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10 December 2008

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Paper presented to Complexity and Nonlinear Dynamics II at the Conference of Biomedical Applications of Micro- and Nano-engineering IV and Complex Systems at RMIT University, Melbourne, Australia, 10 December 2008.

Version 20

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Adaptive interactive profit expectations using small world networks and runtime weighted model averaging

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ABSTRACT

The aim of this paper is to simulate profit expectations as an emergent property using an agent based model. The paper builds upon adaptive expectations, interactive expectations and small world networks, combining them into a single adaptive interactive profit expectations model (AIE). Understanding the diffusion of interactive expectations is aided by using a network to simulate the flow of information between firms. The AIE model is tested against a profit expectations survey. The paper introduces "runtime weighted model averaging" and the "pressure to change profit expectations index" (p^x). Runtime weighted model averaging combines the Bayesian Information Criteria and Kolmogorov's Complexity to enhance the prediction performance of models with varying complexity but a fixed number of parameters. The p^x is a subjective measure representing decision making in the face of uncertainty. The paper benchmarks the AIE model against the rational expectations hypothesis, finding the firms may have adequate memory although the interactive component of AIE model needs improvement. Additionally the paper investigates the efficacy of a tuneable network and equilibrium averaging. The tuneable network produces widely spaced multiple equilibria and runtime weighted model averaging improves prediction but there are issues with calibration.

Keywords: Small World Networks, Agent Based Model, Adaptive, Interactive, Profits, Expectations, Model Averaging, Survey, Australia, Business Cycle.

1. INTRODUCTION

Profit expectations are important because they influence future investment and credit decisions as such they contribute to the business cycle and economic growth.

This paper builds upon Hicks's adaptive expectations[1], Flieth and Foster's interactive expectations[2], Watts and Strogatz's small world networks[3] and the findings of the 'Beer Distribution Game' to simulate the process of profit expectations formation using an agent based model. Adaptive expectations form when a firm changes its future expectations based upon the difference between actualisations and expectations for the current or previous periods. Interactive expectations form when a firm's expectations are affected by the expectations of other firms for the current or previous periods. Understanding the diffusion of interactive expectations is aided by using a network to simulate the flow of information between firms. The paper combines all three components into the adaptive interactive profit expectations (AIE) model.

This is an empirically based study using profit expectations and actualisation indices from the Dun and Bradstreet (D&B) National Business Expectations Survey[4]. These indices are based upon the change in profit expectations and actualisation rather than the level of expected or actual profit. This approach is consistent with Kahneman[5]'s empirically supported observation "the primacy of change over state" but at odds with utility curve theory.

The AIE model is benchmarked against Muth's rational expectations hypothesis[6,7].

Tesfation (2008) lists four objectives of agent–based computation economics: empirical understanding; normative understanding; qualitative insight and theory generation; and methodological advancement. How does this paper contribute to these objectives? The AIE model contributes to empirical understanding by generating a bottom up model of profit expectations. The methodological advancements include: (1) introducing a 'pressure to change profit expectations index', (2) introducing 'runtime weighted model averaging' a variant on Bates and Granger's model averaging[8] to handle models with varying complexity but a fixed number of parameters such as the AIE model, and (3) using calibration and prediction of benchmark models to evaluate the AIE model.

The structure of the paper is as follows. Sec. 2 discusses the methodology for the AIE model and runtime weighted model averaging. Sec. 3 presents the results: an evaluation of runtime weighted model averaging and visualisation of the network and model variance topologies. Sec. 4 discusses the results. Sec. 5 concludes the paper.

2. METHODOLOGY

The AIE model combines the adaptive expectations model[1] and the interactive expectations model[2] extended with small world networks[3] within an agent based model[9].

Supplementing the components above, the AIE model introduces two techniques: (1) a 'pressure to change profit expectations index' (p^x) to replace the probabilistic treatment in the interactive model[2], and (2) 'runtime weighted model averaging' to enhance prediction.

Each run of the AIE model has a unique set of parameters and a model variance. The model variance is the SSE/T between the all–firms profit expectations index of the AIE models and of the D&B profit expectations survey[4] in Fig. 1.

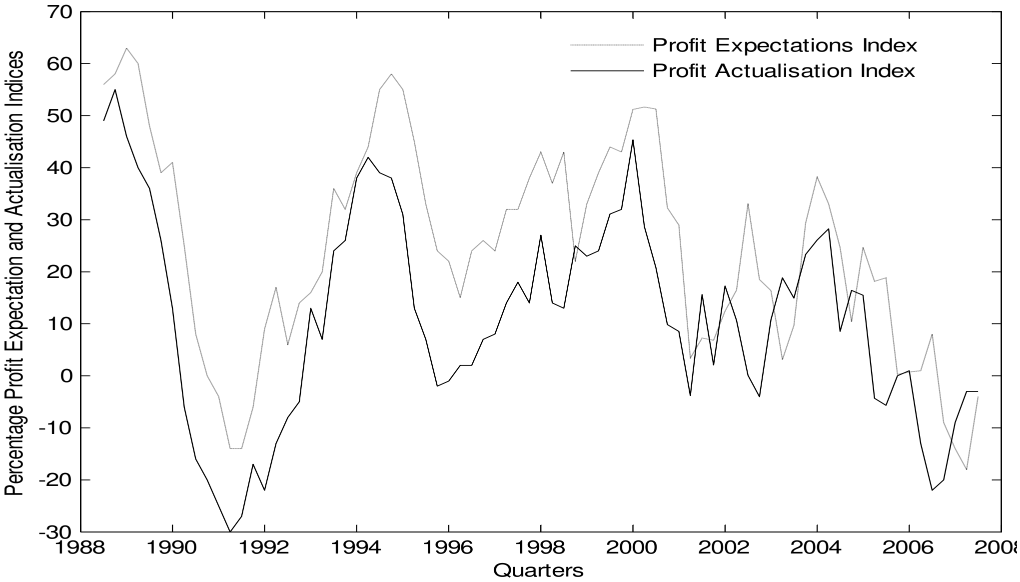


Fig. 1.The Dun and Bradstreet All–firms Profit Expectations and Actual Indices[4]

The model's multiple equilibria are located by finding the runs with low model variance or the local minima. An alternating gradient and limited broad sweep search method is used to find the multiple equilibria in the AIE model. These multiple equilibria are then used in 'runtime weighted model averaging' to enhance prediction.

