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Heterogeneity in stock prices:

A STAR model with multivariate transition function

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Abstract

This paper applies a heterogeneous agent asset pricing model, featuring fundamentalists and chartists, to the price-dividend and price-earnings ratios of the S&P500 index. Agents update their beliefs according to macroeconomic information, as an alternative to evolutionary dynamics. For estimation, a STAR model is introduced, with a transition function depending on multiple transition variables. A procedure based on linearity testing is proposed to select the appropriate linear combination of transition variables. The results show that during periods of favorable economic conditions the fraction of chartists increases, causing stock prices to decouple from fundamentals.

Keywords: Asset pricing, Heterogeneous beliefs, Smooth-transition autoregression

JEL classification: C22, E44, G12

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1 Introduction

Asset pricing models based on the efficient market hypothesis (EMH) have a difficult time explaining the observed dynamics of financial markets. According to these models, asset prices reflect a rational forecast by the market of future cash flows (dividends) generated by the asset and are therefore expected to be smoother than the actual cash flows. However, financial asset prices such as stock prices are historically more volatile than real economic activity including corporate earnings and dividends. Several studies (e.g. LeRoy and Porter, 1981; Shiller, 1981; West, 1988; Campbell and Shiller, 1988, 2001) discuss this excess volatility in financial markets and conclude that stock prices can not be explained by expected dividends alone.

Heterogeneous agent models provide an alternative to the EMH. In these models, the single representative rational agent is replaced by boundedly rational agents who are heterogeneous in beliefs, are not necessarily forecasting future dividends and may switch between trading strategies over time. Hommes (2006) and Manzan (2009) provide surveys of such models in economics and finance. The model in this paper is based on the work by Brock and Hommes (1997, 1998), who introduce a simple analytically tractable heterogeneous agent model with two types of agents: Fundamentalists and chartists. Fundamentalists believe, in accordance with the EMH, that asset prices will adjust toward their fundamental value. Chartists (or trend-followers) speculate on the persistence of deviations from the fundamental value. I use data on the S&P500 index to estimate a heterogeneous agent model in which macroeconomic and financial variables simultaneously govern the agents' switching between strategies. It turns out that during periods of high economic growth, agents switch from fundamentalism to chartism, i.e. loose sight of fundamentals and become more interested in following recent trends in asset prices, which causes asset price bubbles to inflate.

Heterogeneous agent models are typically estimated empirically using regime-switching regression models, with the distinct regimes representing the expected asset pricing processes according to each type of agent. In particular smooth-transition regime-switching models such as the smooth-transition autoregressive (STAR) models (Teräsvirta, 1994) are suitable, as the modeled process is a time-varying weighted average of the distinct regimes. The time-varying weights of the regimes are then interpretable as the fractions of agents belonging to each type.

Recent studies have estimated asset pricing models featuring chartists and fundamentalists for several types of asset prices including exchange rates (Manzan and Westerhoff, 2007; De Jong et al., 2010), option prices (Frijns et al., 2010), oil prices (Reitz and Slopek, 2009; Ter Ellen and Zwinkels, 2010) and other commodity prices (Reitz and Westerhoff, 2007). Boswijk et al. (2007) apply the model by Brock and Hommes (1998) to price-dividend (PD) and price-earnings (PE) ratios of the US stock market, finding that the unprecedented stock valuations observed during the 1990s are the result of a prolonged dominant position of the chartist type over the fundamentalist type.

Agents are in general assumed to switch between strategies based on evolutionary considerations. Boswijk et al. (2007) follow Brock and Hommes (1998) by letting the agents choose their regime based on the realized profits of each type. Alternatively, the switching may be based on relative forecast errors (Ter Ellen and Zwinkels, 2010), or on the distance between the actual and fundamental price (Manzan and Westerhoff, 2007). In this paper, the agents' choice of strategy is not evolutionary, but varies instead over the business cycle. In practice, this means I estimate a STAR model, in which the transition function depends on a linear combination of exogenous or predetermined macroeconomic variables. This framework allows for identifying the macroeconomic conditions under which chartism or fundamentalism dominates the market.

The result that chartism is associated with economic expansion is novel but can be related to existing results in the literature on the effects of the real economy on financial markets. For example, Fama and French (1989), Campbell (2003) and Cooper and Priestley (2009), amongst others, study the variation of risk aversion over the business cycle, and find more risk appetite on financial markets during economic upturns. The interpretation of countercyclical risk premiums is different from this paper. Instead of a rational representative agent becoming less risk averse, I assume that under favorable economic conditions an increasing fraction of agents chooses a more speculative trading strategy by becoming chartist. These findings are, however, not necessarily inconsistent, as chartists are sometimes described as being less risk averse than fundamentalists (Chiarella and He, 2002; Chiarella et al., 2009). Using a cross-section of US stock returns, Chordia and Shivakumar (2002) find that momentum strategies are profitable only during the most expansionary periods of the business cycle. Without making any agent-based interpretations, Spierdijk et al. (2012) use a panel of stock market indices from 18 OECD countries to find that the speed of mean reversion towards the fundamental value accelerates during periods of high economic uncertainty. This result confirms my findings since a high speed of mean reversion implies a high fraction of fundamentalists.

