Robust policy choice under Calvo and Rotemberg pricing

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Robust policy choice under Calvo and Rotemberg pricing

Sebastian Sienknecht*

Abstract
This paper examines the robustness of welfare-based policy choices across the nonlinear Calvo and Rotemberg pricing assumptions. Comparisons between simple interest rate rules turn out to be robust and independent of the price dispersion inherent in the Calvo setting. This robustness is violated if there is a policy alternative that controls for price dispersion.

JEL classification: E30; E52; E58

Keywords: Calvo pricing; Rotemberg pricing; welfare analysis; robustness analysis

1 Introduction

Two common ways to introduce rigidity in prices are the staggering approach by Calvo (1983) and the convex adjustment cost structure by Rotemberg (1982). Both specifications are known to deliver identical first-order dynamics because they share the same New Keynesian Phillips curve and resource constraint. Moreover, Lombardo and Vestin (2007) show that they have different implications when conducting welfare analyses that require a second-order approximation around an inefficient steady state. They find that the Calvo setting delivers lower absolute welfare levels due to the presence of price dispersion. In other words, a given policy is always undesirable under Calvo pricing. However, policy recommendations should be made by comparing one policy strategy against another. Naturally, the question about the robustness of a policy recommendation across both pricing assumptions arises. A policy that is not favored in the Calvo setting in absolute terms may be preferred in relative terms, or vice versa. This opens the possibility that one single policy is recommended under both pricing assumptions.

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Using numerical results in a nonlinear New Keynesian setting I aim to answer the following questions: (i) Are welfare-based policy choices robust to the Calvo and the Rotemberg price rigidity assumptions? (ii) Does price dispersion influence the robustness of a welfare-based policy choice? (iii) Are there policy rules that generate robust policy choices?

2 Model

I use two versions of the standard nonlinear New Keynesian model which differ in the price rigidity assumption only. Agents are households, firms, and the monetary authority. The economy is assumed to respond to a deterministic cost-push shock.

2.1 Households

Identical households lying in an interval of unity mass maximize an expected sum of discounted utilities \( E_t \sum_{k=0}^{\infty} \beta^k \left( \log(C_{t+k}) - (N_{t+k})^{1+\eta}/(1+\eta) \right) \) subject to the periodical resource constraint \( C_t + B_t/P_t = W_tN_t/P_t + R_{t-1}B_{t-1}/P_t + \Pi_t \) and with respect to consumption \( C_t \), labor \( N_t \), and one-period nominal bonds \( B_t \). Further, the household receives a real wage \( W_t/P_t \), gross interests \( R_t \), and firms’ real profits \( \Pi_t \). This problem results in the labor supply schedule \( W_t/P_t = N_t^\eta/C_t \) and the Euler consumption equation \( 1/C_t = \beta E_t[R_t/C_{t+1}\pi_{t+1}] \), with \( \pi_t \) as gross inflation, \( 1/\eta \) as the real wage elasticity of labor supply, and \( \beta \) as a discount factor.

