Hyperinflation, disinflation, deflation, etc.: A unified and micro-founded explanation for inflation

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Abstract

In this paper, I present a unified and micro-founded explanation for various types of inflation without assuming ad hoc frictions or irrationality. The explanation is similar to the conventional inflation theory in the sense that an independent central bank can control inflation and also similar to the fiscal theory of the price level in the sense that a source of inflation lies in the behavior of government. Inflation accelerates or decelerates through the simultaneous optimization of a government and the representative household if their time preference rates are heterogeneous. This inflation acceleration mechanism will be prevented from working if a central bank is truly independent.

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I. INTRODUCTION

There are several types of inflation: hyperinflation, chronic inflation, disinflation, low and stable inflation, and deflation. Different shocks make the paths of inflation deviate from the stable path in various ways. Conventional inflation theory focuses on using monetary policy when inflation deviates from the targeted path (e.g., Svensson 2003). The theory does not, however, sufficiently answer the fundamental question of why severely deviated paths like hyperinflation, chronic inflation, or deflation occasionally occur. In fact, if all of the agents behave rationally, it is hard to explain these phenomena. One of the few explanations is to assume that a government is weak, foolish, or untruthful. This reason has been used to explain chronic inflation. A government can be pressured by interest groups to take an inflationary policy stance and intervene in a central bank's decision-making, and the central bank is then unable to fully commit to its policies, which generates the possibility of chronic inflation (e.g., Kydland and Prescott 1977; Barro and Gordon 1983; Rogoff 1985; Berger et al. 2000). The assumptions of ad hoc frictions or that households or firms are irrational to some extent have been used to explain hyperinflation (e.g, Cagan 1956). For example, hyperinflation can occur only if adaptive expectations or some ad hoc frictions are assumed when large budget deficits are allowed in the well-known Cagan (1956) framework (e.g., Auernheimer 1976; Evans and Yarrow 1981; Kiguel 1989). Neither of these explanations is particularly compelling because they rely on irrational behavior or ad hoc frictions.

The fiscal theory of the price level (FTPL) argues that a problem with conventional inflation theory is that it practically neglects the importance of the government's borrowing behavior in inflation dynamics (e.g., Leeper 1991; Sims 1994, 1998, 2001; Woodford 1995, 2001; Cochrane 1998a, 1998b, 2005).[1] It has been argued that, if a government borrows money without limits, inflation will eventually explode (e.g., Sargent and Wallace 1981). The FTPL implies that, if a government's borrowing behavior is well modeled, the mechanism of severely deviated inflation paths (e.g., hyperinflation, chronic inflation, or deflation) can be explained without assuming ad

[1] See also Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), and Gordon and Leeper (2002).

hoc frictions or irrationality. Most FTPL models have not, however, explicitly modeled the behavior of government in detail. Hence, some critics contend that the theory is fallacious (e.g., Kocherlakota and Phelan 1999; McCallum 2001, 2003; Buiter 2002, 2004; Niepelt 2004).

If, however, the government's borrowing behavior is modeled properly and explicitly, it may be possible to use the FTPL to explain the common mechanism of various types of inflation. My purpose in this paper is to explore this possibility. First, I examine the nature of the government budget constraint in detail and then construct a model that fully incorporates the government's borrowing behavior. The important features of the model are that (1) both the government and the representative household achieve simultaneous optimization and (2) the roles of government and the central bank are explicitly separated. Because of the first feature, I do not need to assume ad hoc friction or irrationality. Moreover, although the roles of government and the central bank are different (as the second feature shows), both government and the central bank are responsible for the development of inflation. These characteristics indicate that the model has characteristics of both conventional inflation theory and the FTPL. Similar to conventional inflation theory, an independent central bank can control inflation by manipulating the nominal interest rate with a target rate of inflation. At the same time, similar to the FTPL, the behavior of government represents a source of inflation.

The model presented here indicates that inflation accelerates or decelerates if the time preference rates of the government and the representative household are heterogeneous. Because a government represents the median of households under a proportional representation system and the economically representative household represents the mean of households, the preferences between them are usually heterogeneous. Given these heterogeneous preferences, it is not possible for both of them to achieve simultaneous optimization if inflation is constant. The model indicates that inflation must accelerate or decelerate for the government and the representative household to be able to achieve simultaneous optimization. Simultaneous optimization is possible because the acceleration or deceleration of inflation changes the government's borrowing behavior. The problem therefore is not one of irrationality or friction but one of preference. Because it is hard for a government to control its own preferences even if it behaves in a fully rational manner, an independent central bank is necessary to prevent this inflation acceleration mechanism from working. The combination of the inflation acceleration mechanism and various degrees of central bank independence gives us a unified and micro-founded explanation for various types of inflation without ad hoc assumptions of friction or irrationality.

The paper is organized as follows. In section II, I examine the nature of the government budget constraint and construct a model that assumes an economically Leviathan government in which the government and the representative household achieve simultaneous optimization. The natures of the simultaneous optimization and the inflation acceleration mechanism are examined in section III. In section IV, I look at how the independent central bank influences inflation dynamics. In section V, I show that the model can provide a unified and micro-founded explanation for various types of inflation. Finally, I offer concluding remarks in section VI.

II. THE MODEL

1. The government budget constraint

The government budget constraint is a key element in the explanation for inflation in this paper. The budget constraint is

$$\dot{B}_t = B_t R_t + G_t - X_t - S_t ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, R_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and S_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time t. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $s_t = \frac{S_t}{P_t}$, where P_t is the price level at time t. Let also $\pi_t = \frac{\dot{P}_t}{P_t}$ be the inflation rate at time t. By dividing by P_t, the budget constraint is transformed to

$\frac{\dot{B}_t}{P_t} = b_t R_t + g_t - x_t - s_t$, which is equivalent to

$\dot{b}_t = b_t R_t + g_t - x_t - s_t - b_t \pi_t = b_t (R_t - \pi_t) + g_t - x_t - s_t$.

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\overline{R}_t \geq E_t \int_t^{t+1} (\pi_s + r_t) ds$ at time *t*, where $\overline{R}_t$ is the nominal interest rate for bonds bought at *t* and r_t is the real interest rate in markets at *t*. Hence, by arbitrage, $\overline{R}_t = E_t \int_t^{t+1} (\pi_s + r_t) ds$ and $\overline{R}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ if r_t is constant such that $r_t = r$ (i.e., if it is at steady state). The nominal interest rate $\overline{R}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between *t* and *t* + *dt*, the government's obligation to pay for the bonds' return in the future increases not by $dt(\pi_t + r)$ but by $dt\left(E_t \int_t^{t+1} \pi_s \, ds + r \right)$. If π_t is constant, then $\pi_t + r = E_t \int_t^{t+1} \pi_s \, ds + r$ and $\pi_t = E_t \int_t^{t+1} \pi_s \, ds$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at *t* have been issued between $t-1$ and *t*. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time *t* is the weighted sum of $\overline{R}_t$ such that $R_t = \int_{t-1}^{t} \overline{R}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r$, where $\overline{B}_{s,t}$ is the nominal value of bonds at time *t* that were issued at time *s*. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between $t-1$ and *t* are not so different from each other, then approximately $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \, ds + r$. To be precise, if the absolute values of π_s for $t-1 < s \leq t+1$ are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$R_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r$ (see Appendix 1).[2] The average nominal interest rate for the total government bonds, therefore, develops by $R_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r$. If π_t is constant, then $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t$; thus, $R_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t$ and $R_t = \pi_t + r$ do not necessarily hold.

2. An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[3] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[4] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[5] For example, if a Leviathan government considers national security

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[2] If the absolute values of π_s for $t-1 < s \leq t+1$ are very large, the weight $\frac{\overline{B}_{t,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ will be much larger than $\frac{\overline{B}_{t-1,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ when π_t is increasing. In this case, R_t will be closer to $\int_{t}^{t+1} \pi_s \, ds + r$ than $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r$.

[3] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991; Cukierman et al. 1992).

[4] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

[5] The government behavior assumed in the FTPL reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a proportional representation system (e.g., Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[6] In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^2} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} > 0$.[7] An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing; that is, it maximizes its expected political utility subject to the budget constraint.

