Macroeconomic fluctuations, regime switching (structural breaks) and impulse responses: Nigerian evidence

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MACROECONOMIC FLUCTUATIONS, REGIME SWITCHING
(STRUCTURAL BREAKS) AND IMPULSE RESPONSES: NIGERIAN EVIDENCE

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Indeed, global output which contracted in 2009, resumed growth in 2010 and although with uncertainties about its sustainability in the world’s three biggest markets: United States, Europe and China. The observed increase in growth was due largely to the stronger-than-expected activity in many emerging economics in the second half of 2010; strong domestic aggregate demand in developing countries (Africa); and the new policy initiatives implemented in the United States of America. However, the sovereign debt crisis in Europe and the policy responses it generated triggered alarm in international financial markets. Similarly, the tightening of monetary policy and efforts to douse the overheated property markets in China, raised concerns about the growth trajectory. On the other hand, the global financial markets which had recovered appreciably by the end of 2009 suffered a major setback during the second quarter of 2010. Here, concerns about the sustainability of the sovereign debt of some European nations triggered financial market worries. Consequently Market volatility increased and risk appetite declined during the period.

While the global economic recovery continues, it remains uneven and subject to downside risks. In particular, a turn for the worse in Europe would have adverse implications for many countries in the region. East and North Africa, with whom some countries in sub-Saharan Africa have strong links, are likely to increase uncertainty about the global economy. For the countries in the region where output is back to its trend path, there are likely to be a new set of policy challenges, including ensuring that excessive domestic demand growth does not spill over into widening macroeconomic imbalances. Yet, to the extent that these effects linger, they could undermine growth further and foster larger macroeconomic imbalances. With relatively few exceptions, monetary policy
rates in the region were reduced as the global recession threatened and have remained at relatively low levels. Because inflation declined sharply as activity showed domestically and commodity prices crashed in 2009, low policy rates were not problematic well into 2010. But with inflation picking up, real rates in many African countries are now either very low or even negative. And with growth back to pre-crisis levels in most cases, this is likely to do little to restrain inflation in the years ahead.
Recently (as at 2010) the Nigerian Gross Domestic Product (GDP) measured at 1990 constant basic prices, was estimated at ₦775.4 billion, indicating a growth rate of 7.9 percent. This exceeded the 7.0 percent recorded in 2009 and the average annual growth rate of 6.7 percent for the period 2006-2010; but lower than the target growth rate of 10 percent for the year. Perhaps, the observed growth in GDP reflected (largely) the sound and stable monetary and fiscal policies; as well as the favorable weather conditions which boosted agricultural output. Again, other drivers of growth included an increase in crude oil production throughout the year; stability in the price and supply of petroleum products; huge investment in infrastructure; building and construction activities; and continued expansion in the telecommunications sub sector. However, the Nigerian economy when measured by the Real Gross Domestic Product (GDP) on an aggregate basis grew by 7.40 percent in the third quarter of 2011 as against 7.86 percent in the corresponding quarter of 2010. The observed 0.46 percentage point decreases in real GDP growth recorded in the third quarter of 2011 was accounted for among other sectors by the decrease in activities in the oil sector. In fact, this drop in crude oil production could be explained by the operational constraints experienced by some of the oil producing companies during the period in question. Consequently, one unwanted characteristic that most Sub-Saharan African economies share is the prevalence and magnitude of output collapses.

Unfortunately, research into output collapses remains largely unexplored and much of the focus of growth studies has been on cross-country analysis, ignoring the volatility of growth patterns. Here, concentrating on average growth rates gives little insight into the growth patterns of an individual country or how the growth rates evolve overtime. Obviously, economic growth/particularly in African economies, undergoes frequent episodes in which the behavior of the series changes significantly. In other words, the
growth paths of such economies are often characterized by large swings and fluctuations. In these periods, the application of linear models seems inappropriate because the changes in regime can alter the long-run growth path of the economy. Basically, an output collapse is defined as a fall in the level of output in an economy that is in excess of 10 percent over a three year period (Darlauf el.al.2004; Beck and Mavro, 2006; Byrne, 2010). Regrettably, output collapses occur more frequently in Sub-Saharan Africa than in the developed and emerging economies. Here, the most common explanation offered is that these collapses take place during periods of intensive civil war; but civil war only partially explains the extent of such collapses among Sub-Saharan nations and Nigeria in particular. Table I.I shows the largest real output drops for Nigeria and sample nations (between 1980 and 2000).

**TABLE I.I OUTPUT DROPS IN NIGERIA AND SAMPLE COUNTRIES**

<table>
<thead>
<tr>
<th>S/N</th>
<th>COUNTRY</th>
<th>TIME PERIOD</th>
<th>OUTPUT DROP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CANADA</td>
<td>1980–1983</td>
<td>5.79</td>
</tr>
<tr>
<td>2</td>
<td>USA</td>
<td>1980–1983</td>
<td>5.37</td>
</tr>
<tr>
<td>3</td>
<td>UK</td>
<td>1980–1983</td>
<td>3.59</td>
</tr>
<tr>
<td>4</td>
<td>MALI</td>
<td>1985–1988</td>
<td>33.94</td>
</tr>
<tr>
<td>5</td>
<td>CAMEROON</td>
<td>1987–1990</td>
<td>33.50</td>
</tr>
<tr>
<td>6</td>
<td>NIGERIA</td>
<td>1997–2000</td>
<td>32.04*</td>
</tr>
</tbody>
</table>

Indeed, countries that endure output collapses display a high level of volatility around their average growth rate. Unless periods of large drops in output are accounted for, little can be inferred through standard quantitative analysis because the dynamics of output following a large collapse can differ significantly from the dynamics of output during stable time periods. Thus, the
generally disappointing growth in Sub-Saharan Africa (particularly Nigeria) over the past decades reflects the difficulties posed by institutional and economic factors. These include the lack of resource endowments; low level of human capital; administrative, legal and institutional framework; stance of financial policies and structural policies that have often been distortionary. These factors coupled with an adverse external environment (with significant declines in terms of trade) have all contributed to hinder sustained economic growth in Nigeria. Unlike studies that have pointed to the significance of these factors in explaining long-run output growth in Nigeria; this paper will pay greater attention to the sources of macroeconomic fluctuations. In otherworld, this paper seeks to document the sources of macroeconomic fluctuations in Nigeria by measuring the relative importance of domestic versus external shocks.