2.2 Calvo pricing

There is a probability of \( 0 < \theta^k < 1 \) that a monopolistic firm lying in the unit interval is forced to hold its price \( P_t(i) \) until period \( t + k \) (see Calvo (1983)). Individual output is then given by \( Y_{t+k|i}(i) \) and the needed labor amount is \( N_{t+k|i}(i) \) for \( k = 0,1,\ldots \). Since households own firms, the stochastic discount factor for future real profits is \( \Delta_{t,t+k} = \beta^k C_t/C_{t+k} \). The task is to maximize the expected sum of discounted real profits \( E_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} \left[ P_t(i)Y_{t+k|i}(i)/P_{t+k} - \mu_{t+k} W_{t+k}N_{t+k|i}(i)/P_{t+k} \right] \) subject to the demand function \( Y_{t+k|i}(i) = (P_t(i)/P_t)^{-\epsilon}Y_{t+k} \) and the production function \( Y_{t+k|i}(i) = \left(N_{t+k|i}(i)\right)^{1-\alpha} \). Aggregate output is a CES function according to \( Y_t = \left( \int_0^1 Y_t(i)(\epsilon-1)/\epsilon \right)^{\epsilon/(\epsilon-1)} \) and the corresponding price index is \( P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} \right)^{1/(1-\epsilon)} \).
variable $\epsilon$ is the elasticity of substitution between goods varieties and $\mu_t$ represents a cost-push variable that forces the resetting firms to set a higher price and reduce production. In the Calvo version of the model, the goods and the labor market are in equilibrium, such that $Y_t(i) = C_t(i)$ and $N_t = \int_0^1 N_t(i)$. Moreover, the bonds market clearing condition requires that $B_t = B_{t-1} = 0$. Upon aggregation a resource consuming price dispersion term $s_t = \int_0^1 \left( \frac{P_t(i)/P_t}{e} \right)^{1/(1-\alpha)}$ arises due to cross-sectional relative price distortions (see Yun (2005)). Table 1 gives the model absent of monetary policy and a process for $\mu_t$. Item 1 makes clear that price dispersion limits consumption possibilities and item 6 shows how it evolves. The variable $x_t = (P_t(i)/P_t)$ is a convenient definition for simulating the nonlinear model. Item 2 is the price index under Calvo pricing. Item 3 is the nonlinear New Keynesian Phillips curve in recursive form which results from the constrained maximization problem of the individual firm stated above.

**Table 1: Resource constraint and Phillips curve under Calvo pricing**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Resource constraint:</td>
<td>$s_t^{\alpha-1}N_t^{1-\alpha} - C_t = 0$</td>
</tr>
<tr>
<td>2. Price index:</td>
<td>$1 - \left( (1 - \theta)x_t^{1-\epsilon} + \theta \pi_t^{\epsilon-1} \right) = 0$</td>
</tr>
<tr>
<td>3. Phillips curve:</td>
<td>$x_t^{1+\alpha(\epsilon-1)} - \left( \frac{\epsilon}{\epsilon-1} \right) \frac{\psi_t}{\phi_t} = 0$</td>
</tr>
<tr>
<td>4. Recursive nominator:</td>
<td>$\psi_t - \mu_t \left( \frac{1}{1-\alpha} \right) N_t^{\eta} C_t^{\frac{1}{1-\alpha}} - \theta \beta E_t \left[ \pi_t^{\frac{\phi_t}{1-\alpha}} \psi_{t+1} \right] = 0$</td>
</tr>
<tr>
<td>5. Recursive denominator:</td>
<td>$\phi_t - 1 - \theta \beta E_t \left[ \pi_t^{\frac{\phi_t}{1-\alpha}} \Phi_{t+1} \right] = 0$</td>
</tr>
<tr>
<td>6. Price dispersion:</td>
<td>$s_t - (1 - \theta)x_t^{\frac{1}{1-\alpha}} - \theta \pi_t^{\frac{1}{1-\alpha}}s_{t-1} = 0$</td>
</tr>
</tbody>
</table>

### 2.3 Rotemberg pricing

Adjustment costs $Q_{t+k}(i) = \varphi[(P_{t+k}(i)/P_{t+k-1}(i)) - 1]^2/2$ with $\varphi > 0$ arise for $k = 0, 1$ when the monopolistic firm decides on $P_t(i)$ (Rotemberg (1982)). Its task is to maximize the expected sum of real profits $E_t \sum_{k=0}^{\infty} A_{t+k}[P_{t+k}(i)Y_{t+k}(i)/P_{t+k} - \mu_{t+k} W_{t+k} N_{t+k}/P_{t+k} - Q_{t+k}]$ subject to $Q_{t+k}(i)$, the demand function $Y_{t+k}(i) = (P_{t+k}(i)/P_{t+k})^{-\epsilon}Y_{t+k}$, and the production function $Y_{t+k}(i) = \left( N_{t+k}(i) \right)^{1-\alpha}$. The labor market is in equilibrium and all firms
set the same price, such that $P_t(i) = P_t$ and therefore $Y(i) = Y_t$. However, the existence of price adjustment costs limits the aggregate consumption possibilities in the economy. Table 2 gives the Rotemberg counterpart of the model presented in Table 1. Item 1 is the resource constraint and item 2 presents the nonlinear New Keynesian Phillips curve resulting from the first-order condition of the monopolistic firm.

