Securitization and moral hazard: Does security price matter? (New version)

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19 May 2011

Online at https://mpra.ub.uni-muenchen.de/38016/
MPRA Paper No. 38016, posted 20 Apr 2012 13:54 UTC
Securitization and moral hazard: Does security price matter?

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Abstract: This article analyses the effect of security price on the behaviour of bank securitization. We present a model of bank securitization in which security price together with liquid constraints create the incentive for banks to originate and sell assets backed securities to investors. Banks have a comparative advantage in locating and screening projects within their locality. Our results show that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broken out in the US in 2007.

Keywords: securitization, security price, moral hazard

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1 Introduction

The financial crisis triggered by the US subprime mortgage sector in 2007 had an unprecedented negative impact on the real economy and on the banking sector. The use of securitization is considered as a central issue, and it has provoked a number of discussions among the academics and regulators. There is now substantial evidence which suggests that securitization, the act of converting illiquid loans into liquid securities, contributed to bad lending by reducing the incentives of lenders to carefully screen borrowers (Dell’Ariccia et al., 2008; Mian and Sufi, 2009; Purnanandam, 2009; Keys et al., 2009). Securitization weakened lenders’ incentives to screen borrowers, exacerbating the potential information asymmetries which lead to problems of moral hazard.

In economic theory, moral hazard is a situation in which a party insulated from risk behaves differently from how it would behave if it were fully exposed to the risk. Zandi (2009) described moral hazard as a root cause of the subprime mortgage crisis. He wrote: "...the risks inherent in mortgage lending became so widely dispersed that no one was forced to worry about the quality of any single loan.” Informational asymmetry in the securitization market is blamed for the origin of moral hazard of the bank (Ho and Sung, 2012). Several research has placed the emphasis on informational asymmetries in securitization markets, which they consider the main cause of the moral hazard of banks, such as lazy monitoring or screening (Dell’Ariccia et al., 2008; Berndt and Gupta, 2009; Purnanandam, 2009; Mian and Sufi, 2009; Keys et al., 2010).

However, one may ask whether informational asymmetry is the only root of evil in the disastrous impact of securitization. As we know from a large literature in information economics, private information can inhibit trade (Akerlof, 1970), lead to a frozen market. But the situation of subprime lending seems different. In many respects, the subprime market experienced a classic lending boom-bust scenario with rapid market growth, loosening underwriting standards, deteriorating loan performance, and decreasing risk premiums. Demyanyk and Hemert(2008)find that the quality of loans deteriorated for six consecutive years before the crisis and that securitizers were, to some extent, aware of it. They provide evidence that the rise and fall of the subprime mortgage market follows a classic lending boom-bust scenario, in which unsustainable growth leads to the collapse of the market. And we can conclude that informational asymmetry is not the only root that leads to moral hazard. In this article, we measure the level of moral hazard as the number that banks invest on low profitable projects, securitize them and sell them at the secondary markets. We attempt to show a price motivation of moral hazard for securitization in banking sector. We try to link these increases in subprime securitization with the increases in security price. Our results show that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broken out in the US in 2007.

Our paper is the first one in the literature to model the relationship between asset price and moral hazard in the processing of securitization. Our results also have important policy implications for regulators. A higher asset price may lead to higher level of moral hazard and asset price bubble is more harmful than we regarded it as before. In this situation, burst the price bubble before the financial crisis is necessary and important to a healthy development of an economy.
The rest of this article proceeds as follows. Section 2 presents the related literature and section 3 presents the basic setup of the model. Then we analyse the equilibriums under two different pricing mechanisms in part four. To draw comparisons, we discuss some evidence of our analysis in the subprime crisis in part five. The section six gives our conclusion.

