Heckscher Ohlin Vanek Theorem: an excess supply approach

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Heckscher Ohlin Vanek Theorem: An Excess Supply Approach

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Abstract

The paper attempts to work out Heckscher Ohlin Vanek theorem with the help of excess supply approach. The study examines trade performance of ten manufacturing industries on a cross section of 46 countries for the year 2009. Factors taken into consideration are primary, secondary and tertiary educated labor, capital stock, arable land. Data sources such as world integrated trade solutions, Barro and Lee database, world development indicators, food and agriculture organisation etc are used to empirically test the theorem. The results suggest that capital stock, higher education and land are the factors which are creating comparative advantage in current trade pattern and further HOV theorem proves out to be still valid in more than 60% of the cases.
1. Introduction: Heckcher Ohlin Vanek theorem is one of the most prominent theorems in entire international trade theories literature. There has been a series of empirical tests performed to check the empirical validity of the same. But still it is one of the basic trade theorems which is widely acceptable in most of the cases. The basic H-O preposition says that countries export the commodities which require abundant industrial agents which are relatively cheap and imports the goods which require the relatively scarce industrial agents. Ohlin was aware of the fact that the differences in relative factor prices that arise because of the differences in relative factor supplies could be offset by relative differences in consumers’ preferences. But they believed that differences in relative factor endowments are more important than differences in relative consumer’s preferences.

He was also concerned about scale economies and qualitative differences in factors. Thus as he was so much concerned with real world conditions that he tried to integrate the factor prepositions framework into a general equilibrium pricing system assuming identical and constant returns to scale in production functions. He assumes that the two countries specialize in the unique set of goods that are cheaper than in the other country. Thus he did not believe in full factor price equalization.

Ohlin’s theory could not relate the ordering of country’s ratio of its endowment of each factor to the world endowment of each factor is connected to the ordering of country’s net exports of each factor to world endowment of each factor.

He although adopted very broad approach to analyze the influences shaping trade patterns, along with the factor endowments such as relative qualities of factors, consumer preferences etc., yet he did not attempt to undertake rigorous empirical tests of the HO preposition. He only relied on historical examples.

Vanek (1968) developed the exact relation. Subsequently, the authors tried to produce the strong predictions about how changes in product prices change factor prices (Stolper-Samuelson theorem) and how output changes as a consequence of relative factor supplies (Rybczynski theorem). Here in this paper our objective is to test the Heckscher Ohlin Vanek theorem which explains Heckscher Ohlin theorem in multi country, multi factor and multi commodity framework and says that if a country’s endowment of a factor relative to world endowment exceeds that country’s share world GDP, then the factor is said to be abundant in that country.
2. The Model and Literature Review: The basic Heckscher Ohlin theorem says that a country will export the commodity which uses its abundant factor intensively and vice versa. Leontief (1953) was the first to confront the HO model with data and found that the capital-labour ratio embodied in U.S. imports was higher than capital-labour ratio embodied in U.S. exports. This was called “Leontief’s Paradox.” Interestingly, Leontief did not mention either Heckscher or Ohlin in his pioneering work as he was explaining the importance of input output framework before group of non-economists.

<table>
<thead>
<tr>
<th>Table 1</th>
<th></th>
<th>Exports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital ($ millions)</td>
<td>$2.55</td>
<td>$3.1</td>
<td></td>
</tr>
<tr>
<td>Labour (person years)</td>
<td>182</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>Capital/Labour ratio ($ person)</td>
<td>$13,700</td>
<td>$18,200</td>
<td></td>
</tr>
</tbody>
</table>

Leontief first measured the amount of capital and labour required for $1 million worth of U.S. exports (provided in table 1 above). This calculation requires that we measure the labour and capital used directly, i.e. in each exporting industry, and also these factors used indirectly, i.e. in the industries that produce intermediate inputs that are used in producing exports. From the first row of Table, we see that $2.5 million worth of capital was used in $1 million of exports. This amount of capital seems much too high, until we recognize that what is being measured is the capital stock, so that only the annual depreciation on this stock is actually used. For labour, 182 person-years were used to produce the exports. Taking the ratio of these, we find that each person employed in producing exports (directly or indirectly) is working with $13,700 worth of capital.

On the Import side, Leontief did not know foreign technology matrix. He simply used the U.S. technology to calculate the amount of labour and capital used in imports (because of the assumption that technologies are the same across countries). Using the U.S. technology to measure the labour and capital used in imports, both directly and indirectly, he arrived at the estimates in the last column of Table: $3.1 million of capital, 170 person-years, and so a capital/labour ratio in imports of $18,200. Remarkably, this is higher than the capital/labour ratio found for U.S. exports. Under the presumption that the U.S. was capital-abundant in 1956, this appears to contradict the HO Theorem.
Explanations for the paradox:

1. U.S. and foreign technologies are not the same.
2. Leontief ignored Land.
3. U.S. exports are intensive in skilled labour.
4. Leamer (1980) performed an alternative test which relies on the “factor content” version of H-O model developed by Vanek (1968). Leamer used HOV theorem to resolve the Leontief paradox.

The HOV theorem predicts that if a country’s endowment of a factor relative to world endowment exceeds that country’s share of world GDP, then we say country is abundant in that factor. Thus it defines the Heckscher Ohlin theorem in multi-country, multi-factor and multi-commodity framework and is a factor content version of the HOS model.

