On the mathematic prediction of economic and social crises: toward a harmonic interpretation of the Kondratiev wave

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On the Mathematic Prediction of Economic and Social Crises: Toward a Harmonic Interpretation of the Kondratiev Wave

By Scott A. Albers[*] and Andrew L. Albers[**]

Abstract: *In Part One of this paper we use the harmonic analogy of a musical octave to analyze mathematic ratios of U.S. real GNP. These ratios are generated by bringing together figures for U.S. real GNP over intervals of time – "spreads of years" – as numerator and denominator in a single fraction.*

Using a range of 7-year to 18-year "spreads," we find that this approach provides strong evidence that American economic history is composed of four 14-year quarter-cycles within a 56 year circuit in the real GNP of the United States, 1869-2007. These periods correlate closely with analysis by Nickolai Kondratiev and provide a framework for predicting an annual steady state rate of growth for the United States falling between 3.4969% and 3.4995% per year.

In Part Two of this paper we provide three postscripts including:

(1) correlations / speculations on the political and social consequences of this model,

(2) simplification / expansion of the geometries implied and

(3) analysis / prediction based upon this approach,

as concluded by a brief afterword.

These post-script refinements narrow the steady state rate of growth predicted to between 3.4969% and 3.4973% per year correlating closely with the 3.4971% rate for annualized quarterly data calculated for Okun's Law, 1947-2007. The size and interconnectedness of world economies, and the virtually exact correlations provided herein, suggest that the dates predicted for future crises will see changes which are unexpectedly global, dramatic and fierce.[1]

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Introduction: The Global Financial Crisis

On March 7, 2012 Professor William Black, Associate Professor of Economics and Law at University of Missouri - Kansas City, summarized in testimony before Congress the economic theory leading to the Global Financial Crisis. (Black, 2012) He states:

Neo-Classical Economic Policies are Criminogenic:
They Cause Control Fraud Epidemics

> Neo-classical economics (has) failed … to develop a coherent theory of fraud, bubbles, or financial crises (Black 2005). It continued to rely on a single methodological approach (econometrics) that inherently produces the worst possible policy advice during the expansion phase of a bubble. …
>
> A lender optimizes accounting control fraud through a four-part recipe. Top economists, criminologists, and the savings and loan (S&L) regulators agreed that this recipe is a "sure thing" – producing guaranteed, record (fictional) near-term profits and catastrophic losses in the longer-term. Akerlof & Romer (1993) termed the strategy: *Looting: Bankruptcy for Profit*. The firm fails, but the officers become wealthy (Bebchuk, Cohen& Spamann 2010). …
>
> The remarkable fact is that economists dominated financial policy and despite the success of the S&L regulators … neo-classical economists continues to ignore even the existence of accounting control fraud. They argued that such frauds could not exist because markets were "efficient." …
>
> The claim that no one could have foreseen the crisis is false. Unlike the S&L debacle, the FBI was far ahead of the regulators in recognizing that there was an "epidemic" of mortgage fraud and that it could cause a financial crisis. The FBI warned in September 2004 (CNN) that the "epidemic" of mortgage fraud would cause a "crisis" if it were not contained. [2], [3]

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paper were first published as a peer-reviewed research article on August 8, 2011 in *The Middle East Studies Online Journal*, H. Karoui, editor, Issue 6, Volume 3, pp. 199-253 at http://www.middle-east-studies.net/?p=22639. For the positions taken and the methods used herein we are alone responsible. This article is comprised of 16,441 words with a 235 word abstract.

[2] At the present time, and in the wake of the Global Financial Crisis, a large body of criticism of macroeconomics and its various models may be cited in support of this view. See e.g. Krugman, 2009: "So here's what I think economists have to do. First, they have to face up to the inconvenient reality that financial markets fall far short of perfection, that they are subject to extraordinary delusions and the madness of crowds. Second, they

This paper argues that a fundamental financial crisis could be expected to take place in 2005 based upon a 56-year cycle in American history of economic meltdowns in 1781, 1837, 1893, 1949 and – subsequently – in 2005.

We argue that the FBI's warning in September 2004 that a financial crisis was imminent correlates to predictions based upon this analysis to within a period of months.[4] This analysis is useful because, in addition to predicting dates for expected crises, it permits an explanation of the U.S. steady-state rate of growth presently calculated at 3.4971% per year for annualized quarterly data, 1947-2007. (Knotek, 2007)

Although this economic approach is of distinctly Russian vintage, in this article it will be applied to the economic history of the United States alone.

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have to admit .. that Keynesian economics remains the best framework we have for making sense of recessions and depressions. Third, they'll have to do their best to incorporate the realities of finance into macroeconomics. … To some economists (the "beauty" of their theories) will be a reason to cling to neoclassicism, despite its utter failure to make sense of the greatest economic crisis in three generations."

See also Solow, 2010. "(W)hen it comes to matters as important as macroeconomics, a mainstream economist like me insists that every proposition must pass the smell test: does this really make sense? I do not think that the currently popular DSGE ("Dynamic Stochastic General Equilibrium") models pass the smell test. They take it for granted that the whole economy can be thought about as if it were a single, consistent person or dynasty carrying out a rationally designed, long-term plan, occasionally disturbed by unexpected shocks, but adapting to them in a rational, consistent way. I do not think that this picture passes the smell test. The protagonists of this idea make a claim to respectability by asserting that it is founded on what we know about microeconomic behavior, but I think that this claim is generally phony. The advocates no doubt believe what they say, but they seem to have stopped sniffing or to have lost their sense of smell altogether."

See also Stiglitz, 2011. "Prediction is the test of a scientific theory. But when subject to the most important test - the one whose results we really cared about - the standard macroeconomic models failed miserably. Those relying on the Standard Model did not predict the crisis; and even after the bubble broke, the Fed Chairman argued that its effects would be contained. They were not. … Monetary authorities allowed bubbles to grow, partly because the Standard Models said there couldn't be bubbles. They focused on keeping inflation low, partly because the Standard Model suggested that low inflation was necessary and almost sufficient for efficiency and growth. They focused on *n*th-order distortions arising from price misalignments that might result from inflation, ignoring the far larger losses that result (and have repeatedly resulted) from financial crises. … (I)t was repeatedly claimed that it would be cheaper to clean up the aftermath of any bubble that might exist than to interfere with the wonders of the market. Thus, while financial markets and regulators have been widely blamed for the crisis, some of the blame clearly rests with the economic doctrines on which they came to rely (Stiglitz 2010a)."

[3] A candid appraisal of graduate education in economics is found at Smith, 2011. "(I)n spite of all the mathematical precision of these (economic) theories, very few of them offered any way to *calculate* any economic quantity. In physics, theories are tools for turning quantitative observations into quantitative predictions. In macroeconomics, there was plenty of math, but it seemed to be used primarily as a descriptive tool for explicating ideas about how the world might work. …

That was the second problem I had with the course: it didn't discuss *how we knew if these theories were right or wrong*. … (E)mpirics were only briefly mentioned, if at all, and never explained in detail. When we learned RBC (real business cycle), we were told that the measure of its success in explaining the data was - get this - *that if you tweaked the parameters just right, you could get the theory to produce economic fluctuations of about the same size as the ones we see in real life*. When I heard this, I thought "You have got to be kidding me!" …

The editors of *Econometrica*, the *American Economic Review*, the *Quarterly Journal of Economics,* and the other top journals are the ones who publish paper after paper on these subjects, who accept "moment matching" as a standard of empirical verification, who approve of pages upon pages of math that tells "stories" instead of making quantitative predictions, etc."

[4] This prediction was made publicly to Senator Max Baucus, Chairman of the Senate Finance Committee, in an early draft of this article entitled "The Coming Panic of 2005" on December 8, 2003.

The Kondratiev Wave

In his 1925 work *The Major Economic Cycles* Nikolai Kondratiev postulated a long-term wave running throughout the economic histories of various western countries of approximately 50 to 60 years.[5] (Kondratiev, 1925) Kondratiev's plan analyzed European and even global patterns of economic development with the thesis that democratic capitalism may possess the tools necessary to save itself from the inevitable self-destruction predicted by Marx and many of his disciples.[6] Kondratiev's original plan (Korotayev & Tsirel, 2010) provided dates for "upswings," "transition periods" and "downswings"[7] which Joseph Schumpeter's 1939 work *Business Cycles* acknowledged as significant to economics. (Schumpeter, 1939)

The academic search for evidence of "long waves" running through the economic history of various nation-states is long standing (Goldstein, 1988) and a central topic of heterodox economics. Indeed a 52-53 year cycle has been described in very extensive detail underlying the global meltdown (Korotayev and Tsirel, 2010) and incorporated into the study of the current revolutionary movements in the Middle East. (Tausch, 2011) As one modern researcher of Kondratiev Waves has remarked, "Altogether I think the idea of 55 year cycles in the behavior of our society is one of the most penetrating and useful in organizing social and economic facts." (Marchetti, 1988:7) However the dating and even existence of these periods are controversial.[8]

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[5] Kondratiev's work originated in the dangerous political context of prior socialist discoveries (Van Gelderen (1913), DeWolff (1924) and Kautsky (1917)) and communist theories (e.g. Marx, Lenin, Trotsky, Stalin) as to the evils of capitalism and the nature of its inevitable demise. (Goldstein, 1988:30-31) Kondratiev's suggestion that democratic capitalism might avoid such demise brought to him the censure of Stalin and death in a prison camp.

Orthodox economics, on the other hand, maintains an enormous breadth of opinion as to whether considerations of political policy must, or must not, be a part of doctrinal discipline. This paper concludes that there is much in Kondratiev's work which is directly applicable to the economic history of the United States, but does so without reference to Marx, et al.

[6] See Goldstein, 1988:30: "The Kondratieff-Trotsky long-wave debate … revolved around the question of the stability of capitalism. Do 'universal crises' threaten the survival of capitalism (as Trotsky thought), or are they only a phase of a more stable capitalist dynamic (as Kondratieff argued)? Kondratieff, like Kautsky, presented a picture of capitalism as more stable over the long term than either Trotsky or Lenin saw it. This parallel between Kondratieff's approach and that of the hated Kautsky may help to explain the very negative reception given to Kondratieff by his fellow Soviet Marxists."

[7] See Goldstein 1988:7. "Long waves (or Kondratieff cycles) are defined by alternating economic phases – an expansion phase (for which I will often use the more convenient term *upswing*) and a stagnation phase (which I will often call the *downswing*). These economic phase periods are not uniform in length or quality. The transition point from an expansion phase to a stagnation phase is called a peak, and that from stagnation to expansion is a trough. The long wave, which repeats roughly every fifty years, is synchronous across national borders, indicating that the alternative phases are a systemic-level phenomenon."

These terms are used in Korotayev and Tsirel, 2010:1-2, et seq. but may hide a diversity of views in light of contrasting research. See e.g. Korotayev and Tsirel, 2010:1-6, Goldstein 1988. See also Coccia, M. 2010:730-738. "(T)here are different long-wave chronologies and certain timings of long waves are often better for some countries but not for the world as a whole... These different cycles "do not have a synchronized rhythm across countries..."

[8] Orthodox economics rejects Kondratiev as a fallacy. See e.g. Rothbard, 1984. See also, e.g. Solomou, 1990:61. "(T)he evidence rejects the Kondratieff wave phasing of post-1850 economic growth. This conclusion is valid for all the national case studies examined here. Whether one takes the 1856-1913 or 1856-1973 a Kondratieff wave phasing can not be supported. … (O)bserved variations do not follow a Kondratieff wave pattern."

Mainstream analysis has focused rather on econometric measurements of other variables, i.e. the stochastic vs. deterministic effects governing the creation of real GNP itself. (See e.g. Nelson and Plosser, 1982) The

Studies in globalization have attempted to merge evolutionary theories[9] with fractal geometry, "emergence," the study of complexity and a host of other mechanisms in explication of Kondratiev Waves. Calls for clarification have followed as to the research methods, dates and theories surrounding "long waves."[10]

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distinction has been important for mainstream economics. (See e.g. Cochrane, 1988: "The distinction between a random walk … and a trend-stationary series … is extreme. Long-range forecasts of a random walk move one for one with shocks at each date, while long-range forecasts of a trend-stationary series do not change at all. There are two related ways to think about a series that lies between these two extremes.")

The significance of this inquiry however may be questionable. (Sowell, 1992: "The fact that postwar GNP series cannot distinguish between a time trend and a unit root model has important implications for theoretical models of the economy. Attention should be given to models where both the policy and theoretical implications of interest are not sensitive to the model of the trend. Ideally we would like a model which implies the same results if the trend is modeled as either a time trend or a unit root. Until such models are developed, further attention should be given to new statistical techniques which focus on discovering the long-run behavior of time series.")

[9] See e.g. Modelski, G. (2008:5) "(There are) two important implications of this evolutionary approach: first, there is reason to believe that an analysis drawing on evolutionary theory lends itself to modeling, simulation, and forecasting. Secondly, such an approach allows us to view globalization as an enterprise of the human species as a whole. … The emphasis is not on broad based accounts of the course of world affairs but, selectively, on processes that reshape the social (including economic, political, and cultural) organization of the human species; processes such as urbanization, economic growth, political reform and world organization, and the making of world opinion; and the innovations that animate these developments.

[10] See e.g. Devezas, T., Corredine, J. (2001) "… Complexity theory and nonlinearity are currently hot topics of interdisciplinary interest among the natural and social sciences, but still fall short of explaining the cyclic and evolutionary dynamics of society. … Although much has been published on K-waves, we must consider:

1 - a comprehensive and embracing theory of Kondratiev economic cycles still needs to be elaborated, while at least four major issues remain to be clarified:

i - why is there disregard among many contemporary economists and social scientists, some of them even stubbornly rejecting the existence of these waves?

ii - what is to be understood about the causality of the phenomenon - not just the mechanisms, but also the underlying causes?

iii- why the half-century beat? and since when? (only after, or even before the Industrial Revolution?, and more: where did the clock come from?).

iv- will there be more Kondratievs? Free-will or determinism? …

3 - The use of new tools of science mentioned above may lead us to a better understanding of the causality of the phenomenon. … But the question remains: is it something endogenous, inherent to social behavior of the human being? Or is there some kind of exogenous causality (external to human beings, even cosmic causes?). The understanding of all the above-mentioned aspects (not only in their economic character, but as a whole physical or social phenomenon), could contribute significantly to futures research, helping us trace the best trajectory through the coming millennium. ..."

A Harmonic Interpretation of the Kondratiev Wave

This paper seeks to establish that a cycle of a fixed 56-year length has a significant impact upon the economy of the United States. The economic history of the United States is the sole topic of this paper inasmuch as:

(1) the United States has not suffered from the invasions and border reductions which have typified virtually all other countries available for consideration, thereby permitting an equivalence between the data generated and the subject studied over the long term,

(2) the economic data pertaining to the United States is long-standing, precise, self-consistent, authoritative and easily available, and

(3) the combination of a single political sovereignty with the right to tax, a national legal jurisdiction of arbitrary finality and a monetary / fiscal policy orchestrated by a single government have been central characteristics of the economic history of the United States from at least 1868.

Frequently the effort is made to assert that the Kondratiev Wave is of international significance. However in this paper we deal only with the United States and no other political body.

We suggest that (1) these conflicts regarding the Kondratiev wave may be traced two common paradigms for economics – physics[11] and biology[12] – and that (2) these conflicts may be brought together in the analogy of musical harmony.[13], [14]

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[11] See e.g. McCauley, 2009:9. "Econophysics, simply stated, means following the example of physics in observing and modeling markets."

[12] See e.g. Alfred Marshall (1842-1924) (1920:19) "The Mecca of the economist lies in economic biology rather than economic dynamics."

See also Nicholas Georgescu-Roegen. (1906-1994) (1977:361) "The term (bio-economics) is intended to make us bear in mind continuously the biological origin of the economic process and thus spotlight the problem of mankind's existence with a limited store of accessible resources, unevenly located and unequally appropriated." (As quoted in Gowdy 1993:149)

See also Devezas, Tessaleno (2001). Tessaleno Devezas, George Modelski, (2003).

[13] A third paradigm for economics which bears on this might be entitled "pure logic." See e.g. Karl Marx and his use of the Hegelian dialectic. "The implications of the dialectic, for both Hegel and Marx, were that all history, and indeed all reality, is a process of development through time, a single and meaningful unfolding of events, necessary, logical, and deterministic; that every event happens in due sequence for good and sufficient reason (not by chance); and that history could not and cannot happen any differently from the way it has happened and is still happening today." (Palmer 1969:498-499).

This approach went far beyond the realm of economics. See Ollman, 1976:53. "Marx's own interest in the physical sciences were sufficiently strong to bring him regularly to the lectures of Liebug and Huxley. Darwin, to whom he wanted to dedicate Capital I, was a constant fascination. And though he never wrote on the physical sciences (other than in letters), there are a number of remarks which indicate clearly his agreement with Engel's dialectical approach to nature. Such, for example, is his claim that the law of transformation from quantity to quality ... provides the basis of molecular theory in chemistry; and elsewhere, referring to the same law, he says, "I regard the law Hegel discovered ... as holding good both in history and in Natural Sciences."

At the opposing end of the political spectrum see also Ludwig von Mises, founder of the Austrian school of economics and its study of "praesxology." (von Mises 1949:32) "Praxeology is a theoretical and systematic, not

In this analogy the physical sequence of moments in time (x-axis) is contrasted with their biologic importance in the development of the human person (y-axis). As this analogy expands to aggregates of many human beings – and particularly with reference to the nation-state – it may be anticipated that this larger dimension of human personality will bear within it the structural characteristics of its members as exhibited in the Kondratiev Wave.

In essence, the Kondratiev Wave is the snowflake, and the human being is the water molecule. Like the electric current which ties the larger snowflake to the associated water molecules in an ever balancing and perfect symmetry of both, so is the causation underlying the Kondratiev Wave one of balancing the energies of the individual with society, and society with the individual. A balancing, harmonic sort of causation is at work here, one in which the smaller forms the seed crystal of the larger but nevertheless congruent society.

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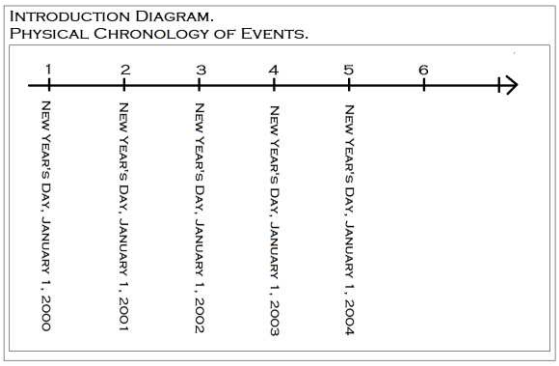
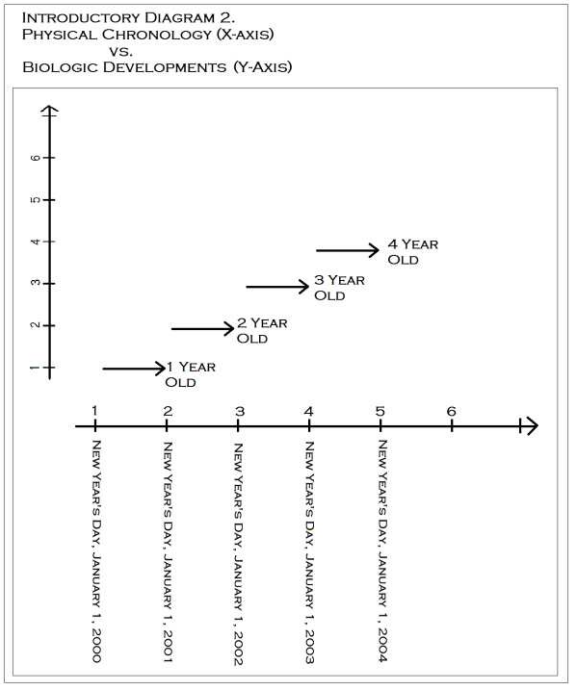
historical, science. Its scope is human action as such, irrespective of all environmental, accidental, and individual circumstances of the concrete acts. ... Its statements … are, like those of logic and mathematics, *a priori*. … They are both logically and temporally antecedent to any comprehension of historical facts." (von Mises 1949:34) "The fundamental logical relations … are primary propositions antecedent to any nominal or real definition. … The human mind is utterly incapable of imagining logical categories at variance with them. No matter how they may appear to superhuman beings, they are for man inescapable and absolutely necessary."

[14] As to requirements for a theory of causation for long waves, see Louca, F. (1999). "According to Kuznets, two conditions had to be met in order to establish the credibility of the Long Wave program: (for the "weak version of the recurrence requirement") one must prove (i) that the oscillations are general, and (ii) that there are either external factors or internal peculiarities within the economic system that create the recurrence (Kuznets, 1940:267). … A stronger version… means that the recurrence must conform to further definitions: a time variation in certain very precise limits and under well definied and stable causal relations – i.e. that the previous phase causes the next phase in the cycle or that sequence not only exists but also that causality can be exhaustively accounted for. This may be called the *strong version of the recurrence requirement*. … Rosenberg and Frischtak prolonged (the debate) by requiring the research programme on Long Waves to indicate a specific form of *causality, timing, recurrence* – precisely what was implied by Kuznets and Lange – and *economy-wide repercussions* of such fluctuations in order to be valid."

To introduce these ideas briefly, let us propose that a child is born at 1:00 a.m., January 1, 2000. On this day the child experiences his first New Year's Day. From this point we may chart the chronologic sequence of his second, third, fourth, etc. New Year's Day, as follows.

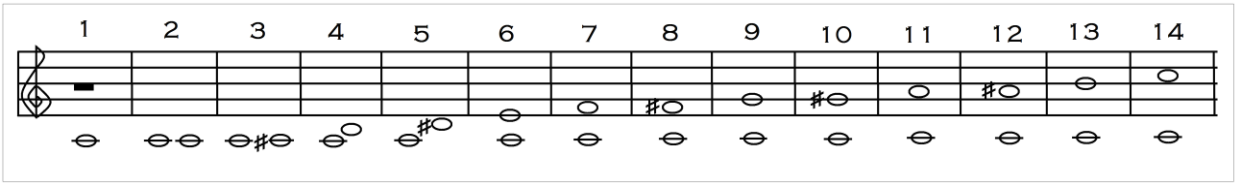


This counting of dates is to be distinguished from the counting of the child's birthdays. To experience one's first birthday party, or second, or third, etc. is a celebration of developmental growth. Each year claimed by a new birthday arrives with the celebration of a new biologic level of accomplishment. This concept of biologic development may be placed along the y-axis as follows.

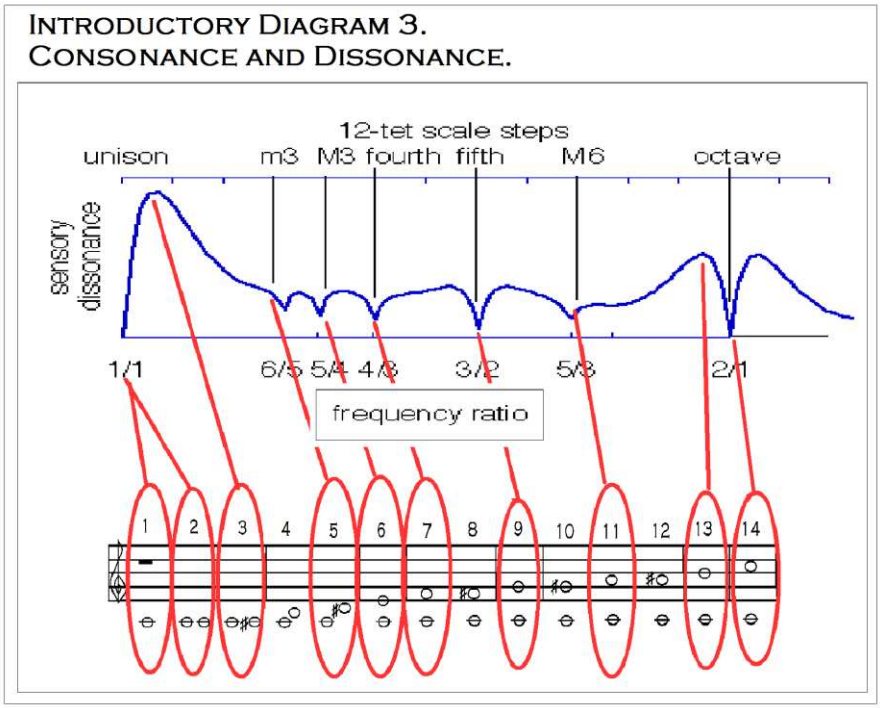


Arranging biologic development along the y-axis biologic growth, as contrasted with the chronologic sequencing of on-going New Year's Days along the x-axis, allows us to see in this simple example the merger of physical and biologic sequences typical of all human life, development and growth.

This ordering of physical dates against biologic development finds a parallel in the study of Pythagorean harmonics. It is well known that Pythagoras first developed the modal system of Western harmony upon noticing that a vibrating string, cut exactly in half, produced a pleasant, melodious sound, whereas even a slight alteration from the division of the string into perfect halves produced dissonant, unpleasant discording sounds. From this a spectrum emerged – the eight tones of the ancient modal scale made famous by Pythagoras, and the thirteen halftones of the modern chromatic scale made famous by J. S. Bach, each based upon the mathematic division of a vibrating string. Upon this modal system the entire spectrum of Western harmony has emerged.



The point in this comparison is that the physical structure of a vibrating string is to be distinguished from the "harmony" which one finds as a subjective individual listening to the relationships which exist in these vibrations as to "consonance" and "dissonance." The "sensory dissonance" (measured below in blue) indicates the level of harmony vs. dissonance for each of the intervals above.



Of importance for this paper, between solitary note Middle C and its octave there exist 14 separate intervals. A similar span of fourteen distinct years of human development may be explored as human development passes through childhood and reaches adolescence.

As demonstrated below, of the 15.6 million "regular secondary school students" in the United States in 2007-2008, 12.5 million (79.7%) were enrolled in school systems which ended primary school at eighth grade and began enrollment in secondary school at ninth grade. This break occurs generally at the age of 14. (total student population of these schools, including 9, 10, 11, 12 grade students in red lettering below).

		Student Population	School System[15]
1.	Total, all secondary schools (post-primary)	16,184,724	24,426
2.	Total, all regular secondary schools	15,680,507	19,264
3.	Grades 7 to 8 and 7 to 9	1,578,163	3,047
4.	Grades 7 to 12	927,888	3,278
5.	Grades 8 to 12	451,656	777
6.	Grades 9 to 12	12,500,341	15,179
7.	Grades 10 to 12	418,850	748
8.	Other spans ending with Grade 12	41,545	378
9.	Other grade spans	266,281	1,409

The en masse separation of primary and secondary education into two completely different school systems tracks the tremendous difference between the end of childhood (in aggregate at the age of 14) and the beginning of adolescence and onset of procreative capabilities (in aggregate at the age of 14). [16] Certainly the popularity of alternative systems to the 9-12 scheme, as measured by student enrollment, leaves little doubt that the preferred transfer date for students from primary to secondary education is at the age of 14. Other ages for transfer to secondary enrollment are less popular by ratios of 13:1, 27:1, 29:1, 46:1 and 300:1.

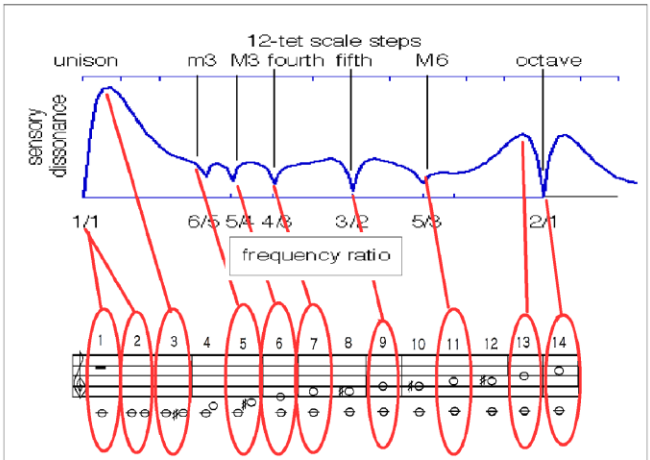
		Student enrollment	Comparative size to enrollment in 9-12 system	
4.	Grades 7 to 12	927,888	1:	13.47
5.	Grades 8 to 12	451,656	1:	27.67
6.	Grades 9 to 12	12,500,341	1:	1
7.	Grades 10 to 12	418,850	1:	29.84
8.	Other spans ending with Grade 12	41,545	1:	300.88
9.	Other grade spans	266,281	1:	46.94

[15] Taken from the Digest of Education Statistics, Table 99, Public secondary schools, by grade span, average school size and state or jurisdiction: 2007-2008, National Center for Education Statistics; and Enrollment of public secondary schools, by state, 2007-2008, collected at the request of the authors from the NCES on Friday, June 10, 2011. Data Set Six and Seven are at the conclusion of this paper.

[16] This approach may parallel studies emphasizing the role of learning in the structure of globalization. See e.g. Marchetti, C. (1980) and Devezas, T., et al. (2008:32) "The framework proposed by Devezas and Modelski opens the door to conceptualizing the emergence of world organization and, more recently of globalization, as a process of systemic learning, which leads in turn to the concept of a learning civilization."

Viewed in aggregate, the 14th year of life may be a fundamental biologic rhythm, one which lays through biologic fertility the economic basis for a 14-year spread in the higher social level of the Kondratiev Wave.

INTRODUCTORY DIAGRAM 4.
CONSONANCE, DISSONANCE AND A 14-YEAR TIME SPAN



PRIMARY SCHOOL	BIRTH AND INFANCY	TO	14TH BIRTHDAY
SECONDARY SCHOOL	14TH BIRTHDAY	TO	28TH BIRTHDAY
EARLY CAREER	28TH BIRTHDAY	TO	42ND BIRTHDAY
MID-CAREER	42ND BIRTHDAY	TO	56TH BIRTHDAY
LATE CAREER	56TH BIRTHDAY	TO	70TH BIRTHDAY
RETIREMENT	70TH BIRTHDAY		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	
15	16	17	18	19	20	21	22	23	24	25	26	27	28	
29	30	31	32	33	34	35	36	37	38	39	40	41	42	
43	44	45	46	47	48	49	50	51	52	53	54	55	56	
57	58	59	60	61	62	63	64	65	66	67	68	69	70	
71	72	73	74	75	76	77	78	79	80	81	82	83	84	

If this is true, then it should be possible to find in these repeated 14-year cycles a pattern of human development over time. These are provided in the graph to the left wherein the human development is separated by periods of 14 years stages of: "Primary School," "Secondary School," "Early Career," Mid-Career," "Late Career" and "Retirement." These stages are the "harmonies" of the economy as we move forward in aggregate through time.

An additional aggregate of human beings is their labor and the production of that labor. Consequently we suggest that there are "harmonies" within this productivity which – like the musical intervals above – occur over time.

The question arises: If this is so, may we demonstrate the "octave" of relationships within the economy, the fundamental building block of economics? If so, does this discovery provide the basis for an endogenous and biologic causality for the Kondratiev Wave, at least as understood within the context of the development of the American economy?

Part One:
Economic Methodology

2. Hypothesis

Our hypothesis is that the 50-60 year Kondratiev Wave is in reality a wave form composed of a number of smaller well-defined parts. Possible wavelengths can be evaluated and distinguished from one another by examining the underlying ratios of real GNP in the United States over various "intervals of years" or "spreads of years" which make up the cycle itself.

3. Methods

1. Prices.

In the first section of this paper we establish a data set for prices in the United States for the period 1801 through 1993. The two data sets which provide this information have a clear splicing multiple of 3. This data set of 193 years is then analyzed by:

a. collecting figures from two United States Federal Government data sets;

b. splicing these figures together into a single data set by way of their "splicing multiple" of 3;

c. placing the figures in centered moving 7-year averages;

d. determining the annual change in these centered moving 7-year averages; and

e. dividing this change in "d." for any given year by the centered moving 7-year average for that year under the heading "Change / Average Inflation."

Gross National Product.

We also establish a coherent and reasonable set of real GNP numbers for the United States for the period 1868 through 2007. This involves:

f. collecting figures from two United States Federal Government data sets;

g. examining the 23 years of overlap between these two data bases, i.e. 1947-1970;

h. choosing the second of two proposed "splicing multiples" and then splicing these data sets into a single data set for the purposes of this paper.

