Government spending in a model where debt effects output gap

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Abstract

In this paper I present a simple model of government spending where the level of government debt affects the output gap. The structure of the economy is specified such that the output gap has a structural part, which is a function of debt. Based on empirical research, the structural part is assigned a specific functional form. The government faces an optimization problem where they attempt to close the output gap. The optimal change in government debt is found by solving a nonlinear equation. Numerical results show that the optimal change in debt has nonlinear behaviour. The solution to the unconstrained problem is an alternating equilibrium, whereas the solution to the constrained problem is a non linear cycle around the government's upper bound of admissible debt.

Keywords: Debt, Macroeconomy, Fiscal, Government Spending, Output Gap, Nonlinear, Numerical Method

1. Introduction

Debt is an important part of economic life, yet it does not receive equal treatment by different schools of thought in macroeconomics. The research presented here adapts an empirical insight about debt into a theoretical framework and places debt at the core of a macroeconomic model. The empirical result from Cecchetti, Mohanty and Zampolli (2011) is that high levels of debt-to-GDP cause GDP growth to slow. My research does not attempt to explain this result, rather I use it as an assumption; specifically, I propose a framework where government debt levels effect output gap and government chooses spending to close the output gap. I find that the results for the optimal debt path are characterized by nonlinear cycles.

Existing research on debt in macroeconomics can be separated into three categories. The examples I use to characterize each category are published after 2008, which reflects the increasing attention that this area is receiving. The first category is policy-focused research, which is concerned with the analysis of data or real countries. This research identifies causal relationships between macroeconomic variables and provides recommendations for practical actions. The book by Mauldin and Tepper (2011) is an outstanding effort in this category that charts the real economic effects of current events in debt markets. They challenge existing frameworks for understanding debt in macroeconomics and highlight the need for better theory in this area.

The second category is empirical-historical, which is concerned with the role of debt in financial crises. An important book that characterizes this category is Reinhart and Rogoff (2009), which uses a case-study approach to identify common mechanisms in the real effects of debt. This book has identified specific conditions that foreshadow crisis and has been a rallying point for people concerned with the impact of large debt. Another important paper in this category is Cecchetti et al. (2011), which uses a statistical approach to establish the effect of large debt burdens on GDP growth. As stated before, my research attempts to adapt this result into a context where debt affects the output gap and the government wants to close the output gap.

The third category is theoretical research, which includes many different models that have a non-trivial role for debt in the economy. The acclaimed authors Cecchetti et al. write: "For a macroeconomist working to construct a theoretical structure for understanding the economy as a whole, debt is either trivial or intractable." (2011, 2), which speaks to the challenge we face. Recently, Eggertsson and Krugman (2011) made a breakthrough that provides a formal derivation of several important ideas that trace back to Irving Fisher, including the idea of a Debt Deflation cycle. This model uses heterogeneous agents who interact in a labour market, which is a big step towards modelling the economy as a whole. In contrast, my research only has a market for government debt and an exogenous GDP variable.

I view the government's choice of fiscal spending as an optimization problem, where the objective is to force the output gap close to zero. A government may want to do this because a positive output gap is associated with an overheating economy and a negative gap is associate with recession - both of which, the government may want to avoid. The government can achieve this objective based on the structure of the economy. The structure is based on the Fisher GDP equations and extended to include a nonlinear effect between government debt and output gap. I propose that the output gap has a cyclic and structural part, where the structural part is the 'debt wedge'. Based on salient features from Cecchetti et al (2011), I show that a cubic function can provide the type of behaviour that we expect from the debt wedge.

In the simplest version of my model, the debt wedge equals zero. In this case I find that $\mathrm{G}^* = \mathrm{t}\,\overline{\mathrm{Y}}$ which says the government spends a fixed portion of potential GDP and runs a balanced budget in expected value. With a change of variables, it is possible to consider the government's optimization problem when the debt wedge is equal to the cubic function mentioned above. In this case, the optimal path for debt is determined by a nonlinear equation. To represent the solution, I present a Policy Rule graphic and simulation of debt over time. The Policy Rule shows how the government should change debt, given current debt levels; the rule is nonlinear and has two points of discontinuity where the behaviour of debt issuance changes drastically. In the unconstrained case, optimal debt path follows an alternating equilibrium where $\Delta D_i = -\Delta D_{i-1}$. However, a more realistic version of this problem imposes a constraint on ΔD_i. In the constrained case the optimal debt path follows a more complicated attractor (equilibrium pattern). For certain parameter values, the attractor occurs around the upper discontinuity in the policy rule, which means the government flirts with 'too much' debt.