The commodity version of the model, which is popularly called the HOS model says that a country trades those goods which intensively use the country’s relatively abundant factors and focuses on whether the industrial units in a country are net exporters or importers, while the HOV model or the factor content model predicts that the quantities of the relatively abundant factors embodied in the commodities which are exported will be greater than the quantities embodied in the import competing commodities.

\[ F_i = A T_i = V_i - s_i V_w \] (1)

In the equation (1) matrix A denote the amount of labour, capital, land and other primary factors needed for one unit of production in each industry.

\[ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \]

Here \( a_{11} \) denotes the amount of labour required for one unit of production in industry 1 and so on. Now to find out the factor content of trade we have,

\[ F_i = A T_i \] (2)

where,

\[ T_i = Y_i - D_i \] (3)

\( Y_i \) matrix denotes the output of each industry and \( D_i \) matrix denotes the demand for each good.

Now, the goal of the HOV model is to relate the factor content of trade to endowments of that country. To do so we have \( AT_i = V_i \) i.e. demand for factors in country equals endowment of
country because of full employment condition. Now consumption vectors of all countries are proportional to each other because of the assumption of homothetic preferences:

\[ D_i = s_iD_{wp} \]  

it follows \[ A_iD_i = s_iAD_{wp} \] i.e. demand related to factor content. Now with the assumption of balanced trade \( s_i \) also equals country \( i \)'s share of world GDP. Equating world consumption to world production, \[ AD_i = s_iAD_{wp} = s_iAY_{wp} = s_iV_{wp} \], so it is proved that

\[ F_i = AT_i = V_i - s_iV_{wp} \] \hspace{1cm} (4)

### 2.1 Possibilities for HOV prediction:

Edward Leamer (1980) restated the Leontief paradox. On the basis of HOV theorem Leamer compared the capital/labour ratio embodied in production and capital labour ratio embodied in consumption and found that the theorem was satisfied.

\[ \frac{K_1}{L_1} \geq \frac{(V_i - F_i)}{(V_i - F_{K})} \] \hspace{1cm} (5)

Where \( K_i \) and \( L_i \) denote capital and labour endowments which are fully embodied in production due to the assumption of full employment and \( K_1 - F_{K} \)

and \( K_i - F_{K} \) denotes factor content of consumption by subtracting content of factors embodied in trade from factors embodied in production.

In 1947, US was a net exporter of both the goods. Therefore, the HOV theory would predict the capital intensity of exports to be greater than the capital intensity of consumption if US was a capital abundant country. Table 2 shows these results.

### Table 2

**Capital Intensity of Production, Consumption and Trade**

<table>
<thead>
<tr>
<th></th>
<th>Production</th>
<th>Net Exports</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital (in million dollars)</td>
<td>328.519</td>
<td>23.450</td>
<td>305.069</td>
</tr>
<tr>
<td>Labour (in million man years)</td>
<td>47.273</td>
<td>1.99</td>
<td>45.23</td>
</tr>
<tr>
<td>Capital/Labour (in million per man)</td>
<td>6.949</td>
<td>11.783</td>
<td>6.737</td>
</tr>
</tbody>
</table>
Maskus\textsuperscript{ii} (1985) was one of the first to go for the complete test of the HOV theorem. He analyzed US trade for the years 1958 and 1972. He compared the factor content of net exports with the relative endowment of these factors. He calculated the factor content of trade and factor endowments with the help of an input-output table for 79 sectors. The results of Maskus tests are presented in the Table 3.

**Table 3**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Weak HOV</th>
<th>Rank Test (Actual/Predicted)</th>
<th>Strong Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1958</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prof Labour</td>
<td>Fails</td>
<td>2/2</td>
<td>38,4</td>
</tr>
<tr>
<td>Other labour</td>
<td>Fails</td>
<td>1/2</td>
<td>76,4</td>
</tr>
<tr>
<td>Capital</td>
<td>Holds</td>
<td>3/1</td>
<td>2,4</td>
</tr>
<tr>
<td>1972</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prof Labour</td>
<td>Holds</td>
<td>1/2</td>
<td>32,6</td>
</tr>
<tr>
<td>Other Labour</td>
<td>Holds</td>
<td>3/3</td>
<td>69,8</td>
</tr>
<tr>
<td>Capital</td>
<td>Holds</td>
<td>2/1</td>
<td>13,0</td>
</tr>
</tbody>
</table>

HOV test using 34 countries of the world

The results show that HOV theorem could not satisfy even weak prediction and performs very poorly in second and strong tests. Thus on the basis of this, Maskus concludes that HOV theorem is not supported by his empirical finding. The reason for this could be too restrictive assumptions of the theorem.

Some other complete test of HOV show tests by Bowen, Leamer and Sveikauskas (BLS, 1987). They proposed two tests:

$$\text{Sgn}(P_i^t) = \text{Sgn} (P_i^t - s_iV_i^w)...........(6)$$

$$F_i^t > F_i^t \iff (P_i^t - s_iV_i^w) > (P_i^t - s_iV_i^w) ...............(7)$$

BLS study considered 12 factors and 23 countries. They computed the amount of each factor embodied in net exports using 1967 U.S. I-O table and country’s factor endowment. The difference between Maskus study and BLS study is that they used production shares instead
of consumption shares so that unbalanced trade is also taken into account. Now the HOV equation becomes:

$$F_i \equiv \Delta T_i = V_i - x_i V_{xp} = V_i - V_{xp} (V_i - UBT) / y_{xp} \ldots \ldots \ldots \ldots \ldots (8)$$

UBT is unbalanced trade of the country and $y_i$ is the GNP of country $i$.