The STAR model is typically univariate, in which the transition between regimes depends on a lag of the dependent variable as in Teräsvirta (1994). Alternatively, the transition function may depend on a single exogenous or predetermined transition variable as in Reitz and Westerhoff (2003), Reitz and Taylor (2008) and Reitz et al. (2011), who study the nonlinear effects of purchasing power parity and central bank policies on exchange rates. In contrast to these studies, I allow for a multivariate transition function depending on multiple exogenous or predetermined transition variables with unknown weights, in order to estimate the nonlinear effects of multiple economic variables simultaneously. Estimating this multivariate STAR model raises two difficulties compared to the univariate STAR: Selection of the transition variables to include, and estimation of their weights. Medeiros and Veiga (2005) and Becker and Osborn (2012) consider estimating STAR models with unknown weighted sums of transition variables, but both are limited to univariate models in which the transition functions depend on linear combinations of different lags of the dependent variable. I propose to apply the linearity test by Luukkonen et al. (1988) to select the transition variables from a large set of information and simultaneously estimate their respective weights in the transition function. The resulting STAR model with multivariate transition function provides a better fit to the PD and PE ratios than linear models and STAR models with a single transition variable do, while the estimates support the idea of a smooth transition between chartism and fundamentalism.

The next section presents the heterogeneous agent model and the STAR specification in more detail. Data descriptions and linearity tests are given in section 3 while section 4 presents estimation results, interpretation and diagnostic checks. Section 5 concludes.

2 The model

In a simple linear present value asset pricing model, consistent with the efficient market hypothesis, the price of a financial asset (P_t) equals the discounted sum of the expected asset price next period and any expected cash flows (dividends, D_{t+1}) paid out on the asset in the coming period (Gordon, 1959). Iterating forward, the price can be expressed as a infinite sum of discounted expected dividends:

$$P_t = \frac{1}{1+r} E_t \left[P_{t+1} + D_{t+1} \right] = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} E_t \left[D_{t+i} \right], \tag{1}$$

in which the constant discount factor is given by $(1+r)^{-1}$. By introducing the dividend growth rate g_t, such that $D_t = (1+g_t)D_{t-1}$, this equation can be rewritten as:

$$\frac{P_t}{D_t} = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} E_t \left[\prod_{j=1}^{i} \left(1+g_{t+j}\right) \right]. \tag{2}$$

According to equation (2), any movements of the PD ratio $\left(\frac{P_t}{D_t}\right)$ can be caused only by time-variation of the discount factor or by changed expectations on future dividend growth rates. Under the assumption of a constant discount factor, an increase in the PD ratio should predict an increase in future dividends and vice versa. However, Campbell and Shiller (2001) argue that neither the PD nor the PE ratio are good predictors for future dividend growth rates. Instead, both valuation ratios work well as a predictor for future stock returns. High valuation ratios predict decreasing stock prices, while low ratios predict increasing prices (Campbell and Shiller, 2001).

The assumption of a constant discount factor is very restrictive. Instead, modern asset pricing models often incorporate a stochastic discount factor (SDF), representing the time-varying risk aversion of a representative agent (Cochrane, 2011). Nevertheless, Campbell and Shiller (1988) show that the finding of excess volatility is robust to several time-varying discount factors, including discount factors based on consumption, output, interest rates and return volatility.

Brock and Hommes (1998) provide an alternative to the present-value relationship (1) and the SDF framework, by allowing asset prices to depend on the expectations of H different types of boundedly rational agents:

$$P_t = \frac{1}{1+r} \sum_{h=1}^{H} G_{h,t} E_t^h \left[P_{t+1} + D_{t+1}\right], \tag{3}$$

with $E_t^h[\cdot]$ representing the beliefs of agent type h. The fraction of agents following trading strategy h at time t is denoted by $G_{h,t}$. For analytical tractability, Brock and Hommes (1998) assume a constant discount factor. This model nests the standard present-value model; when all types have rational beliefs (i.e. $E_t^h[\cdot] = E_t[\cdot]\ \forall h$), model (3) reduces to (1). Boswijk et al. (2007) show that if dividends are specified as a geometric random walk process, model (3) can be reformulated as follows:

$$y_t = \frac{1}{1+r}\sum_{h=1}^{H} G_{h,t} E_t^h[y_{t+1}], \tag{4}$$

in which y_t is defined as the PD ratio in deviation from its fundamental value. The results of Campbell and Shiller (2001) suggest to estimate mispricings in the market as the PD ratio in deviation from its long-run average:

$$y_t = \frac{P_t}{D_t} - \mu, \tag{5}$$

in which $\mu = \frac{1}{T}\sum_{t=1}^{T}\frac{P_t}{D_t}$ represents an estimate of the fundamental value of the PD ratio. y_t gives the size of the bubble in the market, which can be negative as well as positive. The asset is over-valued if $y_t > 0$ and under-valued if $y_t < 0$. The price of the asset P_t can be decomposed in an estimated fundamental value μD_t and bubble $y_t D_t$:

$$P_t = \mu D_t + y_t D_t \tag{6}$$

A widely cited example of model (3) distinguishes two types of agents, fundamentalists and chartists, who are both aware of the fundamental value, but disagree about the persistence of the deviation from this fundamental value. The fundamentalists' strategy is to buy stocks when the market is undervalued and sell when the market is overvalued. They believe in mean reversion; mispricings in the market should disappear over time: $E_t^F[y_{t+1}] = \eta_F y_{t-1}$, with $\eta_F < 1+r$. Chartists (or trend-followers), on the other hand, speculate that the stock market will continue to diverge from its fundamental valuation: $E_t^C[y_{t+1}] = \eta_C y_{t-1}$, with $\eta_C \geqslant 1+r$.

