Applying approximate entropy (ApEn) to speculative bubble in the stock market

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10 April 2012

A Short Note on Approximate Entropy (ApEn)- a Measure to trace Speculative Bubble in Stock Markets

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Abstract

The paper introduces an order statistic, Approximate Entropy (ApEn), to investigate the presence of speculative bubbles in the equity market. In contrast to the traditional duration dependence test, the paper using Approximate Entropy examines three major events of stock market crash in US, Japan, and India. In addition, the paper also investigates the 1997 Asian crisis using weekly data from seven major Asian indices which includes Hong Kong, Malaysia, Singapore, Korea, Taiwan, Indonesia and Japan. The evidences presented in this study show that there are strong "tale-tell" signs which point to a substantially lower level of ApEn during these crash events.

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1. INTRODUCTION

The stock market has often been considered as a primary indicator and a barometer of performance of the entire economy due to its sensitivity to the various economic activities. Among others one of the most important functions of the stock market is to facilitate and encourage capital accumulation by channeling short-term savings into long-term investments and to allocate limited capital to the most valued social usage. The stock prices of an efficient market would hence provide accurate signals for optimal resource allocation in the economy.

The efficient market theory assumes that investors in the market are rational. However, the stock market is often influenced by speculative activities that break down this very assumption that investors always behave rationally. A simple example as in the case of 'herd mentality' explains the fact that investors do act irrationally and occasionally succumb to psychological factors that may lead them to believe that they could earn higher returns, even if their actions might not be considered rational. Therefore, when the market starts being driven by speculation, prices of assets may increase unexpectedly to high levels pushing it much beyond the fundamental values leading to a speculative bubble.

A speculative bubble describes a condition of consistent market overvaluation wherein though the prices of certain assets deviate from their deemed fundamental values yet the investors continue to believe that with high probability the bubble will continue and yield a substantially higher return to compensate the odd of a crash (Kindlerbeger, 1978). Further, when bubbles occur, price increases lead to successively larger increases and as the price reaches a barrier when there can be no further price increase to sustain the demand for the asset, the bubble bursts and the price drops sharply, thus leading to a market crash. In other words, the speculative bubbles are characterized by a situation wherein the investors continue to buy an over valued asset as they anticipate the expansion of the bubble will ensure a greater probability of earning abnormally high returns. This explains the rationality of remaining in the market despite the overvaluation as the higher return earned is expected to compensate the investors for the probability of a crash.

Various techniques have been employed to capture the presence of bubbles. Perhaps the earliest techniques include tests for excess volatility that were employed by Friedman (1953), Baumol (1957), Kohn (1978); Shiller (1981). Other techniques used were tests for bubble premiums by Hardouvelis (1988), Rappoport and White (1993). Test for non-stationarity and cointegration developed by Diba and Grossman (1988) also gained importance in the following years. These techniques however are criticized for their low predictive power and their limitations in the study of speculative bubbles.

In recent times, the duration dependence models have emerged as important technique to study the speculative bubble. In contrast to many traditional tests that look for autocorrelation, skewness and kurtosis to identify bubble, the duration dependence models derived by McQueen and Thorley (1994), provide more discriminatory power by testing for nonlinearity inherent in returns due to bubbles.

As an alternative to the duration based approach this paper develops and introduces order statistic known as Approximate Entropy to investigate the presence of speculative bubbles in the equity market for a sample of moderately large number of countries encompassing both developing and developed economies. The paper adds value to the existing literature in three primary ways. First the paper introduces Approximate Entropy (ApEn), a widely used measure in statistical physics to test the special empirical properties of rational speculative bubble during three known episodes of crash across the various markets like U.S, Japan, and India respectively. The Approximate Entropy (ApEn) proposed by Pincus et al (1991, 2004), has been used in this article to quantify the likelihood of "order" (repetitiveness) in the financial time series. The three episodes of crash considered in the paper involve the US crash in October 1997, the Japan crash in December 1989, and January 2007-08 crash in India. Apart from these episodes we also examine the existence of rational speculative bubble during the Asian crisis in 1997.