The sign test was found to be satisfied for 61% of cases. Rank test showed satisfaction in about 49% of the cases. So both tests seem to show very little empirical support.

Trefler (1993, 1995) used two ways to introduce technological differences. In 1993, he took productivity of factors in different countries by treating U.S factor productivity as benchmark and is normalized at unity.

$\pi^i_k$ denotes the productivity of factor $K$ in country $i$ relative to its productivity in U.S. Now effective endowment of factor $K$ in country becomes $\pi^i_k V^i_k$ while matrix A is same across countries. Now HOV equation becomes,

$$F^i_k = \pi^i_k V^i_k - s^i \sum_{j=1}^{c} \pi^i_j V^i_j \ldots \ldots \ldots \ldots \ldots (9)$$

where $i = 1, \ldots, c$

$K = 1, \ldots, M$

There are M(C-1) equations excluding U.S. and M(C-1) parameters. But the problem here was that with differences in productivity parameters, for almost all datasets, there will be solution for productivities $\pi^i_k$ such that the HOV equation holds with equality i.e. we can’t test the relation between net trade and factor endowments of a country. For this Trefler recommended two methods: first, need to check whether productivity parameters are positive and second, comparison of these parameters to other economic data to evaluate how reasonable these parameters are. For example, Trefler compared the productivity parameters to wages across countries and found them to match quite closely. This led to support Treflers’ extension of HOV model.

In the second way, Trefler allowed the factor requirement matrix to differ across countries. By comparing factor requirement matrix with U.S technology matrix, he arrived at following expression:
This means that \( A^t_i > A^{US}_i \) so that country \( i \) is less productive and requires more labour, capital and other resources for a unit of production relative to U.S. Now HOV equation becomes,

\[
A^t_{Y^t} = A^t_Y - A^t_D = V^t - A^t (s^t D^w) = V^t - A^t \left( s^t \sum_{j=1}^{n} Y_{j} \right)
\]

or

\[
F^{US} = \theta^t A^t = \theta^t V^t - \left[ \sum_{j=1}^{n} \theta^i V^j \right] \quad \ldots \ldots \ldots .. (11)
\]

From the above equation Trefler obtained estimates for \( \theta^t \) and their asymptotic t-statistics. Most countries were found to have a technological development that were significantly less advanced than that in the U.S and correlation between \( \theta^i \) and each country’s GDP per capita relative to U.S was 0.89. This supports the model.

Now comparing original HOV model with Trefler (1993) and Trefler (1995), it was found that for Treflers’ data, the variance of the factor contents relative to the variance of the country endowments turns out to be only 0.032. Trefler refers to this as mystery of missing trade. At the other extreme, when we allow for uniform productivity differences, the \( R^2 \) turns out to be 0.486 i.e. nearly one half of missing trade is explained by this. So he prefers the initial model which allows for uniform productivity differences.

Davis and Weinstein (2001) further provided complete tests of HOV theorem. But he still relied on U.S technology matrix. Now with the availability of this data, Davis and Weinstein made use of it to construct \( A^t_i \) across countries. Here rather than using actual data for \( A^t_i \), Davis and Weinstein estimated the differences in the technology matrices across countries.

\[
V^t - \theta^t \left( \sum_{j} V^j \right) = F^t_{ij} - F^t_{jj} \quad \ldots \ldots \ldots .. (12)
\]

First term on the right hand side, explains the factor content of exports from country \( i \) to all countries and second term on the right hand side shows factor content of imports from all countries. This equals factors used in country \( i \) to produce exports for all countries minus factors used in every country to produce exports to country \( i \).
Baldwin (1971) partially tested the theorem and in his method $T_i$ was regressed on $A_i$ to estimate the relative abundance of each factor and found that Leontief paradox exists. The test was criticized on the ground that $T_i$ should have been regressed on $A_i^{-1}$ not on $A_i$.

Leamer (1984) tested HOV by treating factor endowments $(V_k - x_iV_k^e)$ as data while estimating the elements of $A^{-2}$. Focusing on single industry $j$, and letting the elements of $A^{-2}$ be written as $\beta_{jk}$, the equation is:

$$T_j = \sum_{k=1}^{N} \beta_{jk} \left(V_k^i - x_iV_k^w\right)$$  \hspace{1cm} (13)

Leamer worked with the trade data for 60 countries in two years (1958 and 1975). The results obtained by regressing net trade on factor endowments, it was found that an increase in capital increases the net exports of manufactured goods and same is the case for non-professional and illiterate workers. Increase in most types of lands and professional and technical workers led to decrease in net exports of manufactured goods. Increase in land favored agriculture over industry and increase in professional and technical workers favoured non-traded services over manufacturing. This was testing of Rybczynski effects and of limited usefulness in HOV theorem.

