value falls into $\theta^s \in [0.22, 0.96]$, if there can exist a critical value of $\theta^s > \theta$ such that for all $\theta > 0$, we find the difference as $SW^{bb} > SW^{cb}$, and $SW^{bb} < SW^{cb}$ if $\theta^s > \theta$. Note that in the case of complements, $SW^{bb} > SW^{cb}$ because of the assumption of $1 > \theta > 0$.

Moreover, comparing SW^{cc} with SW^{bc} yields

$$\begin{aligned} SW^{cc} - SW^{bc} &= -\theta^2(4b^4 - 5b^6 + b^8) - \theta(8b^3 - 8b^4 - 10b^5 + 10b^6 + 2b^7 - 2b^8) \\ -4b^2 + 8b^3 + b^4 - 10b^5 + 4b^6 + 2b^7 - b^8 &= -b^2(1-b^2)(4-b^2)(1-b+b\theta)^2 < 0. \end{aligned} \tag{12}$$

Hence, regardless of the nature of goods, we always obtain that $SW^{cc} < SW^{bc}$. Having derived the comparison each social welfare for each critical value of θ, we can find the Nash equilibrium in the contract stage for any given set of private firm's profit and the level of social welfare in the mixed duopoly.

Clearly, there can be sustained a unique SPNE in the contract stage depending on the degree of the inefficiency of the public firm: (B, B) or (C, B). Hence, if the range of critical value falls into $\theta^s \in [0.22, 0.96]$ when the goods are substitutes, choosing Bertrand (respectively, quantity-price) competition is the best for public and private firms if $\theta < \theta^s$ (respectively, $\theta > \theta^s$). Otherwise, if the range of critical value falls into either $\theta^s \in (0, 0.22)$ or $\theta^s \in [0.97, 1)$ when the goods are substitutes, there can be sustained a unique SPNE in the contract stage: (B, B). In addition, regardless of the the value of θ when the goods are complements, there can be also sustained a unique SPNE in the contract stage: (B, B).

As in Tables 1 and 2, for the private firm, choosing Cournot competition is strictly dominated by choosing Bertrand competition, so the private firm never chooses Cournot competition. A unique SPNE of the two-stage game in a mixed duopoly is found and this can be stated in the following proposition:

Proposition 1: *Suppose that the condition $1-\frac{b}{2} > 1-\frac{b}{2-b^2} > 1-b > \theta$ is satisfied, there can be sustained a unique SPNE in the contract stage when the goods are complements: (B, B). On the one hand, if the range of critical value falls into either $\theta^s \in (0, 0.22)$ or $\theta^s \in [0.97, 1)$ when the goods are substitutes, the private and public firm always chooses a Bertrand type of contract. On the other hand, if the range of critical value falls into $\theta^s \in [0.22, 0.96]$ when the goods are substitutes, choosing Bertrand (respectively, quantity-price) competition is the best for* *public and private firms if $\theta < \theta^s$ (respectively, $\theta > \theta^s$) in the first stage.*

By restricting attention to the SPNE of the two-stage game, one significant result can be derived from Proposition 1: Sustaining of a unique SPNE from the private firm's dominant strategy and the degree of public firm's inefficiency thus plays an important role in the derivation of the result, as explained below.

Given the private firm's strategy (i.e., profit maximization), and once its decision variable has been set, the choice of the public firm only depends on its degree of inefficiency. If the degree of inefficiency is sufficiently small or large ($\theta^s \in (0, 0.22)$ or $\theta^s \in [0.97, 1)$) when the goods are substitutes, the total output under Bertrand competition is larger than that under quantity-price competition (i.e., $X^{bb} = x_0^{bb} + x_1^{bb} > X^{cb} = x_0^{cb} + x_1^{cb}$). Thus, welfare improvement under Bertrand competition is possible because the negative effect on social welfare (recall social welfare function from Eq. (3)) is dominated by total output effect. On contrary, if the critical value falls in the middle range ($\theta^s \in [0.22, 0.96]$) when the goods are substitutes, it turns out that depending upon the degree of inefficiency, social welfare under Bertrand competition can be lower or higher than that under quantity-price competition. This also can be understood by comparing total output effect and the negative effect on social welfare. These effects mean that each level of social welfare is comparable[13]. In our framework of a mixed duopoly, if the condition $1 - \frac{b}{2} > 1 - \frac{b}{2-b^2} > 1 - b > \theta$ is satisfied, there can be sustained a unique SPNE in the contract stage when the goods are complements regardless of the degree of inefficiency. However, depending upon the degree of inefficiency, social welfare under Bertrand competition can be lower or higher than that under quantity-price competition.