The structure of this section is as follows. Sec. 2.1 discusses linking the macro level indices with the micro level firms' behaviour and initialising the AIE model. Sec. 2.2 discusses the calculation of p^x. Sec. 2.3 discusses searching for local minima or equilibria in the AIE model. Sec. 2.4 discusses runtime weighted model averaging.

2.1 Linking Macro Indices to Firms' Micro Behaviour

The AIE model starts with and uses macro level all–firms profit indices[4] to assign profit expectations and actualisation levels to individual firms. To do this the profit expectations index is decomposed into the percentage of firms expecting profits to increase, to undergo no change and to decrease, using Eq. (1).

$$\text{Profit Expectations Index} = \text{\% business expecting increases} - \text{\% business expecting decreases} \quad (1)$$

Additionally, the profit actualisation index is decomposed into the percentage of firms whose profits actually increase, undergo no change and decrease, using Eq. (2).

$$\text{Profit Actualisation Index} = \text{\% business with actual increases} - \text{\% business with actual decreases} \quad (2)$$

The decomposition requires the percentage of firms that expect 'no change' in profits from an Australian Bureau of Statistics[10] table, see Fig. 2. This 'no change' dataset is used to represent the D&B[4] 'no change' data for both the profit expectations and actual profits. This 'no change' data is the best that could be found. Each firm *i* at time *t* is assigned a level of expectations ($e_{i,t}$) of 1, 0 or –1 to represent whether they expect profits to increase, to undergo no change or decrease, using the percentage breakdowns. The actualisations ($a_{i,t}$) are assigned similarly. So far these assignments reflect the D&B indices[4]. The calibration period starts just after the phase change in Fig. 2. Sec. 3 discusses this further.

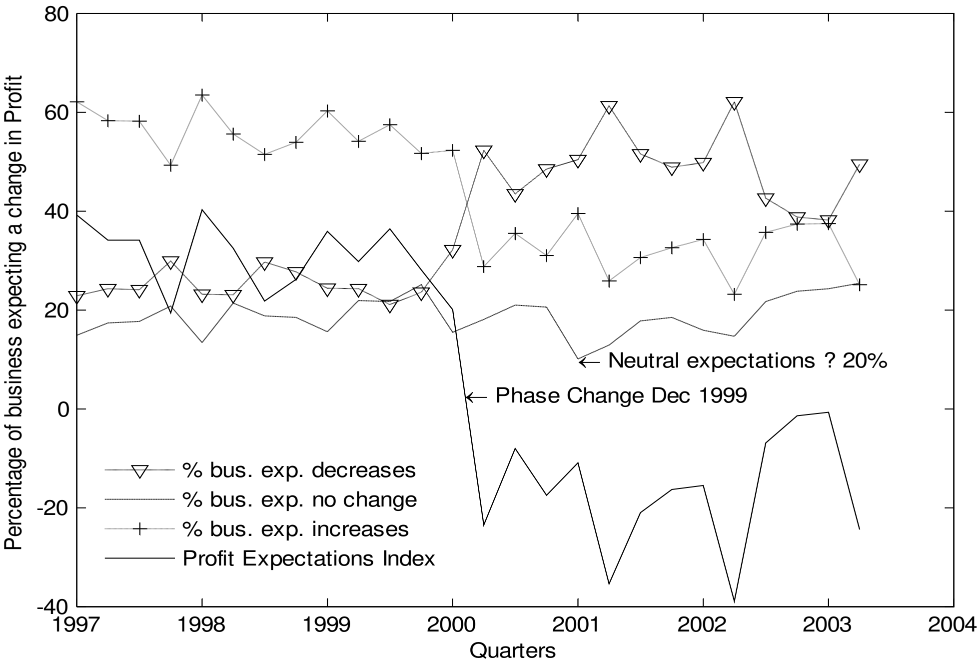


Fig. 2 Components of the Profit Expectations Index and Phase Transition[10]

The first two periods of a dataset from the D&B survey[4] are used to initialise the each firm's level of profit expectations and actual profits. Sec. 2.2 discusses how these firms change their expectations based upon the p^x for each successive period. Once the AIE model calculates the expectations of each firm for each period, the AIE model's expectations index is calculated using Eq. (1). A measure of the goodness of fit of the model run is the model variance between the all–firms profit expectations index of the AIE model's run and D&B[4]. The runs with the lowest model variance are local minima or equilibria. Sec. 2.3 discusses searching for the equilibria and Sec. 2.4 discusses runtime weighted model averaging.

2.2 The Pressure to Change Profit Expectations Index

The $p^x_{i,t}$ is calculated for each firm *i* each quarter *t*. Rather than using a probability to assign a change in expectations to an agent, which is common in the expectations literature[2,11]. This paper introduces the p^x as a subjective measure representing decision making in the face of uncertainty as opposed to a probability. Probability is more useful in representing a known risk. Each agent in the model is subjected to pressure to change their profit expectations. Eq. (6) shows how the maximum and minimum p^x is restricted to 100 and –100 respectively. In addition to the index's suitability to measure decision making under uncertainty, the index more easily handles double jumps in expectations. A double jump in expectations is when a respondent changes from expecting profits to decrease in one quarter to expecting profits to increase in the next quarter, or vice versa, bypassing the intervening 'no change' in expectations. This relaxes Flieth and Foster[2]'s simplifying assumption that no such double jumps would occur over a quarter. Eq. (3) shows the calculation of the p^x, Eq. (3a) for firms who currently expect profits to decrease, Eq. (3b) for firms who currently expect no change on profits and Eq. (3c) for firms who currently expect profits to increase.