By substituting these two beliefs into (4) and allowing the fractions of both agent types to vary over time, the asset pricing process can be described by a smooth-transition autoregressive (STAR) process:

$$y_t = \alpha_F y_{t-1}(1-G_t) + \alpha_C y_{t-1} G_t + \varepsilon_t, \tag{7}$$

with $\alpha_F = \eta_F/(1+r) < 1$ and $\alpha_C = \eta_C/(1+r) \geqslant 1$. The transition function G_t defines the fraction of chartist in the market. The fraction of fundamentalists is in this two-type model is given by $1-G_t$. Although both types use a linear prediction rule, the time-varying fractions of each agent type makes the process nonlinear and, under certain parametrizations, chaotic (Brock and Hommes, 1998).

Boswijk et al. (2007) estimate a variant of this model for both the PD and PE ratio of the S&P 500 index, in deviation from their mean, for the period 1871 to 2003. They follow Brock and Hommes (1998) by letting agents update their beliefs based on the realized profits of each type in the previous period. Under these evolutionary dynamics, agents switch from the less profitable strategy to the more profitable strategy. The transition function therefore becomes a logistic function depending on lagged values of the dependent variable:

$$G_t = \left(1 + exp[-\gamma(\eta_C - \eta_F) y_{t-3}(y_{t-1} - (1+r) y_{t-2})]\right)^{-1}, \tag{8}$$

in which γ represents the intensity of choice of the agents. If $\gamma \to \infty$ all agents choose the strategy that was most profitable in the previous period. On the other hand, if $\gamma = 0$, the fraction of both types is exactly 50% in all periods, independent of the realized profits.

Instead of these evolutionary dynamics, I let the agents base their choice of strategy on macroeconomic and financial information, which can be interpreted as an extension of the agents' information set. Of interest is to find which economic conditions can be associated with each type of agent.

The transition function G_t is a logistic function, as in the logistic STAR (LSTAR) model by Teräsvirta (1994):

$$G_t = (1 + exp[-\gamma(x_t - c)])^{-1}, \tag{9}$$

in which the transition variable x_t is usually a lagged value or lagged difference of the dependent variable, but can be any predetermined or exogenous variable. The transition function may also depend on a linear combination of variables:

$$G_t = (1 + exp[-\gamma(X_t\beta - c)])^{-1}, \tag{10}$$

with $X_t = [x_{1,t} \ldots x_{p,t}]$ and p is the number of included transition variables. For this model; γ, c and β can not be all identified. This problem can be solved by placing a restriction on β. In this paper, the elements of β are restricted to sum to one, so that $X_t\beta$ is a weighted sum of multiple transition variables.

3 Data and linearity tests

Figure 1 shows quarterly data of the PD (left) and PE (right) ratios of the S&P500 index since 1881[1]. These valuation ratios show the level of the S&P500 index relative to the cash flows that the indexed stocks are generating. In particular the path of the PE ratio (right) seems stable or mean-reverting in the long run. Even after reaching record levels around the start of this century, the PE ratio recently dropped again below its average value during the credit crisis in 2009. This latest peak is comparable in size to earlier episodes, most notably the 1920s. For the PD ratio, this pattern is less clear. Due to relatively low dividend payouts by listed firms in recent decades (Fama and French, 2001), the PD ratio climbs during the 1990s

[1]Source: Robert Shiller, http://www.irrationalexuberance.com/index.htm

to much higher levels than during any earlier peaks in the market. Although the model in section 2 is expressed in terms of the PD ratio, I estimate the STAR model with both these valuation ratios as the dependent variable. Earnings are smoothed over a period of ten years, creating the so-called cyclically adjusted PE ratio. Both valuation ratios are taken in deviation from their average value.

I follow the specification, estimation and evaluation cycle for STAR models proposed by Teräsvirta (1994). The specification stage includes the selection of the appropriate lag structure and justification of STAR modeling by testing for linearity. To find the optimal lag length, I estimate linear AR(q) models including up to six lags for both the PD and PE ratio. Table 1 shows the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for all specifications. For both valuation ratios, the AR(1) model is selected as the appropriate specification. The STAR model is therefore estimated with an autoregressive structure of one lag, as in equation (7). At the end of this paper, I verify the sufficiency of this lag structure by submitting the residuals from the final STAR model to a test of serial independence.