This model has several simplifying assumptions. I consider a closed economy where GDP can be divided into private GDP, government spending, and taxes. This simplification overlooks 'multipliers' and intertemporal effects of government spending or taxes. Also, I do not consider the interest rate burden and assume interest rates are zero for government debt. It may be that the interest rate burden is an important mechanism whereby debt has a real effect on the economy, but this is not the focus of my research. My research assumes that the debt wedge is the mechanism and then solves for optimal government spending in this situation.

2. Analysis

In this section I present the original model and results. First, I specify equations that define the economy. These structural equations are combined to obtain an expression for output gap that depends on government spending and debt. In the second section I explore the concept of the debt wedge and specify a nonlinear formula to describe it. The specification of the debt wedge is based on Cecchetti et al. (2011), which provides a different interpretation of 'good effect' from that which the government considers to be a 'good effect'. This tension is an important part of the model. In the third section, I will state the optimization problem which the government faces and explore the solution.

2.1 Structural Equations

To define the structure of the economy in a parsimonious way, my model uses five equations. The first equation states that GDP can be separated into private GDP (Y_i^P), government spending (G_i) and taxes (T_i):

(1) $$Y_i = Y_i^P + G_i - T_i$$

Equation (1) is loosely based on the 'Fisher equations' for GDP, as it is possible to interpret Y_i^P as $C + I + NX$. However, my version of this equation simplifies the role of government spending and tax in an economy. I suppose that all government spending only affects current GDP and there are no multiplier effects. Although reality is more complex than equation (1) suggests, this is a useful place to start because it makes the role of government spending explicit.

(2) $$Y_i^P = \overline{Y_i} + \varepsilon_i + f(D_i, Y_i)$$

Equation (2) is motivated by historical observations of GDP in USA, where GDP moves around potential GDP (FRED 2012). Although I am not able to address the mechanism that causes GDP to stay close to potential GDP in this paper, I do assume that difference between GDP and potential can be described well by a random variable (possibly autoregressive). Equation (2) states that private GDP is equal to potential GDP ($\overline{Y_i}$) plus a random term (ε_i) and the debt wedge ($f(D_i, Y_i)$). The random term should be seen as a cyclical part, whereas the debt wedge is a structural component.

(3) $$D_i = \Delta D_i + D_{i-1}$$

Equation (3) states that the government debt grows according to debt issuance. This means that interest rates do not play a role in the accumulation government debt or interest rates are zero. Current debt (D_i) is equal to change in debt (ΔD_i) plus debt in the prior period (D_{i-1}). This is an unrealistic assumption but it simplifies the change of variables performed in Section 2.3. It is possible to relax this assumption in several ways, which will allow researchers to explore different mechanisms where debt levels affect GDP. These extensions are discussed in the concluding section.

(4) $$\Delta D_i = G_i - T_i$$

Equation (4) states that debt issuance is equal to the government deficit, which simplifies the process whereby a government accumulates debt. There is no mention of monetary policy or unfunded liabilities, just the fiscal budget and tax revenues. When the government runs a surplus, $G_i < T_i$, the government reduces outstanding debt. In the unconstrained optimization problem, I let ΔD_i be free, however, this implies that $G_i < 0$ is admissible. The constrained version of the problem will force government spending to be non-negative ($G_i \geq 0$) which causes change in debt to be bounded below by tax revenues ($\Delta D_i > -T_i$).

$$(5) \qquad T_i = t\, Y_i^P$$

Equation (5) states that the government tax revenues equal a fixed rate (t) of private GDP. It is important that the government collect taxes based on private GDP, not final GDP because Equation (1) states that final GDP depends on government spending and tax revenue. Based on analysis of Federal tax revenue in the USA (FRED 2012), this assumption is fair and $t = 5\%$ is an accurate value.

The definition of the output gap is: $Y_i - \overline{Y_i}$. Using the equations introduced here, this can be simplified to the following expression:

$$(6) \qquad Y_i - \overline{Y_i} = \varepsilon_i + f(D_i, Y_i) + G_i - T_i$$

This expression gives the output gap in terms of government spending and debt. This expression will be used in the government's optimization problem.