2 Related Literature

Our study is mainly related to the literature on the motivation of credit risk transfer. One main motivation for securitization is banks’ perspective on risk management, according to which banks use securitization to transfer or diversify credit risks. Duffee and Zhou (2001) show that a bank can use such swaps to temporarily transfer credit risks of their loans to others, reducing the likelihood that defaulting loans trigger the bank’s financial distress. However, they find that the introduction of a credit-derivatives market is not necessarily desirable because it can cause other markets for loan risk-sharing to break down. Different from their research, Allen and Carletti (2006) show that credit risk transfer can be beneficial when banks face uniform demand for liquidity. Wagner and Marsh (2006)’s analysis suggests that the incentive of banks to transfer credit risk is aligned with the regulatory objective of improving stability. Moreover, they find the transfer of credit risk from banks to non-banks to be more beneficial than credit risk transfer within the banking sector. Another argument is that of the regulatory arbitrage associated with capital requirements. Given that capital is more costly than debt, the retention of a proportion of capital for loans in a balance sheet creates additional cost for banks. By taking this loan off their balance sheet, they can save their capital. Carlstrom and Samolyk (1995) shows that a loan sales market allows a banker having adequate capital to acquire profitable projects originated by a banker whose own capital is insufficient to support the additional risk. Calomiris and Mason (2004) show that the avoidance of capital requirements could be motivated either by efficient contracting or by safety net abuse. They find that securitizing banks set their capital relative to managed assets according to market perceptions of their risk, and seem not to be motivated by maximizing implicit subsidies relating to the government safety net when managing risk. Our paper differs from these as we focus on the moral hazard problem in the securitization processing. Unlike all three of these papers, our model is geared to key features of investors whose behavior are influenced by market sentiment or investor sentiment.

Investor sentiment can come from a variety of sources, such as shifts in psychology, regulatory rules, or demand for a particular asset class that is otherwise unrelated to fundamental payoffs (Kaplan and Stein, 1993; Caballero and Krishnamurthy, 2008; Gorton and Metrick, 2009; Shleifer and Vishny, 2010). Following their idea, we consider the two pricing situations influenced by the investor sentiment which makes our model more intuitive.

Since the paper explores moral hazard under the possibility of overinvestment, it is related to the research on moral hazard. When lending banks sell their loans they no longer bear the full costs of default and so may choose to screen borrowers less than the efficient amount. Such a moral hazard problem could arise if buyers were naive about lender screening incentives or if the benefits of securitization were perceived to be so large that it remained preferable to buy loans despite moral hazard. There is rich research on moral hazard and bank supervision: e.g. Merton (1977), Freixas et al. (2004) and Niinimaki(2009). The paper differs from these articles because it investigates how different level of security price affects moral hazard. The research on moral hazard problem in the process of securitization is vast, however, there is few research that explicitly analyse the link between security price and moral hazard in bank securitization behaviour. Our research concentrates on the effect of price have on the behaviour of banks’ choice of securitised loan. We introduce a two period model to show that
under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious.

3 The basic setting and the benchmark model

3.1 The model setting

The basic set up of the model builds on the work of Bernanke and Gertler (1987), Samolyk (1989), Carlstrom and Samolyk (1995). There is an economy made up of individuals who can be described as bankers, depositors and investors. The economy lasts two periods. There are several segmented markets and there is one banker per region. In each market, there is an unlimited supply of safe projects available to all individuals and each bank also has \( n_i \) local risky investment opportunities. The subscript “\( i \)” denotes different segmented markets. \( n_i \) is different across markets so the banks face different invest opportunities. The safe projects yield a risk free rate of return of \( R^f \) in the next period. The risky project cost $1 in period 0 and yields two possible outcomes in period 1. If the project fails, it will yield \( \theta_L \); If the project success, it will yield \( \theta_H \). The probability that the project success is \( \pi_k \), \( k \in [1, n_i] \). Each local banker ranks the success probabilities of his local projects from high to low, \( \pi_{n_i} < \pi_{n_{i-1}} < \cdots < \pi_1 \). It is assumed that \( \theta_L < R^f < \theta_H \), so that the banker will be willing to hold both safe and risky projects.

In period 0, a banker \( j \) in market \( j \) takes his endowment of bank capital, \( w^b_j \), as given. We assume that \( w^b_j \) is identical across the markets. Without loan sales, bankers invest their endowments and attract local deposits to fund their on-balance-sheet portfolios. In period 0, the representative depositor in each market receives an endowment, \( w_d \), which is identical across markets.