The next step is to test for linearity and simultaneously select the transition variables. I consider a set of financial and macroeconomic indicators as potential transition variables[2]. The first set of indicators is related to the performance of the stock market and includes both dependent variables (PD and PE), monthly returns (RET) and the volatility of the S&P500 index (VOL), defined as the variance of daily returns in each quarter. For the other indicators I follow the choice of variables by Campbell (2003), who uses business cycle indicators, inflation and interest rates to study the cyclical properties of risk premiums. The business cycle indicators considered by Campbell (2003) are real GDP (GDP) and consumption (CON). I

[2] Source: FRED® (Federal Reserve Economic Data)

supplement these indicators with the output gap (*OPG*) and industrial production (*IND*). The inflation rates are the consumer price index (*CPI*) and GDP deflator (*DEF*). The interest rates used by Campbell (2003) are the short-term yield on 3-month US treasury bills (*STY*) and the long-term yield on 10-year US treasury notes (*LTY*). I add to this the 10-year yield on Baa-rated corporate bonds (*CBY*) and construct the term spread ($TSP = LTY - STY$) and the yield spread of corporate bonds over sovereign bonds ($YSP = CBY - LTY$). While the business cycle indicators measure the current state of the economy, these interest rates and spreads contain expectations on future macroeconomic conditions (Bernanke, 1990; Estrella and Mishkin, 1998). *GDP*, *CON*, *IND*, *CPI* and *DEF* are measured in quarter-on-quarter growth rates. *OPG* is a percentage of GDP. For the interest rates and the output gap I look at both levels and first differences (denoted by $\triangle$). These data are not available for the full period of S&P500 data, so the model is estimated using 208 obervations (1960Q1-2011Q4). All variables are standardized (demeaned and divided by their standard deviation), to accommodate numerical estimation of the nonlinear model. For all explanatory variables, I consider both first and second lags, which are therefore predetermined with respect to the dependent variable.

To determine which of these variables are valid transition variables in the STAR model, they are submitted to a linearity test based on a Taylor approximation of the STAR model following Luukkonen et al. (1988). First, I consider the univariate transition function (9). A third-order Taylor approximation of (7) with univariate transition function (9) around $\gamma = 0$ gives:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sum_{i=1}^{3} \phi_{1+i} y_{t-1} x_t^i + e_t. \tag{11}$$

Linearity can now be tested by estimating this Taylor approximation by OLS and testing the null hypothesis $H_o : \phi_2 = \phi_3 = \phi_4 = 0$. Rejection of linearity implies that x_t is a valid transition variable.

Results of the linearity tests are given in Table 2, which shows the test statistics and corresponding P-values. The test statistic is asymptotically $F(n, T-k-n-1)$ distributed under the null, with $T = 208$ (observations), $k = 2$ (unrestricted parameters) and $n = 3$ (restricted parameters). An asymptotically equivalent χ^2-test may be applied here as well, but the F-test has preferable properties in small samples (Teräsvirta et al., 2010). The results in Table 2 show that several variables are valid transition variables.

I consider the LSTAR only, since a logistic transition function follows directly from the logit switching rule in the model by Brock and Hommes (1998). Alternatively, the transition function could be an exponential function as in the ESTAR model. To verify that the LSTAR is the correct model, I apply a sequence of three F-tests based on (11) proposed by Teräsvirta (1994) to choose between both transition functions: $H_{o1} : \phi_4 = 0$, $H_{o2} : \phi_3 = 0 \mid \phi_4 = 0$ and $H_{o4} : \phi_2 = 0 \mid \phi_3 = \phi_4 = 0$. If H_{02} yields a stronger rejection than H_{01} and H_{03}, the ESTAR model is the best choice. Otherwise, the LSTAR model is preferred. Table 2 shows that with most transition variables, the LSTAR (marked by L) is the preferred specification. Teräsvirta (1994) further recommends to estimate the STAR model with the transition variable for which rejection of linearity is the strongest. However, the fact that linearity is rejected for different transition variables suggests to incorporate more than one variable in the transition function.

Allowing for a multivariate transition function, I now propose a similar procedure based on linearity tests to select the appropriate transition variables $X = [x_1 \ldots x_p]$. From substituting $x_t = X_t\beta$ into (11) it becomes clear that this Taylor approximation can not be estimated by OLS if the weights β are unknown. To circumvent this problem, I first estimate β based on a first-order Taylor approximation[3] of (7), with a multivariate transition function (10) around $\gamma = 0$:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-1}(X_t\beta) + e_t, \tag{12}$$

[3]A linearity test based on a first-order Taylor approximation does not allow to choose between a LSTAR and ESTAR, but does provide power against STAR nonlinearity in general, except when the regime switching is in the intercept rather than the autoregressive parameters (Luukkonen et al., 1988).

or:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sum_{i=1}^{p} \theta_i y_{t-1} x_{i,t-1} + e_t, \tag{13}$$

such that $\theta_i = \phi_2 \beta_i$. This Taylor approximation can be estimated by OLS for any set of explanatory variables, after which the OLS estimates $\widehat{\theta}$ and the restriction $\sum_{i=1}^{p} \beta_i = 1$ can be used to derive estimates of β:

$$\begin{aligned}
\theta_i &= \phi_2 \beta_i \\
\sum_{i=1}^{p} \theta_i &= \phi_2 \sum_{i=1}^{p} \beta_i = \phi_2 \\
\widetilde{\beta}_j &= \left(\sum_{i=1}^{p} \widehat{\theta}_i \right)^{-1} \widehat{\theta}_j.
\end{aligned} \tag{14}$$

Selecting the optimal set of transition variables consists of the following steps. First, I estimate (13) for each possible set of one to four transition variables, which never includes more than one variable out of each of the following four groups: (*i*) Stock market indicators, (*ii*) business cycle indicators, (*iii*) inflation rates and (*iv*) interest rates and spreads. This approach limits the number of sets under consideration and, because several variables within each group are highly correlated, it avoids multicollinarity within the transition function. For each set, I then compute $\widetilde{\beta}$, following (14) and perform a t-test on each element of $\widetilde{\beta}$. In trying to avoid selecting an overfitted model, I proceed only with those sets of variables for which all elements of $\widetilde{\beta}$ are significant at the 10% level. For these selected sets, I substitute $x_t = X_t \widetilde{\beta}$ into the third-order Taylor approximation (11) in order to test the null hypothesis $H_o : \phi_2 = \phi_3 = \phi_4 = 0$. Finally, I choose the set of variables yielding the strongest rejection of linearity as the optimal set of transition variables. Table 3 reports the final results of this test procedure. With the selected linear combinations of transition variables, the rejection of linearity is stronger than with any of the single transition variables in Table 2. In both cases the LSTAR model is preferred over the ESTAR.