By contrast, in the study by Singh and Vives (1984) under a purely private duopoly, a dominant strategy exists for both firms that choose Cournot (or Bertrand) competition if the goods are substitutes (or complements). Contrary to finding of Singh and Vives (1984), we show that social welfare is higher under Bertrand than it is under quantity-price competition if the degree of inefficiency is sufficiently small, and vice versa.

Ghosh and Mitra (2010) compared Cournot and Bertrand competition in a mixed oligopoly without an endogenous type of contract when the public firm is equally efficient as a private firm. They demonstrated that despite the ambiguity in price ordering between Bertrand and Cournot competition for private firms, a comparison of quantities and profits yields unambiguous results. In other words, the public firm's output is higher under Cournot competition, whereas the private firm's output is lower in the same circumstances. In addition, the profits of both firms are lower under Cournot competition than they are under Bertrand competition. It should be noted that Ghosh and Mitra (2010) focused on the case of substitutes, however. The present study shows that the total output and social welfare under either Bertrand competition or quantity-price competition can be higher depending on the degree of inefficiency of the public firm; because choosing a Bertrand type of contract for private firm is preferable irrespective of the nature of goods. Moreover, when the public firm is less efficient than the private firms, the present study differs from previous studies of (unionized) mixed oligopolies, in which the private and public

[13] In reality, the services provided by public sectors are generally charged at lower prices than those provided by private counterparts. For example, in vertically differentiated mixed markets, a public firm typically sets its price lower with a lower quality than the private counterparts since it is concerned about social welfare even though the public firm is unprofitable under Bertrand competition. According to Ishibashi and Matsushima (2008) and Glazer and Niskanen (1997), governmental facilities are often small, of poor quality, and overcrowded, and many services provided by public sectors are also provided privately (e.g., public versus private schools and universities; public versus private medical care; public parks versus private golf courses). The reason for the existence of public firms is that they are socially desirable, even though they are unprofitable under either Bertrand or quantity-price competition. Theoretical results of present paper can claim considerable reliability, however, the result of empirical researches needs to be understood restrictively in the light of our theoretical results.

firms can choose to strategically set prices or quantities.

With the equilibrium levels, we are ready to assess the impacts on social welfare. By comparing the profits obtained under either Bertrand or quantity-price equilibrium in a mixed duopoly, the following proposition can be stated:

Proposition 2: *Suppose that the condition* $1 - \frac{b}{2} > 1 - \frac{b}{2-b^2} > 1 - b > \theta$ *is satisfied. In the equilibrium, regardless of the nature of goods, the private firm's profit under quantity-price competition is always higher than that under Bertrand competition.*

Proposition 2 states that the comparison of a private firm's profit holds irrespective of the nature of goods. This is because regardless of the nature of goods, output and price are smaller under a Bertrand contract than under a quantity-price contract (i.e., $p_1^{cb} > p_1^{bb}$ and $x_1^{cb} > x_1^{bb}$). In this case, the ranking of a private firm's profit is not reversed (i.e., $\pi_1^{cb} > \pi_1^{bb}$), which contrasts with the findings of Singh and Vives (1984).

Next, when the public firm is equally efficient as the private firm, we can easily obtain $SW^{bc} > SW^{cc}$ and $SW^{bb} > SW^{cb}$ regardless of the nature of goods[14]. The following corollary can thus be stated:

Corollary 1: *Suppose the public firm is equally efficient as the private firm. In equilibrium, regardless of the nature of goods and the range of critical value* θ^k*, there can be sustained a SPNE in the contract stage of the game: (B,B).*

Corollary 1 suggests that regardless of nature of goods, choosing the Bertrand contract is the best for each firm i, when the competitor's choice of contract is either the price or the quantity contract. This is because there exists a dominant strategy for both public and private firms that choose Bertrand competition, and $X^{bb} > X^{cb}$ and $X^{bc} \equiv x_1^{bc} + x_0^{bc} > X^{cc} \equiv x_1^{cc} + x_0^{cc}$ for the public firm when $\theta = 0$. Thus, in our framework of a mixed duopoly, there can be sustained a unique SPNE of Bertrand competition in the contract stage of the game when the public firm is equally efficient as the private firm. Corollary 1 differs from Choi (2012), which there can be sustained multiple SPNEs (i.e., (B,C) and (B,B) as in the present paper) in contract stage under unionized mixed duopoly if the public firm is equally efficient as the private firm. Moreover, Matsumura and Ogawa (2012) ignored the cost efficiency gap between the public and private firms. If the efficiency gap between the the public and private firms is ignored, choosing Bertrand competition contract is a dominant strategy for both public and private firms.

On the other hand, if condition $1 - \frac{b}{2} > \theta > 1 - \frac{b}{2-b^2} > 1 - b$ is satisfied, then the payoff matrix for the contract game can be represented as shown Table 3. Note that we do not consider the case of complements since the public firm is always active in all competition modes.