2. In the second section of this paper we examine ratios of U.S. real GNP. A ratio of GNP is a numeric fraction which takes as its numerator the real GNP of one year and takes as its denominator the real GNP of an earlier year. The number of years between numerator and denominator is referred to as a "spread of years" or simply a "spread."

We investigated spreads of years between numerator and denominator ranging from a 7-year spread between years to an 18-year spread between years. This range was chosen because it seemed likely to include the most eligible sub-cycles for a Kondratiev Wave of 50-60 years. We thought that if the Kondratiev Wave was in reality seven 7-year sub-cycles, or three 18-year sub-cycles, etc. this range of investigation might demonstrate such a finding.

This requires:

a. creating ratios between years of un-averaged figures U.S. real GNP as taken across spreads of years, (we use spreads of 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18 years),

b. placing them in Excel spread sheets wherein each year of the spread is given a row of the spreadsheet and the number of columns is in inverse proportion to the number of rows,

c. examining the patterns and variances which emerge as to the High, Midrange, Median Average and Low of the ratios generated in both rows and columns, and

d. using the concepts "General Dissonance," "Used General Dissonance," "Acute Dissonance" and "Claimed Dissonance" we determine the best sub-cycle from which to compose the larger, encompassing long wave.

3. In the third section of this paper we delineate which cycle best fits as a sub-cycle within a larger periodic wave.

4. In the fourth section of this paper, we examine the data set to find the fundamental Median Average between GNP values given by this analysis.

A first post-script is added to this paper wherein we correlate social and political changes to the Federal constitution according to the dynamics of this model and further speculate as to the underlying pattern involved.

A second post-script is provided wherein the model is simplified and expanded.

A third post-script provides a final analysis with predictions based upon the model provided.

A brief Afterword concludes this paper.

4. Data

We located two sources for US prices 1800 through 1993.

Series E 135-166, "Consumer Price Indexes (BLS - all items, 1800-1970, and by groups, 1913-1970), pp 210-211, of the book *Historical Statistics of the United States: Colonial Times to 1970, Part 1*, published by the United States Department of Commerce.

The Consumer Price Index of 1997, also published by the United States Department of Commerce, continues this series by dividing the historic series by 3, or a multiple of 1/3.

We located two sources for real US GNP.

Figures for U. S. Real GNP 1869-1970 may be found in the book *Historical Statistics of the United States: Colonial Times to 1970, Part 1*, published by the United States Department of Commerce. Series F 1-5 presents "Gross National Product" for the United States between the years 1869-1970 according to 1958 prices. The years 1869-1878, and 1879-1888 are given with decade averages of 23.1 billion and 42.4 billion dollars respectively.

Figures for U. S. Real GNP 1947-present are collected by the St. Louis Federal Reserve.[17]

Miscellaneous

Each spreadsheet is a mathematic arrangement of the figures given in "Data Set 2 – U.S. Real GNP."

Data Set 3, infra, is a compilation of all "Midrange Minus Median Average" values which are created by the spreadsheets.

Data Set 4, infra, is a summary of all spreadsheets.

Data Set 5, infra, is a mathematic re-arrangement of Data Set 1.

Appendices.

Data Sets 6 and 7, infra, provide secondary school statistics mentioned in the Afterword.

[17] These figures are available at: http://research.stlouisfed.org/fred2/series/GNPC96

5. Procedure

5.1. Section One: Establish Data Set

5.1.a. Collecting Data - Prices

We began with the Consumer Price Index listed in Series E 135-166 of the *Historical Statistics of the United States: Colonial Times to 1970, Part 1* (column 3) and compared this with the Consumer Price Index of 1960–1997. (column 1) The years of overlap clearly reduce the number for the historic series to a precise one-third of its value as the value given for the modern series. (column 2)

5.1.b. Splicing - Prices

We then spliced these two series into a single data set for prices based upon the values given in the historic series. We continued this data set past 1970 by multiplying the modern number by 3 and including this value in the final data set. (column 4)

5.1.c. Centered moving 7-year averages - Prices

We then figured centered moving averages for seven-year periods for the entire series. In this format a price index is averaged for seven sequential years and the average is placed at the middle term, e.g. the price indices for 1870, 1871, 1872, 1873, 1874, 1875, 1876 are averaged and placed as the figure for 1873. The process then continues to the next seven-year series by dropping the first and adding the next year in the chronology and beginning the averaging again. The technical term for this alteration of the data is "smoothing." (column 5)

5.1.d. Annual Changes in running 7-year averages - Prices

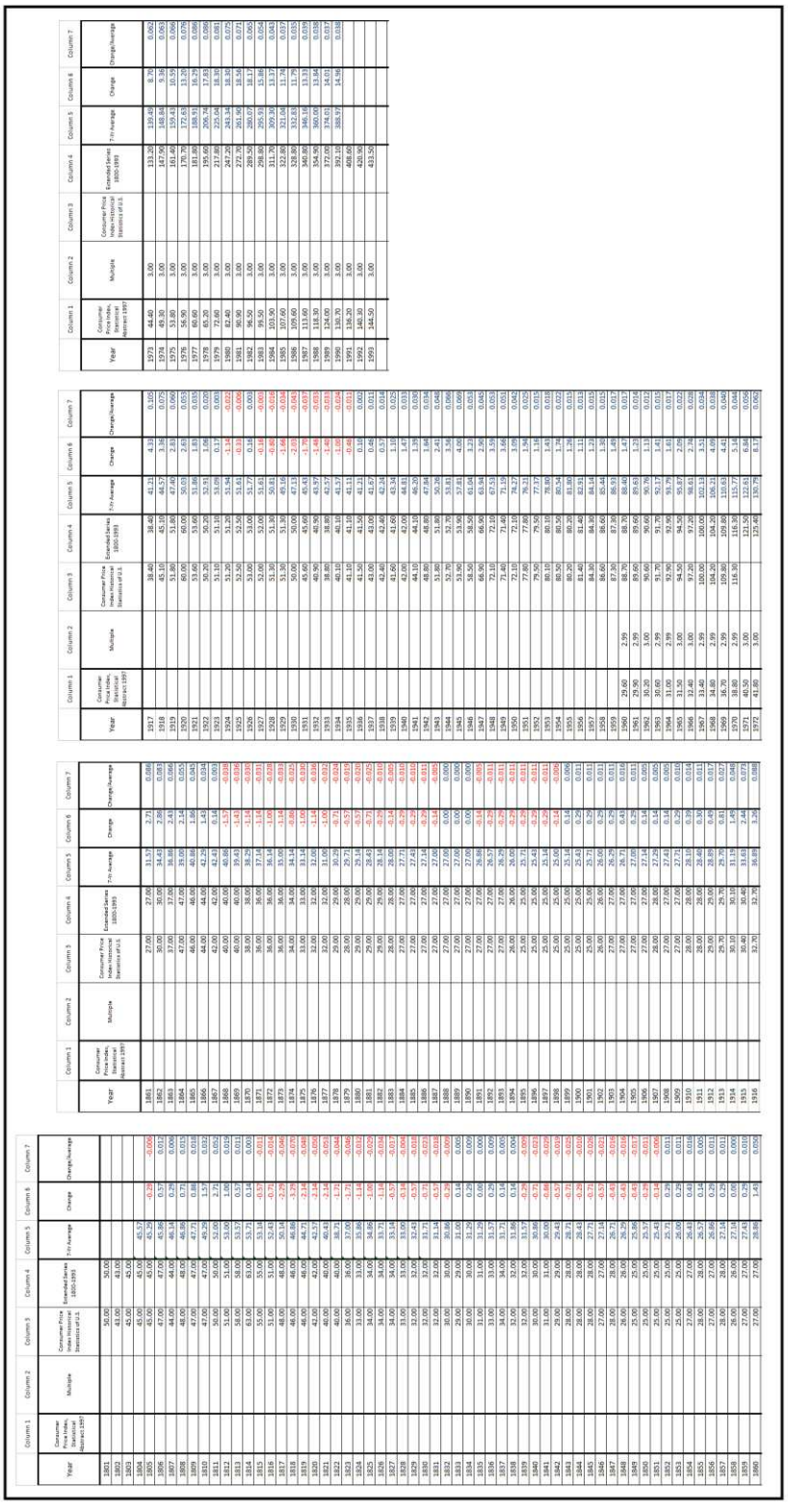
We then found the annual change between 7-year running averages for each year, and placed these next to the centered moving average itself. (column 6)

5.1.e. "Change / Average Inflation" - Prices

We then divided the annual change in 7-year running averages for a given year by the 7-year running average for that year, to be denominated "Change / Average Inflation." In this way the larger numbers for the Consumer Price Index found in later years were brought into conformity with the price patterns of prior years. (column 7)

The resulting "Data Set 1 – Prices" is as follows.

Data Set 1 – Prices.



5.1.f. Collecting Data – US Real GNP.

The United States Department of Commerce has published one set of numbers based upon 1958 prices running extending from 1869 through 1970. (column 2) The St. Louis Federal Reserve has published a different sequence of numbers based upon 2005 prices extending between 1947 through to the present day. (column 7)

Splicing multiples are quite necessary when considering two different series each of which proposes to calculate U.S. Real GNP over different periods of time. To "splice" or to "graft" these two sets together is necessary if an extended series running from 1869 to the present day is to be obtained. There does not exist at the present time such a series published by the United States Government. Consequently our first step in the analysis is to construct such a series as the foundation of this approach.[18]

5.1.g. Dates of overlap – US Real GNP

We considered two possible multiples with which to splice these two series of U.S. Real GNP figures together. The first possible splicing multiple is 5.881696, the average of all 23 multiples between 1947-1970. These are the years during which these two separate series overlap. (column 6) This number is problematic in that there is a clear drift from 1947 through 1970 toward higher multiples. Figures from 1947-1960 range from 5.646318 (1953) to 5.977644 (1958) and average at 5.8239423. Figures from 1961-1970 range a bit higher, i.e. from 5.907649 (1962) to 6.071220 (1965).

A second possible splicing multiple is 5.962552, the average of the final ten years of overlap, i.e. between 1961-1970. This multiple is the one used to splice these series in this paper as it is nearer in time to the eventual cutoff between the series and includes only multiples found in the later and more recent multiples. (column four)

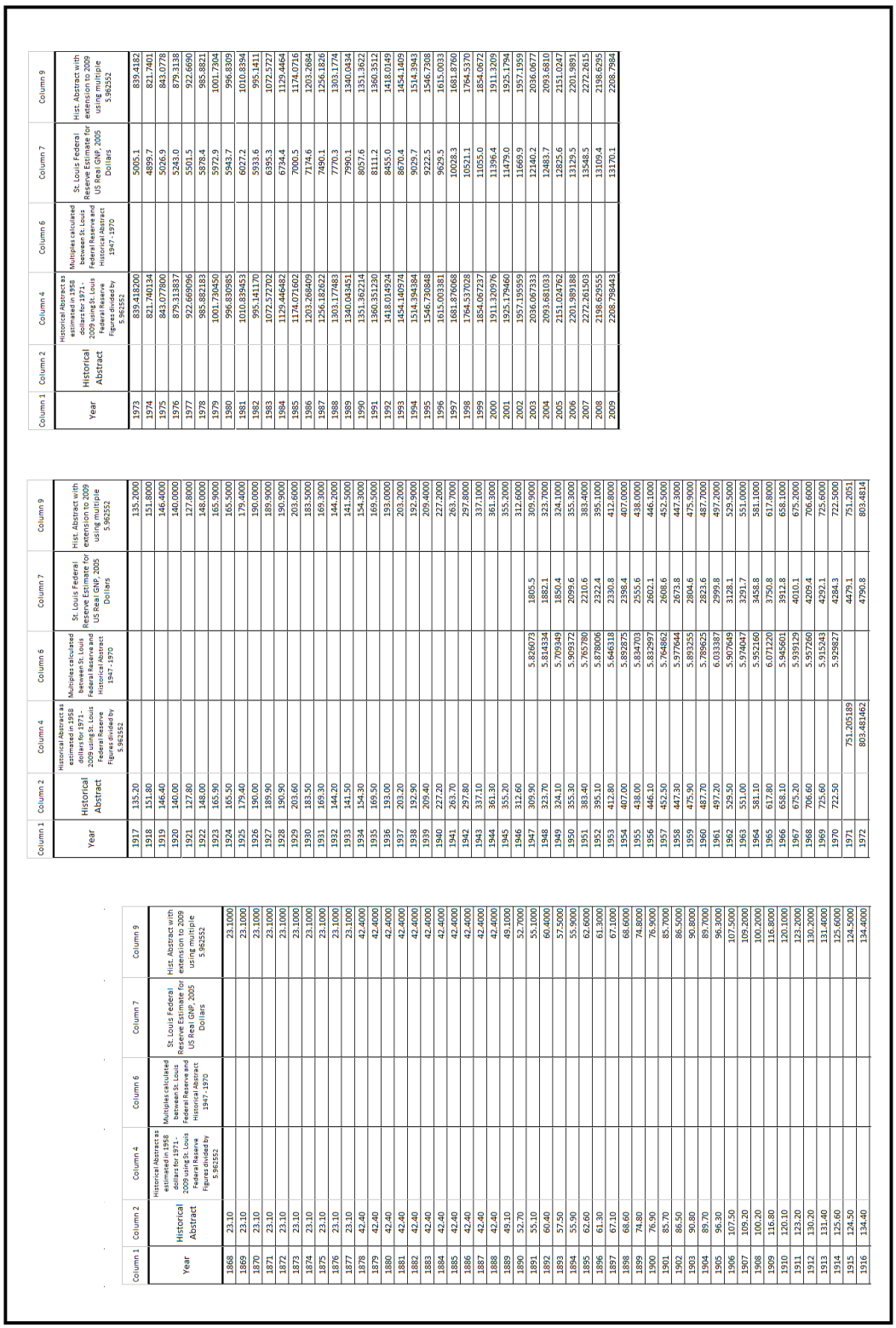
5.1.h. Splicing

For the purposes of the demonstration herein, more elaborate splicing techniques have not been deemed necessary. Data Set 2 figures an extended series for U.S. Real GNP in constant terms from 1868 to 2009. For the purposes of this paper only the second splicing multiple, 5.962552, will be used for calculations. (column 9)

The resulting "Data Set 2 – U.S. Real GNP" is as follows.

[18] See e.g. Cochrane, 1988:902. "The presence of a splice in 1947 also does not drive the result. Every long series of GNP data contains at least one splice. The wide surveys used to construct later data are simply not available for earlier periods, so some projection using a restricted set of industries is unavoidable."

Data Set 2 – U.S. Real GNP.



Year	Historical Abstract	Historical Abstracts as estimated in 1958 dollars for 1971- 2009 using St. Louis Federal Reserve Figures divided by 5.962552	Multiples calculated between St. Louis Federal Reserve and Historical Abstract 1947 -1970	St. Louis Federal Reserve Estimate for US Real GNP, 2005 Dollars	Hist. Abstract with extension to 2009 using multiple 5.962552
1917	135.20				135.2000
1918	151.80				151.8000
1919	146.40				146.4000
1920	140.00				140.0000
1921	127.80				127.8000
1922	148.00				148.0000
1923	165.90				165.9000
1924	165.50				165.5000
1925	179.40				179.4000
1926	190.00				190.0000
1927	189.90				189.9000
1928	190.90				190.9000
1929	203.60				203.6000
1930	183.50				183.5000
1931	169.30				169.3000
1932	144.20				144.2000
1933	141.50				141.5000
1934	154.30				154.3000
1935	169.50				169.5000
1936	193.00				193.0000
1937	203.20				203.2000
1938	192.90				192.9000
1939	209.40				209.4000
1940	227.20				227.2000
1941	263.70				263.7000
1942	297.80				297.8000
1943	337.10				337.1000
1944	361.30				361.3000
1945	355.20				355.2000
1946	312.60				312.6000
1947	309.90		5.826073	1805.5	309.9000
1948	323.70		5.814334	1882.1	323.7000
1949	324.10		5.709349	1850.4	324.1000
1950	355.30		5.969372	2095.6	355.3000
1951	383.40		5.765780	2210.6	383.4000
1952	395.10		5.878006	2322.4	395.1000
1953	412.80		5.646318	2330.8	412.8000
1954	407.00		5.892875	2398.4	407.0000
1955	438.00		5.834703	2555.6	438.0000
1956	446.10		5.832997	2602.1	446.1000
1957	452.50		5.764862	2608.6	452.5000
1958	447.30		5.977644	2673.8	447.3000
1959	475.90		5.893255	2804.6	475.9000
1960	487.70		5.789625	2823.6	487.7000
1961	497.20		6.033387	2999.8	497.2000
1962	529.50		5.907649	3128.1	529.5000
1963	551.00		5.974047	3291.7	551.0000
1964	581.10		5.952160	3458.8	581.1000
1965	617.80		6.071220	3750.8	617.8000
1966	658.10		5.945601	3912.8	658.1000
1967	675.20		5.939129	4010.1	675.2000
1968	706.60		5.957260	4209.4	706.6000
1969	725.60		5.915243	4292.1	725.6000
1970	722.50		5.929827	4284.3	722.5000
1971		751.205189		4479.1	751.2051
1972		803.481462		4790.8	803.4814
1973		839.418200		5005.1	839.4182
1974		821.740134		4899.7	821.7401
1975		843.077800		5026.9	843.0778
1976		879.313837		5243.0	879.3138
1977		922.669096		5501.5	922.6690
1978		985.882183		5878.4	985.8821
1979		1001.730450		5972.9	1001.7304
1980		996.838985		5943.7	996.8309
1981		1010.839453		6027.2	1010.8394
1982		995.141170		5933.6	995.1411
1983		1072.572702		6395.3	1072.5727
1984		1129.446482		6734.4	1129.4464
1985		1174.071602		7000.5	1174.0716
1986		1203.268409		7174.6	1203.2684
1987		1256.182622		7490.1	1256.1826
1988		1303.177483		7770.3	1303.1774
1989		1340.043451		7990.1	1340.0434
1990		1351.362214		8057.6	1351.3622
1991		1360.351230		8111.2	1360.3512
1992		1418.014924		8455.0	1418.0149
1993		1454.140974		8670.4	1454.1409
1994		1514.394384		9029.7	1514.3943
1995		1546.730848		9223.5	1546.7308
1996		1615.003381		9629.5	1615.0033
1997		1681.876068		10028.3	1681.8760
1998		1764.337028		10521.1	1764.3370
1999		1854.067237		11055.0	1854.0672
2000		1911.320976		11396.4	1911.3209
2001		1925.179460		11479.0	1925.1794
2002		1957.195959		11669.9	1957.1959
2003		2036.067333		12140.2	2036.0677
2004		2093.681033		12483.7	2093.6810
2005		2151.024762		12825.6	2151.0247
2006		2201.989188		13129.5	2201.9891
2007		2272.261503		13548.5	2272.2615
2008		2198.629555		13109.4	2198.6295
2009		2208.798443		13170.1	2208.7984

5.2. Section Two: Examine Ratios of un-averaged U.S. real GNP

If the Kondratiev wave is to be found within the economic data of the United States, it is necessary to locate within this wave the fundamental sub-cycles. In this second section of this paper we examine "ratios of U.S. real GNP" in order to determine whether such sub-cycles may be demonstrated empirically.

A ratio of GNP is a numeric fraction which takes as its numerator the real GNP of one year and takes as its denominator the real GNP of an earlier year. The number of years between numerator and denominator is referred to as a "spread of years" or simply a "spread." In order to establish the possible period of the sub-cycle we took ratios of GNP at different spreads of years and placed these ratios in Excel spreadsheets based upon the number of years in the spread.[19]

For every year of the spread we constructed a single row within the spreadsheet. Because the data set is finite, a tighter spread between years results in a larger number of columns, and a broader spread between years results in a reduced number of columns.

We investigated spreads of years between numerator and denominator ranging from a 7-year spread to an 18-year spread. This range was chosen because it seemed likely to include the most eligible sub-cycles for a Kondratiev Wave of 50-60 years. We thought that if the Kondratiev Wave was in reality seven 7-year sub-cycles, or three 18-year sub-cycles, etc. this range of investigation might demonstrate such a finding.

The result of dividing figures for real GNP by one another is a third number, the quotient. The fraction 6/5 represents the mathematic operation of division or $6 \div 5 = 1.2$, in which case the quotient is 1.2.

The spread between years is a measure of the passage of time. When the spread between years is slight, the quotients generated are generally quite close to the number one because the passage of time has been short. One would not expect the real GNP of 1888 to be significantly different than the real GNP of 1889 because only one year has passed between the two dates. Consequently, dividing one figure for real GNP by the other, we would expect to have a result which is close to the number one. When the spread between years is great, a larger period of time is being considered and the quotients generated are usually larger than one.

If a quotient is set as a ratio or proportion to the number one, it copies the proportion first stated as between the numerator and denominator in the first instance. Considering the example above, just as 6 is to 5, so is 1.2 to 1, or set mathematically, $6 : 5 = 1.2 : 1$. These numerators, denominators and quotients are considered "ratios of U.S. real GNP" because we are looking for the common patterns underlying the numbers themselves, the numerators and denominators given for the real GNP of the United States for any given year.

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[19] The data provided by the Federal Government commences with a series of GNP values for the nine year period of 1869-1877 of a single figure, i.e. 23.1. This is followed by an 11-year period of 1878-1888 of a single value, i.e. 42.4. We have extended this series back one year by giving the year 1868 the figure 23.1, thereby permitting the larger spreads to include data series dating back to 1868.

This has been helpful in that it allows the 14-year, 15-year, 16-year, 17-year and 18-year spreads to include both the most antique, as well as the most current data – through 2010 – in their spreadsheets. Given the significance of the 14-year spread as described in this paper, it has been important to use this 1868 value of 23.1 as the beginning point for each spreadsheet in an effort to provide uniformity in this approach.

By way of example, the real GNP of the United States for 2005 divided by that of 1995 represents the division of a numerator by a denominator both of which are stated in the billions of dollars, resulting in a quotient which is the final result of this simple mathematic operation. The term "ratio" suggests a proportion between these two numbers which, no matter how large, over time governs the general existence of the numbers themselves.

A typical Excel spread sheet with this data is as follows:

DIAGRAM 1:
SAMPLE SPREAD SHEET

12 YEAR RATIOS BASED ON ANNUAL REAL GNP; MULTIPLE 5.962552

		1		2		3		4		5		6		7		8		9		10		11		A	B	C	D	E	F	G
		YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP			Maximum Ratio of Row	Minimum Ratio of Row	Spread	Mid-Range Ratio of Row	Median Ratio of Row	Average Ratio of Row	Median Average
1	Year	1880	42.4000	1892	60.4000	1904	89.7000	1916	134.4000	1928	190.9000	1940	227.2000	1952	395.1000	1964	581.1000	1976	879.3138	1988	1303.1774	2002	1957.1959							
	14	1868	23.1000	1880	42.4000	1892	60.4000	1904	89.7000	1916	134.4000	1928	190.9000	1940	227.2000	1952	395.1000	1964	581.1000	1976	879.3138	1990	1351.3622							
	Ratio		1.8354978		1.4245283		1.4850993		1.4983278		1.4203869		1.19015191		1.73899648		1.47076689		1.51118844		1.48203963		1.448313329	1.835498	1.190152	0.645346	1.512825	1.482039	1.505898	1.493969
2	Year	1881	42.4000	1893	57.5000	1905	96.3000	1917	135.2000	1929	203.6000	1941	263.7000	1953	412.8000	1965	617.8000	1977	922.6690	1991	1360.3512	2005	2151.0247							
	14	1869	23.1000	1881	42.4000	1893	57.5000	1905	96.3000	1917	135.2000	1929	203.6000	1941	263.7000	1953	412.8000	1965	617.8000	1979	1001.7304	1993	1454.1409							
	Ratio		1.8354978		1.3561321		1.6747826		1.403946		1.5059172		1.29518864		1.56541524		1.49660852		1.49347523		1.35800131		1.479240904	1.835498	1.295187	0.540311	1.565342	1.493475	1.498496	1.495986
3	Year	1882	42.4000	1894	55.9000	1906	107.5000	1918	151.8000	1930	183.5000	1942	297.8000	1954	407.0000	1966	658.1000	1978	985.8821	1992	1418.0149	2006	2201.9891							
	14	1870	23.1000	1882	42.4000	1894	55.9000	1906	107.5000	1918	151.8000	1930	183.5000	1942	297.8000	1954	407.0000	1966	658.1000	1980	996.8309	1994	1514.3943							
	Ratio		1.8354978		1.3181982		1.9230769		1.412093		1.2088274		1.62388829		1.36668965		1.61695317		1.49807139		1.42252301		1.454019448	1.923077	1.208827	0.714250	1.565952	1.422523	1.522502	1.472512
4	Year	1883	42.4000	1895	62.6000	1907	109.2000	1919	146.4000	1931	169.3000	1943	337.1000	1955	438.0000	1967	675.2000	1979	1001.7304	1993	1454.1409	2007	2272.2615							
	14	1871	23.1000	1883	42.4000	1895	62.6000	1907	109.2000	1919	146.4000	1931	169.3000	1943	337.1000	1955	438.0000	1967	675.2000	1981	1010.8394	1995	1546.7308							
	Ratio		1.8354978		1.4764151		1.7444089		1.3406593		1.1564208		1.99113999		1.29931771		1.54155251		1.48360545		1.43854794		1.469073675	1.991140	1.156421	0.834719	1.573780	1.476415	1.530757	1.503586
5	Year	1884	42.4000	1896	61.3000	1908	100.2000	1920	140.0000	1932	144.2000	1944	361.3000	1956	446.1000	1968	706.6000	1980	996.8309	1994	1514.3943	2008	2198.6295							
	14	1872	23.1000	1884	42.4000	1896	61.3000	1908	100.2000	1920	140.0000	1932	144.2000	1944	361.3000	1956	446.1000	1968	706.6000	1982	995.1411	1996	1615.0033							
	Ratio		1.8354978		1.4457547		1.6345884		1.3972056		1.03		2.50594785		1.234708		1.58394787		1.41074285		1.52178851		1.361377714	2.505548	1.030000	1.475548	1.767774	1.445755	1.559978	1.502866
6	Year	1885	42.4000	1897	67.1000	1909	116.8000	1921	127.8000	1933	141.5000	1945	355.2000	1957	452.5000	1969	725.6000	1981	1010.8394	1995	1546.7308	2009	2208.7984							
	14	1873	23.1000	1885	42.4000	1897	67.1000	1909	116.8000	1921	127.8000	1933	141.5000	1945	355.2000	1957	452.5000	1969	725.6000	1983	1072.5727	1997	1681.8760							
	Ratio		1.8354978		1.5825472		1.7406855		1.0941781		1.1071987		2.51024735		1.27393018		1.60353591		1.39310832		1.44207548		1.31329444	2.510247	1.094178	1.416069	1.802213	1.442075	1.558300	1.500188
7	Year	1886	42.4000	1898	68.6000	1910	120.1000	1922	148.0000	1934	154.3000	1946	312.6000	1958	447.3000	1970	722.5000	1982	995.1411	1996	1615.0033	2010	2270.9907							
	14	1874	23.1000	1886	42.4000	1898	68.6000	1910	120.1000	1922	148.0000	1934	154.3000	1946	312.6000	1958	447.3000	1970	722.5000	1984	1129.4464	1998	1764.5370							
	Ratio		1.8354978		1.6179245		1.7507289		1.2323064		1.0425678		2.02582353		1.43090211		1.61524703		1.37735792		1.42996988		1.287017884	2.025934	1.042568	0.983356	1.534246	1.438902	1.535836	1.483369
8	Year	1887	42.4000	1899	74.8000	1911	123.2000	1923	165.9000	1935	169.5000	1947	309.9000	1959	475.9000	1971	751.2051	1983	1072.5727	1997	1681.8760									
	14	1875	23.1000	1887	42.4000	1899	74.8000	1911	123.2000	1923	165.9000	1935	169.5000	1947	309.9000	1959	475.9000	1971	751.2051	1985	1174.0716									
	Ratio		1.8354978		1.7641509		1.6470588		1.3465909		1.0216998		1.82831858		1.53565666		1.57849359		1.42780274		1.4325157			1.835498	1.021700	0.813798	1.428599	1.535657	1.541779	1.538718
9	Year	1888	42.4000	1900	76.9000	1912	130.2000	1924	165.5000	1936	193.0000	1948	323.7000	1960	487.7000	1972	803.4814	1984	1129.4464	1998	1764.5370									
	14	1876	23.1000	1888	42.4000	1900	76.9000	1912	130.2000	1924	165.5000	1936	193.0000	1948	323.7000	1960	487.7000	1972	803.4814	1986	1203.2684									
	Ratio		1.8354978		1.8136792		1.6931079		1.2731214		1.1661631		1.67720207		1.50664195		1.64749108		1.40569079		1.46645337			1.835498	1.166163	0.669335	1.500830	1.506642	1.548305	1.527473
10	Year	1889	49.1000	1901	85.7000	1913	131.4000	1925	179.4000	1937	203.2000	1949	324.1000	1961	497.2000	1973	839.4182	1985	1174.0716	1999	1854.0672									
	14	1877	23.1000	1889	49.1000	1901	85.7000	1913	131.4000	1925	179.4000	1937	203.2000	1949	324.1000	1961	497.2000	1973	839.4182	1987	1256.1826									
	Ratio		2.1255411		1.7454175		1.5332555		1.3652988		1.1326644		1.59498071		1.53409442		1.68829082		1.39867303		1.47595357			2.125541	1.132664	0.992877	1.629103	1.533258	1.559417	1.546338
11	Year	1890	52.7000	1902	86.5000	1914	125.6000	1926	190.0000	1938	192.9000	1950	355.3000	1962	529.5000	1974	821.7401	1986	1203.2684	2000	1911.3209									
	14	1878	42.4000	1890	52.7000	1902	86.5000	1914	125.6000	1926	190.0000	1938	192.9000	1950	355.3000	1962	529.5000	1974	821.7401	1988	1303.1774									
	Ratio		1.2429245		1.6413662		1.4520231		1.5127389		1.0152632		1.84188696		1.4902899		1.55191709		1.46429313		1.46662699			1.841880	1.015263	0.826624	1.428575	1.490290	1.467932	1.479111
12	Year	1891	55.1000	1903	90.8000	1915	124.5000	1927	189.9000	1939	209.4000	1951	383.4000	1963	551.0000	1975	843.0778	1987	1256.1826	2001	1925.1794									
	14	1879	42.4000	1891	55.1000	1903	90.8000	1915	124.5000	1927	189.9000	1939	209.4000	1951	383.4000	1963	551.0000	1975	843.0778	1989	1340.0434									
	Ratio		1.2995283		1.6479125		1.3711454		1.5253012		1.1026856		1.83094556		1.43714137		1.53003675		1.48999688		1.43665414			1.830948	1.102688	0.728260	1.466816	1.489996	1.467140	1.478568

		A	B	C	D	E	F	G
A	Maximum Ratio of Column	2.125541	1.813679	1.923077	1.525301	1.505917	2.510247	1.738996	1.688291	1.513188	1.521789
B	Minimum Ratio of Column	1.242925	1.318396	1.371145	1.094178	1.015263	1.190152	1.234708	1.470767	1.377350	1.358001
C	Spread	0.882617	0.495283	0.551932	0.431123	0.490654	1.320095	0.504288	0.217529	0.135831	0.163787
D	Range Ratio of Column	1.684233	1.566038	1.647111	1.309740	1.260590	1.850200	1.486852	1.579529	1.445273	1.439895
E	Median Ratio of Column	1.835498	1.582547	1.647059	1.357206	1.132664	1.828319	1.490290	1.578494	1.464293	1.442075
F	Average Ratio of Column	1.765620	1.569539	1.637498	1.366647	1.159150	1.826202	1.451148	1.577074	1.446334	1.447766
G	Median Average	1.800560	1.576033	1.642278	1.381926	1.145907	1.827260	1.470719	1.577784	1.455334	1.444918

	Max. of F - Rows	Min. of F - Rows	Mid-Range of F - Rows	Median of F - Rows	Avg. of F - Rows	Mid-Range + Average/2	Median + Average/2
	2.510247	1.015263	1.762755	1.533296	1.524695	1.643725	1.528996

	Max. of F - Columns	Min. of F - Columns	Mid-Range of F - Columns	Median of F - Columns	Avg. of F - Columns	Mid-Range + Average/2	Median + Average/2
	2.510247	1.015263	1.762755	1.510334	1.524695	1.643725	1.517515

By way of example let us consider Column Four Row One of the 12 year spread. (See Diagram 1, Sample Spread Sheet.) This GNP ratio is 1916 / 1904, representing a spread of 12 years between the numerator and the denominator of the ratio. The US real GNP values for this fraction are 134.4 / 89.7 with a result of 1.49833. This ratio is placed in Column Four Row One in the 12-year spread spreadsheet.