4 Results

The parameter estimates for the STAR model are presented in Table 4. The models are estimated by nonlinear least squares, preceded by a $(p+1)$-dimensional grid search for γ, c and the $(p-1)$ free elements of β to find starting values. The selection criterion in this grid search is the sum of squares of the STAR model, which can be estimated by OLS when γ, c and β are kept fixed. The estimated autoregressive parameters of each regime are denoted by α_1 and α_2, rather than α_C and α_F, because the latter notation implies restrictions on these parameters that I do not impose during estimation.

The top rows of Table 4 show the parameter estimates for the STAR models (7) with univariate transition function (9), using the transition variable for which rejection of linearity is the strongest, which is the first lag of industrial production (IND_{t-1}) for both valuation ratios. Because there is only one transition variable, there are no weights β to estimate. Although both estimated models include a mean-reverting and a trend-following regime, the results are not entirely consistent with the spirit of the heterogeneous agent model by Brock and Hommes (1998), because the intensity of choice parameter γ is so high that the fraction of each type is either zero or one. Contrary to the idea of heterogeneous beliefs these results suggest that the entire population of agents makes the same switch simultaneously.

The bottom rows of Table 4 show the STAR models (7) with multivariate transition function (10). With multiple transition variables, the estimates of γ are lower, in support of a smooth transition between the regimes. In both estimated models, two distinct regimes are identified. Each specification has one autoregressive parameter significantly smaller than one (representing the fundamentalist type), while the other autoregressive parameter is significantly greater than one (representing the chartist type). Interpreting β reveals that chartists are more dominant during periods of economic expansion, while the fraction of fundamentalists increases during economic downturns.

With $y_t = PD_t$, the effect of volatility (VOL_{t-1}) does not seem significant. I keep this transition variable in the model, because excluding it does not improve the fit of the model. Industrial production growth (IND_{t-1}) has a positive coefficient, implying in this case it supports the chartist type. An increase in industrial production causes an increase in the fraction of chartists in the economy. Also the short-term yield on 3-month treasury bills (STY_{t-2}) has a positive coefficient. A high yield on low-risk assets like treasury bills implies low levels of risk aversion, and in this model a high fraction of chartists. With $y_t = PE_t$, the model does not include the exact same set of transition variables, but the results tell a similar story: Chartism is the dominant strategy during expansive periods, signalled by high industrial production growth (IND_{t-1}) and inflation (DEF_{t-2}).

Several measures are applied to evaluate the fit of the STAR model, compared to the fit of an AR(1) model and the linear regression model:

$$y_t = \omega_1 y_{t-1} + X_t \omega_2 + e_t, \tag{15}$$

which includes the same explanatory variables as the STAR model. Table 5 presents, in addition to the R^2, AIC and BIC of all models, the results of a pseudo out-of-sample forecasting exercise. Using an expanding window approach, I estimate all models using a subset of the data (1960Q2-S) and use the estimated models to compute forecasts for period $S+1$. This process is repeated 48 times, creating pseudo out-of-sample forecasts for the period (2000Q1-2011Q4), from which Mean Absolute Errors (MAE) and Root Mean Squared Errors (RMSE) are computed. Due to the high persistency of the valuation ratios, the R^2 of all models including the univariate AR(1) are relatively high. The improved fit of the STAR model over the linear alternatives is small but seems robust to several measures. According to the AIC, BIC and out-of-sample results, the STAR model with multivariate transition function outperforms its linear alternatives as well as the STAR model with a univariate transition function. The result that the STAR model (7)-(10) has a better fit than the linear model (15) implies that the variables in X_t work better in explaining the switching process between mean-reverting and trend-following regimes than they do in explaining the level of PD_t and PE_t, which supports the notion of chartism and fundamentalism. The macroeconomic information is not simply correlated with stock prices but has an effect on the nonlinear adjustment towards the fundamental value. Table 5 also shows the test statistics and bootstrap P-values for the linearity test by Hansen (1996, 1997). Like the linearity tests in section 2, these tests show strong rejections of linearity, with P-values lower than 1%.

