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A THEORY OF ANTITRUST ENFORCEMENT GAME*

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Abstract

We analyze a situation where an antitrust authority delegates to an audit inspector the mission of gathering the sufficient information to condemn a cartel. The authority has two instruments at her disposal: rewarding the inspector with a proportion of the collected fine or providing him with information which enhances the probability of the success of the prosecution. More precisely, we explore the efficiency consequences of a contest between the audit inspector and the cartel. Both of them bid to win the contest by expending efforts. We show that the race issue depends positively on the financial incentives proposed to the inspector but the impact of an increase of the level of the fine, to be paid once an illegal agreement is detected, is ambiguous. Moreover, we show that the optimal combination of the two instruments consists in two regimes. When the marginal cost of providing the relevant information is relatively high, the antitrust authority equally shares the collected fine and does not provide the inspector with any information. Conversely, when this marginal cost is relatively small, the authority uses the two instruments. She has to provide him with the maximum level of information consistent with winning the contest with certainty.

Keywords: Antitrust Enforcement, Collusion, Moral Hazard, Contest.

JEL Classification: L40, K42.
1. Introduction

One of the main difficulties in formulating antitrust policy is the lack of information on possible restrictive agreements and on firms’ generated effort to conceal unlawful actions. In practice the antitrust authorities are subject to two sorts of constraints: limited resources and imperfect information. Because resources are limited, the authorities cannot monitor all the markets and investigate all the firms which are suspected of colluding. The second problem is that markets are rarely transparent. The authorities do not perfectly observe the characteristics and behavior of the firms. This asymmetry of information is the source of adverse selection and moral hazard problems that reduce the efficiency and impact of public interventions.

Moreover, whenever an authority delegates enforcement of audit policy, opportunities for corruption arise. Therefore, in order to limit non benevolent behavior towards firms, the authorities motivate the delegated inspector by paying him a reward per dollar of fine (Becker and Stigler, 1974). Detection of violation of competition laws appears therefore as a complex double moral hazard game in which we explore the efficiency consequences of a contest between a delegated agency and some "collusive firms". We focus on the trade off between the optimal reward rate, the delegated agency’s enforcement effort and firms’ “secret” effort to hide restrictive agreements.

Even though our analysis is done in an antitrust framework, it can be applied to other areas. The basic situation is the following one. Suppose that an authority suspects an agent of infringing a law. It then delegates to an inspector the task of collecting sufficient documentary evidence to convict the agent. The question is then how to give the right incentives to the inspector in order to expend the optimal effort of detection when the agent can hide this documentary evidence. This issue can arise in several contexts. We can cite for example tax evasion, pollution, minimum wage enforcement, cost falsification in insurance, etc.

Our analysis is complementary to the one used in some models of antitrust enforcement. Pénard and Souam (2002) generalize the work of Besanko and Spulber (1989) and analyze the optimal policy from the point of view of the competition authorities in order to efficiently

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1 See for example Milgrom and Robert (1988) and Mookherjee and Png (1995)
deter collusion in a given industry (in a context of asymmetric information between the two parties about the cost of production of the industry). By devoting more resources to control activities, the authorities can increase the probability of detecting collusion and gather the necessary evidence to condemn the firms involved in such activity. There is a trade-off between the number of interventions and their effectiveness. A smaller number of interventions can allow the authorities to devote more resources to each intervention, thereby increasing the probability of success of the interventions that are undertaken. But in doing so, they leave more markets without monitoring. Pénard and Souam (2002) consider that the cost of detecting, prosecuting and fining the guilty firms is exogenous and characterize the best policy, from the authorities’ point of view, within this framework.

Our contribution in this paper can be considered as an attempt to explain the origins of this cost. By opening this black box, we shed light on an important aspect of the antitrust process. We show that the race issue depends on the financial incentives proposed to the delegated agency but does not always depend on the level of the fine to be paid once an illegal agreement is detected. We also emphasize the fact that financial incentives can be substituted by available economic data\(^2\) for the delegated agency (information provision).

The paper is organized as follows. Section 2 presents the model and the assumptions, section 3 analyzes the incentives to undertake efforts and characterizes the optimal antitrust policy in an asymmetric environment. Section 4 shows how should be the optimal combination between the two instruments used in our analysis (financial incentives vs. information provision). Finally, in section 5 we conclude and give some directions for future research.