3. Optimization problems

The optimization problem of an economically Leviathan government is

$$Max\ E_0 \int_0^\infty u_G(g_t, x_t) \exp(-\theta_G t) dt$$

subject to the budget constraint

$$\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t,$$

where u_G is the constant relative risk aversion utility function of the government and θ_G is the government's rate of time preference. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its

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6 It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

7 Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} < 0$. However, the assumption used is not an important issue here because $\frac{x_t \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2}}{\frac{\partial u_G(g_t, x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

expected political utility considering the behavior of the economically representative household that is reflected in R_t in its budget constraint.[8]

In contrast, the economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E_0 \int_0^\infty u_P(c_t, m_t) \exp(-\theta_P t) dt$$

subject to the budget constraint

$$\dot{a}_t = (r_t a_t + w_t + \tau_t) - [c_t + (\pi_t + r_t) m_t] - g_t ,^9$$

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, τ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^2} < 0$, where $f(\bullet)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that lump-sum government transfers (τ_t) is equal to the seigniorage (s_t), and that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption c_t, the real investment $\dot{k}_t$, and the real government expenditure g_t such that $f(k_t) = c_t + \dot{k}_t + g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in section

[8] The model can be used to analyze various inflation phenomena (see Harashima 2004b, 2005, 2006, 2007a, 2007b).

[9] The constraint is equivalent to $\dot{a}_t = (r_t a_t + w_t + \tau_t) - [c_t + (\pi_t + r_t) m_t] - \dot{b}_t - x_t - s_t + b_t (R_t - \pi_t)$.

IV.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

III. THE INFLATION ACCELERATION MECHANISM

1. The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P t) + \lambda_{P,t}[r_t a_t + w_t + \tau_t - c_t - (\pi_t + r_t)m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are

(1) $\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}\exp(-\theta_P t) = \lambda_{P,t}$,

(2) $\dfrac{\partial u_P(c_t, m_t)}{\partial m_t}\exp(-\theta_P t) = \lambda_{P,t}(\pi_t + r_t)$,

(3) $\dot{\lambda}_{P,t} = -\lambda_{P,t} r_t$,

(4) $\dot{a}_t = (ra_t + w_t + \tau_t) - [c_t + (\pi_t + r_t)m_t - g_t]$, and

(5) $\lim_{t\to\infty} \lambda_{P,t} a_t = 0$.

By conditions (1) and (2), $\dfrac{\dfrac{\partial u_P(c_t,m_t)}{\partial m_t}}{\dfrac{\partial u_P(c_t,m_t)}{\partial c_t}} = \pi_t + r_t$, and by conditions (1) and (3),

$-\dfrac{c_t \dfrac{\partial^2 u_P(c_t,m_t)}{\partial c_t^2}}{\dfrac{\partial u_P(c_t,m_t)}{\partial c_t}} \dfrac{\dot{c}_t}{c_t} + \theta_P = r_t$. Hence,

(6) $\theta_P = r_t = r$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t,x_t)\exp(-\theta_G t) + \lambda_{G,t}[b_t(R_t - \pi_t) + g_t - x_t - s_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are

(7) $\dfrac{\partial u_G(g_t,x_t)}{\partial g_t}\exp(-\theta_G t) = -\lambda_{G,t}$,

(8) $\dfrac{\partial u_G(g_t,x_t)}{\partial x_t}\exp(-\theta_G t) = \lambda_{G,t}$,

(9) $\dot{\lambda}_{G,t} = -\lambda_{G,t}(R_t - \pi_t)$,

(10) $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$, and

(11) $\lim_{t\to\infty} \lambda_{G,t} b_t = 0$.

Combining conditions (7), (8), and (9) yields the following equations: $\dfrac{g_t \dfrac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\dfrac{\partial u_G(g_t,x_t)}{\partial g_t}} \dfrac{\dot{g}_t}{g_t} + \theta_G = R_t - \pi_t = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ and $-\dfrac{x_t \dfrac{\partial^2 u_G(g_t,x_t)}{\partial x_t^2}}{\dfrac{\partial u_G(g_t,x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} + \theta_G = R_t - \pi_t =$ $r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ because $R_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r_t$ (as shown in section II). Here, $\dfrac{g_t \dfrac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\dfrac{\partial u_G(g_t,x_t)}{\partial g_t}} \dfrac{\dot{g}_t}{g_t} = 0$ and $\dfrac{x_t \dfrac{\partial^2 u_G(g_t,x_t)}{\partial x_t^2}}{\dfrac{\partial u_G(g_t,x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus, $\theta_G = r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$. Hence, by equation (6),

(12) $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds = \pi_t + \theta_G - \theta_P$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[10]

Equation (12) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then $R_t - r = \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds \neq \pi_t$. This result may seem surprising because it has been naturally conjectured that $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds = \pi_t$ and that $R_t = \pi_t + r$. However, this is a simple misunderstanding because π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{P}_t}{P_t}$, whereas $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds$ roughly indicates a total price change by inflation during a unit period. Equation (12) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds - \theta_G + \theta_P$. When π_t is constant, the equations $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \, ds = \pi_t$ and $R_t = \pi_t + r$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (12) indicates that the equations $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds = \pi_t$ and $R_t = \pi_t + r$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $R_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

2. The law of motion for price

Equation (12) indicates that inflation accelerates or decelerates when the rates of time preference are heterogeneous. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t}\int_{s}^{s+1}\pi_v \, dv \; ds$, then π_t is not constant.

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[10] If and only if $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ at steady state, then the transversality condition (11) $\lim_{t \to \infty} \lambda_{G,t} \, b_t = 0$ holds. The proof is shown in Appendix 2.

Without the acceleration or deceleration of inflation, therefore, equation (12) cannot hold in an economy in which $\theta_G \neq \theta_P$. That is, inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference.

The integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \theta_G + \theta_P$ implies that inflation accelerates or decelerates nonlinearly in the case in which $\theta_G \neq \theta_P$ such that $\pi_t = \pi_0 + y(\theta_G - \theta_P)\exp[z_t \ln(t)]$, where y is a constant and z_t is a time dependent variable. For example, a solution of this integral equation is $\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2$. For a sufficiently small period between $t+1$ and $t+1+dt$, π_{t+1+dt} is determined with $\pi_s \ (t-1 < s \leq t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_v \, dv \, ds = \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_v \, dv \, ds + \pi_{t+dt} - \pi_t$. Suppose that initially $\theta_G = \theta_P$, then θ_G changes at time 0, and θ_G and θ_P are not identical from that time. Because π_t is constant before $t = 0$, then $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds = \pi_0 + \int_{-1}^{0}\int_{0}^{s+1} (\pi_v - \pi_0) dv \, ds$ at $t = 0$. Here, I assume that $\pi_t = \pi_0 + yt$ for $0 \leq t < 1$ for π_t to be continuous (y is a constant). Thus, $\pi_t = \pi_0 + 6(\theta_G - \theta_P)t$ for $0 \leq t < 1$. After $t = 1$, π_t gradually departs from the path of $\pi_t = \pi_0 + 6(\theta_G - \theta_P)t$, upward if $\theta_G > \theta_P$ and downward if $\theta_G < \theta_P$, such that $\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)]$ where $z_t > 1$, so as to hold $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t = \theta_G - \theta_P$. Although a solution of the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \theta_G + \theta_P$ is $\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2$, the path of inflation $\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)]$ approaches the path

(13) $\pi_t = \pi_0 + 3(\theta_G - \theta_P)t^2$

as time passes because of the boundary condition that π_t is constant before $t = 0$.

It should be stressed that inflation must be constant unless $\theta_G \neq \theta_P$, and it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. That is, $\theta_G \neq \theta_P$ bends the path of inflation and makes it nonlinear, which enables inflation to accelerate or decelerate. Equation (13) can be also interpreted as the difference of time preference rates $\theta_G - \theta_P$ at time *t* is transformed to the accelerated or decelerated inflation π_t at time *t*. The many episodes of accelerating and decelerating inflation across countries and time periods imply that the condition in which $\theta_G \neq \theta_P$ is not rare.

3. The mechanism of inflation acceleration

In this subsection, I explore the inflation acceleration (or deceleration) mechanism in greater detail.