In period 1, the bank takes his endowment of bank capital, \( w^b \), as given. We assume that \( w^b \) is identical across all banks. Without loan sales, bankers invest their endowments and attract local deposits to fund their on-balance-sheet portfolios.

Bankers are risk neutral and possess an information technology that enables them to screen the ex-ante quality and monitor the ex-post performance of certain risky projects. Depositors and other bankers cannot observe the quality of a given banker’s projects.

For the purpose of simplicity, we consider the securitization as loan sales. The banker can offer to sell a pool loan projects, \( L_p \), to other banks or investors. The bank is rational and it chooses the \( n \) best projects to hold on-balance-sheet. And securitise the next best project available. The securitised loan, \( L_p = \{n+1, \ldots, n+l_p\} \), can be sold for a per-project price \( P \). At the same time, the bank may purchase a pool of loans, \( L_{p,k} \), from other banks in market. Let \( L_{p,k} = \{n_{k}+1, \ldots, n_{k}+l_{p,k}\} \) denotes the securities the bank buy from other banks. In period 0, the bank maximizes expected period 1 profits of
\[
\Pi = \text{Max} \left\{ \sum_{i=1}^{n} [\pi_i \theta_{H} + (1-\pi_i) \theta_{L}] + R^f s + E(R(L_{p,k}))l_{p,k} - R^d w^d \right\}
\]

(1)

where \( E(R(L_{p,k})) = \frac{1}{l_{p,k}} \sum_{i=1}^{l_{p,k}} E(\pi_{n_{i-1}})(\theta_{H} - \theta_{L}) + \theta_{L} \), \( n \) is the number of risky projects originated and funded on-balance-sheet. And \( s \) is the bank's investment in safe projects. Under a reasonable price, the bank can attract all the depositors’ fund in its region. The bank maximizes Eq. (1) subject to the portfolio-balance constraint,

\[
w^d + w^b + (P(l_p) - 1)l_p = n + s + P(l_{p,k})l_{p,k}
\]

(2)

The left side of Eq. (2) indicates that any proceeds from loan sales net of origination costs, \( (P(l_p) - 1) \), augment bank capital and deposits as a source of funds for on-balance-sheet activities. And the banks face a regulatory constraint in the period,

\[
\theta_L(n + l_{p,k}) + R^f s \geq R^d w^d
\]

(3)

In Eq.(3) means that even if all the projects the bank invests and buys from other banks realise a low return, the bank can still keep solvency. It’s a very rigorous regulation. Because depositors cannot observe the ex post returns on a bank's risky investments, the bank must offer a return on deposits that is not contingent on the return on bank projects. Deposit contracts must also offer a return that is greater than or equal to the opportunity cost of funds. These two considerations imply that

\[
R^d \geq R^f
\]

(4)

There are infinite quantity investors in the markets whose risk taking is influenced by investor sentiment. Sentiment can reflect either biased expectations or institutional preferences and constraints, such as the demand by foreigners or money market funds for securitised loan. For expositional simplicity, it is assumed that whenever an agent is indifferent between a nonzero asset position and not trading at all, the former is chosen for the reason to keep the market share or build a strong client relationship.

### 3.2 The benchmark model: Without securitization

When loan sales and purchases are prohibited, \( l_{p,k} = 0 \) and \( l_p = 0 \). So in the beginning period, the banks contract with depositors and choose \( n \) and \( s \) to maximize Eq.(1), subject to constraints (2), (3) and (4). And we can get our Lemma 1.

**Lemma 1:** When loan sales and purchases are prohibited, the banks maximize utility when:

1. \( \pi_n (\theta_H - \theta_L) + \theta_L = R^d \) if the bank is not constraint and \( \lambda_1 = 0 \)
2. \( \pi_n (\theta_H - \theta_L) + \theta_L = R^d + \lambda_1 (R^d - \theta_L) \) if the bank is constraint and \( \lambda_1 > 0 \)

\( R^d = R^f \)
where the Lagrange multiplier $\lambda_1$ is positive when (3) is binding for the profit-maximizing level of risky investments.