This research study therefore intends to show that the hypothesis of a common stochastic productivity trend has a set of econometric implications that allows us to test for its presence, measure its importance and extract estimates of its realized value. Furthermore, we intend to know how much of cyclical variation in the research data can be attributed to innovations associated with nominal variables explain important cyclical movements in the real variables. Building on a venerable economic tradition, it is hoped that the estimated relationships will separate the regular response of policy to the economy from the response of the economy to policy; and therefore producing a more accurate measure of the effects of the policy changes and shocks. In this way, the proposed model will integrate policy behavior variously with broad macroeconomic aggregates to provide a fuller understanding of the factors underlying the bulk of economic fluctuations in Nigeria. Distinctively, this paper propose to contribute to the current modeling literature, by accommodating regime switching and structural break dynamics in a unified
framework. Unfortunately, current regime switching models are not suitable for capturing instability of dynamics because they assume a finite number of states and that the future is like the past (Song, 2011). On the other hand, structural break models allow the dynamics to change over time. However, they may incur loss in the estimation precision because the past states cannot recur and the parameters in each state are estimated separately. Consequently, an infinite dimension markov switching model is proposed to accommodate both types of model and provide much richer dynamics as well as showing how to globally identify structural breaks versus regime switching as applied to the Nigerian Economy.

1.0 OBJECTIVES OF STUDY

The fundamental aim of this research study is to build a specific structural long run model that incorporates the important features of the Nigerian economy. This objective requires us to carry out the following steps:

(1) To construct new classical based vector autoregression models for Nigeria.

(2) To examine whether business cycles are mainly the result of permanent shocks to productivity.

(3) To identify the role of monetary and fiscal policy during economic fluctuations using variance decomposition, impulse response analysis and persistence profiles (by differentiating the effect of variable specific and system-wide shocks).

(4) And to provide a solution by proposing an infinite dimension markov switching model that incorporates regime switching and structural break dynamics in a unified framework.
2.0 THEORETICAL AND METHODOLOGICAL FRAMEWORK

In a real-business-cycle model with permanent productivity shocks, output $Y_t$ is produced via a constant-return-to-scale Cobb-Douglas production function:

$$Y_t = \lambda t^{1-\theta} N_t^{\theta}$$  \hspace{1cm} (1.1)

Where $k_t$ is the capital stock and $N_t$ represents labor input. Total factor productivity, $\lambda_t$ follows a logarithmic random walk:

$$\log (\lambda_t) = \mu \lambda_t + \log (\lambda_{t-1}) + \xi_t$$  \hspace{1cm} (1.2)

Where the innovations, $(\xi)$ are independent and identically distributed with a mean of 0 and a variance of $\sigma^2$. The parameter $\mu_t$ represents the average rate of growth in productivity; $\xi_t$ represents deviations of actual growth from this average.

Within the basic neoclassical model with deterministic trends, it is familiar that per capital consumption, investment, and output all grow at the rate $\mu_t / \theta$ in steady state. The common deterministic trend implies that the great ratios of investment and consumption to output are constant along the steady-state growth path. When uncertainty is added, realizations of $\xi_t$ change the forecast of trend productivity equally at all future dates:

$$E_t \log (\lambda_{t+s}) = E_{t-1} (\lambda_{t+s}) + \xi_t$$  \hspace{1cm} (1.3)

A positive productivity shock raises the expected long-run growth path: there is a common stochastic trend in the logarithms of consumption, investment, and output. The stochastic trend is $\log ((\lambda_t / \theta)$, and its growth rate is $(\mu_t + \xi_t) / \theta$, the analogue of the deterministic models common growth rate restriction, $\mu_t / \theta$.

With common stochastic trends, the ratios $c_t / y_t$ becomes stationary stochastic processes (Kings et. al. 1991). Thus following Hoffmaister and Roldos (1997), Hoffmaister et. al. (1998); we consider a small open economy that produces an exportable good ($y_x$) and a non-tradable good ($y_n$) using imported intermediate
inputs. Here, optimal production and consumption decisions determine an equilibrium exchange rate \((Q)\) that is used to define total GDP as \(Y_t = Y_x + QY_n\)

Using lower-case letters to denote the logs of upper-case variables, an expression for the (log) of total GDP can be obtained:

\[
Y = \hat{\beta}_1 \hat{\alpha}^x - B_1 \hat{P}_m + r_1 \hat{k}_t + d_1 (k_t \hat{I}_t)
\]

(1.4)

The first two terms in equation (1.4) are supply shocks that enter symmetrically because, as Bruno and Sachs (1985) pointed out, an increase in the price of intermediate inputs \((P_m)\) acts like negative technological progress. For Nigerian economy, changes in \(a_x\) could also be weather-related crop successes (failures). Next, the second term can be decomposed into the world price of intermediate inputs \(P_m\) and the tariff rate \(\lambda\). Basically, this allows us to model supply responses to structural reforms such as trade liberalization as well as the impact of terms of trade shocks (Lee, 1993).