\(^2\) Available economic data correspond for instance to price data or industry output observations or any relevant information which summarizes the behavior of the industry in the past. Collecting this is of course costly.
2. The Model

We consider a two-player game. Player 1 is a delegated antitrust agency called the audit inspector. Player 2 represents some "collusive firms". We assume the game played by the firms to be per se illegal. Indeed, competition policy in most modern economies typically makes price-fixing illegal, even if this was not always the case. 

Moreover, we consider that the inspector can obtain information about potential violations of antitrust laws and can detect a violation with a probability $0 \leq P(e_1, e_2, m) \leq 1$. This probability of detection of a violation of antitrust laws depends on the enforcement effort $e_1$ of the audit inspector, the effort $e_2$ exerted by the collusive firms in order to conceal unlawful actions and on the economic data (information) $m$ available to the audit inspector once he decides to investigate a potential violation case. If the firms are found to have colluded, they are sued and condemned to pay an exogenous amount of fine $F$.

From a social point of view, collusion is bad per se but its deterrence is costly. There then exists a problem of how to give the right incentives to the audit inspector in order to expend effort in detecting price agreements. We assume that the audit inspector is able to determine on the basis of documentary evidence whether explicit collusion has taken place. However, he only wins the contest with certainty when the result is not contested by the "collusive firms".

For the sake of computational simplification, we do assume that the contest itself is modeled as a "lottery auction" whereby the probability of detecting the violation is determined by a unit-logit function:

$$P(e_1, e_2, m) = \frac{(1 + m)e_1}{(1 + m)e_1 + e_2} \quad \text{for each } m \geq 0$$

This is now a traditional approach and is used in different contexts. Both players "bid" to win the antitrust decision. The bid takes the form of efforts expended by the audit inspector:

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3 Historically, collusion was legal in many European countries such as Germany and the Netherlands.
4 The commitment of the authorities is assumed to be credible. This is similar to an extensive literature on random audit in the case of incomplete information. See for example Baron and Besanko (1984) and Reinganum and Wilde (1985).
5 Hereafter, we give more general results when it is possible.
and the "collusive firms" in the contest. Moreover, both players are assumed to be risk neutral.

The antitrust authority motivates the audit inspector by paying him a reward $\gamma$ per unity of collected fine, and by giving him a fixed wage $w$. The enforcement effort generates a cost $\psi_i(e_i)$ for the audit inspector.

The expected utility of the audit inspector is given by:

$$U = (1 - P(e_1; e_2, m))w + P(e_1, e_2, m)(w + \gamma F) - \psi_1(e_1)$$

$$= w + P(e_1, e_2, m)\gamma F - \psi_1(e_1)$$

(2)

We assume that the inspector's objective function is strictly concave in the enforcement effort $e_i$.

Given the effort $e_i$, the firms' problem is to maximize their expected profit net of the cost of concealing the unlawful actions:

$$V = (1 - P(e_1; e_2, m))\pi + P(e_1, e_2, m)(\pi - F) - \psi_2(e_2)$$

$$= \pi - P(e_1, e_2, m)F - \psi_2(e_2)$$

(3)

where $\pi$ is the firms' profit generated by the collusive behavior, and $\psi_2(e_2)$ is the disutility generated by the hidden restrictive agreement, which can consist in the burden cost to reach such agreement and the effort exerted to contest the race issue. $V$ is also supposed to be concave in $e_2$.

3. Characterization of the Equilibrium

Let us now analyze the incentives to expend efforts for the "collusive firms" and the audit inspector. We will also characterize the optimal antitrust policy within the context of asymmetric information considered in this paper.

Let us consider the maximization program of the delegated agency. We assume the inspector to be a leader relatively to the "collusive firms"\(^7\). The audit inspector chooses an

\(^7\) In practice, when a preliminary investigation is launched the delegated agency (for example DGCCRF, Direction Générale de la Concurrence, de la Consommation et de la Répression des Fraudes, in France) expend effort through its inspectors in order to gather evidence of an infringement. Thus firms will thus try or not to hide information about their illegal activity if any. This is why we model the contest as a Stackelberg game in our ex post intervention framework. Nonetheless, our results are still valid under simulataneous contest game as we will mention it hereafter.
enforcement effort $e_i$ in order to maximize the net expected returns given the firms’ reaction function:

$$\text{Max } U = w + + P(e_1, e_2, m) \psi F - \psi_1(e_1)$$

s.t. $e_2 = \text{Arg Max } V = \pi - P(e_1, \bar{e}_2, m) F - \psi_2(\bar{e}_2)$ (4)

In order to exhibit an explicit solution of this program, let us assume that $\psi_i(e_i) = \psi_i e_i$ for i = 1; 2. $\psi_i$ is considered as the marginal cost of increasing the effort from the point of view of the agent i.

