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A Dynamic Model of Auctions with Buy-it-now: Theory and Evidence*

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Abstract

In the ascending-price auctions with Yahoo!-type buy-it-now (BIN), we characterize and derive the closed-form solution for the optimal bidding strategy of the bidder and the optimal BIN price of the seller when they are both risk-averse. The seller is shown to be strictly better off with the BIN option, while the bidders are better off only when their valuation is greater than a threshold value. The theory also implies that the expected transaction price is higher in an auction with an optimal BIN price than one without a BIN. This prediction is confirmed by our data collected from Taiwan's Yahoo! auctions of Nikon digital cameras.

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1 Introduction

An interesting feature of the recent on-line bidding auctions which is absent from the traditional auctions is the existence of the buy-it-now (BIN) option.[1] There are two main explanations of how a buy-it-now benefits the seller. The first is the seller's ability to exploit bidders' time preference (see, e.g., Mathews, 2004). Under this explanation, the bidder is impatient, and is willing to pay a higher price to obtain an object immediately, rather than through a time-consuming bidding process. The seller can then set up a BIN to satisfy this need and thereby makes more profit. The second explanation is that if the bidders are risk-averse, then they will be willing to buy the object with a higher, but fixed, price rather than obtaining the object through the bidding process, which has the risk of either losing to other bidders or, even if they win, paying an uncertain price.[2]

Both explanations imply that auctions with the BIN option will have a greater transaction price than auctions of identical objects but without BIN. However, both explanations are incomplete, because the same reasoning can also be applied to the seller. That is, not only bidders, but also the sellers can be impatient or risk-averse. In either case the sellers will be willing to set a lower BIN price so that the objects can be sold earlier (if they are impatient) or at prices with smaller variation (if they are risk averse).[3] Thus the two mentioned explanations have implicitly assumed that it is the bidders, rather than the sellers, who are impatient or risk averse.[4] When both the seller and the bidders are risk-averse, the function served by BIN for the seller and its

[1] See Lucking-Reiley (2000), Bajari and Hortaçsu (2004), Ockenfels et al. (2007), and Haruvy and Popkowski Leszczyc (2009) for general discussions of internet auctions.

[2] See, e.g., Budish and Takeyama (2001), Reynolds and Wooders (2004).

[3] Assuming bidder's risk neutrality, Mathews and Katzman (2006) show that a risk averse seller has incentive to set a BIN price low enough so that it is exercised with positive probability. Their model, however, is for the eBay-type BIN, in which the bidders are allowed either to place a bid or to buy out with the BIN price before any other bidder has placed a bid. Once a bidder places a bid (higher than the reserve price), the BIN option will disappear, and the auction becomes one without BIN. Our model is a Yahoo!-type BIN, in which the BIN option remains active as long as the auction has not ended.

[4] For an example in the empirical literature, Ackerberg et al. (2006) develop a structural empirical model of auctions for laptops with and without a buy-it-now price to estimate risk aversion and time preference parameters of the bidders with the data collected from eBay. Although their results suggest that the bidders are risk averse, the seller does not have a strategic role.

consequence (especially its effect on transaction prices) deserve further investigation.

In this paper we focus on how both the bidder's and the seller's attitudes toward risk affect the bidder's incentives to buy out and the seller's optimal BIN price in response to the bidders' strategies. We develop a dynamic model of English auction with two bidders (extended to n-bidder case latter) who, at every prevailing price, need to decide whether to continue with bidding or to buy out.[5] Either the bidder or the seller (or both) can be risk-averse. We solve for the optimal buyout strategy of the bidders in closed form. This closed-form solution is in turn used to solve for the optimal BIN price of the seller. We show that under the optimal strategy, the higher a bidder's valuation of the object, the earlier is he willing to buy out the object. Furthermore, the optimal BIN price is an increasing (decreasing) function of the bidders' (seller's) degree of risk-aversion. We also show that whether the winner wins with buy-it-now or competitive bidding depends on the configuration of the bidders' valuations of the object.

In our model, BIN serves two purposes for the seller. First, if the bidders are risk-averse, it can be used as an instrument to exploit their aversion to risk by forcing them to pay a premium in order to avoid the risks in the bidding process. Second, if the seller is risk-averse, it also serves to decrease price risk for the seller himself. This implies that even if the bidders are risk-neutral, the seller still has incentives to offer the buy-it-now option, not to make more profit, but to avoid the riskier outcome of the bidding process. Indeed, our result indicates that the only case in which the seller does not gain from a buy-it-now option is when both the bidders and sellers are risk-neutral. This extends Budish and Takeyama (2001), who show that (when the seller is risk-neutral) the buy-it-now option is of value if and only if the bidders are risk-averse. Our results also extend Hidvegi et al. (2006), who prove several of the same results under non-optimal BIN prices. Although the main body of the paper assumes a two-bidder case, in Section 3 we show that these results continue to hold true in the n-bidder case.

[5] Our model is therefore a Yahoo!-type permanent BIN auction, in which BIN remains an option for the bidders throughout the auction.

Our model also provides several testable empirical predictions. The most important among these predictions is that the expected transaction price of an item will be higher in an auction with optimal BIN price than that of an identical item but without BIN. This prediction is confirmed by the data collected from Taiwan's Yahoo! auctions of Nikon digital cameras. Our empirical results also support certain previously tested results.[6] Note that our choice of Taiwan Yahoo! auction,[7] together with the theoretical model corresponding the Yahoo!-type BIN, is not out of convenience. This is because in the eBay auctions, the buy-it-now option will disappear at the time a bidder places a competitive bid. Therefore, an item in the eBay data which is recorded as a standard auction might actually have started out having a BIN option, and should have been recorded as a BIN auction. A Yahoo!-type BIN therefore has more advantage for empirical investigations.

Recently, there have been many theoretical papers discussing the role of the BIN option in auctions. Mathews and Katzman (2006) propose a theory for the eBay-type buy-it-now, in which all the bidders decide either to buy out the item or to enter the bidding process at the beginning of auction, and the choice is irreversible: If any bidder places a bid, BIN will disappear, and the auction outcome is determined solely by competitive bidding. Reynolds and Wooders (2009) compare Yahoo!- and eBay-type BIN auctions, and show that when the bidders are risk-neutral, the two types of auction have the same expected revenue for the seller. However, when the bidders are risk-averse, the Yahoo! version raises more revenue. Their model assumes constant absolute risk-averse bidders and general distribution of the bidder's valuation. Kirkegaard and Overgaard (2008) propose a multi-unit demand explanation for buy-it-now. They assume two bidders (each demanding two units) and two sequential sellers (each offering one unit), and show that the first

[6] For example, good reputation of the seller increases both the probability of an item being sold and its transaction price (see, e.g., Livingston, 2005).

[7] Yahoo! has ceased its online auction operation in the US, UK and some other European countries. In fact, as we are aware, currently the only three regions in which Yahoo! auctions remain active are Japan, Taiwan, and Hong Kong.

seller can raise his expected revenue by posting a buy-it-now. Our paper is closest to that of Hidvegi et al. (2006). In a model with n bidders and general density and utility functions, they characterize the bidder's optimal bidding strategy under BIN. Among others, they show that the seller is strictly better off when either he or the bidder is risk averse (Theorems 11 and 12). This is consistent with our Propositions 3 and 5. They also show that the bidders can be either worse- or better-off under BIN (Section 3.1). This is consistent with our Proposition 4. Their paper is mainly a theoretical investigation, and does not attempt to derive any testable empirical implication from the model. Our model is partially motivated by trying to derive properties which have empirical implications, and can be tested by auction data.

Compared to the theoretical literature, our paper makes two contributions. First, in the more general contexts of Hidvegi et al. (2006) and Reynolds and Wooder (2009), it is difficult to characterize the optimal BIN price. Our specifications enable us not only to characterize the bidder's optimal buyout strategy and the seller's optimal BIN price, but also to derive their closed-form solutions. This in turn facilitates the computation of the bidder's and seller's expected utilities and the expected transaction price, together with their relation to the degree of risk-aversion of both the bidder and the seller. Most importantly, our derivation is intuitive, showing clearly the cost and benefit of a buy-it-now option for the seller, and how he can design the buy-it-now price to balance its cost and benefit. Furthermore, the clear theoretical predictions on transaction prices also enable us to devise an empirical model to test, and confirm, their validity. Second, since the optimal buy-it-now price cannot be explicitly solved for in the previous literature, the comparison between the buy-it-now auction and the pure auction has to be based on certain "properly chosen" (but possibly suboptimal) buy-it-now prices. As a result, only necessary conditions can be found under which a buy-it-now auction yields higher expected utility for the seller. In our model, the closed-form solution enables us to derive the necessary and sufficient condition. In fact, buy-it-now auction dominates pure auction: Unless both the bidders *and* the sellers are risk-neutral, the seller's utility is strictly higher with buy-it-now when its price is set optimally.[8]

On the empirical side of the literature, in addition to Ackerberg et al. (2006) mentioned above, Dodonova and Khoroshilov (2004) find that a buy-it-now price does influence bidders' behavior and that auctions with a higher buy-it-now price cause bidders to bid more aggressively, which results in higher sale prices. Anderson et al. (2004) use instrumental variables methods to investigate the effect of offering a buy-it-now option on the sale price. Their results indicate that the existence of a buy-it-now price does not have a significant positive correlation with the sale price except in the subsample with posted-price auctions in which the minimum bids are equal to the buy-it-now prices. In this paper, we exclude these auctions from our sample. (But we include these auctions when we test the robustness of our empirical results.) Other differences are mainly due to econometric considerations. Specifically, our sample selection model in the first stage of the two-stage least squares (2SLS) and the instrumental variable strategy are different from theirs. Wang et al. (2008) consider bidder participation costs in the buy-it-now auctions. Their results show that there is a positive effect of offering the buy-it-now option on the expected profits in the auctions where bidder participation costs are high. Popkowski Leszczyc et al. (2009) find that the buy-it-now price has a reference price effect and thus has a positive effect on bidders' willingness to pay for the product. All of these studies use eBay data to analyze the effects of temporary BIN, while our research adopts Yahoo! data to investigate the effects of permanent BIN.

Although the more general specifications in Hidvegi et al. (2006) and Reynolds and Wooders (2009) have prevented them from making predictions regarding the relation between BIN and

[8] Note that the bidders's optimal buyout strategy depends only on their (but not the seller's) degree of risk-aversion. As such, the credit for deriving the bidder's optimal buy-it-now strategy belongs to Hidvegi et al. (2009) and Reynolds and Wooders (2009). Our theoretical contribution lies in results which depend on the derivation of the seller's optimal buy-it-now price in closed-form, together with characterizing its relation to the degrees of risk-aversion, the comparison of transaction prices, and the necessary and sufficient condition under which the seller's welfare improves.

transaction price, it is reasonable to conjecture that, in their model, the expected transaction price will also be higher with BIN, as BIN in general serves as an option for the bidders to avoid risk by paying a premium. Consequently, our empirical study is not an attempt to discriminate between theoretical models, but to lend support to the theoretical hypothesis that BIN offers an option to reduce the price risk of the bidders by raising the transaction price.

2 The Model

A risk-averse seller conducts an English auction to sell an object. Two bidders ($i = 1, 2$) are participating in the auction.[9] The value of the object to bidder i, v_i, is his own private information, but is known to be independently and uniformly drawn from $[0, \bar{v}]$.

A bidder can either buy the object by out-bidding the other bidder, or by buying the objective with the buy-it-now price, v_b, set by the seller at the beginning of the auction.[10] The utility function of a bidder with valuation v is

$$u(v, p) = (v - p)^{\alpha}/\alpha \tag{1}$$

if he buys the objective with price p, and is 0 if he does not buy it; where $0 < \alpha \leq 1$. The value of α denotes the bidder's reverse degree of relative risk-aversion. The smaller the value of α, the

[9] See Section 3 for the n-bidder case.

