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Abstract

In this paper, we assume that a cash-in-advance (CIA) constraint itself depends on relative income, which implies status. This constraint means that agents with higher income are more creditworthy and can make purchases with fewer money holdings. Under this assumption, we construct a one-sector neoclassical growth model and show that there exists a unique steady state that has saddle-path stability without specifying each function. Furthermore, we examine the effects of money growth on capital accumulation. If the status elasticity of CIA constraint is large, the Tobin effect can arise. In contrast, if it is small, the anti-Tobin effect can arise.

Keywords: Cash-in-advance constraint; Status; Money growth; Neoclassical growth model; Tobin/anti-Tobin effect

JEL classification: E41; E52; O42

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1 Introduction

There is a growing macroeconomic literature that examines the effects of inflation (money growth) on capital accumulation. Tobin (1965) regards money as a substitute for capital and concludes that money growth accelerates capital accumulation, known as the Tobin effect. Thereafter, many studies have discussed the effects of money growth in the context of cash-in-advance (CIA henceforth) constraints. For example, Clower (1967) and Lucas (1980) show that if the CIA constraint applies only to consumption, then money growth has no effect on capital accumulation in the long run, known as the superneutrality of money. On the other hand, Stockman (1981) considers a standard neoclassical growth model with the CIA constraint applying to both consumption and investment, and shows that money growth decreases capital accumulation. This is because after a period of high inflation the net rate of return on capital falls. In this case, the level of capital and the money growth rate are negatively correlated. This is referred to as the anti-Tobin effect.

Recent studies on neoclassical growth models with CIA constraints capture the role of status in terms of a social device providing priority in the nonmarket good sector, seen in Cole et al. (1992).[1] For example, Chang et al. (2000), Gong and Zou (2001), and Chang and Tsai (2003) introduce status, defined as capital holdings, into preferences and obtain significant results concerning the effects of money growth on the steady-state level of capital.[2] Under the Clower-Lucas-type CIA constraint, Chang et al. (2000) confirm that money growth and the steady-state level of capital are positively correlated. This is because higher inflation increases the cost of money holdings, so that the agent shifts his/her assets from money to capital, which provides utility. On the other hand, Gong and Zou (2001) employ the Stockman-type CIA constraint and show that whether money growth promotes capital accumulation or not depends on the measure of the agent's desire for status.[3] In a similar vein, Chang and Tsai (2003) find that when the status seeking effect dominates the inflation tax effect, money growth and the steady-state capital stock are positively correlated under the

[1] Zou (1994) interprets utility from capital in terms of the spirit of capitalism, based on Weber (1958), and shows that endogenous growth can arise even if the interest rate is smaller than the time preference rate.

[2] This type of preferences had already been constructed mathematically by Kurz (1968).

[3] Gong and Zou (2001) analyze the effects of money growth in the case of the Clower-Lucas-type CIA constraint and that of the Stockman-type CIA constraint. Under the Clower-Lucas-type CIA constraint, Gong and Zou (2001) obtain the same results as in Chang et al. (2000).

general-type CIA constraint, which means that whole consumption and a positive fraction of investment are purchased using real money balances.

In this paper, on the other hand, we capture status in terms of social credibility when making purchases. Specifically, we embody this concept by assuming that individuals with higher income (higher status) are more creditworthy and can make purchases with fewer money holdings. This setting can be supported by empirical studies: Avery et al. (1987) show that high income individuals use cash and cash plus checks for a smaller fraction of their total transactions than low income individuals; and Wolff (1983), Kessler and Wolff (1991), and Kennickell and Starr-McCluer (1996) find that the fraction of household wealth held in liquid assets decreases with income and wealth. Hence, we assume that the CIA constraint itself depends on relative income, which implies status (hereafter this constraint is referred to as the "CIA-status constraint"). In the present study, we consider a neoclassical growth model with such a CIA-status constraint, which applies to consumption and a fraction of investment, under this setting we clarify how status has an impact on the relationship between the rate of money growth and the steady-state level of capital, as well as a uniqueness of the steady state and its stability.

