Costly Divorce and Marriage Rates

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Abstract

This paper develops a model of choice between marriage and cohabitation to study the effect of divorce costs on marriage decision. The paired agents are heterogeneous, the utility is non-transferable, and break up and divorce decisions are modeled explicitly as unilateral, that is, it takes the decision of only one partner to terminate a relationship. This framework is empirically relevant, since unilateral divorce is legal in many countries, and multiple empirical studies of the effect of changes in divorce laws on divorce rates demonstrate that Coase theorem does not hold (partners cannot bargain efficiently). The model seeks to reconcile the conflicting empirical evidence on the relationship between marriage rates and divorce costs.

Keywords: family economics, marriage, cohabitation, divorce, externalities
JEL classification: D62, D91, J12

1 Introduction

Marriage rates vary significantly across countries. For instance, in 2009 Slovenia had the lowest marriage rate among the OECD countries, 3.17 versus 7.31 for the US.[1] Comparisons with countries outside of the OECD reveal even larger differences in marriage rates around the world. Marriage rates in the developed countries have also declined over time, and most of the debate around the institution of marriage has focused on these trends. Since 1970, the average marriage rate for the twenty seven OECD countries has fallen by almost 40%.[2]

Economists tend to seek the explanation for social phenomena from the point of the cost / benefit analysis. The data indicate that there are vast differences among countries in social, economic, and legal relative costs and benefits of marriage, and that the relative benefits of marriage must have declined over the past few decades.

This paper focuses on the effect of divorce costs on marriage decisions and the resulting relationship between divorce costs and marriage rates. Since cohabitation is often a precursor or even a substitute for legal marriage, agents in the model can choose to cohabit or marry legally. [3] Empirical evidence indicates that higher divorce costs can result in either higher or lower marriage rates and the model seeks to reconcile this evidence.

Rasul [9] presents evidence that the adoption of unilateral divorce in the US has contributed to the decline in marriage rates. Matouschek and Rasul [7] demonstrate that propensity to divorce is lower for couples married after the introduction of unilateral divorce laws in the US. Their paper and the work of Rasul [10] also develop theoretical models of marriage that explain these findings: marriage contract acts as a commitment device and lower exiting costs undermine its ability to serve this

[1] Marriage rate is defined as the number of marriages performed in a given year divided by total population and multiplied by 1000.

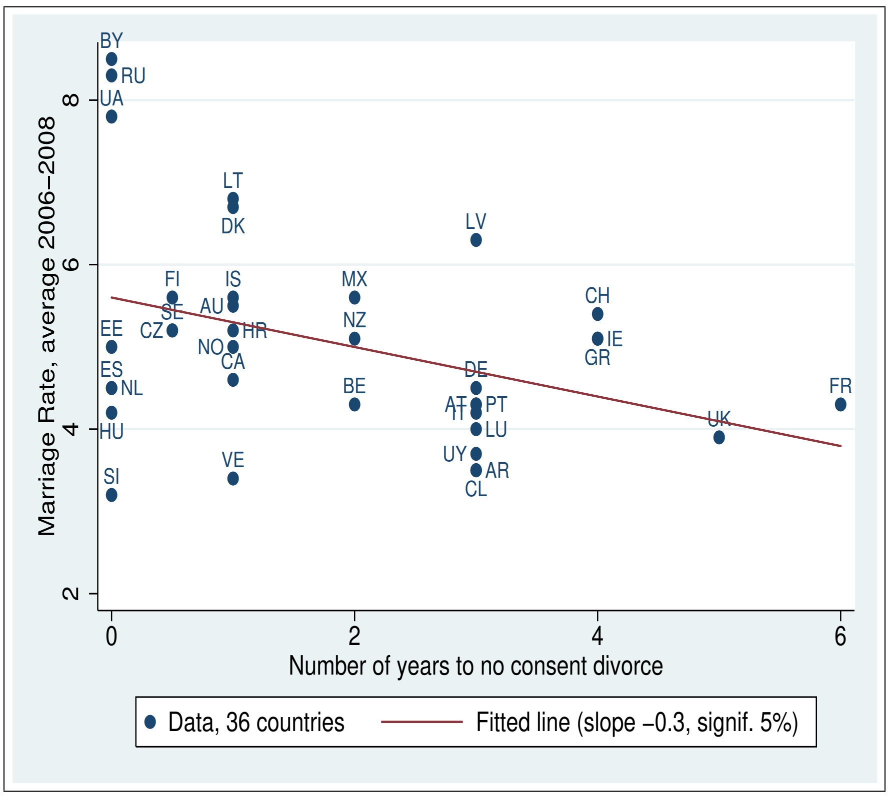
[2] Source: OECD [8]. The exact numbers vary across countries, but the overall trends remain roughly similar.

[3] Stevensen and Wolfers [11] discuss recent trends in cohabitation and marriage.

purpose.

Adoption of unilateral divorce laws can be interpreted as a reduction in costs of obtaining divorce. Figure 1 depicts the relationship between marriage rates across countries with unilateral divorce laws and a different measure of divorce costs: the length of the mandatory separation period before the divorce is legalized. [4] Countries with longer separation requirements are interpreted to have higher divorce costs. Figure 1 shows that marriage rates are higher in countries with lower divorce costs.

Figure 1: Marriage rates and no-consent divorce laws



[4]The sample includes countries with unilateral divorce legislation for which the author was able to obtain information on legal grounds for divorce. There are 36 countries in the sample: Argentina (AR), Australia (AU), Austria (AT), Belarus (BY), Belgium (BE), Canada (CA), Chile (CL), Croatia (HR), Czech Republic (CZ), Denmark (DK), Estonia (EE), Finland (FI), France (FR), Germany (DE), Greece (GR), Hungary (HU), Iceland (IS), Ireland (IE), Italy (IT), Latvia (LV), Lithuania (LT), Luxembourg (LU), Mexico (MX), Netherlands (NL), New Zealand (NZ), Norway (NO), Portugal (PT), Russian Federation (RU), Slovenia (SI), Spain (ES), Sweden (SE), Switzerland (CH), Ukraine (UA), United Kingdom (UK), Uruguay (UY), Venezuela (VE). Note that the US is not on the sample since the divorce law differs by state.

The model in this paper seeks to reconcile this evidence. It combines two incentives to marry: 1) exogenous benefits to marriage, social, financial, or legal, and 2) commitment value of marriage, which is an endogenous outcome of higher separation costs associated with divorce. Single agents are matched in pairs and are free to enter either type of relationship contract or to remain single. The decisions are based on the observed initial match quality and expectations of the future match qualities, and the utility is non-transferable. For every cohabiting or married agent future realizations of the additional match quality shock are random. The agents are heterogeneous: paired agents may receive different realizations of this match quality signal. Paired agents observe their individual realization of the match quality shock, and each agent makes his or her decision on whether to stay in the current relationship or terminate it. The match survives only if both partners choose to preserve it.

The assumption of non-transferable relationship utility is very important. Since Becker, Landes and Michael [1], most of the theoretical literature on marriage and divorce typically assumes that the Coase theorem applies to marital bargaining, so spouses can always reach divorce agreements by the redistribution of welfare. If this assumption is valid, changes in consent versus no-consent divorce laws should have no effect on incidences of divorce and marriage. Empirical studies, however, demonstrate that the change to no-fault unilateral divorce laws has caused a small, but statistically significant increase in the divorce rates in the US and Europe. [5] More recent models of marriage assume that Coasian bargaining is not possible in marriage: for example, Fella et al [3], Rasul [10], and Guha [6]. This paper follows in their footsteps, demonstrating that the inability of partners to efficiently compensate each other when separation is desirable by only one of them plays a crucial role in determining the relationship between divorce costs and the decision to marry.

The non-transferability of utility only matters when the agents are heterogeneous. Otherwise, they make the same decisions and no transfers between partners are ef-

[5] See Friedberg [4] and Wolfers [12] for the US and Gonzalez and Viitanen [5] for Europe.

ficient. To illustrate the effect of the heterogeneity assumption on the model's predictions when the utility is non-transferable this paper considers three cases: 1) the paired agents are homogeneous, that is, they receive the same future realizations of the match quality shock; 2) the agents are heterogeneous and their utility depends only on their own realization of the relationship quality signal, but not their partner's; 3) the agents are heterogeneous and the utility of each partner depends not only on his/her realized match quality, but also on that of the other partner. The first case introduces the framework and shows that when the agents are homogeneous or, alternatively, the utility is perfectly transferable, the model cannot generate a positive relationship between divorce costs and marriage rates. The second case contains the main result of the paper, and the third case is a robustness check.