3.1. Necessity of reconciling heterogeneous discount factors

The sum of the government's real expenditure, the representative household's real consumption, and the real investment is equal to the real output at any time, as shown in the budget constraint of the representative household such that $f(k_t) = c_t + \dot{k}_t + g_t$. Hence, the streams of expenditure and consumption are not determined independently of each other and should be consistent with the stream of the output. However, if the discount factors (θ_G and θ_P) are different, there is no guarantee that the streams are consistent; that is, there is no guarantee that both transversality conditions of the government and the representative household (equations [5] and [11]) are satisfied and that both expected utilities are maximized simultaneously. For example, the expected utility of the representative household is maximized if the point $r = \theta_P$ is at steady state as usual and as equation (6) shows. However, if $\theta_G \neq \theta_P$, it is not rational for the government to stop changing its real expenditures, taxes, and borrowing at the point where $r = \theta_P$. The government's expected utility will increase by changing them even if $r = \theta_P$ because $r = \theta_P \neq \theta_G$. The government's behavior obstructs the optimization of the representative household, but it is completely rational behavior for the government. Therefore, this contradiction of discount factors should be reconciled by some mechanism to make the streams consistent (except for corner solutions) and to make both the government and the representative household able to achieve simultaneous optimization.

The easiest way to achieve a steady state in an economy when discount factors are heterogeneous is to expel the government from markets, but that is impossible. Unless a way is found that enables the government and the representative household to coexist at steady states (other than corner solutions), the economy may break down. For them to be able to coexist at steady states, the government should stop changing its real borrowing at the point where $r = \theta_P \neq \theta_G$. Hence, if there is a mechanism that penalizes the government for having θ_G unequal to θ_P and makes it refrain from changing its real borrowing at the point where $r = \theta_P \neq \theta_G$, coexistence will be possible. Equation (13) indicates that the mechanism for penalizing the government does indeed exist—the acceleration or deceleration of inflation. I explain how this mechanism works in section 3.2.

3.2. Moved up real obligations

Suppose for simplicity that $\theta_G > \theta_P$. Inflation accelerates by equation (13) because of the heterogeneous discount factors. Equation (13) indicates that the government's existing real obligation b_t increases at a higher rate than r by $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t \; (>0)$ because the real obligation increases by $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t = b_t\left(r_t + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t\right) + g_t - x_t - s_t$ as shown in the real government budget constraint. This higher rate of increase in the real obligation by $\int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$, a result of accelerating inflation, indicates the government's penalty for having higher θ_G than θ_P.

Note, however, that the real rate of return on investments in government bonds is always r regardless of the acceleration of inflation because $\int_t^{t+1} \left(\overline{R_t} - \pi_s\right) ds = \int_t^{t+1} \pi_s \, ds + r - \int_t^{t+1} \pi_s \, ds = r$. When inflation accelerates, the increased rate of the government's real obligation b_t at time t is not r, however, but $r + \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ because increases in the real obligation are moved forward in time (or "moved up") by the acceleration of inflation. Figure 1 shows the increases in the real obligation at each time during a unit period for bonds issued at t, $\overline{B_t}$, the real value of which at time t is $\overline{b_t}$ when inflation is accelerating. Because the nominal interest rate $\overline{R_t} = r + \int_t^{t+1} \pi_s \, ds$ indicates that the government's nominal obligation to pay for the bonds' return in the future increases at the constant rate $r + \int_t^{t+1} \pi_s \, ds$ between t and $t+1$, then the real obligation expands at a time-varying rate between t and $t+1$ such that $r + \int_t^{t+1} \pi_s \, ds - \pi_v$ owing to the accelerating rate of inflation π_t. Clearly the line of the increases in the real obligation should slope down to the right as shown in Figure 1. Hence, the rate of increase in the real obligation at time t is higher than r. The increases in the real obligation are moved up by the acceleration of inflation, holding the total increase in the real obligation during a unit period between t and $t+1$ to $r\,\bar{b}_t$.

However, the moved up increases in the real obligation mean that the increases in the real obligation become smaller later in a unit period for each bond. Thus, because bonds issued between t - 1 and t offset each other, does the rate of increase in the real obligation of total government borrowing remain r at any time? It does not, because the degree of moving up increases as inflation accelerates. Figure 2 shows the effect of the increasing moving up. Because the rate of inflation increases by the square of time as shown in equation (13), more increases in the real obligation are moved up as time passes. As a result, the increases in real obligation cannot be indefinitely offset completely by the smaller increases in the real obligation of bonds issued in the past. Figure 3 shows the increases in the real obligation of bonds issued between t - 1 and t at time t (i.e., the increases in the real obligation of $\bar{B}_{t-1+s,t}$ for $0 < s \le 1$, the real value of which at time t is $\bar{b}_{t-1+s,t}$). Here, $\bar{b}_{t-1+s,t}$ is constant at steady state, $\bar{b}$. Hence, the increases in the real obligation of the total government bonds at time t is $r\bar{b}$ plus the area of triangle ABC minus the area of triangle CDE in Figure 3. The area of triangle ABC is larger than the area of triangle CDE because the degree of moving up increases as time passes. Thus the rate of increase in real obligation of the total government borrowing at time t (i.e., $r + \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_s$) is larger than r for any future t indefinitely. The government therefore must continue to face rates of increase in real obligation higher than r by $\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \pi_t$ at all points in the future.

3.3. Optimal behaviors of the government and the representative household

The government optimally plans its streams of future real expenditures, taxes, and borrowings subject to this moved up higher rate of increase in the real obligation for a future *t*. The government stops changing its real expenditures, taxes, and borrowing if the rate of increase in the real obligation $r+\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\ \ ds-\pi_t$ equals the government's time preference rate θ_G. The rate at which the real obligation increases therefore is a crucial variable in determining the government's behavior. As equations (6) and (12) show, equations $\theta_P = r$ and $\theta_G = \ r+\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\ \ ds-\pi_t$ hold simultaneously at steady state. That is, both the government and the representative household can achieve simultaneous optimization. In this sense, the rate at which the real obligation increases is a crucial variable not only for the government but for the entire economy.

The mechanism by which the government stops changing its real borrowing at the point where $r=\theta_P \neq \theta_G$ implies that the government is penalized if it has a higher time preference rate than the representative household. The penalized government is obliged to refrain from changing its real borrowing at the point $\theta_P = r$ facing $\theta_G =$ $r+\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\,ds-\pi_t$. Therefore, with the penalty, $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\,ds-\pi_t$ the contradiction of discount factors between the government and the representative household is reconciled, and the government is penalized by the representative household by its expectation of accelerating inflation so as to prevent the economy from breaking down.

Determining behavior on the basis of the rate at which the real obligation increases, as equations (12) and (13) indicate, is optimal for the government as well as for the representative household. This is still true even if it is assumed that a government can perceive less moved up real obligations (i.e., perceive less penalty) in the sense of a less steep slope in Figure 1 (i.e., the government can behave on the basis of a lower R_t than equation [13] indicates). This is true because, if such a government seeks to exploit the opportunity of higher expected utility by intentionally perceiving a lower rate of increase in the real obligation, then θ_G is always higher than the rate of increase in the real obligation. Therefore, the representative household cannot achieve optimality at the point where $r=\theta_P$. To prevent this consequence, the representative household penalizes the government more heavily by expecting more rapid inflation acceleration than equation (13) shows and makes the rate of increase in the real obligation the government perceives identical to θ_G. Note here that equations (12) and (13) concern only price level changes and are unrelated to real values, and real values are thus unaffected as long as the rate of increase in the real obligation the government perceives is identical to θ_G, irrespective of how the values of R_t and the penalty are perceived by the government. Hence, the government cannot achieve the higher expected utility by intentionally perceiving less moved up real obligations, which will only result in a more rapid acceleration of inflation. Conversely, if the representative household penalizes the government more heavily than equation (13) shows to exploit the opportunity of a higher expected utility, then θ_G is always lower than the rate of increase in the real obligation and thereby the government cannot achieve optimality. To prevent this consequence, the government changes itself to perceive less moved up real obligations and makes the rate of increase in the real obligation identical to θ_G. Hence, the representative household cannot achieve the higher expected utility by penalizing the government more heavily, which also only results in more rapid acceleration of inflation. As a result, even if it is assumed that a government can wilfully perceive less moved up real obligations, in the sense of a less steep slope in Figure 1, equation (13) gives both the government and the representative household the least inflation acceleration for the highest expected utilities. In this sense, the strategy profile that both the government and the representative household do not seek to exploit these opportunities is a Nash equilibrium. Both know this mechanism well and expect inflation to accelerate as equation (13) indicates when they perceive that $\theta_G > \theta_P$.