As a result of this paper, we show that there exists a unique steady state that has saddle-path stability without specifying each function.[4] In addition, the effect of money growth on capital stock changes from negative to positive when the status elasticity of the CIA constraint exceeds the fraction of the liquidity constraint applying to investment expenditure. The intuition is explained through the two effects, that is, the status enhancement effect and the inflation tax effect. The former effect, which can be captured by the status elasticity of the CIA constraint, arises when the agent invests more in order to make the CIA constraint less restricted. On the other hand, the latter effect, which can be measured by the fraction of the liquidity constraint applying to investment expenditure, occurs when the agent purchases investment goods. If the status enhancement effect is sufficiently small (resp. large), the agent attempts to invest less (resp. more) after the policy of raising money growth. This is because the benefit of holding additional capital stock generated from the status enhancement effect is smaller (resp. greater) than the cost of purchasing investment goods caused by the inflation tax effect. Thus, a rise in the rate of money growth depresses (resp.

[4] Although the existing studies on status preferences mentioned above show that there may exist multiple steady states, they focus only on a steady state that has saddle-path stability.

accelerates) capital accumulation.[5]

The remainder of this paper is organized as follows. Section 2 explains the CIA-status constraint in detail and provides the basic framework. Section 3 analyzes the effects of the changes in the status elasticity of the CIA constraint and that in the rate of money growth on capital stock and consumption. Section 4 concludes.

2 Model

2.1 CIA-status constraint

In this paper, we impose a CIA constraint, which itself depends on relative income. We here regard relative income as status. We introduce the ratio of goods which *require* cash to goods which *may require* cash, and denote this ratio by Ω. Taking the observations mentioned in the Introduction into consideration, we assume that Ω depends on the agent's credit when making purchases — the agent's own relative income — and that Ω lies in $(0, 1)$ along the lines of the standard CIA model:

$$0 < \Omega\left(\frac{y}{\bar{y}}\right) < 1, \tag{1}$$

where y and $\bar{y}$ are private income and average income in the economy respectively, and $y/\bar{y}$ stands for the agent's own relative income. In addition to (1), we posit that $\Omega(\cdot)$ is strictly decreasing and strictly concave with respect to $y/\bar{y}$:

$$\Omega'\left(\frac{y}{\bar{y}}\right) < 0, \quad \Omega''\left(\frac{y}{\bar{y}}\right) > 0. \tag{2}$$

We now construct the CIA constraint employed in this paper. From the definition of $\Omega(\cdot)$, we find that

$$\frac{L_c}{L} = \Omega\left(\frac{y}{\bar{y}}\right), \tag{3}$$

where L and L_c are goods which *may require* cash and goods which *require* cash to purchase, respectively. Goods which *may require* cash are defined as

$$L \equiv c + xi, \tag{4}$$

[5] The results of this paper are different from those of Gong and Zou (2001) and Chang and Tsai (2003) in that we provide the condition for the Tobin or the anti-Tobin effect by using the parameters not in preferences but in the CIA-status constraint.

where c is consumption, i is investment, and x, which lies in $[0, 1]$, represents the fraction of the liquidity constraint applying to investment expenditure. Concerning goods which *require* cash to purchase, L_c, the following constraint holds from the definition:

$$m \geq L_c, \tag{5}$$

where m is real money balances defined as nominal money balances divided by the price level. From (3), (4) and (5), we have

$$m \geq \Omega\left(\frac{y}{\bar{y}}\right)(c + xi). \tag{6}$$

2.2 Optimal conditions and dynamic system

The economy is inhabited by a continuum of identical, infinite-lived agents endowed with a unit of labor. The size of the population is constant and is normalized to unity. Each agent consumes a continuum of non-perishable consumption goods produced with a simple neoclassical production technology. We assume that each consumption good is perfectly complementary. The representative agent maximizes the following lifetime utility:

$$\int_0^\infty u(c(t))e^{-\theta t}dt, \tag{7}$$

where $u(\cdot)$ is an instantaneous utility function which satisfies $u'(\cdot) > 0$, $u''(\cdot) < 0$ and the Inada conditions ($\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$). In addition, θ is the rate of time preference.