Table 2: Resource constraint and Phillips curve under Rotemberg pricing

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Resource constraint:</td>
<td>$N^{1-\alpha}_t - C_t - \frac{\phi}{2}(\pi_t - 1)^2 = 0$</td>
</tr>
<tr>
<td>2. Phillips curve:</td>
<td>$\pi_t(\pi_t - 1) - \beta E_t\left[\frac{C_t}{C_{t+1}}\pi_{t+1}(\pi_{t+1} - 1)\right] - \frac{\epsilon}{\phi}\left(\frac{1}{1-\alpha}\right)\mu_t C_t N^{1+\eta}_t + \left(\frac{\epsilon-1}{\phi}\right)N^{1-\alpha}_t = 0$</td>
</tr>
</tbody>
</table>

2.4 Monetary policy

I assume that the monetary authority can choose between three policy strategies: an interest rate rule à la Taylor (1993), a Taylor rule with smoothing, and a Ramsey policy under timeless perspective commitment. The interest rate rule $R_t = (R_{t-1})^{\phi}\left((Y_t/Y)^{\delta}(\pi_t)^{\gamma}\right)^{1-\phi}\beta^{\phi-1}$ embeds the Taylor rule (for $\phi = 0$) and its counterpart with smoothing (for $0 < \phi < 1$), with $\delta$ and $\gamma$ as reaction parameters and $Y = ((1-\alpha)(\epsilon - 1)/\epsilon \mu)^{(1-\alpha)/(1-\eta)}$ as the inefficiently low output level at the steady state. Both interest rate rules are structurally independent of the pricing assumptions. According to Kahn et al. (2003), the Ramsey policy minimizes at period $t = 0$ the absolute welfare loss $V_{f,0}^{abs.} = E_0 \sum_{t=0}^{\infty} \beta^t \left(N_t^{1+\eta}/(1+\eta) - \log(C_t)\right)$, which is measured in negative utility units such that $V_{f,0}^{abs.} > 0$ for $\eta > 1$. The constraints are specific to the pricing assumption and given by the equations of Table 1 for $j = c$ (Calvo pricing) and the equations of Table 2 for $j = r$ (Rotemberg pricing). Moreover, the sets of control variables are $C_t, N_t, \pi_t, \Psi_t, \Phi_t$ for $j = c$ and $C_t, N_t, \pi_t$ for $j = r$. This implies that there are two types of the Ramsey policy which depend on the pricing assumption. The simple interest rate rules are motivated by the fact that they do not control for the price dispersion term present in the Calvo setting. I check whether the policy recommendation between these interest rate rules depends on the pricing
assumption. In contrast, the Ramsey planner governs over the price dispersion term present in the Calvo setting (but not in the Rotemberg setting). I compare the Taylor rule against this policy in order to assess robustness when a price dispersion-controlling policy is involved.