The next ratio in the series, 1917 / 1905, or 135.2 / 96.3, gives the result of 1.40395. This is placed in Column Four Row Two of the 12-year spread spreadsheet.

This continues on for a period of 12 years, i.e. from 1916 through 1927. The final fraction in Column Four Row Twelve is 1927/1915, or 189.9 / 124.5, for a result of 1.5253. This result is placed in Column Four Row Twelve and the series continues on to the next column.

The next column, Column Five, begins in Row One with the ratio 1928 / 1916, for a ratio of 190.9 / 134.4 and a result of 1.42039. This is placed in Column Five Row One and the process continues. Notice that the numerator of the cell in Column Four Row One ("1916 = 134.4") becomes the denominator of the cell immediately to the right, Column Five Row One.

An Excel spread sheet may be generated for any given spread of years using "Data Base 2 – U.S. Real GNP" as its foundation.

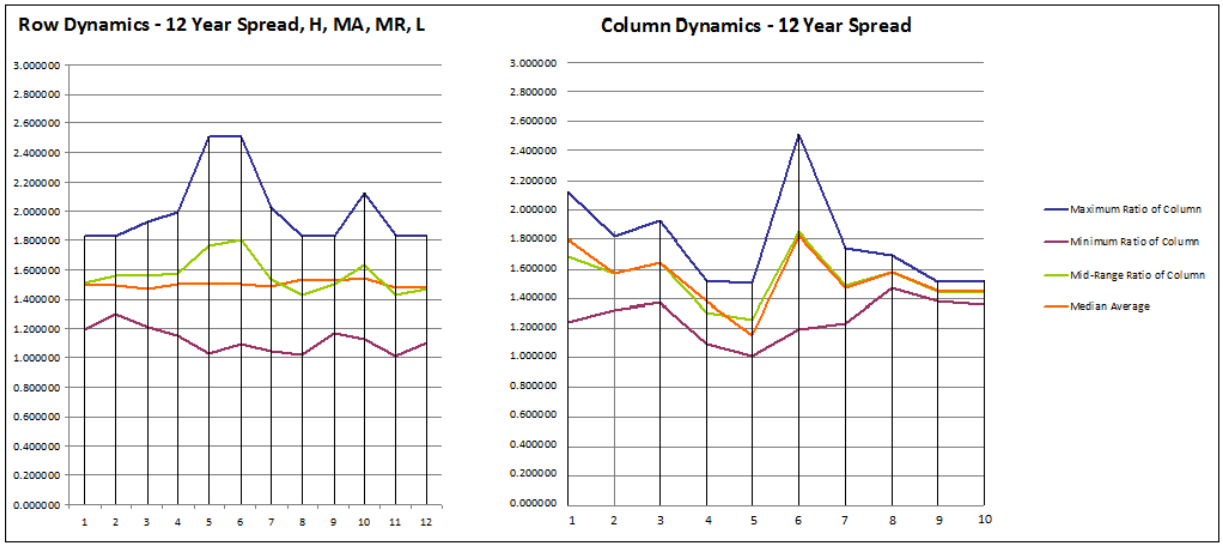
For every Row and for every Column in every spread sheet there exists a High Ratio and a Low Ratio. For example, in the Columns and Rows mentioned previously regarding the 12-year spread, we have the following:

12-year Spread,	High		
Row One	1880/1868	= 42.4/23.1	= 1.8354978
Row Two	1881/1869	= 42.4/23.1	= 1.8354978
Row Twelve	1951/1939	= 383.4/209.4	= 1.8309455
Column Four	1927/1915	= 189.9/124.5	= 1.5253012
Column Five	1928/1916	= 190.9/134.3	= 1.4203869
12-year Spread,	Low		
Row One	1940/1928	= 227.2/190.9	= 1.1901519
Row Two	1941/1929	= 263.7/203.6	= 1.2951866
Row Twelve	1939/1927	= 209.4/189.9	= 1.1026856
Column Four	1921/1909	= 127.8/116.8	= 1.0941781
Column Five	1938/1926	= 192.9/190.0	= 1.0152632

We noticed that High Averages represent ratios which contrast a very dynamic year of growth in the numerator with a previous year of very slow or depressed growth in the denominator. Conversely Low Averages contrast a year of slow or depressed growth in the numerator with a previous year of growth in the denominator.

The full range of these contrasts is as follows as to the 12-year spread.

DIAGRAM 2.
ROW AND COLUMN DYNAMICS FOR 12-YEAR SPREAD



From the above charts it becomes clear that these spread sheets are characterized by "Row Dynamics" and "Column Dynamics." From these dynamics we have calculated four additional points within both the Rows and the Columns of all spreadsheets. These are:

The "Mid-Range." The mid-range is the mid-point lying between the high and low ratios in the sample, i.e. the average of the highest and lowest numbers in the set: "(H + L) / 2".

The "Average" or "Arithmetic Mean." The sample mean is the sum of all the observations divided by the number of observations.

The "Median." The median is that number for which half the data is larger than it, and half the data is smaller. It is also called the 50th percentile. If the data has an odd number of members, the median will be the number in the center of these members; if an even number of members, the median will be the mid-point between the two numbers closest to the center.

The "Median Average." The Median Average is the mid-point between the Median and the Average (Arithmetic Mean). It is figured as: "(Median + Average) / 2" and is the approximation used throughout this paper – in conjunction with the Midrange – as the best estimate of the dynamics within Rows and Columns.

We then compared the High, Midrange, Median Average and Low of Row Dynamics for each Excel spread sheet. The following points are made as to this approach.

1) In every Row there exists a Highest Average of the possible averages in the Row. This Highest Average represents the greatest margin of growth over decline for the time period of that spread for that Row. Conversely the Lowest Average represents the greatest depth of decline over growth for the time period of the spread for that Row.

2) We noted that the Midrange between the Highest Average and the Lowest Average is simply the arithmetic division of the distance between these two. It lies half-way between them in any given row. The Midrange represents the arbitrary balance between these two extremes for that Row in any given spread of years. The Midrange is completely independent of, and unconnected to, the Median Average of the Row, other than the fact that they both include the Highest Average and the Lowest Average in their calculus.

3) The Median Average states the accumulated "weight" of all the ratios in the row. It is unconnected to the Highest Average and the Lowest Average other than it includes both of them as a part of its calculation. It is completely independent of, and unconnected to, the Midrange value and does not take it directly into account in its calculus.

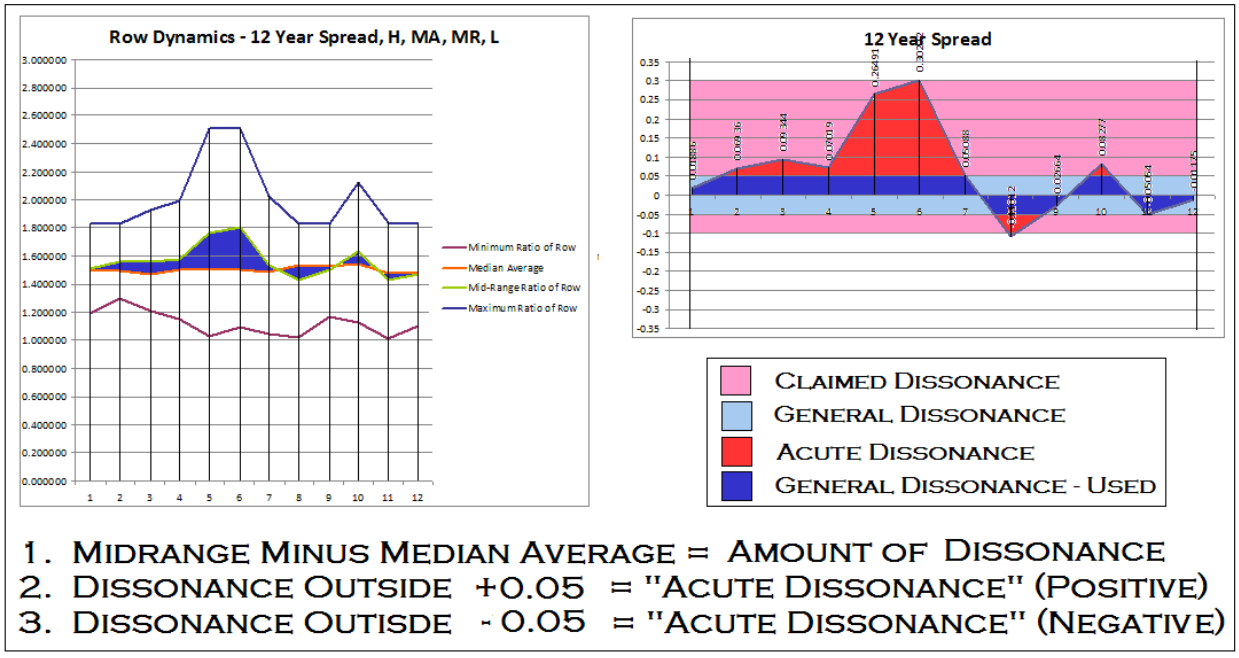
4) When a particular spread of years generates Rows which contain Midrange values and the Median Average values which are quite close to one another, the spread has established a relationship between the most basic ratios of the economy which is balanced and uniform. In the context of our search herein, we use the term "harmonic" to indicate this balance.

5) When a particular spread of years generates Rows which contain Midrange values and Median Average values which are at relatively great distances from one another, the spread has failed to establish a relationship between these basic ratios of the economy. By comparison to the other spreads, the particular spread in question is relatively unbalanced and not uniform. In the context of our search herein, we use the term "dissonant" to indicate this discord, turbulence or lack of harmony.

6) The implication is that when a given spread of years generates Midrange and Median Average values which are proximate to one another and therefore "harmonious" or "balanced," some underlying pattern or overriding logic may be at work to create this harmony as opposed to a random and disconnected set of processes and their resulting discordant and dissonant variables.

Diagram 2, left side, presents the Row Dynamics for the 12-year spread shown in Diagram 1. The x-axis indicates the row of the spreadsheet under consideration. The y-axis represents the figure presented by that row as its High, Low, Midrange or Median Average ratio.

DIAGRAM 3.
DISSONANCE BETWEEN MIDRANGE AND MEDIAN AVERAGE



1. MIDRANGE MINUS MEDIAN AVERAGE = AMOUNT OF DISSONANCE
2. DISSONANCE OUTSIDE +0.05 = "ACUTE DISSONANCE" (POSITIVE)
3. DISSONANCE OUTISDE -0.05 = "ACUTE DISSONANCE" (NEGATIVE)

Diagram 2, right side, presents the graph of the

x-axis = Row of the Spread
y axis = Midrange minus Median Average

When the Median Average is greater than the Midrange, the score is negative; when the Median Average is less than the Midrange, the score is positive. The number along the x-axis again indicates the row of the spread sheet under consideration. The number along the y-axis represents an amount of difference between Midrange and Median Average as found in that row.

The effort to compare systematically the common characteristics of different spreads led us to invent four new terms. Referring to Diagram 2 above these are:

> "General Dissonance." The pale blue area running as a ribbon from left to right represents the notion of a "General Dissonance," i.e. an arbitrary, acceptable distance between Median-Average and Midpoint. When a row possesses a Midrange and a Median Average which are in close proximity to one another, the distance between them will be found within the space designated by pale blue, "General Dissonance." After reviewing all spreads of years, this number has been set at +/- 0.05 in as much as it appears applicable to all spreads of years as general field of activity.
>
> "Used General Dissonance." The amount of dark blue is termed "Used General Dissonance," i.e. that portion of "General Dissonance" which is actually used by the given row in stating the distance between the Midrange and the Median Average, either as a positive or negative amount surrounding y = 0.
>
> "Acute Dissonance." The portion in red represents an "Acute Dissonance." When the distance between Midrange and Median Average falls outside the arbitrarily stated "General Dissonance" the excess is given in red shading. If the distance between the Midrange and the Median Average of a row is great, the "Acute Dissonance" so stated will be signified by large areas of red shading. Lesser amounts of "Acute Dissonance" generate less red shading.
>
> "Claimed Dissonance." The pink portion running as a ribbon from left to right is "Claimed Dissonance," i.e. that volume of spread between the high point of "Acute Dissonance" and the low point of "Acute Dissonance." This is the range of values necessary to accommodate the entire spectrum of variation between these two extreme points.

We then compared all spreads of years, from the 7-year spread to the 18-year spread using the "Midrange Minus Median Average" formula. The data for this formula is as follows.

Data Set 3 -
Values for "Midrange Minus Median Average"

Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average
1.420684	1.257599	0.163085	1.353398	1.325104	0.028295	1.352348	1.347649	0.004699	1.417749	1.402263	0.015486	1.616044	1.452544	0.163500	1.512825	1.493969	0.018856
1.319644	1.279068	0.040576	1.326172	1.350297	-0.024125	1.479789	1.337755	0.142034	1.431993	1.401026	0.030967	1.434208	1.443657	-0.009449	1.565342	1.495986	0.069356
1.290117	1.293580	-0.003462	1.371056	1.322612	0.048444	1.522397	1.345267	0.177131	1.500597	1.395069	0.105528	1.455653	1.443306	0.012347	1.565952	1.472512	0.093440
1.324015	1.260418	0.063598	1.440683	1.344659	0.096024	1.511834	1.364345	0.147489	1.436418	1.398507	0.037911	1.452486	1.449358	0.003128	1.573780	1.503586	0.070195
1.361699	1.287513	0.074185	1.436010	1.382905	0.053106	1.428369	1.360640	0.067729	1.519756	1.416638	0.103118	1.425648	1.455403	-0.029755	1.767774	1.502866	0.264908
1.391717	1.285707	0.106011	1.404050	1.364457	0.039593	1.449965	1.391147	0.058818	1.617628	1.402286	0.215342	1.466204	1.442454	0.023750	1.802213	1.500188	0.302025
1.417749	1.266501	0.151248	1.443362	1.296073	0.147289	1.391472	1.371842	0.019630	1.636934	1.412955	0.223979	1.475706	1.432655	0.043051	1.534246	1.483369	0.050876
			1.427996	1.336802	0.091194	1.408532	1.366651	0.041881	1.520196	1.416338	0.103858	1.544457	1.454296	0.090161	1.428599	1.538718	-0.110119
						1.480915	1.365450	0.115465	1.425644	1.412712	0.012932	1.517064	1.441348	0.075716	1.500830	1.527473	-0.026643
									1.452767	1.410216	0.042551	1.700919	1.460360	0.240559	1.629103	1.546336	0.082767
												1.754719	1.423443	0.331276	1.428575	1.479113	-0.050538
															1.466816	1.478568	-0.011752

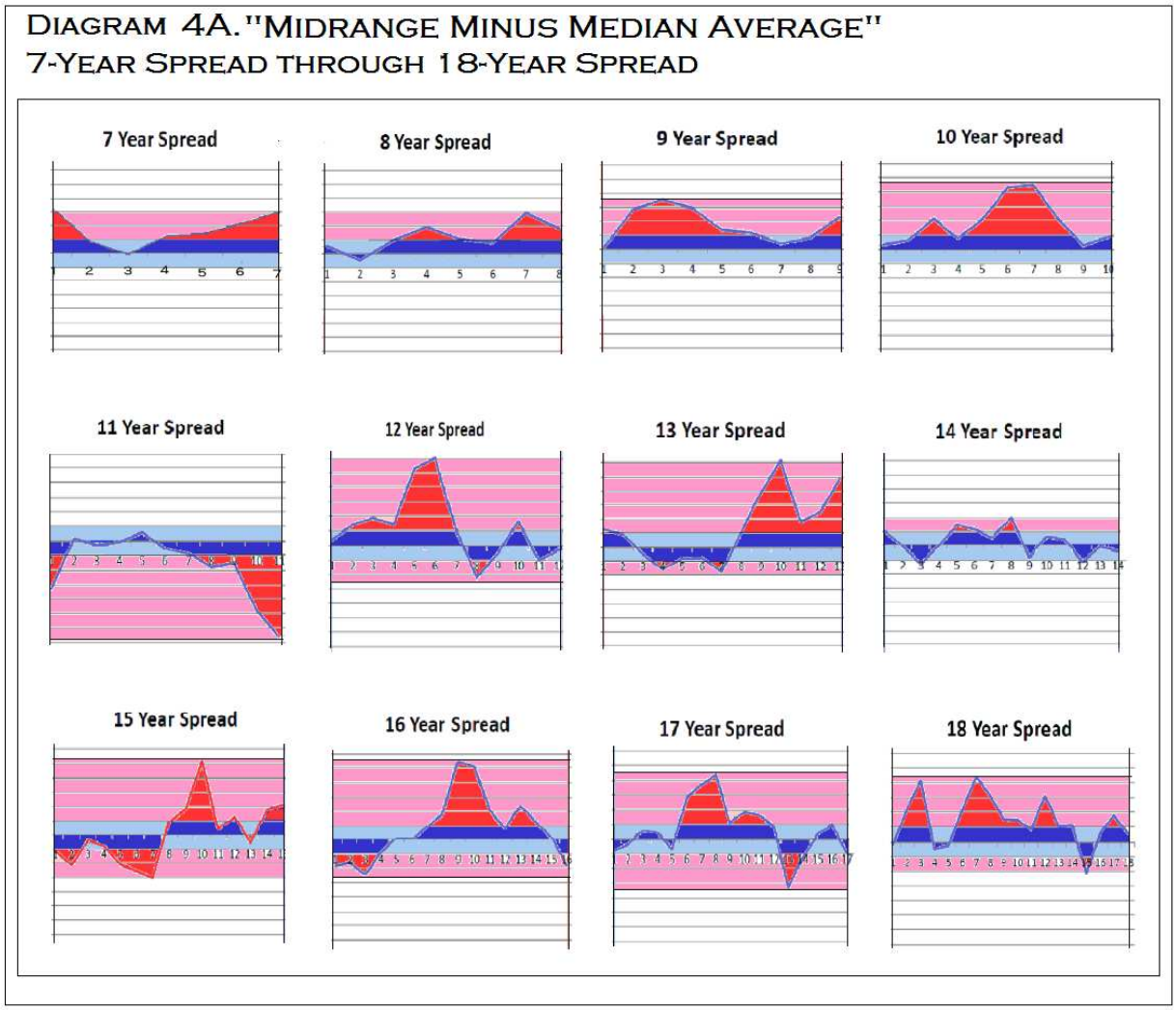
Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average	Mid-Range Ratio of Row	Median Average	Mid-Range Ratio - Median Average
1.609951	1.545540	0.064411	1.606885	1.557662	0.049223	1.621179	1.679915	-0.058736	1.658858	1.751061	-0.092203	1.728583	1.768989	-0.040406	1.825351	1.831532	-0.006182
1.607890	1.562299	0.045592	1.569285	1.575163	-0.005877	1.576343	1.685776	-0.109433	1.668528	1.748435	-0.079907	1.796469	1.814444	-0.017975	1.930369	1.819411	0.110958
1.527502	1.553554	-0.026052	1.546874	1.608772	-0.061898	1.654362	1.673314	-0.018952	1.659563	1.782610	-0.123047	1.847678	1.821362	0.026316	2.037456	1.828777	0.208679
1.499425	1.577735	-0.078310	1.612062	1.614731	-0.002670	1.640450	1.679591	-0.039142	1.709867	1.752882	-0.043015	1.843046	1.823491	0.019555	1.849483	1.870958	-0.021475
1.531646	1.571672	-0.040025	1.666742	1.592803	0.073939	1.607833	1.715428	-0.107595	1.747069	1.746208	0.000861	1.779455	1.814485	-0.035031	1.873463	1.883731	-0.010268
1.531405	1.568648	-0.037243	1.681779	1.622799	0.058980	1.609890	1.737551	-0.127661	1.726322	1.727562	-0.001240	1.898477	1.757082	0.141395	1.968260	1.861660	0.106600
1.494462	1.583419	-0.088957	1.639062	1.612435	0.026626	1.589751	1.739676	-0.149924	1.792382	1.746304	0.046078	1.991667	1.806185	0.185482	2.102114	1.881051	0.221063
1.608823	1.554593	0.054230	1.688880	1.587642	0.101238	1.746050	1.698917	0.047133	1.823744	1.737351	0.086393	2.030735	1.810631	0.220104	2.005640	1.849678	0.155962
1.753446	1.556557	0.196889	1.615660	1.658054	-0.042394	1.787059	1.695541	0.091518	1.993764	1.720864	0.272901	1.932660	1.879113	0.053546	1.914642	1.839490	0.075152
1.855977	1.548413	0.307564	1.675906	1.643953	0.031953	1.985015	1.722769	0.262246	1.979539	1.723974	0.255565	1.955311	1.860958	0.094354	1.999194	1.926329	0.072865
1.568293	1.478486	0.089807	1.600002	1.579006	0.020996	1.651359	1.633455	0.017904	1.733438	1.629703	0.103735	1.778144	1.694639	0.083504	1.806014	1.765595	0.040420
1.624682	1.500146	0.124536	1.541212	1.596952	-0.055740	1.677351	1.612950	0.064401	1.689899	1.647738	0.042161	1.770028	1.729400	0.040628	1.923043	1.767059	0.155984
1.724109	1.478613	0.245496	1.572493	1.577997	-0.005503	1.597130	1.624830	-0.027700	1.736914	1.618567	0.118348	1.629960	1.795614	-0.165654	1.854331	1.802412	0.051918
			1.555823	1.578014	-0.022191	1.698841	1.610624	0.088217	1.679023	1.613727	0.065296	1.702903	1.768209	-0.065306	1.844548	1.789074	0.055474
						1.699456	1.593296	0.106161	1.661449	1.653246	0.008203	1.781891	1.767960	0.013931	1.690109	1.796190	-0.106081
									1.595159	1.685809	-0.090650	1.813021	1.762594	0.050427	1.771745	1.742269	0.029476
												1.723787	1.773292	-0.049504	1.845227	1.756973	0.088254
															1.784212	1.757966	0.026247

Mid-Range Ratio Minus Median Average

7 Year Spread	8 Year Spread	9 Year Spread	10 Year Spread	11 Year Spread	12 Year Spread	13 Year Spread	14 Year Spread	15 Year Spread	16 Year Spread	17 Year Spread	18 Year Spread
0.163085	0.028295	0.004699	0.015486	-0.1635	0.018856	0.064411	0.049223	-0.058736	-0.092203	-0.040406	-0.006182
0.040576	-0.02412	0.142034	0.030967	0.009449	0.069356	0.045592	-0.005877	-0.109433	-0.079907	-0.017975	0.110958
-0.003462	0.048444	0.177131	0.105528	-0.01235	0.093440	-0.026052	-0.061898	-0.018952	-0.123047	0.026316	0.208679
0.063598	0.096024	0.147489	0.037911	-0.00313	0.070195	-0.078310	-0.002670	-0.039142	-0.043015	0.019555	-0.021475
0.074185	0.053106	0.067729	0.103118	0.029755	0.264908	-0.040025	0.073939	-0.107595	0.000861	-0.035031	-0.010268
0.106011	0.039593	0.058818	0.215342	-0.02375	0.302025	-0.037243	0.058980	-0.127661	-0.001240	0.141395	0.106600
0.151248	0.147289	0.019630	0.223979	-0.04305	0.050876	-0.088957	0.026626	-0.149924	0.046078	0.185482	0.221063
	0.091194	0.041881	0.103858	-0.09016	-0.110119	0.054230	0.101238	0.047133	0.086393	0.220104	0.155962
		0.115465	0.012932	-0.07572	-0.026643	0.196889	-0.042394	0.091518	0.272901	0.053546	0.075152
			0.042551	-0.24056	0.082767	0.307564	0.031953	0.262246	0.255565	0.094354	0.072865
				-0.33128	-0.050538	0.089807	0.020996	0.017904	0.103735	0.083504	0.040420
					-0.011752	0.124536	-0.055740	0.064401	0.042161	0.040628	0.155984
						0.245496	-0.0055	-0.027700	0.118348	-0.165654	0.051918
							-0.02219	0.088217	0.065296	-0.065306	0.055474
								0.106161	0.008203	0.013931	-0.106081
									-0.09065	0.050427	0.029476
										-0.0495	0.088254
											0.026247

An important difficulty arises in this regard as each spreadsheet is composed of varying numbers of columns and rows. Consequently the frequency of repetition varies. The 18-year spread is 2.571 longer in duration than is the 7-year spread. This means that – taken to infinity – the 7-year spread may be anticipated to have 2.571 as many columns as the 18-year spread. Conversely, because the number of rows is always finite, the 18-year spread has approximately 2.5 as many rows as the 7-year spread.

In the chart below the number of years in the spread is equalized by stretching the horizontal frame so that all spreads between a 7-year and an 18-year spread take up the same total horizontal space. This balances large spreads (large number of rows, relatively few columns) with the smaller spreads (small number of rows, large number of columns).



One may notice above that some spreads have distinctly lower profiles as to claimed dissonance than the other spreads. We examined this finding in more detail by comparing the numbers generated by these different spreads and associating them with one another in a more systematic way.

Each value given as the sum or difference for equation "Midrange Minus Median Average" may be divided into two parts, i.e. positive and negative values. These parts are further sub-divided by those values for this number which fall close to the y = 0 axis and inside the range of +/- 0.05. This range is referred to as "General Dissonance." Values which fall outside this range are referred to as "Acute Dissonance."

"Claimed Dissonance" locates the High and the Low extremes of the "Midrange Minus Median Average" for a given Row. Once we locate the point at which the Midrange most exceeds the Median Average (High), and the point at which the Midrange is most exceeded by the Median Average (Low), we may draw the y-axis distance between these two extremes (column 13). This is then taken as the boundary of a pink ribbon denoting "Claimed Dissonance" against the y-axis for the entire spread.

"Claimed Dissonance" is a measurement of the extent to which any given spread of years generates turbulence and discord between the Midrange and the Median Average. Like harmonies with discord between them, a high value for Claimed Dissonance indicates that the GNP ratio in question would not function well as a fundamental building block for an economic system, whereas low values for Claimed Dissonance provide the underlying balance necessary.

"The Magic Fraction."

All of these figures fit into the broader scheme of our effort to compare spreadsheets. Toward this end we have developed "the magic fraction," i.e. that fraction which serves as a stretching or shrinking device to accomplish numerically for spreadsheets what stretching and shrinking the horizontal frame of graphs accomplished in Diagram 3.

By way of example, in order to make the distance for "Claimed Dissonance" for the seven year spread equal that of the "Claimed Dissonance" for the 18-year spread, it must expand 2.571 times. If we used the fraction 18/7 we would create this "magic fraction" and thereby "stretch" the data for the seven year spread accordingly.

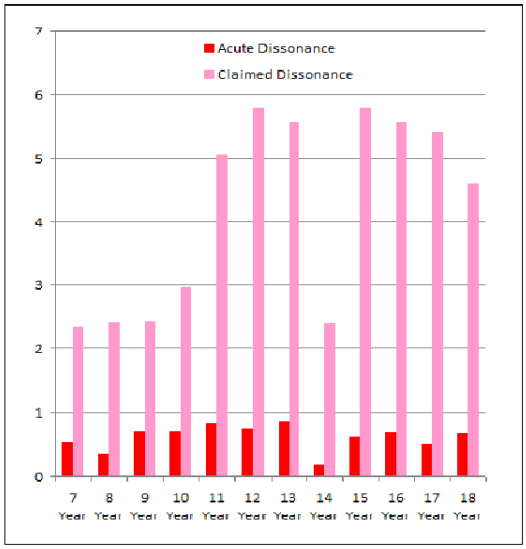
Such a fraction may be used to equalize all figures for all spreadsheets. For example, an "Acute Dissonance" at the 7-year spread sheet exists within a pattern of time which repeats itself 10 times in a 70 year span. An "Acute Dissonance" of an equivalent amount in an 18-year spreadsheet repeats under four times in the same 70 year span. The following fractions were used to multiply the spreadsheet data into numeric representations which would be equivalent.

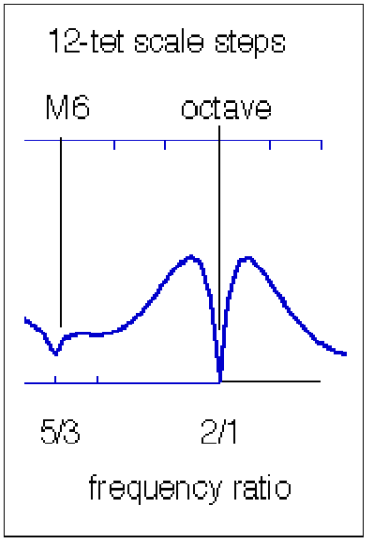
7-year spread x	14/7	2.0000
8	14/8	1.7500
9	14/9	1.5555
10	14/10	1.4000
11	14/11	1.2727
12	14/12	1.6666
13	14/13	1.0769
14	14/14	1.0000
15	14/15	0.9333
16	14/16	0.8750
17	14/17	0.8235
18	14/18.	0.7777

As demonstrated below, a remarkable and unexpected result occurs when a ratio of real GNP possesses a numerator and the denominator separated by 14 years. At this span of time, the level of Acute Dissonance is the least of all ratios (0.151795) and the level of Claimed Dissonance is second-to-least (2.39229). In addition, the spreads of three years before (11, 12, 13) and after (15, 16, 17) the 14-year spread generate the greatest amount of Claimed Dissonance, more than double that of the 14-year spread.

This "piling on" of Claimed Dissonance immediately before and after the 14-year spread is the origin of our selection of the term "dissonant," i.e. the sense that at the 14-year spread an almost acoustic "octave" is sounded against an underlying reality. This is surrounded by discording, conflicting "harmonies" immediately preceding and following this spread which are out-of-harmony with this reality.

DIAGRAM 4B.
"ACUTE DISSONANCE" AND "CLAIMED DISSONANCE" WITH MUSICAL OCTAVE



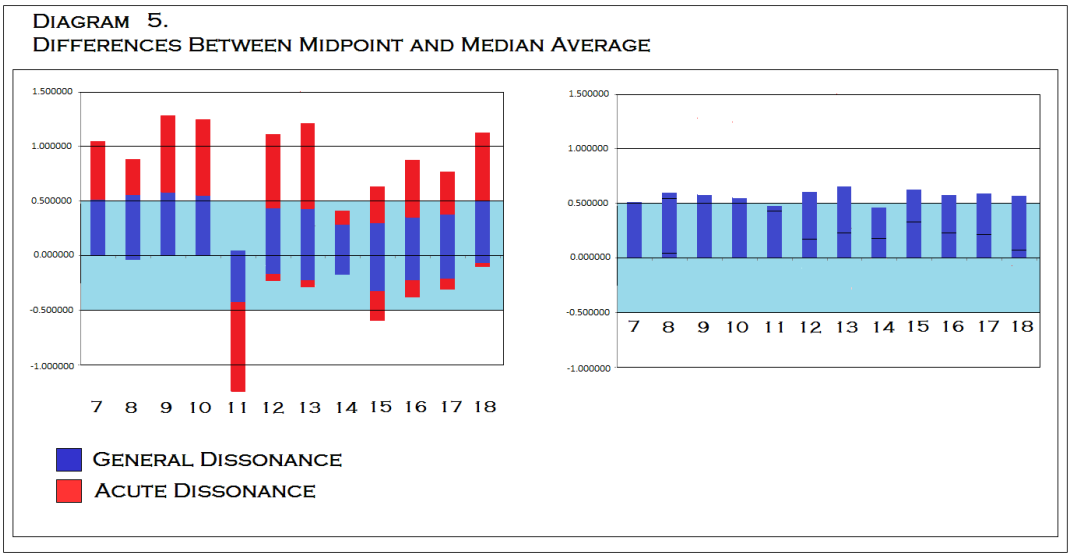


- RED	+ RED		Acute Dissonance	Claimed Dissonance
0	0.539222	7 Year	0.53922225	2.33166
0	0.328323	8 Year	0.32832275	2.39980
0	0.71348	9 Year	0.713480444	2.41405
0	0.702555	10 Year	0.702555	2.95466
-0.82882	0	11 Year	0.828815273	5.05443
-0.07077	0.680828	12 Year	0.751594667	5.77002
-0.07244	0.789312	13 Year	0.861753846	5.55129
-0.01764	0.134157	14 Year	0.151795	2.39229
-0.28313	0.338373	15 Year	0.6214992	5.77038
-0.16258	0.526958	16 Year	0.689539375	5.54327
-0.10785	0.394316	17 Year	0.502165176	5.40061
-0.04362	0.624485	18 Year	0.668103333	4.58002

The suggestion is that just as an octave is created by the equal division of a vibrating string into two harmonic parts, and just as a slight variation from this even division between the perfect center of the vibrating string results in intolerable out-of-tune sense of dis-harmony, so does the use of a 14-year interval between years when measuring GNP values result in great sympathy and proximity between Midrange and Median Average values for the entire economy, unlike every other spread of years. And also like the vibrating string, the most out-of-tune

dissonance occurs immediately surrounding the perfect division of the string, while tapering off as one takes distances further from the center.[20]

If we consider the positive and the negative "General Dissonances" as a combined positive distance (absolute value), we can see that each spread of years comes to approximately the same amount of "General Dissonance" (dark blue columns below).