[10] We are assuming that the reserve price is 0. Although both Hidvegi et al. (2006) and Reynolds and Wooders (2009) assume a positive reserve price, since its value has not been optimized in their models, the effect of reserve price is not important. In general, however, the optimal reserve price has an interesting contrast to BIN, as the former is the lower bound for transaction prices, while the latter is an upper bound. Beyond this, they are quite different. The reserve price represents an attempt by the seller to increase profit, while BIN is an attempt to avoid risk (at least so in our model). Their solution techniques are also different. The solution for the optimal reserve price is characterized by standard first-order conditions, while that for the optimal BIN price requires solving for the bidder's buyout strategy, which is characterized by differential equations. However, since BIN imposes an upper bound on possible transaction prices, and since optimal reserve price will generally be dependent on this upper bound, the optimal BIN and reserve prices should be complementary. We have not formally investigated this in our paper, but as can be seen from Table 3, the average minimum bid for auctions with BIN is substantially higher than that without (NT$ 5,978 vs NT$ 4,991). Another dataset we collected, for US eBay auctions of iPods, also shows a similar pattern. This is a phenomenon which deserves investigating.

more risk-averse is the bidder. The utility function of the seller is assumed to be $\pi(p) = p^{\beta}/\beta$; where $\beta \in (0, 1]$ is the seller's reverse degree of relative risk-aversion. Similarly, the smaller the value of β, the more risk-averse is the seller.[11]

2.1 Equilibrium Buy-it-now Strategy

In this subsection, we derive the optimal buy-it-now strategy of the bidder under a given buy-it-now price. Thus throughout this subsection, we assume that buy-it-now price is fixed at v_b.

Even with a buy-it-now price, a basic result of the standard English auction remains true: It is a dominant strategy for a bidder to stay active in the auction as long as the prevailing price is lower than his valuation of the object. The complication comes from the fact that, at every prevailing price, he now has the additional option to pre-empt his opponent by buying the objective immediately at the buy-it-now price v_b. Since Yahoo! and eBay auctions are essentially second-price auctions, the strategy of a bidder is composed of two independent parts: First is the maximum he is willing to pay, which (as bidding one's valuation is the dominant strategy) is exactly his valuation. Second is the function linking his valuation to the standing price at which he is willing to buy out the item, given the value of the buy-it-now price. We will call the latter his buy-it-now strategy.

Given v_b, let $P(v)$ be the buy-it-now strategy of the bidder whose valuation of the objective is v. That is, a bidder who values the object at v is willing to buy out the object (by paying v_b) when the prevailing price reaches $P(v)$. Reynolds and Wooders (2009) show that $P(v)$ is differentiable and decreasing in v. The latter fact is intuitive, as the greater the value of v, the more willing is the bidder to obtain the object immediately by paying v_b.

As will be shown later, when the valuation of a bidder is high enough, he wants to obtain the

[11] Our assumption on utility function thus implies constant relative risk-aversion. A recent paper (Chiappori, 2006) using the Italian Survey of Household Income and Wealth dataset suggests that individuals do exhibit constant relative risk-aversion.

item so much so that he is willing to buy out the item when the standing price is still 0. In other words, $P(v) = 0$ when v is large enough, implying the bidder buys out immediately after the auction opens. A technical problem arises when both bidders have high valuations, and are both willing to buy out at a zero standing price. In that case there is a discontinuity in the optimal buy-it-now strategy that is well-known in the literature.[12] This discontinuity prevents us from obtaining a closed-form solution, not only for the optimal buy-it-now strategy, but also for the optimal buy-it-now price and the expected transaction price. In order to facilitate our empirical discussion, which requires explicit comparison of transaction prices between cases, we will make the following assumption: If it is the case that both bidders intend to buy out when the standing price is 0, the one with higher valuation will win. Although a strong assumption, it has certain justification. In an on-line auction, an item is put up for auction for a much longer time span than the traditional English auction.[13] Moreover, a bidder need not be present during the whole auction process. A bidder can enter at any time to bid as long as the auction is still open. That means a bidder might miss the chance even if he is willing to buy out when the prevailing price is 0, as he might be absent. A bidder with higher valuation, being one having greater surplus from buying the item, is then more alert to stay online searching for the item. Therefore, he is more likely to be present when auction of the item in question starts, and thus has a greater chance to buy out. Our assumption essentially says that this advantage for the higher valuation bidder is absolute, in that he wins with probability 1. If this assumption is made, then the discontinuity in expected payoff mentioned above will cease to exist, as a bidder wins if and only if his valuation is greater, even if both bidders propose to buy out at price 0.[14]

[12] See, for example, Hidvegi et al. (2006) and Reynolds and Wooders (2009).

[13] In Taiwan's Yahoo! auction site, it can be from 1 to 10 days.

[14] We also analytically solve for the case in which the winning chance is 1/2 for each bidder when both propose to buy out in the beginning. But since there exists no closed-form solution, the comparative statics derivation and price comparison become extremely complicated and burdensome. We therefore use simulation to check for the properties that we derive in Section 2 of the paper. All the results go through. These simulation results can be downloaded from the following website: http://idv.sinica.edu.tw/kongpin/auctionsimulation.nb. The file must be viewed with Mathematica software. Please contact the corresponding author for a pdf printout file. The pdf file,

Proposition 1. *Given any buy-it-now price v_b, the bidder's optimal buy-it-now strategy is: Never buy out if $v \leq v_b$. If $v > v_b$,*

$$P(v) = \begin{cases} 0, & \text{if } v \geq (1 + \frac{1}{\mu^*})v_b; \\ (1+\mu^*)v_b - \mu^* v, & \text{if } v < (1 + \frac{1}{\mu^*})v_b; \end{cases}$$

where μ^* satisfies $(1+\mu^*)^\alpha = 1 + \frac{1}{\mu^*}$.

Proof: See Appendix A.

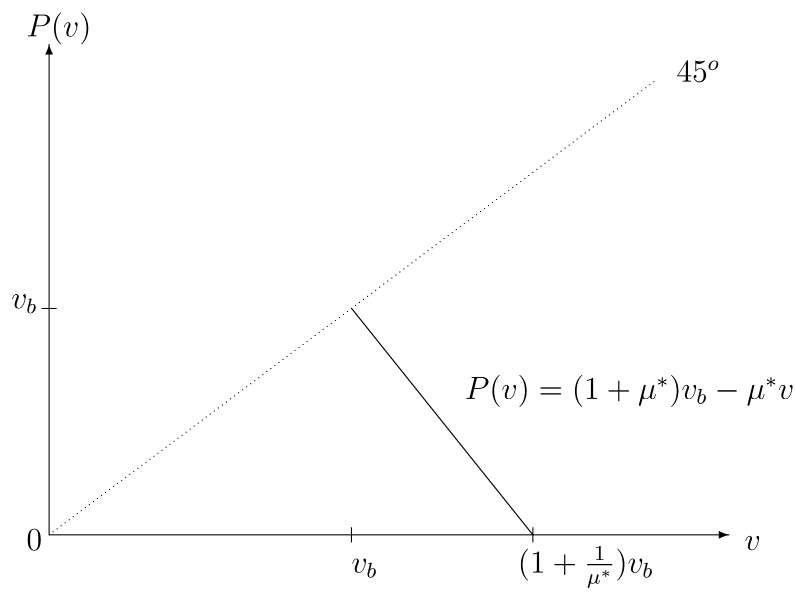

Figure 1: Buyer's optimal buy-it-now strategy.

The graph of $P(v)$ is drawn in Figure 1. It visualizes the relation between a bidder's valuation and the prevailing price at which he wants to buy out. Specifically, if the bidder's valuation is v, then he will be willing to buy out the item when the standing price reaches $(1+\mu^*)v_b - \mu^* v$. The higher a bidder's valuation for the objective, the lower is the prevailing price at which he is willing to buy it out. In particular, if $v \geq (1 + \frac{1}{\mu^*})v_b$, then his valuation is so high that he is willing to buy out the objective right at the beginning of the auction (i.e., when the standing price is 0).

however, is truncated, as the simulation results are too wide to be contained on letter-size paper.

Given the optimal buy-it-now policy, the optimal strategy of the bidder with valuation v is then easy to describe: Stay active as long as the prevailing price is lower than $P(v)$, and buy out the object when price reaches $P(v)$. Note that since a bidder will consider buying out only if $v > v_b$, we know that $v - P(v) = (1+\mu^*)(v - v_b) > 0$. That is, if a bidder will buy out the object, then he will do so before the price reaches his valuation. This also implies that the transaction price cannot be higher than v_b. In other words, by setting v_b as the buy-it-now price, the seller essentially sets v_b as the upper-bound for the possible transaction prices. Moreover, since μ^* is decreasing in α, it implies that the more risk-averse a bidder, the earlier he is willing to buy out the object. This result is fairly intuitive. The more risk-averse a bidder, the less willing he is to face the uncertain outcome of the bidding process. Thus he is more willing to buy it out early.

The solution $P(v)$ also makes it possible to characterize the outcomes of the auction as a function of v_1 and v_2. Note that by the symmetric nature of the equilibrium (and our assumption regarding ties), a bidder will win if and only if his valuation of the object is greater than his opponent's. The question is only whether he will win by out-bidding his opponent or by direct buy-it-now. Since $P(v)$ is the relation between a bidder's valuation and the prevailing price at which he is willing to buy out, bidder i will win by bidding if and only if $v_i > v_j$ and $v_i < (1+\frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. On the other hand, i will win by buy-it-now if and only if $v_i > v_j$ and $v_i > (1+\frac{1}{\mu^*})v_b - \frac{v_j}{\mu^*}$. We can thus characterize the outcomes of the auction as a function of the bidders' valuations of the item in Figure 2. In the figure, regions I and I' depict the case in which the winner wins by out-bidding his opponent. In regions II and II', the winner obtains the object by buy-it-now.

It should be noted that if the buy-it-now price v_b is chosen to be very high (specifically, higher than $\mu^*\bar{v}/(1+\mu^*)$), then the lower part of the line $v_2 = (1+\mu^*)v_b - \mu^* v_1$ will fall outside of $\bar{v}$. In that case this line should have a truncation at $\bar{v}$. That is, for any buy-it-now price v_b the line dividing regions I and II should be $v_2 = (1+\mu^*)v_b - \mu^* v_1$ if $(1+\frac{1}{\mu^*})v_b - \frac{v_2}{\mu^*} \le \bar{v}$, and is $v_1 = \bar{v}$ if $(1+\frac{1}{\mu^*})v_b - \frac{v_2}{\mu^*} > \bar{v}$. But since the optimal buy-it-now price will never be chosen to be greater than $\mu^*\bar{v}/(1+\mu^*)$,[15] we have not considered this additional complication in our derivation of the optimal buy-it-now price.