3 Welfare analyses

The cost-push variable is assumed to evolve as $\mu_t = \mu^{1-\rho} \mu_{t-1}^\rho \exp(e_t)$, where $\mu$ is its steady state counterpart. The parameter $0 \leq \rho < 1$ gives the degree of shock persistence and $e_t$ is a deterministic shock impulse. For the purpose of welfare comparisons, I define the relative welfare measure in percent as $V_{j,0}^{rel} = (V_{j,0}^{abs,(policy\ 1)}/V_{j,0}^{abs,(policy\ 2)}) \times 100$ for $j = c, r$. A percentage value $V_{j,0}^{rel} < 100\%$ indicates that the absolute welfare loss of policy 1 falls below the absolute welfare loss of policy 2 at the pricing regime $j$. Robustness with respect to the policy choice then requires $V_{j,0}^{rel} < 100\%$ or $V_{j,0}^{rel} > 100\%$ for $j = c$ and $j = r$. I assume commonly used parameter values on a quarterly basis and set $\alpha = 0.3, \beta = 0.99, \gamma = 0.3, \delta = 0.3, \epsilon = 6, \eta = 2, \theta = 0.75$, and $\mu = 1$. Note that price dispersion is absent in the Calvo setting when considering a non-persistent cost-push shock ($\rho = 0$). Then, comparisons against the assumption of high persistence ($\rho = 0.95$) allow me to study the role of price dispersion for the robustness of policy choice. For the case of interest rate smoothing, I assume $\phi = 0.8$. Even though I am not analyzing first-order dynamics, the Rotemberg adjustment cost parameter $\phi$ is fixed in order to obtain the same slopes across the log-linear New Keynesian Phillips curves $\pi_t = \beta E_t[\pi_{t+1}] + \kappa_j y_t + \omega_j \mu_t$ for $j = c, r$. The condition $\kappa_c = \kappa_r$ then requires $\varphi = \theta(1 + \alpha(\epsilon - 1))(\epsilon - 1)N^{1-\alpha}/(1 - \theta(1 - \theta))(1 - \alpha)$, with $N = ((1 - \alpha)(\epsilon - 1)/\epsilon \mu)^{1/(1-\eta)}$ as the amount of labor at the steady state.

Result 1: Price dispersion may reverse the policy recommendation but does not affect robustness when considering policy rules that do not control for price dispersion.

The numerical results in Table 3 were obtained by simulating the pricing models under a Taylor rule (policy 1) and a Taylor rule with smoothing (policy 2). In the absence of price dispersion (see (a)), the choice of the Taylor rule is robust across both pricing assumptions. Similarly, the choice of the Taylor rule with smoothing is robust across both pricing assumptions when allowing for price dispersion (see (b)). This demonstrates that even if a given policy is always inferior in the Calvo setting, it may be justifiable when related to another policy.
Table 3: Values in brackets denote absolute welfare. Values in percent represent welfare under the Taylor rule relative to its variant with smoothing.

<table>
<thead>
<tr>
<th></th>
<th>Calvo</th>
<th>Rotemberg</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $\rho = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor rule (sm.)</td>
<td>[32.02112832469623]</td>
<td>[32.02111051791479]</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>99.999920424904133 %</td>
<td>99.999905444569691 %</td>
</tr>
<tr>
<td></td>
<td>[32.021102851626054]</td>
<td>[32.021080774092134]</td>
</tr>
<tr>
<td>(b) $\rho = 0.95$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor rule (sm.)</td>
<td>[32.028106298326946]</td>
<td>[32.026662312798756]</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>100.0032131460892 %</td>
<td>100.000995070231 %</td>
</tr>
<tr>
<td></td>
<td>[32.029135408171918]</td>
<td>[32.026980820204706]</td>
</tr>
</tbody>
</table>

Result 2: Price dispersion reverses the policy recommendation and affects robustness when considering at least one policy that controls for this dispersion.