An intuitive interpretation of the results is found by giving (7) the alternative formulation of an AR(1) process with a time-varying parameter:

$$y_t = \delta_t y_{t-1} + \varepsilon_t, \tag{16}$$

in which $\delta_t = \alpha_1(1 - G_t) + \alpha_2 G_t$, which can be interpreted as an indicator of market sentiment. When $\delta_t \geqslant 1$ the valuation ratio is diverging from its mean, implying that the chartist regime is dominant, while the valuation ratio is mean-reverting when $\delta_t < 1$. Figure 2 offers a graphical evaluation of both estimated models by showing plots of δ_t over time and scatter plots of G_t against $X'_{t-1}\beta$, evaluated at the estimates of the multivariate STAR model. Because of the relatively low value of the intensity of choice parameter γ, both scatter plots on the right side of Figure 2 clearly show a logistic curve. Most of the time, both chartists and fundamentalists are represented in the economy, with δ_t fluctuating around one. In 2001 and again in 2008 the market turned almost completely to the fundamentalist type for a prolonged period, causing the bubble built up in the 1990s to deflate.

Finally, the estimated multivariate models in Table 4 are evaluated with diagnostic checks. Table 6 presents results on tests of serial independence, parameter constancy and no remaining nonlinearity. Eitrheim and Teräsvirta (1996) provide technical details on all three tests.

The test of serial independence test the null hypothesis of no q^{th} order autocorrelation in the residuals. For a q^{th} order test, the resulting test statistic is asymptotically $F(q, T-q-4)$ distributed under the null, with $T = 208$ (sample size). I execute this test for first- up to fourth-order autocorrelation. For both models, the test results give no reason the reject the null hypothesis, confirming the sufficiency of an autoregressive structure of only one lag.

Under the null hypothesis of no time-variation of the parameters in (7) and (10), the parameter constancy test statistic is asymptotically $F(6, T-10)$ distributed. Also this test gives no reason to reject the specification.

The test of no remaining nonlinearity checks whether any variable has a significant non-linear effect on the residuals. This could be the case when a transition variable is omitted, or when these variables have an effect on y_t through some other nonlinear channel. The test statistic is asymptotically $F(3, T-6)$ distributed under the null. This test is repeated for the first lags of all potential transition variables considered in this paper. For the majority of the variables, the null hypothesis of no remaining non-linearity can not be rejected at the 10% level. There are some exceptions, in particular lagged returns (RET_{t-1}), but including these variables in the transition function does not improve the fit of the model. Given that the test is repeated for many variables, it is possible that the rejections are Type I errors. Overall, the results of these diagnostic checks are positive and provide support to the specification of the model.

5 Conclusion

In this paper, I identify two types of agents: fundamentalists and chartists. The presence of chartists, who are predicting trends rather than fundamentals, explains the existence of bubbles in asset prices. To estimate the effects of macroeconomic conditions on the behavior of agents, I propose a STAR model with a multivariate transition function. This STAR model outperforms STAR models with a single transition variable as well as linear alternatives in terms of goodness-of-fit.

Agents are more willing to believe in the persistence of bubbles during times of positive macroeconomic news. Chartists gain dominance during periods of favorable economic conditions, mainly measured by industrial production. The fraction of fundamentalists increases during economic downturns, which encourage agents to re-appreciate fundamentals.

Further research in this area may include an investigation of international stock markets, in order to find whether the switching between chartism and fundamentalism is based on the same factors and occurs simultaneously across countries. In addition, the framework presented in this paper is suitable to find the macroeconomic conditions under which any asset price deviates from some measure of fundamental value. Other possible applications include the deviation of exchange rates from purchasing power parity (see e.g. Rogoff, 1996), or the term structure of interest rates in deviation from the expectations hypothesis (see e.g. Mankiw and Miron, 1986).

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Tables and charts

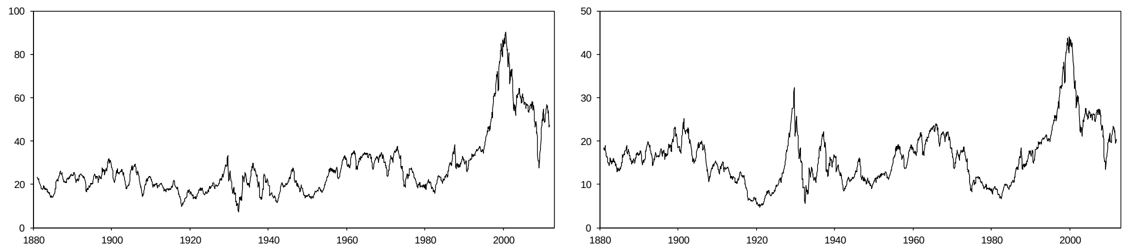
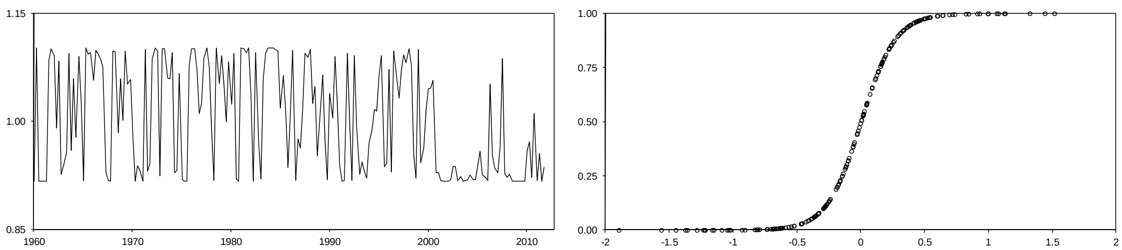


Figure 1: S&P 500 index 1881Q1-2011Q4: price-dividend ratio (left) and price-earnings ratio (right).