Second, most of the empirical studies investigating the speculative bubbles have primarily focused on developed countries. Ironically, assumptions underlying the efficiency hypothesis are more likely to be violated in the case of developing economies than their developed counterparts due to the poor legal and information systems. Therefore, the study of speculative bubbles has gained renewed importance in the context of developing economies. Despite significant developments, which have put these markets almost at par with the best in the world (in terms of their structure, systems and regulation), the markets still have witnessed several bouts of speculative activities in recent times. However, not many of the existing studies have addressed the issue whether the price behavior in many developing markets is consistent with the characteristics of bubbles. Therefore, this paper is a first step towards testing the speculative bubble for a developing economy like India. Particularly in India there have been innumerable speculations about the existence of bubbles in the stock market. In addition to these speculative activities, the Indian stock market has also witnessed various scams and crises leading to irrational price behaviour over the last decade and hence provides a unique opportunity to test the power of ApEn in identifying irregular market movements.

Finally, apart from its contribution to the sparse empirical work, the findings of the paper also provide a valuable policy implication for many developing and emerging countries which are vulnerable to speculative bubble. The paper, therefore, attempts to suggest an early warning system to detect the emergence of bubbles and thereby correct any persisting anomalies in the market. The paper also carries out a series of robustness checks for the findings revealed through this study.

The rest of the paper is organized as follows: Section 2 presents the methodology and the data used in the paper. Section 3 summarizes the results using ApEn. The paper's conclusions are presented in Section 4.

2. DATA AND METHEDOLOGY

2.1 Empirical Model:

In a simple rational speculative bubble model Shiller (1978) and Blanchard and Watson (1982) have argued that the market price of a stock can deviate from its fundamental values under a speculative rational bubble episode so long as the bubble grows at a specific rate. Defining the expected return of a stock as $E_t(R_{t+1}) = (P_{t+1} - P_t)/P_t$, where P_t is the price at time t and E_t denotes the expectation given the information set at time t, in an efficient market condition require that a stock's expected return equals its required rate of return (r_t) i.e. $E_t(R_{t+1}) = (r_{t+1})$. In other words, the competitive equilibrium condition requires current price to be equal to the expected future price discounted by the required rate of return, i.e. $P_t = E_t(P_{t+1})/(1+ r_{t+1})$. Further, Shiller (1978) and Blanchard and Watson (1982) and West (1987) among others observed that any price of the form $P_t = P_t^* + b_t$, where $E_t(b_{t+1})=(1+ r_{t+1})b_t$, is also a solution to equilibrium condition. Therefore, the market price of a stock can deviate from its fundamental value by a rational bubble factor b_t, so long the bubble factor grows at the required rate of return (r_t).

Blanchard and Watson (1982) and McQueen and Thorley (1994) offer a rational speculative bubble process that allows the bubble to grow and burst with bubble satisfying following condition:

$$\begin{aligned} b_{t+1} &= \frac{(1+r_{t+1})b_t}{\pi} - \frac{1-\pi}{\pi}a_0 \quad \textit{with probability } \pi \\ &= a_0 \quad \textit{with probability } (1-\pi) \end{aligned}$$

In other words, in this process bubble grows by the exact amount needed to offset the loss due to a crash with probability (1-π). It is important to note that the model proposed by Blanchard and others suggest a bubble with a long run-up in price followed by a crash with π >1/2.

2.2 Methodology

The first rational speculative bubble model was introduced by McQueen and Thorley (1994) in which they suggested that bubbles lead to explosive price changes which grow each period that it survives. Therefore, they hypothesized that a long run of positive abnormal returns suggests the presence of a bubble if the conditional probability of the run ending is a decreasing function of the duration of the run. [2] In other words, in the rational bubbles, stock prices should deviate from random walk and show evidence of long duration runs of either positive or negative abnormal returns. In this paper we follow the basic argument of the traditional duration dependence test as proposed by McQueen

[2] Since bubbles cannot be negative there is no such restriction placed on runs of negative abnormal returns (McQueen and Thorley, 1994).

and Thorley (1994), but deploy a new measure of "order" to test the speculative bubble hypothesis.

The paper introduces Approximate Entropy (ApEn) as a measure of regularity statistic in order to test speculative bubble in stock markets. The Approximate Entropy estimates the likelihood that "similar" patterns of observations will not be followed by additional "similar" trends. Therefore any time series that contains many repetitive patterns has a relatively small ApEn value when compared to more random ones. This particular ability to trace repetitive patterns in time series has been exploited by the paper to identify the speculative bubble in the data.

The approximate entropy was introduced by Pincus et al (1991, 1997 and 2004) to quantify the creation of information in a time series. Though initially developed for measuring the irregularities of a complex nonlinear system, it has been gradually introduced into finance literature as a measure of market efficiency for both stock and foreign exchange market (Pincus et al, 2004, and Oh et al 2006). However, the paper for the first time attempts to use this statistics to test for speculative rational bubble in the stock markets.