These basic tendencies (β) are, as the name suggests, the tendency for a firm to feel pressure to change to another level of expectations. The basic tendency to increase (β^+), to decrease (β^-) and to neutral (β^0) could be interpreted respectively as optimism, pessimism, or neutral feelings that permeate the economy. Looking at Fig. 1, it appears that there are overly optimistic expectations, because profit expectations exceed profit actualisations most of the time, so one would predict that the basic tendency to increase is greater than the basic tendency to decrease. The AIE model does find this to be the case.

Eq. (3) – Pressure to change profit expectations index

For firm *i* who currently expects profits to decrease ($e_{i,t} = -1$)

The pressure to increase expectations

$$p^x_{i,t} = \beta^+ + \beta^0 + A\,[\,a_{i,t} - e_{i,t}\,] + A_{-1}\,[\,a_{i,t-1} - e_{i,t-1}\,] + I\,[\,(L_{i,t}^+ + L_{i,t}^0) / L\,]^\wedge\delta \qquad (3a)$$

For firm *i* who currently expects no change in profits ($e_{i,t} = 0$)

positive pressure to increase expectations and

negative pressure to decrease expectations

$$p^x_{i,t} = \beta^+ - \beta^- + A\,[\,a_{i,t} - e_{i,t}\,] + A_{-1}\,[\,a_{i,t-1} - e_{i,t-1}\,] + I\,[(\,L_{i,t}^+ / L\,)^\wedge\delta - (\,L_{i,t}^- / L\,)^\wedge\delta] \qquad (3b)$$

For firm *i* who currently expects profits to increase ($e_{i,t} = 1$)

The pressure to decrease expectations

$$p^x_{i,t} = \beta^- + \beta^0 + A\,[\,e_{i,t} - a_{i,t}\,] + A_{-1}\,[\,e_{i,t-1} - a_{i,t-1}\,] + I\,[\,(L_{i,t}^- + L_{i,t}^0) / L\,]^\wedge\delta \qquad (3c)$$

Where

$p^x_{i,t}$ = pressure to change profit expectations index for firm *i* at time *t*

$p^x_{i,t} \in [-100, 100\,]$

β^+ = basic tendency to increase expectations

β^0 = basic tendency to neutral expectations

β^- = basic tendency to decrease expectations

A = adaptive influence this quarter

A_{-1} = adaptive influence last quarter

$a_{i,t}$ = profit actualisation of firm *i* at time *t*

where a decrease, no change or increase is –1, 0 or 1 respectively

$e_{i,t}$ = profit expectations of firm *i* at time *t*

where a decrease, no change or increase is –1, 0 or 1 respectively

I = interactive influence

L = total number of links to a node or firm (2, 4, 6, …, 22)

L^+ = the number of linked firms who expect profits to increase (e = 1)

L^0 = the number of linked firms who expect no change in profits (e = 0)

L^- = the number of linked firms who expect profits to decrease (e = –1)

δ = interactive power (1.0, 1.2, 1.4, …, 3.0)

The interactive influence (I) in Eq. (3) indicates the influence of other firms holding a certain level of profit expectations on the firm. This is adapted from Ref. 2, see Eq. (4).

Eq. (4) shows the Interactive Influence from Ref. 2 to compare with the AIE interactive component in Eq. (3)

For firms who currently expect profits to decrease – the interactive pressure to increase expectations

$$I\,[\,(N^+ + N^0) / N\,]^\wedge 2 \qquad (4)$$

Where

I = interactive influence

N = total number of firms

N^+ = the total number of firms who expect profits to increase

N^0 = the total number of firms who expect no change in profits

δ = interactive power = 2

Eq. (3) differs from Eq. (4) in that it connects the firm via a network rather than assuming total connectivity. Sec. 2.3 discusses the AIE network topology (L and ρ) and parameter ranges. Note to ease comparison between Eq. (3) and Eq. (4) that the variable names in Eq. (4) have been made consistent.

Eq. (4) results in a probabilistic treatment of the whole population's expectations, whereas Eq. (3) considers each firm within a network of interactive influence.

These two differing approaches are appropriate to the situation being studied. Ref. 2 examines interactive expectations using an electoral opinion poll, whereas this paper examines interactive profit expectations among the manufacturing, wholesale and retail divisions. Ref. 2's approach more closely approximates a complete graph as individuals are exposed to regular national media coverage of political events, which includes regular surveys of the voting population. The AIE model's approach more closely resembles a network of interconnected supply chains as firms are linked to one another via orders in expectation of sales in a similar fashion to the 'beer distribution game'. Admittedly, the two approaches are not as black and white as portrayed, but more different shades of grey.

The AIE model borrows the network naming conventions and topology parameters from Ref. 3, the code from Ref. 12, and parameter increments from Ref. 11. This ensures that the design of the AIE model's network builds upon the existing literature. However the results in Sec. 3 show that an alternative network approach is required, a point taken up in the Sec. 4. The AIE model also relaxes the assumption in Ref. 2 that the interactive power in Eq. (4) is two by allowing the power to vary from 1 to 3 by 0.2 increments to test Ref. 2's assumption.

In addition to the network lattice, the AIE model differs from Ref. 2 in that it also incorporates an adaptive expectations influence (A) from Ref. 1. This allows a connection between profit actualisations and profit expectations, which Ref. 2's Interactive Expectations lacks. In Eq. (3a), the parameters A and A_{-1} act as weights in the p^x and the parameters ($a_{i,t} - e_{i,t}$) and ($a_{i,t-1} - e_{i,t-1}$) form a link between the profit actualisation and profit expectations. The AIE model uses the current and last quarter only, assuming a cognitive bias called the recency effect holds. Additionally, the model reflects the fact that a firm lacks full information about the actual profits for the current quarter until the following quarter, so a firm behaving adaptively would use the full information available from last quarter and the partial information available about the current quarter.