The analysis in the paper (cases two and three) demonstrates that when the agents are heterogeneous and the utility is non-transferable, higher costs of divorce may increase the value of marriage and make it preferred to cohabitation even in the absence of additional benefits to marriage. The intuition is as follows. Any relationship survives if and only if both partners chose to maintain it. If one person enjoys the relationship and prefers to stay in it, she would only be able to do so if her partner also prefers not to terminate it. If he is no longer happy in the relationship and chooses to end it, his decision imposes a negative externality on her. Higher costs of terminating legal marriage may induce her partner to choose to preserve the relationship, eliminating the negative externality. Thus, commitment is valuable and divorce costs make marriage a more committed relationship form. If marriage does not carry any additional benefits relative to cohabitation, the model generates a positive relationship between divorce costs and marriage rates, consistent with the evidence on the decline in marriage rates after the adoption of unilateral divorce laws from Rasul [9]. What about the negative relationship between marriage rates and the difficulty of obtaining divorce in Figure 1?

If marriage provides additional utility relative to cohabitation, however small, the predicted relationship between the costs of divorce and marriage rates becomes U-shaped: the marriage rate declines for lower values of divorce costs and increases for the costs of divorce above some threshold value. When the divorce costs are relatively low, the exogenous marriage benefit serves as the main incentive for marriage, and couples that marry for this incentive are discouraged with higher divorce costs. Once the divorce costs are high relative to the marriage benefit, the commitment effect dominates the marriage benefit incentive, and the additional couples choose marriage for the lower likelihood of separation. Thus, the model is capable of reconciling empirical evidence when both incentives to marry are present.

Rasul [10] was the first to demonstrate the commitment value of greater difficulty of obtaining divorce in the mutual consent divorce regime with non-transferable utility and heterogeneous agents. He studies how the move from mutual consent divorce to unilateral divorce affects the marriage market outcomes with a model of choice between marriage and singlehood. The model can explain the observed decline in marriage rates after the adoption of unilateral divorce laws.

Theoretical models of greater commitment in marriage relative to cohabitation have been developed by Wydick [13] and Matouschek and Rasul [7]. They have homogeneous agents and use a repeated prisoner's dilemma game setting to show that marriage can foster cooperation better than cohabitation due to its higher termination costs. The implication is that partners in marriage are more likely to act cooperatively towards each other and behave themselves than the cohabiters, so marriage generates endogenous benefits relative to cohabitation.

Brien, Lillard, and Stern [2] and Matouschek and Rasul [7] have models of choice between cohabitation and marriage with marriage providing an exogenous benefit to both spouses. These models predict a negative relationship between divorce costs and marriage rates.

The rest of the paper is organized as follows. Section 2 describes the model. The solution of the model with homogeneous agents and the results are given in Section

3. Section 4 solves the model with heterogeneous agents and independent utilities and presents the main intuition and the results of the paper. Section 5 explores the implications of assuming that paired agents have interdependent utilities. Section 6 concludes.

2 Model Setup

This section presents the basic framework for analyzing the relationship between divorce costs, marriage benefits, and marriage market outcomes.

Men and women in the model are symmetric.

The model has two periods. In the beginning of period one individuals are matched in pairs and draw the same couple specific match quality shock q from uniform distribution on $[q_L, q_H]$, where $q_L < 0 < q_H$. The match quality determines the relationship utility each agent receives in period one if he / she were to enter a household sharing relationship with the current match partner. Assume that the relationship utility in period one is equal to the realization of match quality q. The utility from remaining single is normalized to zero.

Upon observing q, each agent decides whether to remain single or enter into cohabitation or marriage with the current match partner. Since the initial match quality is the same for both match partners, they make identical decisions.

Single agents remain single in period two and receive the total utility of zero. Cohabiting and married agents receive an additional individual relationship quality shock x from the same distribution. Upon observing the quality shocks, each agent unilaterally decides whether to preserve the relationship or to exit it. If the relationship is preserved, each agent in a couple receives relationship utility of $r(q, x, x_{-})$ in period two, where x is the realization of the agent's own additional match quality shock and x_{-} is that of his/her partner. If the agents separate, each receives the utility of zero in period two. The relationship survives only if both agents make the decision to maintain it.

For what values of q do single agents decide in period one to remain single, cohabit, or marry? Denote the expected value of remaining single when the observed initial match quality is q by $S(q)$, the expected value of cohabiting by $R(q)$, and that of getting married by $W(q)$. There is no discounting between periods.

The expected value of cohabitation and marriage depend on the decisions of agents in period two. Each agent observes the additional shocks to match quality in period two and compares the value from staying together to that of splitting. Assume that for cohabiting agents the cost of breaking up is zero, and that for married agents the cost of obtaining divorce is $d > 0$. Then, each cohabiting agent will choose to stay in the relationship as long as the resulting relationship utility in period two exceeds zero, and each married agent will decide to stay married if the total utility from being married exceeds $-d$.

Assume also that married agents may receive an additional exogenous utility bonus $M \geq 0$ in every period of their marriage.

Next we explore three cases:

1. The agents are homogeneous: both partners receive the same additional match quality shock x. Thus, they make identical decisions and break-up or divorce occurs only if beneficial to both. This case is studied in Section 3.

2. The agents are heterogeneous: the additional match quality shocks are independent draws from the same distribution. Also, for each agent in a couple the utility in period two depends only on the realization of own additional match quality shock x, $\frac{\partial r(q,x,x_{\text{-}})}{\partial x_{\text{-}}} = 0$. Thus, match partners can make different decisions. The relationship is preserved only if both prefer not to terminate it. If one agent decides to end the relationship while the other would prefer to maintain it, this decision imposes a negative externality on the pro-relationship partner. For simplicity, assume that the relationship utility is period two is $r(q, x, x_{\text{-}}) = q - x$. This case is explored in Section 4.

3. The agents are heterogeneous with independent realizations of the additional relationship quality shock in period two and $\frac{\partial r(q,x,x_{-})}{\partial x_{-}} \neq 0$. That is, the relationship utilities are interdependent. Specifically, assume $u = \alpha x + (1-\alpha)\, x_{-}$ and $v = \alpha x_{-} + (1-\alpha)\, x$, where $0.5 < \alpha < 1$. Then, in period two if the couple stays intact the relationship utility of the first agent is $r\,(q, x, x_{-}) = q - u$, and that of the second agent is $r\,(q, x_{-}, x) = q - v$. Section 5 analyzes this case.

In order to solve the model for each of the three cases it remains to specify the distribution for the additional match quality shock x. Assume that it is distributed exponentially with mean $1/\lambda$. The cumulative distribution function is

$$F_x\,(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases} . \tag{1}$$

3 Homogeneous agents

Here we assume that the paired agents are homogeneous, that is, the realized value of the additional relationship quality shock is the same for both agents in a cohabiting or married union, $x = x_{-}$. The relationship utility for paired agents in period two if the agents stay together is $r\,(q, x, x) = q - x$.

The purpose of the analysis in this section is to demonstrate that in the absence of heterogeneity and when the utility is non-transferable, higher divorce costs cannot increase the relative value of marriage. That is, for any positive value of the marriage benefit, higher divorce costs result in lower marriage rates.

We begin by formulating the problem of cohabiting agent, and then proceed to that of married agent.

In period two a cohabiting agent would prefer to stay in the relationship if the relationship utility $r\,(q, x, x_{-}) = q - x \geq 0$. Since both agents in a couple receive the same realization of the match quality shock, the probability of them staying together is $F_x\,(q)$.