Harrigan (1995) took industry outputs as dependent variable than trade. He regressed industry outputs on factor endowments. He took panel data of OECD countries for the period-1970-85 for 10-manufacturing sectors and four factor supplies. Result of his studies was that for each manufacturing industry there is at least one factor with a negative Rybczynski effect indicating that an increase in that endowment would reduce the manufacturing output. These factors were usually skilled or unskilled labour and sometimes land. Conversely, capital has a positive coefficient in all ten regressions.

Using true technology matrices, Hakura (1999) tested the bad performance of the HOV model with the adjusted version where true technology matrices of the countries are used. She specified HOV prediction in bilateral way:

$$T_t = (I - B_i)Q_t - C_t$$  \hspace{1cm} (14)

$$A_t(I - B_i)^{-2}T_t = V_t - A_t(I - B_i)^{-2}C_t$$  \hspace{1cm} (15)

Here above equation has been multiplied with $A_t(I - B_i)^{-2}$ and considered $A_tQ_t = V_t$. 

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Here intermediates are also part of input output matrix. Now, retaining the assumption of identical and homothetic preferences and dropping the assumption of identical technologies, taking the differences in the bilateral model gives,

$$A_1(I - B_1)^{-1}T_2 - \alpha A_2(I - B_2)^{-1}T_2 = V_2 - \alpha V_2 - (A_2(I - B_2)^{-1} - A_1(I - B_1)^{-1})C_2$$

Rearranging the above gives,

$$F_1 - \alpha F_2 = V_1 - \alpha V_2 - (A_2(I - B_2)^{-1} - A_1(I - B_1)^{-1})C_2$$  \[16\]

$$\alpha = \frac{z_2}{z_1}$$ gives size of country 2 relative to country 1, considering identical and homothetic preferences. Here if technologies are same across the countries, the right hand side of the equation would turn out to be zero.

Yong-Seok and Pravin Krishna (2004) empirically tested the approach proposed by Elhanan Helpman with bilateral trade data. The model does not require factor price equalization across countries and also no assumption on preferences. The starting point of model is again the trade prediction.

The relationship can be expressed in equation terms as:

$$\left( w^j - w^i \right) (F^U - F^J) \geq 0$$  \[17\]

where $w^j$ and $w^i$ are the vectors of factor prices in the two trading countries, $F^U$ is the gross import vector of factor content by country j from country i, measured with the help of technological coefficient matrix of the exporting country. Choi and Krishna (2004) implemented equation empirically using data for eight countries (Denmark, France, Germany, Netherlands, United Kingdom, United States, Canada and Korea) and seventeen industrial sectors. Five factors of production, namely capital, production, workers and three groups of non-production workers were taken into account for the year 1980. The results suggest that as entrance of technology and factor price data simply gives an un-normalized numerical sum whose conformance or departure from the theory cannot be easily ascertained, so equation can be written as:

$$\frac{w^j_F^H + w^i_F^J}{w^j_F^U + w^i_F^J} \geq 1$$  \[18\]
The ratio is the sum of importers hypothetical cost of production (using importers factor prices and exporters factor usage) to exporters actual cost of production (using the actual producer’s factor prices and factor usage). The authors found strong empirical support for the bilateral Heckscher Ohlin theorem.

Feenstra-Taylor (2007) have illustrated that instead of taking factor endowments into account it is better to measure effective factor endowments. Effective factor endowments take productivity differences into consideration. In their illustration these authors have considered eight countries, namely USA, China, Japan, India, Germany, UK, France, Canada and the rest of the world and six factors of production which are physical capital, R&D scientists, skilled labour, less skilled labour, illiterate labour and arable land for the year 2000. First they measured factor abundance according to the simple HOV theorem which says that if a country’s factor share is larger than its share of GDP, then the country is said abundant in that factor, and when a country’s factor share is less than its share of GDP, then the country is considered to be scarce in that factor. The results show that USA was abundant in physical capital, R&D scientists and skilled labour while India was scarce in R&D scientists. China is found to be abundant in R&D scientists. The findings seem to contradict HO theorem. Secondly, they have shown that it could be possible that the productivity of factors may not be the same in all the countries. This gives rise to the new concept of effective factor endowment.

One explanation of Leontief paradox could be that labour is highly productive in the U.S. and less productive in the rest of the world. Then the effective labour force in the U.S. is much larger than if we just count people. Effective factor endowment is the factor endowment times its productivity. To determine if a country is abundant in a certain factor, country’s share of that effective factor with share of world GDP should be compared.

If share of an effective factor is less than its share of world GDP, then that country is abundant in that effective factor and if share of an effective factor is less than its share of world GDP, then that country is scarce in that effective factor.

One way to measure the effective R&D scientists is through a country’s R&D spending per scientist. By taking the total number of scientists and multiply that by the R&D spending per scientists gives effective R&D scientists. With these productivity corrections, the U.S. is more abundant in effective R&D scientists and China is lower. Similarly, effective arable land is the actual amount of arable land times the productivity in agriculture. The U.S. has a
very high productivity in agriculture where China has a lower productivity. The U.S. is neither scarce nor abundant in effective arable land.

Now coming to the left hand side of the equation, to measure factor content of trade, Feenstra and Taylor looked at data similar to Leontief. Multiplying his numbers by actual values of US exports and imports gives the values for total exports and imports. Now the values obtained are called the factor content of imports and factor content of exports and taking the difference between the two would give net factor content of exports.