$$y_t = PD_t, X_t = (VOL_{t-1}, IND_{t-1}, STY_{t-2})$$



$$y_t = PE_t, X_t = (IND_{t-1}, DEF_{t-2})$$

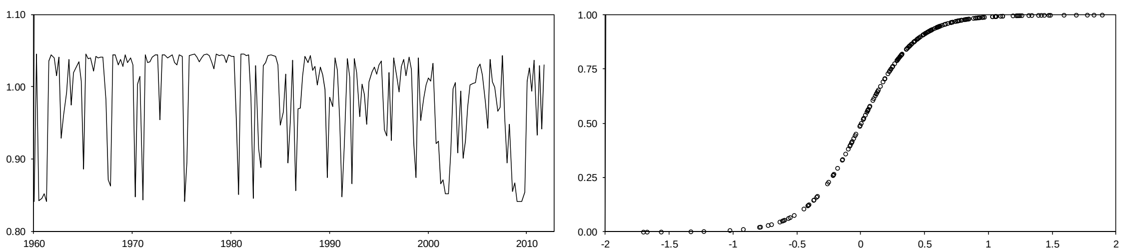


Figure 2 :Regression results: Plot (left) of $\delta_t = \alpha_1(1-G_t) + \alpha_2 G_t$ over time and scatterplot (right) of G_t against $X_t\beta$, evaluated at parameter estimates in Table 4.

TABLE 1. AR(q): Selection criteria

y_t	q:	1	2	3	4	5	6
PD_t	AIC	-699.5	-696.7	-691.2	-686.7	-680.5	-676.7
	BIC	-692.8	-686.7	-677.8	-670.1	-660.6	-653.5
PE_t	AIC	-681.8	-678.1	-672.4	-669.7	-664.9	-662.1
	BIC	-675.2	-668.1	-659.1	-653.1	-645.0	-638.9

Notes: Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) for AR(q) models. Sample size (for $y_t = PD_t$ and $y_t = PE_t$) is 208 observations: 1960Q1-2011Q4.

TABLE 2. Linearity tests: Univariate transition function

	$y_t = PD_t$						$y_t = PE_t$					
lag	$t-1$			$t-2$			$t-1$			$t-2$		
x	F	P	L/E	F	P	L/E	F	P	L/E	F	P	L/E
PD	0.667	0.573	L	0.974	0.406	L	3.359	0.020	E	2.811	0.041	E
PE	0.236	0.871	E	0.282	0.838	L	0.512	0.674	L	0.475	0.700	L
RET	2.407	0.068	E	0.600	0.616	L	2.741	0.044	E	0.266	0.850	E
VOL	1.621	0.186	L	0.818	0.486	L	0.496	0.686	L	0.541	0.655	L
GDP	4.742	0.003	L	0.868	0.459	L	3.495	0.017	L	0.574	0.633	L
CON	2.596	0.054	L	0.873	0.456	L	0.849	0.469	L	0.484	0.694	E
OPG	1.555	0.202	L	0.337	0.799	L	0.483	0.694	E	1.820	0.145	E
$\triangle OPG$	3.847	0.010	L	0.760	0.518	L	3.299	0.021	L	0.614	0.607	L
IND	5.073	0.002	L	2.845	0.039	L	4.358	0.005	L	2.249	0.084	L
CPI	1.119	0.342	L	1.084	0.357	L	1.261	0.289	L	0.732	0.534	L
DEF	2.639	0.051	L	1.201	0.311	L	4.102	0.007	L	1.472	0.223	L
STY	1.139	0.334	L	1.247	0.294	L	1.205	0.309	L	1.339	0.263	L
$\triangle STY$	0.254	0.858	L	1.475	0.223	L	0.162	0.922	L	0.577	0.631	L
LTY	0.238	0.870	L	0.577	0.631	E	0.283	0.838	L	0.833	0.477	L
$\triangle LTY$	0.496	0.686	L	0.565	0.639	L	0.335	0.800	L	0.519	0.670	L
TSP	2.591	0.054	L	2.724	0.045	L	1.476	0.222	E	1.498	0.216	L
CBY	0.128	0.943	E	0.163	0.921	E	0.056	0.982	L	0.071	0.975	E
$\triangle CBY$	0.391	0.760	L	0.076	0.973	L	0.109	0.955	L	0.354	0.787	L
YSP	1.414	0.240	L	1.971	0.119	L	1.375	0.252	L	2.216	0.087	L

Notes: F-test statistics and corresponding P-values for $H_o : \phi_2 = \phi_3 = \phi_4 = 0$ in equation (11), using both first and second lags of several transition variables. L/E refers to the LSTAR or ESTAR model selected by the procedure of Teräsvirta (1994).