Generally, an improvement in the terms of trade or structural reform that removes distortions leads to a positive response in total GDP in the long run. Here, the last two terms are the (log of) capital stock, \(k\), and the (log of) capital/labor ratio, \(k^{-1}\), which respond endogenously to the different shocks.

Again, in order to introduce demand shocks, it is convenient to assume that government spending \((g)\) falls mostly on non tradable goods. Hence, the main effect of a fiscal expansion is to change the composition of demand (and hence production) toward non tradable goods with as ambiguous effect on total GDP.

And given the ambiguity of the impact of government spending on GDP, we do not impose a sign on the long run impact of fiscal policy on GDP; rather we assume that it is small and not very different from zero. Again, we assume that individuals in this economy have access to international capital markets where they borrow an amount, \(D\), at the world interest rate \((r^*)\). Here, the effect of world interest rate shocks is captured by the fourth term in equation (1.4)
because in the long run the marginal productivity of capital (determined by the capital/labor ration) equals $r^*$ under perfect capital mobility. Thus, an increase in world interest rates tends to have a contractionary effect on total GDP as the decrease in the capital/labor ratio is multiplied by a positive coefficients in equation (1.4).

Indeed, the dual nature of the responses of the real exchange rate is the trade balance is well understood: excess demand pressures lead to real exchange rate appreciation and trade deficits. Thus, the long-run response of the (log of) real exchange rate ($q$) to the different shocks in summarized by equation (1.5).

$$q_t = \bar{U}_t a_x \bar{\epsilon} B_2 P_{me} + r_2 k_e + \dot{P}_2 (k_t \bar{\epsilon} I_t)$$

(1.5)

which is the analog of equation (1.4) for the relative price. However, a positive supply shock due to technological progress in the tradable sector (to a good crop) or to trade liberalization (as well as terms of trade improvement) leads to a real exchange rate appreciation under plausible parameter values. Really, this is due to the fact that positive wealth effects of these shocks lead to a higher demand for non tradable goods sector induced by the increase in the relative price of the non traded good. Here, an increase in government spending also leads to a real exchange rate appreciation. Despite having a negative wealth effect, the fact that government spending is biased toward non tradable goods requires an increase in the relative price of non-tradable good to reach a new equilibrium. On one hand, the fiscal expansion leads to a decline in the capital stock/which has a first order effect on the real exchange rate but a negligible effect on the level of total GDP. It also causes a reduction in the trade surplus as the decline of the capital stock leads to lower steady-state level of external debt and interest payments. Again, an increase in world interest rates leads to a larger trade surplus, as the fall in domestic absorption relative to output accommodates the increased interest payments. Conventionally, we assume long run neutrality of money and nominal exchange rate and include in the
model a general unspecified equation for the evolution of the price level. Owing to the different exchange rate regimes, it is difficult to establish whether the evolution of the price level is determined by the money supply, nominal exchange rate or both. Nevertheless, it is likely that the inflation rate will be affected by the other variables of the economic system, either via a direct effect through money demand or through some feedback rule the authorities follow on the chosen nominal anchor.

Methodologically, the structural VAR model uses the long-run properties of the long-run model to recover the underlying economic shocks and estimate their relative importance as well as their cyclical effects. Following Blanchard and Quah (1989) we show how to use the long-run effects from an economic model together with the condition needed for the independence of shocks (orthogonality conditions) to recover or identify the economic shocks from a reduced form model. Essentially, this allows the researcher to leave the short-run dynamics of the model unrestricted. Here, the results can be interpreted either as the result of transitional equilibrium dynamics of capital accumulation and labor supply in response to the economic shocks or as the disequilibrium dynamics implicit in a model with wage/price stickness.

Specifically, our basic structural VAR model contains five variables. WORLD REAL INTEREST RATE, TERMS OF TRADE, OUTPUT, EXCHANGE RATE AND PRICES. This implies that a total of 25 independent restrictions are needed to identify the underlying economic shocks. This model is given as

\[ \Delta x = A(L)i \]  

(1.6)

Where \( \Delta x \) = vector of model variables

\( A(L) \) = matrix of lag polynomials that summarize model dynamics

\( i \) = vector of shocks or innovations
Here, the small open economy assumption provides six restrictions: domestic shocks (supply, fiscal and nominal) do not affect the world interest rate or the country’s terms of trade. Again, the long-run model provides four additional restrictions: fiscal shocks can affect real exchange rate and hence the composition of output between traded and non-traded goods but not the long-run level of output.

The long run neutrality of nominal shocks provides two restrictions such that nominal shocks do not affect output or the real exchange rate and terms of trade shocks do not affect world interest rates in the long run.

Orthogonality of the economic shocks provides the 15 additional restrictions needed to exactly identify the impact of the economic shocks.

To identify trends and their impulse response functions, let \( x_t \) denote an \( nx1 \) vector of time series and the individual series are assumed to be I(1) and to have the world representation:

\[
\Delta X_t = \mu + C(L)\epsilon_t
\]

(1.7)

Where

\[ \epsilon_t = \text{vector of one-step ahead linear forecast errors in } X_t \text{ given information on lagged values of } X_t \]

These \( \epsilon_t \) are serially uncorrelated with a mean of zero and covariance matrix \( \Theta \).

However, it is important that the analysis of co-integration be accompanied by some estimate of the speed with which the economy or the market under consideration return to their equilibrium state, once shocked. (Pesaran and Shin, 1996) such an analysis would be particularly valuable in cases where there is two or more co-integrating relation characterizing equilibrium, possibly in different markets, where we will be able to estimate the relative adjustment speeds of different markets towards their respective equilibrium. Basically, there are four possible methods that can be used to characterize and estimate the
time profile of the effect of shocks on one more co integrating relations. All these approaches are based on explicit relationship that we drive between the co integrating relation and the current and lagged values of the shocks in the convergence to equilibrium.