**Table 4**

<table>
<thead>
<tr>
<th></th>
<th>Exports</th>
<th>Imports</th>
<th>Net Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For $1 million exports</td>
<td>Total exports</td>
<td>For $1 million imports</td>
</tr>
<tr>
<td>Capital ($ Million)</td>
<td>$2.55</td>
<td>$42,600</td>
<td>$3.1</td>
</tr>
<tr>
<td>Labour (person years)</td>
<td>182</td>
<td>3.04 million</td>
<td>170</td>
</tr>
<tr>
<td>Capital/Labour</td>
<td>$14,000</td>
<td>$14,000</td>
<td>$18,200</td>
</tr>
</tbody>
</table>

Here both the factor contents are positive (table 4), we can conclude that US was running a trade surplus.

**3. Data and Data Sources:** The paper goes for complete test on a cross section for the year 2009 on a sample of 46 countries. Ten important manufacturing sectors are selected from the HS classification of trade data. Numbers of industries belonging to one category are combined. The list is provided in Appendix A. The data required for complete test to prove if the HOV theorem applies to world trade is obtained from WITS as above and factor endowments data is procured from different sources. In the present study, we have taken endowment data of five factors. For labour endowment in the form of human capital, the data is obtained from Barro and Lee data set. This data set presents the percentage of different educational attainments by those over 25 years of age in total population. We used these figures to construct our endowment variables. The data in percentages was converted to levels by using relevant population figures. HI stands for the number of people who have graduated from primary school and those who have received some degree of secondary
education without a diploma. H2 stands for those people who graduated from high school and did not continue higher education as well as those who received some years of higher education but did not graduate. H3 represents that part of the population which completed higher education.

For the capital stock data, the perpetual inventory method is used to construct it. Method for constructing capital stock series is provided in Appendix B and data for the year 2009 is received from the capital stock series. Land data is obtained from Food and Agriculture Organisation. The GDP data to estimate factor share is obtained from world development indicators.

4. Methodology: For the complete testing of HOV theorem on world trade, excess supply approach is adopted. As described earlier, the major problem with the HOV testing is the availability of the data and the calculation of technology matrix is another challenge. The crucial assumption for calculating the technology matrix is that it should be a square matrix, i.e. the number of factors should be equal to the number of goods otherwise the inversion of matrix will not be possible. This is an unrealistic assumption as usually numbers of goods are greater than number of factors.

Alternatively, one can use the excess supply function side to prove the theory.\textsuperscript{iv} The establishment of relationship between trade and endowments requires the link between output and endowment. The Rybczynski theorem says that at constant relative goods prices, a rise in the endowment of one factor will lead to a more than proportional expansion of the output in the sector which uses that factor intensively, and an absolute decline of the output of the other good.\textsuperscript{v} To prove this, we begin with the GDP function. The GDP function records the maximal income that a country can achieve if facing the vector $p$ of commodity prices and vector $v$ of factor endowments. According to the accounting identity, the total value of GDP equals the payment made to the primary factors. The payment made to the factors should be such that the cost of production of the goods should not be less than the price of the goods. Therefore, this dual identity can be written as:\textsuperscript{vi}:

$$GDP(p,V) = yp = wV \quad \text{(19)}$$

where $y$ is the vector of commodity output and $w$ is the vector of payments to factors of production. Now the aim is to maximize output $y$ subject to the constraint of endowment and
to minimize \( w \) subject to the constraint that cost should not be more than \( p \). Differentiating GDP with respect to price gives:

\[
\frac{\partial GDP}{\partial p_i} = y_i + \sum_j \frac{\partial y_j}{\partial p_j} p_j = y_i \\
\]  

…………………………………….. (20)

here the totality term under the summation sign vanishes as a condition of maximization. Now differentiating GDP function with respect to endowment gives:

\[
\frac{\partial GDP}{\partial V_j} = w_j + \sum_i V_i \frac{\partial w_i}{\partial V_j} = w_j \\
\]  

……………… (21)

Finally, differentiating GDP function in equation (21) with respect to endowment of factor gives:

\[
\frac{\partial^2 GDP}{\partial p_i \partial V_j} = \frac{\partial y_j}{\partial V_j} \\
\]  

…………………………………….. (22)

And differentiating equation (22) with respect to price of the commodity gives:

\[
\frac{\partial^2 GDP}{\partial p_i \partial V_j} = \frac{\partial w_j}{\partial p_i} \\
\]  

…………………………………….. (23)

Young theorem implies that

\[
\frac{\partial^2 GDP}{\partial p \partial V} = \frac{\partial^2 GDP}{\partial V \partial p} = \frac{\partial y_j}{\partial V_i} = \frac{\partial w_j}{\partial p_i} \\
\]  

…………………………………….. (24)

Samuelson called the relation as “reciprocity relation”. This is the whole explanation of supply side of the economy. Now coming to the demand side it is assumed that tastes are homothetic, thus expenditure on goods is a constant fraction of income.

\[
e(p,u) = \beta GDP_i \\
\]  

…………………………………….. (25)

where GDP is the function of price of goods and factor endowments. Now trade can be written as:
Differentiating equation (26) with respect to endowment provides

$$\frac{\partial T}{\partial V} = \frac{\partial y}{\partial V} - \beta \frac{\partial GDP}{\partial V} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (27)$$