The expected value of cohabiting with the initial relationship quality q is $R(q) = q + \int_0^q (q-x)\, dF_x(x)$. With the exponential cumulative distribution function

$$R(q) = \begin{cases} 2q - \frac{1-e^{-\lambda q}}{\lambda}, & q \geq 0 \\ q, & q < 0 \end{cases}. \tag{2}$$

A married agent with the initial relationship quality q would choose to stay married in period two if the total utility $r(q, x, x_{\_}) + M = q + M - x \geq -d$. Thus, the expected value of marriage is $W(q) = (q+M) + \int_0^{q+M+d} (q+M-x)\, dF_x(x) + [1 - F_x(q+M+d)](-d)$ or

$$W(q) = \begin{cases} 2(q+M) - \frac{1-e^{-\lambda(q+M+d)}}{\lambda}, & q \geq -M-d \\ (q+M) - d, & q < -M-d \end{cases}. \tag{3}$$

To solve the model we need to find the values of q for which matched agents choose marriage in period one for given d and M. Denote the lowest value of q above which matched agents prefer to marry by $\underline{q}$. Recall the model's assumptions of $d > 0$ and $M \geq 0$. Proposition below establishes the following results:

Proposition 1 *Let $\underline{q}$ be the initial relationship quality such that*
i) $\underline{q}$ solves $W(q) = \max\{0, R(q)\}$, where $W(q)$ and $R(q)$ are given by equations (3) and (2) respectively;
ii) For all $q \geq \underline{q}$, $W(q) \geq \max\{0, R(q)\}$.

Then,

(a) For $M = 0$, $\underline{q}$ does not exist. That is, with no additional exogenous benefits marriage is never preferred to cohabitation or singlehood;

(b) For any $M > 0$, $\underline{q}$ is increasing in d.

Proof.

Consider two cases: 1) $R(q) \geq 0$ or, equivalently, $q \geq 0$ and 2) $-M - d \leq q < 0$. Note that when $q < -M - d$ marriage is never chosen, so $\underline{q}$ cannot belong to this range of q values.

1) $q \geq 0$

From $W\left(\underline{q}\right) = R\left(\underline{q}\right)$ obtain $\underline{q} = \frac{1}{\lambda}\left[\ln\left(1 - e^{-\lambda(M+d)}\right) - \ln(2\lambda M)\right]$.

This value is ≥ 0 when M, d, and λ are such that $2\lambda M \leq 1 - e^{-\lambda(M+d)}$.

For part (a) of the proposition, note that the solution exists only if $M > 0$. The solution also satisfies condition ii) of the proposition when M is strictly positive.

To obtain (b), differentiate $\underline{q}$ with respect to d:

$\frac{\partial \underline{q}}{\partial d} = \frac{e^{-\lambda(M+d)}}{1-e^{-\lambda(M+d)}}$.

The derivative is positive for all strictly positive and finite values of M, d, and λ.

2) $q \in [-M - d, 0)$

The cut-off value $\underline{q}$ solves $W(q) = 0$, where $W(q) = 2(q + M) - \frac{1-e^{-\lambda(q+M+d)}}{\lambda}$ (from equation 3). Unfortunately, no closed-form solution can be obtained. Analysis of the function $W(q)$ yields the following:

i) $W(q)$ is strictly convex;

ii) It has a unique minimum that is below zero;

iii) $\lim_{q\to-\infty} W(q) = \lim_{q\to\infty} W(q) = \infty$;

Thus, equation $W(q) = 0$ has two roots. Denote these roots by q' and q'', with $q' > q''$. $W(q) \geq 0$ for $q \in (-\infty, q'']$ and $q \in [q', \infty)$.

Since $W(-M - d) = -2d < 0$, $q'' < -M - d < q'$. Thus, the unique candidate solution for $\underline{q}$ is q'. It is the solution if also $q' < 0$, which is true for all M, d, and λ such that $2\lambda M > 1 - e^{-\lambda(M+d)}$. For (a), note that this condition is never satisfied when $M = 0$.

To show (b), implicitly differentiate equation $W\left(\underline{q}\right)=0$ with respect to d:

$\frac{\partial \underline{q}}{\partial d}=\frac{e^{-\lambda\left(\underline{q}+M+d\right)}}{2-e^{-\lambda\left(\underline{q}+M+d\right)}}>0$ for any $\underline{q}\in\left[-M-d,0\right)$.

■

Marriage rate can be obtained as the fraction of matches with initial relationship quality of at least $\underline{q}$. If F_q denotes the cumulative distribution function for q, then $MR=1-F_q\left(\underline{q}\right)$. The main result of this section follows straightforwardly:

Corollary 2 (Divorce Costs and Marriage Rates: the Case of Homogeneous Agents) *Under the assumptions of the model with homogeneous agents,*

(a) If $M=0$, marriage rate is also zero for any value of divorce cost d;

(b) For any $M>0$, marriage rate is a decreasing function of d.

Figure 2 illustrates the relationship between marriage rates and divorce costs for different values of the exogenous marriage benefit M when $\lambda=0.5$ and q is uniformly distributed on $[q_L,q_H]$, with $q_L=-5$ and $q_H=10$.[6]

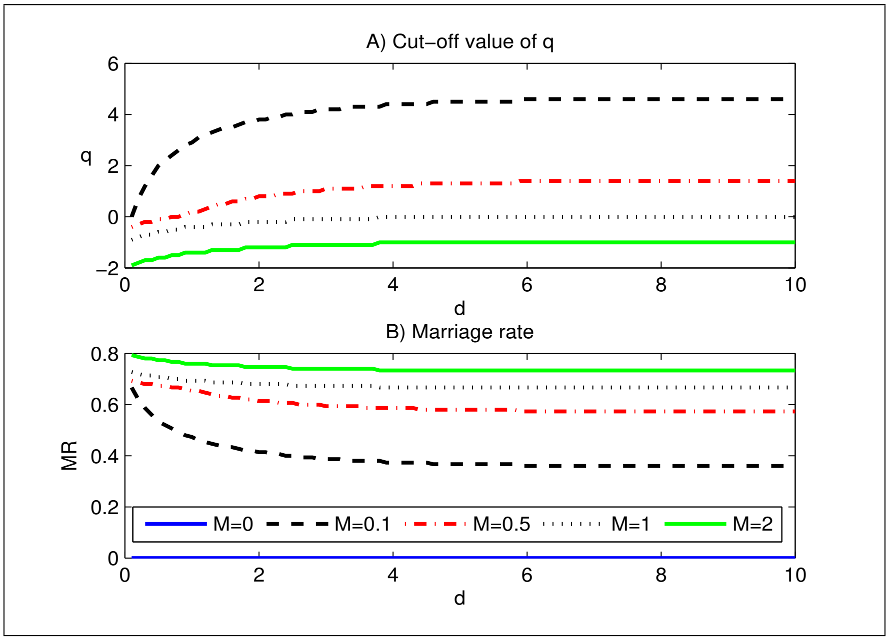
Figure 2 A) depicts the lowest value of q above which matched agents prefer to marry ($\underline{\mathrm{q}}$) as a function of the divorce cost d for $M=0$, $M=0.1$, $M=0.5$, $M=1$, $M=2$. Figure 2 B) shows the respected marriage rates, that is, the fraction of agents that choose marriage in period one.

Observe that without exogenous benefits to marriage ($M=0$), marriage is never optimal. For any $M>0$, agents with higher values of the initial match quality prefer to marry, and more agents marry for higher values of M. For any value of M marriage rates decline in the cost of divorce d.

Figure 3 A) shows the net divorce rate for each value of M as function of divorce cost, where net divorce rate is a fraction of married agents that divorce in period two. The divorce rates are lower with higher cost of divorce.

[6] Note that higher values of λ reduce the likelihood of "bad" relationship utility shocks in period two. The likelihood of staying together in period two is higher for both married and cohabiting couples, so the divorce costs play a relatively smaller role in the couple's decision of what type of union to form.