Proof: Please refer to the appendix.

Lemma 1 shows that the profit-maximizing choice of $n$ is equivalent to the choice of a cutoff success probability for investment in local risky projects. When the regulatory constraint is not binding ($\lambda_1 = 0$), the banker will invest on the risky projects until the expected return from the risky projects equals the return of the safe projects. While if the regulatory constraint is binding ($\lambda_1 > 0$), the banker will invest at the cutoff that the expected return from the risky projects equals the return of the safe projects and the constraint value.

From $\pi_n(\theta_H - \theta_L) + \theta_L = R^d$, we can get $\pi_n = \frac{R^d - \theta_L}{\theta_H - \theta_L}$. Given the results in Lemma 1 and Eq.(2), we can calculate $n$ yields the associated constraint on the on-balance-sheet funding of risky projects,

$n \leq \frac{R^f w^b}{R^f - \theta_L} \equiv n_c$

$n_c$ is the maximum number of risky projects that the banker's capital can support. Facing a draw of project opportunities, the bank invests in the best local projects available subject to this constraint. When $n_c < n_f$ where $R_{n_f} = R^f$, the bank can’t fund all risky project opportunities in his locality which is not socially optimal. Similarly, if $n_c > n_f$, the bank have some fund left to invest on the safe projects. In next section, we will show that under low pricing mechanism the economy can achieve social optimal through securitization by the constraint bank and the moral hazard problem is not so serious while under high pricing mechanism the economy tend to be over investment through securitization and the moral hazard problem is serious.

4 Equilibria under two pricing mechanism

For the purpose of simplicity, we just consider two banks: constrained bank $i$ and unconstrained bank $j$. We consider two pricing scenarios between bank $i$ and bank $j$. Different liquidity constraints mean that beneficial exchange can occur via a loan sale by constrained bank $i$ to unconstrained bank $j$ as long as

$E(R(L_{p,k})) / R^f \geq P(L_{p,k}) \geq 1$

Here we assume $E(R(L_{p,k})) > R^f$. In either scenario, the problem is that of calculating the equilibrium exante expected utility of the banks for any given price, and then studying the bankers’ optimal asset choice.

We assume there is no pure arbitrage, i.e. the banker cannot buy and sell loans at the same time which means that the banker cannot identify and buy good risky projects while sell the rubbish on his balance sheet. In such sense, if $l_{p,k} > 0$, then $l_p = 0$. And if $l_{p,k} = 0$, then $l_p \geq 0$.

4.1 Price formation mechanism
Price of securitized loan is determined by the choice of the infinite quantity investors whose risk taking is influenced by investor sentiment. Investor sentiment can come from a variety of sources, such as shifts in psychology, regulatory rules, or demand for a particular asset class that is otherwise unrelated to fundamental payoffs (Shleifer and Vishny, 2010). For example, if some investors such as foreigners, insurance companies, or money market funds demand AAA-rated bonds for reasons beyond the fundamental economics of payoffs, and are willing to pay substantially more for such bonds than for almost equally safe bonds, we think of this as investor sentiment (Caballero and Krishnamurthy, 2009; Gorton and Metrick, 2009; Shleifer and Vishny, 2010). Such demand can be fueled by loose monetary policy or by evidence of a default history (Kaplan and Stein, 1993) or when the price appreciation of houses made mortgage defaults relatively rare (similar as the subprime crisis).

At the other end of the spectrum, bad fundamental news can cause investors to dump securities when they lose confidence in their valuation models (Caballero and Krishnamurthy, 2008). Except the sentiment, speculative trading plays a role in determining the asset price, Mei et al.(2009) find that trading caused by investors’ speculative motives can help explain a significant fraction of the price difference between the dual-class shares. But the pricing forming mechanism is very similar to the investor sentiment. We consider the two extreme pricing mechanisms influenced by the investor sentiment:

1. $P(L_{p,k}) = 1$ (Pessimistic investor sentiment)
2. $P(L_{p,k}) = E(R(L_{p,k}))/R^f$ (Optimistic investor sentiment)

In fact, the price can be reduced below 1 when the market broke down and can be increased above $E(R(L_{p,k}))/R^f$ which we call it as “price bubble”. For simplicity, we exclude these two situations.