From equation (26),

$$\frac{\partial T}{\partial V} = \frac{\partial y}{\partial V} - \beta w_i \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (28)$$

Multiplying equation (28) with \( y_j \) affords

$$\frac{\partial T_i}{\partial V_i} y_j = \frac{\partial y_j}{\partial V_i} y_j - \beta w_i \frac{V_i}{y_j} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (29)$$

In case of no trade, consumption equals production:

$$e(p,V) = \beta GDP = y_i \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (30)$$

therefore,

$$\beta = \frac{y_i}{GDP} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (31)$$

Substituting the expression (13) in expression (11) furnishes:

$$\frac{\partial T_i}{\partial V_i} y_j = \frac{\partial y_j}{\partial V_i} y_j - w_i \frac{V_i}{GDP(p,V)} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (32)$$

Where \( \frac{w_i V_i}{GDP(p,V)} \) is the share of factor \( i \) in national income and \( \frac{\partial y_j}{\partial V_i} y_j = \frac{\partial \ln y}{\partial \ln V} \) is the percentage increase in output of \( j \) due to a 1% increase in the endowment of factor \( i \). So it is the Rybczynski elasticity of output of \( j \) with respect to the endowment \( i \). Rybczynski effects can also be obtained by translog production function. The second term on the RHS shows the percentage increase in GDP caused by a 1% increase in the endowment of \( i \). With the
homothetic preferences; it is also the percentage increase in demand for each good. Equation (32) has the implication that an increase in the endowment of a factor increases the production of goods which uses it intensively and thus its consumption. The theory is generalized as a correlation between factor intensities, endowments and net trade level.

5. Results and Discussion: The result of complete test for 46 countries is shown below.

5.1 Trade and Endowments: The equation to be estimated for a particular industry is

\[ T^{ci} = \phi 0 + \phi H_1^c + \phi H_2^c + \phi H_3^c + \phi K^c + \phi Land^c + \phi R & DSci^c \] ........(33)

Where \( T^{ci} \) stands for trade of country \( c \) in industry \( i \). \( H_1^c, H_2^c, H_3^c \) are three categories for human capital endowments. \( K \) stands for capital stock and land (1000 hectares) in country \( c \). The estimated coefficients for each industry for the year 2009 are shown in the Table 5. A positive coefficient indicates that particular endowment creates comparative advantage and negative coefficient shows that particular endowment creates comparative disadvantage. The explanation of negative coefficient also shows that an increase in particular endowment increases the domestic demand for that good more than its production. Table 5 depicts the contribution of different factors in ten major industries. Industries are clubbed into one category from HS classification.

<table>
<thead>
<tr>
<th>Industries</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>Capital</th>
<th>Land</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mineral products</td>
<td>-0.33</td>
<td>-0.07</td>
<td>-0.57</td>
<td>-0.35</td>
<td>2.82</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.04)**</td>
<td>(0.06)</td>
<td>(0.10)**</td>
<td>(0.18)*</td>
<td>(0.37)**</td>
<td></td>
</tr>
<tr>
<td>Chemical products</td>
<td>-0.15</td>
<td>0.14</td>
<td>0.45</td>
<td>0.80</td>
<td>0.04</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(0.07)**</td>
<td>(0.12)</td>
<td>(0.20)**</td>
<td>(0.36)**</td>
<td>(0.72)</td>
<td></td>
</tr>
<tr>
<td>Plastics and rubber products</td>
<td>-0.18</td>
<td>0.60</td>
<td>0.25</td>
<td>0.93</td>
<td>-0.02</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(0.05)**</td>
<td>(0.08)**</td>
<td>(0.14)*</td>
<td>(0.24)**</td>
<td>(0.48)</td>
<td></td>
</tr>
<tr>
<td>Leather products</td>
<td>0.16</td>
<td>-0.78</td>
<td>0.40</td>
<td>0.28</td>
<td>0.96</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.03)**</td>
<td>(0.05)**</td>
<td>(0.07)**</td>
<td>(0.13)**</td>
<td>(0.26)**</td>
<td></td>
</tr>
<tr>
<td>Wood industry</td>
<td>-0.08</td>
<td>0.15</td>
<td>0.14</td>
<td>0.30</td>
<td>2.42</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.12)</td>
<td>(0.20)</td>
<td>(0.36)</td>
<td>(0.72)**</td>
<td></td>
</tr>
</tbody>
</table>
The results reveal that in all the industries, the coefficient of capital stock is positive and significant except in stone/glass and mineral products. This shows that the rise in overall capital endowment enhances exports of particular industry and creates comparative advantage. The coefficient of H1 is positive for leather, stone/glass, textiles and machinery and for the rest of the industry it is negative. The secondary educational level also proves to be positive and significant for almost 50% of the cases. Here it is notable that higher education is creating comparative advantage in almost all industries except in mineral products. Land endowments are also creating comparative advantage for the world trade.

5.2 Factor Shares: As derived earlier, the impact of factor endowments on trade can be split into the impact of endowment on production and consumption. In this section, we estimate factor shares by following equations:

\[ GDP^c_i = \pi_0 + \pi H_1^c + \pi H_2^c + \pi H_3^c + \pi K^c + \pi Land_i^c + \pi R & DSci^c \] ....(34)

The results of the estimations are provided in the Table 6.