Eq. (5) – Determining the pressure level at which to change expectations

For firms who currently expect profits to decrease, (5a)

determining the pressure level to increase expectations

if random (p^+) < $p^x_{i,t}$ then $e_{i,t+1} = 0$

the firm increases expectations one level

if random ($p^{++} - p^+$) < ($p^x_{i,t} - p^+$) then $e_{i,t+1} = 1$

the firm increases expectations two levels

For firms who currently expect no change in profits (5b)

determining the pressure level to increase or decrease profit expectations

if $p^x_{i,t} > 0$ and if random(p^+) < abs($p^x_{i,t}$) then $e_{i,t+1} = 1$

the firm increases expectations one level

if $p^x_{i,t} < 0$ and if random(p^-) < abs($p^x_{i,t}$) then $e_{i,t+1} = -1$

the firm decreases expectations one level

For firms who currently expect profits to increase (5c)

The pressure to decrease expectations

if random (p^-) < $p^x_{i,t}$ then $e_{i,t+1} = 0$

the firm decreases expectations one level

if random ($p^{--} - p^-$) < ($p^x_{i,t} - p^-$) then $e_{i,t+1} = -1$

the firm decreases expectations two levels

Where

p^+ = the pressure level at which a firm increases profit expectations by 1 level

p^{++} = the pressure level at which a firm increases profit expectations by 2 levels

p^- = the pressure level at which a firm decreases profit expectations by 1 level

p^{--} = the pressure level at which a firm decreases profit expectations by 2 levels

ei,t+1 = profit expectations the firm holds next quarter

Eq. (5) shows how the p^x in conjunction with a random number generator and the '*pressure levels to change expectations*' (p^+, p^{++}, p^- and p^{--}) determines the level of expectations the firm holds for the next quarter ($e_{i,t+1}$).

The random function in Eq. (5) reports a random integer greater than or equal to 0, but strictly less than the pressure to change level. The random function uses a flat distribution (Ref. 9).

The profit expectations index for the next quarter is calculated from the number of firms holding positive and negative expectations for next quarter as per Eq. (1). These values are aged and the process is repeated for each quarter to form a single run. At the end of the run, the model variance between the all–firms profit expectations of D&B[4] and of the AIE model is calculated. What has been described is the process for a single run to find the model variance for a single set of parameter values. Sec. 2.3 discusses the process used to search the parameter space for local minima of model variance or equilibria.

Eq. (6) – Setting the maximum and minimum p^x to 100 and –100 respectively

$$100 = \beta^+ + \beta^0 + I + 2 * [A + A_{-1}] \quad (6)$$

Eq. (6) shows how the weights in the p^x are set to 100. The constraint allowed the elimination of one parameter from the parameter sweeping; the basic tendency neutral (n) was chosen for elimination. In Eq. (3), because the parameters $a_{i,t}$, $e_{i,t}$, $a_{i,t-1}$ and $e_{i,t-1}$ can all take the values 1, 0 or –1, this can result in doubling the weight of A or A_{-1} on the p^x. The factor of two in Eq. (6) reflects this. Additionally, the parameter β^- proved to be redundant and eliminated by setting it to zero.

2.3 Searching the parameter space for local minima or equilibria

This section discusses the search for minima or equilibria in the AIE model. The search for the lowest model variance between the profit expectations index of the AIE model and of the D&B[4] survey combines the gradient method with a limited broad sweep to prevent the gradient method becoming lodged on a local minimum and to reduce the risk of missing other local minima, which may be equally plausible solutions to a global minimum. These equally plausible equilibria become candidates for inclusion in runtime weighted model averaging discussed in Sec. 2.4. Additionally the limited broad sweep provides for visualisation, see Sec. 3.2.

Each run is defined by the eleven parameters: β^+, I, L, δ, A, A_{-1}, ρ, p^+, p^{++}, p^- and p^{--}. The gradient and limited broad sweeps method involves setting an initial value for the 11 parameters. The 11 parameter values to initialise the gradient method are based upon reason and assumptions. Each parameter value is allowed to vary plus or minus one increment: $\beta^+ \pm 1$, $I \pm 1$, $L \pm 2$, $\rho \pm 0.1$, $\delta \pm 0.2$, $A \pm 1$, $A_{-1} \pm 1$, $\rho \pm 1$, $p^+ \pm 1$, $p^{++} \pm 1$, $p^- \pm 1$ and $p^{--} \pm 1$. This gives 3^{11} parameter combinations or runs. The minimum parameter values are $L = 2$, $\delta = 1$ and $\beta^+ = \beta^0 = I = A = A_{-1} = \rho = p^+ = p^{++} = p^- = p^{--} = 0$. The condition in Eq. (6) determines β^0. The gradient method is repeated until a local minimum is found. The parameter values from the local minimum are used in a limited broad sweep. To make a limited broad sweep, the pressure levels to change expectations (p^+, p^{++}, p^- and p^{--}) are held constant. The ranges for other parameters are $\beta^+ \pm 5$, $I \pm 5$, L = (2, 4, 6, …, 22), δ = (1.0, 1.2, 1.4, …, 3.0), ρ = (0, 0.1, 0.2, …, 1), $A \pm 5$, $A_{-1} \pm 5$. This gives 11^6 parameter combinations or runs. The parameters from the run with the minimum model variance in the limited broad sweep are used to initialise the next gradient method search. The gradient method and limited broad sweep are repeated until a global minimum is found.