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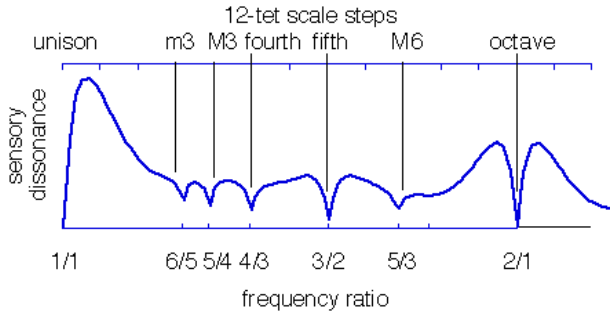
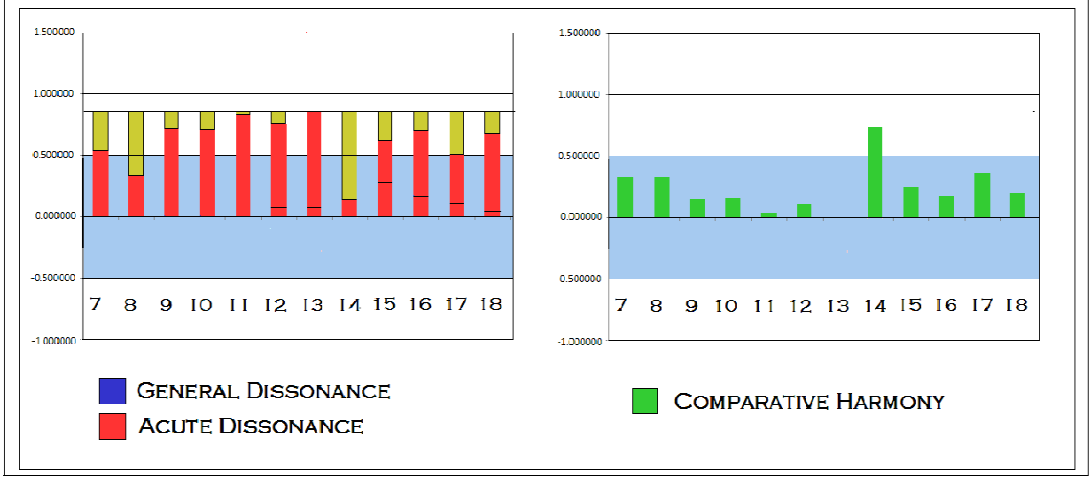
[20] See e.g. William Sethares, Relating Tuning and Timbre, *Experimental Musical Instruments*: "To explain perceptions of musical intervals, Plomp and Levelt note that most traditional musical tones have a spectrum consisting of a root or fundamental frequency, and a series of sine wave partials that occur at integer multiples of the fundamental. Figure 2 depicts one such timbre. If this timbre is sounded at various intervals, the dissonance of the intervals can be calculated by adding up all of the dissonances between all pairs of partials. Carrying out this calculation for a range of intervals leads to the dissonance curve. For example, the dissonance curve formed by the timbre of figure 2 is shown below in figure 3.



Observe that this curve contains major dips at many of the intervals of the 12 tone equal tempered scale. The most consonant interval is the unison, followed closely by the octave. Next is the fifth, followed by the fourth, the major third, the major sixth, and the minor third. These agree with standard musical usage and experience. Looking at the data more closely shows that the minima do not occur at exactly the scale steps of the 12 tone equal tempered scale. Rather, they occur at the "nearby" simple ratios 1:1, 2:1, 3:2, 4:3, 5:4, and 5:3 respectively, which are exactly the locations of notes in the "justly intoned" scales (see Wilkinson). Thus an argument based on tonal consonance is consistent with the use of just intonation (scales based on intervals with simple integer ratios), at least for harmonic timbres."

On the other hand, if we look to the amount of "Acute Dissonance" which goes above and beyond the general dissonance of these two points we have the following. The diagram on the left represents the amount of dissonance created by the spread (absolute value), and the diagram on the right represents the amount of harmony of the spread, i.e. the difference between the greatest level of dissonance (13 year spread) and the year in question.

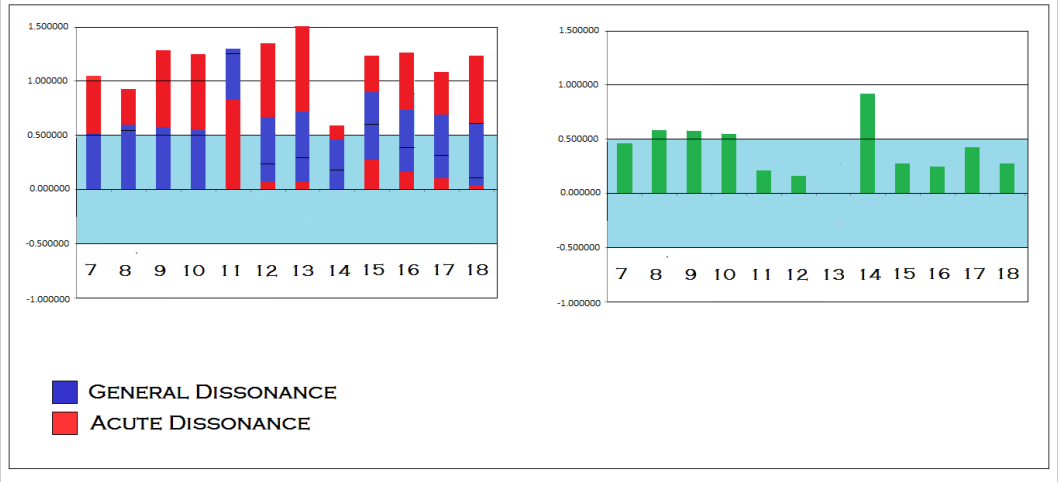
DIAGRAM 6.
DIFFERENCES BETWEEN MIDPOINT AND MEDIAN AVERAGE



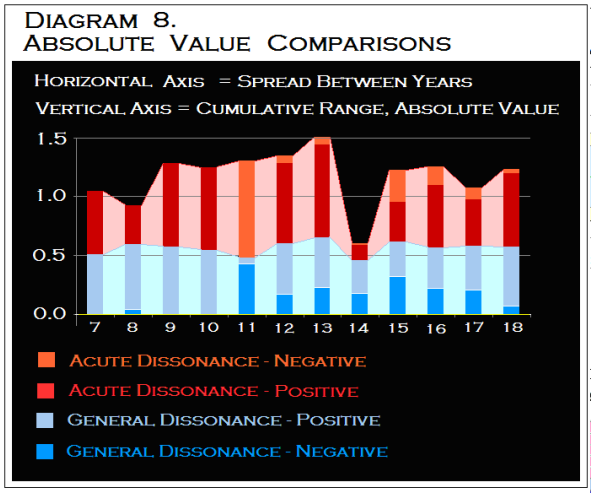
In both charts, the relative lack of dissonance in the 14 year spread, or conversely the striking harmony of the 14 year spread, is quite clear.

If we look at the combined total of these dissonances, we have an even stronger representation of that portion wherein harmony resides, as opposed to measurements of other spreads.

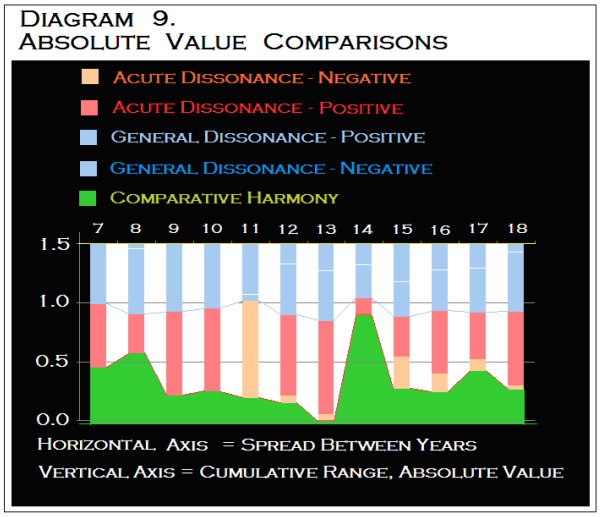
DIAGRAM 7.
DIFFERENCES BETWEEN MIDPOINT AND MEDIAN AVERAGE



The charts above track the level of harmony/dissonance for twelve different spreads between years. It is quite clear that again the 14-year spread provides the most harmony and the least dissonance. Like a place on a ball bat where the "acoustics" of the bat provide a "sweet spot" where it is best to hit a baseball, the span of 14 years seems to bring with it a natural "sweet spot" in the harmonics of the economy.



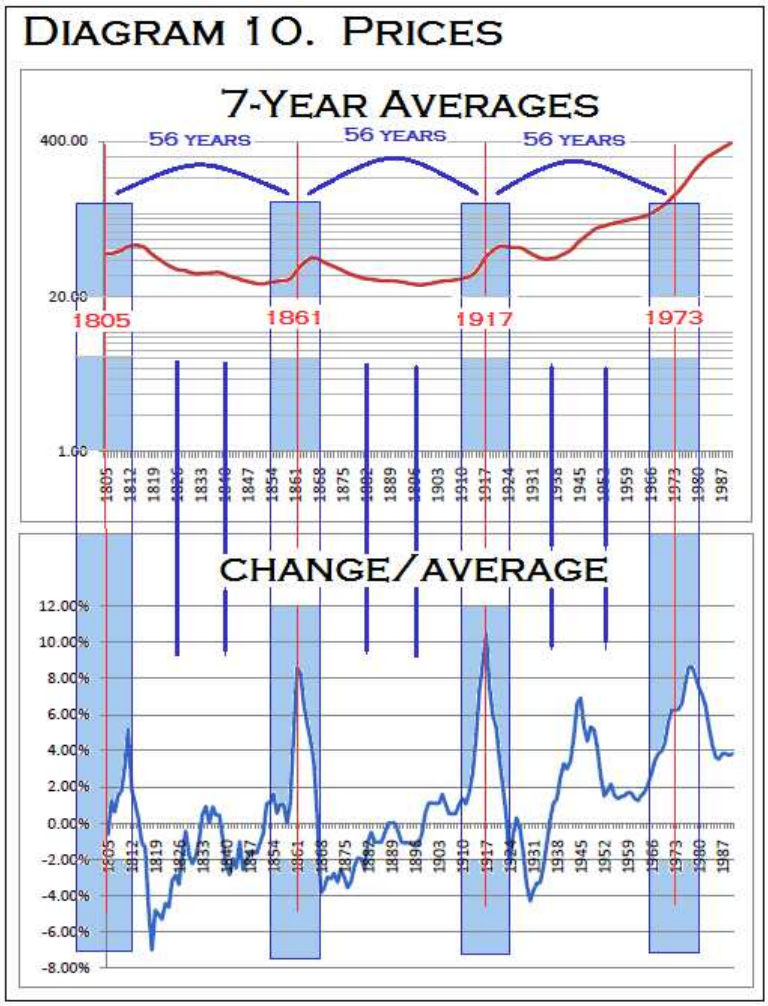
By simply flipping the comparison, we can see the preferred harmony brought on by a 14 year spread between years with very little acute dissonance.[21]



[21] The significance of a 14-year spread between years as a defining characteristic of the American economy finds at least tentative support in spectral analysis. Note that in both charts provided, the 14-year span is the most significant point of balance between the two charts, no matter how adjusted. (as taken from Korotayev and Tsirel, 2007:10) "As is easily seen in Figure 2A in both spectra one can detect distinctly the Kondratieff cycle (its period equals approximately 52-53 years), however, the cycle with a period of 13-14 years is detected even more distinctly. In the study by Claude Diebold and Cedric Doliger (2006, 2008) this wave is tentatively identified with Kuznets "swings." … Estimates of the length of Kuznet cycles will vary: here, 13-15 years but we note below estimates by others of 15-25 and later give our own estimate of 17-18 which agrees rather well with the original Kuznets' estimate."

5.3 Section Three: Evaluate Period of Long Wave

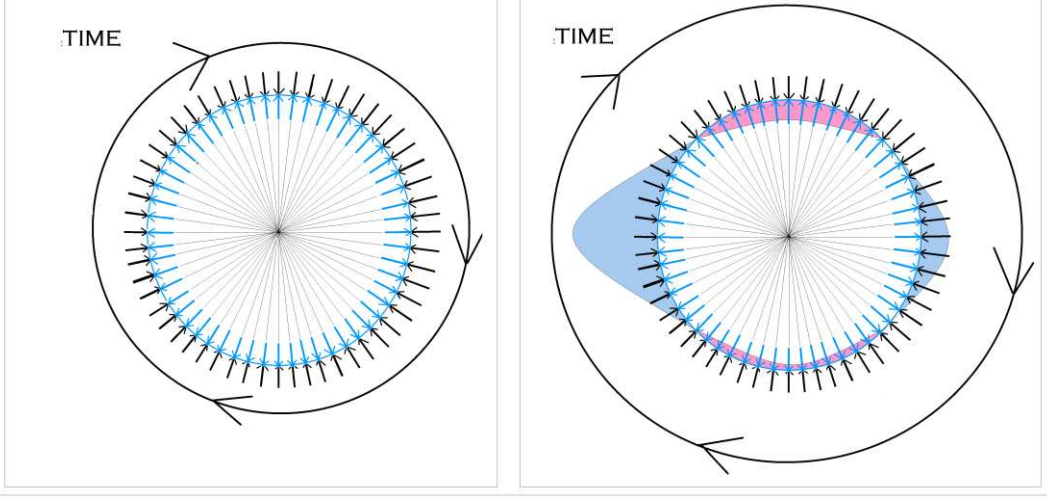
Having established that a 14-year sub-period may be important in the evaluation of the Kondratiev wave, we examined the price indexes for the United States between 1800 and 1994. The figures from "Data Set 1 – Prices" are stated below (1) in 7-year running averages (red line, top graph, semi-logarithmic scale), and (2) the change between a given year's seven-year average as divided by the average itself (blue line, bottom graph). The lower graph permits us to see the increasingly large inflationary price index values of later years (post-1966) as placed in a more consistent relationship with the preceding values of the series.



We noted in the above that the 56 year period (14 x 4 = 56) between peaks at 1861 through 1917 suggests the possibility that similar periods of time might connect other peak points of inflation. If a 14-year span (blue rectangles above) is drawn around the years 1805, 1861, 1917 and 1973 (each of which is separated by periods of 56 years), virtually all inflationary peaks are contained in a single model.

As this relates to the productive capacity represented by US real GNP, if we divide a circle into 56-year rays, all things being equal, as the arrows of production move outward to meet the expectation of GNP per year (arrows of radii moving out from the center of the circle) this production should be met by uniform resistances (arrows moving toward the center of the circle) which balance the natural increase of production exactly.

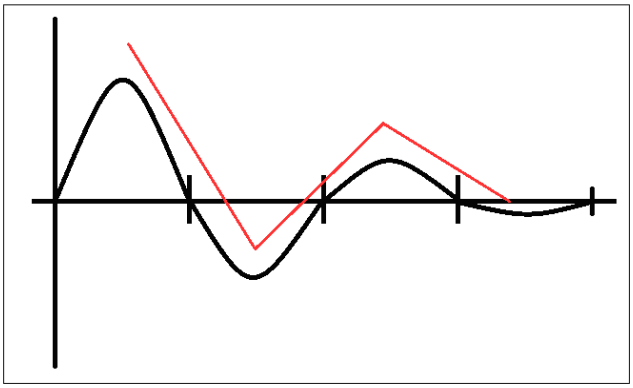
DIAGRAM 11.
STEADY STATE RATE OF GROWTH VS. WAVES OF GROWTH



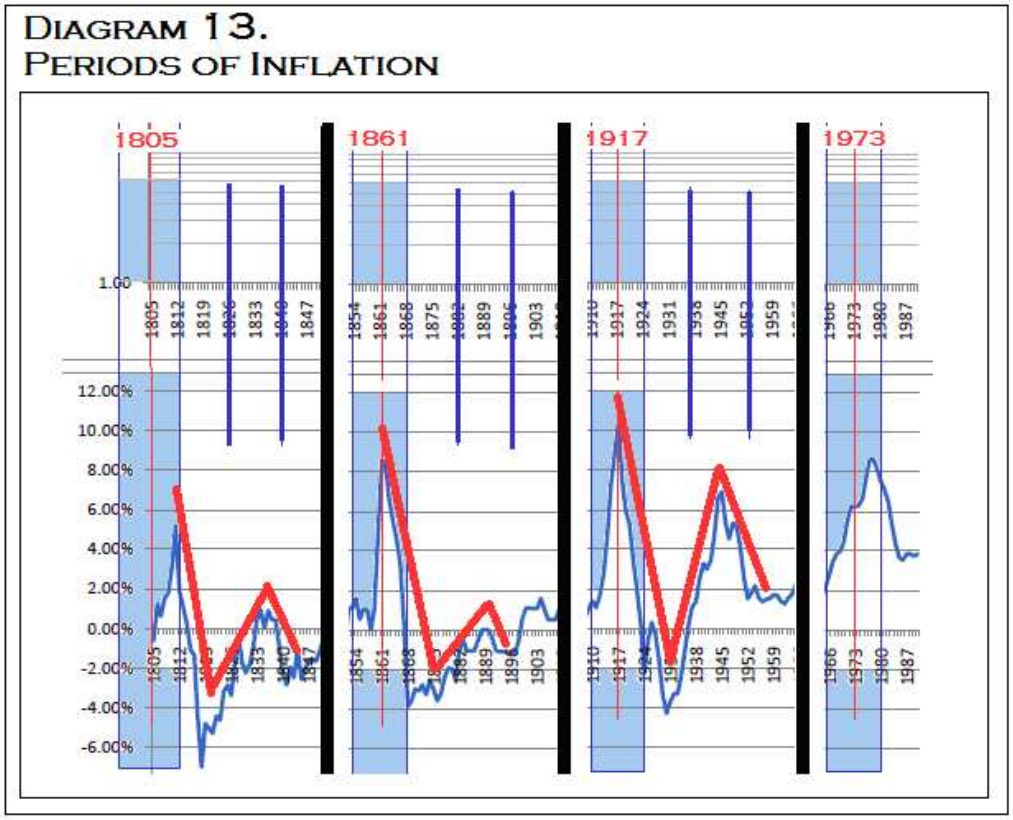
However if a particular period of time fails to offer uniform resistance to production, or if the strength of production for some reason is particularly strong, the inherent productivity of the citizenry will create a bulge in productivity which must then be balanced out by a depression at some other time in the course of the circuit. Only in this fashion can a constant of growth be maintained in the face of unequal strengths of production and resistance to production. A wave must then develop over time during which this bulge will even out as time goes on until the next unexpected opportunity for unusual productivity.

If this dampening wave is placed along an x-axis, we have the following.

DIAGRAM 12.
DAMPENING WAVE



The dampening wave has been noticed three times in the course of American economic history in consideration of prices.



Regarding the above chart, and as mentioned at the beginning of the paper, we concern ourselves here exclusively with the United States and the discovery of strong evidence that a Kondratiev Wave appears to have significant impact upon the US economy. A long-standing issue regarding Kondratiev Waves is the causation of the wave itself. This debate centers largely upon the "exogenous" vs. "endogenous" nature of the cycle. (see footnotes 6, 7 and 11)

From the "exogenous" point of view, it is difficult to understand how events which occur with an apparently chaotic randomness outside the United States can affect the American economy with dependable regularity.

From the "endogenous" point of view, although a form of biologic regularity might be granted to the American economy, it remains difficult to explain how such internal developments might affect with the same regularity international events over which the United States has no control whatsoever.

There can be no question that political events in Europe and throughout the world have had much to do with the inauguration of these cycles. Nor can there be serious question that the relationship between the economic development of the United States and that of Europe must be explored. The problem appears to be that two distinct yet interacting levels of economic life must be considered, one national (American) and one European. These concerns are dealt with in our separate paper entitled "*On Revolution and the Cultural Development of Europe: Toward a European "System of Movement* ." (unpublished at this time)

We present as persuasive a 14 x 4 = 56 year cycle as found between the inauguration of the American Civil War and the entry of the United States into the First World War. As these relate to the "exogenous" / "endogenous" debate, the following points may be made.

1. The American Civil War began on April 12-13, 1861.

2. The First World War began in Europe on July 28, 1914.

3. 56 years after the inauguration of the American Civil War, almost to the day, the United States entered the First World War on April 2, 1917.

One can explore the "endogenous" vs. "exogenous" nature of the 56-year period by considering the price patterns within the United States leading up to the First World War. As taken from Data Set One, these are:

Year	Price Index	Change from previous year	
1910	28.00		
1911	28.00	+0.0	
1912	29.00	+1.0	
1913	29.70	+0.7	
1914	**30.10**	**+0.4**	**World War I Between European States**
1915	**30.40**	**+0.3**	
1916	**32.70**	**+2.3**	
1917	**38.40**	**+5.7**	**United States Enters World War I**
1918	**45.10**	**+6.7**	
1919	**51.80**	**+6.7**	
1920	60.00	+8.2	
1921	53.60	- 6.4	
1922	50.20	- 3.4	
1923	51.10	+0.9	

One can see from the above that the inauguration of World War I in Europe in 1914 did not impact dramatically upon the price structure of the United States. Examining the United States' price structure for the years of European conflict 1914, 1915 and 1916 (in blue) changes of 0.4 + 0.3 + 2.3 = 3.0 may be noted.

The American entry into World War I in 1917 is associated with a spike in prices for the years 1917, 1918 and 1919 (in red) for a total of 5.7 + 6.7+ 6.7 = 19.1, over six times the cumulative changes of the previous three years. This would indicate that the domestic decision to enter World War I had far more to do with the resulting inflation than did the existence of the war in Europe itself.

The "exogenous" aspects of the analysis simply admit that at a European level, a vast war was occurring into which the United States ultimately was drawn. The "endogenous" aspects of the analysis insist that the United States was governed by its own internal development as to whether and when to join the conflict.

A similar point may be made with regard to the Vietnam War. Below are contrasted the steadily casualty counts for American soldiers 1956-1980 (as taken from the National Archives at http://www.archives.gov/research/military/vietnam-war/casualty-statistics.html) with the change in price index from Data Set 1 which exceed the fraction 1.06 (highlighted in red).

We see below that the Vietnam War was not a strong inflationary factor throughout the years of its most ferocious conflict when the annual casualty count exceeded 1,000, i.e. between 1965-1971 (also highlighted in red). At no time during this period did the price index exceed a multiple of 1.06 over the previous year.

On the other hand as of 1973, a year when the annual casualty count had diminished to less than 200, the inflation rate suddenly increased by no less than a multiple of 1.06 for nine of the following ten years.

In a fashion similar to 1917, inflation during this period is associated with the United States passing through a particular phase of its development and is not directly connected with the previous existence of the War in Vietnam.

	Casualty count	Current year / Previous year	Price Index	Current year / Previous year
1956-1960	9		88.70	
1961	16	+ 1.77	89.60	1.0101
1962	52	+ 3.25	90.60	1.0111
1963	118	+ 2.26	91.70	1.0121
1964	206	+ 1.74	92.90	1.0130
1965	**1,863**	**+ 9.04**	94.50	1.0172
1966	**6,143**	**+ 3.29**	97.20	1.0285
1967	**11,153**	**+ 1.81**	100.00	1.0288
1968	**16,592**	**+ 1.48**	104.20	1.0420
1969	**11,616**	**+ 0.70**	109.80	1.0537
1970	**6,081**	**+ 0.52**	116.30	1.0591
1971	**2,357**	**+ 0.38**	121.50	1.0447
1972	641	+ 0.27	125.40	1.0320
1973	168	+ 0.26	**133.20**	**1.0622**
1974	178	+ 1.05	**147.90**	**1.1103**
1975	161	+ 0.90	**161.40**	**1.0912**
1976	77	+ 0.47	170.70	1.0576
1977	96	+ 1.24	**181.80**	**1.0650**
1978	447	+ 4.65	**195.60**	**1.0759**
1979	148	+ 0.33	**217.80**	**1.1134**
1980	26	+ 0.17	**247.20**	**1.1349**
1981			**272.70**	**1.1031**
1982			**289.50**	**1.0616**
1983			298.80	1.0310
1984				

Placing in red inflation rates exceeding a multiple of 1.06 or greater from the previous year, we have:

Year	Price Index	Change from previous year	Year	Price Index	Change from previous year	Year	Price Index	Change from previous year
						1961	89.60	+1.01
						1962	90.60	+1.01
						1963	91.70	+1.01
						1964	92.90	+1.01
						1965	94.50	+1.01
1854	27		1910	28.00		1966	97.20	+1.02
1855	28	1.03	1911	28.00	+1.00	1967	100.00	+1.02
1856	27	0.96	1912	29.00	+1.03	1968	104.20	+1.04
1857	28	1.03	1913	29.70	+1.02	1969	109.80	+1.05
1858	26	0.92	1914	30.10	+1.01	1970	116.30	+1.05
1859	27	1.03	1915	30.40	+1.00	1971	121.50	+1.04
1860	27	1.00	**1916**	**32.70**	**+1.07**	1972	125.40	+1.03
1861	27	1.00	**1917**	**38.40**	**+1.17**	**1973**	**133.20**	**+1.06**
1862	**30**	**1.11**	**1918**	**45.10**	**+1.17**	**1974**	**147.90**	**+1.11**
1863	**37**	**1.23**	**1919**	**51.80**	**+1.14**	**1975**	**161.40**	**+1.09**
1864	**47**	**1.27**	**1920**	**60.00**	**+1.15**	1976	170.70	+1.05
1865	46	0.97	1921	53.60	+0.89	**1977**	**181.80**	**+1.06**
1866	44	0.95	1922	50.20	+0.93	**1978**	**195.60**	**+1.07**
1867	42	0.95	1923	51.10	+1.01	**1979**	**217.80**	**+1.11**
						1980	**247.20**	**+1.13**
						1981	**272.70**	**+1.10**
						1982	**289.50**	**+1.06**
						1983	298.80	+1.03

56 years separates dates along a horizontal line. Given the striking inflationary trends noticed below the above horizontal line, we conclude that a 56-year Kondratiev Wave has much to offer in the analysis of decisions "endogenously" considered by the United States, while acknowledging the importance of the world wide "exogenous" factors which compel these decisions to be made.

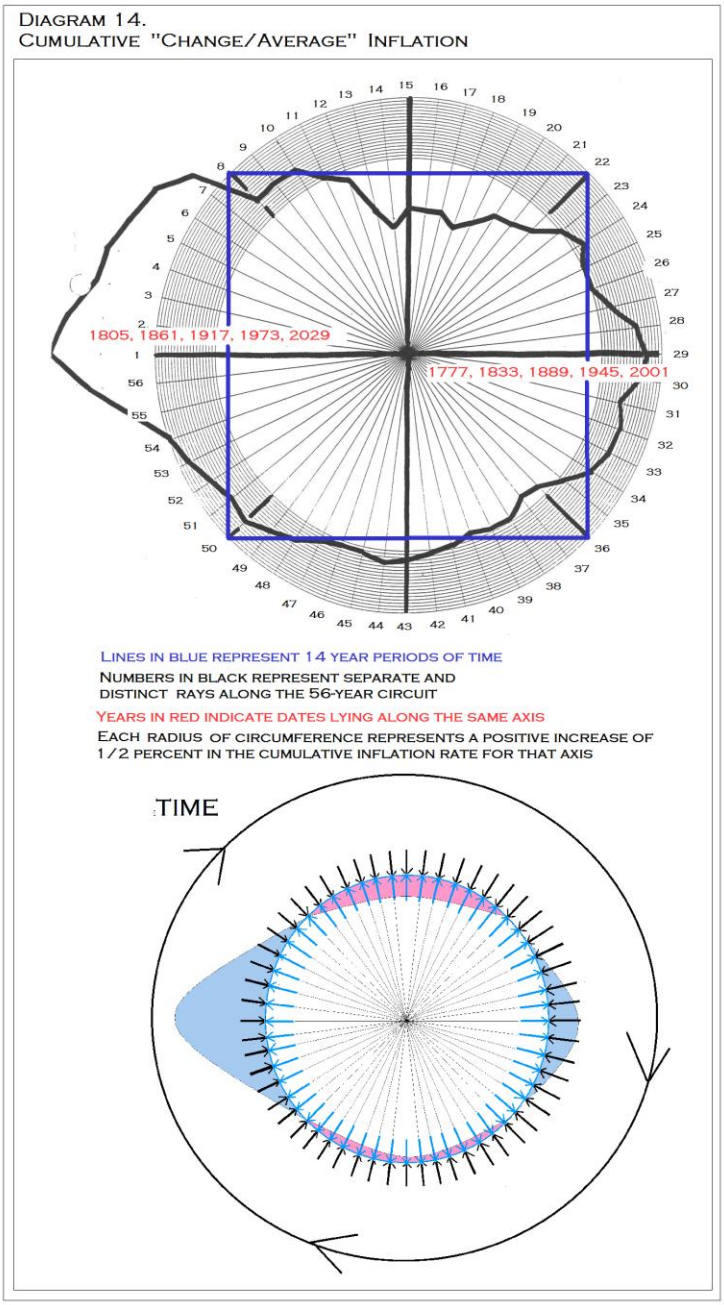
We then placed all change/average inflation (lower graph above) along a 56-year circuit shown below. In the following diagram 9 o'clock represents the midpoint of the cumulative average of all inflation along a 56 year cycle as contained within the blue rectangles above. (This is marked as "Year One" in Data Set 4.) 3 o'clock represents the midpoint of the cumulative average of all inflation rates 28 years later. (Line 29 in Data Set 4)

Data Set 5 - Inflation: Cumulative Averages.