The obvious method has been the impulse response approach (Sims, 1980) use to estimate the time profile of the effect of particular shocks on the co integrating relations. But such an analysis is subject to the major criticism that the estimate impulse response function are not unique and depend on the way the shocks in underlying VAR model are orthogonalised. An alternative approach is to consider the resultant time profile of the effect not a system wide shock on the co integrating relation the resultant time profile will be unique and do not require the prior orthogonalization of the shocks.

Following Lee and Pesaran (1993), we measure the impact of system-wide shock on the co integrating relation by their persistence profile, defined as the scaled difference between the condition variances of the n step and the (n-1) step ahead forecast, and viewed as a function of n, the forecast horizon. This measure readily capture the essential difference that exists between co integrated and non co integrating relation, and provide unique time profile of the effect of shocks to the co integrating relation, in the case of relation between I (1) variable that are not co integrated, the effect of a shock persists forever, while in the case of coin grated relation the impact of shock will be transitory and eventually disappear as the economy returns to its steady trend its long run equilibrium

Specifically, we assume the following augmented vector autoregressive model:

\[ Z_t = a_0 + a_1 t + a \Delta Z_{t-1} + Q W_t + U_t \quad t = 1, 2, \ldots, n \]   

\[ = A g_t + U_t \]
Where $Z_t$ is an $m \times 1$ vector of jointly determined dependent variable and $W_t$ is a $q \times 1$ vector of deterministic or exogenous variable. The statistical framework for the co-integrating VAR is the following general vector error correction mode (VECM)

$$\Delta Y_t = a_0y + a_1y Y_{t-1} + \Gamma_{iy} \Delta Z_t + \Omega_y W_t + \varphi \quad t = 1, 2, \ldots, n \quad (1.9)$$

Where $Z_t = (Y_t, X_t')$, $Y_t$ is an $M_y \times 1$ vector of jointly determined (endogeneous) $I(1)$ variables.

$X_t = M_x \times 1$ vector of exogenous $I(1)$ variables.

$$\Delta X_t = a_0x + \sum \Gamma_{ix} \Delta Z_t - \xi + \Psi x W_t + V_t \quad (1.10)$$

$W_t = q \times 1$ vector of exogenous/deterministic $I(0)$ variables, excluding the intercepts and / or trends.

However, the disturbance vectors and $\varphi$ and $v_t$ satisfy the following assumptions:

$$U_t = \begin{pmatrix} \varphi \\ \text{iid}(0, \Sigma) \\ v_t \end{pmatrix} \quad (1.11)$$

where $\Sigma$ is a symmetric positive definite matrix and the disturbances in the combined model $U_t$ are distributed independently of $W_t$.

$$E(U_t/W_t) = 0 \quad (1.12)$$

The intercept and the trend coefficients $a_{0y}$ and $a_{1y}$ are $M_y \times 1$ vector; $U_y$ is the long-run multiplier matrices of order $M_y \times M$,

where $M = M_x + M_y$: $\Gamma_{iy}, \Gamma_{2y}, \Gamma, \Gamma_{p-1}$, $y$ are $M_y \times M$

Coefficient matrices capturing the short-run dynamic effect: and $\Omega_y$ is the $M_y \times q$ matrix of coefficient on the $I(0)$ exogenous variable.

Indeed, (1.9) allow for a sub-system approach in which the $M_x$ vector of random variable, $X_t$ is the forcing variable, or common stochastic trend in the
sense that the error correction term do not enter in the sub-system for $X_t$. Thus our co integrating analysis allows for contemporaneous and short-term feedbacks from $Y_t$ to $X_t$ but require that no such feedback are possible in the long-run forcing variable of the system. The co integrating VAR analysis concerned with the estimation of (1.9) when the rank of the long-run multiplier matrix, $\mathfrak{U}$, could at most be equal to $M_y$. Therefore, rank deficiency of $\mathfrak{U}$ can be represented as

$$H_r: \text{Rank } (\mathfrak{U} y) = r \leq M_y$$

Which follows that $\mathfrak{U} y = \tilde{\mathfrak{U}} y \beta$ where $\tilde{\mathfrak{U}}$ and $y$ are $M_y \times r$ and $M \times r$ matrices, each with full column rank, $r$. in the case where $\mathfrak{U} y$ is rank deficient, we have $Y_t \sim I(1)$, $\Delta Y_t \sim I(0)$, and $\beta Z_t \sim 1(0)$. The $rx1$ trend stationary relations, $\beta Z_t$ are referred to as the co integrating relation, and characterize the long run equilibrium (steady state) of the VECM (1.9).

It is however, important to recognize that in the case where the VECM (1.9), contains deterministic trends (i.e. $a_i y = 0$), in general there will also be a linear trend in the co integrating relations. Thus, combining the equation system (1.9) and (1.10), we have

$$\alpha Z_t = a_0 + a_1 t - Z_{t-1} + \mathfrak{U} Z_{t-1} + \mathfrak{Q}_i Z_{t-1} + \alpha Z_{t-1} + \mathfrak{Q}_y W_t + U_t \quad (1.13)$$

for $t = 1, 2, \ldots, n$, where

$$Z_t = \begin{pmatrix} Y_t \\ X_t \end{pmatrix}, \quad U_t = \begin{pmatrix} \mathfrak{U} y \\ \mathfrak{V} y \end{pmatrix}, \quad a_0 = \begin{pmatrix} a_0 y \\ a_0 x \end{pmatrix}, \quad a_1 = \begin{pmatrix} a_1 y \\ a_1 x \end{pmatrix},$$