The following section provides a brief summary of algorithm to estimate ApEn for a time series of stock index data. Given a sequence S_N, containing N absolute returns from index series (S_i), $U_i = S_{i+1} - S_i$, two input parameters are defined such as, m, r to compute ApEn (S_N, m, r), where m defines the pattern length and r reflects the similarity criterion.

To compute the approximate entropy, ApEn, of a time series S_N , first the series of vectors of length m, $v(n)=[U(n), U(n+1),...U(n+m-1)]^T$ is derived from the signal sample S(n). The distance D(i,j) between two vectors v(i) and v(j) is defined as the maximum difference in the scalar components of v(i) and v(j). Then $N^{m,r}(i)$, i.e., the number of vectors j (with $j \leq N-m+1$) such that the distance between the vectors v(j) and the generic vector v(i) (with $i \leq N-m+1$) is lower than r, $D(i,j) \leq r$, is computed where r reflects the similarity criterion. The probability, $C^{m,r}(i)$, to find a vector which differs from v(i) less than the distance r, is defined as:

$$C^{m,r}(i) = \frac{N^{m,r}(i)}{(N-m+1)}$$

And the logarithmic average over all the vectors of the $C^{m,r}(i)$ probability is defined as :

$$F^{m,r} = \frac{\sum_{i=1}^{N-m+1} \ln(C^{m,r}(i))}{N-m+1}$$

ApEn is given by:

$$ApEn^{m,r} = F^{m,r} - F^{m+1,r}$$

Therefore, ApEn of a time series S_N measures the logarithmic likelihood that runs of patterns of length m that are close to each other will remain close in the next incremental comparisons, m+1. And hence the fact that a greater likelihood of remaining close (high regularity) produces smaller ApEn values (low regularity produces higher ApEn values), has been exploited to test the presence of rational speculative bubble in this paper.

Extant empirical literature contends that market during bubble would experience long runs of similar patterns of returns during the growing and the deflating phases of bubble. Most of the existing duration dependence models exploit these patterns through a testable hypothesis of negative duration in runs of positive abnormal returns (Mc Queen and Thorley, 1994). Following this, the paper argues that in the presence of a rational speculative bubble the return shows runs of repetitive patterns both before and after the crash, thereby leaving a “tell tale” sign in the return series. Therefore, the paper hypothesizes that an episode of rational speculative bubble should be associated with relatively low values of ApEn when compared to the average historical value. The paper develops several measures to observe the characteristic of ApEn during the crash periods. The first measure of rational speculative bubble involves testing an empirical pattern in ApEn using a 12 weeks discrete windows of returns data. In these tests, a low level of ApEn during specific episode of crash would provide a “tell tale” signs of speculative bubbles in the data. The second measure involves a yearly measure to examine the empirical patters in ApEn before, during and after the crash.

2.2 DATA

The paper tries to develop an alternative statistical measure to traditional duration dependence test in order to analyze speculative bubble across multiple countries. We investigate the speculative bubble for multiple episodes of market crashes which includes the US crash in October 1997, the Japan crash in December 1989, and 2007 crash in India. Apart from these episodes we also examine the claims of rational speculative bubble during the Asian crisis for Hong Kong, Thailand, Taiwan, Malaysia, Singapore, and South Korea in 1997. For this purpose the weekly data of the primary stock market from the respective countries have been used for the paper. Since the purpose of the paper is to study the crash, pre and post crash period data have also been analyzed. Usually the decade of the crash has been identified as the sample period for our study. More specifically, for the crash in the US, Japan, and India, we have considered the weekly data for 1981-90, 1984-1994, 1998-2008 respectively. Further to study the Asian crisis we have considered the weekly data for 1991-2003, except Malaysia which has data for 1994-2003.

As argued by McQueen and Thorley (1994), our choice of weekly data over daily data is based on the fact that the findings of bubble could be contaminated by the high signal-to-noise ratio in daily returns. On the other hand, weekly returns may be appropriate over other alternative of monthly tests as it may lack power given the relatively short data series used in the study.

Once the time series is determined, we have considered the incremental series, S_{i+1}-S_i and several alternatives such as [(S_{i+1}/S_i)-1] and the log-ratio series [ln (S_{i+1}/S_i)], to estimate the ApEn. Since findings of the paper are robust to these alternatives, we report only the results for the incremental series.