The budget constraint of the representative agent is

$$\dot{m}(t) = f(k(t)) - c(t) - i(t) - \pi(t)m(t) + \tau(t), \tag{8}$$

where π is the rate of inflation and k is capital stock. Output is produced using a neoclassical production function, $f(\cdot)$, satisfying $f'(\cdot) > 0$, $f''(\cdot) < 0$ and the Inada conditions ($\lim_{k\to 0} f'(k) = \infty$, $\lim_{k\to\infty} f'(k) = 0$). The law of motion of capital stock is given by

$$\dot{k}(t) = i(t). \tag{9}$$

For simplicity, the depreciation rate of capital is assumed to be zero. In (8), τ is the seigniorage that the agent receives from the monetary authority as a lump-sum transfer:

$$\tau(t) = \phi m(t), \tag{10}$$

where ϕ is the constant, time-invariant money growth rate. By using ϕ, the nominal money supply, M, is expressed as

$$M(t) = M(0)e^{\phi t} \ , \quad \text{given } M(0) > 0. \tag{11}$$

Assuming a representative agent and a neoclassical technology, from (6), we find that the CIA-status constraint becomes

$$m \geq \Omega\left(\frac{f(k(t))}{f(\bar{k}(t))}\right)(c + xi). \tag{12}$$

The representaive agent maximises (7) subject to (8), (9) and (12). In what follows, we drop the time index from the endogenous variables. To derive the necessary conditions for an optimum, we set up the following current-value Hamiltonian function:

$$H = u(c) + \lambda\big[f(k) - c - i - \pi m + \tau\big] + \mu\big[\, i \,\big] + \eta\left[m - \Omega\left(\frac{f(k)}{f(\bar{k})}\right)(c + xi)\right],$$

where λ and μ are the shadow prices of real money balances and the capital stock, respectively, and η is the Lagrange multiplier associated with the CIA-status constraint (12). The first-order conditions for optimization are given as follows:

$$u'(c) = \lambda + \eta\Omega\left(\frac{f(k)}{f(\bar{k})}\right), \tag{13a}$$

$$\mu = \lambda + x\eta\Omega\left(\frac{f(k)}{f(\bar{k})}\right), \tag{13b}$$

$$\dot{\mu} = \theta\mu - \lambda f'(k) + \eta\Omega'\left(\frac{f(k)}{f(\bar{k})}\right)\frac{f'(k)}{f(\bar{k})}(c + xi), \tag{13c}$$

$$\dot{\lambda} = \lambda(\pi + \theta) - \eta. \tag{13d}$$

and the transversality conditions for k and m are

$$\lim_{t\to\infty} e^{-\theta t}\mu k = 0, \tag{14}$$

$$\lim_{t\to\infty} e^{-\theta t}\lambda m = 0. \tag{15}$$

Equation (13a) implies that the marginal utility of consumption equals the marginal cost of consumption, which is the marginal utility of having an additional unit of real money balances. Equations (13b) and (13c) together describe that the evolution of capital over time, where the last term on the left-hand side in (13c) represents the marginal benefit from the higher income position (i.e. higher status). This benefit implies that the agent gets relatively higher credibility and can make purchases with fewer money holdings. Equation (13d) implies that the marginal value of real money balances equals the marginal cost.