$$\mathfrak{U} = \begin{pmatrix} \mathfrak{U} y \\ \mathfrak{Q}_i \\ 0 \end{pmatrix}, \quad \mathfrak{Q}_i = \begin{pmatrix} \mathfrak{Q}_i y \\ \mathfrak{Q}_i x \end{pmatrix}, \quad \mathfrak{Q}_y = \begin{pmatrix} \mathfrak{Q}_y y \\ \mathfrak{Q}_y x \end{pmatrix}$$
which is the vector error correction form of (1.8). In the case where $\Psi$ is mark
deficient, the solution of (1.13) involves common stochastic trends, and is given
by
\[
Z_t = Z_o + b_o t + b_1 \{ t ( t + 1)/2 \} + C(1) S_t + C^* (L) (h_t \tilde{h}_o)
\] (1.14)
Where $h_t = Q W_t + U_t$ (1.15)
$S_t = \mathcal{G} U_i, t = 1, 2, \ldots$ (1.16)
$b_o = C (1)a_o + C^* (1) a_1$ (1.17)
$b_1 = C (I) a_1$ (1.18)
$C(L) = C(L) + (I-L)C^*(L)$ (1.19)
\[
C^*(L) = \mathcal{G} \sum_{i=0}^{2/5} C_i L^i
\]
where $L$ is the one period lag operator and the $M \times M$ matrices, $C_i^*$, are
obtained recursively from
\[
C_i^* = C_{i-1}^* \tilde{U}_p, i = 1, 2, \ldots (1.20)
\]
With $C^*o = \text{Im} \tilde{C}(1) C_i^* = 0, i < 0$, and
\[
\Psi C(1) = 0 = C(1) \Psi (1.21)
\]
The matrices $\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_p$ are the coefficient matrices in the VAR form of (1.13),
and in term of $\Psi, \mathcal{A}_2$ and $\mathcal{A}_{p-1}$ are given by
\[
\tilde{U}_1 = \mathcal{A}_m \Psi + \mathcal{A}
\]
$\tilde{U}_i = \mathcal{A}_{i-1} \mathcal{A}_i \mathcal{A}_{i+1} I = 2, 3, \ldots, p-1$
$\tilde{U}_p = \mathcal{A}_{p-1}$
From solution (1.14) it is clear that, in general, $Z_t$ will contain a quadratic trend.
When $a_1 \neq 0$, the quadratic trend disappears only if $C(1) a_i = 0$, otherwise the
number of independent quadratic trend terms in the solution of $Z_t$ will be equal
to the rank of $c (1)$. And hence depends on the number of cointegrating
relations. Since rank $\{ C(1) = m-r $ and without some restrictions on the trend
coefficients, \( a_1 \) \( \mathcal{U}_t \) the solution (1.14) has the unsatisfactory property that the nature of the trend in \( Z_t \) varies with the assumed number of the cointegrating relations. This outcome can be avoided by restricting the trend coefficients namely, by setting \( a_1 = \Pi_r \) and using (1.18) and (1.12), we have
\[
b_1 = C(1)a_i = c(1) \mathcal{U}_t = 0
\]
and the VECM in (1.13) becomes
\[
\Delta Z_t = a_0 - \Pi (Z_t - 1 - r_t) + \sum \Gamma_i \Delta Z_{t-i} + \Psi W_t + U_t \quad (1.22)
\]
Using (1.14), the co integrating relations, \( \mathbf{b}'Z_t \), can also be derived in terms of the shocks \( U_t \), \( i = 0, 1, 2, \ldots \), and the current and past values of \( 1(0) \) exogenous values.
Pre-multiplying (1.14) by \( \mathbf{b}' \) and bearing in mind the co integrating restrictions \( \mathbf{b}'C(1)=0 \), we obtain
\[
\mathbf{b}'Z_t = \mathbf{b}'Z_0 + (\mathbf{b}'b_0) + \mathbf{b}'C^*(L)(h_t \mathbf{r} h_0) \quad (1.23)
\]
using (1.17) we also have
\[
\mathbf{b}'b_0 = \mathbf{b}'C^*(1)a_1 \quad (1.24)
\]
and hence when \( a_1 \neq 0 \), the co integrating relations, \( \mathbf{b}'Z_t \) in general, contain deterministic trends; which do not disappear even if \( a_1 \) is restricted. Indeed, when \( a_1 = \Pi_r \), the coefficients of the deterministic trend in the co integrating relations are given by
\[
\mathbf{b}'b_0 = \mathbf{b}'C^*(1) \mathcal{U}_t
\]
but using the result in (1.23), we have
\[
\mathbf{b}'Z_t = \mathbf{b}'Z_0 + (\mathbf{b}'r)t + \mathbf{b}'C^*(L)(h_t \mathbf{r} h_0) \quad (1.25)
\]
A test of whether the co integrating relations are trended can be carried out by testing the following restrictions:
\[
\mathbf{b}'r = 0 \quad (1.26)
\]
Referred to as co-trending restrictions.
The computation of the impulse response function for the co integrating VAR model can be based on the VECM (1.13), which combines the equation system for \( Y_t \) and \( X_t \) given by (1.9) and (1.10), respectively. The solution of the combined model is given by (1.14) and the orthogonalized impulse response function of the effect of a unit shock to the \( i \)th variable at time \( t \) in (1.13) on the \( j \)th variable at time \( t+N \) given by

\[
OI_{ijN} = e_{ij}^j (C(1) + C_{N^*}) T e_i \tag{1.27}
\]

Where \( T \) is a lower triangular matrix such that \( \Theta = TT' \), \( e_i \) is the selection vector and \( (I) \) and \( C^*N \) are defined by relations (1.19) to 1.21). This implies that the effects of shocks on individual variable in a co integrating VAR model do not die out and persist forever.