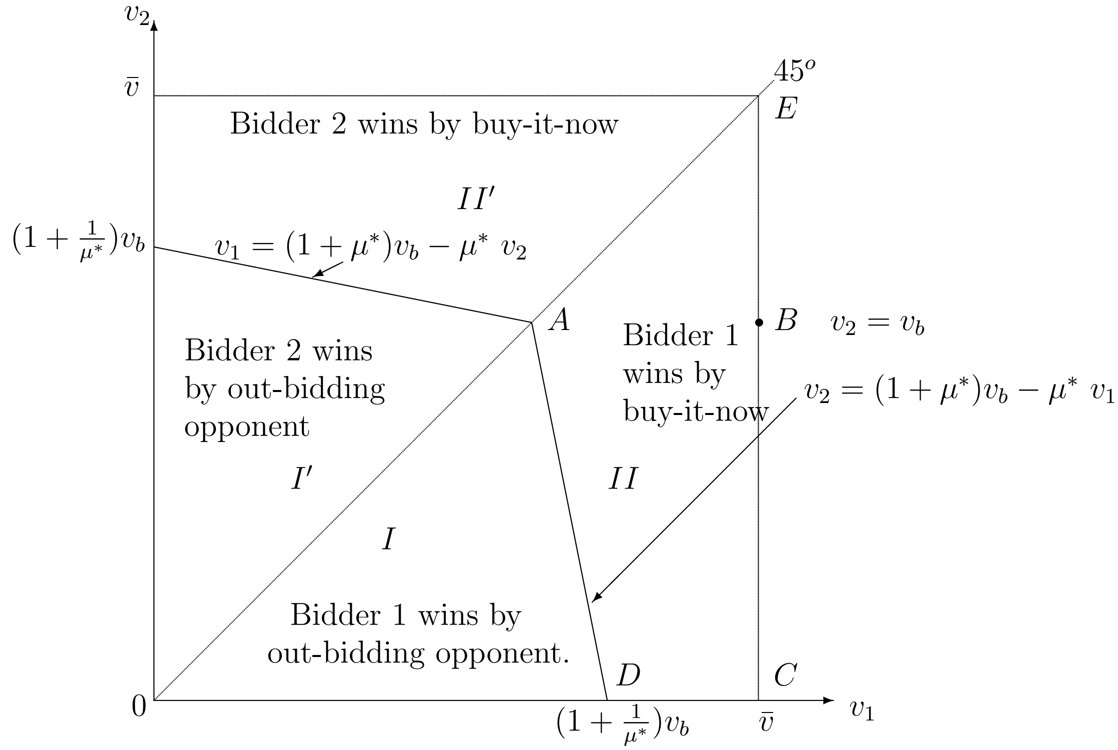

Figure 2: Outcomes of bidding as a function of bidders' valuations.

The strategic effect of a buy-it-now option on the seller's revenue can be seen very clearly in Figure 2. Take the case when bidder 1 eventually wins (i.e., the region OEC). Without a buy-it-now option, bidder 1 will win by paying bidder 2's valuation, v_2. With a buy-it-now, there are three possible outcomes to consider. First, the outcomes in region OAD are the same as the case without buy-it-now: Bidder 1 wins by paying bidder 2's valuation v_2. Second, in region ABCD, bidder 1 wins by paying the buy-it-now price v_b. Note that in this region $v_2 < v_b$. What would have been sold with price v_2 is now sold with a higher price v_b. The seller thus gains by setting up a buy-it-now price in this region. Third, in region ABE, bidder 1 pays the buy-it-now price,

[15] The reason for this is that when v_b rises beyond v_b^* by a little, the increase in the region where the seller's expected utility decreases is of one order greater than the increase in the region where his expected utility increases. As a result, his expected utility has a local maximum at v_b^*. Given that $\beta \leq 1$ (so that the seller's marginal expected utility can change sign at most once between v_b^* and $\bar{v}$), and (as will be shown later) that the seller has lower expected utility at $v_b = \bar{v}$ than at v_b^*, the optimal value of v_b cannot be greater than $\mu v^*/(1+\mu^*)$.

v_b, to win the item, but now $v_b < v_2$. This is the region in which the seller actually loses with the buy-it-now option. The optimal buy-it-now price of the seller must thus balance the latter two types of outcomes; that is, to maximize the expected revenue from region ABCD net of the expected loss from region ABE.

Figure 2 also shows clearly that a buy-it-now option reduces the risk that both the bidders and the seller face in the bidding process. Again, consider the case in which bidder 1 eventually wins, i.e., region OEC. In region OAD, the outcomes of bidding (and thus the uncertainty faced by the bidders and the seller) are the same regardless of whether there is a buy-it-now option, since in both cases bidder 1 wins by paying bidder 2's valuation v_2. In region AECD, if there is no buy-it-now option, bidder 1 will win by paying bidder 2's valuation v_2, which is uncertain. However, with a buy-it-now option, bidder 1 will win by paying a fixed price v_b. Obviously, the price risk faced by both the bidders and the seller is reduced by the buy-it-now option.

2.2 Optimal Buy-it-now Price

Given the outcomes of the auction depicted in Figure 2, it is straightforward to compute the expected utility of the seller under any buy-it-now price v_b:

$$\pi(v_b) = \frac{2}{\bar{v}^2}\Bigg\{\int_0^{v_b}\int_0^{v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^* v_1}\frac{v_2^\beta}{\beta}dv_2dv_1 + \frac{v_b^\beta}{\beta}\left[\frac{\bar{v}^2}{2} - \frac{v_b}{2}\left(1+\frac{1}{\mu^*}\right)v_b\right]\Bigg\}; \tag{2}$$

where the first two terms in the braces are profits from region I, and the third term is that from region II. $\pi(v_b)$ can be shown to be equal to

$$\frac{v_b^\beta}{\beta}\left[1 - \frac{\beta(3+\beta)(1+\mu^*)}{(\beta+1)(\beta+2)\mu^*}\cdot\left(\frac{v_b}{\bar{v}}\right)^2\right]. \tag{3}$$

The seller chooses the values of v_b to maximize $\pi(v_b)$. The first-order condition for v_b is

$$\frac{\partial\pi}{\partial v_b} = v_b^{\beta-1}\left[1 - \frac{(3+\beta)(1+\mu^*)}{(\beta+1)\mu^*}\ \left(\frac{v_b}{\bar{v}}\right)^2\right] = 0. \tag{4}$$

This implies that $v_b^* = \sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\bar{v}$.

Some properties of the optimal buy-it-now price deserve to be discussed. First, since v_b^* is an increasing function of μ^*, which in turn is decreasing in α, we know that the optimal buy-it-now price is increasing in the degree of the bidder's degree of risk-aversion. This is an intuitive result, since one purpose of setting up a buy-it-now price is to make more profit by exploiting the aversion of bidders to the uncertainty of whether he will win or, if he wins, the uncertainty of the price he needs to pay. What is surprising is that even if the bidders are risk-neutral, there is still incentive for the seller to set a non-trivial buy-it-now price (i.e., a buy-it-now price lower than $\bar{v}$). This can be seen clearly from the fact that when $\alpha = 1$, we have $\mu^* = 1$, $v_b^* = \sqrt{\frac{1+\beta}{2(3+\beta)}}\bar{v} < \bar{v}$ and that the difference in the seller's expected utility in equation (6) is strictly positive for $\beta < 1$.[16] This is in contrast to the conventional wisdom that the reason for the buy-it-now price is to satisfy the bidder's desire to avoid risks.[17] Again, the intuition for this is actually quite clear. In the case when both bidder and seller are risk-averse, the buy-it-now price serves two purposes for the seller. On the one hand, it can be used to exploit the bidder's aversion to risk and increase the seller's revenue. On the other hand, it can also be used as a way to avoid risk for the seller. Therefore, even if the bidders are risk-neutral, the seller still has incentives to evoke the buy-it-now option, not to increase revenue, but to reduce his own risk.

Second, since v_b^* is increasing in β, it means that the optimal buy-it-now price is decreasing in the seller's degree of risk-aversion. The reason behind this is transparent. The more risk-averse the seller, the more he abhors the uncertainty brought about by the result of the competitive bidding between the bidders. He is then more willing to set up a lower, but fixed and certain, buy-it-now price to avoid price risk. We summarize these in Proposition 2.

[16] If the seller sets $v_b > \bar{v}$, then the buy-it-now price is redundant.

[17] For example, in Budish and Takeyama (2001), a buy-it-now price benefits the bidder only if the bidders are risk neutral. Since the seller can always set an impossibly high buy-it-now price to make the auction equivalent to one without buy-it-now, this implies that only if the bidders are risk-neutral will the buy-it-now price have any function.

Proposition 2. *The seller's optimal buy-it-now price is* $v_b^* = \sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\bar{v}$*. It is increasing (decreasing) in the bidder's (seller's) degree of risk-aversion. In particular, even if the bidders are risk neutral, the seller might still gain from the buy-it-now option.*

It might be instructive to compare the optimal BIN price with the inequality restriction imposed on BIN price in Theorems 9, 11 and 12 of Hidvegi et al. (2006). That restriction is equivalent to requiring $v_b \geq \bar{v}/2$ in our model. However, the optimal BIN price in our model, v_b^*, has a minimum of $\bar{v}/\sqrt{6}$, which occurs when the bidder is risk neutral and the seller is extremely risk averse (i.e., $\alpha = 1$ and $\beta = 0$). This means that in order to guarantee that the seller is better off with BIN, it suffices that $v_b \geq \bar{v}/2$. But in order for the seller to gain the most from BIN, sometimes it requires that v_b be smaller than $\bar{v}/2$. This is especially so when the seller is very risk averse and the bidder is relatively risk neutral.

By plugging v_b^* into (3), we can compute the expected utility of the seller under the optimal buy-it-now price to be

$$\pi(\beta) \equiv \frac{2}{\beta(2+\beta)} \left(\sqrt{\frac{\mu^*(1+\beta)}{(1+\mu^*)(3+\beta)}}\, \bar{v} \right)^{\beta}. \tag{5}$$

On the other hand, the expected utility of the seller without buy-it-now option is

$$\pi^0(\beta) \equiv \frac{2}{\bar{v}^2} \int_0^{\bar{v}} \int_0^{v_1} \frac{v_2^{\beta}}{\beta} dv_2 dv_1 = \frac{2\bar{v}^{\beta}}{\beta(\beta+1)(\beta+2)}.$$

The difference in expected utility is thus

$$\pi(\beta) - \pi^0(\beta) = \frac{2\bar{v}^{\beta}}{\beta(\beta+1)(\beta+2)} \left[\left(\frac{\mu^*}{1+\mu^*} \right)^{\frac{\beta}{2}} (1+\beta)^{\frac{\beta}{2}+1} (3+\beta)^{-\frac{\beta}{2}} - 1 \right]. \tag{6}$$

Let $\Phi(\mu^*, \beta)$ be the term in the brackets of (6). It is easy to see that Φ is increasing in μ^*. Moreover, $\Phi(1, \beta) = (\frac{1}{2})^{\beta}(1+\beta)^{\frac{\beta}{2}+1}(3+\beta)^{-\frac{\beta}{2}} - 1$, which is increasing in β initially, then decreasing in β. Note that $\Phi(1, 0) = \Phi(1, 1) = 0$, implying that $\Phi(1, \beta) \geq 0$ for all $\beta \in (0, 1]$. By the fact that Φ is an increasing function of μ^*, we know that $\Phi(\mu^*, \beta) \geq 0$ for all $\beta \in (0, 1]$ and $\mu^* \geq 1$.

That is, the expected utility of the seller is always greater with the buy-it-now option. Moreover, $\Phi(\mu^*, \beta) = 0$ only if $\mu^* = 1$ (i.e., $\alpha = 1$) and $\beta = 1$, meaning that the expected utility of the seller is strictly higher with buy-it-now unless both the bidders and the seller are risk-neutral. We thus have the main proposition of this paper:

Proposition 3. *Suppose the seller sets the buy-it-now price optimally. Then if both the bidders and the seller are risk-neutral, then the seller's expected revenue is the same as without buy-it-now. If either the seller or the bidder is risk-averse, the seller's expected utility is strictly higher with buy-it-now.*

Although we assume two bidders in Proposition 3, it is also true in the general n-bidder case (see Section 3).[18]

It is important to note that Proposition 3 is true only if the buy-it-now price is set optimally. If the buy-it-now price is not set optimally, the seller's expected utility might actually be lower than when there is no buy-it-now. For example, as we will later show in Step 1 of the proof of Proposition 5 (Appendix B), the seller's expected profit is strictly lower when the buy-it-now price is set below the optimal, and is redundant if it is set above the optimal. Given that the auction platform charges the sellers for posting buy-it-now, this implies that the seller's profit is strictly lower than without buy-it-now if the buy-it-now price is anything other than the optimum. This in turn implies that if a seller does not know exactly the distribution of the bidders' valuations (which is likely to be true in reality), he might refrain from posting buy-it-now simply because of the fear of losing profit. This explains why, despite the fact that Proposition 3 shows that posting a buy-it-now is the seller's dominant strategy, in reality there are a substantial number of auctions which do not post buy-it-now. The same explanation also implies that more experienced

[18] Our result that a risk-neutral seller is strictly better off with BIN when the bidders are risk-averse is consistent with Theorem 11 of Hidvegi et al. (2006); and our result that the seller is strictly better off with BIN when he is risk averse is consistent with Theorem 12 of Hidvegi et al. (2006). Their results are proved with general utility and density functions, but not under the optimal BIN price.

sellers, who might know the density function for the bidders' valuations more precisely, and are more likely to figure out the optimal BIN price, are more likely to adopt the buy-it-now option. In Section 4 of the paper's empirical part, we will use the seller's experience to endogenize his choice to post buy-it-now. Moreover, since the sellers use BIN to reduce risk, and since the sellers with more items to sell can more easily diversify price risk, we will expect the sellers with more items to be less likely to auction their items using BIN. Therefore, in our empirical model, we use the number of a seller's listings as another instrumental variable to endogenize the choice of BIN.