Figure 2: Homogeneous agents: Cut-off values of q and Marriage rate

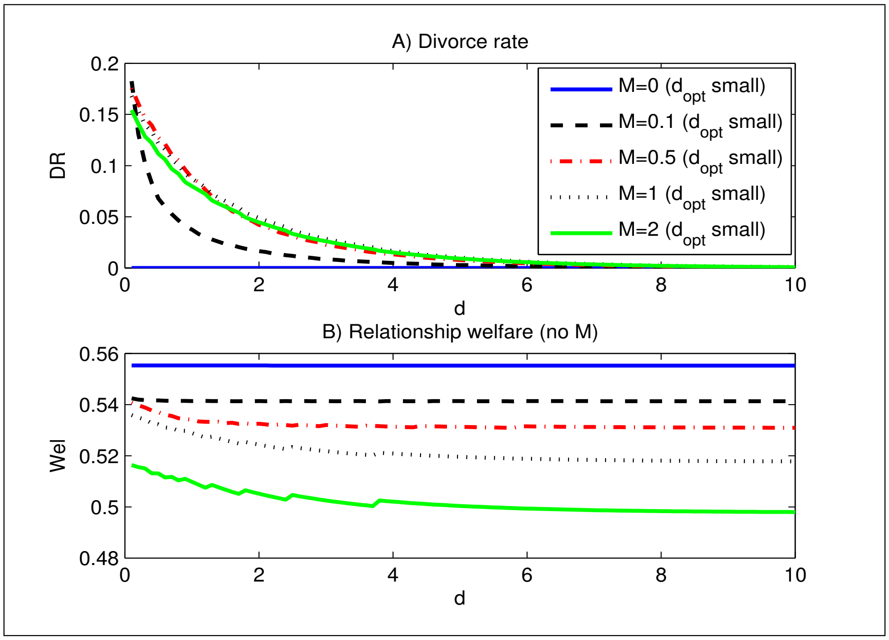


Average relationship welfare is depicted in Figure 3 B). It is computed without the exogenous benefit M, since it is obvious that adding a positive constant to utility increases it, and we are interested in comparing the welfare from the matches for various levels of M and d. Observe that higher exogenous marriage benefits reduce relationship welfare, since matches of lower quality result in marriages. The average welfare also declines with d, since the divorce costs are incurred with positive probability in any marriage. For any value of M, the welfare maximizing divorce cost is zero.

4 Heterogeneous agents: Independent utilities

In this section the agents are heterogeneous: in period two each paired agent draws an additional match quality shock from the same distribution, and the draws are independent. The relationship utility of each agent in period two depends only on the agent's own realization of the match quality shock x and is independent of the

Figure 3: Homogeneous agents: Divorce rates and Average relationship welfare



value received by his/her partner $x_{\text{-}}$, specifically, $r(q, x, x_{\text{-}}) = q - x$. Upon observing own quality shock, each agent unilaterally decides whether to stay in the relationship or to exit it. The relationship is preserved only if both partners decide to maintain it. Thus, exiting decision by one agent may impose a negative externality on his/her partner if the partner would prefer to stay in the relationship.

The purpose of this section is to demonstrate that when the utility is non-transferable and the agents are heterogeneous, positive divorce costs may help eliminate the externality and increase the value of marriage. Thus, for some values of marriage benefits and divorce costs marriage rate can increase in the cost of divorce.

Paired agents in period two draw independent realizations of the additional relationship quality shock x from the same distribution $F_x(x)$ from (1). They remain paired only if both partners prefer to stay together. Thus, the expected value of cohabiting with initial match quality shock q is $R(q) = q + F_x(q) \int_0^q (q - x)\, dF_x(x)$, and that of being married is $W(q) = (q + M) + F_x(q + M + d) \int_0^{q+M+d} (q + M - x)\, dF_x(x) + \left[1 - F_x(q + M + d)^2\right](-d)$.

With the exponential cumulative distribution function

$$R(q) = \begin{cases} q + \left(1 - e^{-\lambda q}\right)\left[q - \frac{1-e^{-\lambda q}}{\lambda}\right], & q \geq 0 \\ q, & q < 0 \end{cases}. \tag{4}$$

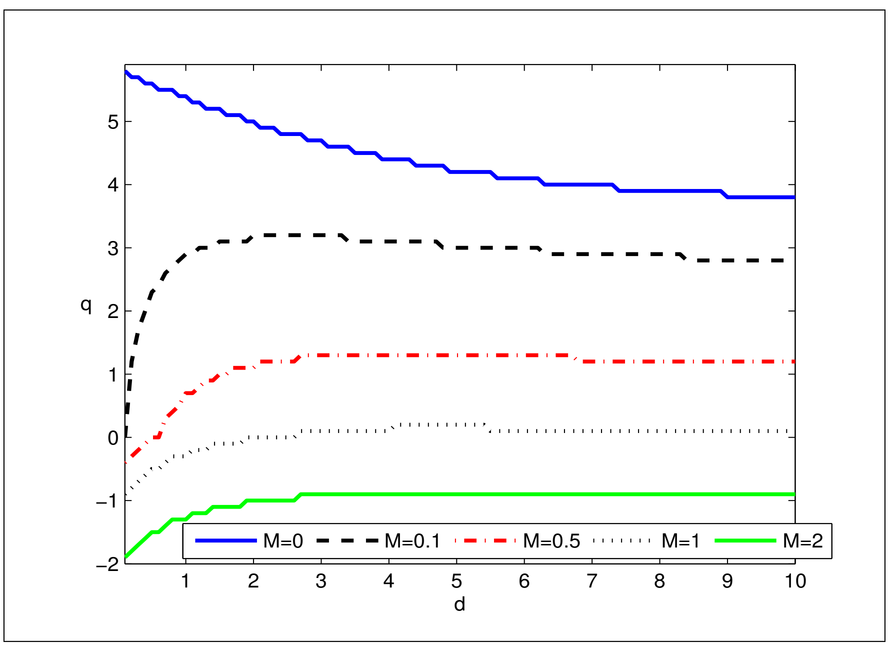
and

$$W(q) = \begin{cases} q + M - d + \left(1 - e^{-\lambda(q+M+d)}\right)\left[q + M + d - \frac{1-e^{-\lambda(q+M+d)}}{\lambda}\right], & q \geq -M-d \\ q + M - d, & q < -M-d \end{cases}. \tag{5}$$

Solving the model involves finding the values of q such that matched agents choose marriage in period one for given d and M.

Figure 4 shows the cut-off values of the initial relationship quality q above which matched agents prefer to marry as a function of the divorce cost d for several values of M when $\lambda = 0.5$.

Figure 4: Heterogeneous agents with independent utilities: Cut-off values of q



First, note that for $M = 0$, the cut-off value of q is lower for higher values of divorce costs, that is, when the divorce cost is larger it induces marriage for lower match quality couples. The analysis in the previous section with homogeneous agents demonstrated that in the absence of additional exogenous benefits, marriage is never preferred to cohabitation. This is no longer the case when the agents are heterogeneous with independent additional relationship quality shocks: marriage is preferred by matched agents with higher initial match quality and the fraction of couples choosing marriage increases with higher divorce cost. This is the commitment effect of divorce costs.

For high values of the marriage benefit M (here it is for $M > 1$) the relationship between the cut-off value of q and the cost of divorce is reversed. Agents with lower initial match qualities choose marriage, and their main incentive is obtaining the exogenous marriage benefit. These couples are discouraged by higher divorce costs. The commitment effect of higher divorce costs is dominated when the exogenous marriage benefit is sufficiently large.

For intermediate values of the marriage benefit (here for $M \in (0, 1]$) the relationship between the cut-off values of q and the cost of divorce is bell-shaped. For small divorce costs, couples with matches of lower quality marry to obtain the exogenous benefit M. Since they divorce with positive probability, higher divorce costs reduce the relative value of marriage, resulting in higher cut-off values of q for marriage. When d is sufficiently large, the commitment effect dominates. Higher divorce costs help eliminate the break-up externality and induce marriage for lower match quality couples.

The following proposition formally establishes these results:

Proposition 3 *Let $\underline{q}$ be the initial relationship quality such that*
i) $\underline{q}$ solves $W(q) = \max\{0, R(q)\}$, where $W(q)$ and $R(q)$ are given by equations (5) and (4) respectively;
ii) For all $q \geq \underline{q}$, $W(q) \geq \max\{0, R(q)\}$.

Then,

(A) For $M = 0$, 1) $\underline{q} \geq 0$ and 2) $\underline{q}$ is decreasing in d;

(B) $\exists \underline{M} > 0$, such that for any $M > \underline{M}$, $\underline{q}$ is increasing in d;

(C) For $M \in (0, \underline{M}]$, $\underline{q}$ can be increasing or decreasing in d.