An alternative approach would be to consider the effect of system-wide shocks or variable-specific shocks on the co integrating relations, \( \hat{b}' Z_t \) rather on the individual variables in the model. The effect of shocks on co integrating relations is bound to die out, and their time profile contains useful information on the speed of convergence of the model to its co integrating (or equilibrium) relations. Consider first the time profile of the effect of a unit shock to the variable in \( Z_t \) on the \( j \)th cointegrating relation, namely \( \hat{b}' Z_t \) using (1.23), we have

\[
OI_i(\hat{b}' Z_{t,N}) = \hat{b}' A_N T e_i \tag{1.27}
\]

For \( I = 1,2,\ldots,m, j = 1,2,\ldots,r, N = 0,1,2,\ldots \). Which give the responses of a unit change in the \( i \)th orthogonalized shock (\( =\tilde{\alpha}_{ui} \)) on the \( j \)th co integrating relation \( \hat{b}' Z_t \). The corresponding generalized impulse responses are given by

\[
GI_i(\hat{b}' Z_{t,N}) = \hat{b}' A_N \frac{\Theta e_i}{\tilde{\alpha}_{ui}} \tag{1.28}
\]

For \( i = 1,2,\ldots,m, j = 1,2,\ldots,r, \) and \( N = 0,1,2,\ldots \)
But given the ambiguities that surround the impulse response analysis with respect to variable specific shocks, we consider the effect of system-wide shocks on co integrating relations. Such a time profile, referred to as the persistence profile of the effect of system-wide shocks on the jth co integrating relationship is given by

$$h(\beta_j Z_t N) = \beta_j Z_t N \sum A_i$$

(1.29)

for $j = 1, 2, \ldots, r$, and $N = 0, 1, 2, \ldots$. The value of this profile is equal to unity on impact, but should tend to zero as $N \to \infty$, if $\beta_j$ is indeed a co integrating vector. The persistence profile, $h(\beta_j Z_t N)$ viewed as a function of N provides important information on the speed with which the effect of system wide shocks on the co integrating relation, $\beta_j Z_t$, disappears even though these shocks generally have lasting impacts on the individual variables in $Z_t$.

Indeed, empirical data observation may have shown that the growth path of Nigeria displays different behavior during periods of stable growth and periods of collapse. In order to capture this asymmetry, we propose a two-state markov-switching model with time-varying transition probabilities. This regime-switching model combines two or more sets of model parameters into one system. And the regime the system is likely to be in a certain time determines which set of parameters (coefficients) should be applied. Conventionally, a two-state switching model takes the form:

$$Y_t = \begin{cases} x(t) x b_1, & s(t) = 1 \\ x(t) x b_2, & s(t) = 2 \end{cases}$$

(1.30)

Where $s(t)$ denotes the state the economy is in at time $t$. On one hand, the first set of parameter estimates apply to the observed independent variables when the system is in state. On the other hand, the second set applies when the system can be extended to incorporate any number of regimes. Here, $s(t)$ is determined by a markov chain which itself depends on a transition matrix. The transition
matrix gathers the probabilities that one particular state is followed by another particular state. In modeling the regime changes, it is assumed that at some point in the sample, the mean value of the growth rate will shift to another value, for this project, this is expected to occur when a country moves from a stable to a collapse regime. Yet the probability of being in a particular regime is inferred from the data. Here, the two state Markov switching model is estimated using quality data on real GDP for the period of 1960: Q1 to 2010 Q4. Notably in using Markov switching models, the first challenge is to determine the true number of regimes. In fact, the idea behind regime switching models is that the parameters of the underlying data generating process of the observed time series vector variable, the probability of being in a certain state, and if there is insufficient information in the series, the regime classification will be weak. Consequently, badly parameterized switching models may not be an improvement over models that do not allow for switching (Byrne, 2010; Song, 2011). Finally, air research method is equally proposing an infinite dimension Markov switching model that incorporates regime switching and structural break dynamics in a unified framework. Here, recuing states are allowed to improve estimation and forecasting precision; and an unknown number of states is embedded in the infinite dimension structure and estimated endogenously to capture the dynamic instability.

2.1 UNIT ROOT AND COINTEGRATION TESTING MODEL

Conventionally, the most widely applied test in working with multivariate time series, are residual based test. Here, the null hypothesis of no cointegration is tested against the alternative that the relation is cointegrated in the sense of Engle and Granger (1987) as originally suggested by Granger (1981). Consequently, most researchers start a cointegration analysis with the usual Augmented Dickey. Fuller (ADF) test, and proceed only if the statistic rejects
the null of no cointegration. Here, if the model is indeed cointegrated with a one-time regime shift in the cointegrated vector, the standard ADF test may not reject the null and the investigator will falsely conclude that there is no long-run relationship. In fact, Gregory et.al (1994) has shown that the power of the conventional ADF test falls sharply in the presence of a structural break. However, if the proposed tests of this paper are employed, there is a better chance of rejecting the null hypothesis leading to a correct model formulation. Here, the proposed new tests are complementary to those of Hansen (1992a) and Quintos and Phillips (1993), which developed tests of the hypothesis of time variance of the coefficients of a cointegrating relation. Specifically, their null hypothesis is Engle-Granger cointegration while proposed test null hypothesis is no cointegration. On the other hand, the test of Hansen and Quintos-Phillips are best viewed as specification tests for the Engle-Granger cointegration model. On the other hand, the newly proposed tests for cointegration are best viewed as pre-tests akin to the conventional residual-based cointegration tests (see Gregory and Hansen, 1996a).