In this paper, the ApEn is estimated with embedding dimension, m = 2 and the similarity measure, r = 20% of the standard deviation of the time series, similar to the previous work (Pincus et al 2004). To test the robustness of our findings several values of m and r are attempted. However, as our finding remains invariant to different choices of m and r, we report only ApEn (2, 20) following the convention.

3. RESULTS

Preliminary Observations

Table 1 and 2 provide the preliminary descriptive statistics of return data during the crash decades. Descriptive statistics presented suggest some signs of irregularities in the data: First, the negative significant skewness, with a few exceptions (Japan, Malaysia and Singapore) suggesting unusually large negative returns associated with crash is often considered as a tell-tale sign of bubble in observed returns. Second, consistent with the fat tail due to the mixing of distribution as the bubble grows, all the stock returns are leptokurtic. Finally, the rational speculative bubble often suggests a strong positive autocorrelation due to the fact that returns tend to be positive as the bubble grows. For weekly returns statistics presented in tables 1 and 2 suggest a significant autocorrelation for several sample periods indicating further tell-tale sign for speculative bubble in the data.

However, as these return characteristics such as autocorrelation, skewness, or kurtosis can also be attributed to other anomalies, we need a more discriminatory statistics to provide more reliable evidence for bubbles[3]. Therefore, the paper introduces ApEn as an alternative statistic to analyze speculative bubble in the data.

[3] For example, time-varying risk premiums (e.g. Fama and French, 1988), and non-synchronous trading (e.g. Lo and MacKinlay, 1990a) could also induce autocorrelation while skewness could result from asymmetric fundamental news. Further, the leptokurtosis in stock return could be a consequence of arrival of information in batches (e.g. Tauchen and Pitts, 1983).

Table 1: Summary Statistics of weekly log returns on national stock indexes.

Returns	SENSEX (1998-2007)	S&P 500 (1981-1990)	Hang Seng (1991-2000)	Nikkei (1989-1998)
T (No of weekly Return)	522	521	521	519
Mean	0.0014	0.0007	0.0013	-0.0005
Standard Deviation	0.0152	0.0096	0.0166	0.0127
Skewness	-0.3736	-0.5761	-0.5363	0.0912
(SE) derived as $(6/T)^{1/2}$	0.1072	0.1073	0.1073	0.1075
Excess-Kurtosis	4.7468	6.418	5.7643	4.3522
(SE) derived as $(24/T)^{1/2}$	0.2144	0.2146	0.2146	0.2150
autocorrelation				
ρ1	0.036	-0.006	0.039	-0.040
ρ2	0.017	0.060	0.081	0.037
ρ3	0.039	-0.043	-0.037	0.021
ρ4	-0.041	0.031	-0.001	0.025
Ljung –Box Q5	16.081	10.444	10.671	11.221
(p-value)	0.097	0.402	0.384	0.341
Jarque -Bera	78.5048	282.4408	190.8569	40.2625
(p-value)	0.0000	0.0000	0.0000	0.0000

Table 2: Summary Statistics of weekly log returns on national stock indexes

Returns	KLSE (1994-2003)	STRAITS (1991-2000)	KOSPI (1991-2000)	SET (1991-2000)	TWSE (1991-2000)
T (No of weekly Return)	518	521	522	522	522
Mean	-0.0004	0.0004	-0.0003	-0.0007	0.0000
Standard Deviation	0.0166	0.0137	0.0195	0.0192	0.0172
Skewness	0.4053	-0.4476	-0.0964	0.4283	-0.5432
(SE) derived as $(6/T)^{1/2}$	0.1073	0.1073	0.1072	0.1072	0.1072
Excess-Kurtosis	10.143	14.8596	4.6574	4.6702	5.9657
(SE) derived as $(24/T)^{1/2}$	0.2152	0.2146	0.2144	0.2144	0.2144
autocorrelation					
ρ1	0.036	0.064	-0.056	0.075	-0.050
ρ2	0.085	0.030	0.052	0.141	0.090
ρ3	-0.010	0.198	0.087	0.042	0.091
ρ4	0.111	-0.196	0.048	-0.014	0.010
ρ10	-0.091	0.017	0.058	-0.017	-0.029
Ljung –Box Q5	25.55	31.073	13.877	19.426	12.064
(p-value)	0.004	0.001	0.179	0.035	0.281
Jarque -Bera	1115.434	3070.657	60.5582	76.632	216.9743
(p-value)	0.0000	0.0000	0.0000	0.0000	0.0000

Next, the paper analyses trends in ApEn during the crash period using a 12 weeks discrete observation window. The trends observed in table 3 shows a significant drop in

ApEn (2, 20%) during the crash period confirming the “tell-tale” sign of a crash. A low ApEn during the crash period indicates a high likelihood of repetitive patterns in return both during, pre and post crash zone. Further, the paper looks for more discriminating evidence to establish the incidence of bubble during the crash periods.