Since the agents are assumed to be symmetric and the size of the population is unity, in equilibrium the level of the agent's capital stock is equal to the average level of capital stock in the economy:

$$k = \bar{k}. \tag{16}$$

Additionally, in equilibrium the goods market clears, and money demand is equal to money supply:

$$\dot{k} = f(k) - c, \tag{17}$$

$$\dot{m} = (\phi - \pi)m. \tag{18}$$

We assume that the CIA constraint is always binding in equilibrium, as is common in the CIA literature. Thus, from (12) and (16) we get

$$m = \Omega(1)(c + xi). \tag{19}$$

From (9) and (13a)-(19), we obtain the following dynamic system:[6]

$$\dot{c} = -D^{-1}\left[(1-x)\left\{\lambda\left(\phi - \frac{xf'(k)\dot{k}}{xf(k)+(1-x)c}\right) - \frac{(u'(c)-\lambda)}{\Omega(1)}\right\} + \lambda f'(k) - \theta x u'(c) + \xi(u'(c)-\lambda)\left\{xf(k)+(1-x)c\right\}\frac{f'(k)}{f(k)}\right], \tag{20a}$$

$$\dot{\lambda} = \theta\lambda - \frac{(u'(c)-\lambda)}{\Omega(1)} + \lambda\left\{\phi - \frac{xf'(k)\dot{k} + (1-x)\dot{c}}{xf(k)+(1-x)c}\right\}, \tag{20b}$$

$$\dot{k} = f(k) - c, \tag{20c}$$

where

$$D \equiv \left\{xu''(c) - \frac{(1-x)^2\lambda}{xf(k)+(1-x)c}\right\} < 0,$$

$$\xi \equiv -\frac{\Omega'(1)}{\Omega(1)} > 0.$$

Note that ξ expresses the elasticity of the CIA constraint with respect to status.[7]

[6]See Appendix A.1 for the derivation of the dynamic system.

[7]In what follows, ξ is referred to as the status elasticity of the CIA constraint.

2.3 Steady state and stability

In this subsection, we consider a steady state and its stability. In a steady state, the economy is characterized by $\dot{c} = \dot{\lambda} = \dot{k}$ $(= \dot{m}) = 0$. We then obtain

$$f'(k) = \theta + (\theta+\phi)\Omega(1)\left[x\theta - \frac{\xi\left\{xf(k) + (1-x)c\right\}f'(k)}{f(k)}\right], \tag{21a}$$

$$f(k) = c. \tag{21b}$$

Equation (21a) states that when the CIA constraint itself depends on status ($\xi > 0$), the steady-state level of capital is not determined only by the constant rate of time preference (i.e. the modified golden rule ($f'(k) = \theta$) does not hold), even if the CIA constraint applies only to consumption ($x = 0$). In this case, the level of capital hinges on money growth, that is, the superneutrality of money is not valid. In the next section, we conduct the analysis concerning the effects of changes in the status elasticity of the CIA constraint and that of money growth on capital stock and consumption.

From (21a) and (21b), we have

$$f'(k^*) = \frac{\left\{1 + x(\theta+\phi)\Omega(1)\right\}\theta}{1 + \xi(\theta+\phi)\Omega(1)}. \tag{22}$$

Since the right-hand side of (22) is constant and the production function, $f(\cdot)$, satisfies concavity and the Inada conditions, we have a unique solution, k^*, which represents the steady-state level of capital. Additionally, substituting k^* into (21b) yields the steady-state level of consumption, c^*.

Next, we consider the stability of the steady state. Linearizing the dynamic system (20a)-(20c) around the steady state, we obtain the following relationships between the three characteristic roots, g_1, g_2 and g_3:[8]

$$\begin{aligned} g_1 + g_2 + g_3 = &-\frac{(1-x)\xi f'(k^*)u'(c^*)}{D^* f(k^*)} \\ &+ \frac{(f'(k^*)+\phi)u''(c^*)}{D^*}\left\{\frac{1}{(\theta+\phi)\Omega(1)} + x\right\} > 0, \end{aligned} \tag{23a}$$

$$g_1 g_2 g_3 = -\frac{f'(k^*)f''(k^*)u'(c^*)}{D^*\Omega(1)} < 0, \tag{23b}$$

where

$$D^* \equiv \left[xu''(c^*) - \frac{(1-x)^2 u'(c^*)}{\left\{1 + (\theta+\phi)\Omega(1)\right\}f(k^*)}\right] < 0.$$

[8]See Appendix A.2 for the linearization of the dynamic system.