Hereafter, we first look for the optimal effort chosen by the firms in order to conceal unlawful actions. The following lemma gives the collusive firms’ effort level expended in the contest game depending on the effort expended by the audit inspector.

Lemma 1
The optimal firms’ effort to hide restrictive agreements is given by:

$$e_2^* = \sqrt{\frac{(1 + m)e_1 F}{\psi_2} - (1 + m)e_1} \text{ if } e_1 \leq \frac{F}{(1 + m)\psi_2}$$

$$e_2^* = 0 \text{ otherwise } .$$

Proof.
Let us consider the first order condition when maximizing the firms’ profit. This is given by:

$$V'(e_2) = \frac{\partial}{\partial e_2} (\pi - P(e_1, e_2, m) F - \psi_2(e_2)) = -\frac{\partial P}{\partial e_2}(e_1, e_2, m) - \psi_2 = 0$$

if the optimum is an interior solution. Using the definitions of the functions, one can easily see that this is equivalent to:

$$\frac{(1 + m)e_1 F}{((1 + m)e_1 + e_2)^2} = \psi_2$$

which gives a positive value for $e_2^* = \sqrt{\frac{(1 + m)e_1 F}{\psi_2} - (1 + m)e_1} \text{ if } e_1 \leq \frac{F}{(1 + m)\psi_2} .$

If this last inequality is not verified, we have $V'(e_2) < 0$ for every $e_2$. The optimal solution is then $e_2^* = 0$. (Q.E.D.)
The firms’ reaction function is continuous but not monotonic in the enforcement effort. This non monotonicity is of some interest in our context. For comparatively low levels of contest effort $e_1$ by the audit inspector, the ”collusive firms” respond to increases in that level by increasing their own concealing effort $e_2$. Beyond some threshold level, however they react to further increases by decreasing their own effort. Moreover, let us remark that the firms ”fight their corner” more strongly when penalty $F$ is high, than when it is low. The converse is true for the marginal cost of effort $\psi_2$.

Let us now derive the optimal enforcement effort of the inspector in the contest. This is the purpose of the following lemma.

**Lemma 2**

*The optimal inspector’s effort is given by*:

$$
e_1^* = \left(\frac{\gamma}{2\psi_1}\right)^2 F \left(1 + m\right)\psi_2 \quad \text{if} \quad \gamma \leq \text{Min} \left(\frac{\psi_2}{\psi_2 \left(1 + m\right)}, 1\right) = \gamma(m)
$$

$$
e_1^* = \frac{F}{\left(1 + m\right)\psi_2} \quad \text{otherwise}
$$

**Proof.**

As lemma 1 stated it, when $e_1 \leq \frac{F}{\left(1 + m\right)\psi_2}$ we have $e_1^* = \frac{\left(1 + m\right)e_1 F}{\psi_2} - \left(1 + m\right)e_1$

The probability of detection is thus:

$$
P(e_1, e_2^*, m) = \frac{(1 + m)e_1}{(1 + m)e_1 + e_2^*} = \sqrt{\frac{(1 + m)\psi_2}{F}} e_1
$$

and the expected utility of the inspector is:

$$
U = w + \alpha P(e_1, e_2^*, m) F - \psi_1 e_1 = w + \gamma \sqrt{\psi_2 (1 + m)F} e_1 - \psi_1 e_1
$$
We then have an interior optimum at

\[ e_1^* = \left( \frac{\gamma}{2\psi_1} \right)^2 F \left( 1 + m \right) \psi_2 \]  

when this is less than

\[ \frac{F}{(1 + m)\psi_2} \] .

One can easily show that this the case when: \( \gamma \leq \text{Min} \left( \frac{2\psi_1}{\psi_2(1 + m)}, 1 \right) \).

When \( e_1 \geq \frac{F}{(1 + m)\psi_2} \); the effort exerted by the "collusive firms" is null (\( e_2^* = 0 \)).

We then have \( U = w + \gamma F - \psi_1 e_1 \) . It is straightforward to see that the expected utility \( U_1 \) decreases with \( e_1 \) . The optimal solution is then to set \( e_1 \) at its minimum possible level (i.e.

\[ e_1 = \frac{F}{(1 + m)\psi_2} \] .