Table 3: Trends in ApEn (2, 20%) during the crash events.

US Crash 1987	
11th May -27th July 1997	0.4682
3rd Aug - 19th Oct 1987*	0.3871
2nd Nov - 11th Jan 1988	0.7455
Japan Crash Dec 1989	
4th Sep 89- 6th Nov 89	0.6762
13th Nov - 29th Jan 90*	0.3296
5th Feb -23rd April 90	0.427
India Crash 2007-08	
27th Aug - 12th Nov 2007	0.5544
19th Nov - 4th Feb 2007*	0.3871
11th Feb - 28th April 2008*	0.2197
5th May - 30 June 2008	0.3296

Note: * indicates the week containing the crash

Crash Event Evidence:

In order to examine the trends in approximate entropy an observation window of seven years is considered around the known crash period. The results presented in table 3 are based on the normalized ApEn (2, 20%) where the normalization is done with respect to the lowest value of ApEn during the sample period. Three years of post and pre crash period along with the crash is reported in table 4. There is a significant evidence of a decline on ApEn during and pre crash period. All the major crashes considered in our sample shows a very similar pattern in ApEn and figure 1 visually corroborates this trend for all the cases. However, it is important to note that for S&P, ApEn based analysis also points out an episode of speculative activities during 1990.

Table 4: Trends in normalized ApEn (2, 20%) during the crash

	S &P (1987)	NIKKEI (1989)	SENSEX (2007)
T-3	1.066893512	1.130691607	1.101134601
T-2	1.047018792	1.080072196	1.142437352
T-1	1.065810842	1.068258266	1.093951514
CRASH	1.002706674	1	1
T+1	1.084834893	1.156862745	
T+2	1.031706751	1.077693002	
T+3	1	1.251210107	

A similar analysis is carried out for Asian crisis of 1997 and the results are reported in Table 5. Seven major Asian indices are considered for this analysis which includes Hong Kong, Malaysia, Singapore, Korea, Taiwan, Indonesia and Japan. The normalized trend in ApEn reported in Table 5 (figure 2), however does not show a consistent pattern during the Asian crisis. The countries like Indonesia, Singapore and Taiwan experience a significant drop in ApEn during 1997 crash, while, Hong Kong, Japan and Malaysia have experienced a major dip during 1994 or 1995. These results are consistent with the earlier findings that Asian stock returns do not conform to the predictions of the rational speculative bubbles model (Chan et al 1998).