2.4 Runtime Weighted Model Averaging

$$BIC(k) = \log \sigma^2 + (k \log n) / n \quad (7)$$

Where

k = the number of parameters in the model

n = sample size

σ^2 = model variance

$$BIC^* = \log \sigma^2 + (ct \log n) / n \quad (8)$$

Where

* denotes a modification to representing complexity that is using *ct* to replace *k* in Eq. (7)

t = the time for the model to run

c = some constant to be determined by experiment

$$\log K \approx -(n/2)\, BIC \quad (9)$$

Where

$\approx$ denotes approximately

Modified Bayes Factor from Eq. (8) and Eq. (9).

$$K^* \approx \sigma^{-n} n^{-ct/2} \quad (10)$$

This section discusses the derivation of the runtime weighted model averaging and the reason why existing weighting methods are inappropriate for the AIE model.

Eq. (7) shows Greene[13]'s version of the Bayesian Information Criteria (BIC). The BIC is inappropriate to form model averaging weights for the AIE models because the BIC definition of complexity is inapplicable as the number of free parameters in the AIE model is constant but the complexity of the model varies greatly by altering two parameters: (1) the probability of a link being rewired and (2) the number of links in a network. Together, they provide for 121 levels of complexity or network structures. The 121 structures are the product of the 11 settings for the number of links in the network and 11 settings for the probability of a link being rewired. Levin[14]'s *Kt* complexity provides a more suitable and alternative complexity measure. Levin complexity makes the assumption that Universal Turing machines are able to simulate each other in linear time to retain invariance with Kolmogorov complexity (Ref. 15). The time for an AIE model to run becomes a proxy for complexity. Each of the 121 network structures require different running times; generally the more links in the network the longer the running time; intuitively more complex. The probability of a link being rewired has the general effect of making the running time longer; again intuitively more complex. Eq. (8) shows the complexity component of the BIC formula in Eq. (7) replaced with the Levin[14]'s *Kt* complexity; *t* is the model runtime and the constant *K* renamed *c* is determined by experiment. This constant *c* will vary according to the speed of the computer running the AIE model, using the same computer to measure the runtime for all the versions of the AIE model would prevent this problem. Alternatively each computer could be benchmarked using the runtime of the least complex AIE model. This runtime on each computer becomes the unit time for each computer, allowing for a quasi universal constant *c*.

Eq. (11) shows the Bayes factor from Eq. (10) used to form a weight for each model.

$$w_m = \frac{\sigma^{-n}_m \; n\text{^}(-ct_m/2)}{\Sigma^M_i \, \sigma^{-n}_i \; n\text{^}(-ct_i/2)} \quad (11)$$

Where

w_m = weight for each model m

M = the number of models

Eq. (9) shows Ref. 16's observation that the BIC gives a rough approximation to the logarithm of the Bayes factor (K), which is easy to use and does not require evaluation of the prior distribution.

The derivation of the weight in Eq. (11) assumes that theorem 2 of Levin[14]'s complexity is *Kt* when it is *Kt + c*. However Eq. (11) can be derived from either form of Levin's complexity; the simpler form aids clarity.

3. RESULTS

This section provides the results comparing the AIE model over a short calibration period against the benchmark model, the rational expectations hypothesis. The short calibration period is March 2000 to December 2006. The period starts after the phase transition seen in Fig. 2. The prediction period is March 2006 to June 2007. The forthcoming paper[17] compares calibrating the AIE model over a longer and a shorter period. Ref. 17 finds that calibration using the shorter period provides more accurate predictions, concluding that the economy makes sufficient structural changes during a phase transition to make calibrations over both sides of a phase transition inaccurate for prediction. Consequently this paper only addresses the short calibration.

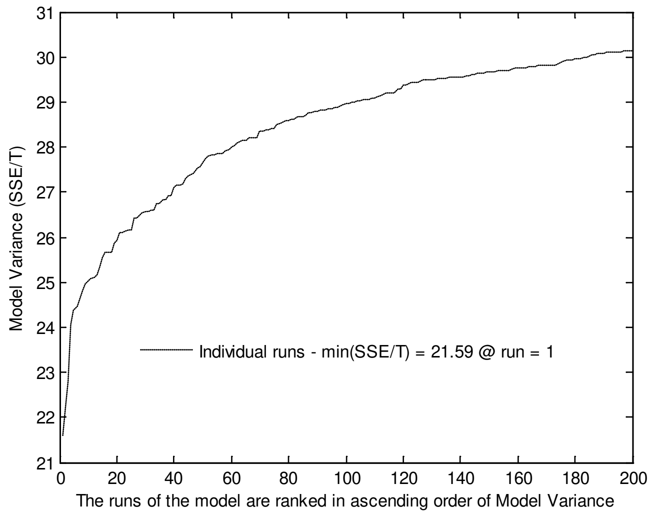


Fig. 3 The 200 runs with the lowest model variance

3.1 AIE model calibrated over the short period

Fig. 3 shows the 200 runs with the lowest model variance ranked in order of ascending model variance.

Table 1 shows the parameter values for the five runs with the lowest model variance in ascending order. Noteworthy is the widely spaced equilibria. This is consistent with the multiple equilibria modelled in Ref. 2.

Table 1 Parameter values for the five runs with the lowest model variance (SSE/T)

Run	SSE/T	δ	ρ	L	β^+	I	A	A_{-1}	p^+	p^{++}	p^-	p^{--}
1	21.59	1.2	0.6	16	3	27	13	18	45	117	48	122
2	22.25	1.8	0.9	22	5	24	10	19	45	117	48	122
3	22.80	2.8	1	12	4	30	9	18	45	117	48	122
4	24.11	1.4	0.3	22	4	28	12	22	45	117	48	122
5	24.44	1.8	0.8	8	4	30	9	19	45	117	48	122

Notable is that p+ and p++ are smaller in magnitude than p– and p– – respectively. This is consistent with the all–firms profit expectations indices being greater than the actualisation indices seen in Fig. 1. Note also that the values of L, δ and ρ are widely spread. This is consistent with Fig. 8, Fig. 9 and Fig. 10 showing widely spread minimums.