(Q.E.D)

Lemma 2 explicitly gives us the inspector’s enforcement effort as a function of the rate of reward \( \gamma \) . Any change in the reward system affects the enforcement effort of the inspector. It is worth noting that for low values of \( \gamma \) this effort increases with the level of the information \( m \) and the marginal cost of the firms ‘expending effort \( \psi_2 \) while for high levels of \( \gamma \), the contrary happens. Moreover, this effort intuitively increases with the level of the fine \( F \) and decreases with \( \psi_1 \).

The following proposition characterizes the optimal firms’ effort and derives the likelihood of detecting restrictive agreements in the context of our race model.

**Proposition 3**

i). *The optimal collusive firms’ effort to conceal unlawful action is defined by:*

\[ e_2^* = \frac{\gamma(1 + m)F}{4\psi_1^2} \left( 2\psi_1 - \gamma(1 + m)\psi_2 \right) \text{ if } \gamma \leq \gamma(m) \]

\[ e_2^* = 0 \text{ otherwise} \]

ii). *The probability of detecting unlawful agreement is given by:*

\[ P^*(\gamma, m) = \frac{(1 + m)\psi_2}{2\psi_1} \gamma \text{ if } \gamma \leq \gamma(m) \]

\[ P^*(\gamma, m) = 1 \text{ otherwise}. \]
Proof.

If $\gamma \leq \gamma(m)$, we have $e_1^* = \left(\frac{\gamma}{2 \psi_1}\right)F(1 + m)\psi_2$ and $e_2^* = \sqrt{\frac{(1 + m)\psi_1 F}{\psi_2} - (1 + m)\psi_1}$. A simple substitution of the value of $e_1^*$ in the second formula gives the equilibrium value of $e_2^* = \frac{\gamma(1 + m)F}{4 \psi_1^2}(2\psi_1 - \gamma(1 + m)\psi_2)$. Moreover, the probability of detection can be written as a function of only $\gamma$ and $m$, $P^*(\gamma, m) = \frac{(1 + m)\psi_2}{2 \psi_1}$. Otherwise, when $\gamma \geq \gamma(m)$, the optimum level of the firms’ effort is null and $e_1^* = \frac{F}{(1 + m)\psi_2}$. The probability of detection is thus equal to 1 and the authorities are certain to win the race issue. (Q.E.D)

Let us remark that in equilibrium a marginal increase in the reward rate $\gamma$ induces an increase in the probability of detection. Moreover, as the intuition suggests the probability of detection increases with $m$ and $\psi_2$ and decreases with $\psi_1$.

However, it is important to notice that the likelihood of detection is independent of the fine level $F$ in our context. This could be seen as a source of divergence with ex ante deterrence models à la Becker which would recommend to put the level of fine as high as possible.\footnote{In a different enforcement model, Malik (1990) shows that the costs associated with avoidance activities, that reduce the probability of being caught and fined, imply that it is not necessarily optimal to set fines for offenses as high as possible.} This quite counterintuitive result can be explained by the fact that even though an increase in the level of the fine will give more incentives to the audit inspector to expend effort to find evidence of an illegal activity, the “colluding firms” are also incited to enhance their effort to win the race (the firms ”fight their corner” more vigorously). The latter effect exactly counterbalances the former in our example.

This explains why the contest issue depends on the financial incentives proposed to the audit inspector while it does not depend on the level of the fine. This interesting result still holds under a simultaneous contest.

What can we say from a general point of view about this result? Of course, our extreme result is due to the choice of a logit function for the probability of detection. In general, the
derivative of the probability of detection w.r.t. the fine, \( \frac{\partial P^*}{\partial F} \), in equilibrium is quite complicated in our case and its sign is a priori ambiguous.

