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A dynamic efficiency wage model with learning by doing

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Abstract

We propose a dynamic efficiency wage model with learning by doing. By taking into account the change in the stock of workers' knowledge, firms set efficiency wages such that the effort–wage elasticity is not in general equal to one.

Keywords: Solow condition; Effort function; Learning

JEL classification: J41

1. Introduction

The efficiency wage literature (Akerlof and Yellen, 1986) stipulates that there is a direct and increasing relation between the wage paid by firms and the effort provided by workers: the greater the wage, the larger the effort. The model is in general *static* and maximizing profit firms set an efficiency wage such that the effort–wage elasticity is equal to unity. This is referred to as the Solow condition (see Solow, 1979).

It has been argued, however, that this condition does not hold in general. In their introduction, Akerlof and Yellen (1986) stipulate that the effort–wage elasticity is less than one. Different reasons have been given in the literature. Schmidt-Sørensen (1990) has introduced fixed employment costs per worker (such as for example employer-provided health insurance) in the profit function; Pisairo (1991) has taken into account specific taxes on labor; Rasmuswamy and Rowthorn (1991) have dropped the assumption of the labor augmenting production function and have used a general one; Lin and Lai (1994) have proposed a dynamic model with (external) turnover costs; and Jellal and Zenou (1999) have focused on the role of uncertain job matching on workers' effort. They have all shown that the Solow condition does not hold and in general the effort–wage elasticity is less than one.

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In this paper, we develop a *dynamic* model of efficiency wage where firms take into account both static and dynamic gains of high wages. We assume that there is some *learning by doing* so that workers accumulate a stock of knowledge which allows them to increase their effort. This is in the same spirit as in Arrow (1962), since learning by doing is here the steady improvement engendered by the experience of producing.

We show that our dynamic efficiency wage can be greater, lower or equal to the static one (for which the elasticity of effort with respect to wage is unitary) depending on a trade off between productivity and static and dynamic gains. We believe that the work relation is a long run one so that firms must induce workers' effort at each period of time.

The remainder of the paper is as follows. The next section describes the basic model. In Section 3, we develop the dynamic analysis and focus on the steady-state solution. Section 4 concludes.

2. The basic model and notations

Consider an infinitely lived representative firm and N homogeneous workers. As it is standard in the efficiency wage literature, the firm takes into account the role of the wage offer as an incentive to increase labor efficiency. Let us denote by $e(w)$ the effort function and w the wage offer, with $e'(w) > 0$ and $e''(w) < 0$. In this context, the production function can be written as:

$$Y_t = F(X(A_t, e(w_t)), N_t) \quad (1)$$

where Y_t is the output at time t , N_t is the number of workers hired at time t , $X(\cdot)$ corresponds to the efficiency function per worker which depends on the knowledge (specific or not) A_t accumulated at time t and on the flow of effort $e(w_t)$. We assume that the production function is such that:¹

$$\begin{aligned} F'_X(\cdot) > 0, \quad F'_N(\cdot) > 0, \quad X'_A(\cdot) > 0, \quad X'_e(\cdot) > 0 \\ F''_X(\cdot) < 0, \quad F''_N(\cdot) < 0, \quad X''_A(\cdot) < 0, \quad X''_e(\cdot) < 0 \\ \frac{\partial^2 F}{\partial w^2} \frac{\partial^2 F}{\partial N^2} > \left(\frac{\partial^2 F}{\partial w \partial N} \right)^2 \end{aligned}$$

so that $F(\cdot)$ is both increasing and concave in w and N .² It is important to observe that this production function is quite different than the one used in general in the efficiency wage model (see Solow, 1979) in which: $Y = F(e(w)N)$. Indeed, in the standard static approach, wages have a direct and instantaneous effect on effort, and thus on the output. Here, we want to emphasize the fact that effort can vary from one period to another so that the control variable w_t used by the firm must induce workers at each

¹In terms of notation, $F'_X = \partial F / \partial X$. We use the same notation for all partial derivatives.