	Col. 1		Col. 2		Col. 3		Col. 4		Col. 5	Col 6.
Axis	Year		Year		Year		Year		Average %	Cumulative %
1	1805	-0.63%	1861	8.60%	1917	10.50%	1973	6.24%	6.1765%	24.71%
2	1806	1.25%	1862	8.30%	1918	7.53%	1974	6.29%	5.8409%	23.36%
3	1807	0.62%	1863	6.59%	1919	5.97%	1975	6.64%	4.9539%	19.82%
4	1808	1.52%	1864	5.49%	1920	5.25%	1976	7.65%	4.9799%	19.92%
5	1809	1.80%	1865	4.55%	1921	3.53%	1977	8.62%	4.6222%	18.49%
6	1810	3.19%	1866	3.38%	1922	2.00%	1978	8.62%	4.2970%	17.19%
7	1811	5.22%	1867	0.34%	1923	0.32%	1979	8.13%	3.5028%	14.01%
8	1812	1.89%	1868	-3.85%	1924	-2.20%	1980	7.52%	0.8402%	3.36%
9	1813	1.07%	1869	-3.62%	1925	-0.64%	1981	7.09%	0.9731%	3.89%
10	1814	0.27%	1870	-2.99%	1926	0.30%	1982	6.49%	1.0181%	4.07%
11	1815	-1.08%	1871	-3.08%	1927	-0.30%	1983	5.36%	0.2254%	0.90%
12	1816	-1.36%	1872	-2.77%	1928	-1.57%	1984	4.32%	-0.3451%	-1.38%
13	1817	-4.56%	1873	-3.27%	1929	-3.37%	1985	3.66%	-1.8843%	-7.54%
14	1818	-7.01%	1874	-2.51%	1930	-4.30%	1986	3.54%	-2.5715%	-10.29%
15	1819	-4.79%	1875	-3.02%	1931	-3.74%	1987	3.85%	-1.9253%	-7.70%
16	1820	-5.03%	1876	-3.57%	1932	-3.31%	1988	3.85%	-2.0184%	-8.07%
17	1821	-5.30%	1877	-3.23%	1933	-3.29%	1989	3.75%	-2.0169%	-8.07%
18	1822	-4.43%	1878	-2.36%	1934	-2.41%	1990		-3.0640%	-9.19%
19	1823	-4.63%	1879	-1.92%	1935	-1.11%	1991		-2.5561%	-7.67%
20	1824	-3.19%	1880	-1.96%	1936	0.24%	1992		-1.6351%	-4.91%
21	1825	-2.87%	1881	-2.51%	1937	1.10%	1993		-1.4281%	-4.28%
22	1826	-3.39%	1882	-1.02%	1938	1.35%	1994		-1.0174%	-3.05%
23	1827	-1.72%	1883	-0.51%	1939	2.54%	1995		0.1012%	0.30%
24	1828	-0.43%	1884	-1.03%	1940	3.28%	1996		0.6065%	1.82%
25	1829	-1.76%	1885	-1.04%	1941	3.00%	1997		0.0652%	0.20%
26	1830	-2.25%	1886	-1.05%	1942	3.43%	1998		0.0430%	0.13%
27	1831	-1.83%	1887	-0.53%	1943	4.80%	1999		0.8133%	2.44%
28	1832	-0.93%	1888	0.00%	1944	6.61%	2000		1.8947%	5.68%
29	1833	0.46%	1889	0.00%	1945	6.92%	2001		2.4598%	7.38%
30	1834	0.91%	1890	0.00%	1946	5.29%	2002		2.0674%	6.20%
31	1835	0.00%	1891	-0.53%	1947	4.54%	2003		1.3345%	4.00%
32	1836	0.90%	1892	-1.08%	1948	5.31%	2004		1.7132%	5.14%
33	1837	0.45%	1893	-1.09%	1949	5.14%	2005		1.5003%	4.50%
34	1838	0.45%	1894	-1.10%	1950	4.15%	2006		1.1681%	3.50%
35	1839	-0.90%	1895	-1.11%	1951	2.55%	2007		0.1777%	0.53%
36	1840	-2.31%	1896	-1.12%	1952	1.50%	2008		-0.6476%	-1.94%
37	1841	-2.86%	1897	-1.14%	1953	1.81%	2009		-0.7269%	-2.18%
38	1842	-1.94%	1898	-0.57%	1954	2.16%	2010		-0.1164%	-0.35%
39	1843	-2.49%	1899	0.57%	1955	1.54%	2011		-0.1275%	-0.38%
40	1844	-1.01%	1900	1.12%	1956	1.34%	2012		0.4875%	1.46%
41	1845	-2.58%	1901	1.11%	1957	1.46%	2013		-0.0020%	-0.01%
42	1846	-2.11%	1902	1.10%	1958	1.52%	2014		0.1717%	0.52%
43	1847	-1.60%	1903	1.09%	1959	1.71%	2015		0.3973%	1.19%
44	1848	-1.63%	1904	1.60%	1960	1.66%	2016		0.5461%	1.64%
45	1849	-1.66%	1905	1.06%	1961	1.37%	2017		0.2572%	0.77%
46	1850	-1.12%	1906	0.53%	1962	1.24%	2018		0.2175%	0.65%
47	1851	-0.56%	1907	0.52%	1963	1.53%	2019		0.4987%	1.50%
48	1852	1.11%	1908	0.52%	1964	1.72%	2020		1.1177%	3.35%
49	1853	1.10%	1909	1.03%	1965	2.18%	2021		1.4351%	4.31%
50	1854	1.62%	1910	1.37%	1966	2.78%	2022		1.9252%	5.78%
51	1855	0.54%	1911	1.06%	1967	3.44%	2023		1.6783%	5.04%
52	1856	1.06%	1912	1.68%	1968	3.85%	2024		2.1973%	6.59%
53	1857	1.05%	1913	2.74%	1969	3.99%	2025		2.5948%	7.78%
54	1858	0.00%	1914	4.76%	1970	4.44%	2026		3.0688%	9.21%
55	1859	1.04%	1915	7.26%	1971	5.58%	2027		4.6289%	13.89%
56	1860	4.95%	1916	8.83%	1972	6.25%	2028		6.6763%	20.03%

The circumference of each circle represents a positive increase in the cumulative change/average figure of 1/2 percent (for example, a change/average cumulative amount of 1805 + 1861 + 1917 + 1973 lying directly at 9 o'clock). Points found within the interior of the smallest circumference represent negative figures by a comparable amount.

The blue square below represents the four 14-year segments of time set forth in Diagrams 10 and 13. The blue rectangles (previously given) are represented by the vertical left line segment (below). Taken together 4 x 14 periods of time create the 56 year circuit of time of this model. Note that the Great Depression of 1929-1940 is part of the deep indentation between axis 8 and 22, i.e. at the top horizontal of the blue square and interior to the smallest radii.

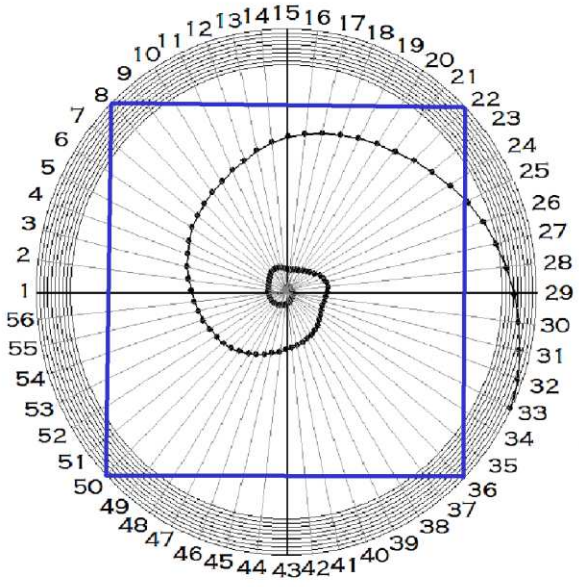


5.4 Section 4. Find fundamental average of the set

We then placed the U.S. real GNP figures given in "Data Set 2 – U.S. Real GNP" in a 56 year circuit, with the four 14-year quarter cycles indicated in blue, to create the spiral below. The center of the spiral, beginning at axis 9 = 1869, represents the real Gross National Product for that year of 23.10 billion dollars in 1958 prices. The Gross National Product for subsequent years in real terms are given along each axis respectively, with each circle of circumference representing ten billion dollars of real GNP in 1958 prices. Each row of the 14-year spreadsheet is represented by a "cross" within the spiral, beginning with Row 1 at the diagonal of the square, and moving to Row 8 at the horizontal and vertical axes of the square. The ratios of the spread sheet are simply the relative distances from the center of different points along the spiral as they relate to other points along the cross within the spiral.

DIAGRAM 15. THE "GNP SPIRAL"

	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015



	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1856	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

As can be seen from the following enlargement of the 14-year spreadsheet, we then:

(1) figured the average for each row of the spreadsheet for a total of 14 averages (Column F),
(2) figured the Median (1.617735) and Average (1.619446) of Column F, and
(3) figured a final Median Average for the entire spreadsheet of 1.618590.

In all spreadsheets this set of calculations is termed a "circle analysis." This nomenclature refers to the arrangement of Row Averages as points along the circumference of a circle, each one counted equally and but once toward a final Median Average of the spreadsheet.

DIAGRAM 16.

14 YEAR RATIOS BASED ON ANNUAL REAL GNP; MULTIPLE 5.962552

		1		2		3		4		5		6	
		YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP
1	Year	1882	42.4000	1896	61.3000	1910	120.1000	1924	165.5000	1938	192.9000	1952	395.1000
	14	Est. *	23.1000	1882	42.4000	1896	61.3000	1910	120.1000	1924	165.5000	1938	192.9000
	Ratio		1.8354978		1.4457547		1.959217		1.3780183		1.1655589		2.0482115
2	Year	1883	42.4000	1897	67.1000	1911	123.2000	1925	179.4000	1939	209.4000	1953	412.8000
	14	1869	23.1000	1883	42.4000	1897	67.1000	1911	123.2000	1925	179.4000	1939	209.4000
	Ratio		1.8354978		1.5825472		1.8360656		1.4561688		1.1672241		1.9713467
3	Year	1884	42.4000	1898	68.6000	1912	130.2000	1926	190.0000	1940	227.2000	1954	407.0000
	14	1870	23.1000	1884	42.4000	1898	68.6000	1912	130.2000	1926	190.0000	1940	227.2000
	Ratio		1.8354978		1.6179245		1.8979592		1.4592934		1.1957895		1.7913732
4	Year	1885	42.4000	1899	74.8000	1913	131.4000	1927	189.9000	1941	263.7000	1955	438.0000
	14	1871	23.1000	1885	42.4000	1899	74.8000	1913	131.4000	1927	189.9000	1941	263.7000
	Ratio		1.8354978		1.7641509		1.7566845		1.4452055		1.3886256		1.6609784
5	Year	1886	42.4000	1900	76.9000	1914	125.6000	1928	190.9000	1942	297.8000	1956	446.1000
	14	1872	23.1000	1886	42.4000	1900	76.9000	1914	125.6000	1928	190.9000	1942	297.8000
	Ratio		1.8354978		1.8136792		1.63329		1.5199045		1.559979		1.4979852
6	Year	1887	42.4000	1901	85.7000	1915	124.5000	1929	203.6000	1943	337.1000	1957	452.5000
	14	1873	23.1000	1887	42.4000	1901	85.7000	1915	124.5000	1929	203.6000	1943	337.1000
	Ratio		1.8354978		2.0212264		1.4527421		1.6353414		1.6556974		1.3423317
7	Year	1888	42.4000	1902	86.5000	1916	134.4000	1930	183.5000	1944	361.3000	1958	447.3000
	14	1874	23.1000	1888	42.4000	1902	86.5000	1916	134.4000	1930	183.5000	1944	361.3000
	Ratio		1.8354978		2.0400943		1.5537572		1.3653274		1.9689373		1.2380293
8	Year	1889	49.1000	1903	90.8000	1917	135.2000	1931	169.3000	1945	355.2000	1959	475.9000
	14	1875	23.1000	1889	49.1000	1903	90.8000	1917	135.2000	1931	169.3000	1945	355.2000
	Ratio		2.1255411		1.8492872		1.4889868		1.2522189		2.0980508		1.3398086
9	Year	1890	52.7000	1904	89.7000	1918	151.8000	1932	144.2000	1946	312.6000	1960	487.7000
	14	1876	23.1000	1890	52.7000	1904	89.7000	1918	151.8000	1932	144.2000	1946	312.6000
	Ratio		2.2813853		1.7020873		1.6923077		0.9499341		2.1678225		1.5601408
10	Year	1891	55.1000	1905	96.3000	1919	146.4000	1933	141.5000	1947	309.9000	1961	497.2000
	14	1877	23.1000	1891	55.1000	1905	96.3000	1919	146.4000	1933	141.5000	1947	309.9000
	Ratio		2.3852814		1.7477314		1.5202492		0.9665301		2.190106		1.6043885
11	Year	1892	60.4000	1906	107.5000	1920	140.0000	1934	154.3000	1948	323.7000	1962	529.5000
	14	1878	42.4000	1892	60.4000	1906	107.5000	1920	140.0000	1934	154.3000	1948	323.7000
	Ratio		1.4245283		1.7798013		1.3023256		1.1021429		2.0978613		1.6357739
12	Year	1893	57.5000	1907	109.2000	1921	127.8000	1935	169.5000	1949	324.1000	1963	551.0000
	14	1879	42.4000	1893	57.5000	1907	109.2000	1921	127.8000	1935	169.5000	1949	324.1000
	Ratio		1.3561321		1.8991304		1.1703297		1.3262911		1.9120944		1.7000926
13	Year	1894	55.9000	1908	100.2000	1922	148.0000	1936	193.0000	1950	355.3000	1964	581.1000
	14	1880	42.4000	1894	55.9000	1908	100.2000	1922	148.0000	1936	193.0000	1950	355.3000
	Ratio		1.3183962		1.7924866		1.4770459		1.3040541		1.8409326		1.6355193
14	Year	1895	62.6000	1909	116.8000	1923	165.9000	1937	203.2000	1951	383.4000	1965	617.8000
	14	1881	42.4000	1895	62.6000	1909	116.8000	1923	165.9000	1937	203.2000	1951	383.4000
	Ratio		1.4764151		1.8658147		1.4203767		1.2248342		1.886811		1.6113719
A	Maximum Ratio of Column		2.385281		2.040094		1.959217		1.635341		2.190106		2.048212
B	Minimum Ratio of Column		1.318396		1.445755		1.170330		0.949934		1.165559		1.238029
C	Spread		1.066885		0.594340		0.788887		0.685407		1.024547		0.810182
D	Mid-Range Ratio of Column		1.851839		1.742925		1.564773		1.292638		1.677832		1.643120
E	Median Ratio of Column		1.835498		1.779801		1.520249		1.326291		1.840933		1.635519
F	Average Ratio of Column		1.801155		1.780123		1.582953		1.313233		1.735392		1.616954

Est. * The data for 1868, 23.10, has been taken from the estimate given in the Historical Abstract for the years 1869 - 1877. This enables Column 1, 1882 - 1895, to be added into the graph with the minimum of difficulty in figuring averages for both rows and columns. This amount is repeated throughout the period, 1869 - 1877, and appears to represent a reasonable estimate for the Real GNP of 1868.

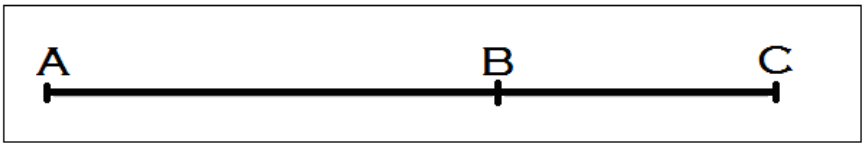
DIAGRAM 16.

14 YEAR RATIOS BASED ON ANNUAL REAL GNP; MULTIPLE 5.962552

7		8		9		10		A	B	C	D	E	F
YEAR	GNP	YEAR	GNP	YEAR	GNP	YEAR	GNP	Maximum Ratio of Row	Minimum Ratio of Row	Spread	Mid-Range Ratio of Row	Median Ratio of Row	Average Ratio of Row
1966	658.1000	1980	996.8309	1994	1514.3943	2008	2198.6295						
1952	395.1000	1966	658.1000	1980	996.8309	1994	1514.3943						
	1.66565426		1.51471038		1.51920882			2.048212	1.165559	0.882653	1.606885	1.516960	1.614648
1967	675.2000	1981	1010.8394	1995	1546.7308	2009	2208.7984						
1953	412.8000	1967	675.2000	1981	1010.8394	1995	1546.7308						
	1.63565891		1.49709627		1.53014495			1.971347	1.167224	0.804123	1.569285	1.556346	1.612417
1968	706.6000	1982	995.1411	1996	1615.0033								
1954	407.0000	1968	706.6000	1982	995.1411								
	1.73611794		1.4083514		1.62288875			1.897959	1.195789	0.702170	1.546874	1.620407	1.618355
1969	725.6000	1983	1072.5727	1997	1681.8760								
1955	438.0000	1969	725.6000	1983	1072.5727								
	1.656621		1.47818729		1.56807646			1.835498	1.388626	0.446872	1.612062	1.612349	1.617114
1970	722.5000	1984	1129.4464	1998	1764.5370								
1956	446.1000	1970	722.5000	1984	1129.4464								
	1.61959202		1.56324761		1.56230256			1.835498	1.497985	0.337513	1.666742	1.562775	1.622831
1971	751.2051	1985	1174.0716	1999	1854.0672								
1957	452.5000	1971	751.2051	1985	1174.0716								
	1.66012177		1.5629175		1.57917728			2.021226	1.342332	0.678895	1.681779	1.607259	1.638339
1972	803.4814	1986	1203.2684	2000	1911.3209								
1958	447.3000	1972	803.4814	1986	1203.2684								
	1.79629197		1.49756846		1.58844103			2.040094	1.238029	0.802065	1.639062	1.571099	1.653772
1973	839.4182	1987	1256.1826	2001	1925.1794								
1959	475.9000	1973	839.4182	1987	1256.1826								
	1.76385417		1.49649198		1.53256334			2.125541	1.252219	0.873322	1.688880	1.514528	1.660756
1974	821.7401	1988	1303.1774	2002	1957.1959								
1960	487.7000	1974	821.7401	1988	1303.1774								
	1.68492946		1.58587539		1.50186452			2.281385	0.949934	1.331451	1.615660	1.635402	1.680705
1975	843.0778	1989	1340.0434	2003	2036.0677								
1961	497.2000	1975	843.0778	1989	1340.0434								
	1.69565125		1.58946588		1.5194043			2.385281	0.966530	1.418751	1.675906	1.596927	1.690979
1976	879.3138	1990	1351.3622	2004	2093.6810								
1962	529.5000	1976	879.3138	1990	1351.3622								
	1.66064929		1.53683725		1.5493115			2.097861	1.102143	0.995718	1.600002	1.592543	1.565470
1977	922.6690	1991	1360.3512	2005	2151.0247								
1963	551.0000	1977	922.6690	1991	1360.3512								
	1.67453539		1.47436535		1.58122748			1.912094	1.170330	0.741765	1.541212	1.627881	1.566022
1978	985.8821	1992	1418.0149	2006	2201.9891								
1964	581.1000	1978	985.8821	1992	1418.0149								
	1.69657907		1.43832097		1.55286739			1.840933	1.304054	0.536879	1.572493	1.594193	1.561800
1979	1001.7304	1993	1454.1409	2007	2272.2615								
1965	617.8000	1979	1001.7304	1993	1454.1409								
	1.62144772		1.451629		1.56261439			1.886811	1.224834	0.661977	1.555823	1.586993	1.569035
	1.796292		1.589466		1.622889								
	1.619592		1.408351		1.501865			Max. of F - Rows	Min. of F - Rows	Mid-Range of F - Rows	Median of F - Rows	Avg. of F - Rows	Mid-Range + Average/2	Median + Average/2
	0.176700		0.181114		0.121024			1.690979	1.561800	1.626389	1.617735	1.619446	1.622918	1.618590
	1.707942		1.498909		1.562377									
	1.665654		1.497096		1.552867			Max. of F - Columns	Min. of F - Columns	Mid-Range of F - Columns	Median of F - Columns	Avg. of F - Columns	Mid-Range + Average/2	Median + Average/2
	1.683407		1.506790		1.555007			1.801155	1.313233	1.557194	1.616954	1.619446	1.588320	1.618200

	Max. of F - Rows	Min. of F - Rows	Mid-Range of F - Rows	Median of F - Rows	Avg. of F - Rows	Mid-Range + Average/2	Median + Average/2
Circle Analysis	1.690979	1.561800	1.626389	1.617735	1.619446	1.622918	1.618590
Square Analysis	1.690979	1.561800	1.626389	1.617114	1.619126	1.622758	1.618120

This number 1.618590, the final Median Average of rows[22], is 0.034% greater than the constant phi, 1.6180339… This constant, sometimes referred to as "the Golden Mean," "the Golden Ratio" or "the Golden Section," was defined circa 300 b.c. by Euclid of Alexandria, as follows:



> A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.[23], [24]

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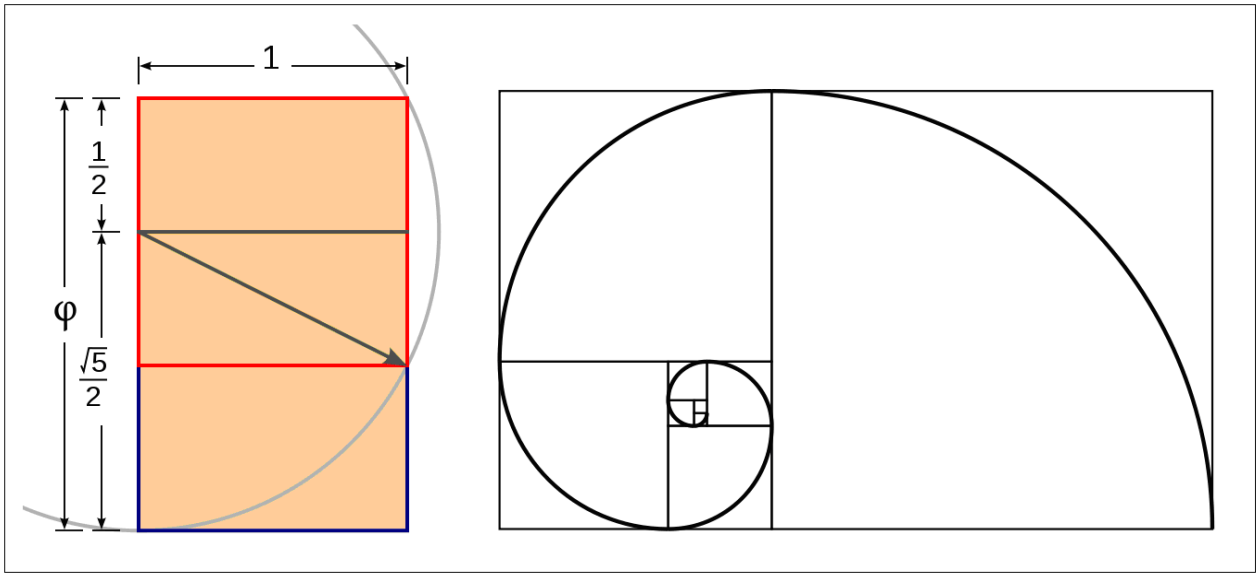
[22] As mentioned in the text, a "circle analysis" counts each average of rows (column F) a single time toward a final Median Average for the entire spreadsheet. A "square analysis" counts the first row twice, and arrives at a slightly different number, one which is 0.0053% in proximity to the Golden Mean. A further discussion of the rationales underlying "circle analysis" and "square analysis" is placed in the Second Post-script to this article.

[23] Euclid of Alexandria, Elements, Book VI, Definition 3, circa 300 b.c.. A broad array of texts may be suggested describing the well-known associations between the Golden Mean and patterns discovered in Nature. See e.g. Livio, 2002; Skinner, 2006; Hemenway, 2005.

[24] Geometrically, the proportion of 1: φ may be created by the following construction. A spiral may be obtained from this construction as follows. This spiral and its relationship to the economy of the United States has been one of the central points of this paper.



If line segment AB is set to 1, and if the line segment AC is in a Golden Mean relationship to AB, then line segment AC will equal 1.6180339… This finding can be checked by creating the following graph wherein we:

a) indicate the spread between years which generates the ratio (presented below in the "# of years" first column),

b) set forth the Median Average for all ratios generated for any given spread of years (second column below),

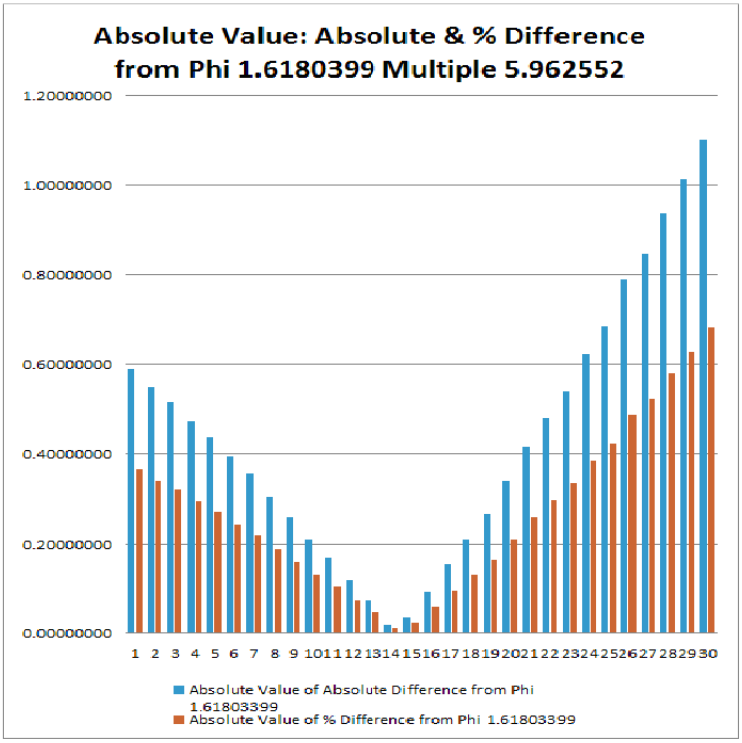
c) figure the "absolute difference" and the "percentage difference" of these different Median Averages from phi (3rd and 4th columns below), and finally

d) state these differences as absolute values (5th and 6th columns below).

This data is summarized in the bar graph below this data. This graph demonstrates that Median Average generated by a 14-year spread between years are closest to 1.6180339…, = phi, or the Golden Mean.

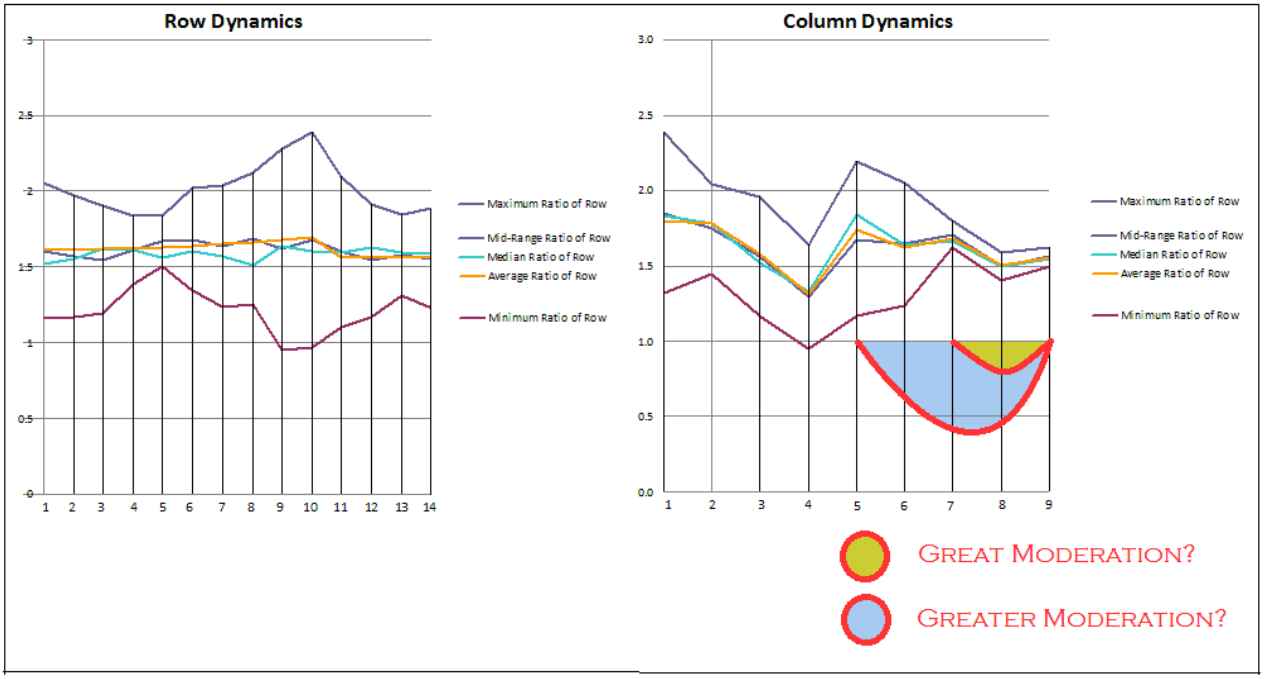
DIAGRAM 17.
COMPARATIVE DIFFERENCES:
MEDIAN AVERAGES VS. 1.61803399

# of Years	Median Aveage	Absolute Difference from Phi 1.61803399	% Difference from Phi 1.61803399	Absolute Value of Absolute Difference from Phi 1.61803399	Absolute Value of % Difference from Phi 1.61803399
1	1.03086043	0.58717356	36.2893216%	0.58717356	36.2893216%
2	1.06996068	0.54807331	33.8727936%	0.54807331	33.8727936%
3	1.10353672	0.51449727	31.7976802%	0.51449727	31.7976802%
4	1.14504076	0.47299323	29.2325895%	0.47299323	29.2325895%
5	1.18247232	0.43556167	26.9191915%	0.43556167	26.9191915%
6	1.22633118	0.39170281	24.2085649%	0.39170281	24.2085649%
7	1.26388505	0.35414894	21.8876084%	0.35414894	21.8876084%
8	1.31520833	0.30282566	18.7156551%	0.30282566	18.7156551%
9	1.36070905	0.25732494	15.9035558%	0.25732494	15.9035558%
10	1.40916235	0.20887164	12.9089775%	0.20887164	12.9089775%
11	1.44965664	0.16837735	10.4062924%	0.16837735	10.4062924%
12	1.50019982	0.11783417	7.2825524%	0.11783417	7.2825524%
13	1.54501537	0.07301862	4.5127988%	0.07301862	4.5127988%
14	1.60189961	0.01613438	0.9971593%	0.01613438	0.9971593%
15	1.65125029	-0.03321630	-2.0528801%	0.03321630	2.0528801%
16	1.70936280	-0.09132881	-5.6444307%	0.09132881	5.6444307%
17	1.77052591	-0.15249192	-9.4245191%	0.15249192	9.4245191%
18	1.82742627	-0.20939228	-12.9411549%	0.20939228	12.9411549%
19	1.88097935	-0.26294536	-16.2509171%	0.26294536	16.2509171%
20	1.95675154	-0.33871755	-20.9338960%	0.33871755	20.9338960%
21	2.03196341	-0.41392942	-25.5822452%	0.41392942	25.5822452%
22	2.09620235	-0.47816836	-29.5524302%	0.47816836	29.5524302%
23	2.15690921	-0.53887522	-33.3043204%	0.53887522	33.3043204%
24	2.23755840	-0.61952441	-38.2887142%	0.61952441	38.2887142%
25	2.30123214	-0.68319815	-42.2239677%	0.68319815	42.2239677%
26	2.40625778	-0.78822379	-48.7149093%	0.78822379	48.7149093%
27	2.46439399	-0.84636000	-52.3079247%	0.84636000	52.3079247%
28	2.55145856	-0.93342457	-57.6888107%	0.93342457	57.6888107%
29	2.62813943	-1.01010544	-62.4279492%	1.01010544	62.4279492%
30	2.71795717	-1.09992318	-67.9789908%	1.09992318	67.9789908%



As noted at the outset of this paper, the final Median Average for the 14-year spread of 1.618590 was generated as a result of the following Row Dynamics, a pattern which had the least "Used General Dissonance," the least "Acute Dissonance" and the second-to-least "Claimed Dissonance" of all spreads considered. As can be clearly seen below, and unlike the other spreads considered, when a high average of the row is reached it is immediately balanced by a low as determined from the approximate midpoint of the Golden Mean. In addition, as time has passed the American economy has steadily narrowed its focus to precisely this same single point.[25]

DIAGRAM 18.
ROW AND COLUMN DYNAMICS: 14-YEAR SPREAD



[25] The last two columns of the Column Dynamic graphic represent a time period stretching from the end of Column 7 (1979) through the end of Column 9 (2007). During this period of time the economic volatility of previous years markedly narrowed. Although hailed at the time as "The Great Moderation" and a possible sign of progress in economic understanding (e.g. Bernanke, 2004), post-Global Financial Crisis this view has come under attack. (e.g. Chomsky, 2011) The same graphic demonstrates that a marked narrowing of volatility began two columns prior to 1979, i.e. beginning with the end of Column 5 (1951), named here "The Greater Moderation" by way of comparison. (See "Second Post-script. Correlations and Speculations." for additional material on this point.)