IV. CENTRAL BANK INDEPENDENCE

1. The power of a central bank to control a government

In the previous sections, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them may be merely partially independent. Furthermore, in the conventional models of inflation, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional models of inflation show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in the models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (13) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if equation (13) is added to conventional models, inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop by equation (13), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop as equation (13) indicates. That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

Consider a conventional discrete-time inflation model with a backward-looking Phillips curve. It consists of an aggregate supply function, $\pi_{t+1} = \pi_t + \alpha_x x_t + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1} + \varepsilon_{t+1}$; an aggregate demand function, $x_{t+1} = \beta_x x_t + \boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1} - \beta_r (r_t - r) + \eta_{t+1}$; and a Taylor-type instrument rule for a central bank, $i_t = \bar{\gamma} + \gamma_\pi (\pi_t - \pi^*) + \gamma_x x_t$ (e.g., Svensson 2003); where x_t is the output gap; $\boldsymbol{\omega}_t$ is a column vector of exogenous variables; r_t is the real interest rate; r is the real interest rate at steady state; i_t is the nominal interest rate; π^* is the target rate of inflation; α_x, β_x, β_r, $\bar{\gamma}$, γ_π, and γ_x are constant coefficients; $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ are row vectors of constant coefficients; ε_t and η_t are i.i.d. shocks with zero mean; and $\varepsilon_0 = 0$ and $\eta_0 = 0$. The real interest rate is defined as follows: $r_t \equiv i_t - \pi_{t+1|t}$, where $\pi_{t+1|t}$ is the rate of inflation that is expected in period t for period $t+1$, and

$\bar{\gamma} = \pi^* + r$ as is usually assumed. It is assumed that $r_{t+s|t} = r$ for any s (s = 1,2,3,…). Consider also an extended model that combines the above conventional model with equation (13). Here, the equation $r_t = \theta_P + \mu_t = r + \mu_t$ holds at equilibrium with random shocks, where μ_t is i.i.d. shocks with zero mean and $\mu_0 = 0$. Thus, equation (13) in a discrete-time model with random shocks can be expressed as $\pi_{t+1} = \pi_0 + 3(\theta_G - r)(t+1)^2 - 3\sum_{v=1}^{t+1} v^2 \mu_v + \xi_{t+1}$ where ξ_t is an i.i.d. shock with zero mean and $\xi_0 = 0$. The extended model that includes this discrete-time version of equation (13) indicates that approximately

$$(14) \quad \theta_G - r = \frac{1 - \beta_x}{3\left[(1 - \beta_x)t^2 + 2t + 1\right]}\left(\pi^* - \pi_0\right),$$

which implies that either the target rate of inflation (π*) or the time preference rate of government (θ_G) needs to be an endogenous variable (see Appendix 3).

A central bank will be regarded as truly independent if θ_G is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate according to the Taylor-type instrument rule in the above extended model. If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $R_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_v \, dv \, ds + r = \theta_G + \pi_t$ to $R_t = \theta_G + \pi_t + \psi$ $(\psi > 0)$ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than θ_G by the extra rate ψ.[11] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating, the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time

[11] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.

passes because of the gap between the accelerating inflation rate and the target rate of inflation widens by equation (13) and γ_x in Taylor-type instrument rules is usually larger than unity, say 1.5. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to one that is consistent with the target rate of inflation.

2. The necessity of an independent neutral organization to control the government's preferences

Equation (13) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the

Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical research indicates that the rate of time preference negatively correlates with permanent income (e.g., Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable. The degree of independence has varied across countries and time periods, which may offer an explanation for why various types of inflation have been generated. The level of central bank independence is therefore very important in explaining the nature of various types of inflation.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous shocks to both the government and the representative household because they are planned solely by the central bank. When a shock on θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by, for example, resetting θ_G, π_0, and t in equation (13).

V. TYPES OF INFLATION

1. The basic mechanism

The path of inflation consists of three movements: acceleration, constant, and deceleration. All types of inflation can be explained simply by combining these three movements. Equation (13) indicates that, if $\theta_G > \theta_P$, inflation accelerates; if $\theta_G = \theta_P$, inflation is constant; and if $\theta_G < \theta_P$, inflation decelerates. Hence, various types of inflation can be explained by the magnitude of θ_G in relation to θ_P. For example, hyperinflation can be explained by $\theta_G > \theta_P$, chronic inflation by a combination of periods in which the relationship between θ_G and θ_P sporadically changes from $\theta_G > \theta_P$ to $\theta_G = \theta_P$ and back again, disinflation by a combination of periods in which alternately $\theta_G < \theta_P$ and $\theta_G = \theta_P$, low and stable inflation by $\theta_G = \theta_P$, and deflation by $\theta_G < \theta_P$.

To cover all types of inflation, various combinations of periods in which $\theta_G > \theta_P$, $\theta_G = \theta_P$, and $\theta_G < \theta_P$ are necessary, which implies that θ_G should be time variable and change frequently. However, preferences do not seem to change frequently by their nature. If a government does not change its preference frequently, only three types of inflation— hyperinflation ($\theta_G > \theta_P$), stable inflation ($\theta_G = \theta_P$), and deflation ($\theta_G < \theta_P$)—would usually be observed. In reality, chronic inflation and disinflation and their variants have not been rare, which suggests that, other than the government itself, there is another important force that changes θ_G. A central bank also can control θ_G to the extent of its degree of independence. Hence, the types of inflation varies according to relative magnitudes of θ_G and θ_P and the degree of central bank independence.

2. The mechanisms of various types of inflation

2.1. Hyperinflation

Equation (13) implies that, if θ_G is significantly higher than usual, hyperinflation will be generated in a very short period.[12] Faced with significantly higher θ_G, people expect extremely high inflation and inflation explodes as equation

[12] When θ_G is significantly high, the weight $\dfrac{\overline{B}_{t,t}}{\int_{t-1}^{t} \overline{B}_{v,t}\, dv}$ in R_t is much larger than $\dfrac{\overline{B}_{t-1,t}}{\int_{t-1}^{t} \overline{B}_{v,t}\, dv}$ and thus R_t will be closer to $\int_t^{t+1} \pi_s \, ds + r$ than $\int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \ \ ds + r$. Hence, the path of inflation will be closer to $\pi_t = \pi_0 + 2(\theta_G - \theta_P)t$ than $\pi_t = \pi_0 + 3(\theta_G - \theta_P)t^2$.

(13) indicates. The value of money $\frac{1}{\int_0^t \pi_s ds} = \frac{1}{\pi_0 + 3(\theta_G - \theta_P)\int_0^t s^2 ds} = \frac{1}{\pi_0 + (\theta_G - \theta_P)t^3}$ rapidly declines to nil.[13] What factors would contribute to a significantly higher θ_G than usual? Hyperinflation has often been observed when governments were very fragile and unstable, for example, after a defeat in war or after a revolution. Germany after WWI, Japan and Hungary after WWII, and Russia after the collapse of the Soviet Union are typical examples of hyperinflation. If a government is fragile and unstable, not only households but the government itself will anticipate that the regime may soon collapse. If the probability of the end of regime is very high, it is likely that the government will behave very myopically (e.g., Fisher 1930; Yaari 1965). The government will not put a high value on the future, but it will struggle to survive in the present. It is not likely to listen to the advice of a central bank. The very fragile and unstable government's considerably myopic behavior will cause extremely high inflation expectations and then hyperinflation by equation (13). This explanation appears more natural than Cagan's (1956) hyperinflation model, in which it is suggested that hyperinflation basically occurs irrespective of the fragility or stability of the government.