Given the optimal buy-it-now price and the auction outcomes summarized in Figure 2, we can also compute the expected transaction prices for the case when the seller posts the optimal buy-it-now price and the case when he does not. The expected transaction price in the case without buy-it-now can be easily computed to be $\bar{v}/3$. The average transaction price, when buy-it-now price is v_b, is

$$\begin{aligned}&\frac{2}{\bar{v}^2}[\int_0^{v_b}\int_0^{v_1} v_2 dv_2 dv_1 + \int_{v_b}^{(1+\frac{1}{\mu^*})v_b}\int_0^{(1+\mu^*)v_b-\mu^* v_1} v_2 dv_2 dv_1 + \frac{v_b}{2\mu^*}(\mu^*\bar{v}^2-(1+\mu^*)v_b^2)]\\ &= 2[\frac{v_b}{2} - \frac{(1+\mu^*)v_b^3}{3\mu^*\bar{v}^2}]. \end{aligned} \tag{7}$$

when $v_b = v_b^*$, the difference in their expected transaction prices is thus

$$\frac{1}{3}[\bar{v} + \left(\frac{2(1+\beta)}{3+\beta} - 3\right) v_b^*]. \tag{8}$$

As a result, the expected transaction price is greater with a buy-it-now option if and only if

$$\mu^* > \frac{(3+\beta)^3}{6\beta(6+\beta)+22}. \tag{9}$$

Note that $\mu^* \in [1,\infty)$ and $(3+\beta)^3/[6\beta(6+\beta)+22] \in (1, 27/22]$. That means the expected transaction price without buy-it-now can be greater only if μ^* falls in the narrow interval $[1, 27/22]$. (Note that in this case the average transaction price with buy-it-now still has a good chance to be greater.)[19] An important special case is when the seller is risk-neutral (i.e., $\beta = 1$). In that

[19] For example, assuming that both α and β are uniformly distributed on $(0, 1]$, then we can show that the probability that $\mu^* \geq 27/22$ (i.e., $\mu^* > (3+\beta)^3/[6\beta(6+\beta)+22]$ for *all* possible values of β) is 0.744.

case the right-hand side of (9) is 1, and (9) will hold for sure: when the seller is risk neutral, the expected transaction price will be higher in an auction with optimal BIN price than one without. This case is important because the sellers usually have many items for sale to diversify their risks, and are in general close to being risk-neutral. The upshot regarding transaction price is therefore that except in the extreme case when α is very close to 1 and β very close to 0 (i.e., the bidder is almost risk-neutral and the seller is very risk-averse), the expected transaction price is greater when there is a buy-it-now option.[20] Since in the on-line auctions, whether to set up a buy-it-now price is the option of the seller, if we look at auctions with identical objects, there will be ones that go with buy-it-now prices and those go without. The empirical implication for this fact is that the average transaction price for items posted with the buy-it-now options (but not necessarily sold with buy-it-now price) will be greater than those without. Also note that, as we will show in Proposition 5 that the results of Proposition 3 continue to hold for the n-bidder and general density function case, this empirical prediction is also true in the more general case.

Finally, we investigate the effect of the buy-it-now option on the bidder's expected utility. Although the buy-it-now option might seem to help the bidders by offering them an option to buy the item with a fixed price, it actually also serves as the seller's instrument to increase the competition between the bidders. With the buy-it-now option, the bidders not only have to compete in the bidding process, but have to compete in buy-it-now. The seller thus extracts more rent (Proposition 3) at the bidders' expense in the auction. Recall (see Figure 2) that a buy-it-now price actually helps the bidders with high valuations, because it enables them to buy the items at the buy-it-now price rather than risk bidding into high prices. Consequently, a bidder who has a very high valuation will benefit from buy-it-now. Buy-it-now is thus a mixed blessing

[20] Note that when $\beta = 1$, the average transaction price is exactly the expected utility of the seller, $\pi(1)$. Although Proposition 1 shows that $\pi(1) \geq \pi^0(1)$, this *does not* imply that the transaction price is always higher with buy-it-now. It only shows that average transaction price is higher with buy-it-now *when the seller is risk-neutral*. In order to make the appropriate comparison, we have to show it for the general case when β is not necessarily 1. More specifically, when the seller is sufficiently risk averse, it can happen that his expected utility is higher, while at the same time the expected transaction price is lower, under the optimal BIN price. In this case the seller simply sacrifices income in exchange for reduction in risks.

for the bidders. On the one hand, it allows them to reduce price risk during the auction. On the other hand, it also increases the competition. Whether it benefits the bidders will therefore depend on the characteristics of the bidders and the sellers, as the following proposition shows. Since a bidder with valuation $v < v_b^*$ will never use the BIN option, his expected utility is the same regardless whether there is BIN. We therefore consider only the case when $v \geq v_b^*$.

Proposition 4. *Consider a bidder with valuation $v \geq v_b^*$. He has higher expected utility in a BIN auction if and only if $v > v_c(\alpha, \beta)\bar{v} \equiv \sqrt{(\frac{\mu}{1+\mu})(\frac{1+\beta}{3+\beta})}\bar{v}/(1-(\alpha+1)^{\frac{-1}{\alpha}})$, which is greater than $(1+\frac{1}{\mu})v_b^*$.*

The proof of Proposition 4 is tedious and is omitted.[21]

Proposition 4 shows that unless a bidder has high valuation, or the seller is very risk averse, he is worse off with the buy-it-now option. Note that if α is close enough to 1 and β is close enough 0 (i.e., if the seller is sufficiently risk-averse and the bidder is close to risk-neutral), then $v_c(\alpha, \beta) > 1$, implying that the inequality in Proposition 4 cannot hold. That is, regardless of their valuations, the bidders must be worse off under buy-it-now. Moreover, since $v_c(\alpha, \beta)$ is increasing in β, it implies that the more risk-averse the seller, the more likely the bidder will gain from buy-it-now. The general conclusion regarding welfare is therefore that buy-it-now benefits the sellers for sure, but whether it benefits the bidders depends on their valuations and degree of risk aversion, and how risk averse the seller is. If the bidder has high valuation and/or the seller is more risk averse relative to the bidders, then buy-it-now benefits the bidders. If the seller is more risk neutral relative to the bidders and/or the bidder has low valuation, then buy-it-now will reduce the bidder's expected utility.

[21] The proof can be downloaded from http://idv.sinica.edu.tw/kongpin/proposition4.pdf.

3 Extension

Our results in Section 2 have been derived under the 2-bidder case. In this section we show that, in the n-bidder case, our main results (in Proposition 3) remain true.

We first derive the differential equation that the optimal buy-it-now strategy must satisfy. Let x be the order-statistic of the second-highest valuation among n bidders. Then at price p, if the auction has not ended yet, it must be that $x \in [p,\, v(p)]$.[22] Simple application of order-statistic implies that the density function for x is $(n-1)x^{n-2}/(v(p)^{n-1} - p^{n-1})$. By the same reasoning as in Section 2.1 we can show that total change of utility in postponing buy-it-now from p to $p + dp$ is

$$du = \frac{(v-p)^\alpha}{\alpha} \frac{(n-1)p^{n-2}dp}{v(p)^{n-1} - p^{n-1}} + \frac{(v-v_b)^\alpha}{\alpha}\Big(\frac{(n-1)v(p)^{n-2}dv(p)}{v(p)^{n-1} - p^{n-1}} - \frac{(n-1)p^{n-2}dp}{v(p)^{n-1} - p^{n-1}}\Big).$$

That $v(p)$ is optimal implies that $\frac{du}{dp} = 0$:

$$\big[(v-p)^\alpha - (v-v_b)^\alpha\big]p^{n-2} = -(v-v_b)^\alpha v^{n-2}\frac{dv}{dp}. \tag{10}$$

In general, differential equation (10) does not have a closed-form solution. However, if we assume that $\alpha = 1$, then (10) becomes separable:

$$\big[v_b - p\big]p^{n-2}dp = -(v-v_b)v^{n-2}dv.$$

Integrating both sides we have

$$\frac{v_b p^{n-1}}{n-1} - \frac{p^n}{n} + C = \frac{v_b v^{n-1}}{n-1} - \frac{v^n}{n}; \tag{11}$$

where C is a constant. The boundary condition requires that $v(v_b) = v_b$. Therefore $C = 0$. Define

$$g(p,\, v_b) \equiv \frac{v_b p^{n-1}}{n-1} - \frac{p^n}{n},\;\; 0 \le p \le v_b, \;\text{and}$$
$$h(v,\, v_b) \equiv \frac{v_b v^{n-1}}{n-1} - \frac{v^n}{n},\;\; v_b \le v \le \frac{n v_b}{n-1}.$$

[22] $v(p)$ is the inverse of the buy-it-now function $P(v)$.

Then (11) implies that

$$g(p,\, v_b) = h(v,\, v_b), \text{ i.e.,}$$

$$P(v) = g^{-1}\big(h(v,\, v_b),\, v_b\big).$$

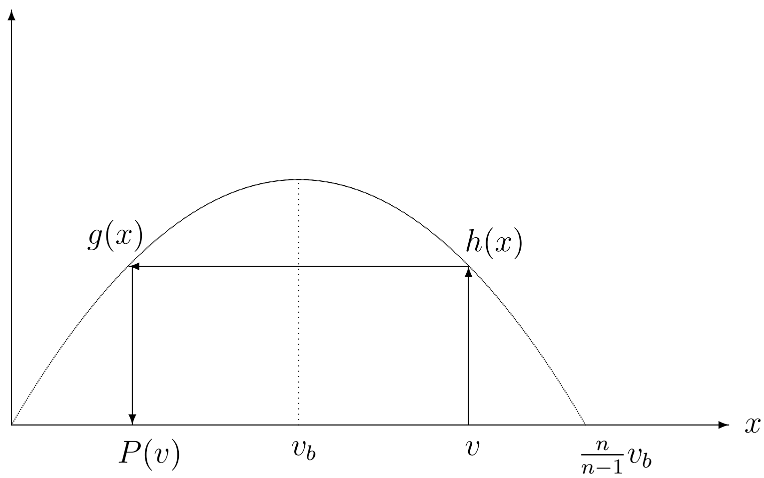

Figure 3: Illustration of the determination of $P(v)$

Figure 3 depicts the determination of $P(v)$ or, equivalently, $v(p)$.[23] The optimal buy-it-now strategy is no longer a straight line as in the 2-bidder case, but can be easily seen to be concave. Going through the same reasoning as in Section 2, Figure 4 (as an analogy Figure 2) summarizes the possible outcomes of the auction for an arbitrary buy-it-now price v_b. In the figure, $v_{(n)}$ is the order statistic of the highest valuation among n bidders, and $v_{(n-1)}$ the second-highest. Note that since $v_{(n)} \geq v_{(n-1)}$ by definition, no auction outcome will fall above the 45^0 line as in the case of Figure 2. We can now state the analogy to Proposition 3:

Proposition 5. *If $\alpha = \beta = 1$, then the seller's expected utility under the optimal buy-it-now price is the same as when he does not post a buy-it-now option. If, however, either $\alpha < 1$ or $\beta < 1$, then the seller's expected utility is strictly higher with optimal buy-it-now price.*

Proof: See Appendix B.[24]

[23] Note that $g(\cdot)$ is defined over $[0, v_b]$ and $h(\cdot)$ over $[v, \frac{n}{n-1}v_b]$.