Equations (23a) and (23b) together indicate that the dynamic system has one negative and two positive eigenvalues. Since consumption, c, and the shadow price of real money balances, λ, are jumpable variables and capital stock, k, is a state variable, the steady state exhibits the saddle-point stability.

Proposition 1. *In a neoclassical growth model in which status is incorporated into the CIA constraint, there exists a unique steady state that is saddle-path stable.*

3 Effects on capital stock and consumption

3.1 Changes in the status elasticity of the CIA constraint

From (21b) and (22), we can show that changes in the status elasticity of the CIA constraint positively affect both the steady-state level of capital stock and that of consumption:

$$\frac{dk^*}{d\xi} = -\frac{\{1 + x(\theta+\phi)\Omega(1)\}\,\theta(\theta+\phi)\Omega(1)}{\{1+\xi(\theta+\phi)\Omega(1)\}^2 f''(k^*)} > 0, \tag{24a}$$

$$\frac{dc^*}{d\xi} = -\frac{\{1 + x(\theta+\phi)\Omega(1)\}\,\theta(\theta+\phi)\Omega(1)f'(k^*)}{\{1+\xi(\theta+\phi)\Omega(1)\}^2 f''(k^*)} > 0. \tag{24b}$$

Note that when the CIA constraint is more elastic with respect to status, the effect of holding additional capital stock on the relaxation of the CIA constraint becomes larger. Because of this, the benefit of investment becomes higher through status enhancement. Therefore, an increase in the status elasticity of the CIA constraint induces greater demand for investment, so that the steady-state level of capital increases. Consequently, it follows that output and consumption increase in the long run.

3.2 Effects of money growth

In this subsection, we clarify how money growth affects the steady-state level of capital stock and that of consumption in our framework.

From (21b) and (22), we have

$$\frac{dk^*}{d\phi} = -\frac{(\xi - x)\theta\Omega(1)}{\{1-(\theta+\phi)\Omega'(1)\}^2 f''(k^*)}, \tag{25a}$$

$$\frac{dc^*}{d\phi} = -\frac{(\xi - x)\theta\Omega(1)f'(k^*)}{\{1-(\theta+\phi)\Omega'(1)\}^2 f''(k^*)}. \tag{25b}$$

Equations (25a) and (25b) indicate that when the status elasticity of the CIA constraint, ξ, is greater (resp. smaller) than the fraction of investment expenditure financed by real money balances, x, an increase in the rate of money growth, ϕ, induces an increase (resp. a decrease) in capital stock and an increase (resp. a decrease) in consumption. The positive (resp. negative) relationship between the rate of money growth and capital stock can be considered as the presence of the Tobin (resp. the anti-Tobin) effect. Additionally, when $\xi = x$, money growth has no impact on capital stock and consumption, that is, the superneutrality of money holds.

These results can be interpreted through the relationship between the benefit of holding additional capital stock and the cost of purchasing investment goods. Here, the benefit is generated from the status enhancement effect, which arises when the agent makes the CIA constraint less restricted by investing more. This benefit can be captured through the parameter ξ. On the other hand, the cost is caused by the inflation tax effect, which occurs when the agent purchases investment goods after the policy of raising money growth induces higher inflation. This cost can be measured by the parameter x.