Equation (13) also implies another type of hyperinflation. Even if θ_G is not significantly high, hyperinflation will eventually be observed if no action is taken when there are relatively large positive values of $\theta_G - \theta_P$. The hyperinflation observed in some countries in South America for the past several decades—sometimes called "modern hyperinflation"—may be examples of this type of hyperinflation. The situation in which relatively large positive values of $\theta_G - \theta_P$ are left as they are implies that a central bank is only somewhat independent. The combination of a more myopic government than usual and a dependent central bank will generate this type of hyperinflation.

Equation (13) indicates that hyperinflation is not caused by the growth of money

[13] If θ_G is significantly high and thus the path of inflation is close to $\pi_t = \pi_0 + 2(\theta_G - \theta_P)t$, then the value of money declines by $\frac{1}{\int_0^t \pi_s ds} = \frac{1}{\pi_0 + 2(\theta_G - \theta_P)\int_0^t s\, ds} = \frac{1}{\pi_0 + (\theta_G - \theta_P)t^2}$.

(i.e., not by seigniorage) but by the unusually myopic preference of a government combined with a scarcely independent central bank. This view is consistent with the conclusions of Sargent and Wallace (1973) and Fischer et al. (2002). They conclude that causation runs from inflation to money growth during hyperinflation, and that once high inflation has been triggered, monetary policy has typically been accommodative, as equation (14) implies. The explanation is also consistent with Sargent's (1982) view that a credible change in policies, preferably embedded in legal and institutional changes, could bring a hyperinflation to an end at a very small cost. Sargent (1982) implies that the main cause of hyperinflation is the behavior of government. Equation (13) indicates that, if the incumbent government is replaced with or changes itself into a government that has a much lower rate of time preference and/or if the authority to set and keep the target rate of inflation is delegated to a truly independent neutral organization that is obliged to stabilize the price level, high inflation expectations soon subside and the ongoing hyperinflation will be brought to an end at a small cost.

Equation (13) also indicates that the mechanism of hyperinflation can be explained without any ad hoc assumption of irrationality or friction, whereas Cagan's (1956) well-known hyperinflation model needs the assumption of adaptive expectations or some ad hoc frictions if large budget deficits are allowed in the model (e.g., Auernheimer 1976; Evans and Yarrow 1981; Kiguel 1989). Equation (13) indicates that hyperinflation is nothing more than a consequence of a deep parameter (i.e., the time preference rate of government) taking various values, and no additional or special mechanism is necessary to explain it.

2.2. Chronic inflation

Chronic inflation occurs when relatively high rates of inflation are sustained for a relatively long period. Many industrialized countries experienced chronic inflation in the 1960s and 1970s, and this period is often called the Great Inflation. Equation (13) indicates that chronic inflation will be observed if there is a combination of sporadic periods in which $\theta_G > \theta_P$ and regular periods in which $\theta_G = \theta_P$. Once a positive $\theta_G - \theta_P$ is allowed (even for a short period), equation (13) indicates that inflation will start to accelerate. The acceleration will stop when $\theta_G = \theta_P$ is restored. However, the higher rate of inflation and higher inflation expectations are retained because $\theta_G < \theta_P$ is necessary to lower inflation.

The combination of sporadic periods in which $\theta_G > \theta_P$ and regular periods in which $\theta_G = \theta_P$ implies that a central bank is not sufficiently independent. This type of central bank cannot sufficiently control θ_G and will sometimes fail to prevent the occurrence of a situation in which $\theta_G > \theta_P$ because $\theta_G > \theta_P$ is generally true by nature. Moreover, because of insufficient independence, the central bank usually will not be able to force the government to lower θ_G so far as $\theta_G < \theta_P$ even if $\theta_G < \theta_P$ is necessary for inflation to decline. As a result, the combination of a relatively more myopic government and an insufficiently independent central bank can generate chronic inflation.

Once the condition in which $\theta_G > \theta_P$ is allowed, the target rate of inflation needs to be raised by equation (14) unless $\theta_G < \theta_P$ is forced sufficiently lower. Clarida et al. (2000), Favero and Rovelli (2001), and Dennis (2001) conclude that the target rate of inflation in the pre-Volker era was much higher than that in the Volker-Greenspan era. Equation (14) suggests that the reason why the central banks at the time set high inflation targets is not because they deliberately committed the "crime" of high inflation. Instead, they were forced to raise the target rates of inflation because they were not sufficiently independent.[14]

Even if only partially independent, a central bank will not give up its attempts to force a government to lower θ_G so that $\theta_G < \theta_P$ and to prevent the already high rate of inflation from exploding. In some cases, the central bank may succeed in achieving periods in which $\theta_G < \theta_P$. Sporadic changes among periods in which $\theta_G > \theta_P$, $\theta_G = \theta_P$, and $\theta_G < \theta_P$ will make the path of inflation zigzag, that is, the path of inflation will have several or more trend breaks during periods of chronic inflation. Several or more

[14] Kydland and Prescott's (1977) and Barro and Gordon's (1983) well-known explanation for chronic inflation needs exceptionally large or successive negative supply shocks and thus needs internationally common such shocks to explain the international aspect of the Great Inflation. It is hard to find such shocks in many industrialized countries during the Great Inflation. The explanation in this paper can explain the international aspect of the Great Inflation without assuming such shocks because it concerns only the attitudes of the government and the central banks. The governments and central banks in most industrialized countries during the Great Inflation seem to have assumed common, respective attitudes because the economic policies conducted in the United States were often imitated by other countries.

trend breaks suggest that inflation looks like a random walk during periods of chronic inflation (see, e.g., Barsky 1987; Evans and Watchel 1993).

2.3. Disinflation

Disinflation occurs when there is a gradual transition from a high rate of inflation to a low and stable rate of inflation, but the decline does not reach deflation. A typical episode was experienced in many industrialized countries in the 1980s after the Great Inflation. Equation (13) indicates that disinflation will be observed when the condition of $\theta_G < \theta_P$ is gradually adjusted to one in which $\theta_G = \theta_P$ as the rate of inflation declines to a low and stable rate.

A truly independent central bank is necessary for disinflation because θ_G must be gradually shifted from $\theta_G < \theta_P$ to $\theta_G = \theta_P$. A government will not be able to discipline itself to keep $\theta_G < \theta_P$ because the opposite condition ($\theta_G > \theta_P$) is generally true by nature. In contrast, this gradual adjustment can be easily implemented by a truly independent central bank because they can force the government to keep $\theta_G < \theta_P$, and the central bank can gradually tune the target rate of inflation as well as θ_G as inflation cools down. Eventually the rate of inflation will land softly at a low and stable rate.

In the above disinflation path, there is often a point at which a central bank abruptly becomes truly independent. Taylor (2001, 2002) emphasizes the importance of changes in economic and political leadership as a cause of the Great Inflation by quoting Milton Friedman, who argued that the Great Inflation was a fundamentally political, not economic, phenomenon and that Ronald Reagan ended the Great Inflation by accepting a severe recession without bringing pressure on the Federal Reserve to reverse course. Similarly, Meltzer (2005) emphasizes the large role of political decision-making during the Great Inflation and concludes that the Federal Reserve was better able to control inflation during the administrations of Presidents Eisenhower and Reagan rather than those of Presidents Johnson, Carter, or Nixon. In other words, keeping the independence of the central bank is the key to stabilizing inflation. This view is consistent with the explanation for disinflation offered in this paper.

Gradual adjustments of θ_G suggest that the path of inflation may have several or more trend breaks during disinflations, particularly if θ_G is adjusted on a step-by-step basis, and then inflation may look like a random walk during disinflations. However, if θ_G is adjusted very smoothly, without any signs of a trend break, inflation may look less like a random walk.

2.4. Low and stable inflation

Equation (13) indicates that if π_0 initially is low and $\theta_G = \theta_P$ is maintained, then a low and stable rate of inflation will be sustained. A truly independent central bank is necessary for low and stable inflation because it forces the government to keep $\theta_G = \theta_P$ completely and indefinitely. Once the central bank succeeds in achieving the condition $\theta_G = \theta_P$ continuously at the target rate of inflation (i.e., inflation is stabilized at the target rate), the central bank need not frequently adjust θ_G anymore. The path of inflation will therefore have only a few small-scale trend breaks, which suggests that inflation will look less like a random walk (see, e.g., Barsky 1987; Evans and Watchel 1993; Cogley and Sargent 2002; Levin and Piger 2002).