### Table 6

<table>
<thead>
<tr>
<th>Industry</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>Capital</th>
<th>Land</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Textile industry</td>
<td>0.19</td>
<td>-0.81</td>
<td>0.38</td>
<td>0.11</td>
<td>0.59</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.01)***</td>
<td>(0.02)***</td>
<td>(0.04)***</td>
<td>(0.07)</td>
<td>(0.14)***</td>
<td></td>
</tr>
<tr>
<td>Stone/Glass industry</td>
<td>0.003</td>
<td>-0.58</td>
<td>0.67</td>
<td>-0.12</td>
<td>1.86</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.08)***</td>
<td>(0.12)***</td>
<td>(0.23)</td>
<td>(0.45)</td>
<td></td>
</tr>
<tr>
<td>Metal industry</td>
<td>-0.16</td>
<td>0.02</td>
<td>0.16</td>
<td>0.90</td>
<td>3.34</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(0.05)***</td>
<td>(0.09)</td>
<td>(0.15)</td>
<td>(0.26)***</td>
<td>(0.54)***</td>
<td></td>
</tr>
<tr>
<td>Machinery</td>
<td>0.05</td>
<td>-0.64</td>
<td>0.86</td>
<td>0.72</td>
<td>1.03</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.02)**</td>
<td>(0.03)***</td>
<td>(0.06)***</td>
<td>(0.11)***</td>
<td>(0.22)***</td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>0.11</td>
<td>0.17</td>
<td>0.81</td>
<td>1.49</td>
<td>0.86</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.05)**</td>
<td>(0.08)**</td>
<td>(0.13)***</td>
<td>(0.23)***</td>
<td>(0.46)*</td>
<td></td>
</tr>
</tbody>
</table>

*Significant at 10%;**Significant at 5%;***Significant at 1%;Values in the parenthesis show standard error.
Using the estimated coefficients, we can estimate output of each industry by putting in average endowment to the following expression:

\[ q^{ci} = (\phi_0 + \pi_0) + (\phi_1 + \pi_1 / G^c)H_1^c + (\phi_2 + \pi_2 / G^c)H_2^c + (\phi_3 + \pi_3 / G^c)H_3^c + (\phi_K + \pi_K / G^c)K^c + (\phi_{\text{Land}} + \pi_{\text{Land}} / G^c)\text{Land}^c \]

We have taken into account only significant coefficients for the calculation. Using the estimates of \( q^{ci} \) we prepared technological coefficient matrix \( a_{ij} \) by \( V_i / q_i \).

5.3 Testing of HOV Theory: In the empirical testing of HOV, we have run correlations between endowments and trade for each industry analysed above. Data for only forty-six countries are taken into account because of limited availability of all variables. Net trade is weighted by the average of technological coefficients. Next the average of the difference between actual endowment and world endowment multiplied by ratio of country’s GDP to world GDP is calculated. The correlation results for each industry are provided in Table 7.

Table 7

<table>
<thead>
<tr>
<th>Industries</th>
<th>Correlation Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mineral products</td>
<td>0.30</td>
</tr>
<tr>
<td>Chemical products</td>
<td>0.81</td>
</tr>
<tr>
<td>Plastics and rubber products</td>
<td>0.68</td>
</tr>
<tr>
<td>Leather products</td>
<td>-0.23</td>
</tr>
<tr>
<td>Wood industry</td>
<td>0.61</td>
</tr>
<tr>
<td>Textile industry</td>
<td>-0.42</td>
</tr>
<tr>
<td>Stone/glass industry</td>
<td>0.20</td>
</tr>
<tr>
<td>Metal industry</td>
<td>0.94</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.89</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Results suggest that HOV theorem applies to more than 50% of cases and in almost all the cases the results are positive. This proves that HOV theorem is still valid.

6. Conclusion: HOV testing is done for world trade along with India through complete test. The complete test results reveal that in world trade pattern, the stock of capital and secondary and higher educated labour is the major source of comparative advantage. This shows that world trade and production patterns seem to increase the requirement for more educated labour force. HOV theorem comes out to be applicable to world trade pattern and the study gives useful insights about what are the factors which are playing a crucial role in determining the world production pattern. In summary, it can be said that factor endowments of a country play a dominant role in determining the trade pattern of that country. Thus, it is important to make policies regarding improvement of education level and technical skills, etc.
References


Bin, X. And W. Li. 2007.’Trade, Technology and China’s Rising Skill Demand.’ *Economics of Transition* 16(1): 59-84.