Consequently, we outline the standard single-equation cointegration model and generalize it to allow for both a regime and trend shift under the alternative hypothesis. Here, the observed data are

\[ Y_t = (Y_{1t}, Y_{2t}), \]

Where \( Y_{1t} \) is real-valued \( Y_{2t} \) is an m-vector

Thus the standard model of cointegration with a trend and no structural change is

\[ Y_{1t} = \mu + b_0 + \bar{U}^T Y_{2t} + e_t \quad t = 1, \ldots, n \]

Where \( Y_{2t} \) is I(1)
And \( e_t \) is I (0)
Indeed, the motivation for these tests is that there may be occasions in which the investigator may wish to test that cointegration holds over some period of
time, but then shifts to a new long-run relationship. In fact, the timing of this shift is treated as unknown. However, the most general kind of structural change considered in Gregory and Hansen (1996a, 1996b) permits changes in the intercept \( \mu \) and changes to the slope coefficients \( \beta \) but not the trend coefficient \( \beta \). Thus, to model the structural change, we define the dummy variable:

\[
\delta_t = \begin{cases} 
0, & \text{if } t \leq [n\tau] \\
1, & \text{if } t > [n\tau] 
\end{cases}
\]  

(I.I c)

Where the unknown parameter \( I\hat{\tau}(0,1) \) denotes the (relative) timing of the change point and \([\ ]\) denotes integer part. Yet, the regime and trend shift alternative is \( Y_t = \mu_1 + \mu_2 + \beta_1 + \beta_2 + \phi + \bar{U}_1^T y_t + \bar{U}_2^T y_{2t} + \epsilon_t \) for \( t = 1, \ldots, n \) (I.I D).

In this case \( \mu_1, \beta_1 \) and \( \beta_1 \) are the intercept, slope coefficients and trend coefficient respectively before the regime shift and \( \mu_2, \beta_2 \) and \( \beta_2 \) are the corresponding changes after the break. It is therefore common in time series regression to test the null of no cointegration against the alternative in equation (I.I D). Yet, a potential pitfall to this strategy is when there is some regime shift as in equation (I.I D), the distributional theory to graduate the residual-based test is not Gregory et al. (1996) have shown that the rejection frequency of the ADF test falls dramatically in the presence of a break in the cointegrating vector. Therefore, to test against alternative (I.I D), we define the innovation vector

\[
U_t = \alpha y_t
\]

Its cumulative process

\[
S_t = X^t \mathbf{0} + \epsilon_t, \text{ (so } y_t = y_0 + S_t \text{) and its long-run variance}
\]

\[
\Omega = \lim_{n \to \infty} E S_n S_n^T
\]

When \( U_t \) is covariance stationary \( \Omega \) is proportional to the spectral density matrix evaluated at the zero frequency. Here, the null hypothesis of no cointegration is
that equation (I.I B) holds with $e_t = I (1)$. And this implies that $q > 0$, with
distributional details at Gregory and Hansen (1996). We therefore compute the
cointegration test statistic for each possible regime shift $I ET$, and take the
smallest value (large negative value) across all possible break points, in
principle, the set $T$ can be any compact subset of (0,1). Practically, it will need
to be small enough so that all of the statistics can be calculated. However, a
standard choice in the literature is $T = (0.15, 0.85)$ and for computational
purposes the test statistic is computed for each break point in the interval
$([0.15, 0.85])$. For each $I$, estimate equation (I.I D) by OLS, yielding the
residual $\varepsilon_t$. Here, the subscript $i$ on the residuals denotes the fact that the
residual sequence depends on the choice of change point $i$. Thus, from these
residuals, calculate the first-order serial correlation coefficient as

$$
\rho_{\varepsilon} = \frac{\sum_{t=1}^{n} \varepsilon_t \varepsilon_{t+1}}{\sum_{t=1}^{n} \varepsilon_t^2}
$$

(I.I E)

Consequently, the Phillips (1987) test statistics are formed using a bias-
corrected version of the first order serial correlation coefficient. Defining the
second-stage residuals as

$$
\hat{V}_{t} = \varepsilon_t - \rho_{\varepsilon} \hat{U}
$$

(I.I F)

Then, the correction involves the following estimate of a weighted sum of
autocovariances

$$
\Sigma_0 = \sum_{j=1}^{M} W_{j} \frac{r_{\hat{U}}(j)}{M}
$$

(I.I G)