2.5 Deflation

Equation (13) indicates that, if the condition in which $\theta_G < \theta_P$ continues over time, deflation will eventually occur. Nevertheless, deflation will be rarely observed because generally $\theta_G > \theta_P$ by nature and because it is unlikely that a central bank would dare to attempt deflation and set a target rate of deflation.[15] In fact, among the industrialized countries, only Japan in the 1990s and 2000s experienced deflation after World War II.

How can deflation occur if $\theta_G > \theta_P$ by nature and a central bank exerts itself to hold $\theta_G = \theta_P$ for a positive target rate of inflation? A huge negative shock that greatly widens the output gap may temporarily make the price level decline, but that would not necessarily be regarded as deflation because deflation means a successive decline of the price level. The possibility for deflation arises when a shock raises θ_P to some

[15] In the case of the gold standard that prevailed before World War II, deflation may be observed relatively more frequently because the gold standard indicates that the target rate of inflation is zero.

extent.[16] It may rarely happen, but if θ_P becomes higher and θ_G stays constant, then it is possible for the condition $\theta_G < \theta_P$ to occur. If this condition is left unchanged, then deflation will be observed by equation (13). The higher θ_P means a lower level of consumption at steady state, and a recession as well as a deflation will generally be observed if θ_P is raised. Nevertheless, if the central bank raises θ_G so that $\theta_G \geq \theta_P$ immediately after the shock, deflation will be prevented. However, if the central bank does not respond quickly, deflation will occur.

Once deflation takes root, it is very difficult even for a truly independent central bank to control θ_G and reverse the deflation because of the zero bound of the nominal interest rate. As shown in section IV, an independent central bank controls θ_G by manipulating the nominal interest rate with the extra rate ψ. Thus, if the central bank cannot manipulate the nominal interest rate because of the zero bound, it also cannot control θ_G. The central bank may advise the government to raise its preference θ_G so far as $\theta_G > \theta_P$ in order to reverse the deflation, but it cannot force the government to make $\theta_G > \theta_P$.[17] Furthermore, if the deflation deepens to a point where the real interest rate is compelled to exceed the marginal productivity, the economy cannot achieve a stable equilibrium anymore. The Great Depression in the 1930s may have been such a case, whereas Japan in the 1990s may have narrowly averted such a situation.[18]

Ahearne et al. (2002) argue that, to prevent deflation like the one experienced in Japan in the 1990s, both monetary and fiscal stimulus should go beyond the levels conventionally implied. For example, if the Bank of Japan lowered short-term interest rates by a further 200 basis points at any time between 1991 and early 1995, deflation

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[16] Since the era of Böhm-Bawerk and Fisher, the rate of time preference has been naturally regarded as time variable. See, for example, Böhm-Bawerk (1889), Fisher (1930), and Uzawa (1968).

[17] Deflation may continue for a long period even if the incumbent government is replaced as long as the time preference rates of the median and the representative households became nearly equal due to the shock that raised θ_P and $\theta_G \cong \theta_P$ is kept. However, if the time preference rate of the median household is raised similarly to θ_P due to the shock, then a replacement of government would reverse deflation because the newly elected government will have the same high rate of time preference as the raised time preference rate of the median household. The election of President Franklin D. Roosevelt in 1933 may have been such a case. Nevertheless, even if deflation is reversed, the other problems caused by a raised θ_P will remain.

[18] In the early 1930s, the ex post real interest rate in the United States was roughly 10% (e.g., Bernanke 1995), whereas that in Japan in the 1990s was generally less than 5% (e.g., Ito 2003).

could indeed have been avoided. This view implies that there are unusual incidents behind deflation, and it seems consistent with the argument that θ_P is unusually high in case of deflation.[19] Equation (13) suggests that, to prevent deflation, it is necessary to raise θ_G above the unusually high θ_P as soon as possible by imposing an unusually large negative extra rate ψ. Thus, deflation will be prevented if a central bank acts quickly and decisively, probably as the Federal Reserve under Chairman Greenspan attempted during the recession in the early 2000s. However, because shocks that make $\theta_G - \theta_P$ largely negative seem to occur rarely, even a truly independent central bank may fail to respond quickly enough to such a shock owing to a lack of experience.

VI. CONCLUDING REMARKS

In this paper, I explore the mechanism that generates various types of inflation. A model that explicitly incorporates the government's borrowing behavior was constructed. The model indicates that, if the time preference rates of an economically Leviathan government and the representative household are heterogeneous, they cannot achieve simultaneous optimization unless inflation accelerates or decelerates. The preferences are usually heterogeneous because the government and the representative household represent different households (i.e., the median and mean household, respectively). Since this inflation acceleration or deceleration path is optimal for the government as well as the representative household, both of them expect inflation to accelerate or decelerate if their time preference rates are heterogeneous. Conversely, it is not until their time preference rates are heterogeneous that inflation can accelerate or decelerate. This inflation acceleration mechanism is a result of the simultaneous optimization and does not require any ad hoc friction or irrationality. Thus, the mechanism provides a micro-foundation for the explanation for various types of inflation.

The problem in controlling inflation therefore is neither irrationality nor friction, but preference. Controlling one's own preferences is very difficult without help of an

[19] Harashima (2004a) estimates that θ_P in Japan rose roughly 2% at the end of the 1980s just before the deflation during the 1990s and 2000s.

independent neutral organization, whether for a person or a government. The government uses a central bank as such an independent neutral organization to control the government's preferences. Central banks have different amounts of independence and different degrees of power to control the government's preferences, which in turn generates the various paths of inflation. Hence, the various types of inflation can be explained by the inflation acceleration mechanism and the various degrees of central bank independence, giving us a unified and micro-founded explanation for the various types of inflation.

The most important point, in terms of price stability, that the explanation implies is the necessity of a truly independent central bank. A government usually has different preferences from the representative household and, more importantly, can barely control its own preferences even if it is fully rational. If a government is left without some neutral organization to help control inflation, the risk of considerable acceleration of inflation exists. A truly independent central bank is therefore necessary to rein in inflation.

APPENDIX

1. The condition for approximately identical weights

If b_t is constant (e.g., if it is at steady state), the weights for $t-1<s\le t$ are $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t}\overline{B}_{v,t}\,dv}=\frac{1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv}{\int_{t-1}^{t}\left[1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}$; thus,

$$R_t=r+\int_{t-1}^{t}\int_s^{s+1}\pi_v\,dv\left(\frac{1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv}{\int_{t-1}^{t}\left[1+(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}\right)ds=r+$$

$$\int_{t-1}^{t}\left[\int_s^{s+1}\pi_v\,dv-\frac{\int_s^{s+1}\pi_v\,dv\left\{\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv\right\}}{1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}\right]ds\cdot$$

Here, the absolute value of $\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds$ $-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv$ is $|\pi_{s1}-\pi_{s2}|$ at the maximum, where π_{s1} is the largest and π_{s2} is the smallest of π_s for $t-1<s\le t+1$, and the absolute value of $1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds$ is $|1+\pi_{s1}|$ at the minimum if $\pi_{s1}\le -1$ or if $\pi_{s2}<-1<\pi_{s1}$ and $|1+\pi_{s1}|<|1+\pi_{s2}|$, otherwise it is $|1+\pi_{s2}|$ at the minimum. In addition, $\pi_{s2}\le\int_s^{s+1}\pi_v\,dv\le\pi_{s1}$. Hence, the absolute value of

$$\frac{\int_s^{s+1}\pi_v\,dv\left\{\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds-(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)-\int_{t-1}^{s}(\pi_v+r)dv\right\}}{1+\int_{t-1}^{t}\left[(t-s)\left(\int_s^{s+1}\pi_v dv+r\right)+\int_{t-1}^{s}(\pi_v+r)dv\right]ds}$$ is

$\frac{|\pi_{s1}||\pi_{s1}-\pi_{s2}|}{|1+\pi_{s1}|}$ or $\frac{|\pi_{s1}||\pi_{s1}-\pi_{s2}|}{|1+\pi_{s2}|}$ at the maximum, and it is negligibly smaller than the absolute value of $\int_s^{s+1}\pi_v\,dv$ if the absolute values of π_s for $t-1<s\le t+1$ are sufficiently smaller than unity because $\pi_{s2}\le\int_s^{s+1}\pi_v\,dv\le\pi_{s1}$, and thus approximately

$$R_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r \; .$$

2. The transversality condition

By equations (6) and (12), $R_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r = \theta_G + \pi_t$ and thus $R_t - \pi_t = \theta_G$ at steady state. Substituting the equation $R_t - \pi_t = \theta_G$ and equation (12) into conditions (3) and (4) and solving both differential equations yield the equation $\lambda_{G,t} \, b_t = -\exp\left[\left(g_t - x_t - s_t\right) \int \frac{1}{b_t} dt + C^{\#} \right]$ at steady state, where $C^{\#}$ is a certain constant. Therefore, it is necessary to satisfy $g_t - x_t - s_t < 0$ and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$ for the transversality condition (5) to hold.