²We have:

$$\frac{\partial F}{\partial w} = \frac{\partial F}{\partial X} \frac{\partial X}{\partial e} e'(w) > 0$$

$$\frac{\partial^2 F}{\partial w^2} = e'(w) \left[\frac{\partial^2 F}{\partial X^2} \left(\frac{\partial X}{\partial e} \right)^2 + \frac{\partial^2 X}{\partial e^2} \frac{\partial F}{\partial X} \right] + \frac{\partial F}{\partial X} \frac{\partial F}{\partial X} e''(w) < 0.$$

period of time. We also want to put forward the fact that there is some *learning by doing* in the efficiency function and thus in the output: the more knowledge is accumulated, the greater is the efficiency and the output. Indeed, knowledge is growing in time, and learning is the product of experience. In order to be more precise, we assume that productivity (or knowledge) per worker is given by the following evolution equation:

$$\dot{A}_t = G(e(w_t)) - \delta A_t \quad (2)$$

where $\delta > 0$ is the rate of depreciation of the stock of knowledge. We assume that:

$$G'_e > 0, \quad G''_e < 0$$

so that $G(\cdot)$ is increasing and concave in w .³ Eq. (2) means that there are two aspects that affect the evolution of the stock of knowledge. On one hand, the wage policy at each period induces workers to be more efficient and thus to learn more. On the other, and independently of the wage policy, the stock of knowledge depreciates over time because workers forget a part of what they have learned. Thus, Eq. (2) implies that change in the stock of knowledge of the worker in the immediate future depends partially on his effort and on the rate of depreciation of the stock of knowledge.

3. The steady-state equilibrium

In this context, the wage rate has an impact on current effort and thus on productivity through the function $F(\cdot)$ as well as on the intertemporal efficiency through $G(\cdot)$. Therefore, a firm would like to set a wage policy that incorporates both static and dynamic gains. For that, the firm solves the following problem:

$$\begin{aligned} \max_{w_t, N_t} \int_0^{+\infty} [F(X(A_t, e(w_t)), N_t) - w_t N_t] e^{-\rho t} dt \\ \text{s.t.} \begin{cases} \dot{A}_t = G(e(w_t)) - \delta A_t \\ A(0) = A_0 \geq 0 \end{cases} \end{aligned} \quad (3)$$

where ρ is the subjective discount rate. We have typically here an optimal control problem where the control variables are w_t and N_t and the state variable is A_t . If we denote by λ_t the costate variable associated with A_t , the Hamiltonian function H can be expressed as:

$$H = F(X(A_t, e(w_t)), N_t) - w_t N_t + \lambda_t (G(e(w_t)) - \delta A_t) \quad (4)$$

³Indeed, we have:

$$\frac{\partial G}{\partial w} = \frac{\partial G}{\partial e} e'(w) > 0$$

$$\frac{\partial^2 G}{\partial w^2} = \frac{\partial^2 G}{\partial e^2} [e'(w)]^2 + \frac{\partial G}{\partial e} e''(w) < 0$$

The first two terms of the RHS of (4) correspond to the immediate profit from the choice of w_t and N_t while the last term is the value of the ‘investments’ that are affected by w_t and N_t . According to the maximum principle of optimal control theory, the control variables w_t and N_t must be chosen so as to maximize H at each date, given the current values of the state and costate variables, A_t and λ_t . If one assumes continuous differentiability of the Hamiltonian, this implies that:

$$\frac{\partial H}{\partial w_t} = \frac{\partial F}{\partial X} \frac{\partial X}{\partial e} e'(w_t) - N_t + \lambda_t G'(\cdot) e'(w_t) = 0 \quad (5)$$

$$\frac{\partial H}{\partial N_t} = \frac{\partial F}{\partial X} - w_t = 0 \quad (6)$$