[24] In fact Proposition 5 is true even when the density function of the bidder's valuation is a general function,

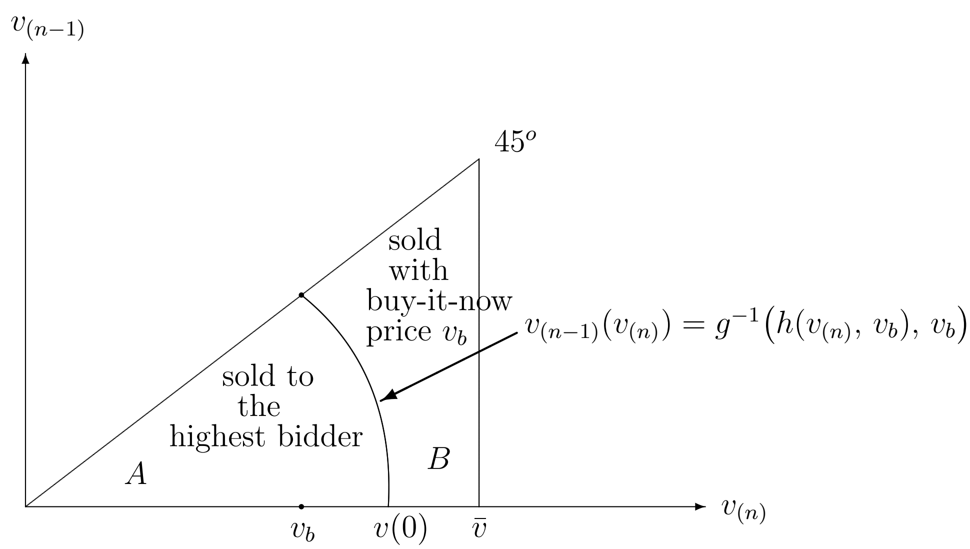

Figure 4: Summary of auction outcomes

Although Proposition 5 generalizes Proposition 3 by relaxing assumptions on the number of bidders and the form of the density function, the assumption that the bidder has a constant relative risk-aversion utility function turns out to be important for our results in Propositions 2 and 4, both of which rely heavily on the closed-form solution of the optimal BIN price. However, as is explained in Section 1, if BIN is mainly used to reduce risks for both the bidders and the seller, then it is reasonable to conjecture that Proposition 2 continues to hold in the more general context, i.e., the optimal BIN price will be increasing (decreasing) in the degree of the bidder's (seller's) risk-aversion.

4 Empirical Study

In this section, we perform an empirical test to examine the implications of our theoretical model. Specifically, we test whether the average transaction price in auctions with the BIN option is higher than that in those selling identical objects but without BIN.

instead of a uniform density. The proof requires only simple adaptation of notations. However, we still need the uniform density assumption to derive a closed-form solution for the optimal BIN price.

4.1 Data Description

The data are collected from Taiwan's Yahoo! auction site during the period from April 3, 2008 to August 2, 2008.[25] Yahoo! is the largest Internet auction site in Taiwan in terms of sales revenue during this period. Our data contain observations (items) for the auctions of Nikon Coolpix digital compact cameras. After deleting 6 outliers[26] and 19 SLR cameras, we are left with 1,272 listings. The data contain 4 categories: L, P and S series, and other types. The last category contains models that are no longer in production. The classification of existing models into L, P, and S series is meaningful, because the suggested prices provided in the website by Taiwan's Nikon official dealer are highest for series P, followed by S and then L.

When listing an item on the Yahoo! Auction platform, a seller always has to choose an appropriate product category and provide product descriptions. The seller also needs to decide a minimal bid[27] and the duration of the listing. Important to our research, the seller has an option of posting a buy-it-now price. Different from some other auction platforms (such as eBay), the buy-it-now option is always available to bidders for the entire duration of a listing.[28] In addition, a seller can set a secret reserve price, which is not observable to bidders. A bid is accepted when it is higher than the secret reserve price.

The listing fee for any item on Taiwan's Yahoo! platform is NT$3. Setting the optional buy-it-now price costs an additional fee of NT$1. Besides, setting the optional secret reserve price incurs a fee of NT$5. All of these fees are very small comparing to the average transaction price (NT$4,268) of our sample.

[25] One U.S. dollar roughly equaled NT$32 during this period.

[26] These 6 items are listed by the same seller who, unsatisfied with the prices, raised the reservation prices before the action ended to be so high (NT$100,000) so that the items were not sold.

[27] The lowest possible minimal bid is NT$1.

[28] Given this permanent buy-it-now feature, the Yahoo! data has an advantage for empirical study in that a buy-it-now auction remains one until it closes. On the contrary, since the buy-it-now option is temporary in eBay, the data collected from eBay might contain items which are recorded as standard auctions but in fact have started as buy-it-now auctions.

Among the 1,272 listings, 618 have the buy-it-now options, while the remaining 654 do not. There are a substantial number of auctions in which the sellers set the buy-it-now prices equal to minimum bids. They are essentially fixed-price listings. Among the 618 listings with buy-it-now options, 487 are fixed-price listings and 131 are auctions with buy-it-now options.[29] Overall, 38% of the listings in the data set are listed with fixed prices, 10% are auctions with the buy-it-now options, and the remaining 51% are standard auctions. For most of our empirical study, we exclude these fixed-price listings because our focus is the effect of buy-it-now options on auctions.[30] Thus, the sample size in the empirical analysis is 785, including 131 auctions with the BIN options, and 654 without. Nonetheless, we include the 487 fixed-price listings in the robustness checks at the end of this section.

Table 1 displays the bidding outcomes of our sample. Among the 1,272 listings, 447 result in a sale, and 825 items remain unsold. Furthermore, among the 447 items that are eventually sold, 163 are listed as fixed-price, 45 are auctions with buy-it-now options, and 239 are standard auctions. Among the 825 items that remain unsold, 324 are fixed price listings, 86 are auctions with buy-it-now options, and 415 are standard auctions.

Among the 45 buy-it-now auctions which result in a sale, 27 are sold with buy-it-now while the other 18 are sold to winning bids. The average transaction price for the former is NT$8,930, and that for the latter is NT$4,047. (The average transaction price for the total 45 buy-it-now auctions is NT$6,977.) As a comparison, the average transaction price for the 239 successful standard auctions is NT$3,758 and the average price for the 163 successful fixed-price listings is NT$5,899. The fact that the average transaction price of auctions with a buy-it-now option is substantially higher than that of the auctions without a buy-it-now option seems to provide evidence for the implication of our theory. However, the choice of listing with the buy-it-now

[29] The average buy-it-now price of the 487 fixed-price listings is NT$6,746. The average buy-it-now price for the 131 auctions with buy-it-now options is NT$7,695 and their average minimal bid is NT$5,978.

[30] This differentiates our empirical sample from that of Anderson et al. (2004). But we include those fixed price auctions in the sample when testing the robustness of our results.

option is a seller's endogenous decision to maximize his/her expected payoff. Consequently, auctions with and without buy-it-now options could potentially be quite different. Our empirical analysis attempts to control for the endogeneity resulting from the selection of the buy-it-now option, and estimate the effect of adding the buy-it-now option in an auction on the transaction price.

4.2 Empirical Specification

In our main regression equation, we are interested in the effect of using the buy-it-now option on the transaction price ($Price$):

$$Price = \alpha BIN + \beta \mathbf{x} + \varepsilon, \tag{12}$$

where the dummy variable BIN indicates whether the seller chooses the buy-it-now option, the vector $\mathbf{x}$ is the observed listing characteristics, and ε captures unobserved characteristics. However, we can observe the transaction price only if an auction results in a sale. Using only the observations with a successful transaction to estimate the coefficients would have selection bias. Consequently, we follow Heckman (1979)'s probit selection model to determine whether a listing results in a successful sale.[31]

$$Trade = \mathbf{1}\{\gamma \mathbf{w} + \nu > 0\}, \tag{13}$$

where the dummy variable $Trade$ indicates whether a listing results in a sale, $\mathbf{w}$ is the observed listing characteristics, and ν represents unobserved characteristics. Assume (ε, ν) has a joint normal distribution with mean zero and the variance of ν is one. Note that we observe $Price$ if and only if $Trade = 1$.

Definitions and summary statistics for the variables used in our empirical study are listed in Table 2. We exclude fixed-price listings in computing summary statistics in this table. The

[31] See Livingston (2005) for the reasons for using the sample selection model rather than the censoring model such as the Tobit model. In his paper, however, the purpose is to study the effects of a seller's reputation on bidders' participation decisions and on the decision of how much to bid.

transaction price $Price$ is equal to the buy-it-now price if it is sold with buy-it-now, and is the value of the winning bid otherwise.[32] The average value of $Price$ is NT$4,268, with large variation across listings. Only 36% of listings result in a sale, so we use the trade equation (13) to control for the selection bias of observing a successful transaction.

The key variable in our test is the buy-it-now dummy (BIN). It equals one if the auction has a buy-it-now option; and is zero otherwise. This dummy is used to test whether the average transaction price for items with the buy-it-now option is higher than that without one, for auctions which result in sale. If our theory is correct, it should have a positive coefficient. Table 3 records the summary statistics for listings with and without the buy-it-now options separately. Most variables do not have a significant difference between these two columns. Nonetheless, sellers who list with a buy-it-now option tend to have higher reputation scores.

For other explanatory variables, both the vector $\mathbf{x}$ in (12) and $\mathbf{w}$ in (13) include $Reputation$ (seller's reputation, which is the rating scores accumulated from past transactions), New (a dummy variable with the value one if the item is new; and is zero otherwise), three product type dummies ($TypeL$, $TypeP$, $TypeS$), $BidderNo$ (a proxy variable to measure the number of potential bidders), and $Duration$ (the length of listing duration). In addition, we include $MiniBid$ (minimum price to start bidding, set by the seller) as a regressor in the trade equation (13).

Many previous works on the impact of a seller's reputation on the transaction price of the Internet auction indicate a positive effect. For example, Livingston (2005) uses the probit model and sample selection model to show that the probabilities that the auction receives a bid, that it results in a sale, and the amount of the winning bid all increase substantially with the first few positive reports of the seller. Nonetheless, the marginal returns to additional positive reports are severely decreasing. (See also Houser and Wooders (2006) among many others.) Given this result,

[32] The transaction price excludes shipping charge.

in order to control for the effect of reputation on transaction price and transaction probability, we incorporate a reputation variable in our regression.

We use a dummy variable New to control for the quality difference between a new camera and a used one. Similarly, we include three dummy variables ($TypeL$, $TypeP$, and $TypeS$) to control the difference between the types of cameras, with the "other type" as the base.

The number of potential bidders faced by a listing affects its transaction price.[33] For example, when the camera maker has an intense advertisement campaign, demand is higher for items listed at that period, and there should be more potential bidders. Similarly, during the holiday season, demand is higher and we should expect a higher transaction price. On the contrary, when more cameras are listed at the same time, each listing attracts fewer potential bidders. Nevertheless, the number of potential bidders is unobservable because not every potential bidder submits a bid or makes a buy-it-now purchase. We can only observe the number of bidders who *actually* place a bid and/or make a buy-it-now purchase. To account for this problem, we follow Yin (2007) to construct a proxy variable $BidderNo$ which measures the number of potential bidders. Specifically, $BidderNo$ is the average number of observed bidders in 20 other listings with closest beginning time in our sample. The intuition is that, for items listed in similar time periods, they are subject to similar demand shocks. By using this variable to control for the common demand shock, the error term ε in the price equation (12) captures the shocks specific to each listed item. For example, ε has a higher value if the seller provides better product description on the auction website. Since common demand shocks are captured by the proxy variable $BidderNo$, it is reasonable to assume that ε is independent across listings.