2.2 Debt Wedge

"Our conclusion is that, at low levels, debt is good... But, at high levels, private and public debt are bad." Cecchetti et al. (2011, 5)

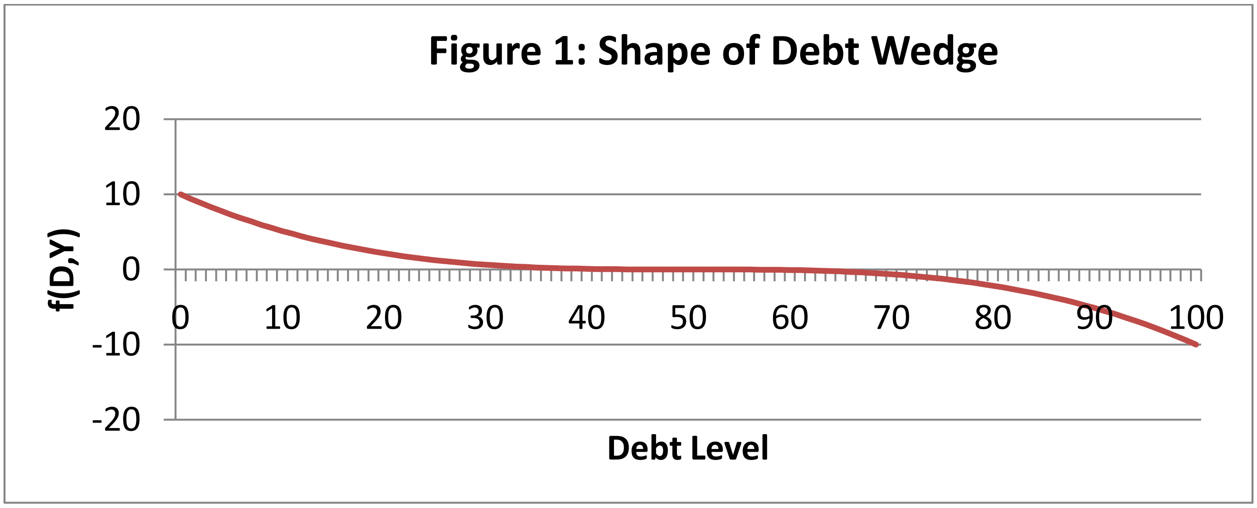
This quote motivates my concept of $f(D_i, Y_i)$, the debt wedge function. I approach this as a curve fitting exercise to match the sign and magnitude of the function to the salient information in the quote. Whereas Cecchetti et al. (2011) interpret 'good' as an increase to GDP growth, which is a statement about the change in GDP, I will interpret 'good' in terms of the level of GDP. I suppose that 'good' means that GDP increases, which implies that the output gap increases. Since this a certain increase, it must be reflected in the structural part not the cyclical (random) part of output gap. Since I assume the structural part is driven only by the debt wedge, the structural part must be positive if it has increased the output gap. To understand the causation, please consider everything else fixed: when debt is decreased to a 'low level', the GDP increases, the output gap increases, and the structural part increases and becomes positive. This means the debt wedge is positive when debt-to-GDP is at 'low levels'. By similar logic, the debt wedge is negative when the debt-to-GDP is at 'high levels'.

Please recall that $f(D_i, Y_i)$ is the structural part of the output gap, so it is possible that observed output gap takes any particular value because of ε_i, the cyclical part. As $f(D_i, Y_i)$ changes, the probability distribution and expected value of the output gap changes.

Next, I will discuss what qualifies as 'high levels' of debt-to-GDP. Cecchetti et al. report that when debt-to-GDP increase beyond 85% it causes GDP growth to slow (2011, 21). They do not report a threshold for a 'low level' of debt-to-GDP. Although the location of these thresholds are important, I will not assume a specific threshold value. Instead, I use a continuous function where the debt wedge is close to zero for an interval, then increases dramatically in the directions described above outside the interval. Many functions (polynomials with odd exponents) can provide this particular behaviour and I propose the following formula for the debt wedge:

$$(7) \qquad f(D,Y) = a\,Y\,\left(\frac{D}{Y} - b\right)^3$$

This formula is a useful representation for several reasons. The formula makes sure that debt-to-GDP is the variable that determines the sign of the debt wedge, which is important based on the results from Cecchetti et al. (2011). If $a < 0$ then the sign will match the signs described above (large D/Y causes $f(D_i, Y_i) < 0$). The location parameter b determines what qualifies as high or low levels of D/Y, which is an important feature. Also, the debt wedge appears in the equation for private GDP so it should be proportional to GDP; this is built into the function because of the coefficient aY . Finally, although this nonlinear function does not offer analytic solutions to the government's problem, it is easy to work with numerically.