Fig. 4 shows the effect of varying the runtime weighted model averaging constant *c* against the model variance during the calibration phase. Noteworthy is that the runtime weighted model averaging constant of zero gives the lowest model variance of 20.00, which means in this case that during the calibration phase the runtime component of the weighting is redundant. The solid line in Fig. 5 shows the model averaging of the profit expectations index for $c = 0$. The dotted line in Fig. 5 shows the profit expectations index for the run with the lowest model variance whose parameters are given in Table 1. The dashed line in Fig. 5 shows the D&B profit expectations index; this is the index the model is simulating. The model averaging decreases the model variance during the calibration phase from 21.59 to 20.

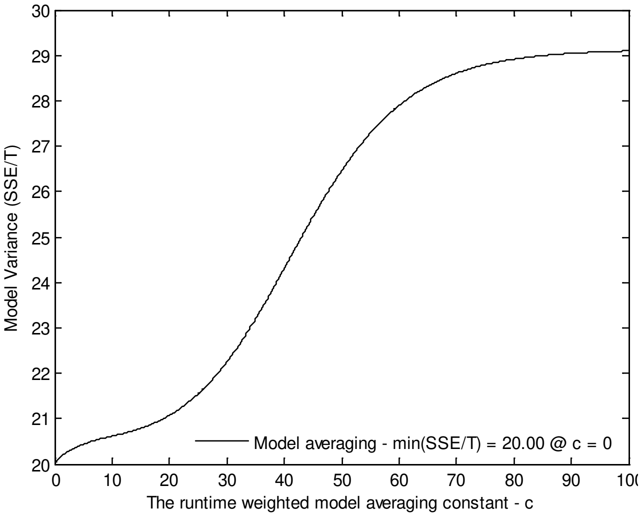


Fig. 4. Finding the optimal runtime weighted model averaging constant c

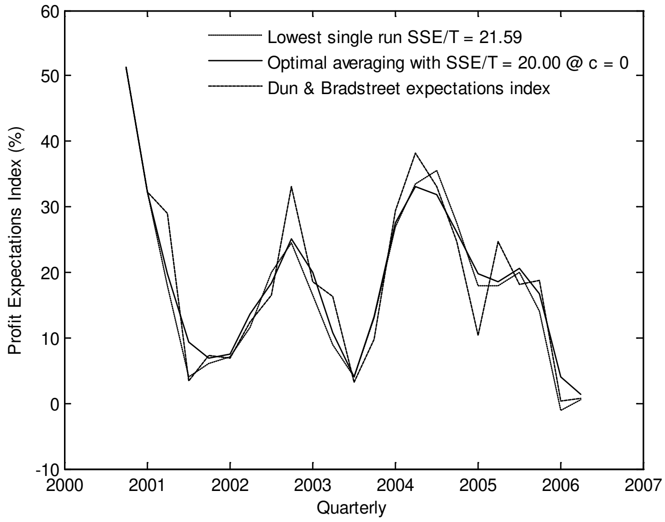


Fig. 5. Comparing the Calibration of the AIE model against the D&B Index.

Fig. 6 shows the prediction of the profit expectation index using a *c = 0*. The model averaging has decreased the model variance from 90.63 to 75.12. Fig. 7 evaluates the model averaging in the prediction and shows that increasing *c* to 0.6 decreases the model variance from 75.12 to 74.70. The discussion takes up this point.

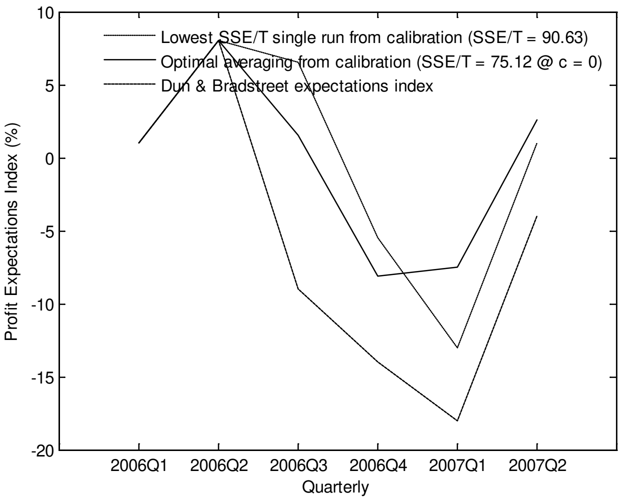


Fig. 6 Prediction based upon the calibration

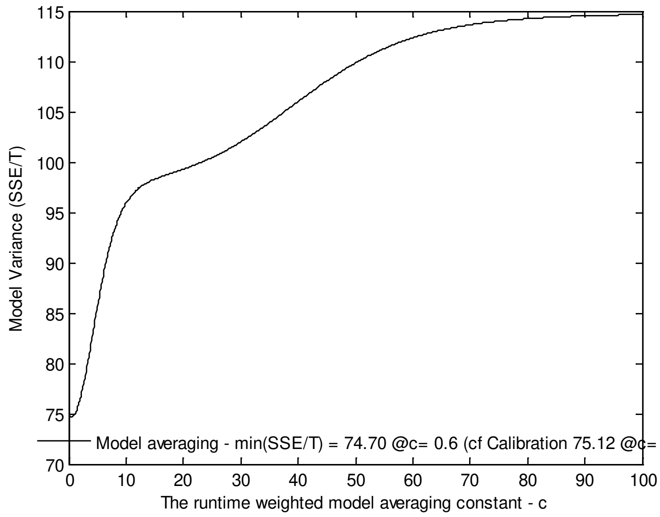


Fig. 7 Evaluating the prediction using runtime weighted model averaging

Table 2 shows the AIE model benchmarked against rational expectations hypothesis. The prediction of the AIE model based on the single run from the calibration with the lowest model variance is slightly smaller than the rational expectations hypothesis. The AIE prediction using runtime weighted model averaging reduced the model variance further.

Table 2 Benchmarking the AIE model against the rational expectations hypothesis using the model variance (SSE/T).