Here, by condition (4), $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t}$ at steady state. Hence, if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} = 0$ at steady state, then b_t is constant; thus, $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \infty$. Thereby, the transversality condition holds. However, if $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} < 0$ at steady state, then b_t diminishes to zero and transversality condition (5) cannot hold because $g_t - x_t - s_t < 0$. If $\frac{\dot{b}_t}{b_t} = \theta_G + \frac{g_t - x_t - s_t}{b_t} > 0$ at steady state, then $\lim_{t \to \infty} \frac{\dot{b}_t}{b_t} = \theta_G$; thus, b_t increases as time passes and $\lim_{t \to \infty} \int \frac{1}{b_t} dt = \frac{C^{\#\#}}{\theta_G}$, where $C^{\#\#}$ is a certain constant. Therefore, transversality condition (5) also cannot hold.

3. Derivation of equation **(14)**

By the Taylor-type instrument rule ($i_t = \bar{y} + \gamma_\pi \left(\pi_t - \pi^*\right) + \gamma_x x_t$) and the equation $r_t \equiv i_t - \pi_{t+1|t}$, then $\pi_t = \pi^* + \frac{1}{\gamma_\pi}\left(r_t + \pi_{t+1|t}\right) - \frac{\gamma_x}{\gamma_\pi} x_t - \frac{\bar{y}}{\gamma_\pi}$, and by substituting the aggregate supply function ($\pi_{t+1} = \pi_t + \alpha_x x_t + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1} + \varepsilon_{t+1}$), then $\pi_t = \pi^* + \frac{r_t + \pi_t + \alpha_x x_t + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t} - \bar{y} - \gamma_x x_t}{\gamma_\pi}$.

Therefore, $x_t = \frac{1}{\alpha_x - \gamma_x}\left[(\gamma_\pi - 1)\pi_t - \gamma_\pi \pi^* - r_t + \bar{\gamma} - \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t}\right]$. By substituting the aggregate demand function ($x_{t+1} = \beta_x x_t + \boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1} - \beta_r (r_t - r) + \eta_{t+1}$), then $(\gamma_\pi - 1)\pi_{t+1} - \gamma_\pi \pi^* - r_{t+1} + \bar{\gamma} - \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+2|t+1} =$ $\beta_x\left[(\gamma_\pi - 1)\pi_t - \gamma_\pi \pi^* - r_t + \bar{\gamma} - \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t}\right] + (\alpha_x - \gamma_x)\left[\boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1|t} - \beta_r (r_t - r) + \eta_{t+1}\right]$, and by substituting the discrete-time inflation acceleration mechanism ($\pi_{t+1} = \pi_0 + 3(\theta_G - r)(t+1)^2 - 3\sum_{v=1}^{t+1} v^2 \mu_v + \xi_{t+1}$), then $(\gamma_\pi - 1)\left[\pi_0 + 3(\theta_G - r)(t+1)^2 - 3\sum_{v=1}^{t+1} v^2 \mu_v + \xi_{t+1}\right] - \gamma_\pi \pi^* - (r + \mu_{t+1}) + \bar{\gamma} - \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+2|t+1} =$

$$\beta_x\left\{(\gamma_\pi - 1)\left[\pi_0 + 3(\theta_G - r)t^2 - 3\sum_{v=0}^{t} v^2 \mu_v + \xi_t\right] - \gamma_\pi \pi^* - (r - \mu_t) + \bar{\gamma} - \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t}\right\} + (\alpha_x - \gamma_x)\left(\boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1|t} - \beta_r \mu_t + \eta_{t+1}\right).$$

Thus, $3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right]\theta_G = \left\{3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right] + (1 - \beta_x)\right\}r$

$$+ (1 - \beta_x)\left[\gamma_\pi \pi^* - \bar{\gamma} - (\gamma_\pi - 1)\pi_0\right] + (\gamma_\pi - 1)\left(3\sum_{v=1}^{t+1} v^2 \mu_v - \xi_{t+1}\right) + \mu_{t+1} + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+2|t+1}$$

$$- \beta_x\left[(\gamma_\pi - 1)\left(3\sum_{v=0}^{t} v^2 \mu_v - \xi_t\right) + \mu_t + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t}\right] + (\alpha_x - \gamma_x)\left(\boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1|t} - \beta_r \mu_t + \eta_{t+1}\right);$$ thus,

$$\theta_G - r = \frac{(1 - \beta_x)\left[r + \gamma_\pi \pi^* - \bar{\gamma} - (\gamma_\pi - 1)\pi_0\right] + (\alpha_x - \gamma_x)\left(\boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1|t} - \beta_r \mu_t + \eta_{t+1}\right)}{3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right]}$$

$$+ \frac{(\gamma_\pi - 1)\left(3\sum_{v=1}^{t+1} v^2 \mu_v - \xi_{t+1}\right) + \mu_{t+1} + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+2|t+1} - \beta_x\left[(\gamma_\pi - 1)\left(3\sum_{v=0}^{t} v^2 \mu_v - \xi_t\right) + \mu_t + \boldsymbol{\alpha}_z \boldsymbol{\omega}_{t+1|t}\right]}{3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right]}.$$

Here, it is assumed for simplicity that the exogenous variables $\boldsymbol{\omega}_t$ play limited roles for inflation and output gaps; thus, $\boldsymbol{\alpha}_z$ and $\boldsymbol{\beta}_z$ are near zero and approximately $\boldsymbol{\alpha}_z\left(\boldsymbol{\omega}_{t+2|t+1} - \beta_x \boldsymbol{\omega}_{t+1|t}\right) + (\alpha_x - \gamma_x)\boldsymbol{\beta}_z \boldsymbol{\omega}_{t+1|t} = 0$. Hence, $\theta_G - r = \frac{(1 - \beta_x)\left[r + \gamma_\pi \pi^* - \bar{\gamma} - (\gamma_\pi - 1)\pi_0\right]}{3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right]}$

$$+ \frac{(\gamma_\pi - 1)\left(3\sum_{v=1}^{t+1} v^2 \mu_v - \xi_{t+1}\right) + \mu_{t+1} - \beta_x\left[(\gamma_\pi - 1)\left(3\sum_{v=0}^{t} v^2 \mu_v - \xi_t\right) + \mu_t\right] - (\alpha_x - \gamma_x)(\beta_r \mu_t - \eta_{t+1})}{3(\gamma_\pi - 1)\left[(t+1)^2 - \beta_x t^2\right]}$$ approximately.

Since η_t, μ_t, and ξ_t are i.i.d. shocks with zero mean, then by taking expectations of both sides of the equation, $\theta_G - r = \frac{(1 - \beta_x)\left[r + \gamma_\pi \pi^* - \bar{\gamma} - (\gamma_\pi - 1)\pi_0\right]}{3(\gamma_\pi - 1)\left[(1 - \beta_x)t^2 + 2t + 1\right]}$ and thus $\theta_G - r =$ $\frac{1 - \beta_x}{3\left[(1 - \beta_x)t^2 + 2t + 1\right]}\left(\pi^* - \pi_0\right)$ approximately because $\bar{\gamma} = \pi^* + r$.