Proof.

Let

$$D(t) = \left(1 - e^{-\lambda t}\right)\left[t - \frac{1 - e^{-\lambda t}}{\lambda}\right] = \left(t - \frac{1}{\lambda}\right) + \frac{e^{-\lambda t}}{\lambda}\left(2 - \lambda t - e^{-\lambda t}\right) \tag{6}$$

for any real $t \geq 0$.

Let $\Delta(q) = W(q) - \max\{0, R(q)\}$. From (4), (5), and (6)

$$\Delta(q) = \begin{cases} (M - d) + D(q + M + d) - D(q), & q \geq 0 \\ (q + M - d) + D(q + M + d), & q \in [-M - d, 0] \\ (q + M - d), & q \leq -M - d \end{cases} . \tag{7}$$

Then, $\underline{q}$ is the value of initial relationship quality q such that 1) $\Delta\left(\underline{q}\right) = 0$ and 2) for all $q \geq \underline{q}$, $\Delta(q) > 0$.

Note that for any $q \leq -M - d$, $\Delta(q) < 0$. Thus, in what follows, this case is not considered.

Claim (A): For $M = 0$, 1) $\underline{q} \geq 0$ and 2) $\underline{q}$ is decreasing in d, that is, its derivative with respect to d is negative: $\underline{q}_d(d) < 0$.

For any real $t \geq 0$, let

$$v_0(t) = e^{-t}\left(2 - t - e^{-t}\right), \tag{8}$$

Then, from (6) and (8),

$$D(t) = \left(t - \frac{1}{\lambda}\right) + \frac{1}{\lambda} v_0 (\lambda t) \tag{9}$$

The properties of function $v_0(t)$ are explored in the Appendix. We use these properties to prove parts 1) and 2) of Claim (A):

1) Show that $\underline{q} \geq 0$. Suppose not, i.e., $\underline{q} \in [-d, 0)$. Then, from (9) and (7),$\Delta(q) = \left(2q - \frac{1}{\lambda}\right) + \frac{1}{\lambda} v_0 (\lambda (q + d)) < 0 \ \forall q < 0$. This is because $v_0(t) \leq 1$ for any $t \geq 0$ (See Appendix). Thus, $\underline{q} < 0$ does not exist.

2) Consider $\underline{q} \geq 0$ and show that $\underline{q}_d(d) < 0$.
For $M \geq 0$, $\underline{q}$ solves $\Delta(q) = (M - d) + D(q + M + d) - D(q) = 0$. Equivalently, $\Delta(q) = 2M + \frac{1}{\lambda} [v_0 (\lambda (q + M + d)) - v_0 (\lambda q)] = 0$.
When $M = 0$, $\Delta(q) = 0$ is equivalent to $v_0 (\lambda (q + d)) = v_0 (\lambda q)$.
From the properties of $v_0(t)$ (See Figure 11 A) in the Appendix), conclude that $\underline{q} \in \left(\frac{t_0}{\lambda}, \frac{t_1}{\lambda}\right)$ and $\underline{q}_d(d) < 0$, where t_0 solves $v_0(t) = 0$ and t_1 solves $v_0'(t) = 0$.

Next we show claim (C) and use the result to establish claim (B).

Claim (C): For any fixed value of $M > 0$, $\underline{q}_d(d)$ can be positive, negative, or zero.

Let $\underline{q}(d)$ be the root of equation $\Delta(q, d) = 0$ for any fixed $M \geq 0$. That is, $\Delta\left(\underline{q}(d), d\right) = 0$. Fully differentiate this equation with respect to d to find the derivative $\underline{q}_d(d)$:

$$\underline{q}_d(d) = -\frac{\Delta_d\left(\underline{q}(d), d\right)}{\Delta_q\left(\underline{q}(d), d\right)},$$

where Δ_q and Δ_d denote the derivatives of $\Delta(q)$ with respect to q and d, respectfully.

By differentiating (7) for $q \geq -M - d$, obtain

$$\Delta_q = \begin{cases} D'(q+M+d) - D'(q), & q \geq 0 \\ 1 + D'(q+M+d), & q \in [-M-d, 0] \end{cases}$$

and

$$\Delta_d = D'(q+M+d) - 1, \; q \geq -M-d.$$

Differentiate (6) to obtain $D'(t) = 1 + e^{-\lambda t}\left(\lambda t - 3 + 2e^{-\lambda t}\right)$, $\forall t \geq 0$.

Let

$$v_1(t) = e^{-t}\left(t - 3 + 2e^{-t}\right). \tag{10}$$

Then,

$$D'(t) = 1 + v_1(\lambda t). \tag{11}$$

Using (11) obtain

$$\Delta_q = \begin{cases} v_1(\lambda(q+M+d)) - v_1(\lambda q), & q \geq 0 \\ 2 + v_1(\lambda(q+M+d)), & q \in [-M-d, 0] \end{cases} \tag{12}$$

and

$$\Delta_d = v_1(\lambda(q+M+d)), \; q \geq -M-d. \tag{13}$$

The properties of function $v_1(t)$ are explored in the Appendix.

To determine the signs of Δ_q and Δ_d, consider the two cases: 1) $\underline{q} \in [-M-d, 0)$ and 2) $\underline{q} \geq 0$:

1) $\underline{q} \in [-M-d, 0)$

$\Delta_q = 2 + v_1\left(\lambda\left(\underline{q} + M + d\right)\right) > 0$ since $v_1(t) \geq -1 \; \forall t \geq 0$ (See Figure 11 B) in the Appendix).

$\Delta_d = v_1\left(\lambda\left(\underline{q}+M+d\right)\right)$, thus,

$$\Delta_d\ is \begin{cases} \leq 0, & \underline{q} \in \left[-M-d, \frac{t_1}{\lambda}-M-d\right] \\ \geq 0, & \underline{q} \in \left[\frac{t_1}{\lambda}-M-d, 0\right) \end{cases},$$

where t_1 solves $v_1(t) = 0$.

2) $\underline{q} \geq 0$

First, show that $\underline{q} < \frac{t_1}{\lambda}$.

As previously established, $\forall q \geq 0$, $\underline{q}$ solves $\Delta(q) = 2M + \frac{1}{\lambda}\left[v_0\left(\lambda\left(q+M+d\right)\right) - v_0\left(\lambda q\right)\right] = 0$. Equivalently,

$$2M\lambda = v_0\left(\lambda\underline{q}\right) - v_0\left(\lambda\left(\underline{q}+M+d\right)\right). \tag{14}$$

Suppose $\underline{q} \geq \frac{t_1}{\lambda}$. For all $t \geq t_1$, $v_0(t)$ is an increasing function, so $v_0\left(\lambda\left(\underline{q}+M+d\right)\right) > v_0\left(\lambda\underline{q}\right)\ \forall \underline{q} \geq \frac{t_1}{\lambda}$.

Thus, equation 14 cannot hold and $\underline{q} < \frac{t_1}{\lambda}$.

Next, determine the signs of Δ_q and Δ_d for all $\underline{q} \geq 0$.

Analysis for Δ_d remains as before.

For $\underline{q} \in \left[0, \frac{t_1}{\lambda}\right)$, $\Delta_q = v_1\left(\lambda\left(\underline{q}+M+d\right)\right) - v_1\left(\lambda\underline{q}\right) > 0$.

Combining the two cases, obtain

$$\underline{q}_d(d)\ is \begin{cases} \geq 0, & \underline{q} \in \left[-M-d, \frac{t_1}{\lambda}-M-d\right] \\ \leq 0, & \underline{q} \in \left[\frac{t_1}{\lambda}-M-d, \frac{t_1}{\lambda}\right) \end{cases}.$$

Claim (B): $\exists \underline{M} > 0$, such that for any fixed $M > \underline{M}$, $\underline{q}_d(d) > 0$.