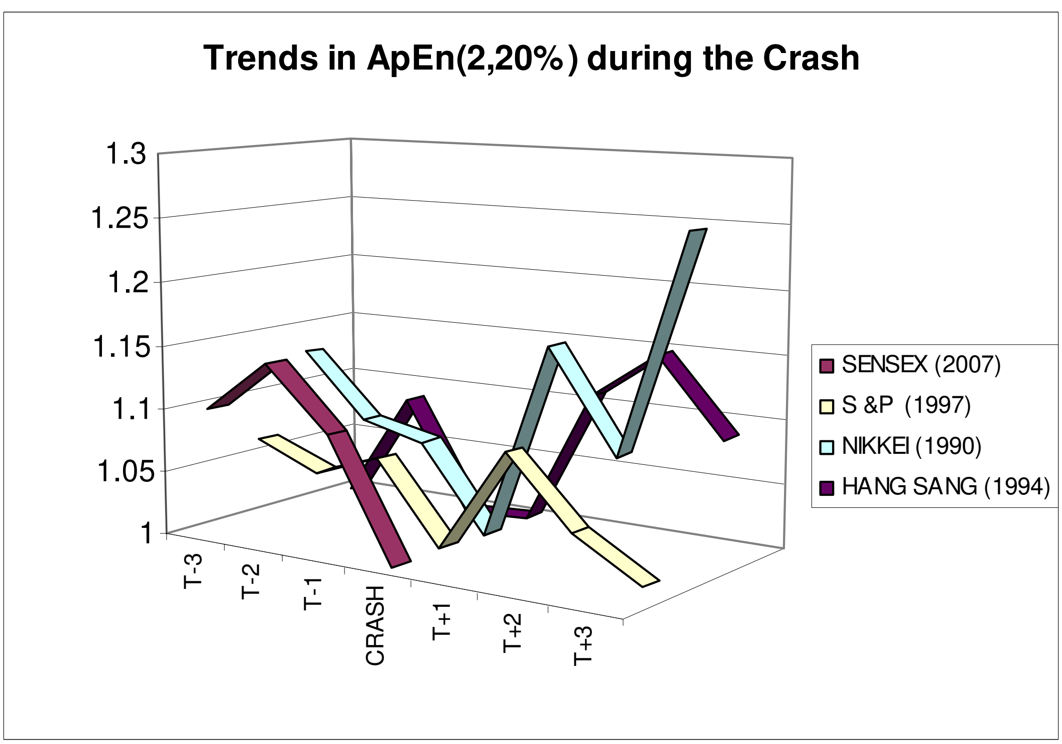

Fig 1: Trends in normalized ApEn (2, 20%) during the crash.

Table 5: Trends in normalized ApEn (2, 20%) during the crash

	Hang Seng	KLSE	SET	KOSPI	TWSE	NIKKEI	STRAITS
T-3	1.0000	1.1009	1.0596	1.1187	1.0442	1.0000	1.0192
T-2	1.1053	1.0000	1.1084	1.0637	1.1187	1.0526	1.0000
T-1	1.1439	1.1059	1.1820	1.1804	1.0376	1.1683	1.0188
CRASH (1997)	1.0845	1.1813	1.0804	1.2047	1.0000	1.0342	1.0098
T+1	1.1750	1.0978	1.0000	1.0000	1.0416	1.1151	1.1061
T+2	1.0207	1.1395	1.1457	1.2379	1.1446	1.1626	1.0190
T+3	1.1001	1.1192	1.0496	1.0954	1.1527	1.1198	1.0656

A series of robustness tests have been carried out using various duration window (m) and tolerance level (r) to check the stability of our conclusion. The trends in ApEn with different values of m that are presented in table 6A and B[4] corroborates our conclusions.

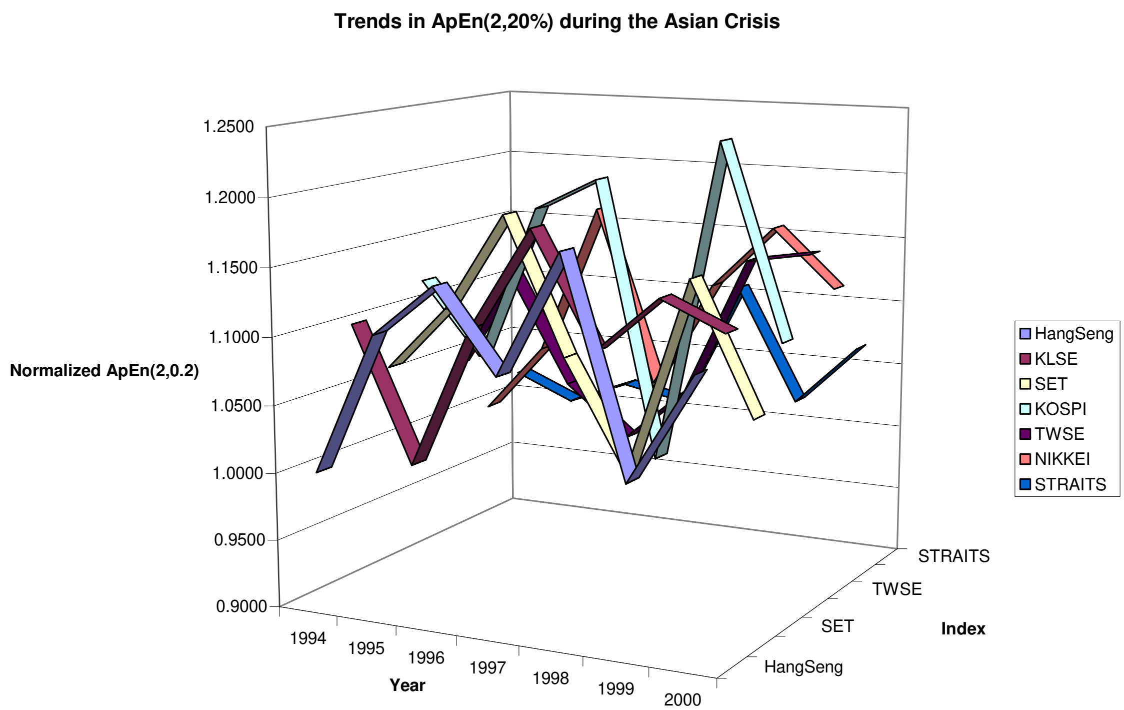

Fig 2: Trends in normalized ApEn (2, 20%) during the 1997 Asian crash

A Case Study of a developing Market, India:

As mentioned earlier the study of speculative bubbles has gained renewed importance in the context of developing economies. Despite significant developments, which have put these markets almost at par with the best in the world (in terms of their structure, systems and regulation), the markets still have witnessed several bouts of speculative activities in recent times and hence provides an unique opportunity to test the power of ApEn in identifying speculative bubble episodes from other anomalies driven by forces such as scams or changes in political scenarios.

[4] A similar exercise is carried out for different values of r. Since our findings are consistent across various values of r and m we only report one set of results.

Table 6A: Trends in normalized ApEn (2, 20%) during the crash

Year	SENSEX				S&P500				HANGSENG				NIKKEI			
	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8
1	1.36	1.35	1.28	1.30	1.20	1.19	1.17	1.16	1.24	1.22	1.23	1.21	1.41	1.40	1.43	1.39
2	1.26	1.22	1.23	1.27	1.32	1.32	1.30	1.26	1.35	1.34	1.33	1.28	1.31	1.31	1.32	1.27
3	1.47	1.41	1.43	1.42	1.32	1.31	1.28	1.25	1.24	1.19	1.15	1.11	1.53	1.50	1.45	1.36
4	1.33	1.32	1.28	1.23	1.38	1.35	1.32	1.25	1.24	1.24	1.26	1.28	1.23	1.20	1.21	1.26
5	1.35	1.33	1.36	1.33	1.35	1.34	1.29	1.26	1.37	1.32	1.30	1.25	1.30	1.32	1.25	1.24
6	1.45	1.41	1.38	1.33	1.38	1.38	1.34	1.29	1.42	1.36	1.30	1.31	1.44	1.36	1.34	1.29
7	1.35	1.34	1.28	1.30	1.30	1.27	1.26	1.26	1.35	1.29	1.27	1.22	1.27	1.27	1.27	1.22
8	1.40	1.32	1.28	1.25	1.40	1.42	1.42	1.35	1.46	1.46	1.43	1.46	1.37	1.35	1.32	1.31
9	1.34	1.29	1.29	1.22	1.33	1.32	1.31	1.29	1.27	1.24	1.22	1.16	1.43	1.42	1.41	1.40
10	1.23	1.23	1.24	1.25	1.29	1.28	1.21	1.19	1.37	1.36	1.35	1.31	1.38	1.31	1.27	1.28

Note: **SENSEX (1998-2007), S&P500 (1981-1990), HANGSENG (1991-2000), NIKKEI (1991-2000)

Table 6B: Trends in normalized ApEn (2, 20%) during the crash

Year	KLSE				STRAITS				KOSPI				SET				TWSE			
	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8	m=2	m=4	m=6	m=8
1	1.35	1.34	1.31	1.25	1.36	1.35	1.36	1.30	1.30	1.28	1.22	1.23	1.30	1.29	1.29	1.24	1.40	1.37	1.35	1.31
2	1.23	1.21	1.16	1.17	1.38	1.37	1.34	1.36	1.19	1.18	1.13	1.11	1.31	1.30	1.23	1.18	1.49	1.45	1.43	1.39
3	1.36	1.31	1.32	1.28	1.29	1.28	1.28	1.32	1.45	1.42	1.41	1.40	1.33	1.32	1.32	1.31	1.25	1.21	1.26	1.25
4	1.45	1.41	1.38	1.36	1.35	1.29	1.25	1.19	1.32	1.29	1.30	1.28	1.34	1.36	1.29	1.30	1.34	1.33	1.30	1.29
5	1.35	1.38	1.33	1.31	1.32	1.28	1.29	1.24	1.26	1.21	1.25	1.24	1.40	1.33	1.35	1.33	1.43	1.39	1.43	1.37
6	1.40	1.34	1.32	1.28	1.35	1.29	1.27	1.23	1.40	1.40	1.36	1.37	1.49	1.47	1.47	1.45	1.33	1.34	1.24	1.22
7	1.37	1.34	1.36	1.33	1.33	1.33	1.33	1.33	1.43	1.44	1.43	1.40	1.36	1.31	1.27	1.24	1.28	1.25	1.19	1.17
8	1.36	1.36	1.32	1.25	1.46	1.44	1.44	1.38	1.18	1.15	1.13	1.16	1.26	1.27	1.27	1.27	1.33	1.28	1.30	1.32
9	1.27	1.32	1.26	1.24	1.35	1.29	1.24	1.20	1.47	1.44	1.38	1.32	1.44	1.42	1.39	1.32	1.46	1.43	1.39	1.37
10	1.36	1.35	1.38	1.37	1.41	1.36	1.34	1.32	1.30	1.23	1.24	1.21	1.32	1.30	1.33	1.32	1.48	1.46	1.41	1.36