We expect that the listing duration has a positive effect on the transaction price and transaction probability because a longer duration is likely to attract more bidders to submit a bid. Hence, we include the variable $Duration$ as a regressor.

[33] See Hu and Shum (2007) for the nonparametric identification and estimation of first price auction models with an unknown number of potential bidders. As for the ascending auction models, see Song (2004).

Minimum bid is the lowest possible bid set by the seller for a potential bidder to start the bidding process. Higher minimum bid may lower the chance that an auction results in a sale by discouraging a potential bidder from placing a bid. But minimum bid should affect only a potential bidder's participation decision, not the decision of how much to bid. As can be seen from the theoretical part in Section 2, there may be observed and unobserved factors that affect the participation decision, but not the fact that a bidder is willing to buy the object as long as price is lower than his valuation. Thus, minimum bid is only used as an exclusion restriction in the trade equation (13).

A seller's decision regarding whether to list an item with the buy-it-now option might be correlated with unobserved characteristics ε. For example, a seller's experience about using the Yahoo! platform might affect the quality of product description (captured by ε) and the buy-it-now decision at the same time. We thus need to find the instrumental variables which affect the seller's listing decision, but not the level of transaction price.

As has been explained in the two paragraphs following Proposition 3, we expect a more experienced seller to be more likely to place the buy-it-now option, as he is more likely to figure out the optimal buy-it-now price. The first instrumental variable we use to endogenize the seller's decision to list an item with BIN is therefore his experience. In the paper, we use the number of days since a seller joined Yahoo! auction as a measure of his experience. Moreover, as BIN is used by the sellers to reduce risk, we will expect the sellers with more items to sell to be less likely to adopt BIN, as they can more easily diversify price risk through transaction of many items. We therefore use the number of listings of the seller as the second instrumental variable to endogenize his decision of whether to post buy-it-now.[34] We therefore use two instrumental variables as proxies for the seller's decision to list a buy-it-now option: the number of days since

[34] In a dataset collected by the second author from eBay iPod auctions, it is also documented that a seller who has a greater number of iPods to sell in the period studied is less likely to post items with BIN. See Chen et al. (2011).

the seller joined Taiwan Yahoo! as a member ($Member$) and the number of listings posted by the sellers in our data ($ListingNo$). The identification assumption is that these two variables are uncorrelated with the error term ε in the price equation (12).

We follow the method proposed by Wooldridge (2010, p.809–813) to estimate the model coefficients (α, β, and γ) in two steps. In the first step, estimate γ in the trade equation (13) by running a probit regression of $Trade$ on $\mathbf{w}$ using all observations. Denote the estimator as $\hat{\gamma}$ and compute the estimated inverse Mills ratios, $\hat{\lambda} = \phi(\mathbf{w}\hat{\gamma})/\Phi(\mathbf{w}\hat{\gamma})$, where ϕ and Φ are the standard normal density and distribution functions, respectively. In the second step, we use the observations of successful listings to estimate the linear equation

$$Price = \alpha BIN + \beta \mathbf{x} + \rho\hat{\lambda} + error,$$

by the 2-Stage Least Squares (2SLS) method with instruments ($Member, ListingNo$). We obtain estimates for the coefficients α and β in the price equation (12). In order to obtain the correct standard errors in this two-step procedure, we use the bootstrap method. Also, the χ^2 statistic for the Anderson-Rubin test of joint significance of endogenous regressors in the 2SLS is 11.56, which has a p-value 0.0031. This suggests that our proposed instruments $Member$ and $ListingNo$ are relevant to the endogenous variable BIN. Furthermore, the Sargan statistic for overidentification is 0.009 with p-value equal to 0.926. We cannot reject the hypothesis of overidentification. The choice of these instruments appears to be valid. As for the weak instruments test statistic, the Cragg-Donald F-stat is 15.61. This is greater than the usual critical value 10.

4.3 Transaction Price and Buy-it-now

Table 4 presents the results of our estimation. The second column is the estimated marginal effects for the probit model of the trade equation (13). The estimated coefficients for the price equation (12), with various specifications, are shown in the remaining four columns. The third column (labeled as “no IV”) ignores the potential endogeneity of the dummy variable BIN.

The coefficients are estimated by the standard Heckman's two-step procedure. To account for the endogeneity of BIN, we perform 2SLS in the second step to obtain coefficients in the price equation (12) from the fourth to the sixth column. The fourth column (labeled as "IV-1") is our preferred specification, using $Member$ and $ListingNo$ as instruments in the second stage. Column Five (labeled as "IV-2") is similar to IV-1, but modifies the definition of $BidderNo$. Instead of defining it as the average number of observed bidders in the 20 other listings with the closest beginning time, we define it as the average number of observed bidders in the 10 other listings with the closest beginning time. Finally, the last column ("IV-3") is the same as IV-1 except dropping $Duration$ as a regressor.

For the trade equation (13), the minimal bid $MiniBid$ of a listing has a strong negative effect on the probability of a successful trade. This is consistent with intuition. In addition, we find that both the seller's reputation score and the number of potential bidders have significantly positive effects for a successful trade. The estimation results for the price equation (12) shown in Table 4 indicate that the transaction price for a listing with the buy-it-now option is significantly higher. The estimated effect of the buy-it-now option is substantially higher when endogeneity is taken into account. The t-ratio of Hausman test of endogeneity is -1.76 with a p-value 0.079, rejecting the null hypothesis of no endogeneity at the 10% level.

As for price regression, the reputation score on Yahoo! Auction has a significantly positive effect on price. This is consistent with findings in the literature. The prices of new items are higher although the difference is not statistically significant. Consistent with the suggested prices listed by Nikon's official dealer, type P has the highest prices and type L has the lowest transaction prices. We also find the proxy for the potential number of bidders has a positive effect, consistent with our conjecture that higher demand results in higher transaction prices. We find that the duration of a listing has almost no effect on transaction prices. Finally, a significantly positive coefficient on the inverse Mills ratio indicates the importance of accounting for the sample selection problem.

In summary, our empirical study confirms our theoretical predictions that an item listed with buy-it-now options is on average sold at a higher price.

4.4 Robustness Checks

As we point out in Subsection 4.1, many sellers choose the minimal bid to equal to the buy-it-now price. These listings are essentially fixed-price listings. We exclude these listings in the above estimation. To check the robustness of our results, we include them in the sample and re-do our empirical analysis. We treat all listings with a buy-it-now price as one category $BIN = 1$, regardless of its minimal bid. In other words, fixed-price listings are also treated as an auction with a buy-it-now option. On the other hand, the other category $BIN = 0$ still consists of listings under a standard auction. Consequently, the sample size is 1,272 with $BIN = 1$ for 618 observations and $BIN = 0$ for the other 654 observations. The estimation results are presented in Table 5. The results are qualitatively very similar to those in Table 4. Even though the magnitude of the effect of BIN becomes smaller, they are still significantly positive after we account for the endogeneity of the buy-it-now decision.

5 Conclusion

In this paper we propose a dynamic model of auction with a buy-it-now option, in which both the seller and bidders are risk-averse. We completely characterize the optimal bidding strategy of the bidder and the optimal buy-it-now price of the seller. The seller is shown to benefit from the buy-it-now option from two sources. He can either use it to exploit the bidder's aversion to price risk in the bidding process, or to reduce price risk in the bidding process for himself. The buy-it-now option benefits the seller even if the bidders are risk-neutral. In fact, the only case in which BIN has no effect on the seller's welfare is when both the bidders and the seller are risk neutral. Since buy-it-now is used as an instrument to intensify the competition between the bidders, unless a bidder has a high valuation of the item or the seller is very risk averse, the bidders are worse off under the optimal BIN auctions.

Our model predicts that the expected transaction price of an identical object will be higher in the buy-it-now auction than in the standard auction, if the buy-it-now price is set at its optimal. This prediction is confirmed by our empirical study based on sample selection model with the consideration of endogeneity problem.

An interesting option in auctions that is omitted from our model is the reserve price. In contrast to the buy-it-now price, which essentially sets an upper-bound on the possible transaction prices, the reserve price sets a lower bound. Moreover, like the buy-it-now price, the reserve price can also serve as a strategic instrument for the seller, and will almost surely have a non-trivial interaction with the optimal buy-it-now price. When a reserve price consideration is incorporated, our model becomes substantially complicated, and requires major modification. But it also points to an interesting direction for future research.

Appendix A: Proof of Proposition 1

Let $v(p)$ be the inverse function of $P(v)$. It relates the prevailing price p with bidder's valuation v, who at p is just willing to buy out the object. That is, a bidder with valuation $v(p)$ is just willing to obtain the object by paying the buy-it-now price, when the prevailing price reaches p. Similarly, if a bidder with valuation v is willing to buy the object, when the prevailing price is p, then a bidder with valuation $v' > v$ will be even more willing to do so at that moment. This implies that $v(p)$ is a decreasing function.

Suppose that both bidders are still active at the moment when the prevailing price is p. This implies that the valuations of both bidders are greater than p, which in turn implies that the possible valuations of any bidder must be distributed on $[p, \bar{v}]$. Moreover, by definition of $v(p)$, any bidder with valuation $v > v(p)$ would have bought the object before the price has risen to p. The fact that this object has not been bought at price p implies that the bidder's possible valuations cannot lie in $(v(p), \bar{v}]$. As a result, both bidders' valuations must lie in $[p, v(p)]$. In other words, if both bidders are still active when the prevailing price is p, then (by Bayes rule) any bidder's possible valuations of the object must be distributed uniformly on $[p, v(p)]$.

Consider the decision of a bidder (whose valuation is v) at the moment when the prevailing price is $p < v$. If he buys the object immediately with buy-it-now price v_b, his utility will be $u(v, v_b) = (v - v_b)^\alpha/\alpha$. If instead he holds out and waits until price is $p + dp$ to buy the object, then he will face three possible outcomes. First, his opponent buys the object while he waits. Second, his opponent drops out between p and $p+dp$. Third, neither of the above happens so that he eventually buys out the object when the prevailing price is $p + dp$. Whether the bidder should buy the object immediately (by paying v_b), or waits until $p + dp$, depends on the difference of the utility between an immediate buy-it-now and the combined expected utility under the three possible outcomes of waiting until $p + dp$.

Figure 5 depicts the possible intervals at which outcomes 1 and 2 occur. When the valuation of the bidder's opponent lies in $[v(p)+dv(p), v(p)]$,[35] then his opponent will buy out the object while he waits. This is the first outcome we mentioned above, which occurs with probability $\frac{-dv(p)}{v-p}$, and his utility is 0. Similarly, if his opponent's valuation lies in $[p, p+dp]$, then his opponent will drop out while he waits, and he will win the bidding with price p. This is the second outcome mentioned, which occurs with probability $\frac{dp}{v-p}$, and his utility is $(v-p)^\alpha/\alpha$. Under the third outcome, which occurs with probability $1-\left(-\frac{dv(p)}{v-p}+\frac{dp}{v-p}\right)$, his utility is $(v-v_b)^\alpha/\alpha$.

Figure 5: Possible outcomes of waiting.