Where

$$
\hat{V}_{(j)} = \frac{1}{n} \sum_{t=j+1}^{n} V_{t-j \hat{U}}
$$

(I.I H)
and \( M = M(n) \) is the bond width number selected so that \( M \to D \). Here, the kernel weights \( W(.) \) need to satisfy the standard conditions for spectral density estimator and the estimate of long-run variance of \( V_t \) is
\[
\hat{\Lambda}^2 = r\hat{\Lambda}(0) + 2\hat{\Lambda} \tag{I.I I}
\]
The bias-corrected first order serial correlation coefficient estimate is therefore given by
\[
P^*_{\hat{\Lambda}} = \sum_{t=1}^{n-1} \frac{(\hat{\Lambda} \hat{v}_t + 1\hat{u} - \hat{\Lambda})}{\sum_{t=1}^{n-1} \hat{v}_t^2} \tag{I.I J}
\]
The Phillips test statistics can therefore be written as
\[
Z_{\hat{\Lambda}}(\hat{\Lambda}) = n \left( P^*_{\hat{\Lambda}} - 1 \right) \tag{I.I K}
\]
\[
Z_t(\hat{\Lambda}) = n \frac{(\hat{\Lambda}^2 - 1)}{S_t} \tag{I.I L}
\]
With
\[
S_t^2 = \frac{\hat{\Lambda}^2}{\sum_{t=1}^{n-1} \hat{v}_t^2}
\]
On the other hand, the augmented Dickey-Fuller (ADF) statistic is calculated by regressing \( \hat{\varepsilon}_t \) upon \( \hat{\varepsilon}_{t-1}, \hat{\varepsilon}_{t-1}, \ldots, \hat{\varepsilon}_{t-k} \) for some suitably chosen lag truncation \( K \). Thus, the ADF statistic is the \( t \)-statistic for the regressor \( \hat{\varepsilon}_{t-1} \), which is denoted by \( \text{ADF}(\hat{\varepsilon}_{t-1}) = \text{tstat}(\hat{\varepsilon}_{t-1}) \) \( \tag{I.I M} \)
Operationally, the test statistics are the smallest values of the above statistics, across all values of \( \hat{\varepsilon}_T \) and we focus on the smallest values since small values of the test statistics provide against the null hypothesis. The resultant test statistics are
\[
Z_{\hat{\varepsilon}}^* = \inf_{\hat{\varepsilon}_T} Z_{\hat{\varepsilon}}(\hat{\varepsilon}) \tag{I.I N}
\]
\[
Z_{\hat{\varepsilon}}^* = \inf_{\hat{\varepsilon}_T} Z_t(\hat{\varepsilon}) \tag{I.I O}
\]
It is important to note that in applying these tests that the null hypothesis, like the usual residual-based cointegration, is no cointegration. Therefore, a rejection of the null hypothesis does not imply that there is a break in the cointegrating vector since the tests will have power against a time-invariant cointegrating relation. And to discern between these kinds of alternatives, we suggest using one of the structural change test (with a null hypothesis of cointegration) proposed in Hansen (1992); Serena and Pernon (2001) as well as Lee and Straicich (2003)

3.0 DATA SOURCES AND COLLECTION

Indeed, the major sources of data for the proposed study will be the published data of the central bank of Nigeria as well as the unpublished data from the various ministries, parastatals and agencies in Nigeria and other supplementary sources of data will include the statistical publications of the Worldbank, United Nations and International Monetary Fund. Here, efforts will be made to collect quarterly data on the variables of the research study (starting 1960Q1 and ending 2010Q4). We shall also search the various intranets, extranets and internet websites accordingly.

4.0 RESEARCH RESULTS AND DISSEMINATION

Our research result is expected to contribute to public policy making for central bank of Nigeria. Federal Ministry of Finance, National Planning Commission, National Assembly committees and Allied Ministries. This is in addition to contributing to existing knowledge and future research in the economics of fluctuation. We intend also to publish our research output accordingly
(especially in REPEC and SSRN network outlets) as well as disseminating to the various professional economists and policy makers in the region. Again, we shall present and discuss our research findings before the various academic and professional economists network meetings.

5.0 STUDY DURATION AND BUDGET

This study is expected to be carried out within a period of eighteen months. In the first six months, we shall be concerned with literature exploration, collection and review. In the subsequent three months, the research study will be modeled accordingly. In another six months, the model data will be collected and analyzed using computing technology. In the last six months, the study will be completed and submitted to the funding agency as appropriate.

Operationally, the expected costs of the research project are as follows:

(A) PERSONNEL COSTS (PRINCIPAL RESEARCHER AND SUPPORT STAFF) = $5,000.00

(B) FIELD WORK COST (LITERATURE SEARCH, DATA GATHERING AND COMPUTING RESOURCES) = $7,000.00

(C) MATERIALS AND SUPPLIES [OFFICE MATERIAL, PRINTING AND COMMUNICATIONS] = $2,500.00

(D) FINAL REPORTS REPRODUCTION (PRODUCTION AND DISSEMINATION) = $2,500.00

(E) MISCELLANEOUS EXPENSES = $1,000.00
6.0 TENTATIVE STUDY OUTLINE

SECTION ONE: INTRODUCTION

(1.1) RESEARCH OF STUDY
(1.2) OBJECTIVES OF STUDY
(1.3) SIGNIFICANCE OF STUDY
(1.4) STUDY COVERAGE
(1.5) ORGANIZATION OF STUDY

SECTION TWO: ECONOMIC STRUCTURES AND PERFORMANCE

(2.1) GLOBAL ECONOMY PERSPECTIVES
(2.2) AFRICAN (REGIONAL) ECONOMY OUTLOOK
(2.3) NIGERIAN ECONOMY REVIEW

SECTION THREE: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

(3.1) THEORETICAL LITERATURE
(3.2) EMPIRICAL LITERATURE
(3.3) THEORETICAL FRAMEWORK

SECTION FOUR: METHODOLOGICAL FRAMEWORK

(4.1) RESEARCH METHOD
(4.2) MODEL SPECIFICATION (VECTOR AUTO-REGRESSION)
(4.3) MODEL ESTIMATION AND SOLUTION (REGIME
SWITCHING/ STRUCTURAL BREAKS)

(4.4) DATA NEEDED AND SOURCES

SECTION FIVE: ANALYSIS OF RESULTS

(5.1) UNIT ROOTS AND COINTEGRATION ANALYSIS
(5.2) FORECAST ERROR VARIANCE DECOMPOSITION
(5.3) IMPULSE RESPONSE ANALYSIS
(5.4) PERSISTENT PROFILE ANALYSIS
(5.5) DYNAMIC MACROFORECASTS
(5.6) SENSITIVITY ANALYSIS
(5.7) POLICY IMPLICATIONS

SECTION SIX: SUMMARY CONCLUSIONS AND RECOMMENDATIONS

(6.1) SUMMARY AND CONCLUSION
(6.2) POLICY RECOMMENDATION
(6.3) STUDY LIMITATIONS
(6.4) FUTURE RESEARCH SUGGESTIONS

NOTES, BIBLIOGRAPHY AND APPENDIXES

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