In order to simplify the analysis and show this ambiguity, let us consider the case of a simultaneous contest. In this case, the first order conditions are given: 

\[
\frac{\partial P}{\partial e_1} F = \psi_1 \quad \text{and} \quad -\frac{\partial P}{\partial e_2} F = \psi_2.
\]

At the optimum, the derivative of the probability w.r.t. the fine is given by

\[
\frac{\partial P^*}{\partial F} = \frac{\partial P}{\partial e_1} \cdot \frac{\partial e_1^*}{\partial F} + \frac{\partial P^*}{\partial e_2} \cdot \frac{\partial e_2^*}{\partial F}.
\]

By using the first order conditions, we can deduce that:

\[
\frac{\partial e_1^*}{\partial F} = -\frac{\frac{\partial P}{\partial e_1}}{\frac{\partial^2 P}{\partial e_1^2} F} \quad \text{and} \quad \frac{\partial e_2^*}{\partial F} = -\frac{\frac{\partial P}{\partial e_2}}{\frac{\partial^2 P}{\partial e_2^2} F}.
\]

This finally gives:

\[
\frac{\partial P^*}{\partial F} = -\left( \frac{\psi_1}{\gamma F} \right)^2 - \left( \frac{\psi_2}{F} \right)^2 = -\frac{1}{\gamma^2 F^3} \left( \psi_1^2 \frac{\partial^2 P}{\partial e_2^2} + \psi_2^2 \frac{\partial^2 P}{\partial e_1^2} \right).
\]

In this kind of literature the following properties are generally assumed: \( \frac{\partial^2 P}{\partial e_1^2} < 0 \) and \( \frac{\partial^2 P}{\partial e_2^2} < 0 \) (i.e. decreasing marginal returns to \( e_1 \) from the inspector’s perspective and to \( e_2 \) from the cartel’s perspective). The sign of \( \frac{\partial P^*}{\partial F} \) is thus given by the sign of

\[ \psi_1^2 \frac{\partial^2 P}{\partial e_2^2} + \gamma^2 \psi_2^2 \frac{\partial^2 P}{\partial e_1^2} \]

Depending on the different parameters and on the levels of the second derivatives, \( \frac{\partial P^*}{\partial F} \) can be positive, null or negative. We thus would like to emphasize the fact that once a decision of delegation is taken, the impact of an increase of the fine on the probability of a successful issue is a priori ambiguous. So, from an ex ante point of view putting the fine at its maximum level could be not optimal in certain cases and optimal in other ones. The firms’ effort is a concave function in the reward rate \( \gamma \). The more the financial incentives increase, the more the inspector’s enforcement effort increases and as a consequence, firms’ effort increases to reach a maximum level. Nevertheless, after the corresponding threshold, the inspector’s enforcement effort is so high that it is not in the interest of the firms to pursue their effort to contest the issue. They will therefore choose to lessen their effort when \( \gamma \) is too high. It is interesting here to make a parallel with the way this effort varies with the inspector’s effort as seen before. Concerning the probability of detection \( P^*(\gamma,m) \), let us mention that there exists a cut-off level which determines the efficiency of the detection (the certainty of the race issue) as shown in figure 3. Moreover, this cut-off reward share is a decreasing function of the economic data at the disposal of the inspector \( \left( Min\left( \frac{2\psi_1}{\psi_2(1+m)}1 \right) = \gamma(m) \right) \). Therefore for all positive m; available economic data can be considered as a substitute to the reward share proposed to the inspector. The question of the optimal mix of these two instruments is analyzed in the next section.

4. Monetary Incentives versus Information Provision

For a given level of the maximal fine \( F \) chosen by the legislator, one can ask the question of which combination (monetary incentives to the inspector ; provision of information) should the antitrust authority choose. This issue raises a practical problem which can be found in other contexts: which welfare function to use? Many alternative solutions are possible. In our context with an ex post intervention, one can think that a possible objective should be to maximize the expected amount of collected fine net of the reward given to the inspector and
of the cost of providing him with the information (supposed to be linear \( \psi, m \)), since the illegal activity has already taken place.

The net expected collected fine is given by:

\[
E(\text{Collected Fine}) = (1 - \gamma)P'(\gamma, m)F - \psi, m
\]

The following proposition gives the optimal combination from the antitrust authority’s point of view and shows that the solution is quite simple and intuitive.

**Proposition 4**

*The optimal combination consists possibly in two different regimes.*

1. When the marginal cost of providing the information is relatively high: \( \psi_3 > \frac{\psi_2}{8\psi_1} F \), the best policy is to share the fine equally with the audit inspector: \( \gamma^* = \frac{1}{2} \), and not to provide him with any information (first regime).

2. Conversely, when the marginal cost is relatively small: \( \psi_3 < \frac{\psi_2}{8\psi_1} F \), a second regime is optimal: give the highest share consistent with a probability of detection equal to unity and choose the level \( m^* \) accordingly.