In Figure 1, I show the debt wedge for a range of values of D_i. The GDP is set equal to 100, so the values of D_i and $f(D_i, Y_i)$ can be interpreted as percentages of GDP. The Figure uses parameter values $a = -0.8,\ b = 0.5$. Although $a < 0$ must hold, the magnitude of a is subject to negotiation and may be 'too large' in this calculation. The parameter b was chosen so that debt wedge becomes large and negative for $D/Y > 85\%$, as stated in Cecchetti et al. (2011).

2.3 Unconstrained Optimization Problem

When the government chooses their spending level, G, they do not know the value of ε_i. This means they do not know the level of private GDP or the output gap. To facilitate this decision making under uncertainty, I suppose they behave as in the following optimization problem:

(8) $$\mathfrak{L} = \min_{G} \sum_{i=1}^{N} [E_i(Y_i - \overline{Y_i})]^2$$

$$s.t.\, Y_i - \overline{Y_i} = \varepsilon_i + f(D_i, Y_i) + G_i - T_i$$

The first order condition amounts to solving for G such that the expected output gap equals zero at each time i. The expectation $E_i()$ denotes the government's expectation, based on information known at time i $(\overline{Y_i}, \overline{Y_{i-1}}, D_{i-1}, Y_{i-1})$.

For the simple case where there is no debt wedge, $f(D_i, Y_i) = 0$, and the government expects zero cyclical disturbance, $E_i(\varepsilon_i) = 0$, the optimal spending is $G^* = E_i(T)$, which gives $G^* = t\,\overline{Y}$. This preliminary result shows that it is optimal for the government to run a balanced budget, under expectation, if the government expects zero cyclical disturbances in output gap and there is no debt wedge. I consider this to be an encouraging result that it is somewhat intuitive. However, debt is not a genuine choice variable in this situation because $f(D_i, Y_i) = 0$.

If debt is to have any impact in this model, we cannot have $f(D_i, Y_i) = 0$. Based on the formula introduced in Section 2.2 I will show how the optimization problem can be rewritten in terms of debt. The choice variable will be change in debt and the state variables will be past debt, past GDP.

By definition, $f(D_i, Y_i) = f(\Delta D_i + D_{i-1}, Y_i)$ and $G_i - T_i = \Delta D_i$. The government knows D_{i-1} and $Y_{i-1}$when it decides ΔD_i, so I assume that $E_i[f(\Delta D_i + D_{i-1}, Y_i)]= f(\Delta D_i + D_{i-1}, Y_{i-1})$ which will be written as $f(\Delta D_i | D_{i-1}, Y_{i-1})$. This notation is meant to emphasize that ΔD_i is the only variable that is not known when the government makes their action. In fact, the government chooses ΔD_i. This assumption requires that the government uses past GDP (Y_{i-1}) to approximate current GDP (Y_i) in decision making. Proceeding in this way, it is possible to rewrite Equation (6) as:

$$(9) \qquad E_i(Y_i - \overline{Y_i}) = E_i(\varepsilon_i) + \; a\, Y_{i-1}\left(\frac{\Delta D_i + D_{i-1}}{Y_{i-1}} - \; b\right)^3 + \Delta D_i$$

The first order condition for the government problem is still $E_i(Y_i - \overline{Y_i}) = 0$. For $E_i(\varepsilon_i) = 0$, $a = -0.8$, $b = 0.5$, the optimal change in debt ΔD_i depends on prior debt and GDP. The government chooses ΔD_i to solve Equation (10):

$$(10) \qquad 0 = \Delta D_i - \; 0.8Y_{i-1}\left(\frac{\Delta D_i + D_{i-1}}{Y_{i-1}} - \; 0.5\right)^3$$

To describe the solution to Equation (10) when ΔD_i is unconstrained I present Figure 2.