		Calib-ration	Pred-iction
AIE Model short calibration	Single run	21.59	90.63
	Model Averaging	20.00	75.12
Rational Expectations Hypothesis		200.76	93.00

3.2 Visualisation to evaluate finetuning the network topology

This section shows how varying the network topology (L and ρ) and interactive power (δ) affects the model variance. L, ρ and δ determine the interactive component of the p^x.

Fig. 8, Fig. 9 and Fig. 10 show how varying the network topology affects the model variance for δ = 1.0, 1.2 and 1.4 respectively. The dark patches are the low model variance values and the white patches the high model variance values. Thinking of dark green valleys and the white tops of mountains is a helpful analogy. The figures show multiple equilibria or minima. The minimum in Fig. 9 shows the run from Fig. 5 with the lowest model variance at 21.59; Table 2 shows the parameters values for the five runs with the lowest model variance, including run 1 from Fig. 5.

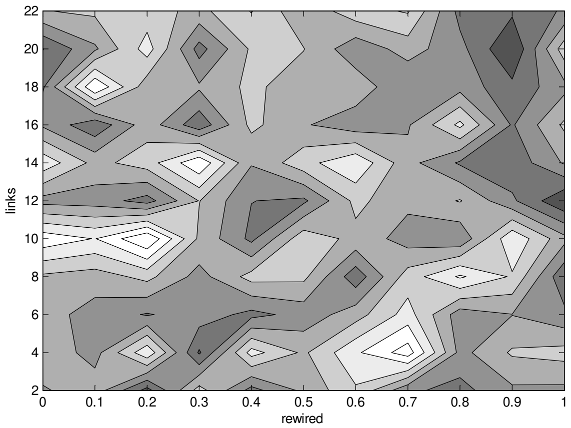


Fig. 8 SSE/T for various L and ρ for δ = 1.0

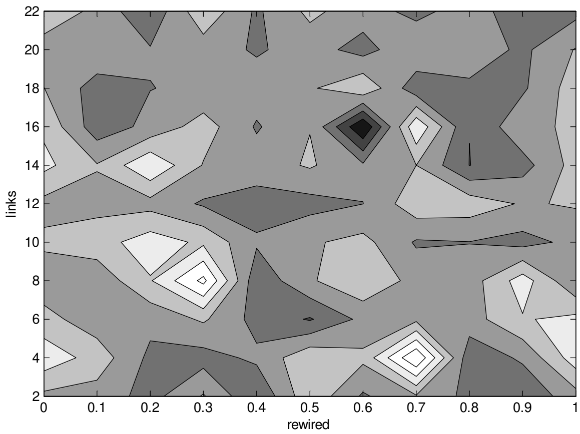
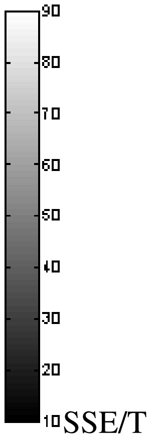


Fig. 9 SSE/T for various L and ρ for δ = 1.2



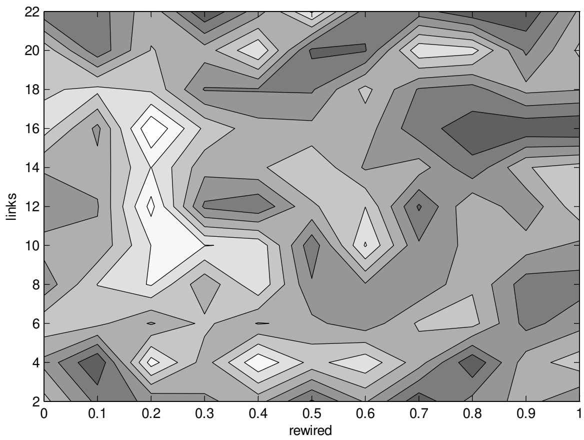


Fig. 10 SSE/T for various L and ρ for δ = 1.4

3.3 Model averaging across unique network topologies improves predictive power

In Sec. 3.1, the runtime weighted model averaging technique is applied to the 200 runs with the lowest model variance. As noted these 200 runs would contain multiple equilibria for the same network topology. This section takes the single run with the lowest model variance for each of 71 of the network topologies and applies the model averaging techniques. Table 3 shows that the predictive performance is greatly enhanced more than when simply taking the 200 runs with the lowest model variance. All model averaging techniques in Table 3 reduce the model variance over the single run for calibration and prediction; the single run model variance is 21.39 and 114.88 respectively.

Table 3 Model averaging comparison using 71 models each having unique network topologies.

Model Averaging Method	Calibration		Prediction	Evaluation	
	SSE/T	c or runs	SSE/T	SSE/T	c or runs
Single Run	21.39	1 run	114.88		
Optimal Calibration	18.27	8 runs	46.93	46.93	8 runs
Runtime Weighted	18.83	c = 3	58.61	58.26	c = 4
Bayes Factor (Benchmark)	18.93		63.88		
Simple (Benchmark)	20.48		64.26		

Fig. 11 shows the optimal calibration model averaging technique. The dashed line in Fig. 11 shows first stage in the technique, which involves ranking the models in ascending order of model variance. The solid line in Fig. 11 shows the second stage in technique, which involves model averaging the first two models, model averaging the first three models and so on until a model average for all 71 models is calculated. Fig. 11 shows that model averaging the first 8 models minimises the model variance. Fig. 12 evaluates the performance of the optimal calibration techniques. The prediction of each model is averaging as described in Fig. 11, while maintaining the rank order from Fig. 11. The number of models to average to minimise the model variance is 8 for both the calibration and evaluation.