**Proof:** See the appendix.

This proposition sheds some light on how the optimal combination between a monetary reward and an information provision should be chosen by the antitrust authority while delegating the gathering of evidence of anticompetitive practices. In this perspective, it is interesting to see that even if it is always possible in our framework to make sure that the authority surely wins the contest, it is not always in her interest to do that. This is basically due to the fact that when the marginal cost of providing the information is relatively high, the authority will not use this instrument. Thus she has only one instrument at her disposal: rewarding the audit inspector. But it is too costly to be sure to win the contest. The authority could do better: equally share the collected fine with the audit inspector without providing him with any information. In this case, the probability of detection is strictly less than 1: the race issue is uncertain.
Conversely, when the marginal cost of providing the information is relatively small, the authority will use both instruments. She will then provide the audit inspector with the maximum level of information consistent with winning the contest with certainty (the probability of detection equals unity). It then appears that the substitution between the two instruments is quite intuitive in our framework even though the way it should be designed is asymmetric relatively to the two instruments. Indeed, the authority should always give monetary incentives to the audit inspector while in some cases provision of information should not be used since it is too costly. Under this framework, it seems important to use both instruments if the antitrust authority wants to be sure to win the contest. Rewarding the audit inspector only is not sufficient to reach this target.

5. Conclusion

In this paper, we developed a strategic model of pure antitrust enforcement. We have studied the impact of a two-dimension effort game where the task of proving that an agent is guilty is delegated to an audit inspector. The antitrust authority can enhance the probability of detection of anticompetitive practices by using two instruments: rewarding the audit inspector and providing him with information. Our results are two-fold. First, we show that in our framework the outcome of the contest game (probability of winning the contest by the authority) depends positively on the financial incentives proposed to the inspector. However, the impact of an increase of the fine is a priori ambiguous on the probability of success. This last result seems to be quite general at least for the kind of rewarding scheme used in this article.

Second, we showed the existence of two regimes concerning the optimal combination between the two instruments (financial reward and information provision). Roughly speaking, the first regime consists in equally sharing the fine with the audit inspector and not providing him with information. In this case, the probability of the detection is strictly less than one which means that the firms are not completely deterred from hiding the relevant information and contesting the race issue. This is the case when the marginal cost of providing information is relatively high. Conversely, the second regime consists in using the two instruments. In this case, the authority can increase the probability of detection adequately by providing the audit inspector with information (whose marginal cost is relatively small). She still uses financial incentives. In this case, the reward is just sufficient to completely deter the firms from contesting the race issue. We think that an interesting development for our analysis
would be to couple the two approaches: ex ante intervention (deterrence models à la Becker) and the ex post intervention as developed in the present article. It would be interesting to shed some light on the potentially important divergence between these two kinds of intervention particularly in the antitrust process where at least some discretion is given to the authorities in order to challenge anticompetitive agreements between normally competing firms. It seems to be an exciting challenge to reconcile these approaches within a single model. Another interesting topic for future research is to analyze how different regimes of fines\textsuperscript{10} would change our results within the new mixed approach proposed. Finally, it would also be interesting to apply our analysis to the other domains mentioned in the introduction.

References


\textsuperscript{10}See Souam (2001) for an analysis in terms of deterrence and efficiency of the European regime where the fine to be paid is at maximum equal to a proportion of the gross revenue and the American regime where the fine can be trebled w.r.t. the damaged caused to the consumers.
The net expected collected fines are given by:
\[ E(Collected\ Fine) = EF = (1 - \gamma)P^*(\gamma, m)F - \psi, m \]

Two cases are then possible.

**Case 1:** \(0 \leq m \leq 2 \frac{\psi_1}{\psi_2} - 1\). In this case, \(\gamma(m) = 1\) and \(EF = (1 - \gamma)\frac{1 + m \psi_2}{2 \psi_1} F - \psi, m\).

The optimal solution for the authority in terms of sharing is then \(\gamma^* = \frac{1}{2}\) which gives an expected fine of \(EF = \frac{1 + m \psi_2}{8 \psi_1} F - \psi, m\). It is then easy to see that if \(\psi_3 > \frac{\psi_2}{8\psi_1} F\) the best solution is \(m^* = 0\) and conversely the best solution is to take the highest possible level for \(m\), i.e. which verifies \(\gamma^* = \frac{1}{2} = \frac{\psi_1}{2 \psi_2 (1 + m)}\). This gives \(m^* = \frac{4 \psi_1}{\psi_2} - 1\).