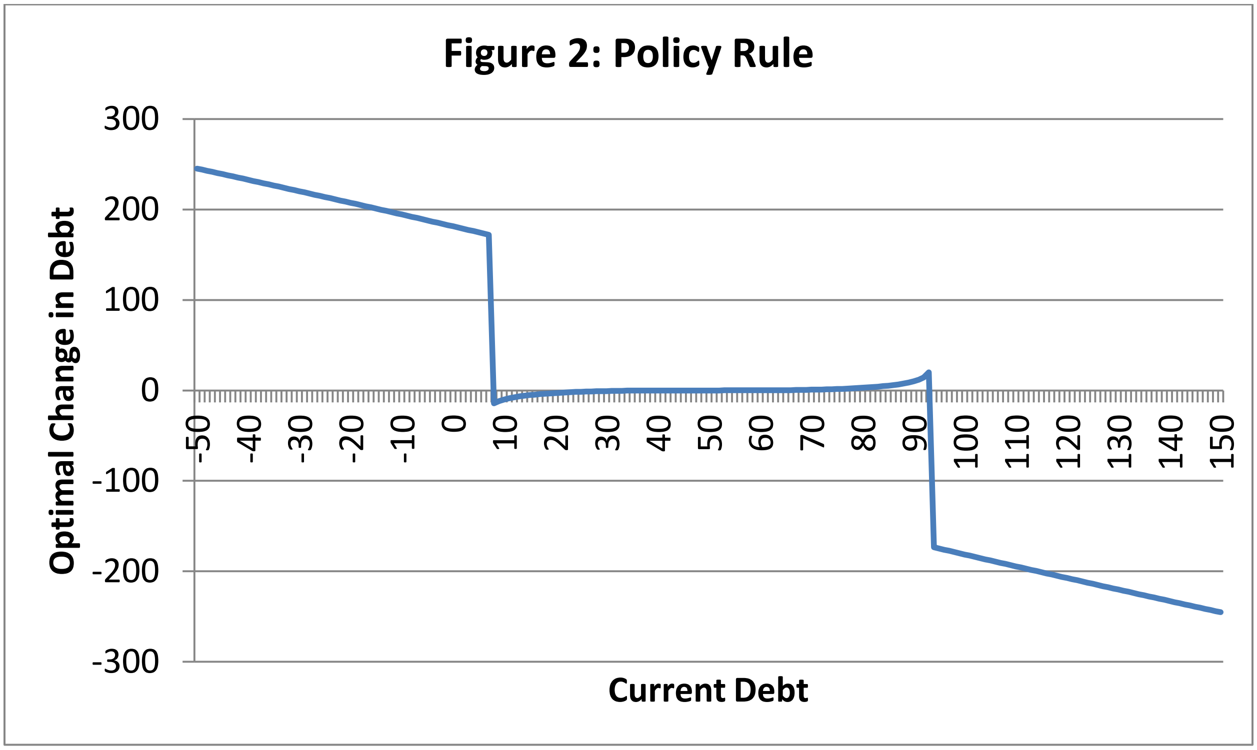


Figure 2: Policy Rule

Figure 2 shows the optimal ΔD_i for a range of values of D_i when change in debt is unconstrained. As in Figure 1, GDP is $Y_{i-1} = 100$, scale parameter $a = -0.8$, and location parameter $b = 0.5$. This figure is referred to as a 'Policy Rule' because it suggests what the government should do next, given current situation.

Figure 2 shows that the optimal debt policy has nonlinear features. When debt is large, the optimal change in debt is large and opposite sign. When debt is small, the optimal change in debt is also small, but destabilizing: if $\frac{D_{i-1}}{Y_{i-1}} > 0.5$ then $\Delta D_i^* > 0$ which causes debt to move towards the upper critical point, where the policy is discontinuous. As debt increases past the critical point, the optimal government reaction is a drastic decrease in debt. The optimal decrease is so large that the new debt level is below the lower critical point. In Table 1, I report a series of values of debt that show the alternating equilibrium. I discuss the intuition about this result below Table 1.

Table 1: Simulated Debt Path			
Time	Debt level	Change in Debt	Debt Burden??
0	0	0	?
1	181	181	?
2	-103	-284	?
3	207	310	?
4	-108	-315	?
5	208	316	?
6	-109	-317	?
7	208	317	?
8	-109	-317	?
9	208	317	?