### Appendix A

#### Clubbing of Industries into single category

<table>
<thead>
<tr>
<th>Industries</th>
<th>Industries According to HS Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mineral Products</strong></td>
<td>Salt, Sulphur, Earth &amp; Stone, Lime &amp; Cement (25), Ores, slag and Ash (26), Minerals, Fuel oils, waxes and Bituminous subs (27)</td>
</tr>
<tr>
<td><strong>Chemicals and Allied Industries</strong></td>
<td>Inorganic Chemicals, Organic/Inorganic compounds of precious metals and isotopes (28), Organic Chemicals (29), Pharmaceutical Products (30), Fertilizers (31), Tanning or Dyeing extracts, Dyes, pigments, Paints &amp; varnishes, Putty, &amp; Inks (32), Oils &amp;Resinoids, Perfumery, Cosmetic or toilet preparations (33), Soaps, Waxes, Scouring products, Candles, Modeling pastes, Dental waxes (34), Albuminoidal sub, Starches, Glues, Enzymes (35), Explosives, Matches, Pyrotechnic products (36), Photographic or Cinematographic goods (37), Miscellaneous chemical products (38)</td>
</tr>
<tr>
<td><strong>Plastics/Rubbers</strong></td>
<td>Plastics &amp; articles thereof (39), Rubbers &amp; articles thereof (40)</td>
</tr>
<tr>
<td><strong>Leather Industry</strong></td>
<td>Raw hides &amp; skins &amp; leather (41), Articles of leather, saddlery &amp; harness, travel goods, Handbags, Articles of gut (42), Furskins&amp; artificial fur manufactures (43)</td>
</tr>
<tr>
<td><strong>Wood and Wood Products</strong></td>
<td>Wood &amp; articles of wood, Wood charcoal (44), Cork &amp; articles of cork (45), Manu. Of straw, esparto, or other plaiting materials, Basketware and Wickerwork (46), Pulp of wood, waste &amp; scrap of paper (47), Paper &amp; paperboard, articles of paper pulp (48), Printed books, newspapers, pictures, manuscripts, typescripts &amp; plans (49)</td>
</tr>
<tr>
<td><strong>Textile Industry</strong></td>
<td>Silk, inc. Yarns &amp; woven fabrics thereof (50), Wool &amp; fine or coarse animal hair, inc. Yarns &amp; woven fabrics thereof (51), Cotton, inc. Yarns &amp; woven fabrics thereof (52), Veg. Textile</td>
</tr>
<tr>
<td>Fiber/Textile</td>
<td>Articles of stone, plaster, Cement, asbestos, mica or similar materials (68), Ceramic products (69), Glass &amp; glassware (70), Pearls, stones, prec. Metals, imitation jewelry, coins (71)</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Stone/Glass</td>
<td></td>
</tr>
<tr>
<td>Metals</td>
<td>Iron &amp; steel(72), articles of iron or steel (73), copper &amp; articles thereof(74), nickel &amp; articles thereof (75), aluminum&amp; articles thereof (76), lead &amp; articles thereof (78), zinc &amp; articles thereof (79), tin &amp; articles thereof (80), base metals nesoi, cermets, articles etc.(81), tools, spoons &amp; forks of base metal (82), miscellaneous articles of base metal (83)</td>
</tr>
<tr>
<td>Machinery and Electrical</td>
<td>Nuclear reactors, boilers, machinery &amp; mechanical appliances, computers (84), electrical machinery &amp; equip. &amp; parts, telecommunications equip., sound recorders, television recorders (85)</td>
</tr>
<tr>
<td>Transportation</td>
<td>Railway or tramway locomotives, rolling stock, track fixtures &amp; fittings, signals (86), vehicles other than railway or tramway rolling stock (87), aircraft, spacecraft, &amp; parts thereof (88), ships, boats, &amp; floating structures (89)</td>
</tr>
</tbody>
</table>
Appendix B

We measure the capital stock series in the form

\[ K(t) = K(t-1) + I(t) - D(t), \] \( (A1) \)

where \( K(t) \) is the real capital stock at period \( t \), \( I(t) \) is the real gross fixed investment, and \( D(t) \) is the real capital depreciation allowance.

We calculate the initial stock" by,

\[ K(1) = \frac{I(0)e^\theta}{\theta}, \] \( (A2) \)

where \( I(0) \) and \( \theta \) are the estimated coefficients of the constant term and time in the following form, by ordinary least squares estimation:

\[ \ln I(t) = C + \theta \text{Time} \] \( (A3) \)

The estimation is that (1) the capital stock in the first period is the sum of all past investments:

\[ K(1) = \int_{t=0}^{1} I(t)dt \] \( (A4) \)

and (2) the investment series may be approximated by an exponential time trend:

\[ I(t) = I(0)e^{\theta t} \] \( (A5) \)

Inserting equation (A5) into equation (A4) yields equation (A2). Taking natural logarithms of equation (A5), we obtain equation (A3) where the constant term c is \( \ln I(0) \)

\[ \text{Baldwin Book} \]

\[ \text{Maskus provided three nonparametric methods to test the HOV theorem. First one is weak HOV prediction: it only compares the sign of the right hand and left hand side of the equation. So, if country is abundant in a factor, it should export it and import it is country is scarce in that factor. Second method is the rank test. Factors which are abundant have to be exported relatively more than less abundant ones and the third method is the strong HOV prediction. It tests whether the extent of net exports is consistent with the extent of world consumption. Under the assumption of balanced trade, relative consumption of each good in the US has to be equal to the rest of the world.} \]

\[ \text{Feenstra and Taylor in their book on International Trade (2007), chapter-IV on Heckscher Ohlin model have illustrated measurement of effective factor endowment using productivity adjustments.} \]

\[ \text{Harrigan (1995)} \]

\[ \text{Wikipedia} \]

\[ \text{Ethier, Higher Dimensional Issues in Trade Theory} \]