TABLE 3. Linearity tests: Multivariate transition function

y_t	X_t	β_1	β_2	β_3	F	P	L/E
PD_t	$(VOL_{t-1}, IND_{t-1}, STY_{t-2})$	0.20	0.54	0.26	7.98	4.7×10^{-5}	L
PE_t	(IND_{t-1}, DEF_{t-2})	0.67	0.33	.	7.79	6.0×10^{-5}	L

Notes: Optimal set of transition variables X_t in terms of the highest F-test statistics and lowest P-values for $H_o : \phi_2 = \phi_3 = \phi_4 = 0$ in equation (11), with $x_t = X_t\beta$. L/E refers to the LSTAR or ESTAR model selected by the procedure of Teräsvirta (1994). The elements of β are estimated based on equations (13)-(14)

TABLE 4. Parameter estimates for STAR model

y_t	X_t	α_1	α_2	γ	c	β_1	β_2	β_3
PD_t	IND_{t-1}	0.948	1.098	80.44	0.375	.	.	.
		(0.010)	(0.021)	(52.79)	(0.012)	.	.	.
PE_t	IND_{t-1}	0.898	1.019	1244	-0.371	.	.	.
		(0.016)	(0.011)	(1247)	(2.148)	.	.	.
PD_t	$(VOL_{t-1}, IND_{t-1}, STY_{t-2})$	0.917	1.101	7.452	0.123	-0.012	0.721	0.291
		(0.017)	(0.026)	(2.572)	(0.089)	(0.076)	(0.077)	(0.040)
PE_t	(IND_{t-1}, DEF_{t-2})	0.841	1.045	4.739	-0.372	0.656	0.344	.
		(0.036)	(0.023)	(1.873)	(0.135)	(0.069)	(0.069)	.

Notes: NLS parameter estimates for model (7) with univariate transition function (9) or multivariate transition function (10). Standard errors in parenthesis. All estimated models include a constant, which are not significantly different from zero and are therefore not reported.

TABLE 5. Goodness of fit

y_t	X_t	model	R^2	AIC	BIC	MAE	RMSE	F^{lin}	P (boot)
PD_t	.	AR(1)	0.966	-699.5	-692.8	1.317	1.526	.	.
PD_t	IND_{t-1}	Linear	0.966	-697.5	-687.5	1.321	1.532	.	.
PD_t	IND_{t-1}	STAR	0.970	-718.0	-704.7	1.292	1.490	23.81	0.002
PD_t	$(VOL_{t-1}, IND_{t-1}, STY_{t-2})$	Linear	0.967	-699.1	-682.4	1.323	1.538	.	.
PD_t	$(VOL_{t-1}, IND_{t-1}, STY_{t-2})$	STAR	0.971	-723.3	-710.0	1.283	1.490	29.79	0.001
PE_t	.	AR(1)	0.963	-681.8	-675.2	0.943	1.227	.	.
PE_t	IND_{t-1}	Linear	0.963	-679.9	-669.9	0.946	1.231	.	.
PE_t	IND_{t-1}	STAR	0.966	-696.1	-682.7	0.919	1.196	19.06	0.003
PE_t	(IND_{t-1}, DEF_{t-2})	Linear	0.965	-686.1	-672.8	0.940	1.216	.	.
PE_t	(IND_{t-1}, DEF_{t-2})	STAR	0.967	-701.1	-687.8	0.904	1.193	24.62	0.002

Notes: Measures of goodness of fit of the STAR models from Table 4, a linear AR(1) model and the linear models (15) including the same explanatory variables as the STAR. Mean Absolute Errors and Root Mean Square Errors are computed from 48 pseudo out-of-sample forecasts for 2000Q1-2011Q4. The F-test for linearity by Hansen (1996, 1997) tests $H_o : \alpha_1 = \alpha_2$ in the STAR model. The corresponding bootstrap P-value is computed based on 10.000 replications.

TABLE 6. Diagnostic tests

	y_t	PD_t		PE_t	
	X_t	$(VOL_{t-1}, IND_{t-1}, STY_{t-2})$		(IND_{t-1}, DEF_{t-2})	
		F	P	F	P
Serial independence:	1^{st}	1.380	0.242	1.327	0.251
	2^{nd}	0.804	0.449	0.805	0.448
	3^{rd}	0.921	0.432	1.683	0.172
	4^{th}	0.846	0.498	1.250	0.291
Parameter constancy:		1.225	0.295	1.529	0.170
No remaining nonlinearity:	PD_{t-1}	1.210	0.307	4.195	0.007
	PE_{t-1}	0.389	0.761	2.974	0.033
	RET_{t-1}	4.878	0.003	4.816	0.003
	VOL_{t-1}	2.267	0.082	0.651	0.583
	GDP_{t-1}	0.835	0.476	0.943	0.421
	CON_{t-1}	0.639	0.591	0.326	0.807
	OPG_{t-1}	0.425	0.735	0.445	0.721
	$\triangle OPG_{t-1}$	0.635	0.593	0.837	0.475
	IND_{t-1}	0.126	0.945	0.645	0.587
	CPI_{t-1}	1.231	0.299	1.478	0.222
	DEF_{t-1}	2.131	0.097	4.832	0.003
	STY_{t-1}	0.090	0.966	0.616	0.605
	$\triangle STY_{t-1}$	0.277	0.842	1.730	0.162
	LTY_{t-1}	0.778	0.508	0.459	0.711
	$\triangle LTY_{t-1}$	0.200	0.896	0.886	0.449
	TSP_{t-1}	1.192	0.314	1.283	0.281
	CBY_{t-1}	0.811	0.489	0.472	0.702
	$\triangle CBY_{t-1}$	0.577	0.631	0.164	0.920
	YSP_{t-1}	0.469	0.704	0.048	0.986

Notes: F-test statistics and corresponding P-values for first- to fourth-order serial independence, parameter constancy and no remaining non-linearity (Eitrheim and Teräsvirta, 1996)