4.2 Equilibrium under low price ($P(L_{p,k}) = 1$)

In this pricing mechanism, the purchase bank accrues all the profits from the transaction and we call it as buyer’s market. The loan purchaser obtains the maximum rents from the transaction. Facing the security price, the bank maximizes (1), subject to the constraints (2) and (3). By solving the problem, we can get the optimal condition for the unconstraint bank:

$$\pi_{m+k}(\theta_H - \theta_L) + \theta_L = R^d$$  \hspace{1cm} (5)

with $l_{p,k} > 0$ and $l_p = 0$. And the optimal condition for the constraint bank is $l_p > 0$ and $l_{p,k} = 0$ (Please refer to the appendix).

A constrained bank will sell loan while unconstrained banks purchase loans, they will continue to do so until profits are maximized. Constrained banks continue to fund $n_l$ loans on-balance-sheet and sells loan to unconstrained banks and investors. The unconstraint banks cannot verify the profitable remaining projects, based on his assessment, he expects to receive a pool of projects in which the marginal project included has an expected return equal to the risk-free rate. We can summary in Proposition 1.

**Proposition 1** When security price is low, the constrained bank continues to fund loans on-balance-sheet and sells projects to unconstrained bank and investors. The unconstrained bank will buy loans
from the constrained banks until regulation constraints bind. The two types of banks can share risk with each other and the moral hazard problem is not serious.

Proof: Please refer to the appendix.

This pricing scenario implies a relatively simple equilibrium allocation. Since loan sales only occur so that on average the efficient number of projects are invested in, the number of projects invested in by an unconstrained bank will in general equal $(n_c + l_{p,k})$. The economy can achieve social optimal,

$$n_c + l_{p,k} + n_c \approx N^*$$

on balance sheet projects of unconstraint banks on balance sheet projects of constraint banks

### 4.3 Equilibrium under high price ($P(L_{p,k}) = E(R(L_{p,k})) / R^f$)

In this section we assume the investors have optimistic investor sentiment and the investors’ expected return of the securitised loan is higher, i.e., $E(R(L_{p,k})) > R^f$. In this pricing scenario, the sell bank accrues all the profits from the transaction and we call it as seller’s market. Similarly as the last section, facing the security price, the bank maximizes (1), subject to the constraints (2) and (3). Solving the problem, we can proof that regardless of whether the regulation constraint binds or not, the optimal condition for the unconstraint bank is

$$\frac{\partial L}{\partial l_p} = R^f (E(R(L_{p,k})) / R^f - 1) > 0$$

which means the more loan sales the more profit the unconstraint bank can get.

**Proposition 2** When security price is high, both the constrained bank and the unconstrained bank continues to fund loans on-balance-sheet and sells projects to investors. The banks tend to security high risky projects that are not socially optimal. The two types of banks cannot share risk with each other and the moral hazard problem is serious.

Proof: Please refer to the appendix.

The assessment of the expected return on unfunded projects is inferred from observed portfolio behaviour. Therefore, if $P(L_{p,k}) = E(R(L_{p,k})) / R^f > 1$ , unconstrained bankers may have the incentive to mimic constrained bankers. This would involve funding some unprofitable local projects in order to appear to have received a good draw of investment opportunities.

Since loan sales occur for both types of banks, both the constraint banks and the unconstraint banks tend to over investment on low profit project and sell it at the secondary market. The economy cannot achieve social optimal,

$$n_c + l_p^u + n_c + l_p^u \equiv N_H^r > N^*$$

on balance sheet projects of unconstraint banks on balance sheet projects of constraint banks

### 4.4 Comparison

Without securitization, the constraint banks can only invest in $n_c$ project and with $(n_f - n_c)$ socially profitable projects uninvested. The number of invested projects including both the constraint banks’ and unconstraint banks’ is $N_c$ which is not socially optimal. Under the low pricing mechanism, the
number of invested projects in the economy is around socially optimal number $N_*$. The level of moral hazard is low. Under the high pricing mechanism, the number of invested projects in the economy is $N^H$ which is above the socially optimal number $N_*$. The banks in the economy tend to over invest on low profitable projects which is not socially optimal. The difference between $N^H$ and $N_*$, $(N^H - N_*)$ can be considered as the level of the moral hazard.