Let $\underline{q}(M)$ be the root of equation $\Delta(q, M) = 0$ for any fixed $d > 0$. As before, can obtain the derivative of $\underline{q}(M)$ with respect to M by fully differentiating this equation with respect to M:

$$\underline{q}_M(M) = -\frac{\Delta_M\left(\underline{q}(M), M\right)}{\Delta_q\left(\underline{q}(M), M\right)},$$

The derivative of $\Delta(q)$ with respect to M is

$$\Delta_M = 1 + D'(q + M + d) = 2 + v_1(\lambda(q + M + d)) > 0,\ q \geq -M - d.$$

Since $\Delta_q > 0\ \forall \underline{q} \in \left[-M - d, \frac{t_1}{\lambda}\right)$, conclude that $\underline{q}_M(M) < 0$.

$\Delta(-M - d) = -2d$ and $\Delta(q)$ is continuous and strictly increasing with respect to M. As M gets larger, the root of $\Delta(q) = 0$ shifts to the left, closer to $(-M - d)$.

Thus, there exists $\underline{M}$ such that $\underline{q}(\underline{M}) = \frac{t_1}{\lambda} - \underline{M} - d < 0$ and for $M > \underline{M}$, $\underline{q} \in \left[-M - d, \frac{t_1}{\lambda} - M - d\right)$ and $\underline{q}_d(d) > 0$.

■

Marriage rate is the fraction of matches with initial relationship quality of at least $\underline{q}$. As before, let F_q denote the cumulative distribution function for q, then $MR = 1 - F_q\left(\underline{q}\right)$. The following corollary establishes the main result of this section:

Corollary 4 (Divorce Costs and Marriage Rates: the Case of Heterogeneous Agents) *Under the assumptions of the model with heterogeneous agents,*

(a) If $M = 0$, marriage rate is increasing in divorce cost d;

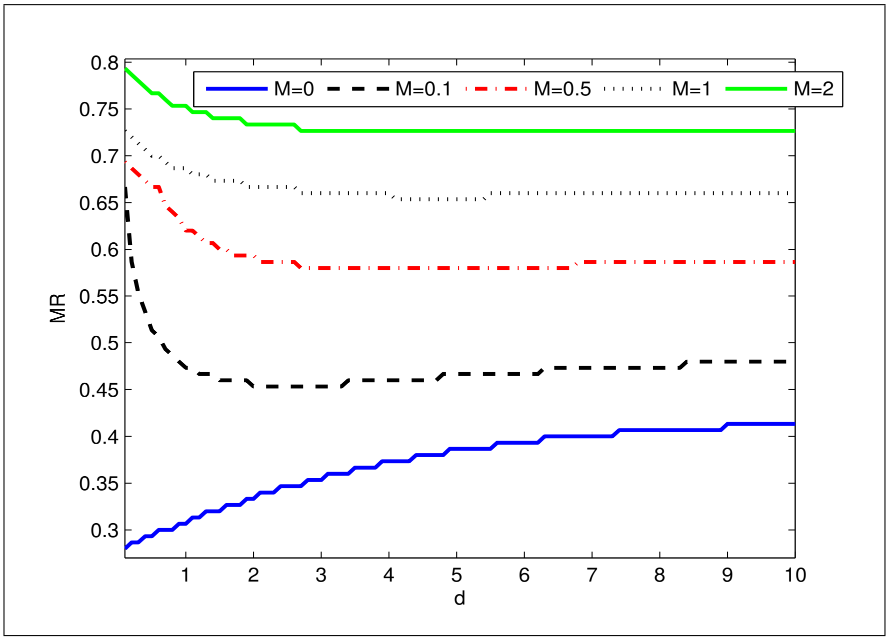
(b) $\exists \underline{M} > 0$, such that for any $M > \underline{M}$, marriage rate is decreasing in d;

(c) For $M \in (0, \underline{M}]$, marriage rate can be decreasing or increasing in d.

Figure 5 depicts the relationship between marriage rate and divorce costs for $\lambda = 0.5$, $q_L = -5$, and $q_H = 10$.

This simple model suggest the following explanation of the empirical evidence. Rasul [9] finds that marriage rates in the US decline after reduction in divorce costs.

Figure 5: Heterogeneous agents with independent utilities, continuous case: Marriage rate

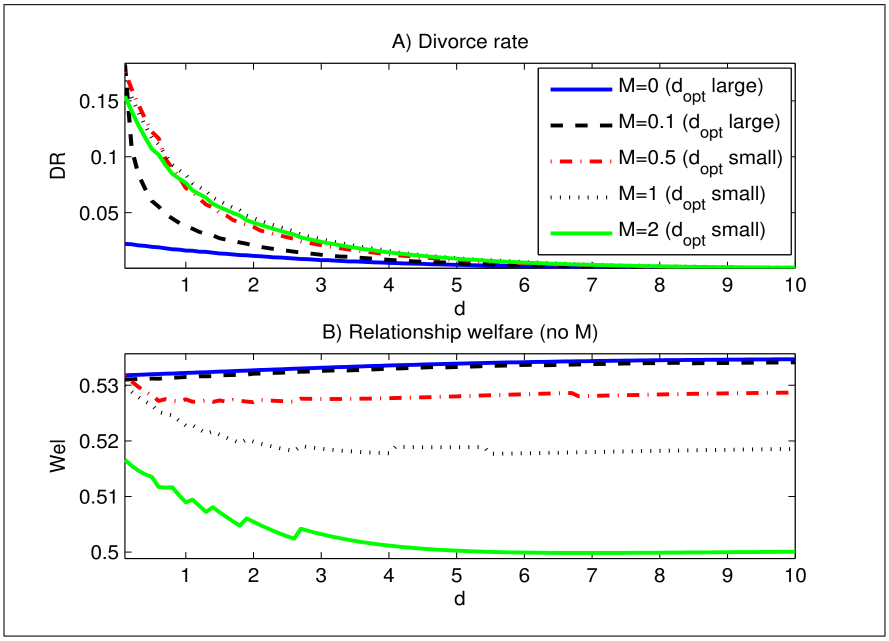


This evidence is consistent with the commitment model of marriage. Here the commitment effect dominates when the exogenous benefits to marriage are low relative to the cost of divorce.

The relationship between marriage rates and the measure of difficulty of obtaining divorce across countries is, however, of the opposite sign. The exogenous benefits part of the marriage decision story appears to be more relevant.

Figure 6 A) shows the net divorce rate as a decreasing function of the divorce cost d. Figure 6 B) depicts the average relationship welfare for different values of M. As before, it is lower for higher values of M since matches of lower quality result in marriage. The total welfare maximizing divorce cost depends on the value of M. For small values of the exogenous marriage benefit, $M = 0$ and $M = 0.1$, the welfare is increasing in d, so the optimal divorce cost is equal to the highest value in the given range, $d = 10$. Higher divorce costs help eliminate the negative break-up externality, increasing the expected value of marriage and welfare. For higher values of M the commitment effect of higher divorce costs is weaker, thus, the welfare-maximizing

Figure 6: Heterogeneous agents with independent utilities, continuous case: Divorce rates and Average relationship welfare



divorce cost is small, just above zero.

We conclude that when the agents are heterogeneous, the relationship utility is non-transferable, and the cost of divorce is high relative to the marriage benefit, decreasing the divorce costs can result in lower marriage rates. When the divorce costs are relatively low, the marriage rate is a decreasing function of the divorce costs for any positive marriage benefit.

The assumption of heterogeneous agents with independent utilities, just like the assumption of homogeneous agents, may be too strong. In the next section we relax this assumption and solve the model for the case of heterogeneous agents with interdependent utilities.

5 Heterogeneous agents: Interdependent utilities

Consider any two agents comprising a cohabiting or married couple in period two. Refer to these agents as agent 1 and agent 2. The additional match quality shock of agent 1 in period two is x, and that of agent 2 is $x_{\_}$. As before, both x and $x_{\_}$ are drawn independently from the same exponential distribution with mean $1/\lambda$.

Let $u = \alpha x + (1-\alpha) x_{\_}$ and $v = \alpha x_{\_} + (1-\alpha) x$, where $0.5 < \alpha < 1$. The relationship utility of agent 1 in period two is $r(q, x, x_{\_}) = q - u$ and that of agent 2 is $r(q, x_{\_}, x) = q - v$. Thus, the additional utility obtained by each agent in period two is a linear combination of the agent's own match quality shock and that of his or her partner, with a larger (in absolute value) weight assigned to the own relationship quality shock.