The total expected utility of waiting until $p+dp$ to buy out is thus

$$\frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha}+\left[1+\frac{dv(p)}{v-p}-\frac{d\,p}{v-p}\right]\frac{(v-v_b)^\alpha}{\alpha}. \tag{14}$$

The total change in utility of waiting until $p+d\,p$ to buy, instead of buying now, is

$$du=\frac{dp}{v-p}\frac{(v-p)^\alpha}{\alpha}+\frac{dv(p)-dp}{v-p}\frac{(v-v_b)^\alpha}{\alpha}. \tag{15}$$

For the function $v(p)$ to be the optimal buy-it-now strategy, it must be the case that $\frac{du}{dp}=0$, i.e., the first-order condition must hold at any p. This implies that

$$(v-p)^\alpha-(v-v_b)^\alpha=-(v-v_b)^\alpha\,\frac{dv}{dp}. \tag{16}$$

Let $y=v-v_b$ and $x=v_b-p$, then $\frac{dv}{dp}=-\frac{dy}{dx}$, and equation (16) becomes

$$(x+y)^\alpha-y^\alpha=y^\alpha\frac{dy}{dx}. \tag{17}$$

It is difficult to directly solve for equation (17), but the boundary condition and the fact that (17) is homogeneous of degree α on both sides supply a clue. Since $v(v_b)=v_b$,[36] the solution of

[35] Since $dv(p)=v'(p)dp$ and $v'(p)<0$, it follows that $dv(p)<0$.

[36] If the current price is v_b, and buy out price is v_b, then it must be optimal to buy immediately.

(17) must pass through $(x, y) = (0, 0)$. We then conjecture that the solution of (17) is linear. Let $x = \mu y$, then (17) becomes

$$(1 + \mu)^{\alpha} = 1 + \frac{1}{\mu}. \tag{18}$$

Denote μ^* as the solution of (18). Figure 6 then depicts how μ^* is determined. It can be

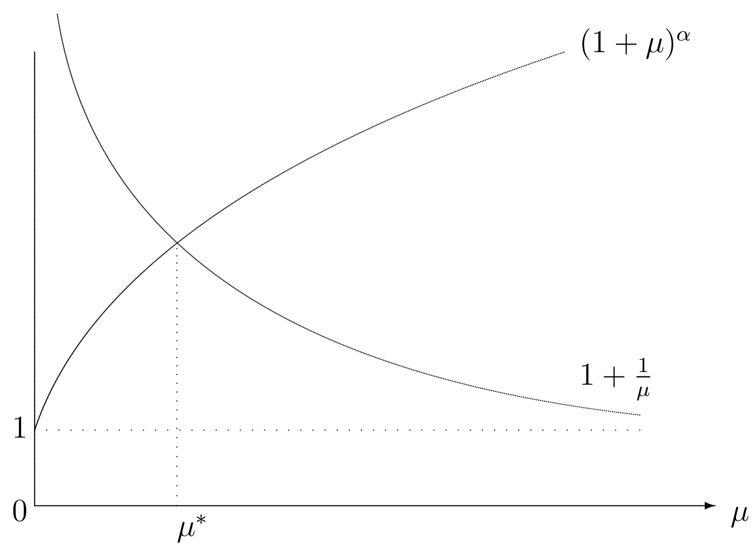

Figure 6: Determination of μ^*.

shown easily that $\mu^* \geq 1$ and that μ^* is decreasing in α. In particular, $\mu^* = 1$ when the bidders are risk neutral ($\alpha = 1$). We thus have $x = \mu^*\ y$. Substituting for $y = v - v_b$ and $x = v_b - p$ we eventually have

$$v(p) = (1 + \frac{1}{\mu^*})v_b - \frac{p}{\mu^*}. \tag{19}$$

Solving for the inverse of the function $v(p)$ we have

$$P(v) = (1 + \mu^*)v_b - \mu^*\ v. \tag{20}$$

The uniqueness of solution is proved in Hidvegi (2006), Mathews and Katzman (2006), and Reynolds and Wooders (2009).

Appendix B: Proof of Proposition 5

We separate the proof into 4 steps. In Step 1, we show that when $\alpha = \beta = 1$, the seller's expected utility under the optimal buy-it-now price is the same as when there is no buy-it-now option. In Step 2, we show that the seller's expected utility is strictly higher with buy-it-now when $\alpha = 1$ and $\beta < 1$, even when the buy-it-now price is set (not optimally) at the same value in Step 1 (i.e., optimal value for the case when $\alpha = \beta = 1$). In Step 3, when $\alpha = 1$ and $\beta < 1$, we show that the seller's expected utility is even higher when the buy-it-now price is set optimally. In Step 4, we show that given any $\beta < 1$, if the value of α decreases from 1, then the bidder will change his buy-it-now strategy so that the seller's expected utility is higher, even if he sets the same optimal buy-it-now price as when $\alpha = 1$. Naturally, his expected utility is even higher when he optimally sets the optimal buy-it-now price corresponding to the case when $\alpha < 1$.

To economize the use of notations, in the proof we will (without loss of generality) normalize the problem by assuming that $\bar{v} = 1$, so that $v_i \in [0, 1]$ for all $i = 1, \ldots, n$. That is, the distribution of every bidder's valuation lies on [0, 1] uniformly. Also, we use b to denote buy-it-now price. (In view of normalization, this actually means $b = \frac{v_b}{\bar{v}}$.) In the case when the seller's (bidder's) degree of risk-aversion is β (α), let $\pi(\alpha, \beta)$ be the seller's expected utility when there is no buy-it-now option. Similarly, let $\pi_B(\alpha, \beta, b)$ be the seller's expected utility when but-out price is b. We use $p_{\alpha\beta}$ to denote the seller's optimal buy-it-now price when the bidder's and the seller's degrees of risk-aversion are α and β, respectively. The strategy of our proof stated above is then, in Step 1, to show that $\pi_B(1, 1, p_{11}) = \pi(1, 1)$, i.e., buy-it-now offers no value for the seller when both are risk-neutral. In Step 2, we show that $\pi_B(1, \beta, p_{11}) > \pi(1, \beta)$ when $\beta < 1$. As a result, it must be that $\pi_B(1, \beta, p_{1\beta}) > \pi(1, \beta)$. In Step 3, we show that, when $\alpha < 1$ and $\beta < 1$, $\pi_B(\alpha, \beta, p_{1\beta}) > \pi(\alpha, \beta)$. Finally, in the fourth step, we show that $\pi_B(\alpha, \beta, p_{\alpha\beta}) > \pi(\alpha, \beta)$, which completes our proof.

Step 1 ($\alpha = \beta = 1$): From Figure 4 (note that in the normalized case, $v_b = b$) it is easy to see that, for any b,

$$
\begin{aligned}
&\pi_B(1,1,b) - \pi(1,1) \\
&= \int_b^1 \int_{v(v_{(n-1)})}^1 \big(b - v_{(n-1)}\big) n(n-1) v_{(n-1)}^{n-2} d_{(n)} dv_{(n-1)} \\
&\quad + \int_b^1 \int_{v_{(n-1)}}^1 \big(b - v_{(n-1)}\big) n(n-1) v_{(n-1)}^{n-2} d_{(n)} dv_{(n-1)} \\
&= \int_b^1 \big(1 - v(v_{(n-1)})\big)\big(b - v_{(n-1)}\big) n(n-1) v_{(n-1)}^{n-2} dv_{(n-1)} \\
&\quad + \int_b^1 \big(1 - v_{(n-1)}\big)\big(b - v_{(n-1)}\big) n(n-1) v_{(n-1)}^{n-2} dv_{(n-1)} \\
&= \int_{v(b)}^{v(0)} (1-v)(b - v_{(n-1)}) n(n-1) v_{(n-1)}^{n-2} \Big(- \frac{dv_{(n-1)}}{dv}\Big) dv \\
&\quad + \int_b^1 (1 - v_{(n-1)})(b - v_{(n-1)}) n(n-1) v_{(n-1)}^{n-2} dv_{(n-1)} \\
&= \int_{v(b)}^{v(0)} (1-v)(b - v_{(n-1)}) n(n-1) v_{(n-1)}^{n-2} \frac{(v-b)}{b - v_{(n-1)}} \frac{v^{n-2}}{v_{(n-1)}^{n-2}} dv \\
&\quad + \int_b^1 (1 - v_{(n-1)})(b - v_{(n-1)}) n(n-1) v_{(n-1)}^{n-2} dv_{(n-1)} \\
&= \int_b^{v(0)} (1-v) n(n-1)(v-b) dv \\
&\quad + \int_b^1 (1 - v_{(n-1)})(b - v_{(n-1)}) n(n-1) v_{(n-1)}^{n-2} dv_{(n-1)} \\
&= \int_{v(0)}^1 (1 - v_{(n-1)})(b - v_{(n-1)}) n(n-1) v_{n-1}^{n-2} dv_{(n-1)};
\end{aligned} \tag{21}
$$

where the third equality is by change of variable, and the fifth equality comes from the fact that $v(b) = b$. It is easy to see that (21) is negative because $v_{(n-1)} > b$ for all $v_{(n-1)} \in [v(0), 1]$. That means $\pi_B(1,1,b) - \pi(1,1) < 0$ unless b is chosen optimally, i.e., $b = p_{11}$. In that case $v(0) = 1$ and $\pi_B(1,1,p_{11}) = \pi(1,1)$.[37] We thus show that when $\alpha = \beta = 1$, unless the seller chooses the buy-it-now price b optimally (in which case the expected profits of the seller are the same with or without buy-it-now price), his profit is strictly lower with the buy-it-now option.

Step 2 ($\alpha = 1$, $\beta < 1$): From Step 1 we know that $\pi_B(1,1,p_{11}) - \pi(1,1) = 0$, which, by substituting

[37] Note that our reasoning also implies that buy-it-now is redundant when b is set higher than its optimum p_{11}.

b in (21) by p_{11}, implies that

$$\int_0^{p_{11}}\int_{v(v_{(n-1)})}^1 p_{11}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}v_{(n-1)}+\int_{p_{11}}^1\int_{v_{(n-1)}}^1 p_{11}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}$$
$$=\int_0^{p_{11}}\int_{v(v_{(n-1)})}^1 v_{(n-1)}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}$$
$$+\int_{p_{11}}^1\int_{v_{(n-1)}}^1 v_{(n-1)}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}. \tag{22}$$

We want to show that $\pi(1,\beta)<\pi_B(1,\beta,p_{11})$. For that purpose, we only need to compare their relative value in region B of Figure 4, as $\pi(1,\beta)$ and $\pi_B(1,\beta,p_{11})$ have the same value in region A. The expected utility of the seller in region B of Figure 4, without the buy-it-now option, is

$$\Big[\int_0^{p_{11}}\int_{v(v_{(n-1)})}^1 \frac{v_{(n-1)}^\beta}{\beta}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}$$
$$+\int_{p_{11}}^1\int_{v_{(n-1)}}^1 \frac{v_{(n-1)}^\beta}{\beta}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}\Big]\frac{1}{\text{Prob(region B)}}$$
$$<\frac{1}{\beta}\Big[\int_0^{p_{11}}\int_{v(v_{(n-1)})}^1 v_{(n-1)}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}$$
$$+\int_{p_{11}^1}^1\int_{v_{(n-1)}^1}^1 v_{(n-1)}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}\Big]^\beta\frac{1}{\text{Prob(region B)}}$$
$$=\frac{1}{\beta}\Big[\int_0^{p_{11}}\int_{v(v_{(n-1)})}^1 p_{11}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}$$
$$+\int_{p_{11}}^1\int_{v_{(n-1)}}^1 p_{11}n(n-1)v_{(n-1)}^{n-2}dv_{(n)}dv_{(n-1)}\Big]^\beta\frac{1}{\text{Prob(region B)}}$$
$$=\frac{p_{11}^\beta}{\beta};$$

where the inequality comes from Jensen's, and the first equality from (22). The last term above, $\frac{p_{11}^\beta}{\beta}$, is exactly the seller's expected utility when the item is sold under the buy-it-now price p_{11}. We therefore have $\pi(1,\beta)<\pi_B(1,\beta,p_{11})$. In other words, when the seller is risk-averse and the bidder risk-neutral, even if the buy-it-now price is p_{11} (rather than the optimal $p_{1\beta}$), the seller's expected utility is higher with the buy-it-now option. However, by the very definition that $p_{1\beta}$ is the optimal buy-it-now price given the degrees of risk-aversion 1 and β, it must be that $\pi_B(1,\beta,p_{11})\le\pi_B(1,\beta,p_{1\beta})$. We therefore have $\pi(1,\beta)<\pi_B(1,\beta,p_{1\beta})$, which completes

Step 2.