Table 4 gives results obtained by simulating the model under a Taylor rule (policy 1) and the Ramsey policy (policy 2). In the absence of price dispersion (see (a)), the robustness of the policy recommendation is preserved. The Ramsey planner is unable to minimize absolute welfare losses by controlling price dispersion in the Calvo model such that the absolute welfare loss is almost identical to the (dispersion-free) Rotemberg setting. This inability generates a comparative advantage of the Taylor rule indicated by the relative welfare drop below the one-hundred percent threshold. By assuming price dispersion in the Calvo model (see (b)), the Ramsey planner is able to reduce absolute welfare such that the relative welfare measure indicates its comparative advantage. In contrast, the Rotemberg setting does not entail price dispersion and the relative welfare measure indicates that the Taylor rule should be preferred. The policy choice is not robust across the pricing assumptions and depends on the presence of price dispersion.
Table 4: Values in brackets denote absolute welfare. Values in percent represent welfare under the Taylor rule relative to the Ramsey policy.

<table>
<thead>
<tr>
<th></th>
<th>Calvo</th>
<th>Rotemberg</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $\rho = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramsey policy</td>
<td>[32.021281913876152]</td>
<td>[32.021258827542127]</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>99.999440802368312%</td>
<td>99.999443952372545%</td>
</tr>
<tr>
<td></td>
<td>[32.021102851626054]</td>
<td>[32.021080774092134]</td>
</tr>
<tr>
<td>(b) $\rho = 0.95$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramsey policy</td>
<td>[32.027418478121746]</td>
<td>[32.027339650997995]</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>100.0053608131150%</td>
<td>99.99879610991111%</td>
</tr>
<tr>
<td></td>
<td>[32.029135408171918]</td>
<td>[32.026980820204706]</td>
</tr>
</tbody>
</table>

4 Concluding remarks

The purpose of this contribution was to determine if welfare-based policy choices are robust across the widely used pricing assumptions of Calvo and Rotemberg. I further examined the effect of price dispersion on the policy choice. I find that price dispersion only plays an important role for the robustness of a policy recommendation if one of the policies controls for price dispersion. This policy will tend to dominate in terms of welfare against any other policy. If the policies in question do not control for price dispersion, the relative welfare measures will indicate the same policy recommendation. When comparing relative welfare performances between simple interest rate rules, price dispersion does not play any role in the policy decision. In this case there is no uncertainty for the policy-maker across both pricing assumptions.

References


Simulation codes

Software: Dynare 4.2.5 and Matlab® R2010b

File format: Dynare .mod files

// This file embeds the interest rate rules and the Ramsey policy in the model under Calvo pricing. Default: interest rate rules. For the Ramsey policy, use /**/ where indicated

// Y: real output, N: labor hours, C: real consumption, mu: shock variable, R: nominal gross interest rate, Util: periodical utility, V: recursive utility, lambda_2 to lambda_7: Lagrange multipliers of the Ramsey problem

var        Y N Pi C mu R s x Psi Phi Util V /*lambda_2 lambda_3 lambda_4 lambda_5 lambda_6 lambda_7*/;
// Remove /**/ for the Ramsey policy
varexo     e;
parameters alpha beta eta theta phi rho delta gamma epsilon
          mu_SS Y_SS N_SS C_SS R_SS x_SS s_SS Psi_SS Phi_SS Util_SS V_SS;
alpha=0.3;
beta=.99;
eta=2;
theta=0.75;
phi=0; // (adjust interest rate smoothing parameter)
rho=0; // (adjust shock persistence parameter)
delta=.125;
gamma=1.5;
epsilon=6;

%%% Steady state values %%%

mu_SS=1;
Y_SS=((1-alpha)/(mu_SS*(epsilon/(epsilon-1))))^((1-alpha)/(1+eta));
N_SS=((1-alpha)/(mu_SS*(epsilon/(epsilon-1))))^(1/(1+eta));
C_SS=Y_SS;
R_SS=1/beta;
x_SS=1;
s_SS=1;
Psi_SS=(mu_SS*(1/(1-alpha))*(N_SS^eta)*(C_SS^(1/(1-alpha))))
    //(1-theta*beta);
Phi_SS=1/(1-theta*beta);
Util_SS=log(C_SS)-((1/(1+eta))*N_SS^(1+eta));
V_SS=(1/(1-beta))*Util_SS;
model;