5 Evidence from the subprime crisis

While securitization has revolutionized fixed income markets and brought billions of dollars of revenues to the banks, for many investors, even some institutional investors, this process can be opaque and filled with problems of asymmetric information and moral hazard. Our main results concludes from the theory model is that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious.

The explosive growth of the private, non–government-sponsored enterprise (GSE) backed mortgage-backed securities (MBS) market lies at the heart of the 2007–2009 global financial crisis. This market both fueled and was fuelled by the expansion of subprime credit and the housing boom(He et al., 2011). Our results fit the subprime crisis well in some aspects.

The market for MBS may have tilted toward scenario “Optimistic investor sentiment” particularly for large issuers when this new and rapidly growing market boomed between 2004 and 2006. Figure 1 plots the median fraction of AAA tranches of MBS, sorted by issuing year and issuer size. The median fraction of financing in AAA tranches sold by large and small issuers is quite similar in 2000 (just above 96 percent for the median deal) but then trends downward for both groups of securities as the housing and MBS markets grow.

The gap increases over time, peaking at about 10 percentage points in 2006, the height of the boom. Moreover, the incentive toward favoritism ought to be stronger during market booming periods.

Notes: “Big issuer” means that the market share of the issuer falls into the top 10 percent of the market share distribution in that year; “Small issuer” refers to the rest of the sample.
Figure 1 Fraction of AAA rating securities

Notes: The price history starts from the month of origination until the security stops trading or April 2009 (whichever comes first).
Source: He et al. (2011)

Figure 2 Media Monthly prices of MBS

Nonetheless, the frantic pace of asset price increases could not be sustained indefinitely, and when prices slipped, the financial feedback loops worked in reverse. By the end of 2006, housing sales had weakened, prices were declining and mortgage payments fell behind which we can consider as “Pessimistic investor sentiment”. Mounting losses continued in 2008, bringing down Bear Stearns, Lehman Brothers, Merrill Lynch and AIG, and the icy breezes spread across Europe. The federal government took over AIG and renationalized Fannie Mae and Freddie Mac (dealers in over half of all secondary mortgages). Subprime securitization plummeted (Figure 3).

Figure 3 Securitization rates for home mortgages

Note: Conforming mortgages are those below a limit set by Fannie and Freddie; FHA/VA are mortgage programs administered by FHA/VA.
Source: Inside MBS and ABS.

6 Conclusions

This paper has shown that under the buyer’s market pricing mechanism the banks with different liquidity constraints can share the risk and the moral hazard problem is not serious; but under the seller’s market pricing mechanism the banks have the incentive to conduct strategic securitization and the moral hazard problem is serious. Our main idea has been supported by the subprime crisis broken out in the US in 2007.
Our paper complements and contributes to the growing literature on the anatomy of the housing crisis and the role of MBS markets in this crisis, as well as incentive problems of financial service industries generally. Our results also have important policy implications for regulators. A higher asset price may lead to higher level of moral hazard and asset price bubble is more harmful than we regarded it as before. In this situation, burst the price bubble before the financial crisis is necessary and important to a healthy development of an economy.

A limitation of the framework is the restriction of omitting the information asymmetry. For studying the interaction of risk-sharing and information-based trading like securities, it is too restrictive. However, there is large range of research in this area (Campbell and Kracaw, 1980; Fulghieri and Lukin, 2001; Inderst and Mueller, 2006; etc.). Another limitation of the work is the pricing mechanism. For some technology reasons, we cannot model the price as detail as Rahi (1996) which shows that the resulting equilibrium is fully revealing, with all private information being transmitted through prices. We use the investor sentiment instead this process.