Then, the expected value of cohabiting for agent 1 (similar for agent 2) with the initial relationship quality q is

$R(q) = q + F_{u,v}(q,q) \int_0^q (q-u) \frac{f_u(u)}{F_u(q)} du,$

and that of being married is

$W(q) = (q+M) + F_{u,v}(q+M+d, q+M+d) \int_0^{q+M+d} (q+M-u) \frac{f_u(u)}{F_u(q+M+d)} du + [1 - F_{u,v}(q+M+d, q+M+d)](-d),$

where $F_{u,v}(u,v)$ is the joint cumulative distribution function of u and v, $f_u(u)$ is the marginal density function of u, and $F_u(u)$ is the marginal cumulative distribution function of u.

To find the joint distribution of u and v let the joint probability density function of independent x and $x_{\_}$ be

$$f_{x,x_{\_}}(x, x_{\_}) = \begin{cases} \left(\lambda e^{-\lambda x}\right)\left(\lambda e^{-\lambda x_{\_}}\right), & \text{if } x \geq 0,\ x_{\_} \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Define $A = \{(x, x_{\_}) :\ f_{x,x_{\_}}(x, x_{\_}) \geq 0\}$ and

$B = \{(u,v) :\ u = \alpha x + (1-\alpha) x_{\_},\ v = \alpha x_{\_} + (1-\alpha) x,\ \forall (x, x_{\_}) \in A\}.$

Then, the joint probability density function of u and v is

$$f_{u,v}(u,v) = \begin{cases} \frac{\lambda^2}{2\alpha-1} e^{-\lambda(u+v)}, & \text{if } (u,v) \in B \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

and the marginal probability density function and the cumulative distribution function for u are, respectively,

$$f_u(u) = \begin{cases} \frac{\lambda}{2\alpha-1} \left(e^{-\lambda \frac{u}{\alpha}} - e^{-\lambda \frac{u}{1-\alpha}} \right), & \text{if } u \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$F_u(u) = \begin{cases} \frac{\alpha\left(1-e^{-\lambda \frac{u}{\alpha}}\right)-(1-\alpha)\left(1-e^{-\lambda \frac{u}{1-\alpha}}\right)}{2\alpha-1}, & \text{if } u \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Unfortunately, analytical solution cannot be obtained in this case. The model is solved numerically for $\lambda = 0.5$ and three values of the utility interdependency parameter α: 0.6, 0.75, and 0.9.[7]

Figure 7 presents the cut-off values of the initial relationship quality q above which the agents prefer to marry for each value of α and different values of the marriage benefit M: $M = 0$, $M = 0.1$, $M = 0.5$, $M = 1$, and $M = 2$. Figure 8 compares the marriage rates for different values of α and M.

[7] The results are robust to changes in the parameter values. As previously mentioned, higher values of λ reduce the likelihood of adverse relationship utility shocks in the second period. With higher values of λ the divorce costs play a relatively smaller role in the paired agents' choice of marriage versus cohabitation, since they are less likely to separate in either case.

Figure 7: Heterogeneous agents and interdependent utilities: Cut-off values of q

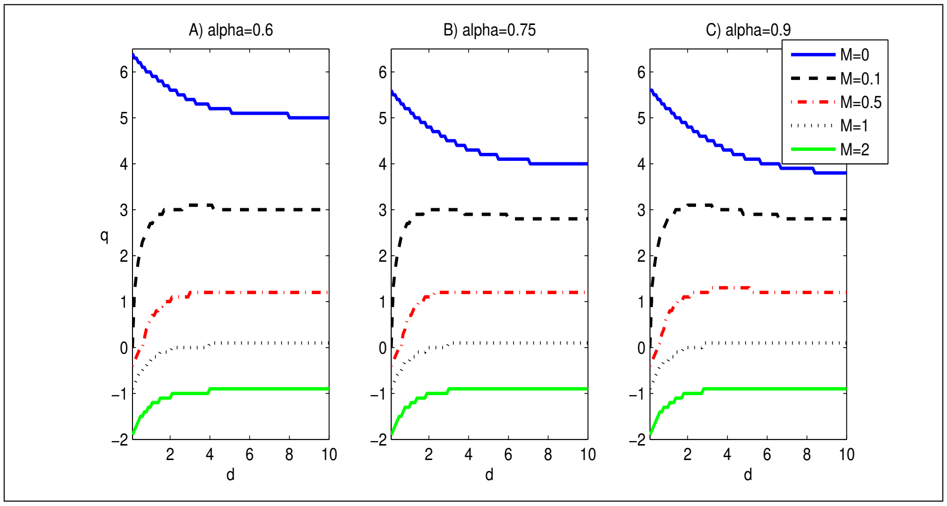
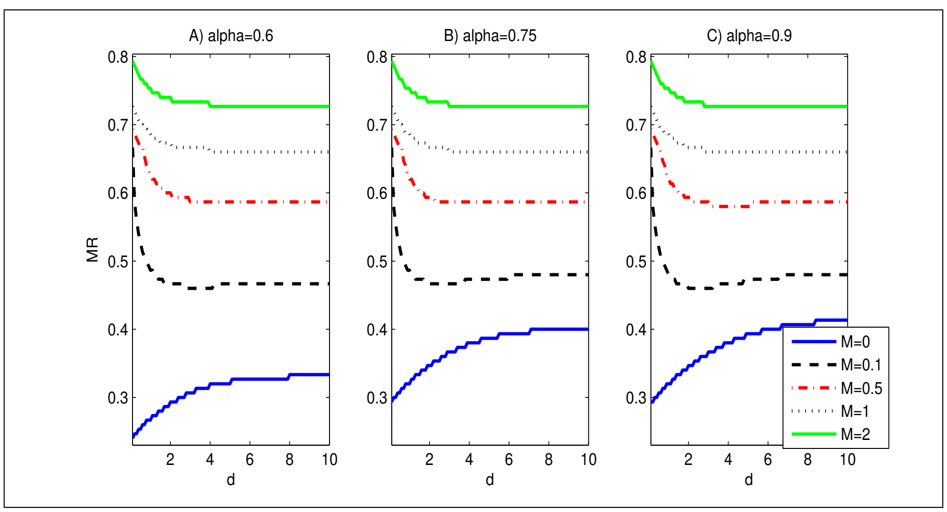


Figure 8: Heterogeneous agents and interdependent utilities: Marriage rate



Notice that even for a large degree of utility interdependency ($\alpha = 0.6$) the cut-off value of q is decreasing and the marriage rate is increasing in the cost of divorce d when $M = 0$. The U-shaped relationship between the marriage rate and the cost of divorce is also present for small values of the marriage benefit and becomes stronger as the utilities become less interdependent ($M = 0.1$ and $M = 0.5$ on Figure 8).

Marriage rates are higher when the utilities of paired agents are less interdependent, since the value of eliminating the negative break-up externality is higher, inducing couples with lower values of the initial relationship quality to marry.

Figure 9 shows that the net divorce rate is a decreasing function of the divorce cost for any value of the exogenous marriage benefit M. Figure 10 depicts the mean relationship welfare of agents for different values of M and α and the welfare-maximizing cost of divorce. As before, the graph shows the welfare without the marriage benefit, so for any value of α the relationship welfare is lower for higher values of M since couples with lower initial match quality choose marriage. The optimal divorce cost, however, is chosen so as to maximize the total welfare, taking into account the marriage benefit M. Observe that for any α the highest relationship welfare is achieved when M is small and d is large.