Note: **KLSE (1994-2003), STRAITS (1991-2000), KOSPI (1991-2000), SET (1991-2000), TWSE (1991-2000)

There are two episodes of crash in the Indian market that attract special attentions: The first episode considered by the common press as a potential bubble occurred between late 2007 and early 2008 during which several bouts of extreme movements were witnessed. The first incident during this period occurred on October 17, 2007, when the Sensex plunged by 1,743 points, the largest fall in a single day in our sample period. The Sensex hit a low of 17,307.90 points within minutes of opening and trading was suspended in the market for an hour. The markets had crashed on the wake of Securities and Exchange Board of India's (SEBI) proposal to tighten the rules for purchase of shares and bonds in Indian companies through the participatory note (PN) route. However, the Sensex recovered sharply from the day's low (17,308 points) and touched an intra-day high of 18,841 points - up 1,533 points (8.9%) from the day's low. The Sensex finally ended with a loss of 336 points (1.8%) at 18,716.Further, on Jan 21, 2008 Sensex saw the highest ever loss of 1,408 points at the end of the session, the biggest ever loss in the absolute term and also the first ever four digit loss for the index at close. The Sensex recovered to close at 17,605 points. On the following day, the Sensex saw its biggest intra-day fall when it hit a low of 15,332, down 2,273 points. Trading was suspended for one hour at the Bombay Stock Exchange after the benchmark Sensex crashed to a low of 15,576.30 within minutes of opening, crossing the circuit limit of 10 per cent.

The second episode of interest occurred on May 17, 2004 when the Sensex dropped by 565 points, its third biggest fall ever, to close at 4,505. The Sensex witnessed its second-biggest intra-day fall of 842 points, twice attracting suspension of trading due to change in political scenario as coalition government with the help of communist parties came to power.

These two episodes of market crashes provide a unique opportunity to test the power of ApEn as the underlying factors influencing these market crashes are very different and ApEn should be able to discriminate between these two episodes. As we have hypothesized in this paper, we should expect substantially lower level of ApEn during the 2007-08 crash as these events are often identified by the common press as bubble. In contrast, 2004 crash is purely driven by political events and might not have any speculative component to it.

Table 7 reports the normalized trend in ApEn (2, 20%) during these two episodes of crash and as expected ApEn during 2007 is substantially lower than the 2004 value, and it is also lowest in the decade providing a strong "tale-tell" sign of speculative bubble during 2007-2008.

Table 7: Trends in normalized ApEn (2, 20%) for Sensex

1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
1.113	1.024	1.196	1.085	1.102	1.184	1.101	1.142	1.093	1

4. CONCLUSION

The paper brings out certain regularities characterized by low ApEn level during many of the major crash events in the several markets including both developed and emerging economies. In addition, the paper also investigates the 1997 Asian crisis using weekly data for seven major Asian indices which includes Hong Kong, Malaysia, Singapore, Korea, Taiwan, Indonesia and Japan. Further, using a case study of SENSEX we also demonstrate the ability of ApEn to differentiate the speculative bubble from other market crashes.

Though this study primarily focuses on crash events, it can be extended to provide an early warning system that would enable investors and agents to be aware of the realities of price movements in the stock market and prevent them from attributing all increasing stock prices to the fundamentals. An early detection of an evolving bubble characterized by a steep fall in ApEn within a very short interval would immensely help both investors and policy makers as they can intervene and correct the market anomalies instantaneously, thereby preventing an otherwise inevitable market crash.

Finally, it is important to note that ApEn has certain weaknesses due to its dependence on sequence length (m) and its poor self consistency. Though the paper has addressed many of these shortcomings through specific robustness tests, further research using other alternative statistical measures in the spirit of ApEn (e.g., Sample Entropy) would provide a more robust understanding of the rational speculative bubbles.

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