The intuition is as follows. Suppose that the economy is in the steady state initially, and that the rate of money growth rises. When $\xi > x$, the status enhancement effect dominates the inflation tax effect. At that time, the benefit of holding additional capital stock generated from the status enhancement effect is greater than the cost of purchasing investment goods caused by the inflation tax effect. Thus, the net rate of return on capital in utility (consumption) terms will increase.[9] Then, the agent shifts his/her demand from consumption to capital stock in order to enhance his/her status and to make the CIA constraint less restricted.[10] As a result, a rise in the money growth rate promotes capital accumulation, which leads to a rise in output and that in consumption in the long run.

In contrast, when $\xi < x$, the inflation tax effect dominates the status enhancement effect. Then, the benefit of holding additional capital stock generated from the status enhancement effect is smaller than the cost of purchasing investment goods caused by the inflation tax effect. Therefore,

[9] Actually, as the third effect, there is the inflation tax effect on purchasing current and future consumption goods. When $\xi = x = 0$, we can extract only this third effect. However, Stockman (1981) focuses on this case and shows that the net rate of return on capital in utility (consumption) terms is unaffected by higher inflation. Thus, we ignore this third effect.

[10] Note that the agent obtains the higher net rate of return on capital by investing more and enhancing his/her status.

the net rate of return on capital in utility terms will decrease, so that the agent shifts the demand from capital stock to consumption. Thus, an increase in money growth depresses the steady-state level of capital, which leads to a decrease in output and that in consumption in the long run.

Finally, when $\xi = x$, the status enhancement effect and the inflation tax effect cancel each other out. Then, the net rate of return on capital in utility terms does not change. In this case, there is no demand shift between capital stock and consumption. Namely, accelerating money growth has no effect on the steady-state level of either capital stock or consumption.

Proposition 2. *In a neoclassical growth model in which status is incorporated into the CIA constraint, if the status elasticity of the CIA constraint is greater (smaller) than the fraction of the investment expenditure financed by real money balances (i.e.* $\xi > (<) x$*), then a rise in the money growth rate increases (decreases) the steady-state level of capital and that of consumption. If* $\xi = x$*, then the superneutrality of money still holds.*

4 Conclusion

This paper has investigated a neoclassical growth model with a CIA constraint which itself depends on relative income, which implies status. This CIA constraint means that agents with higher income are more creditworthy and can make purchases with fewer money holdings. Under this assumption, we have examined how status, which affects the CIA constraint, has an impact on the relationships between money growth and both capital stock and consumption, as well as a uniqueness of the steady state and its stability.

Under the CIA-status constraint, we have shown that (i) there exists a unique steady state that has saddle-path stability without specifying each function, and (ii) the Tobin or the anti-Tobin effect or the superneutrality of money arise depending on the degree of status elasticity of the CIA constraint. Especially, when the status elasticity of the CIA constraint is greater than the fraction of investment expenditure financed by real money balances, the agent shifts his/her demand from consumption to capital stock after the policy of raising money growth induces higher inflation. This is because the benefit of holding additional capital stock generated from the status enhancement effect is greater than the cost of purchasing investment goods caused by the inflation tax effect in this case. Through this process, the higher money growth rate leads to an increase in the steady-state level of capital and that of consumption.

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Appendix

A.1 Derivation of the dynamic system

This appendix presents the derivations of the dynamic system (20a)-(20c).

Taking the logarithms of both sides of (19) and differentiating them with respect to time, we have

$$\frac{\dot{m}}{m} = \frac{xf'(k)\dot{k} + (1-x)\dot{c}}{xf(k) + (1-x)c}. \tag{26}$$

From (13a), (13d) and (18), it follows that

$$\dot{\lambda} = \theta\lambda - \frac{(u'(c) - \lambda)}{\Omega(1)} + \lambda\left(\phi - \frac{\dot{m}}{m}\right). \tag{27}$$

Substituting (26) into (27), we obtain (20b).