Furthermore, the optimal control theory (see, e.g., Kamien and Schwartz, 1981) imposes two necessary conditions (that are also sufficient since H is concave in A_t , w_t and N_t) that the costate variable λ must satisfy. The first one is the Euler equation which states that:

$$\dot{\lambda}_t = (\rho + \delta)\lambda_t - \frac{\partial F}{\partial X} \frac{\partial X}{\partial A_t} \quad (7)$$

and the second one consists of the following transversality condition:

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_t A_t = 0 \quad (8)$$

In this paper, we are interested in the *steady-state solution* of this problem. The stationary state to this growth model is when both A_t (the state variable) and the shadow value λ_t (costate variable) are constant, i.e., $\dot{A} = 0$ and $\dot{\lambda} = 0$. It is important to observe that there is a unique (steady-state) solution to this dynamic system which is the saddle point (A^*, λ^*) solution of the system $\dot{A} = 0$ and $\dot{\lambda} = 0$. Indeed, from (5) and (6), we get w and N as a function of A and λ . By inserting these two values in (2) and (7), we obtain a two-dimensional system of differential equation in A and λ . By using the initial condition $A(0) = A_0 \geq 0$ and the transversality condition (8) which determines a terminal condition, there are just enough boundary conditions to determine a unique solution to the dynamic system; it will be the one that converges asymptotically to the stationary state (A^*, λ^*) .⁴

At the steady-state solution, we have (drop the time index):

$$\dot{A} = G(e(w)) - \delta A = 0 \quad (9)$$

$$\dot{\lambda} = (\rho + \delta)\lambda - \frac{\partial F}{\partial X} \frac{\partial X}{\partial A} = 0 \quad (10)$$

Thus, by using (10), we can rewrite (5) as:

$$\frac{\partial F}{\partial X} \frac{\partial X}{\partial e} e'(w) + \left(\frac{1}{\rho + \delta} \right) \frac{\partial F}{\partial X} \frac{\partial X}{\partial A} G'(\cdot) e'(w) = N \quad (11)$$

⁴It can further be shown that this steady-state equilibrium (A^*, λ^*) is *locally stable*. Indeed, by linearizing the two equations $\dot{A} = 0$ and $\dot{\lambda} = 0$ around (A^*, λ^*) , we can write down the Jacobian matrix evaluated at (A^*, λ^*) and show that its determinant is negative.

Eq. (11) has the following intuitive interpretation. The first term of the LHS is the static marginal gain of raising wages and the second term is the intertemporal marginal gain. The RHS of (11), N , is of course the marginal cost of a wage increase. Thus, when the firm has to decide its wage policy, it faces a trade off between its marginal gain (both today and tomorrow) and its marginal cost. The interesting feature here is that the firm must take into account that the flow of effort varies over time. Now, by using (6) and (9), and after some manipulations, (11) becomes:

$$\frac{X}{F} \frac{\partial F}{\partial X} \frac{\partial X}{\partial e} \frac{e(w)}{X} \frac{e'(w)}{e(w)} + \left(\frac{\delta}{\rho + \delta} \right) \frac{X}{F} \frac{\partial F}{\partial X} \frac{A}{X} \frac{\partial X}{\partial A} \frac{e(w)G'(e)}{G(e)} \frac{e'(w)}{e(w)} w = \frac{N}{F} \frac{\partial F}{\partial N} \quad (12)$$

Let us define the following elasticities:

$$\begin{aligned} \eta_A &= \frac{\partial X}{\partial A} \frac{A}{X} > 0, & \eta_e &= \frac{\partial X}{\partial e} \frac{e(w)}{X} > 0 \\ \eta_X &= \frac{\partial F}{\partial X} \frac{X}{F} > 0, & \eta_N &= \frac{\partial F}{\partial N} \frac{N}{F} > 0 \\ \theta_G &= \frac{\partial G}{\partial e} \frac{e(w)}{G(e)} > 0, & \varepsilon_w &= \frac{\partial e}{\partial w} \frac{w}{e(w)} > 0 \end{aligned}$$

where η_A and η_e are the elasticity of efficiency with respect to knowledge and to effort, respectively; η_X and η_N represent the elasticity of the production with respect to efficiency and to employment, respectively; θ_G is the elasticity of knowledge accumulation with respect to effort and finally, ε_w is the (standard) elasticity of effort with respect to wage. By using these definitions, we can rewrite (12) as:

$$\varepsilon_w \left(\eta_e \cdot \eta_X + \frac{\delta}{\delta + \rho} \eta_A \cdot \eta_X \cdot \theta_G \right) = \eta_N \quad (13)$$