Table 3: Contract Game when $1 - \frac{b}{2} > \theta > 1 - \frac{b}{2-b^2} > 1 - b$

Firm 1

		C	B
Firm 0	C	SW^{cc}, π_1^{cc}	SW^L, π_1^L
	B	SW^L, π_1^L	SW^L, π_1^L

[14] From Eqs. (10) and (12), we obtain that regardless of the nature of goods, $SW^{cc} < SW^{bc} \Leftrightarrow -4c^2 + 8c^3 + c^4 - 10c^5 + 4c^6 + 2c^7 - c^8 < 0$ and $SW^{bb} > SW^{cb} \Leftrightarrow 4c^2 - 8c^3 + c^4 + 6c^5 - 3c^6 > 0$ when $\theta = 0$.

Comparing π_1^{cc} with π_1^L, and SW^{cc} with SW^L, we find that, when the goods are substitutes,

$$\pi_1^{cc} - \pi_1^L = 4 - 4b - 3b^2 + 2b^3 + 2b^4 - b^5 - \theta(8 - 4b - 8b^2 + 2b^3 + 4b^4 - b^5) + \theta^2(4 - 4b^2 + 2b^4) \tag{13}$$

$$SW^{cc} - SW^L = 8 - 16b + 6b^2 + 4b^3 - 2b^4 - \theta(16 - 16b + 4b^3) + 8\theta^2, \tag{14}$$

respectively. By directly applying Eqs. (13) and (14) to a discriminant, we have the roots, θ for $c \in (1, 0)$ because it is a second-order polynomial of θ and the minimum value is attained from $(4 - 4b^2 + 2b^4) > 0$ in Eq. (13) and $8 > 0$ in Eq. (14) when the goods are substitutes. Thus, Table 4 provides two positive roots: One root is always same between $\pi_1^{cc} - \pi_1^L$ and $SW^{cc} - G^L$.

Table 4: Roots for θ with c when $1 - \frac{b}{2} > \theta > 1 - \frac{b}{2-b^2} > 1 - b$

	$SW^{cc} - SW^L$ if $c \in (0, 1)$		$\pi_1^{cc} - \pi^L$ $c \in (0, 1)$	
c	θ^*	θ^w	θ^*	θ^π
0.01	0.995	0.9850005	0.995	0.9949995
0.1	0.95	0.8505	0.95	0.9494975
0.3	0.85	0.5635	0.85	0.835895192
0.5	0.75	0.3125	0.75	0.68
0.7	0.65	0.1215	0.65	0.444488533
0.9	0.55	0.0145	0.55	0.131357977
0.99	0.505	0.0001495	0.505	0.010391895

Noting that the critical value θ^π is always larger than θ^w, the graph of $\pi_1^{cc} - \pi_1^L$ and $SW^{cc} - G^L$ is shown in Figure 1.

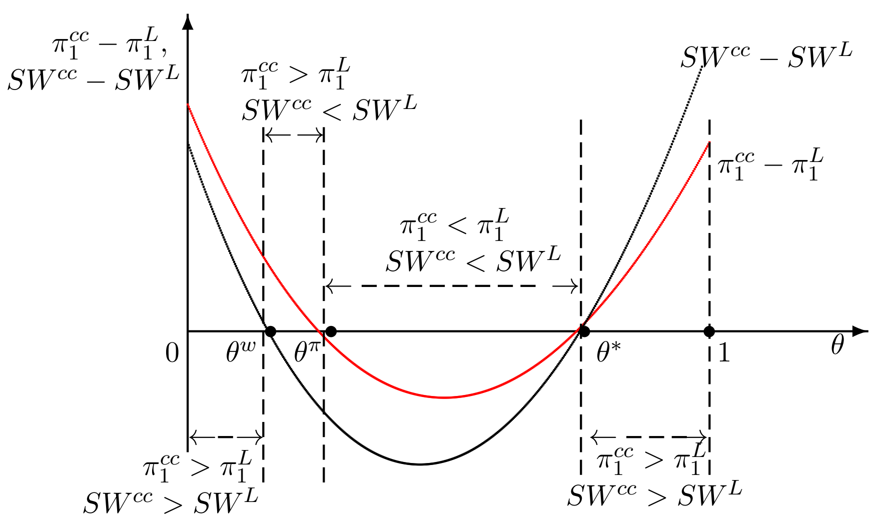

Figure 1. Graph of $\pi_1^{cc} - \pi_1^L$ and $SW^{cc} - SW^L$

Clearly, there can be sustained multiple SPNEs in the contract stage depending on the degree of the inefficiency of the public firm if $1 - \frac{b}{2} > \theta > 1 - \frac{b}{2-b^2} > 1 - b$. Hence, when the goods are substitutes and the range of critical value falls into either $\theta \in (0, \theta^w)$ or $\theta \in (\theta^*, 1)$, there can be sustained multiple SPNEs: (C,C) or (B,B). Otherwise, when the goods are substitutes and the range of critical value falls into either $\theta \in (\theta^w, \theta^\pi)$ or $\theta \in (\theta^\pi, \theta^*)$, there can be sustained multiple SPNEs by limit-pricing of the private firm, i.e., multiple SPNEs: (B,C), (B,B); (B,C), (B,B) and (C,B). Consequently, if $1 - \frac{b}{2} > \theta > 1 - \frac{b}{2-b^2} > 1 - b$, multiple SPNEs in a mixed duopoly are found and this can be stated in the following proposition:

Proposition 3: *Suppose that the goods are substitutes and the condition* $1-\frac{b}{2} > \theta > 1-\frac{b}{2-b^2} > 1-b$ *is satisfied, there can be always sustained multiple SPNEs in the contract stage: (B, B). (i) If* $\theta \in (0, \theta^w)$ *or* $\theta \in (\theta^*, 1)$*, then SPNEs are (C,C), (B,B), (ii) if* $\theta \in (\theta^w, \theta^\pi)$*, then SPNEs are (B,C), (B,B), (iii) if* $\theta \in (\theta^\pi, \theta^*)$*, then SPNEs are (B,C), (B,B) and (C,B). Commonly, choosing Bertrand competition always exists in the first stage.*

The fact that the Cournot and Bertrand competition coexist if the degree of the inefficiency of the public firm is sufficiently small or large differs from Matsumura and Ogawa (2012) and Choi (2012). The intuition of Proposition 2 is as follows. As Zanchentttin (2006) suggested, the three forms of competition on prices and market share affect different effects. Total output and the private firm's output under limit-pricing equilibrium are larger than those outputs than Cournot competition (i.e., $X^{cc} = x_1^{cc} + x_0^{cc} < x_1^L$ and $x_1^{cc} < x_1^L$). This tends to make to increase the market share of the private firm and social welfare higher under limit-pricing equilibrium than under Cournot equilibrium. On the other hand, when firms are asymmetric, the price under Cournot competition is higher than under limit-pricing equilibrium and this tends to reduce more. However, when θ is sufficiently small, the private firm obtains higher profit with higher market share under Cournot competition than under limit-pricing equilibrium. This makes total output higher with increasing social welfare. However, as θ increases, limit-pricing reduces the total output achieving higher market share. This effect makes the private firm's profit higher under limit-pricing equilibrium than under Cournot competition. Finally, if θ is sufficiently large, the private firm's output is decreased under limit-pricing equilibrium while increased under Cournot competition. Hence, market share effect of the private firm gets stronger and its price is lower than the public firm's when the efficiency gap between the public and private firms increases. Concerning industry profits, note that if θ is increased, a move from Cournot to limit-pricing competition shifts production from the private firm to the less efficient public firm, leading to greater total output. Wang (2008) argued that private firms always choose a Cournot contract in a two-stage game continues to hold in the enlarged parameter space when goods are substitutes, while our paper in the mixed duopoly suggests multiple SPNEs in the contract stage depending upon the degree of the public firm's inefficiency.

5 Concluding Remarks

In the present study, we investigated a differentiated mixed duopoly in which private and public firms can choose to strategically set prices or quantities when the public firm is less efficient than the private firm. A choice of strategic variables was proposed endogenously in the first stage. For the case of a mixed duopoly, when the Singh and Vives assumption of positive primary outputs holds, (i) Bertrand competition or quantity-price competition can occur depending on the degree of public firm's inefficiency when the goods are substitutes. (ii) regardless of its inefficiency, there can be always sustained Bertrand competition when the goods are complements. (iii) the ranking of a private firm's profit is not reversed. However, if we relax the parameter restriction imposed implicitly by Singh and Vives to allow for a wider range of cost asymmetry, there can be always sustained multiple subgame Nash perfect equilibria in the contract stage by each critical value of the public firm's inefficiency. Consequently, depending upon public firm's inefficiency leads the private firm to use the strategic commitment of Bertrand competition. This result contrasts with the findings of Singh and Vives (1984), who found that the dominant strategy for each private firm in a purely private duopoly is to choose either a quantity or a price contract[15].

[15] Our results differ from Matsumura and Ogawa (2012) who ignored the nonnegativity conditions of the public firm's output and the cost efficiency gap between the public and private firms in solving the problems.

We conclude by discussing the limitations of the present study. For example, it may be important to extend Singh and Vives's (1984) or Zachenttin's (2006) framework by assuming no ex-ante commitment over the type of contract that each firm offers to consumers. We have not extended the model to consider a situation in which there exists a wider range of cost and demand asymmetries, which have already been investigated by previous studies of a purely private duopoly. Moreover, we did not extend our results by considering nonlinear demand structures. An extension of our model in these directions would offer an avenue for future research.

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