%%% Resource constr. & Phillips curve under Calvo pricing (Table 1) %%%

(s^(alpha-1))*(N^(1-alpha))-C=0; // (Resource constraint)
1-((1-theta)*(x^(1-epsilon))+theta*(Pi^(epsilon-1)))=0; // (Price index)
x^((1+alpha)*(epsilon-1))
  -(epsilon/(epsilon-1))*(Psi/Phi)=0; // (Phillips curve)
Psi-mu*(1/(1-alpha))*(N^eta)*(C^(1/(1-alpha)))
  -theta*beta*(Pi(+1)^(epsilon/(1-alpha)))*Psi(+1)=0; // (Recursive nom.)
Phi-1-theta*beta*(Pi(+1)^(epsilon-1))*Phi(+1)=0; // (Recursive denom.)
s-(1-theta)*(x^(-epsilon/(1-alpha))
  -theta*(Pi^(epsilon/(1-alpha)))*s(-1)=0; // (Price dispersion)

%%% Euler consumption equation %%%

1/C=beta*R*(1/(C(+1)*Pi(+1)));

%%% Production function %%%

Y=(N/s)^(1-alpha);

%%% Shock process %%%

mu=(mu_SS^(1-rho))*(mu(-1)^rho)*exp(e);

%%% Interest rate rules %%%

R=(R(-1)^phi)*(((Y/Y_SS)^delta)*(Pi^gamma)^(1-phi))*(beta^(phi-1));

%%% Ramsey policy (first-order conditions) %%%

// Remove /**/ for the Ramsey policy and comment interest rate rules

/*/ 
1/C-lambda_2
  -lambda_5*mu*((1/(1-alpha))^2)*N^eta)*(C^((alpha/(1-alpha))))=0;
-N^eta+lambda_2*(1-alpha)*(s^(alpha-1))*(N^(-alpha))
  -lambda_5*mu*(1/(1-alpha))*eta*(N^(eta-1))*(C^((1/(1-alpha))))=0;
\[-\lambda_3 (1-\theta)(1-\epsilon) x^{-\epsilon} + \lambda_4 \left(\frac{1+\alpha(\epsilon-1)}{1-\alpha}\right) x^{\left(\frac{1+\alpha(\epsilon-1)}{1-\alpha}\right)-1} + \lambda_7 (1-\theta) \frac{\epsilon}{1-\alpha} x^{-\left(\frac{\epsilon}{1-\alpha}\right)-1} = 0;\]

\[-\lambda_2 (\alpha-1) s^{\alpha-2} N^{1-\alpha} + \lambda_7 - \beta \lambda_7 + 1 \theta \Pi^{(\epsilon/(1-\alpha))} = 0;\]

\[-\lambda_3 \theta (\epsilon-1) \Pi^{\epsilon-2} - \lambda_5 (-1) \theta \frac{\epsilon}{1-\alpha} \Pi^{(\frac{\epsilon}{1-\alpha})-1} \Psi - \lambda_6 (-1) \theta (\epsilon-1) \Pi^{\epsilon-2} \Phi - \lambda_7 \theta \frac{\epsilon}{1-\alpha} \Pi^{(\frac{\epsilon}{1-\alpha})-1} s(-1) = 0;\]

\[-\lambda_4 \left(\frac{\epsilon}{\epsilon-1}\right) \frac{1}{\Phi} + \lambda_5 - \lambda_5 (-1) \theta \Pi^{\epsilon/(1-\alpha)} = 0;\]

\[-\lambda_4 \left(\frac{\epsilon}{\epsilon-1}\right) \frac{\Psi}{\Phi^2} + \lambda_6 - \lambda_6 (-1) \theta \Pi^{\epsilon-1} = 0;\]