Proposition 1. *The long run optimal efficiency wage w^* is given by:*

$$\varepsilon_w = \frac{e'(w^*)w^*}{e(w^*)} = \frac{\eta_N}{\eta_X \left(\eta_e + \theta_G \left(\frac{\delta}{\delta + \rho} \right) \eta_A \right)} \quad (14)$$

We are now in position to discuss whether the Solow condition (i.e., $\varepsilon_w = 1$) is valid.

Proposition 2. *From Proposition 1, we have the following equivalence:*

$$\varepsilon_w \geq 1 \Leftrightarrow \frac{\eta_N}{\eta_X} \geq \eta_e + \theta_G \left(\frac{\delta}{\delta + \rho} \right) \eta_A \quad (15)$$

The interpretation of (15) is quite easy. The LHS of the inequality, $\eta_N/\eta_X = (F'_N)/(F'_X)(N)/(X)$, is the ratio of the elasticity of productivity of output while the RHS is the global gain of incentive (η_e is the static gain emanating from the flow of effort and

$$\theta_G \left(\frac{\delta}{\delta + \rho} \right) \eta_A,$$

the discounted dynamic gain). If the gain of productivity is greater than the global gain of incentive

then firms have no interest to set high wages ($\varepsilon_w > 1$), whereas wages are naturally high ($\varepsilon_w < 1$) if we have the reverse inequality.

Another interesting implication of Proposition 2 is that *the dynamic efficiency wage (14) can be greater, equal or lower than the static one* (in which $\varepsilon_w = 1$). This result contradicts that of Barnerji and Gupta (1997) where the dynamic efficiency wage is always greater than the static one or stated differently where $\varepsilon_w < 1$ (see their Proposition 1). This is due to the fact that they focus on a different issue, namely the problem of health accumulation in LDCs. In their paper, the efficiency function is $h(t) = h[w(t), e(t), \alpha_1]$, where α_1 is a parameter and effort $e(\cdot)$ is identified with health, and the evolution equation is $\dot{e} = g[w(t), \alpha_2] - \delta e$ where α_2 is another parameter. Therefore, in Barnerji and Gupta (1997) a dynamic high wage is needed since health accumulation depends drastically on it.

We have, therefore, obtained a general result that formalizes in a dynamic context the statement of Stiglitz (1987). He indeed argues that many of the results in the efficiency wage theory depend crucially on the existence of some region(s) where an increase in the wage leads to more than proportionate increases in the work effort. We show that there are also some regions where a wage raise generates an effort lower or equal to the initial wage increase (a wage for which the elasticity of effort with respect to wage is greater than 1 is obviously lower than a wage characterized by an elasticity less or equal to 1).

It is now interesting to discuss the following special cases. If $\eta_A = 0$, which means that production requires simple tasks of routine without requiring qualification and accumulation of knowledge, and $\eta_e = \eta_N = \eta_X = 1$, we are back to the Solow case with $\varepsilon_w = 1$. If $\eta_A = 0$ and $\eta_X = 1$, we obtain exactly the result of Rasmuswamy and Rowthorn (1991) in which:

$$\varepsilon_w \geq 1 \Leftrightarrow \frac{\eta_N}{\eta_e} \geq 1$$

This shows that our analysis is quite general so that it can encompass other papers as special cases.

4. Conclusion

The aim of this paper is to highlight the fact that the work relation is a long run one so that firms must induce workers' effort at each period of time. In a dynamic framework with a learning by doing process, we generalize the Solow condition (static efficiency wage) in which the wage elasticity is equal to one and show that our dynamic efficiency wage can be greater, equal or lower than the static one.

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