Figure 9: Heterogeneous agents and interdependent utilities: Divorce rate

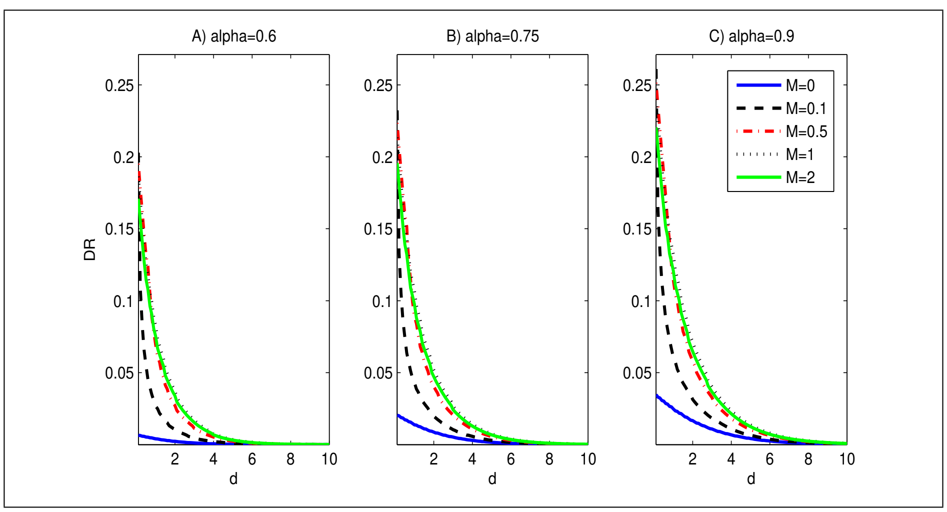
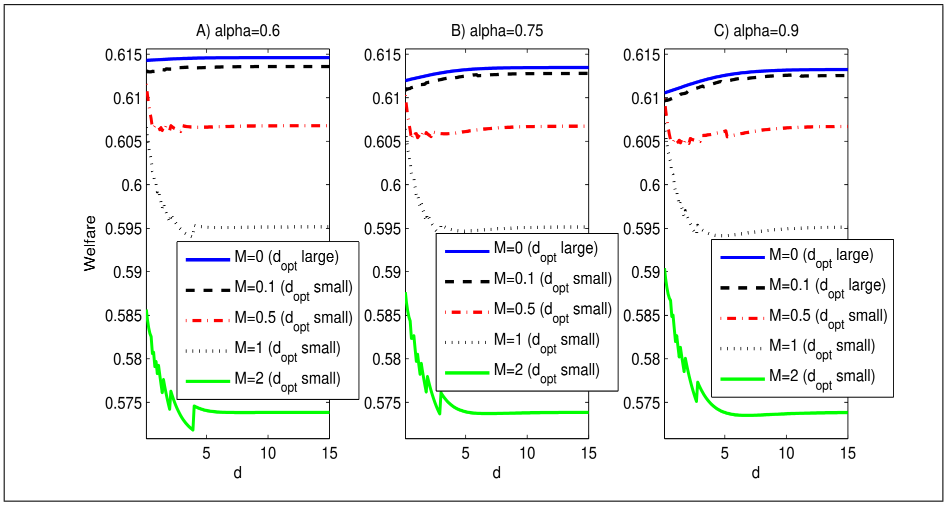


Figure 10: Heterogeneous agents and interdependent utilities: Average welfare



The analysis in this section demonstrates that even a small degree of heterogeneity (low values of α) affects the decision to marry and alters the relationship between the cost of divorce and marriage rates when the utility is non-transferable. Explaining the empirical evidence on this relationship does not require the assumption of heterogeneous agents with independent utilities; the assumption of some degree of independency and utility non-transferability is sufficient as long as the additional benefit to marriage is not too large.

6 Conclusion

References

[1] Gary Becker, Elisabeth Landes, and Robert Michael. An economic analysis of marital instability. *Journal of Political Economy*, 85:1141–1187, 1977.

[2] Michael Brien, Lee Lillard, and Steven Stern. Cohabitation, marriage, and divorce in a model of match quality. *International Economic Review*, 47(2):451–494, 2006.

[3] Giulio Fella, Paola Manzini, and Marco Mariotti. Does divorce law matter? *Journal of the European Economic Association*, 2(4):607–633, 2004.

[4] Leora Friedberg. Did unilateral divorce raise divorce rates? evidence from panel data. *American Economic Review*, 88(3):608–628, 1998.

[5] Libertad Gonzalez and Tarja Viitanen. The effect of divorce laws on divorce rates in europe. *European Economic Review*, 53:127–138, 2009.

[6] Brishti Guha. Sex ratios, divorce laws and the marriage market. Paper no. 28, SMU Economics and Statistics Working Paper Series, 2010.

[7] Niko Matouschek and Imran Rasul. The economics of the marriage contract: Theories and evidence. *Journal of Law and Economics*, 51:59–110, 2008.

[8] Society at a Glance. OECD Social Indicators, 2005, 2006, 2011.

[9] Imran Rasul. The impact of divorce laws on marriage. Unpublished manuscript, University of Chicago, Graduate School of Business, 2005.

[10] Imran Rasul. Marriage markets and divorce laws. *Journal of Law and Economics*, 22(1):30–69, 2006.

[11] Betsey Stevenson and Justin Wolfers. Marriage and divorce: Changes and their driving forces. *Journal of Economic Perspectives*, 21(2):27–52, 2007.

[12] Justin Wolfers. Did unilateral divorce laws raise divorce rates? a reconciliation and new results. *American Economic Review*, 96(5):1802–1820, 2006.

[13] Bruce Wydick. Grandma was right: Why cohabitation undermines relational satisfaction, but is increasing anyway. *KYKLOS*, 60(4):617–645, 2004.

Appendix

Recall functions

$v_0(t) = e^{-t}(2 - t - e^{-t})$

and

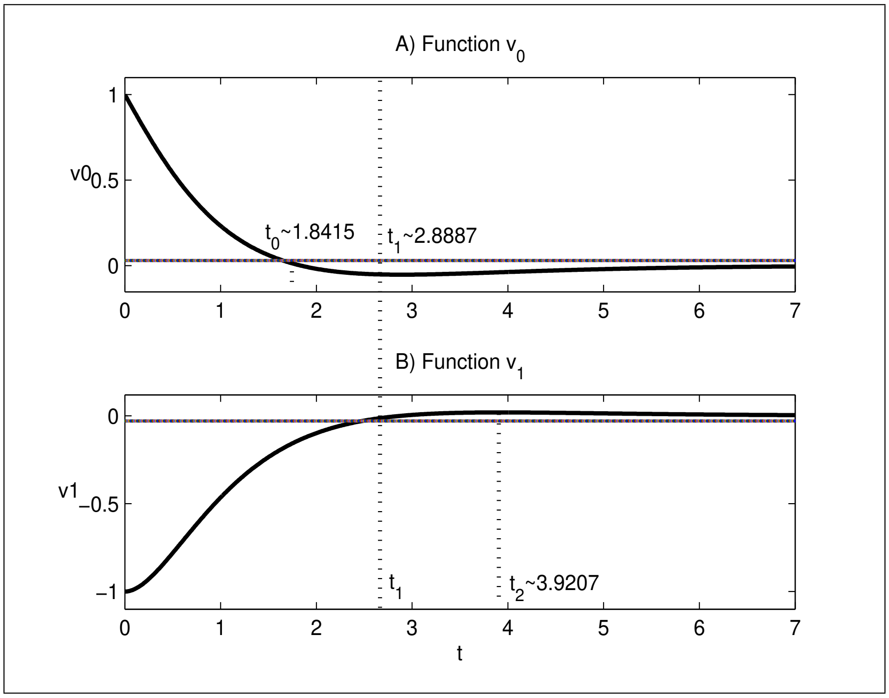
$v_1(t) = e^{-t}(t - 3 + 2e^{-t})$

from equations (8) and (10), respectively.

Functions $v_0(t)$ and $v_1(t)$ are used in establishing the results in Proposition 3, so it is useful to consider their properties. Figure 11 plots these functions.

Observe the following:

Figure 11: Functions $v_0(t)$ and $v_1(t)$



- $v_0(0) = 1$; $v_0(t_0) = 0$, $v_0(t) > 0$ for $t < t_0$, and $v_0(t) < 0$ for $t > t_0$, $t_0 \sim 1.8415$. As $t \to \infty$, $v_0(t) \sim -0$. Function $v_0(t)$ has a unique minimum at t_1, where t_1 solves $v_0'(t) = v_1(t) = 0$, and $t_1 \sim 2.8887$, $v_0(t_1) \sim -0.0526$.
- $v_1(0) = -1$; $v_1(t) < 0$ for $t < t_1$, and $v_1(t) > 0$ for $t > t_1$. As $t \to \infty$, $v_1(t) \sim +0$.Function $v_1(t)$ has a unique maximum at $t_2 \sim 3.9207$, $v_1(t_2) \sim 0.02$.