References


Appendix

Proof of the Lemma 1

From Eq.(2), we can get \( w^d = n + s - w^b \). Substituting it into Eq.(1) and (2), we can construct the Lagrange function:

\[
L_1(n, s) = \sum_{i=1}^{n} [\pi_i \theta_H + (1-\pi_i) \theta_L] + R^f s - R^d (n + s - w^b) - \lambda_1 [R^d (n + s - w^b) - \theta_L n - R^f s]
\]

where \( \lambda_1 \) is the Lagrange Multiplier.

First order condition for \( n \) and \( s \):

\[
n : \frac{\partial L_1(n,s)}{\partial n} = \pi_n (\theta_H - \theta_L) + \theta_L - \lambda_1 (R^d - \theta_L) \leq 0 \quad \text{with} \quad \frac{\partial L_1(n,s)}{\partial n} = 0 \quad \text{if} \quad n > 0
\]

For \( n > 0 \), we can get \( \frac{\partial L_1(n,s)}{\partial n} = 0 \) which shows

\[
\pi_n (\theta_H - \theta_L) + \theta_L + \lambda_1 \theta_L = R^d (1 + \lambda_1) \quad \text{i.e.}
\]

\[
\pi_n (\theta_H - \theta_L) + \theta_L = R^d + \lambda_1 (R^d - \theta_L) \quad \tag{A_1}
\]

\[
s : \frac{\partial L_1(n,s)}{\partial s} = R^f - R^d - \lambda_1 (R^d - R^f) \leq 0 \quad \text{i.e.} \quad R^f (1 + \lambda_1) \leq R^d (1 + \lambda_1)
\]

For \( 1 + \lambda_1 > 0 \), we can get \( R^f \leq R^d \) which together with Eq.(4) \( R^d \geq R^f \) shows that

\[
R^f = R^d
\]

Proof of proposition 1

As \( P(l_p) = P(l_{p,k}) = 1 \), Eq.(2) can be simplified as:

\[
w^d + w^b = n + s + l_{p,k}
\]

We can construct the Lagrange function:

\[
L_2 = \sum_{i=1}^{n} [\pi_i \theta_H + (1-\pi_i) \theta_L] + R^f s + E(R(L_{p,k})) l_{p,k} - R^d (n + s + l_{p,k} - w^b)
- \lambda_2 [R^d (n + s + l_{p,k} - w^b) - R^f s - \theta_L (n + l_{p,k})]
\]

where \( \lambda_2 \) is the Lagrange Multiplier.
First order condition with respect to $n, s$ and $l_{p,k}$:
\[
 n, s: \frac{\partial L_n}{\partial n} = \pi_n(\theta_H - \theta_L) + \theta_L - R^d - \lambda_2(R^d - \theta_L) \leq 0 \text{ with } \frac{\partial L_n(n,s)}{\partial n} = 0 \text{ if } n > 0
\]

We can get similar results as in Lemma 1
\[
 \pi_n(\theta_H - \theta_L) + \theta_L + \lambda_2 \theta_L = R^d(1 + \lambda_2)
\]
The optimal $s$ also shares the same result as in lemma 1, i.e. the optimal choice of the safe projects is $R^f = R^d$.
\[
l_{p,k} : \frac{\partial L_{p,k}}{\partial l_{p,k}} = \pi_{n+k}(\theta_H - \theta_L) + \theta_L - R^d - \lambda_2(R^d - \theta_L) \leq 0 \text{ i.e.}
\]
\[
 \pi_{n+k}(\theta_H - \theta_L) + \theta_L(1 + \lambda_2) - R^d(1 + \lambda_2) \leq 0 \text{ with } \frac{\partial L_{p,k}}{\partial l_{p,k}} = 0 \text{ if } l_{p,k} > 0
\]

If the regulation constraint doesn’t bind, i.e. $\lambda_2 = 0$, the bank maximizes his utility when
\[
 \pi_{n+k}(\theta_H - \theta_L) + \theta_L - R^d = 0 \text{ and } l_{p,k} > 0.
\]
So the optimal condition for the unconstraint bank is
\[
 \pi_{n+k}(\theta_H - \theta_L) + \theta_L = R^d \quad (A2)
\]
with $l_{p,k} > 0$ and $l_p = 0$.