To figure the annual increase implied by the GNP Spiral, we may use the formula for simple interest compounded annually…

$$FV = PV\ (1+r)^t$$

… ; state a present value (PV) of $1,000,000; a time period (t) of 14 years; and the future value (FV) as given below in proportion to the varying numbers derived in the GNP Spiral. These assumptions give us the following interest rates (r).

	Future Value		Interest rate
x= Circle Analysis:	$1,618,590	interest rate is:	3.4995226
x= Golden Mean:	$1,618,033	interest rate is:	3.4969781

These "interest rates" are the annual "rates of growth" necessary to obtain the various proportions of the GNP Spiral over time, [26], [27]

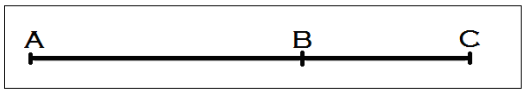
[26] At least one reference – albeit atavistic – may be cited in support of a similarity between the large number of designs found in Nature which incorporate the Golden Mean (the galactic spiral, the Chambered Nautilus, seed pods of various plants, aspects of DNA, etc.) and the almost biologic dynamism of the GNP Spiral presented herein. (See e.g. Kahn, 1961:425) "(I)t … seems likely that Stalin's caution (regarding antagonism toward the United States) did not stem from fear of the atomic bomb as a decisive weapon. What alarmed him about the United States was Detroit – not (the Strategic Air Command)! He appears to have felt very strongly that no sensible government tangles with a nation with a GNP of $300 billion a year. Luckily we had both assets – the bomb and the GNP – so that any difference between U.S. and Soviet calculations was not crucial."

[27] A surprisingly eclectic reading list may be constructed on possible parallels to the 56-year cycle suggested herein. These include: (1) the circular arrangement of 56 "Aubrey holes" at Stonehenge, (Cleal, et al. 1995); (2) price fluctuations predicted in 1875 by an Ohio farmer (Benner 1875); (3) business cycles of 56-years (Funk 1933); (4) astrologic cycles generally connected to the orbit of Saturn (Williams 1947, 1959, 1982); (5) an "energy use cycle" of 56-years (Stewart 1989); (6) the "Joseph Cycle" (Sim 2008) and (7) a compendium of geologic, weather, financial and other information (McMinn 2006, 2007, 2011). The Jewish festival *Birkat Hakhammah* "Blessing of the Sun" takes place every 28 years, most recently April 8, 2009. See also Tompkins (1976:282) "Hunab Ku, sole source of movement and measure, symbolized the universe for the Maya in the form of a circle with an inscribed square. The circle was the symbol of the infinite, the spiritual; the square of the material. Hunab Ku was thus a universal dynamism or that which motivates and stimulates life in its total manifestation as spirit and matter, the all in one."

Conclusion

Referring once again to the definition of the Golden Mean, we have:

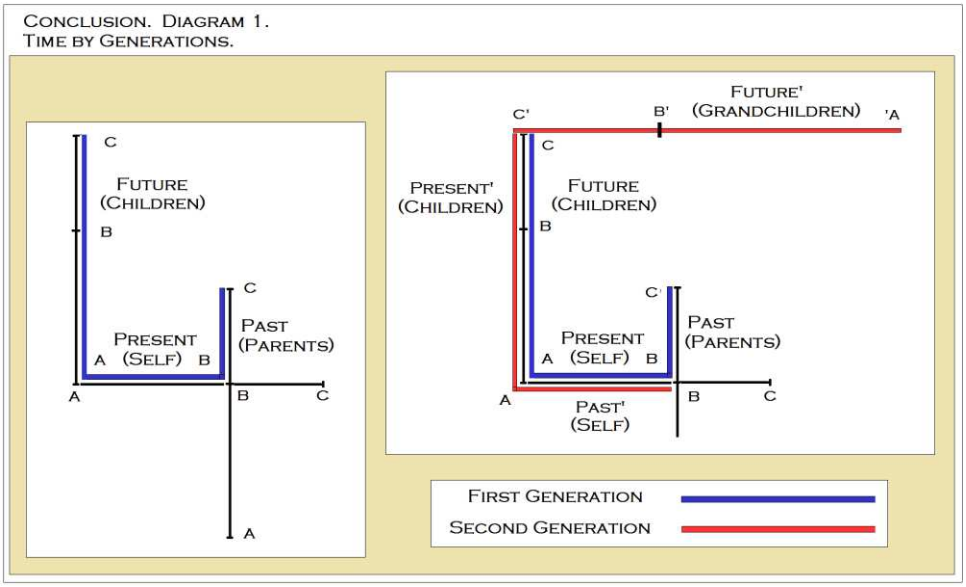
> A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.



As described in this article, and in connection with the economic progress of the United States, *the Golden Mean appears to tie the past (line segment BC) to the present (line segment AB) to the future (line segment AC) in a self-consistent and harmonic fashion*. It is a mathematic statement of the historic identity of the United States itself, as moving from date to date in a coherent, repeating manner as connected to a 14-year spread between years and as nested as a quarter-cycle within a 56-year circuit of social time.

The 14-year interval of time which lays the foundation for the 14-year spread between numerator and denominator in ratios of GNP, like the musical interval of an octave, provides a framework within which this evolution of GNP may take place. Like the octave, it lays the essential mathematic relationship of the entire spectrum of harmonies of growth. This coincides with the 50-60 year period given by Kondratiev as the basis for his model.

There is at least a poetic similarity between the division of a line segment into past-present-future and the familial context underlying society itself wherein one's parents (past) give birth to one's self (present) as continued through one's children (future). Inasmuch as each stage of this familial expansion of self begins with the onset of reproductive capacities at age 14, the GNP Spiral / classic Kondratiev Wave may form as a parallel to an underlying biologic pattern.

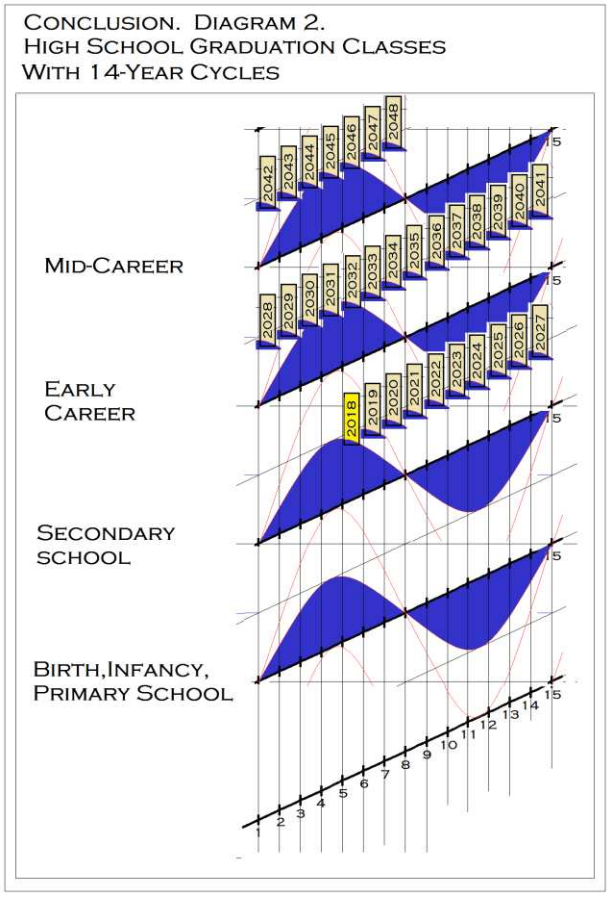


It appears to be very likely that this underlying geometry of "generational time" lays the foundation for the strict cyclical element of the Kondratiev Wave, one which is biologically driven but upon which an enormous host of other economic, social and political relationships float interconnectedly.

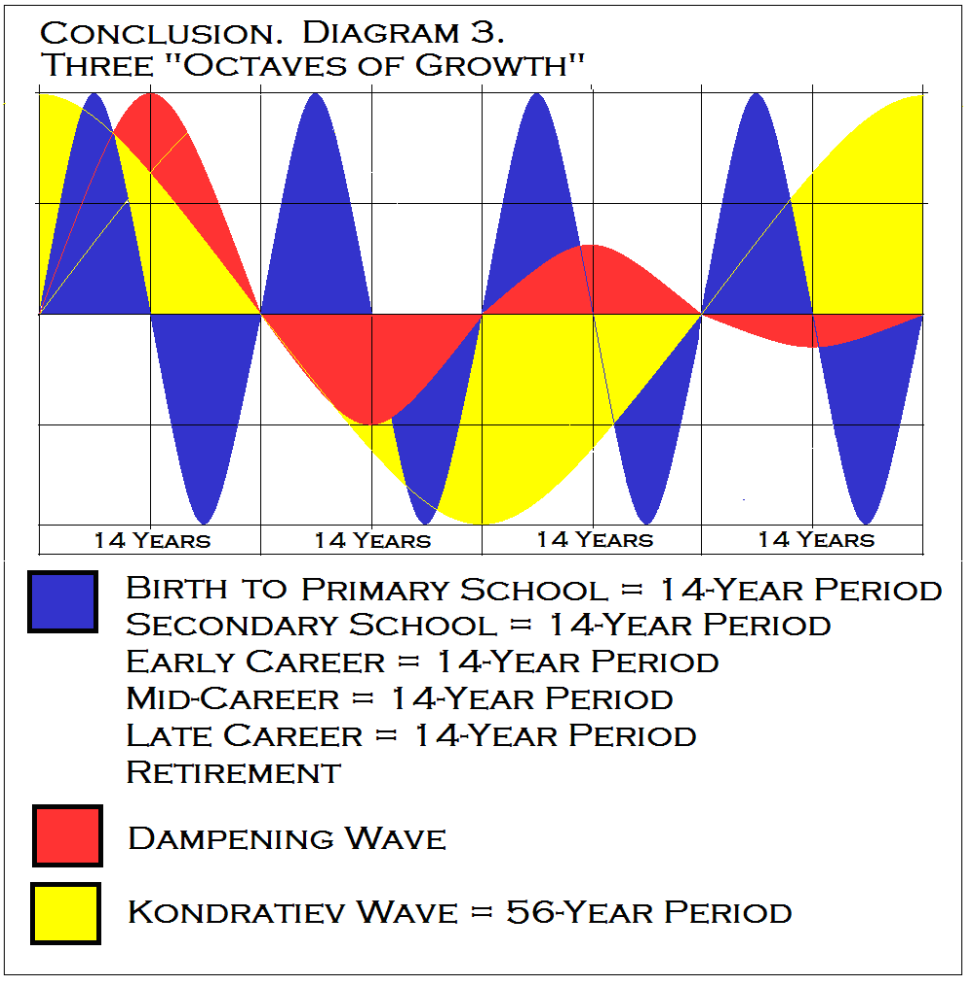
One might bear in mind the sheer force of life which continually bears on this dynamic. If we imagine that this "life force" of the economy may be viewed physically at the graduation of a high-school class, we can see that the force of these repetitive 14-year periods is not limited to a single family unit but rather constitutes a continuing host of waves, each breaking into the future as a new, highly charged and hopeful high school graduation class.

Returning to the hypothetical child born on January 1, 2000, we can watch the cumulative force of this development. Below we see a straight-line development over time as represented by each high school class graduation date, beginning with the graduation date of said child at 2018 (in highlighted yellow below). Every graduation class possesses a 14-year wavelength sustaining it. And each class is like the others in that the persons graduating begin the ascent through the careers which they choose.

As a single life goes through the sequential 14-year periods of Primary School, Secondary School, Early Career, Mid-Career, Late Career and Retirement which are themselves complemented by similar high school class graduations, we have the following.



As presented below, it would appear that the fundamental "octave" of life is the motion leading from birth to reproductive capacity (in blue), as encompassed by the dampening price wave described in Diagrams 11, 12, 13, and 14 (in red), and as further encompassed within the largest 56-year octave of the entire Kondratiev cycle as described in Diagrams 14 and 15 (in yellow).



The intermediate "octave" of price change (in red) transforms the biologic human octave (in blue) into the larger 56-year octave of the Kondratiev Wave (in yellow).

It is to the consideration of this intermediate octave which we now turn.

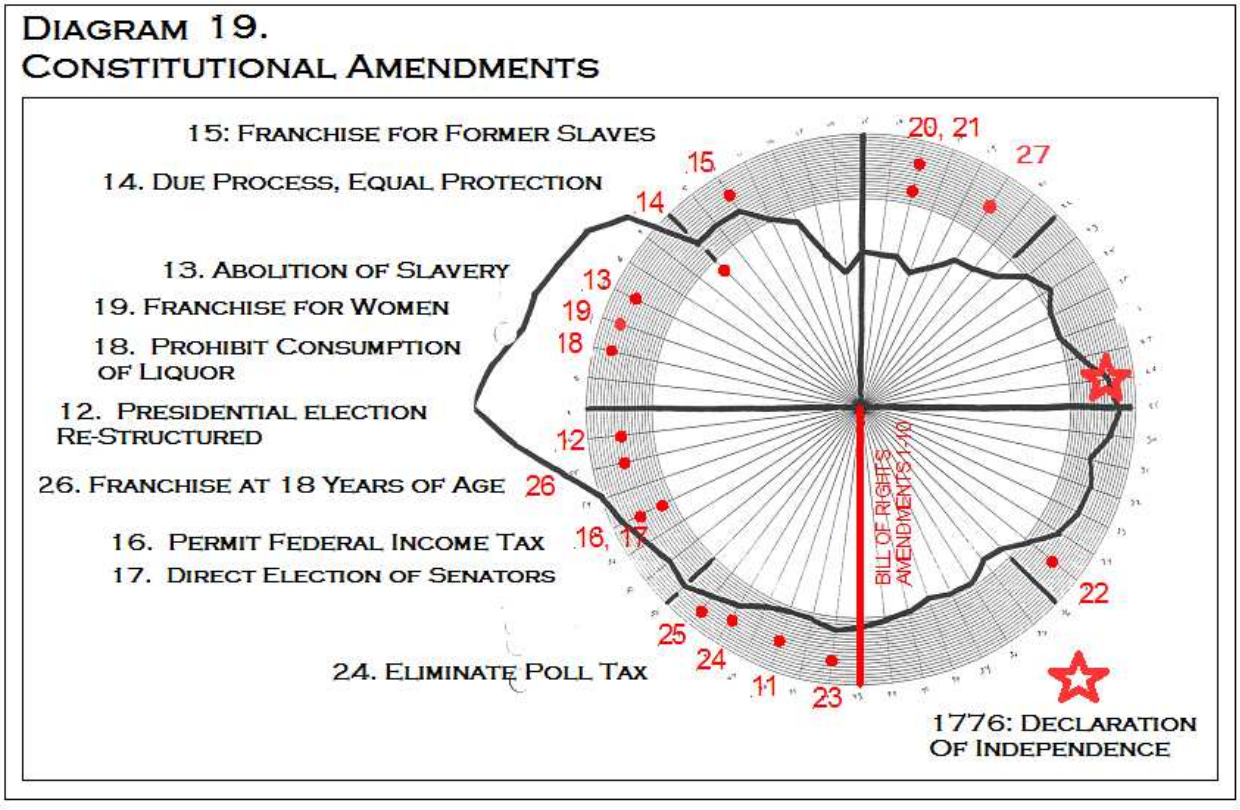
Part Two: Post-scripts

First Post-script. Correlations and Speculations.

Part One.

A major conclusion reached by Kondratiev was that democratic capitalism was capable of avoiding the decline and disintegration predicted by Marx through its ability to correct the worst abuses of capitalism over time. In this vein, the significance of this 56-year cycle may be extended beyond the realm of economics if we correlate the dates of political events with their respective axes in this circuit.

For example if we place on the various axes of the 56-year circuit the dates of the Amendments to the United States Constitution we have the following distribution of significant changes to the legal foundation of the United States. It is immediately apparent that a far greater number of amendments have been adopted toward the left hand side of the circuit than have been adopted during the right hand side.



Let us first discount the Bill of Rights as falling on the exact dividing line between the left and right sides of this circuit (enacted December 15, 1791). If we consider only the remaining amendments we may note that in addition to a numeric difference, a qualitative difference also exists between the right-hand and left-hand sides of the circuit. Falling within a ten-year span before and after "Year 1" (9 o'clock) are amendments:

(1) to give former slaves the franchise (Am. 15, axis 10=1870),
(2) to require "due process of law" and "equal protection" (Am 14, axis 8=1868),
(3) to abolish slavery (Am. 13, axis 5=1865),
(4) to permit women the franchise (Am. 19, axis 4=1920),
(5) to prohibit the consumption of liquor (Am. 18, axis 3=1919),
(6) to re-structure the election of Presidents and Vice-Presidents (Am. 12, axis 56= 1804),
(7) to permit 18 year old citizens the franchise (Am. 26, axis 54=1971),
(8) to permit the imposition of income taxes (Am. 16, axis 53=1913),
(9) to require the direct election of senators (Am. 17, axis 53= 1913), and
(10) to eliminate poll taxes as a requirement to voting (Am. 24, axis 48=1964).

Only two constitutional amendments fall within a ten year span of "Year 29," i.e. 3 o'clock. Amendment 22 restricts a president from serving more than 2 terms in office (axis 31=1951) and enshrines in law a tradition begun by George Washington 154 years earlier when in 1797 he refused to run for a third term in office. Amendment 27 prohibits laws affecting Congressional salary from taking effect until the beginning of the next session of Congress. This amendment was proposed September 25, 1789 and enacted 203 years later in May 1992.

We might also consider the two remaining Amendments on the right hand side of the cycle. Both enacted in 1933, Amendment 20 determined the dates of term commencements for Congress and the President and Amendment 21 repealed the federal prohibition on consumption of alcohol. Amendment 20 was a purely administrative amendment and Amendment 21 returned the country to a well-established social norm.

It is of course possible to take any data set and superimpose upon it a spiral of any sort. The list of Amendments to the Federal Constitution is useful in this analysis because:

(1) each Amendment carries with it a specific date of adoption, thereby making placement in the cycle non-controversial,

(2) each Amendment engages the entire United States by virtue of the centrality of the Federal Constitution and the difficulties posed in their adoption,

(3) each Amendment declares in the clearest possible terms what is intended, albeit this interpretation remains subject to further interpretation by the courts, and

(4) each Amendment remains an influence upon continued American development. In many cases these Amendments are intended to direct the process of the economic future of the American people away from evils previously experienced (slavery, disenfranchisement of African-Americans, women and persons of draft age, resistance to federal taxation of income, addiction to alcohol, unjust use of governmental powers, etc.)

It should be borne in mind that, while the use of other data sets may contest the significance of this cycle, at this point we attempt simply to understand this model, explore the origin of the Golden Mean within the American economy and consider the sort of "balancing" which permits it.

The numerous amendments on the left-hand side of the circuit above should be contrasted with one of the most fundamental documents of American economic history occurring on the right-hand side of the circuit, the Declaration of Independence of 1776. This document makes clear that the colonists did not perceive themselves as setting forth upon some new and novel declaration of rights. Rather they viewed themselves as collectively determined to continue to enjoy rights which they already possessed.

Regarding George III the colonists declared in their first five grievances:

He has refused his assent to laws, the most wholesome and necessary for the public good.

He has forbidden his governors to pass laws of immediate and pressing importance, unless suspended in their operation till his assent should be obtained; and when so suspended, he has utterly neglected to attend to them.

He has refused to pass other laws for the accommodation of large districts of people, unless those people would relinquish the right of representation in the legislature, a right inestimable to them and formidable to tyrants only.

He has called together legislative bodies at places unusual, uncomfortable, and distant from the depository of their public records, for the sole purpose of fatiguing them into compliance with his measures.

He has dissolved representative houses repeatedly, for opposing with manly firmness his invasions on the rights of the people.

The remainder of the Declaration of Independence describes in ever expanding detail the list of wrongs done by the king to his colonists. Each of these royal acts or omissions justified – at least in the minds of the signatory colonists – an immediate separation of the colonies from the crown in protection of long-held rights, customs and privileges.

The correlation between Amendments to the Federal Constitution and the 56-year circuit envisioned by this model provides support for the proposition that the circuit itself is an important part of the underlying social fabric of the United States and its political economy. The Amendments are not scattered uniformly around the spiral but rather are grouped almost entirely on the left-hand side. These Amendments generally alter American political life in quite dramatic ways. Amendments to the right of the cycle are very few and generally intended to honor and fix firmly past traditions and social mores.

The discovery of this "bi-polarity" of American political life suggests the possibility that that the four 14-year segments of time which have been used as the foundation of this circuit may themselves have importance. If this is granted we may now expand this model into an understanding of the underlying nature of the political economy of the United States over time.

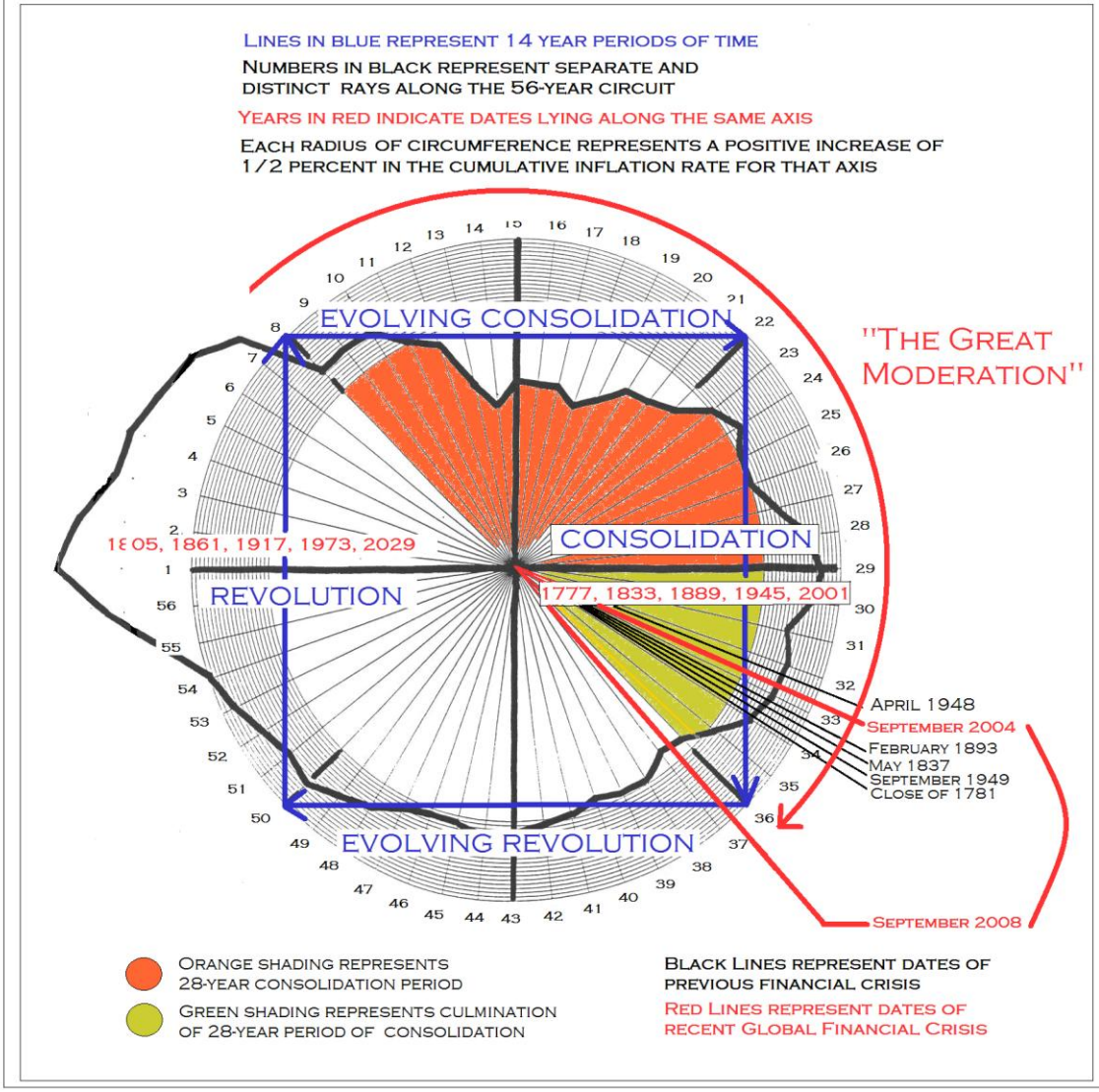
Part Two

We may now speculate as to the nature of the right-left division underlying the GNP Spiral. This will conclude the final step of our analysis of American Economic History.

For the purposes of this paper regarding American economic history, let us define a "*Belief-system*" as the constellation of ideas surrounding any principle of governance: a monarchy, the bourgeoisie, slavery, the relationship of labor to capital, etc. Second, let us define the term "*Revolution*" as a period of time when significant portions of a time-honored belief-systems are destroyed and when new and largely untried belief systems are inaugurated. Third, let us define in contradistinction to "Revolution" the term "*Consolidation*" as an opposing historical period in which honor or reverence are given to relatively recent belief-systems in a manner calculated to preserve and prolong them. It would appear that the left half of the circuit is "revolutionary" in character, while the right half is "consolidating" in character in the context of historic American belief systems.

In light of the numerous constitutional amendments adopted on the left-hand side of the circuit, and the virtual lack thereof on the right-hand side, let us label each of the segments of American History as follows:

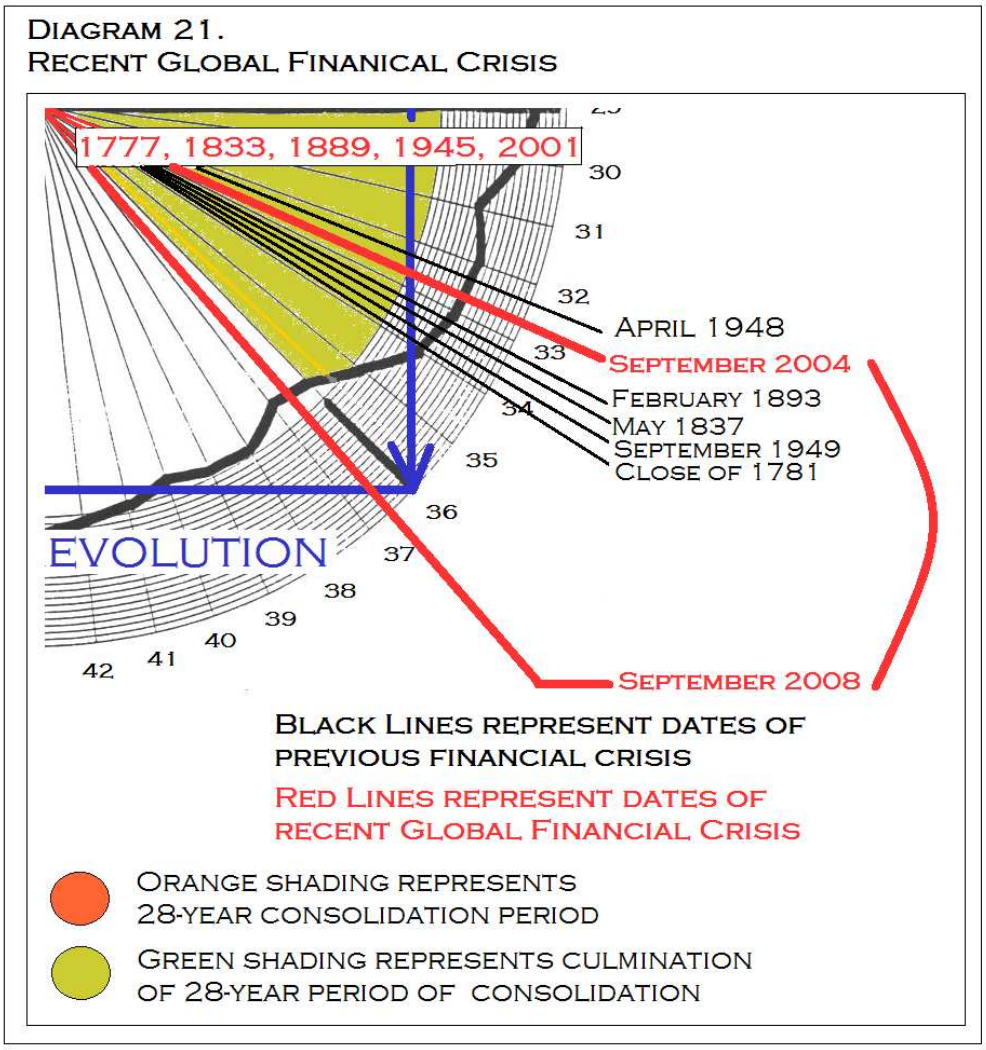
Diagram 20. "Evolving Consolidation" and "Consolidation"



Note in the above that as each period of consolidation has come to its close, the United States has very predictably experienced a complete meltdown of the economy. This occurred most recently in September through December of 2008, the last months of the terms of George W. Bush. Prior events of similar magnitude are:

1. The collapse of the colonial economy, circa 1781,
2. The Panic of 1837,
3. The Panic of 1893 and
4. The Marshall Plan of 1948 and the events of 1949.

Two unusual characteristics of the recent global meltdown should be pointed out. These are (1) the difficulty of "dating" the recent crisis, and (2) the delay of the expected time of crisis. Let us consider these important points briefly.



Each of the previous dates of "meltdown" clearly corresponded with events between axes 33 and 34. A description of these crises may be given simply by citing textbooks of American History.

Colonial meltdown of 1781

"In 1764 Parliament had outlawed paper money in the colonies altogether. Independence ended this restriction, and both the Continental Congress and the states printed large amounts of money during the Revolution, with inflationary results. To cite some examples, the Continental dollar became utterly worthless by 1781, and Virginia eventually called in its paper money at 1,000 to 1."[28]

Panic of 1837

"In 1836 the second United States Bank automatically came to the end of its checkered career and the country under the inspiration of the new democracy entered an epoch of "wild cat" finance. The very next year (May, 1837), a terrible business depression fell like a blight upon the land, bringing as usual more suffering to farmers and mechanics than to the "rich and wellborn"; but this calamity was likewise attributed by the masses to the machinations of the money power rather than to the conduct of their hero, President Jackson. Nothing would induce them to retrace their steps. For three decades a union of the South and West prevented a restoration of the centralized banking system. Not until the planting statesmen withdrew from Congress and the storm of the Civil War swept minor gusts before it were the ravages wrought by Jackson repaired by the directors of affairs in Washington."[29]

Panic of 1893

"The (Cleveland) Administration was not three months old when a series of bank failure and industrial collapses inaugurated the panic of (February) 1893. The treasury's gold reserve was depleted by an excess of imports and by liquidation of American securities in London after a panic there. Gold was subject to a steady drain by the monthly purchase of useless silver required by the Silver Purchase Act of 1890, and by the redemption of greenbacks which by law were promptly reissued and formed an "endless chain for conveying gold to Europe."[30]

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[28] John A Garraty, *The American Nation, A History of the United States*, Harper-American Heritage Textbook, p. 144.

[29] Charles A. Beard, Mary R. Beard, *The Rise of American Civilization*, New Edition, Macmillan Company, New York., p. 570-571.

[30] Garraty, p. 795.

Reviewing the same axes for the years 1948-1949, we have, in addition to the creation of the Marshall Plan to rebuild post-war Europe (April 1948), the following:

> 1949
> In 1949 a business recession occurred and prices declined slightly. (p. 819) … Further alarmed by the news, released in September 1949, that the Russians had produced an atomic bomb, Congress appropriated $1.5 billion to arm NATO and in 1951 General Eisenhower was recalled to active duty and placed in command of all NATO forces. (p. 785) … This (civil war in China) resulted in the total defeat of the nationalists; by the end of 1949 Mao ruled all China and Chiang's shattered armies had fled to sanctuary on the island of Formosa, now called Taiwan. This loss of over half a billion souls to communism caused an outburst of indignation in the United States and deeply divided the American people. Critics claimed that Truman had not backed the nationalists strongly enough and that he had stupidly underestimated both Mao's power and his dedication to the cause of world revolution. (p. 786)[31]

The recent Global Financial Crisis began when, in September 2004, the FBI reported that it had uncovered widespread fraud in the home mortgage market (axis 32). The date of this FBI report precedes the axes of the above mentioned crises, i.e. 1781, 1837, 1893 and 1948-1949, by a matter of months. However, and unlike previous crises, action to correct these frauds was not undertaken and the final implosion was delayed for four years, i.e. to September 2008, two months before the election of Barack Obama. Public reaction, not unlike previous moments along axis 33, has been extremely suspicious about the timing and origin of this world-wide panic. [32]

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[31] Garraty, p. 786.