Next, combining (13a) and (13b) gives

$$\mu = xu'(c) + (1-x)\lambda. \tag{28}$$

Differentiating (28) with respect to time, we have

$$\dot{\mu} = xu''(c)\dot{c} + (1-x)\dot{\lambda}. \tag{29}$$

From (9), (13c), (16), (17), (28) and (29), we obtain

$$\begin{aligned} xu''(c)\dot{c} = -\Big[&\lambda f'(k) + (1-x)(\dot{\lambda} - \theta\lambda) - \theta x u'(c) \\ &+\xi(u'(c) - \lambda)\frac{f'(k)}{f(k)}\left\{xf(k) + (1-x)c\right\}\Big]. \end{aligned} \tag{30}$$

Combining (20b) and (30) yields (20a).

Since (20a), (20b) and (20c) include only c, λ and k, they constitute the full of dynamics of c, λ and k.

A.2 Linearization around the steady state

Linearizing the dynamic system represented by (20a)-(20c) around the steady state, we obtain

$$\begin{bmatrix} \dot{c} \\ \\ \dot{k} \\ \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial \lambda} \\ \\ \frac{\partial \dot{k}}{\partial c} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial \lambda} \\ \\ \frac{\partial \dot{\lambda}}{\partial c} & \frac{\partial \dot{\lambda}}{\partial k} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix} \begin{bmatrix} c - c^* \\ \\ k - k^* \\ \\ \lambda - \lambda^* \end{bmatrix}, \tag{31}$$

where each element of the coefficient matrix is

$$\begin{aligned}
\frac{\partial \dot{c}}{\partial c} &= -(D^*)^{-1} \left[\frac{(1-x)f'(k^*)\left\{x\lambda^* + \xi(u'(c^*) - \lambda^*)\right\}}{f(k^*)} \right. \\
&\qquad \left. - \frac{\left\{(1-x) + (\xi f'(k^*) - x\theta)\Omega(1)\right\} u''(c^*)}{\Omega(1)} \right], \\
\frac{\partial \dot{c}}{\partial k} &= -(D^*)^{-1} \left[\{\xi u'(c^*) + (1-\xi)\lambda^*\} f''(k^*) \right. \\
&\qquad \left. - \frac{(1-x)\{x\lambda^* + \xi(u'(c^*) - \lambda^*)\}\{f'(k^*)\}^2}{f(k^*)} \right], \\
\frac{\partial \dot{c}}{\partial \lambda} &= -(D^*)^{-1} \left[\frac{(1-x)(1+\phi\Omega(1))}{\Omega(1)} + (1-\xi)f'(k^*) \right], \\
\frac{\partial \dot{k}}{\partial c} &= -1, \\
\frac{\partial \dot{k}}{\partial k} &= f'(k^*), \\
\frac{\partial \dot{k}}{\partial \lambda} &= 0, \\
\frac{\partial \dot{\lambda}}{\partial c} &= \frac{x\lambda^* f'(k^*)}{f(k^*)} - \frac{u''(c^*)}{\Omega(1)} - \frac{(1-x)\lambda^*}{f(k^*)} \cdot \frac{\partial \dot{c}}{\partial c}, \\
\frac{\partial \dot{\lambda}}{\partial k} &= -\frac{x\lambda^* \{f'(k^*)\}^2}{f(k^*)} - \frac{(1-x)\lambda^*}{f(k^*)} \cdot \frac{\partial \dot{c}}{\partial k}, \\
\frac{\partial \dot{\lambda}}{\partial \lambda} &= \theta + \phi + \frac{1}{\Omega(1)} - \frac{(1-x)\lambda^*}{(1-x)c^* + xf(k^*)} \cdot \frac{\partial \dot{c}}{\partial \lambda}.
\end{aligned}$$

Note that TrJ and $DetJ$ (J is the coefficient matrix in (31)) are equal to $g_1 + g_2 + g_3$, and $g_1 g_2 g_3$, respectively.

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