*/

%%% Utility %%%

Util = log(C) - \left(\frac{1}{1+\eta}\right) N^{1+\eta};

V = \beta V(+1) + Util;

end;

initval;

Y = Y_SS;
N = N_SS;
P_i = 1;
C = C_SS;
mu = \mu_SS;
R = R_SS;
Util = Util_SS;

V = V_SS;
s = s_SS;
x = x_SS;
Phi = Phi_SS;

Psi = Psi_SS;
end;

steady;

check;

endval;

Y = Y_SS;
N = N_SS;
P_i = 1;
C = C_SS;
mu = \mu_SS;
R = R_SS;
Util = Util_SS;
V = V_SS;
s = s_SS;
x = x_SS;
Phi = Phi_SS;

Psi = Psi_SS;
end;
steady;

shocks;
var e;
periods 1;
values 0.01;
end;
simul(periods=10000);

%%% Absolute welfare %%%
format long;
Vo=V(2); // (Utility in the initial period (V(1) is the steady state))
V_abs=-Vo

%%% Plotting (for checking purposes only) %%%

per=41;
jj=0:per-1;
mu_rel=((mu-mu_SS)/mu_SS)*100;
Y_rel=((Y-Y_SS)/Y_SS)*100;
Pi_rel=(Pi-1)*100;
N_rel=((N-N_SS)/N_SS)*100;
C_rel=((C-Y_SS)/C_SS)*100;
R_rel=((R-R_SS)/R_SS)*100;

figure(1);
subplot(3,2,1)
plot(jj,mu_rel(2:per+1));
title('mu')

subplot(3,2,2)
plot(jj,Y_rel(2:per+1));
title('Y')

subplot(3,2,3)
plot(jj,Pi_rel(2:per+1));
title('Pi')

subplot(3,2,4)
plot(jj,N_rel(2:per+1));
title('N')

subplot(3,2,5)
plot(jj,C_rel(2:per+1));
title('C')

subplot(3,2,6)
plot(jj,R_rel(2:per+1));
title('R')

// This file embeds the interest rate rules and the Ramsey policy in the
// model under Rotemberg pricing. Default: interest rate rules. For the
// Ramsey policy, use /**/ where indicated
// Y: real output, N: labor hours, C: real consumption, mu: shock
// variables, R: nominal gross interest rate, Util: periodical utility,
// V: recursive utility, lambda_2 and lambda_3: Lagrange multipliers of the
// Ramsey problem

var Y N Pi C mu R Util V /*lambda_2 lambda_3*/;
// Remove /**/ for the Ramsey policy

varexo e;

parameters alpha beta eta phi rho delta gamma epsilon
mu_SS Y_SS N_SS C_SS R_SS Util_SS V_SS theta psi;

alpha=0.3;
beta=.99;
eta=2;
phi=0; // (adjust interest rate smoothing parameter)
rho=0; // (adjust shock persistence parameter)
delta=.125;
gamma=1.5;
epsilon=6;

%%% Steady state values %%%

mu_SS=1;
Y_SS=((1-alpha)/(mu_SS*(epsilon/(epsilon-1))))^((1-alpha)/(1+eta));
N_SS=((1-alpha)/(mu_SS*(epsilon/(epsilon-1))))^(1/(1+eta));
C_SS=Y_SS;
R_SS=1/beta;
Util_SS=log(C_SS)-((1/(1+eta))*N_SS^(1+eta));
V_SS=(1/(1-beta))*Util_SS;

%%% Rotemberg parameter as a function of the Calvo parameter %%%

theta=0.75;
psi=(theta/((1-theta*beta)*(1-theta)))*((1+alpha*(epsilon-1))/
    (1-alpha))*(epsilon-1)*N_SS^(1-alpha); /((1-alpha))*(epsilon-1)*N_SS^(1-alpha);

model;