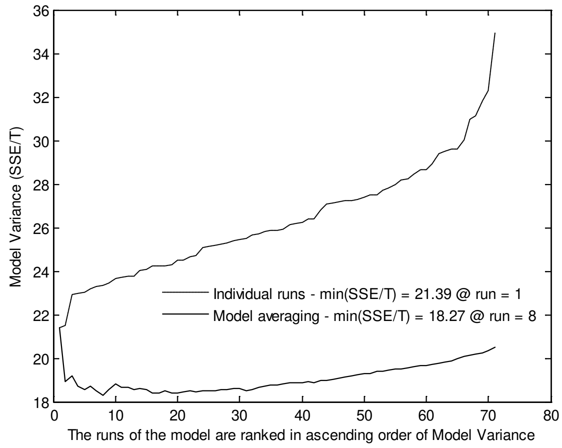


Fig. 11. Optimal Calibration: Finding the optimal number of runs to model average

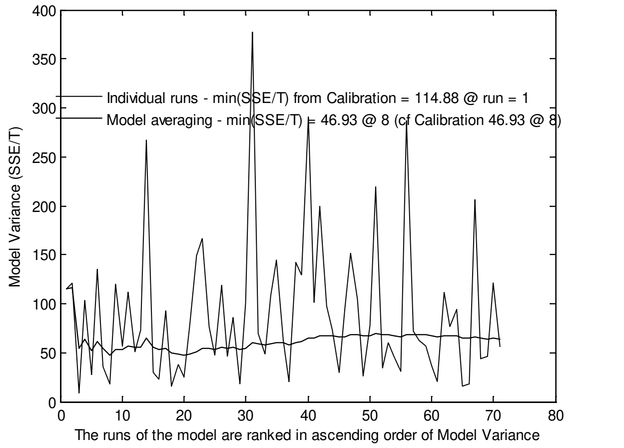


Fig. 12. Optimal Calibration: Evaluating the predictive performance

Table 3 shows that the prediction of the optimal calibration technique has the lowest model variance of all the model averaging techniques. The prediction of the runtime weighted technique produced the second lowest model variance. The evaluation of the runtime weighted technique finds that $c = 4$ would gave a lower model variance in the prediction than the $c = 3$ from the calibration. The prediction of the Bayes Factor technique has the third lowest model variance. The Bayes factor is a benchmark for the runtime weighted technique. It is the runtime weighted technique less the Levin's runtime component or the BIC less the number of parameters (K) component. The prediction of the simple model averaging techniques has the highest model variance. The simple model averaging averages all 71 models giving each an equal weight.

4. DISCUSSION

4.1 AIE Model

The visualisation in Sec. 3.2 provides a clearer picture of the network topology problem in the interactive component of the AIE model. The current AIE model uses a 200 node ring lattice network whose topology is controlled by two parameters: L and ρ. This approach is based upon the literature[3,11,12]. Sec. 3.2 demonstrates multiple equilibria in the model. Many combinations of L and ρ can be calibrated to find a low model variance value. Finetuning the network failed to identify a unique solution; in fact the multiple equilibria are quite disparate. This suggests that the method requires some form of restriction on the network parameterisation. Additionally, any form of simple ring lattice may be unable to represent the interactive network. This is an avenue for further research. However Sec. 3.3 shows that model averaging using the run with the lowest model variance from each network topology improves predictive performance.

The primary motivation for the AIE model is to capture emergence from the endogenous factors. However to do so may require allowance for exogenous factors other than actual change in percentage profits used in the current AIE model. Further research involves identifying the most significant exogenous factors for incorporation into the AIE model, such as a change in interest rate.

4.2 Model averaging

All the model averaging techniques decreased the model variance for both the calibration and the prediction. The runtime weighted model averaging in Sec. 3.3 shows that including some penalty for complexity in prediction is useful. Combining the optimal calibration and runtime weighted model averaging techniques may reduce the model variance of predictions further. This is left for further research.

The method outlined in this paper for finding the runtime weighted model averaging constant c proves suitable when the run with the lowest model variance from each network topology is used. The following alternative method for finding c is left for further research. The alternative uses models that have differing numbers of parameters, calculating their BIC and runtime then using these two measures to find a suitable value for c.

5. CONCLUSION

5.1 The AIE model

The AIE model provides an explanatory description of profit expectations formation, with a smaller model variance for the calibration and predictive benchmarks using the model averaging techniques. However the rational expectations hypothesis has a high model variance, so is not a particularly stringent benchmark to surpass. This means that the AIE model requires more stringent benchmarks and improvement before it is ready to investigate policy implications.

A major constraint on improving the AIE model is the number of parameters that can be tested, so a focus is determining which parameters to include and how to get the best use out of the parameters. These are considerations for traditional mathematical economics also, but the relative times for testing models are hours compared to weeks for agent based models. Simulated annealing may reduce calibration times, which is left for further research.

The interactive component of the AIE model may be improved by increasing the interactive memory and/or replacing the aggregate model with a divisional model whose interactive links between firms of differing division have magnitudes based upon an output–input table.

Beinhocker[18]'s three factors of emergence provide a useful framework to structure the reason why parameters are included in a model: (1) exogenous shocks, (2) participants' behaviour and (3) institutional structure. This paper has identified the following corresponding items for further research: exogenous shocks, the inclusion of the change in interest rates, see D&B[4]; participant behaviour, see Yu[19]'s dynamic cognitive model; and institutional structure, using a disaggregated interactive network and incorporating an input–output table. These changes feature in the forthcoming papers.

6. ACKNOWLEDGEMENTS

I would like to thank Professor Duncan Ironmonger for providing the D&B National Business Expectations Survey in a readily useable format, saving me considerable time. Additionally, I would like to thank Professor John Foster, Dr. Phil Wild and Dr. Rodney Strachan for their feedback and advice in preparing this paper. I would also like to thank Dr. David Green for introducing me to distributive computing and his prompt help with the high performance computing facility without which this paper would have taken too many years to complete. Additionally, I would like to thank Luis Izquierdo for introducing me to NetLogo programming. Finally, I would like to thank the Australian Research Council's Centre for Complex Systems for providing funding.

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