[32] See e.g. House Bill 3995, presented by Representative Kaptur, November 3, 2009:

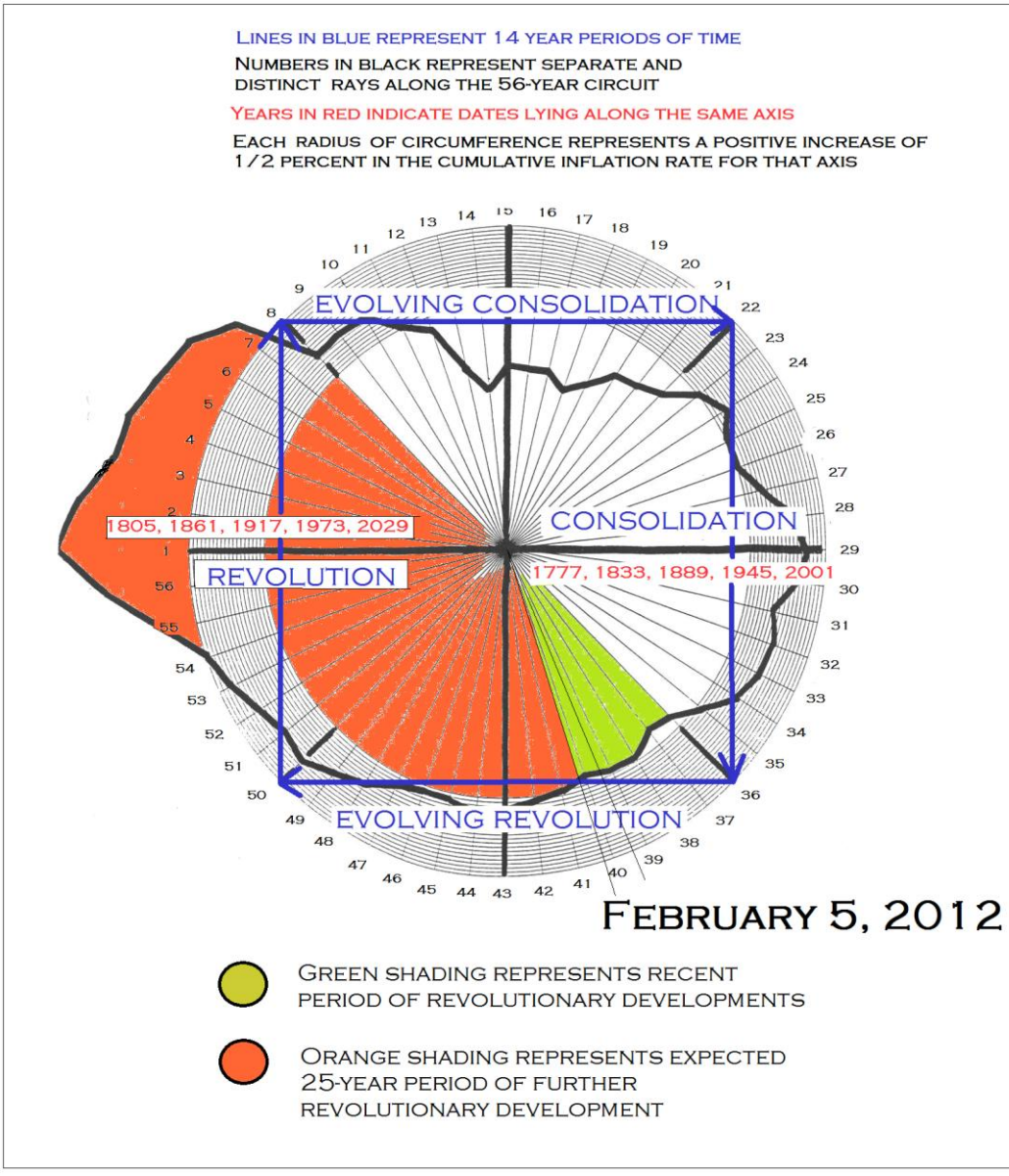
"(4) Fraud also played a decisive role in the Savings and Loan crisis (of the late 1980s and early 1990s). The FBI and Justice Department made prosecuting those elite frauds among its highest priorities. This took a massive commitment of FBI resources, but it produced the most successful prosecution of an epidemic of elite fraud in history--over 1,000 `priority' felony convictions of senior insiders, according to Professor William K. Black in his book `The Best Way to Rob a Bank is to Own One'.

(5) However, the FBI, because of its crippling personnel limitations, has been unable to assign sufficient FBI agents to investigate the current global financial crisis. The FBI identified the mortgage fraud `epidemic' in congressional testimony in September 2004. It had so few white-collar crime specialists available, however, that it was able to assign only 120 special agents to mortgage fraud cases--less than one-eighth the agents it found essential to respond adequately to the huge, but far smaller, Savings and Loan crisis.

(6) Given the magnitude of the financial crisis of 2008 and the resulting losses and billions of taxpayer dollars spent to keep the financial system from collapsing, the FBI should have no less than 1,000 agents to address corporate, securities, and mortgage fraud located across the country, and, in addition, more forensic experts and Federal prosecutors to uncover the crimes committed and bring the perpetrators to justice."

To conclude our speculation as to the nature of this circuit brings us to a discussion of the current events of today. We are, today, at the dividing line between green and orange in the graph below.

DIAGRAM 23.
"EVOLVING REVOLUTION" AND "REVOLUTION"



The green portion of the above represents the beginning of an evolving revolutionary trend starting in 2008.

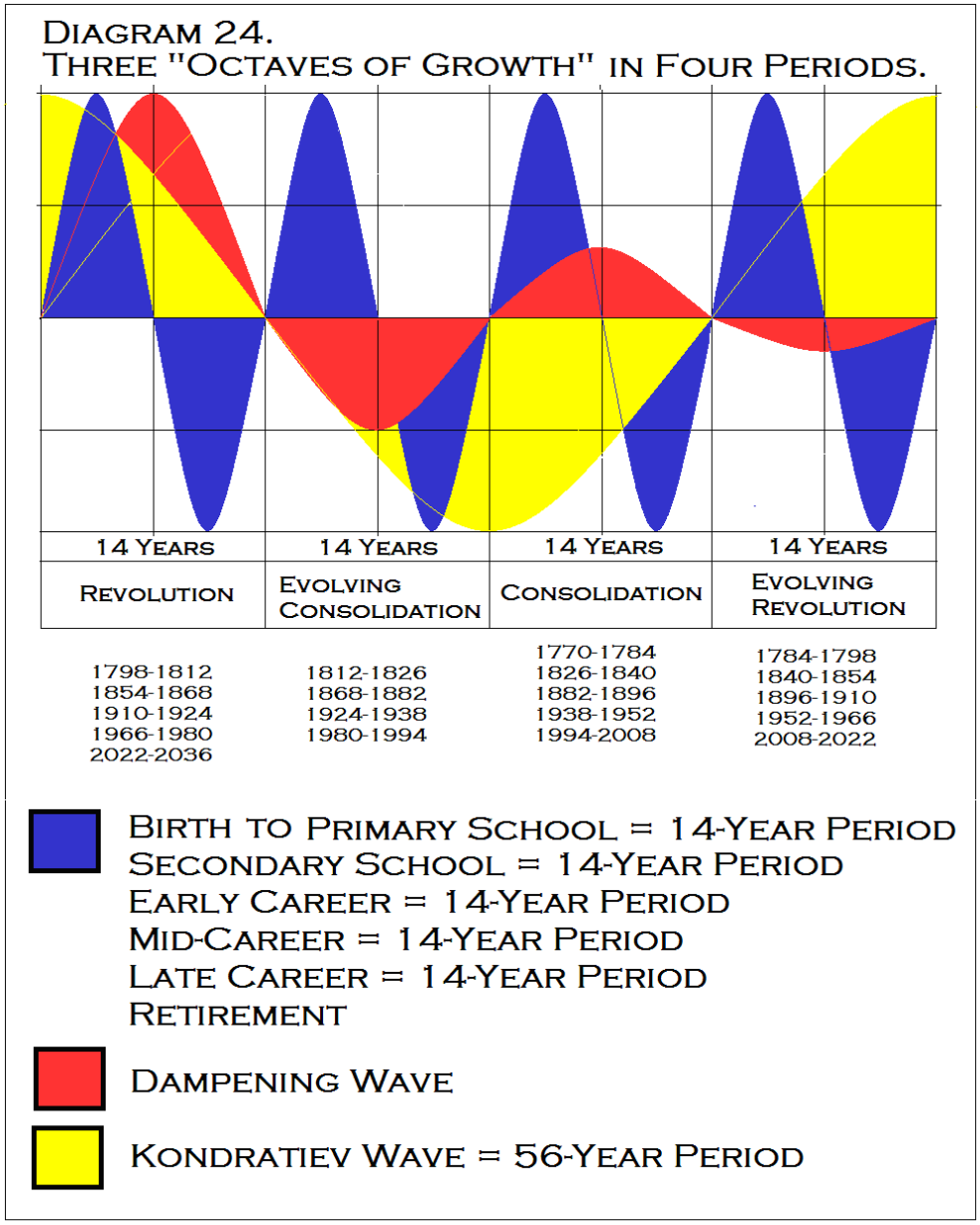
This green section correlates to an impressive extent with the current difficulties faced by the United States in the Middle East. Note that as of the date of the publication of this article, the United States has attempted to deal with a number of revolutionary changes throughout the Arab world.

These have included but are not limited to: Tunisia, Egypt, Libya, Bahrain, Yemen, Syria, Morocco and Algeria. These events have become known popularly as "The Arab Spring." Chronologically, these were preceded by the 2009 Revolution in Iran. They have been joined since that time by protests, revolts and crackdowns in Tibet, China, England and Greece as well as a painful sovereign debt crisis in Europe with additional austerity measures generally anticipated. The fact that these events are taking place at the very beginning of the "Evolving Revolution" segment of American economic history may presage much greater events to come.

A strong correlation between the onset of inflation and the axes of this period has been described by this model. The graph above demonstrates the historic inflationary rise which typically accompanies this period of American economic history.

The amount of orange given in the above development towards revolution represents inflation, the strength of which emerges most dramatically along the left-pointing axis at nine o'clock. These years represent very difficult times in the history of the United States – the coming of the war with Britain in 1812 during which the White House, the Capitol, the Library of Congress and the Treasury were burned to the ground (1814); the American Civil War beginning in 1861 ending in the assassination of President Lincoln in 1865; the First World War beginning for the United States in 1917; and the OPEC Embargo of 1973. This axis brings revolutionary times of great uncertainty, a forced re-reading of America's place in world history.

As presented below, it would appear that the fundamental "octave" of life is the motion leading from birth to reproductive capacity (in blue), as contained within the broader "octave" of 28-year periods of Evolving Revolution to Revolution and Evolving Consolidation to Consolidation (in red), all of which are encompassed within the largest 56-year octave of the entire Kondratiev cycle (in yellow).

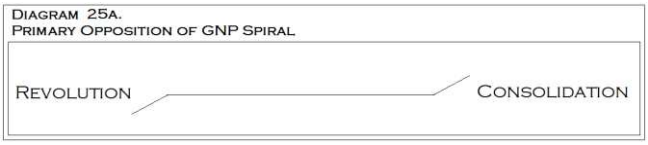


It would further appear that the basic reproductive expectations of life are channeled into the Kondratiev Wave via the willingness of human beings to alter their environment over specific periods of time.

We turn next to a simplification of this model which may permit these separate wavelengths to be coordinated.

Second Post-script. Simplification, Expansion.

Our presentation of the social balance of the economic history of the United States has been based upon a pattern of two essential parts. First we have proposed a distinct and complete separation of periods of Consolidation and Revolution, indicated by what will be named a "Primary Opposition." The purpose of stating this opposition formally is to convey the idea of an absolute or unequivocal difference between two separate and distinct things.



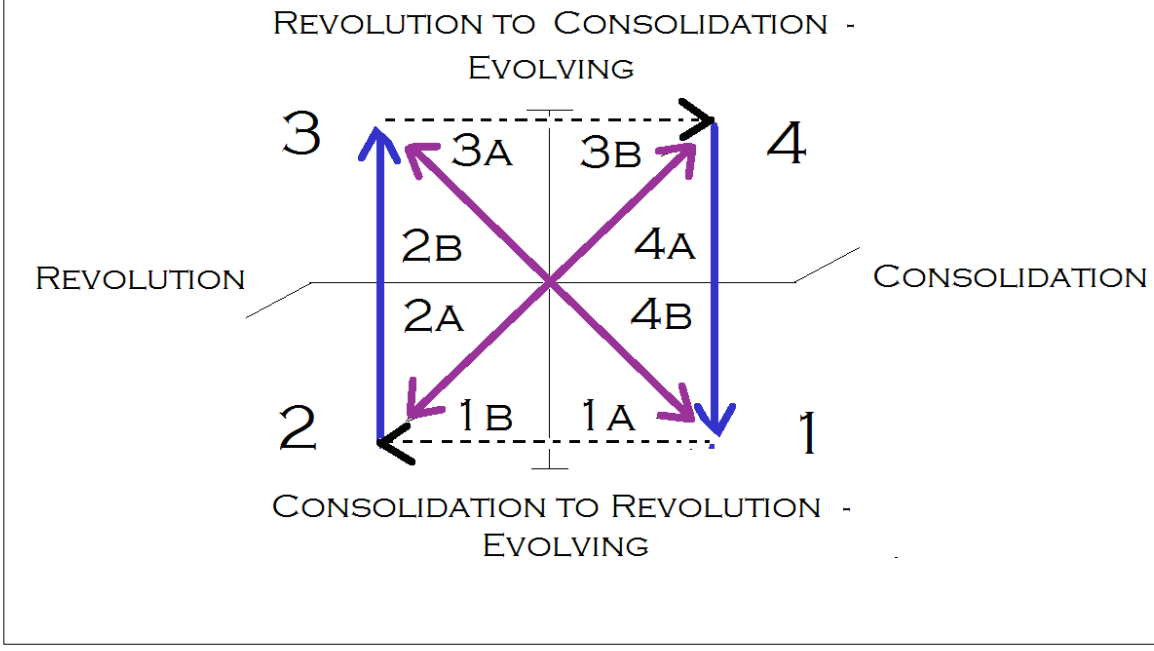
Second, we have contrasted this first division of a 56-year cycle with two additional periods of time wherein an evolutionary or incremental development occurs joining these first two intractable opposites. The addition of this second type of opposition is named a "Secondary Opposition."

Together these two oppositions create a square of tension wherein four central points are brought out. These are:

(1) the point at which Consolidation ends and Evolving Revolution begins,
(2) the point at which Evolving Revolution ends and Revolution begins,
(3) the point wherein Revolution ends and Evolving Consolidation begins and
(4) the point at which Evolving Consolidation ends and Consolidation begins.

The notion that a geometric square is at play in the economic history of the United States arises from the force of these oppositions.

DIAGRAM 26A.
MAP OF GNP SPIRAL - DEFINED CATEGORIES



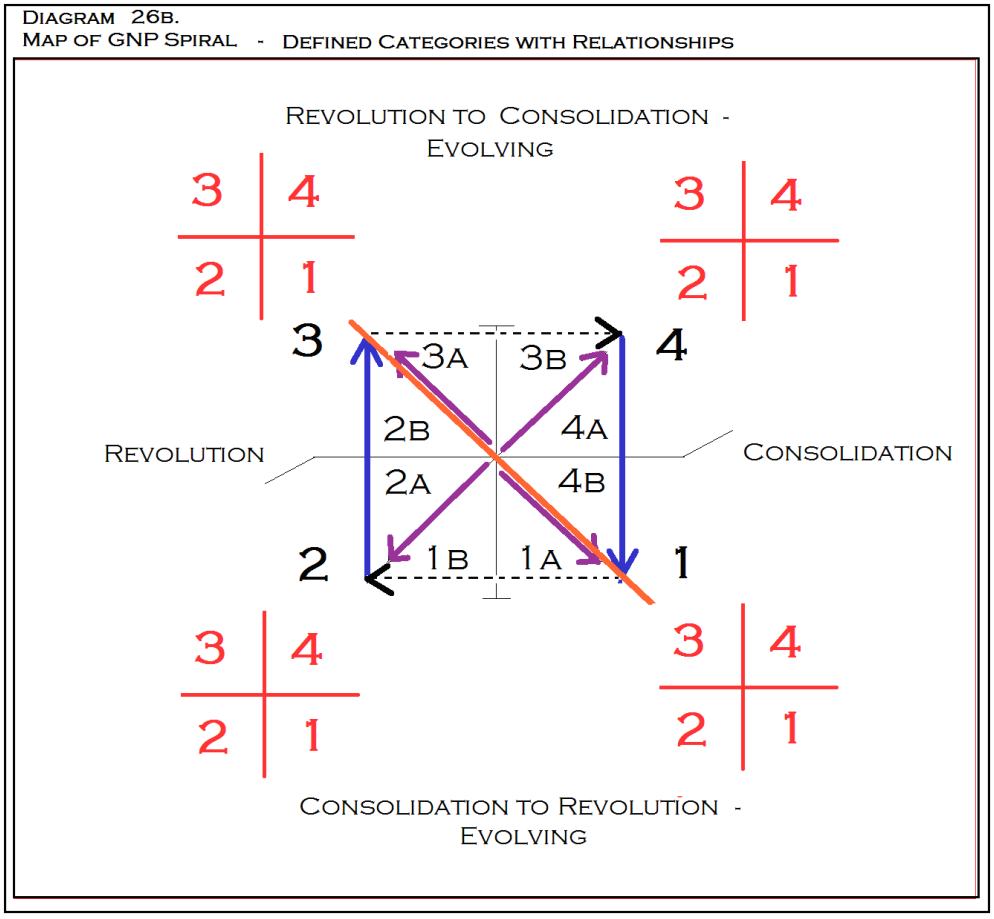
In order to map the square implied by the GNP Spiral, the placement of blue lines below indicates diametrically opposing ideas (Revolution, Consolidation) as separated by an impossible and intractable gulf of opposition and which extend themselves over a period of time.

The placement of black dotted lines below represents that gulf, as traversed by incremental adjustments over time (Evolving Revolution, Evolving Consolidation).

The orange line repeats the separation of the model into equal halves as noted in the foregoing article at length.

Finally, these oppositions give rise to the four corners of a square of relationships (numbers 1, 2, 3, and 4 in black), which in turn have relationships with the other corners of the square (numbers in red which repeat the 1, 2, 3, 4 pattern).

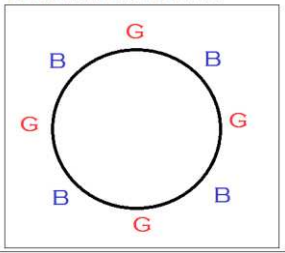
The result is a simple "map" of what might be termed the "logic" or the "social psychology" of the United States as it creates a balanced and productive political economy over time. This "square" of relationships balances the productive capacity of the United States as generated by a 14-year octave of generational development supporting the Golden Mean and its place as a fundamental figure within the economy.



As a result of these relationships, we must consider how the geometry of a square may impact on the analysis of the data we have presented in the main paper.

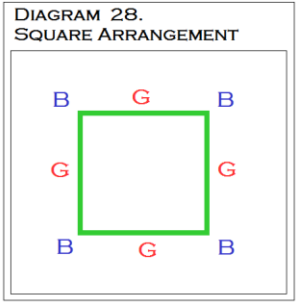
Let us imagine that an elementary school teacher has a class of four girls and four boys. It would be easy to picture her taking her class outside to the playground, placing them side by side, boy-girl-boy-girl, and arranging them in a circle. They might stand as follows in the geometric figure.

DIAGRAM 27.
CIRCLE ARRANGEMENT



We could also imagine the teacher arranging them in a square. The geometric order might be as follows:

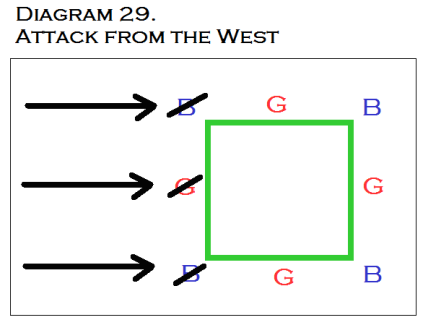
DIAGRAM 28.
SQUARE ARRANGEMENT



Now let us imagine that the same group of boys and girls are sent to war as men and women. In combat the groups are arranged in the same "square" of relationships with 100 yards between soldiers.

We may imagine for the purposes of demonstration that the enemy attacks from the west and kill all soldiers closest to the wave of the attack whilst the others escape. After battle, the enemy must necessarily count 2 male soldiers killed and one female soldier killed. Let us presume that the death count is the only knowledge the enemy has of our military. Consequently any conclusions they come to about our forces are based only upon their knowledge of persons killed.

DIAGRAM 29.
ATTACK FROM THE WEST



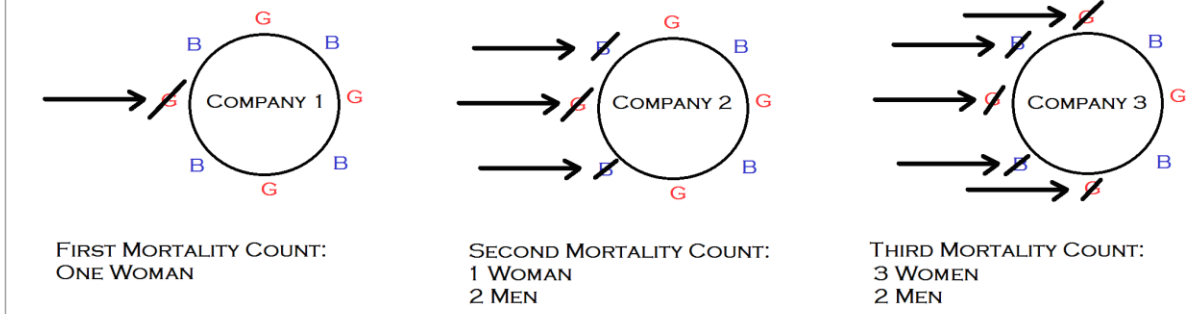
We may further imagine that the enemy repeatedly attacks other companies from the north, east, south and west, with the same dynamics in the persons killed. In each case the mortality count is 2 male soldiers killed and one female soldier killed. Based simply upon an analysis of soldiers killed in battle, the enemy could easily come to a number of incorrect conclusions, i.e.:

(1) there are twice as many men in the company as women, or
(2) women are twice as good as evading death as are men, or
(3) men are one half as courageous as women.

In short, a number of false conclusions could be reached if the geometry of the arrangement of the company remains unknown and the only knowledge available comes from the body count after attacks.

On the other hand if the companies are arranged in circles, and if the enemy attacks as before, the enemy would now be much more likely to count even numbers of men and women killed, over all.

DIAGRAM 30.
ATTACK OF CIRCULAR FORMATION



FIRST MORTALITY COUNT:
ONE WOMAN

SECOND MORTALITY COUNT:
1 WOMAN
2 MEN

THIRD MORTALITY COUNT:
3 WOMEN
2 MEN

The enemy might also note that whenever they capture an entire unit, they always find equal numbers of men to women.

The fundamental lesson of this example is that when one takes averages of things which occur in geometric formations, one must understand the geometry of the formation to take a correct average.

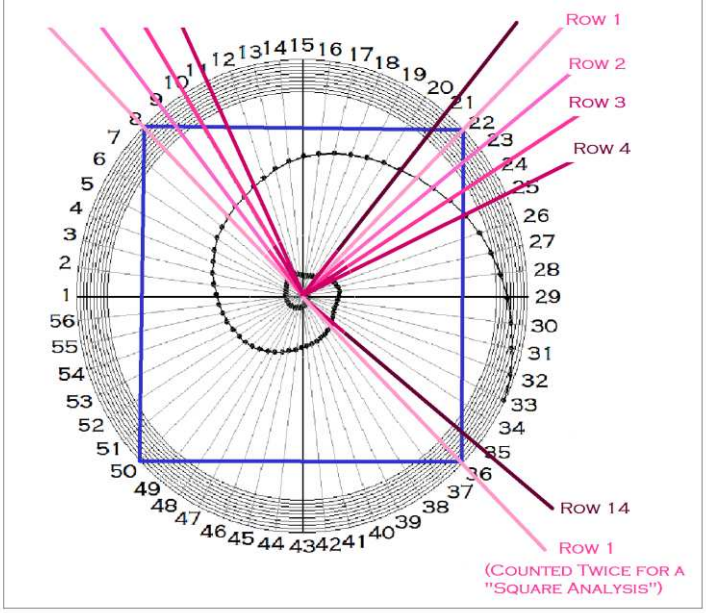
It might also be pointed out that as the numbers of soldiers increases per company the significance of this insight fades. As demonstrated below, as the numbers increase in the company, the ratio of men to women killed in battle approaches a 1:1 ratio without regard to the square vs. circle formation. Referring to a square formation, the significance of the difference between a "square" and a "circle" geometric configuration is as follows:

Total soldiers	Soldiers per side	Men per side	Total fatality count per side men to women	Significance of difference
8	3	2	2:1	2
16	5	3	3:2	1.5
24	7	4	4:3	1.3333…
32	9	5	5:4	1.25
40	11	6	6:5	1.2
48	13	7	7:6	1.1666…
56	15	8	8:7	1.1428…

The association between geometry and ratio affects our analysis because, in essence, the Kondratiev wave proposes that we are in some sort of spiral version of history. According to the mathematic strategies of this paper, this spiral occurs as based upon four sets of 14-year periods of real GNP, for a total of 56 years in the circuit.

We have listed the ratios of un-averaged real GNP at 14-year spreads in an Excel spread sheet. The first date, the ratio of 1882 / 1868, is placed in Column One Row One and presents the diagonals of the square figured as underlying the entire spiral itself, as follows:

DIAGRAM 31.
"CIRCLE ANALYSIS" VS. "SQUARE ANALYSIS"



If we assume that all final row ratios within a spread sheet are of equal importance, we must count each ratio equally in a final average of fourteen rows. This may be referred to as a "circle analysis" because – like the points of the circumference of a circle – all are equidistant from the center and none possess any particular or obvious significance over the others. Under this analysis, we have figured a final average for all rows under the 14-year spread of 1.618590, or 0.034% greater than phi.

On the other hand if the development of American GNP is a *square* of relationships the corners of the square of ratios must be figured twice. The double-counting of this corner point is in a situation similar to that of the soldier standing at the corner of the square whose faces forces coming from two directions rather than one.

However as we consider this fifteenth year as an additional date in the line from corner to corner of all ratios, we must notice that this fifteenth ratio is simply the first row (which gives the diagonal of the square of ratios) counted twice. All of the diagonals of the square are contained in that single, first row in the Excel spreadsheet.

If the diagonal ratios of the 14-year spread sheet are included twice in the calculation of the final Median Average of the figures, we have the following comparisons to the Golden Mean.

DIAGRAM 32.
PROXIMITIES TO THE GOLDEN MEAN

Splicing Multiple 5.962552		Proximity to 1.61803399	
Rows:		Absolute	Percentage
Median Average (Circle)	1.618590	+0.000556	+0.034%
Median Average (Square)	1.618120	+0.000086	+0.0053%
Columns			
Median Average	1.618200	+0.000167	+0.0098%

As noted previously, to figure the annual increase implied by the GNP Spiral, we may use the formula for simple interest compounded annually…

$$FV = PV\ (1+r)^t$$

… ; state a present value (PV) of $1,000,000; a time period (t) of 14 years; and the future value (FV) as given below in proportion to the varying numbers derived in the GNP Spiral. These assumptions give us the following interest rates (r).

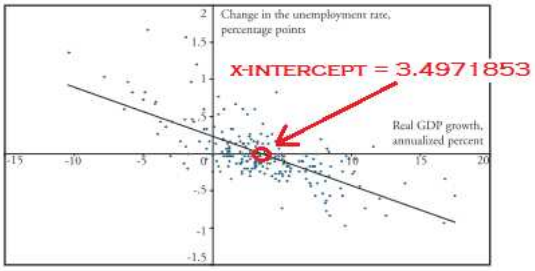
	Future Value		Interest rate
x= Circle Analysis:	$1,618,590	interest rate is:	3.4995226
x= Square:	$1,618,120	interest rate is:	3.4973756
x= Golden Mean:	$1,618,033	interest rate is:	3.4969781

The above "rates of growth" may be contrasted with one of the central empirical regularities of mainstream economics, i.e. Okun's Law. This rule proposes a roughly 3:1 ratio between increases in real GNP and decreases in the rate of unemployment in the economy of the United States. A trend line may be devised for quarterly data between the second quarter of 1948 and the second quarter of 2007 which gives the slope of this relationship as:

$$y = .23094 + -0.066036x$$

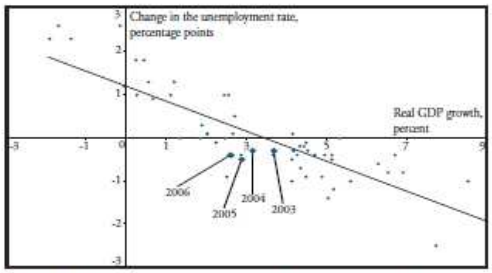
A "steady state" rate of economic growth may be figured for the x-intercept, i.e. that rate of growth which occurs when there is no change in the rate of employment. (y = 0). Using the above equation and trend line, this x-intercept is 3.4971853. (Knotek, 2007, with additional correspondence by the author)[33]

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

As these figures relate to the annual rate of growth necessary to sustain all values investigated above we have:

Analysis:	Future Value	Promixity to Phi	Rate:	Comparison to Okun's x-intercept at 3.4971853
Circle:	$1,618,590	1.00034424	3.4995226	1.000668337
Columns:	$1,618,200	1.00010321	3.4977411	1.000158927
Square:	$1,618,120	1.00005376	3.4973756	1.000054415
Okun's Law x-axis:	$1,618,078	1.00002781	3.4971853	1
Golden Mean:	$1,618,033	1	3.4969781	0.999940752

When this "steady state" rate of growth under Okun's Law is placed among the "rates of growth" calculated by the GNP Spiral, the x-intercept generates a future value in proximity to the Golden Mean of 2.7/100,000 parts, closer than all other values.

[33] "Data Set Five" contains the figures supporting these charts and is found as an Appendix to this paper.

The question arises as to whether Okun's Law can be used as evidence of the presence of the Golden Mean in this context. One may argue that because we measures GNP data herein, and because Okun's Law measures the same data, that it should not be surprising that the steady state rate of growth given by the x-intercept of Okun's Law for quarterly data (which deliberately excludes changes in the rate of unemployment) would be the same as the "Golden Mean" rate of growth.

To test this argument we took the Median Average of each spreadsheet and multiplied it by $1,000,000 to obtain an appropriate "Future Value" for the interest rate equation above. (Figures given are "circle analyses" in as much as only even numbered spreads possess "square analysis" possibilities, and the 14-year spread is the even-numbered spread most proximate to the Golden Mean.)