%%% Resource constr. & Phillips curve under Rotemberg pricing (Table 2) %%%

N*(1-alpha)-C-(psi/2)*(Pi-1)^2=0; // (Resource constraint)
Pi*(Pi-1)-beta*C/C(+1)*Pi(+1)*(Pi(+1)-1)
    -(epsilon/psi)*(1/(1-alpha))*mu*C*(N^(1+eta))
    +(epsilon-1)/psi)*N^(1-alpha)=0; // (Phillips curve)

%%% Euler consumption equation %%%

1/C=beta*R*(1/(C(+1)*Pi(+1)));

%%% Production function %%%

Y=N^(1-alpha);
%%% Shock process %%%

\[\mu = (\mu_{SS}^{1-\rho}) \cdot (\mu(-1)^\rho) \cdot \exp(e)\]

%%% Interest rate rules %%%

\[R = (R(-1)^\phi) \cdot (((Y/Y_{SS})^{\delta})(\Pi^{\gamma}))^{(1-\phi)} \cdot (\beta^{\phi-1})\]

%%% Ramsey policy (first-order conditions) %%%

// Remove /**/ for the Ramsey policy and comment interest rate rules

/*
1/C-\lambda_2
 -\lambda_3*beta*(1/C(+1))*Pi(+1)*(Pi(+1)-1)
 +\lambda_3(-1)\cdot(C(-1)/C^2)*Pi*(Pi-1)
 -\lambda_3*(\epsilon/\psi)*(1/(1-\alpha))\cdot\mu*N^{(1+\eta)}=0;

-N^\eta+\lambda_2*(1-\alpha)*(N^{(-\alpha)})
 -\lambda_3*(\epsilon/\psi)*(1/(1-\alpha))\cdot\mu*C*(1+\eta)*N^\eta
 +\lambda_3*(((\epsilon-1)/\psi)*(1-\alpha)*N^{(-\alpha)})=0;

-\lambda_2*\psi*(\Pi-1)
 +\lambda_3*(2*\Pi-1)
 -\lambda_3(-1)\cdot(C(-1)/C)*(2*Pi-1)=0;
*/

%%% Utility %%%

Util = \log(C) - ((1/(1+\eta)) \cdot N^{(1+\eta)});
V = \beta \cdot V(+1) + Util;
end;

initval;
Y = Y_{SS};
N = N_{SS};
Pi = 1;
C = C_{SS};
\mu = \mu_{SS};
R = R_{SS};
Util = Util_{SS};
V = V_{SS};
end;

steady;
check;
endval;
Y = Y_{SS};
N = N_{SS};
Pi = 1;
C = C_{SS};
\mu = \mu_{SS};
R = R_{SS};
Util = Util_{SS};
V = V_{SS};
end;

steady;
shocks;
var e;
periods 1;
values 0.01;
end;

simul(periods=10000);

%%% Absolute welfare %%%
format long;
Vo=V(2); // (Utility in the initial period (V(1) is the steady state))
V_abs=-Vo

%%% Plotting (for checking purposes only) %%%
per=41;
jj=0:per-1;

mu_rel=((mu-mu_SS)/mu_SS)*100;
Y_rel=((Y-Y_SS)/Y_SS)*100;
Pi_rel=(Pi-1)*100;
N_rel=((N-N_SS)/N_SS)*100;
C_rel=((C-C_SS)/C_SS)*100;
R_rel=((R-R_SS)/R_SS)*100;

figure(1);
subplot(3,2,1);
plot(jj,mu_rel(2:per+1));
title('mu')
subplot(3,2,2);
plot(jj,Y_rel(2:per+1));
title('Y')
subplot(3,2,3);
plot(jj,Pi_rel(2:per+1));
title('Pi')
subplot(3,2,4);
plot(jj,N_rel(2:per+1));
title('N')
subplot(3,2,5);
plot(jj,C_rel(2:per+1));
title('C')
subplot(3,2,6);
plot(jj,R_rel(2:per+1));
title('R')