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Figure 1 ***Increases in the real obligation of*** $\bar{b}_t$

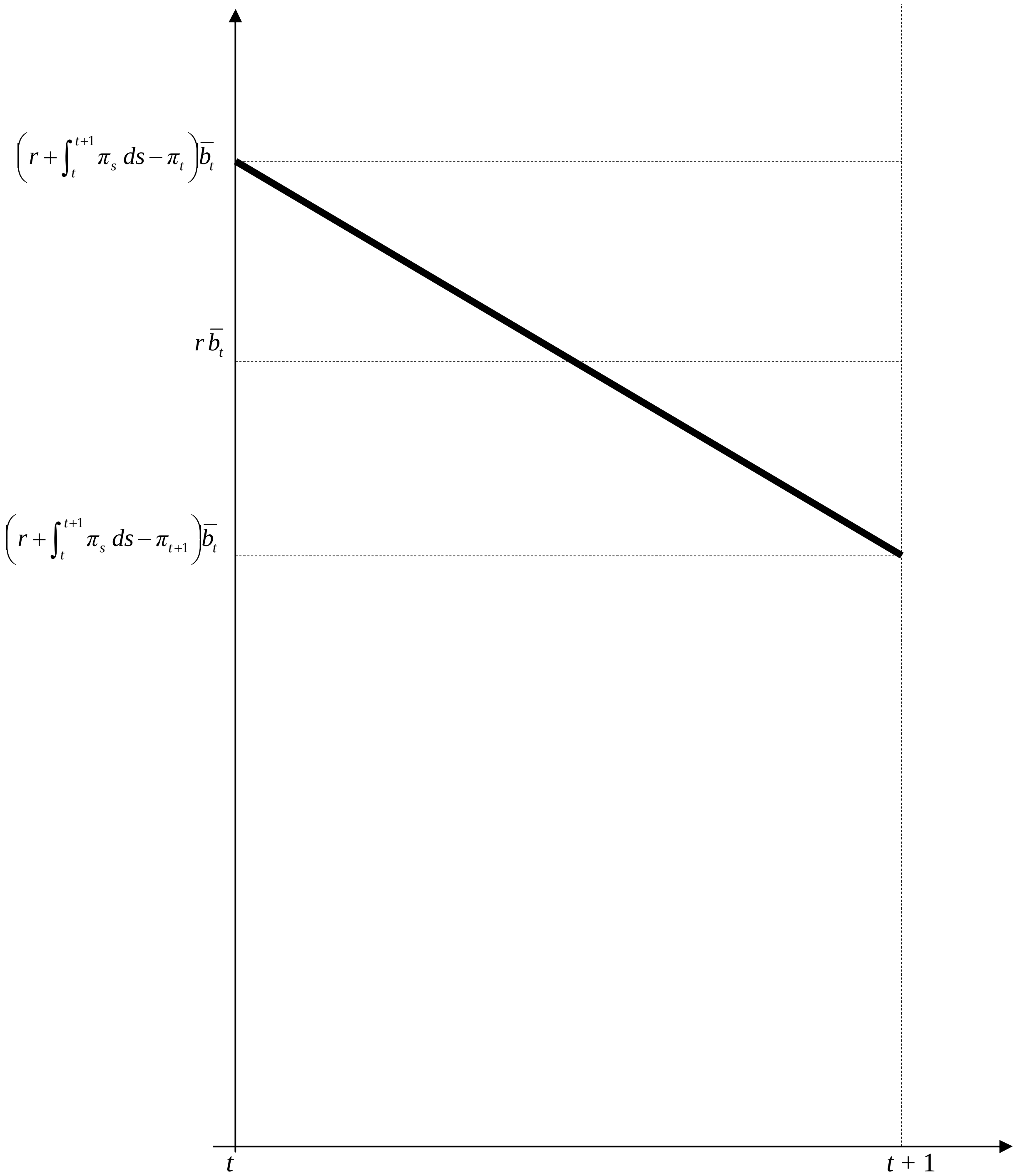

Figure 2 ***Increases in the real obligation of*** $\bar{b}_t$, $\bar{b}_{t+1}$, ***and*** $\bar{b}_{t+2}$

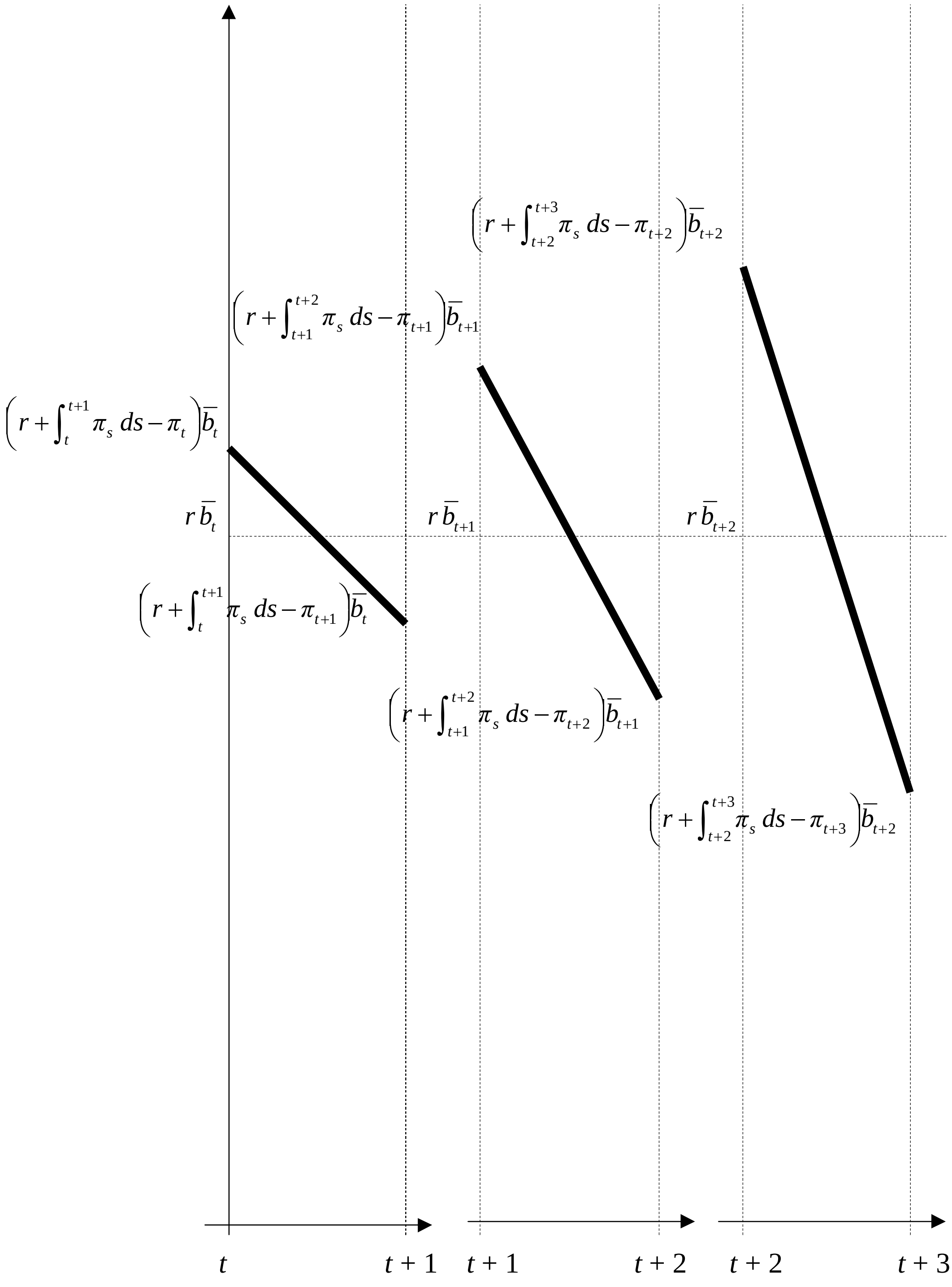

Figure 3 ***Increases in the real obligation of*** $\bar{b}_{t-1+s,t}$ $(0 < s \le 1)$ ***at time t***