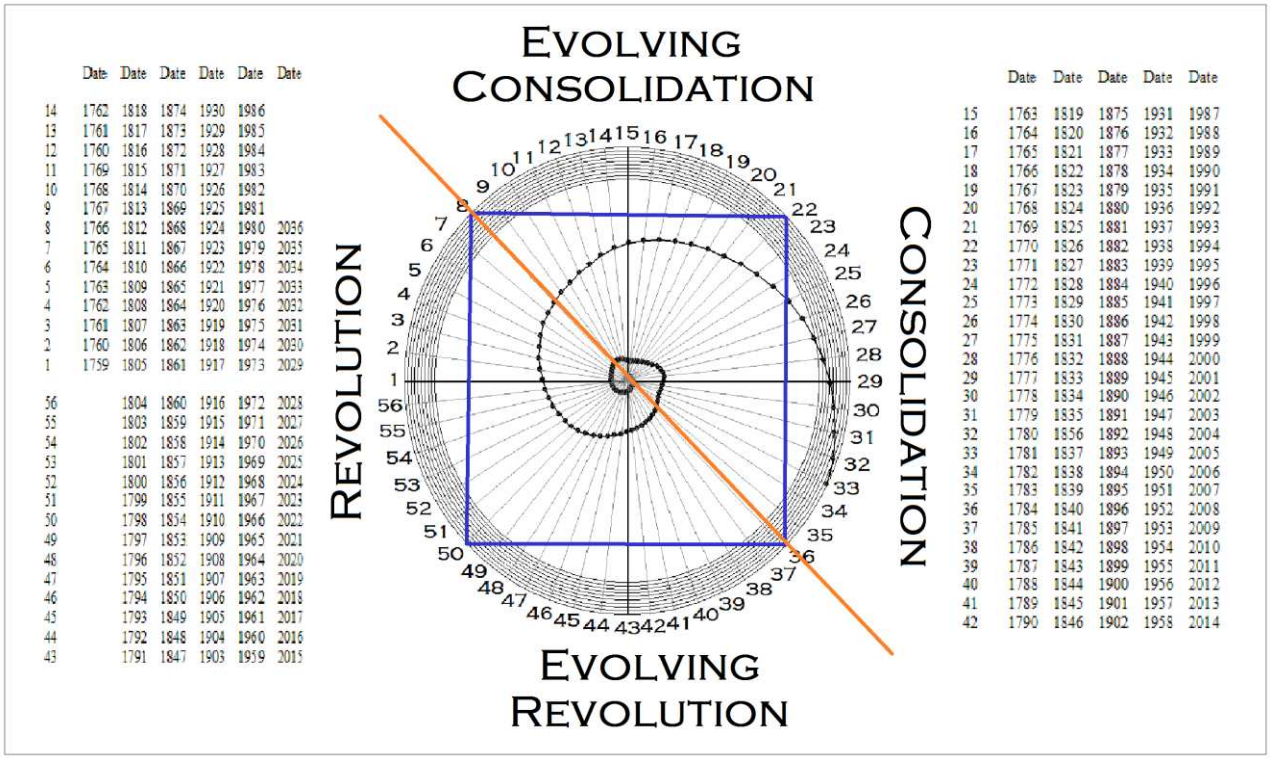
We also took the steady state rate of growth given by the quarterly data for Okun's Law as a rate for the same equation (r = 3.4971853) and used the spread of years for each spreadsheet for the time period (t = number of years in spreadsheet) of the same equation.

If the argument is valid there should be no difference between these two results. As can be seen below, proximities between these two numbers are closest at the "square analysis" of the 14-year spread (0.00259%, see Postscript Two), and become *progressively* more distant as one considers increases or decreases in the number of years in the interval between years – "the spread" – from this point.

Spread	Median Average	Future Value (Median Avg. X $1,000,000)	Future Value (r=3.4971853, t = years in spread)	Row/Okun	Percentage Difference
7-year	1.292308	$1,292,308	$1,272,037	1.0159	+1.59%
8 year	1.334588	$1,334,588	$1,316,522	1.0137	+1.37%
9 year	1.385800	$1,385,800	$1,362,563	1.0170	+1.70%
10 year	1.431250	$1,431,250	$1,410,215	1.0149	+1.49%
11 year	1.470320	$1,470,320	$1,459,533	1.007390	+0.73%
12 year	1.528996	$1,528,996	$1,510,575	1.012194	+1.21%
13 year	1.569588	$1,569,588	$1,563,403	1.003956	+0.39%
14 year					
Circle	1.618590	$1,618,590	$1,618,078	1.000316	+0.031%
Column	1.618200	$1,618,200	$1,618,078	1.0000753	+0.00753%
Square	1.618120	$1,618,120	$1,618,078	1.0000259	+0.00259%
Phi	1.618033	$1,618,033	$1,618,078	- 0.0000279	- 0.00279%
15 year	1.674863	$1,674,863	$1,674,665	1.0001182	+0.011%
16 year	1.735887	$1,735,887	$1,733,231	1.0015323	+0.153%
17 year	1.796057	$1,796,057	$1,793,846	1.0012325	+0.123%
18 year	1.846446	$1,846,446	$1,856,580	- 0.00546	- 0.546%

Third Post-script. Analysis and Prediction.

Renewed interest in the Kondratiev Wave, or Long Wave, has followed the recent global financial crisis. It is possible that the scholarship which has been generated by the Long Wave theory over the past century may be important to consider in evaluating this model and its presentation of American economic history.



	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015

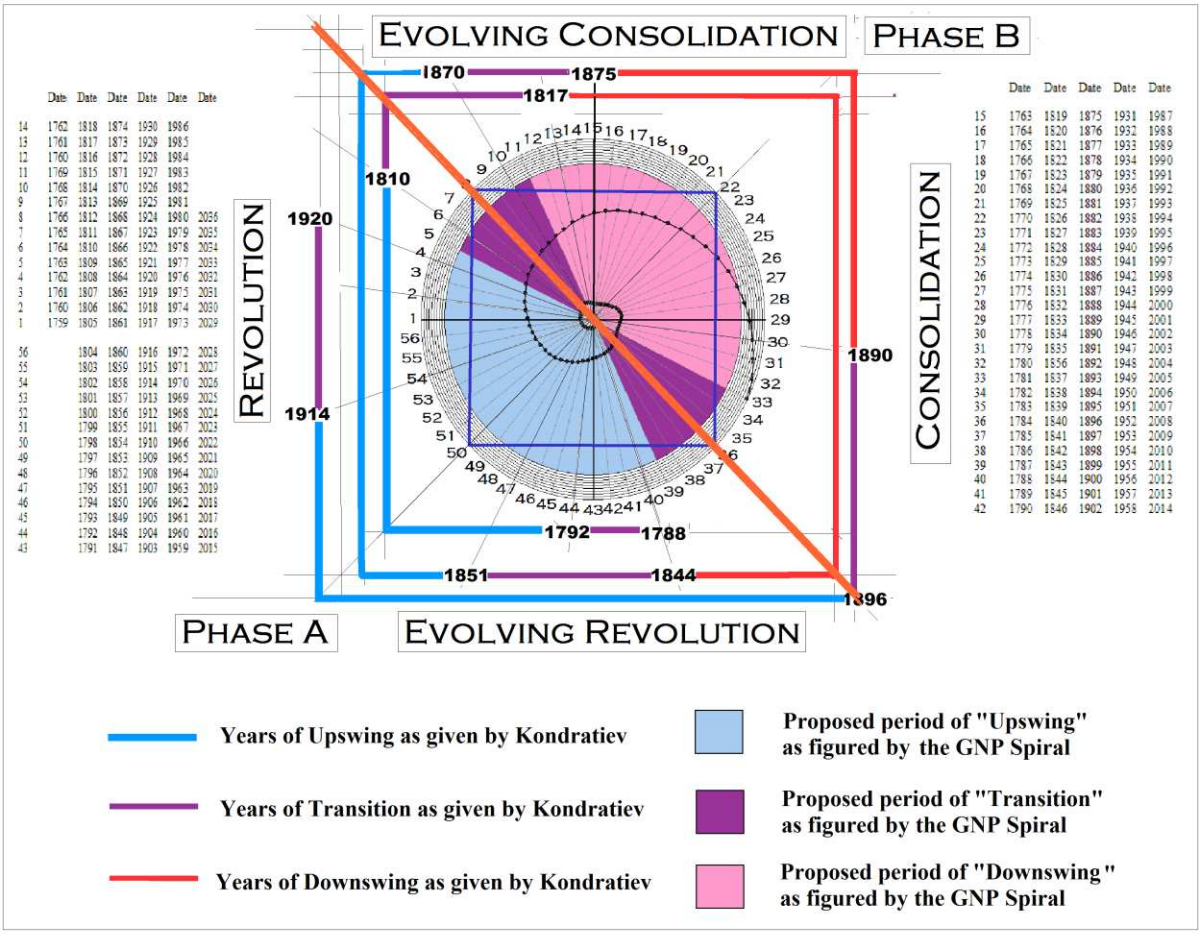
	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1856	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

Moreover the discovery of the Golden Mean at the intersection of price and productivity in the United States in a strict 56-year cycle permits us to evaluate from a more neutral and objective point of view a great deal of research on Kondratiev Waves, at least as it pertains to the American economy.

The plan of the classic Kondratiev wave can easily be superimposed upon the GNP Spiral as follows. A 22-year Phase A "upswing" period is given below by the area marked in blue, a 22-year Phase B "downswing" period is given below by the area marked in red, and two 6-year "transition periods" between these two phases are given by the area marked in purple.

An orange line separates Phase A from Phase B, as an identical orange line in the GNP Spiral separates periods of "Evolving Revolution" and "Revolution" from "Evolving Consolidation" and "Consolidation." Surrounding this model is a square-shaped timeline wherein the dates actually given by Kondratiev for these different periods are presented in the same color scheme for "upswing," "downswing" and "transition." (Korotayev and Tsirel, 2010)

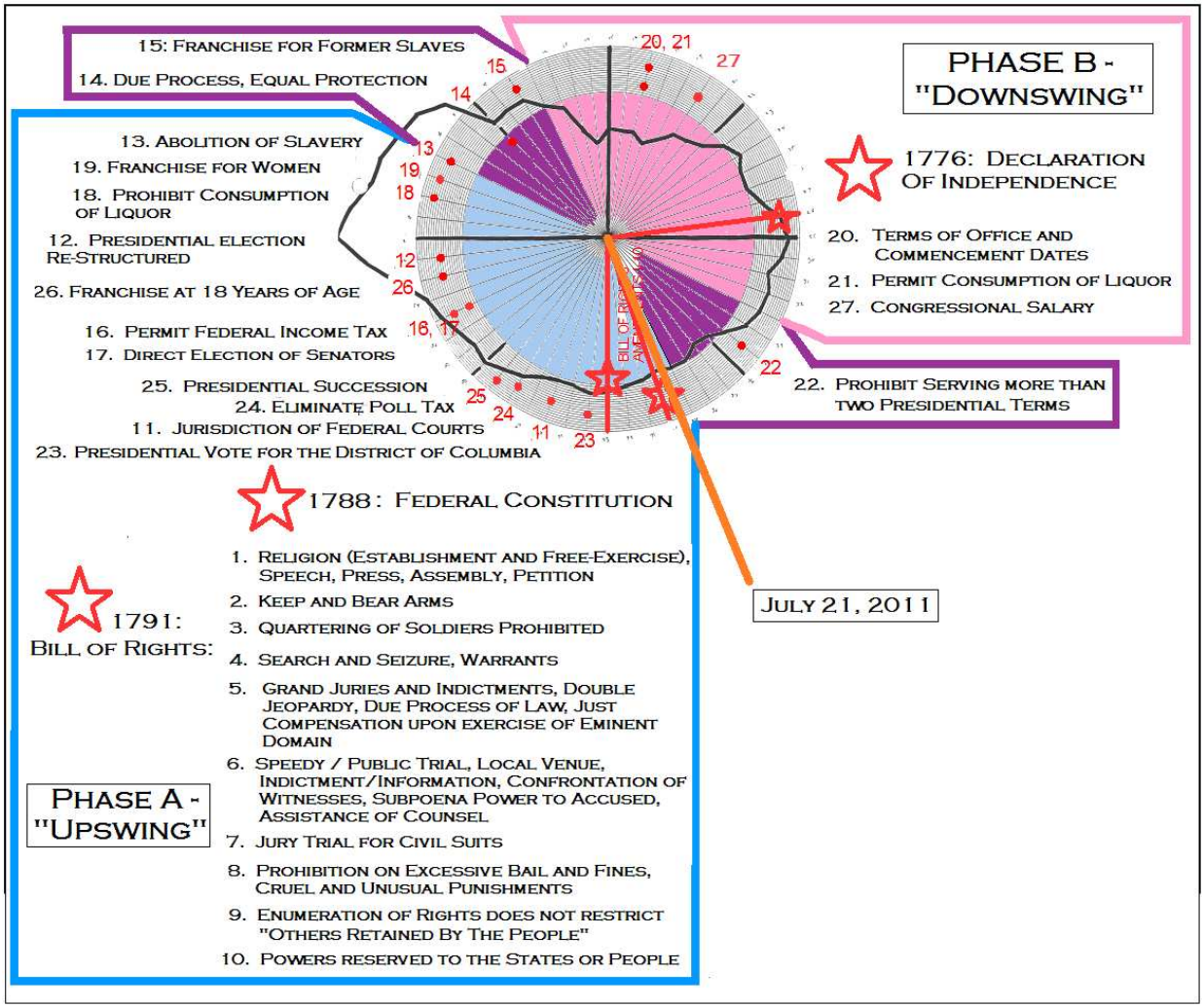
DIAGRAM 80.
KONDRATIEV WAVE AS COMBINED WITH THE GNP SPIRAL



The coloration of the square-shaped timeline surrounding the spiral provides the dates actually given by Kondratiev for periods of Phase A "upswing," Phase B "downswing" and "transition" in blue, red and purple respectively.

In short the square timeline represents the Kondratiev wave as it relates to the GNP Spiral and the circular shading represents the GNP Spiral as it relates to Kondratiev wave.

When this GNP Spiral – Classic Kondratiev scheme is resolved into a pattern of inflation and Amendments to the United States Constitution, the Federal Constitution of 1788, the Bill of Rights (first ten Amendments) of 1791, and 11 additional Constitutional Amendments fall within the upswing of the phase, a total of 21 Amendments. Only 3 Amendments are found in the downswing phase, a ratio of 7:1. As noted previously, the quality of the Amendment is impacted as well. Those falling in the blue shaded area are far more fundamental to American constitutional law than those in the red shaded area. Moreover the transition periods form an interesting unit. Amendment 22, prohibiting a single individual from serving more than two presidential terms, was aimed (by Republicans) at the four elections won by (Democrat) President Roosevelt. The 13th, 14th and 15th Civil War Amendments were clearly intended to consolidate Abolitionist, Western and Northern gains against the Southern slave holding class. A la Kondratiev, "Phase A" Amendments were often the victories of hard-fought battles wherein the people of the United States did, indeed, save themselves from demise.



We can make the following predictions based upon the overall dynamics of this scheme. These are:

1. A 56-year circuit of time characterizes the growth of the United States as composed by four 14-year periods or eight seven year sub-periods. These sub-periods may be named:

1a. Early Evolving Revolution
1b. Late Evolving Revolution
2a. Early Revolution
2b. Late Revolution
3a. Early Evolving Consolidation
3b. Late Evolving Consolidation
4a. Early Consolidation
4b. Late Consolidation

2. The presence of the Golden Mean over this 56 year period permits us to estimate that the steady state rate of growth of production – that rate of production during which no change occurs in the rate of unemployment – lies within a narrow range of values between 3.4969% to 3.4995% per year, over the long term. Annualized quarterly data for Okun's Law agree with this estimate, while annual data for Okun's Law chart the steady state rate of production at 3.455%. This annual trendline and x-intercept is inconsistent with the propositions of this paper. The annualized quarterly trendline may be preferred however inasmuch as there are four times as many data points from which to figure the x-intercept for annualized quarterly data as there exist for annual data. Nevertheless the discrepancy must be acknowledged and may be interesting in its own right.

3. As society develops and changes over time, this steady state rate of growth is maintained in the face of differing rates of political activity, unemployment, production and inflation. High rates of out-of-control inflation are typical of period 2b, Late Revolution. The next period of Late Revolution and its associated out-of-control inflation may be anticipated to occur between the years 2029-2036.

4. As a consequence of the uncontrolled and high rates of inflation during periods of Late Revolution, it may be anticipated that the square described will require a balancing on the opposite side of the square. This brings about a complete meltdown of the economy toward the end of a phase of great conservatism in period 4b, Late Consolidation. This recent period of Late Consolidation and the resulting Global Financial Crisis which occurred in the closing months of 2008 may be expected to re-occur between the years 2057-2064.

5. One outcome of a period of Late Consolidation is that political activity of an increasingly revolutionary type may be expected to follow. These periods occur during the 1b and 2a stages of this model, Late Evolving Revolution and Early Revolution. In American history these periods are often ones of great internal war, social stress and Amendments to the Federal Constitution. Although the early rumbles of these expected developments may be heard today in the Arab Spring and elsewhere, these coming and more dramatically revolutionary periods will commence in 2015-2022 and strengthen considerably throughout the period 2022-2029. These developments will take on additional strength in period 2b, Late Revolution. The prolonged and sustained strain on the value system of the citizens of the United States during these periods of revolutionary change typically results in an inability to price either their own services or that of others with highly inflationary results.

6. The creativity of the legal novelties of Revolutionary periods may be expected to be balanced by the same square of tension in a period of legal suppression and oppression. These will commence at the opposite side of the square, to wit periods 3b and 4a, Late Evolving Consolidation and Early Consolidation respectively. These will occur in 2036-2043 and 2043-2050 respectively.

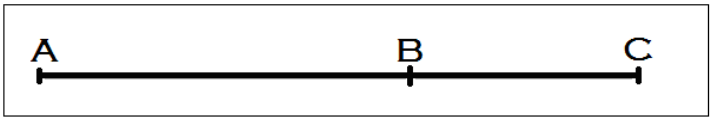
Most immediately, we are on the brink of passing from the Early Evolving Revolution to Late Evolving Revolution. This should take place in 2015.

In so far as the entire planet has demonstrated its interconnectedness with the most recent Global Meltdown, the future change taking place in 2015 may be anticipated to radically alter the very image of global life together, and with perhaps even more force.

Afterword.

Referring once again to the definition of the Golden Mean, we have:

> A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.



We have proposed that "*the Golden Mean appears to tie the past (line segment BC) to the present (line segment AB) to the future (line segment AC) in a self-consistent and harmonic fashion*. It is a mathematic statement of the historic identity of the United States itself, as moving from date to date in a coherent, repeating manner as connected to a 14-year spread between years and as nested as a quarter-cycle within a 56-year circuit of social time."

Ultimately the GNP Spiral may suggest not simply an economic model, but a biologic one as well. Just as honeybees create hexagonal cells within a honeycomb without a conscious awareness of the geometric connections which these constructions have to mathematics, so too might American citizens create and/or associate themselves with the politics, economics, inflation rates and production necessary to ensure the harmonic continuity of their lives from one year to the next, as measured from the onset of their own reproductive identity at the age of 14.

The presentation of social sciences in this way is not an entirely new or novel concept.

E. O. Wilson (1994:328), founder of the study of sociobiology and an early researcher in the connections between the animal and human levels of biology, commented on his efforts in his autobiography as follows:

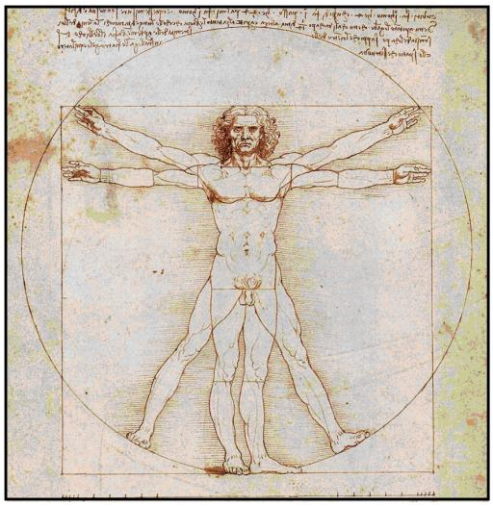
> Perhaps I should have stopped at chimpanzees when I wrote the book ("*Sociology: The New Synthesis"*). Many biologists wish I had.
>
> Still I did not hesitate to include Homo sapiens (in the study of socio-biology), because not to have done so would have been to omit a major part of biology. By reverse extension, I believed that biology must someday serve as part of the foundation of the social sciences. I saw nothing wrong with the nineteenth-century conception of the chain of disciplines, in which chemistry is obedient to but not totally subsumed by physics, biology is linked in the same way to chemistry and physics, and there is a final, similar connection between the social sciences and biology. Homo sapiens is after all a biological species. History did not begin 10,000 years ago in the villages of Anatolia and Jordan. It spans the 2 million years of the life of the genus Homo. Deep history - by which I mean biological history - made us what we are, no less than culture.

Wilson has extended these ideas into the realm of human consciousness in his book *Consilience: The Unity of Knowledge*. He states categorically (1998:8):

> The greatest enterprise of the mind has always been and always will be the attempted linkage of the sciences and humanities. The ongoing fragmentation of knowledge and resulting chaos in philosophy are not reflections of the real world, but artifacts of scholarship.

If this perspective holds true, then it is at least possible that further research into the relationship between the Kondratiev Wave and the Golden Mean – a mathematic proportion of well-known biologic and botanic significance – may ultimately connect the study of economics and politics to much broader vistas of scientific interest. A recent popular article brings forward the interesting historic contrast between the circle and square analysis presented herein and the importance of the distinction between these two geometric forms in the mind of Leonardo DaVinci.

> Ancient thinkers had long invested the circle and the square with symbolic powers. The circle represented the cosmic and the divine; the square, the earthly and the secular. Anyone proposing that a man could be made to fit inside both shapes was making a metaphysical proposition: The human body wasn't just designed according to the principles that governed the world; it *was* the world, in miniature. This was the theory of the microcosm, and Leonardo hitched himself to it early in his career. "By the ancients," he wrote around 1492, "man was termed a lesser world, and certainly the use of this name is well bestowed, because … his body is an analogue for the world."[34]



Scott Albers and Andrew Albers
March 30, 2012

[34] Toby Lester, "The Other Man," *Smithsonian Magazine*, Washington, D.C., February 2012, p. 9. Photograph of drawing by Leonardo DaVinci "Vitruvian Man," in the public domain.

DATA SET SIX.
SCHOOL ENROLLMENT - STUDENT POPULATION

Enrollment of public secondary schools, by state, 2007-08

State	Total	Regular	Junior high	7 to 12	8 to 12	Grades 9-12	10 to 12	11,12-12	Other sec.
Total	16,184,724	15,680,507	1,578,163	927,888	451,656	12,500,341	418,850	41,545	266,281
Alabama	224,711	223,040	20,696	31,465	4,638	153,011	11,021	181	3,699
Alaska	41,004	39,078	7,907	3,433	672	28,726	266	0	0
Arizona	350,928	344,460	47,571	9,488	3,536	279,380	10,038	291	624
Arkansas	177,098	175,870	29,801	35,288	2,801	64,323	29,260	1,305	14,320
California	2,155,154	2,045,990	286,060	67,486	1,280	1,790,115	0	46	10,167
Colorado	253,235	244,201	27,213	8,358	205	207,613	5,787	99	3,960
Connecticut	197,194	183,550	20,092	3,330	5,877	166,038	424	9	1,424
Delaware	40,916	34,271	4,395	116	30,589	5,381	0	0	435
Dist. of Columbia	20,962	18,465	1,218	471	0	19,137	39	0	97
Florida	780,816	763,609	14,554	19,705	27,172	715,591	929	1,709	1,156
Georgia	472,846	467,357	7,694	1,616	3,032	447,166	3,880	89	9,369
Hawaii	63,118	62,939	8,996	5,531	0	48,591	0	0	0
Idaho	94,705	89,494	19,170	5,036	11	54,213	14,112	0	2,163
Illinois	695,769	681,319	64,033	19,151	4,440	589,359	9,091	1,517	8,178
Indiana.....	365,073	363,830	43,486	43,844	673	271,861	3,217	296	1,696
Iowa	171,477	167,360	15,323	18,821	0	126,224	7,571	21	3,517
Kansas	165,490	165,368	24,758	17,523	1,237	115,495	6,462	0	15
Kentucky	207,811	203,021	14,123	9,891	3,962	175,945	2,589	171	1,130
Louisiana	189,919	185,751	19,412	25,680	48,368	89,446	4,921	0	2,092
Maine	65,668	65,618	6,000	2,237	435	56,709	101	0	186
Maryland	280,768	264,881	13,151	1,028	3,431	259,507	272	486	2,893
Massachusetts	319,336	282,426	19,064	21,082	5,328	273,541	0	93	228
Michigan	591,680	557,118	51,998	28,354	21,176	446,232	29,987	2,477	11,456
Minnesota	314,250	299,280	29,259	62,591	9,874	179,606	25,598	2,674	4,648
Mississippi	148,111	148,021	13,128	21,902	3,634	97,600	7,313	362	4,172
Missouri.....	326,470	323,794	38,486	34,044	145	231,179	11,790	622	10,204
Montana	60,355	60,254	13,853	0	0	46,502	0	0	0
Nebraska	112,050	112,034	12,148	28,492	1,749	68,898	165	31	567
Nevada	131,671	126,175	10,970	481	3,660	114,025	1,640	812	83
New Hampshire	70,844	70,844	4,431	0	0	65,765	0	0	648
New Jersey	465,666	438,730	39,712	21,929	2,386	383,611	8,770	1,462	7,796
New Mexico	114,391	111,108	14,395	2,570	950	80,999	9,582	0	5,895
New York	913,079	860,711	50,934	78,176	5,997	729,745	23,676	337	24,214
North Carolina ...	415,325	412,194	13,676	1,409	621	390,498	1,705	355	7,061
North Dakota	38,626	38,617	4,218	11,846	157	16,207	3,770	947	1,481
Ohio	637,089	633,721	62,066	61,315	55,112	441,585	6,376	681	9,954
Oklahoma	199,392	198,585	24,753	0	0	137,023	22,176	4,140	11,300
Oregon	193,303	190,205	18,368	8,481	760	165,054	619	21	0
Pennsylvania	649,436	632,017	64,052	87,655	12,163	425,120	37,389	7,065	15,992
Rhode Island	50,061	47,096	4,033	993	0	44,434	30	0	571
South Carolina ...	221,608	221,526	15,855	4,841	755	188,676	8,388	90	3,003
South Dakota	41,607	41,026	3,921	29	0	37,657	0	0	0
Tennessee	288,904	286,784	12,577	9,050	15,846	246,916	2,268	484	1,763
Texas	1,429,301	1,392,149	181,487	37,720	97,344	1,041,501	10,433	10,566	50,250
Utah	215,405	207,270	71,658	10,861	11,688	46,890	59,578	783	13,947
Vermont	33,156	33,140	2,225	8,677	0	22,254	0	0	0
Virginia	410,561	409,423	26,999	13,741	15,399	348,628	2,971	0	2,823
Washington	357,904	341,744	45,437	19,713	43,622	219,623	21,300	146	8,063
West Virginia	83,502	82,971	4,491	5,684	136	70,499	2,643	12	37
Wisconsin	305,036	301,274	20,879	15,321	533	258,193	5,941	1,165	3,004
Wyoming	31,943	30,798	7,437	1,433	262	18,049	4,762	0	0

SOURCE: U.S. Department of Education, National Center for Education Statistics, Common Core of Data, 2007-08.

DATA SET SEVEN.
SCHOOL ENROLLMENT - SCHOOL TYPE

Table 99. Public secondary schools, by grade span, average school size, and state or jurisdiction: 2007-08

State or jurisdiction	Schools, by grade span									Vocational schools\2\	Average number of students per school\3\	
	Total, all secondary schools	Total, all regular secondary schools\1\	Grades 7 to 8 and 7 to 9	Grades 7 to 12	Grades 8 to 12	Grades 9 to 12	Grades 10 to 12	Other spans ending with grade 12	Other grade spans		All secondary schools	Regular secondary schools\1\
1	2	3	4	5	6	7	8	9	10	11	12	13
United States	**24,426**	**19,264**	**3,047**	**3,278**	**777**	**15,179**	**748**	**378**	**1,019**	**1,409**	**706**	**816**
Alabama	414	314	34	96	19	226	28	3	8	73	681	709
Alaska	84	65	16	20	3	43	2	0	0	3	494	601
Arizona	667	470	76	36	7	527	10	3	8	166	696	729
Arkansas	393	360	59	134	8	127	42	1	22	24	484	494
California	2,449	1,495	342	321	42	1,679	25	13	27	76	901	1,355
Colorado	410	344	61	60	1	274	7	1	6	5	619	710
Connecticut	261	195	35	12	11	184	11	2	6	17	756	941
Delaware	46	34	7	1	27	10	0	0	1	6	952	1,008
District of Columbia	38	30	6	3	1	26	1	0	1	5	549	606
Florida	668	475	20	67	30	488	9	19	35	51	1,276	1,667
Georgia	435	392	11	14	8	350	7	2	43	3	1,137	1,201
Hawaii	53	52	11	9	0	33	0	0	0	0	1,191	1,210
Idaho	231	154	40	47	1	115	24	0	4	11	440	593
Illinois	1,007	802	150	67	19	634	11	57	69	55	745	847
Indiana	439	420	75	89	1	265	1	1	7	29	853	866
Iowa	449	381	48	80	1	302	9	4	5	0	392	450
Kansas	392	387	58	81	4	239	8	0	2	1	430	432
Kentucky	465	240	30	43	24	295	12	9	52	126	586	803
Louisiana	310	263	41	49	68	125	18	0	9	6	637	709
Maine	153	124	15	10	2	115	9	0	2	27	525	533
Maryland	277	208	20	6	8	213	2	6	22	24	1,065	1,270
Massachusetts	370	315	33	36	6	293	0	1	1	39	860	894
Michigan	1,082	745	102	96	37	664	64	39	80	55	569	749
Minnesota	894	482	63	298	40	391	57	32	13	11	405	625
Mississippi	321	226	29	60	8	188	26	2	8	89	652	658
Missouri	684	587	80	204	1	350	21	11	17	63	548	557
Montana	352	348	180	1	0	171	0	0	0	0	172	173
Nebraska	329	325	28	181	1	116	1	1	1	0	360	361
Nevada	134	111	23	7	8	87	2	5	2	1	998	1,158
New Hampshire	106	106	18	0	0	85	0	0	3	0	681	681
New Jersey	503	401	60	40	8	352	18	7	18	55	930	1,094
New Mexico	230	200	39	30	7	137	9	0	8	2	527	572
New York	1,059	980	89	132	10	722	24	3	79	29	862	878
North Carolina	516	486	26	10	7	439	6	5	23	10	830	867
North Dakota	186	179	11	105	2	56	3	1	8	6	215	216
Ohio	1,015	928	131	142	80	605	9	17	31	75	664	684
Oklahoma	564	560	84	0	0	417	45	3	15	0	354	355
Oregon	302	270	30	41	12	211	7	1	0	0	620	679
Pennsylvania	815	720	101	162	13	449	59	9	22	87	863	875
Rhode Island	75	52	9	4	0	59	2	0	1	12	795	906
South Carolina	275	222	24	14	5	210	14	3	5	40	974	983
South Dakota	270	257	80	1	1	188	0	0	0	0	164	166
Tennessee	345	308	24	27	18	248	13	10	5	22	868	924
Texas	2,158	1,482	316	215	109	1,185	37	47	249	1	702	948
Utah	305	219	85	45	23	68	48	12	24	8	715	934
Vermont	72	56	8	19	0	30	0	0	15	15	582	592
Virginia	385	343	33	6	36	272	3	0	35	31	1,183	1,197
Washington	574	388	83	67	53	327	24	9	11	11	652	885
West Virginia	130	116	10	19	1	93	2	3	2	31	657	715
Wisconsin	631	561	69	60	4	434	14	36	14	8	495	543
Wyoming	103	86	24	11	2	62	4	0	0	0	310	358
Bureau of Indian Education	21	21	2	5	0	14	0	0	0	0	---	---
DoD, domestic	7	7	2	0	0	5	0	0	0	0	476	476
DoD, overseas	32	32	2	13	0	17	0	0	0	0	453	453
Other jurisdictions												
American Samoa	6	5	0	0	0	5	1	0	0	1	---	---
Guam	0	0	0	0	0	0	0	0	0	0	---	---
Northern Marianas	6	6	1	1	0	4	0	0	0	0	727	727
Puerto Rico	398	368	191	28	1	3	158	0	17	27	529	518
U.S. Virgin Islands	10	8	5	0	0	5	0	0	0	1	803	896

---Not available.
\1\Excludes vocational, special education, and alternative schools.
\2\Vocational schools are also included under appropriate grade span.
\3\Average for schools reporting enrollment data. Enrollment data were available for 22,800 out of 24,426 public secondary schools in 2007-08.
NOTE: Includes schools with no grade lower than 7. Excludes schools not reported by grade level, such as some special education schools for the disabled. DoD = Department of Defense.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Common Core of Data (CCD), "Public Elementary/Secondary School Universe Survey," 2007-08. (This table was prepared September 2009.)

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Digest of Education Statistics, Table 99, Public secondary schools, by grade span, average school size and state or jurisdiction: 2007-2008, National Center for Education Statistics; and Enrollment of public secondary schools, by state, 2007-2008, collected at the request of the authors from the NCES on Friday, June 10, 2011.