**Case 2:** \(m \geq 2 \frac{\psi_1}{\psi_2} - 1\). In this case, \(\gamma(m) = \frac{2}{\psi_2 (1 + m)}\). If \(\gamma \geq \gamma(m), P^*(\gamma, m) = 1\) and \(EF = (1 - \gamma)F - \psi, m\). It is then optimal to take the lowest possible value for \(\gamma, i.e. \gamma^* = \gamma(m) = \frac{2}{\psi_2 (1 + m)}\). The expected fine is then \(EF = \left(1 - \frac{\psi_1}{\psi_2 (1 + m)}\right) F - \psi, m\).

Conversely, when \(\gamma \leq \gamma(m)\) we have \(\gamma^* = \frac{1}{2}\) if \(\frac{1}{2} \leq \frac{\psi_1}{\psi_2 (1 + m)}\), i.e. when \(m \leq \frac{4 \psi_1}{\psi_2} - 1\).
Finally, we see that only two configurations are possible at the optimum:

1. The first configuration consists in sharing equally the fine between the authority and the audit inspector, $\gamma^* = \frac{1}{2}$. The expected fine is $EF = \frac{1 + m \psi_2}{8 \psi_1} F - \psi_3 m$. If $\psi_3 > \frac{\psi_2}{8 \psi_1} F$ the best solution is $m^* = 0$ ($EF = \frac{1}{8 \psi_1} \psi_2 F$) and conversely $m^* = 4 \frac{\psi_1}{\psi_2} - 1$ and $EF = \frac{1}{2} F + \psi_3 - 4 \frac{\psi_1 \psi_3}{\psi_2}$. 

2. In the second configuration, $\gamma^* = \gamma(m) = \frac{\psi_1}{\psi_2} \frac{2}{1 + m}$. In this case, $EF = \left(1 - \frac{\psi_1}{\psi_2} \frac{2}{1 + m}\right) F - \psi_3 m$.

The best solution is then: $m^* = \max \left\{ \sqrt{\frac{2 \psi_1}{\psi_2} \psi_3} \frac{F}{\psi_3} - 1, \frac{2 \psi_1}{\psi_3} - 1 \right\}$ and $EF = F + \psi_3 - \sqrt{\frac{8 \psi_1 \psi_3}{\psi_2} F}$ in the first case and $\gamma^* = 1$ in the second case which $EF = -\psi_3 m^* = -\psi_3 \left(2 \frac{\psi_1}{\psi_3} - 1\right)$.

Let us now make a final comparison between the two regimes to analyze the optimal combination of $(\gamma, m)$. It turns out that when $\psi_3 \leq \frac{\psi_2}{8 \psi_1} F$, the first regime gives an expected fine of $EF_1 = \frac{1}{2} F + \psi_3 - 4 \frac{\psi_1 \psi_3}{\psi_2}$ and the second $EF_2 = F + \psi_3 - \sqrt{\frac{8 \psi_1 \psi_3}{\psi_2} F}$.

When $\frac{\psi_2}{2 \psi_1} F \leq \psi_3 \leq \frac{\psi_2}{2 \psi_1} F$, $EF_1 = \frac{\psi_2}{8 \psi_1} F$ and $EF_2 = F + \psi_3 - \sqrt{\frac{8 \psi_1 \psi_3}{\psi_2} F}$.

Finally, when $\frac{\psi_2}{2 \psi_1} F < \psi_3$ we have $EF_1 = \frac{\psi_2}{8 \psi_1} F$ and $EF_2 = -\psi_3 \left(2 \frac{\psi_1}{\psi_3} - 1\right)$. 
It is easy to see that when the cost of gathering the information is relatively high (i.e. \( \frac{\psi_2}{2\psi_1} F < \psi_3 \)) the first regime is better (share equally the fine and do not provide any information). Conversely when the cost is relatively small (i.e. \( \frac{\psi_2}{2\psi_1} F > \psi_3 \)), the second regime is better (give the highest share compatible with a probability of detection equals to the unity and choose the level \( m^* \) accordingly). Finally, for intermediate values (\( \frac{\psi_2}{2\psi_1} F < \psi_3 < \frac{\psi_2}{2\psi_1} F \)) it is easy to show that the first regime dominates since we have supposed that \( 2\frac{\psi_1}{\psi_2} > 1 \).