Table 1 shows the alternating equilibrium where $\Delta D_i = 317 = -\Delta D_{i-1}$. This pattern occurs when change in debt is unconstrained and the parameter values are fixed, as in Figure 2. This simulation shows that the government settles into the attractor very quickly and the cycle is characterized by huge swings in debt level.

To better understand the intuition of this result, recall that the change in debt plus the debt wedge after the change in debt has to equal zero based on Equation (10). Imagine that D_{i-1} is a large positive value, then the optimal change ΔD_i^* would be a large negative value because this would cause the new debt wedge to become positive. The optimal change in debt has the same sign as the debt wedge; when current debt levels are beneficial, the policy recommends to increase debt and vice versa. In this way, the optimal debt path follows a nonlinear cycle.

2.4 Constrained Optimization Problem

Government spending should be non-negative. Based on Equation (4), this suggests that $\Delta D_i \geq -T_i$ should hold. It is possible to impose this constraint onto the numerical solution of the optimization problem. For simulation, I assume $t = 5\%$ and ${Y_i}^P = 100$. Figure 3 shows the Policy Rule when ΔD_i is constrained ($\Delta D_i > 5\%$).

<< Figure 3: Policy Rule for constrained Problem >>

This Policy Rule shows that ΔD_i is bounded below, as was assumed. Although the function is still discontinuous, the nonlinear behaviour for large values of debt is very different. It is not possible for the alternating equilibrium to characterize this solution because large decreases in debt are not possible. In fact, the policy rule reveals that the equilibrium cycle for debt will be an attractor around the upper discontinuity point. This is indeed the result that is found by simulation of the debt path.

<< Table 2: Time step, debt, change in debt, debt wedge>>

Table 2 shows the optimal debt path for initial debt value $D_0 = 0$ with fixed GDP $Y_i = 100$. This simulation shows that the equilibrium cycle for debt is an attractor around the upper discontinuity point. This cycle occurs as time passes for any starting point other than $D_0 = 0.5$. If $D_0 = 0.5$ then the debt wedge equals zero and the solution to Equation (10) is $\Delta D_i = 0$ for all time. Note that $D_i = 0.5$ is not a stable equilibrium because the Policy Rule advises the government to move away from $D_i = 0.5$.

The results of this constrained model show that the optimal path for debt settles into a cycle where it moves back and forth across a line in the sand. This line is the discontinuity in the Policy Rule. As seen in the unconstrained problem, when debt is above the discontinuity the optimal response is to decrease debt by a large amount; in this sense, when debt is above the discontinuity there is 'too much' debt and the debt wedge causes GDP to underperform potential. In the constrained version, the government can only decrease debt by a small amount so the amount of debt never gets far from this point of discontinuity and the government repeatedly faces the problem of having too much debt.

3 Conclusion

This research has established a model where debt is an important economic variable. The model shows how an empirical result can be adapted to provide an assumption for a theoretical model. The paper introduced the concept of a debt wedge and explained how a formula can be used to represent the concept in an analytic model. The model provides a new way to understand government spending, based on the output gap, and through numerical simulation, the model has been shown to give a variety of predictions.

It is possible to extend the results presented here in several ways. The model should be revised to include interest rates. This could be done by adding non zero interest rates to the model proposed here, or even adding interest rates that depend on the amount of debt. An interesting topic that deserves close scrutiny is the fact that the government must pay interest on outstanding debt; for countries in crisis, this interest expense becomes a large part of government expense. However, it is important to state this question in a model of an open economy where the government can conduct monetary policy; the ability to print money and the currency in which the debt is denominated are crucial factors in such a situation.

Another simple extension to this paper is to have two types of government spending: welfare and infrastructure. Whereas the welfare spending would be consumed in the same period, the infrastructure would have a lasting, positive effect on GDP. The government's optimization problem would be more complicated in such a model, but it is important to understand the interplay between debt, short term spending, and long term investment.

The results of this paper show that the path of debt is characterized by nonlinear cycles. In the unconstrained case, the debt path follows a dramatic alternating equilibrium. This is unrealistic because it requires the government change from a debtor to a creditor each time step. In the constrained case, the debt path follows a subtle cycle around the upper discontinuity in the Policy Rule. This point of discontinuity is where the government has too much debt so the results suggest the government will regularly incur too much debt, then reduce the debt burden and repeat. Although the scope of these results are limited by the structural assumptions, the paper does demonstrate the rich models and results that are available when considering debt in macroeconomic context.

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