Step 3 ($\alpha < 1$, $\beta < 1$, $b = p_{1\beta}$): First note that $p_{\alpha\beta}$ is a decreasing function of α. That is, the more risk-averse a bidder, the lower the threshold price at which he wants to evoke buy-it-now. We first show that $\frac{dp}{dv}$ will be flatter when α increases: From (10) we have

$$\begin{aligned}\frac{dp}{dv} &= -\frac{(v-v_b)^\alpha v^{n-2}}{(v-p)^\alpha - (v-v_b)^\alpha p^{n-2}} \\ &= \frac{-1}{\left(\frac{v-p}{v-v_b}\right)^\alpha - 1}\left(\frac{v}{p}\right)^{n-2},\end{aligned}$$

which is increasing in α. But since dp/dv is negative, this implies that the curve $v_{(n-1)}(v_{(n)})$ will become flatter as α increases. Since $p(v_b) = v_b$ for any buy-it-now price v_b, this means that $v_{(n-1)}(v_{(n)})$ will rotate clockwisely as α decreases from 1. If we keep buy-it-now price unchanged at $p_{1\beta}$ while α is reduced, the probability at which the item will be sold under buy-it-now will increase. (Region D in Figure 7, which was sold to the highest bidder when $\alpha = 1$, will now be sold with buy-it-now when α decreases.) This in turn implies not only that the item is sold at a higher price,[38] but also that the seller's expected utility in region D increases (this is because the price in region D is a constant, $p_{1\beta}$, and is therefore riskless). We therefore have

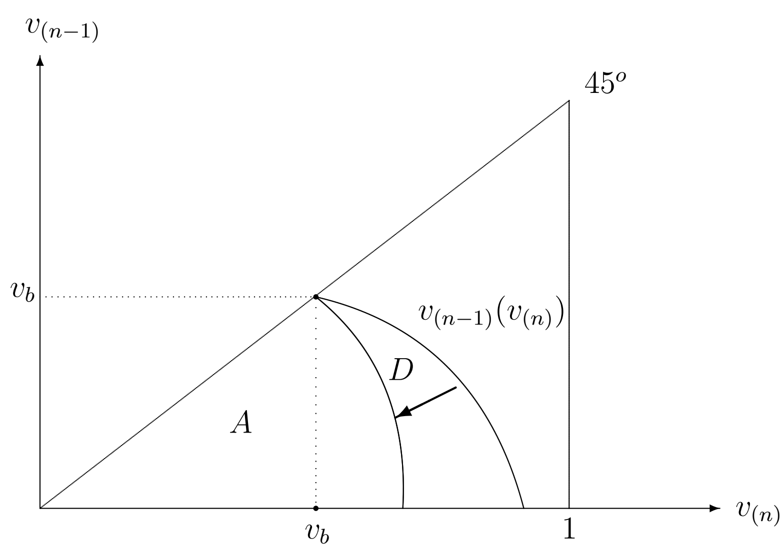

Figure 7: Effect of a decrease in α on auction outcomes

[38] Note that $p_{1\beta}$ is greater than the value of any $v_{(n-1)}$ in this region.

$\pi_B(\alpha, \beta, p_{1\beta}) < \pi_B(\alpha, \beta, p_{1\beta})$ if $\alpha < 1$.

Step 4 ($\alpha < 1$, $\beta < 1$, $b = p_{\alpha\beta}$): By the very definition of buy-it-now price, π_B will be even greater when the buy-it-now price is set optimally at $p_{\alpha\beta}$, rather than $p_{1\beta}$. That is, $\pi_B(\alpha, \beta, p_{1\beta}) \leq \pi_B(\alpha, \beta, p_{\alpha\beta})$. We thus complete our proof.

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Table 1: Summary of Bidding Outcomes

- All listings (1,272)
  - Auctions without buy-it-now option (654)
    - Auctions resulting in a sale (239) → Average transaction price: NT$3,757.73
    - Auctions not resulting in a sale (415)
  - Auctions with buy-it-now option (131)
    - Auctions resulting in a sale (45)
      - Items sold with buy-it-now (27) → Average transaction price: NT$8,930.37
      - Items sold to winning bids (18)
        - Average transaction price: NT$4,046.50
        - Average buy-it-now price: NT$5,410.50
    - Auctions not resulting in a sale (86) → Average buy-it-now price: NT$7,784.97
  - Fixed price listings (487)
    - Auctions resulting in a sale (163) → Average transaction price: NT$5,899.41
    - Auctions not resulting in a sale (324) → Average buy-it-now price: NT$7,171.97

Notes: Numbers in parentheses represent the numbers of observations in each category.

Table 2: Summary Statistics of All Variables, Excluding Fixed-Price Listings

Variables	Definition	Mean (Std. Dev)
Price	Transaction price (NT$ 1,000)	4.268
		(3.193)
Trade	A dummy variable with the value one if the	0.362
	auction results in a sale; zero otherwise.	(0.481)
BIN	A dummy variable with the value one if the	0.167
	auction has buy-it-now option; zero otherwise.	(0.373)
Reputation	Seller's feedback rating in Yahoo! Auction	0.858
	(1,000 points)	(1.953)
New	A dummy variable with the value one if the	0.575
	item is new; zero otherwise.	(0.495)
TypeL	A dummy variable with the value one if the	0.135
	item is of L-series; zero otherwise.	(0.342)
TypeP	A dummy variable with the value one if the	0.121
	item is of P-series; zero otherwise.	(0.326)
TypeS	A dummy variable with the value one if the	0.566
	item is of S-series; zero otherwise.	(0.496)
BidderNo	Number of observed bidders in 20 other auctions	0.869
	in our sample with closest beginning time	(0.569)
Duration	Length of listing set by the seller (days)	9.341
		(1.992)
MiniBid	Minimum bid of the listing (NT$ 1,000)	5.156
		(3.447)
Member	Number of days since the seller joined Yahoo!	1.187
	Auction as a member (1,000 days)	(0.684)
ListingNo	Number of listings by the seller in our sample	12.757
		(15.591)

Notes: The mean and standard deviation of *Price* are calculated from successful listings.

Table 3: Summary Statistics with and without Buy-It-Now Option

Variables	Listings with Buy-It-Now	Listings without Buy-It-Now	Fixed-Price Listings
Price	6.977	3.758	5.899
	(4.113)	(2.711)	(2.408)
Trade	0.344	0.365	0.335
	(0.477)	(0.482)	(0.472)
Reputation	2.702	0.489	2.323
	(4.056)	(0.695)	(2.590)
New	0.641	0.561	0.906
	(0.481)	(0.497)	(0.293)
TypeL	0.076	0.147	0.166
	(0.267)	(0.354)	(0.373)
TypeP	0.145	0.116	0.197
	(0.353)	(0.321)	(0.398)
TypeS	0.641	0.550	0.674
	(0.481)	(0.498)	(0.491)
BidderNo	0.887	0.866	0.674
	(0.525)	(0.578)	(0.566)
Duration	9.708	9.268	9.523
	(1.439)	(2.078)	(1.771)
MiniBid	5.978	4.991	6.746
	(3.880)	(3.332)	(2.603)
Member	1.184	1.188	1.091
	(0.698)	(0.682)	(0.623)
ListingNo	12.168	12.875	38.341
	(14.523)	(15.804)	(35.896)

Notes: (a) Standard deviations are given in parentheses. (b) The mean and standard deviation of *Price* are calculated from successful listings.

Table 4: Regression Results: Excluding Fixed-Price Listings

	Selection	Main Regression Equation			
	Trade	*Price*			
	Probit	no IV	IV-1	IV-2	IV-3
BIN		1.399**	3.185**	3.121**	3.137**
		(0.612)	(1.247)	(1.245)	(1.247)
Reputation	0.213***	0.518***	0.342*	0.355*	0.342*
	(0.037)	(0.168)	(0.201)	(0.201)	(0.202)
New	0.009	0.820	1.026	1.039	1.027
	(0.192)	(0.718)	(0.729)	(0.724)	(0.730)
TypeL	-0.518*	-1.497*	-1.519*	-1.511*	-1.532*
	(0.280)	(0.799)	(0.816)	(0.810)	(0.810)
TypeP	0.012	1.867**	1.961*	1.961*	1.978*
	(0.311)	(1.088)	(1.069)	(1.063)	(1.070)
TypeS	-0.102	0.242	0.246	0.250	0.267
	(0.213)	(0.585)	(0.594)	(0.589)	(0.589)
BidderNo	0.283**	0.711*	0.729*	0.363	0.753*
	(0.122)	(0.394)	(0.391)	(0.286)	(0.391)
Duration	0.007	-0.037	-0.051	-0.059	
	(0.034)	(0.101)	(0.104)	(0.103)	
MiniBid	-0.260***				
	(0.030)				
Mills Ratio		5.750***	5.681***	5.618***	5.670***
		(1.003)	(0.992)	(0.976)	(0.990)
Constant	0.356	-2.379*	-2.401*	-1.941	-2.888***
	(0.379)	(1.328)	(1.327)	(1.238)	(0.944)
R-squared	0.144	0.787	0.759	0.759	0.759

Notes: (a) Standard errors are calculated by 1,000 bootstrap draws and given in parentheses. (b) Superscripts ***, **, and *represent significance at 1%, 5%, and 10%, respectively. (c) Sample size is 785. (d) Marginal effects are shown in the Probit estimation.

Table 5: Regression Results: Including Fixed-Price Listings

	Selection	Main Regression Equation			
	Trade	*Price*			
	Probit	no IV	IV-1	IV-2	IV-3
BIN		-0.027	2.138**	2.036*	2.149**
		(0.181)	(1.060)	(1.047)	(1.058)
Reputation	0.177***	0.580***	0.406***	0.409***	0.403***
	(0.024)	(0.114)	(0.146)	(0.145)	(0.146)
New	0.048	0.385	0.124	0.145	0.061
	(0.167)	(0.594)	(0.593)	(0.584)	(0.589)
TypeL	-0.649***	-2.097***	-2.053***	-1.970***	-1.956**
	(0.241)	(0.781)	(0.782)	(0.765)	(0.765)
TypeP	0.367	2.037**	1.788**	1.795**	1.890**
	(0.254)	(0.808)	(0.804)	(0.791)	(0.799)
TypeS	-0.054	0.285	0.257	0.261	0.342
	(0.198)	(0.611)	(0.610)	(0.599)	(0.600)
BidderNo	0.465***	1.397***	1.355***	0.850***	1.384***
	(0.094)	(0.363)	(0.362)	(0.263)	(0.363)
Duration	-0.040	-0.146*	-0.139	-0.149*	
	(0.027)	(0.088)	(0.087)	(0.087)	
MiniBid	-0.254***				
	(0.025)				
Mills Ratio		6.669***	6.260***	6.158***	6.220***
		(0.845)	(0.917)	(0.888)	(0.914)
Constant	0.620	-2.514**	-2.676**	-2.029*	-3.987***
	(0.314)	(1.170)	(1.180)	(1.097)	(0.928)
R-squared	0.138	0.818	0.728	0.736	0.727

Notes: (a) Standard errors are calculated by 1,000 bootstrap draws and given in parentheses. (b) Superscripts ***, **, and *represent significance at 1%, 5%, and 10%, respectively. (c) Sample size is 1,272. (d) Marginal effects are shown in the Probit estimation.