If the regulation constraint binds, i.e. $\lambda_2 > 0$, together with $\pi_{n+k}(\theta_H - \theta_L) + \theta_L - R^d = 0$ we can get
\[
 \pi_{n+k}(\theta_H - \theta_L) + \theta_L(1 + \lambda_2) - R^d(1 + \lambda_2) < 0 \text{ and } l_{p,k} = 0. \text{ So the optimal condition for the constraint bank is } l_{p,k} = 0 \text{ and } l_p \geq 0.
\]

**Proof of proposition 2**

As $P(l_p) = P(l_{p,k}) = E(R(L_{p,k}))/R^f$, Eq.(2) can be simplified:
\[
w^d + w^b + (E(R(L_{p,k}))/R^f-1)l_p = n + s + E(R(L_{p,k}))/R^f l_{p,k}
\]

We can construct the Lagrange function:
\[
L_3 = \sum_{i=1}^{n} \left[ \pi_i \theta_H + (1 - \pi_i) \theta_L \right] + R^f s + E(R(L_{p,k}))l_{p,k} - R^d \left[ n + s + E(R(L_{p,k}))/R^f l_{p,k} - w^b - (E(R(L_{p,k}))/R^f-1)l_p \right] - \lambda_3 \left[ R^d(n + s + E(R(L_{p,k}))/R^f l_{p,k} - w^b - (E(R(L_{p,k}))/R^f-1)l_p) - R^f s - \theta_L(n + l_{p,k}) \right]
\]
where $\lambda_3$ is the Lagrange Multiplier.
First order condition with respect to $l_p$ and $l_{p,k}$:

$$l_p: \frac{\partial L}{\partial l_p} = R^d (E(R(L_{p,k}))) / R^f - 1 + \lambda_3 R^d (E(R(L_{p,k}))) / R^f - 1 \leq 0 \text{ with } \frac{\partial L}{\partial l_p} = 0 \text{ if } l_p > 0$$

If the regulation constraint doesn’t bind, i.e. $\lambda_3 = 0$, the optimal condition for the unconstraint bank is

$$\frac{\partial L}{\partial l_p} = R^d (E(R(L_{p,k}))) / R^f - 1 > 0$$

which means the more loan sales the more profit the unconstraint bank can get.

If the regulation constraint binds, i.e. $\lambda_3 > 0$, the optimal condition for the unconstraint bank is

$$\frac{\partial L}{\partial l_p} = (\lambda_3 + 1) R^d (E(R(L_{p,k}))) / R^f - 1 > 0$$

which also means that the more loan sales the more profit the constraint bank can get.

$$l_{p,k}: \frac{\partial L}{\partial l_{p,k}} = \pi_{n+k} (\theta_H - \theta_L) + \theta_L - [\pi_{n+k} (\theta_H - \theta_L) + \theta_L] \frac{R^d}{R^f} - \lambda_3 [\pi_{n+k} (\theta_H - \theta_L) + \theta_L] \frac{R^d}{R^f} \leq 0$$

with $\frac{\partial L}{\partial l_{p,k}} = 0$ if $l_{p,k} > 0$

If the regulation constraint doesn’t bind, i.e. $\lambda_3 = 0$, $[\pi_{n+k} (\theta_H - \theta_L) + \theta_L] (1 - \frac{R^d}{R^f}) = 0$ and $l_{p,k} > 0$.

If the regulation constraint binds, i.e. $\lambda_3 > 0$, $[\pi_{n+k} (\theta_H - \theta_L) + \theta_L] (1 - \frac{R^d}{R^f} - \lambda_3) < 0$ and $l_{p,k} = 0$.

So under the high pricing mechanism, both the constraint banks and the unconstraint banks tend to over investment on low profit project and sell it at the secondary market.