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Concern for Relative Position, Rank-Order Contests, and Contributions to Public Goods*

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Abstract

We study the consequences of concern for relative position and status in a public good economy. We consider a group of agents who are engaged in a contest for position whereby a set of rewards are distributed according to relative status. The extent of concern for rewards, together with the relative magnitude of rewards, will have an impact on agents' willingness to contribute to public goods. Depending on the nature of prizes, i.e. whether higher private good consumption is rewarded or punished, the contest for relative position will either exacerbate or ameliorate the free-riding problem inherent in public good environments. In addition to examining the implications of concern for relative position, we also consider how an appropriate scheme of rewards might be designed to induce more efficient levels of public good.

1 Introduction

The objective of this paper is to study the consequences of concern for relative position and status in an economy with public goods. We consider a group of agents who are engaged in a contest for position whereby a set of rewards are distributed according to relative status. The extent of concern for rewards, together with the relative magnitude of rewards, will have an impact on agents' willingness to contribute to public goods. Depending on the nature of prizes, i.e. whether higher private good consumption is rewarded or punished, the contest

*I would like to thank my advisor Andrew Postlewaite and George Mailath for very helpful comments and advice. I would also like to thank Murat Sertel and the participants of the Seminar in Economic Design and Management at Boğaziçi University (Fall 1996) where this paper was presented for helpful comments.

for relative position or status will either exacerbate or ameliorate the free-riding problem inherent in public good environments. In addition to examining the implications of concern for relative position, we also consider how an appropriate scheme of rewards might be designed to induce more efficient levels of public good.

The notion that individuals might value their relative standing or status in a society has recently received some renewed attention in economics. The related literature goes back to Veblen [35] who argued that wealthy individuals engaged in conspicuous consumption of certain goods and services in order to signal their wealth, which in turn conferred on them higher social status which they valued. Duesenberry [17] explored the economic implications of concern for status, and studied its impact on the consumption and saving behavior. Hirsch [22] pointed out the role of social status in the context of growth, and argued that social scarcity implied by the relative nature of social rewards resulted in crowding and rent seeking which limits growth. More recently, Frank [20], Robson [33], Cole, Mailath, and Postlewaite [12], Fershtman and Weiss [19], Bakshi and Chen [6], among others, studied the impact of introducing status into agents' utility functions on various economic behavior and outcomes such as consumption, risk taking behavior, stock price volatility, and economic growth.¹

Including concern for relative position and status directly into agents' utility functions, while in accord with the sociological approach which insists on the importance behavior dictated by social norms, begs the question, however, of why individuals should care about them in the economic sense. Cole, Mailath, and Postlewaite [12] provided a model where they demonstrated that the existence of goods and decisions which are not allocated or made through markets, can endogenously generate an economic concern for relative position and status. They argued that the status of an agent can be interpreted as a ranking device whereby a society solves the problem of allocating certain nonmarketed goods. If, as in their paper, higher wealth implies higher status, and higher status in turn implies a higher valued match in marriage, then agents will have a rational reason to care about their relative position in the economy and to adjust their economic behavior accordingly. One can consider nonmarket decisions other than a matching decision, and similarly argue that concern for relative wealth endogenously creates incentives that will effect economic variables. Note that concern for relative wealth here is not put into the utility function directly, but arises indirectly because final consumption of (both marketed and nonmarketed) goods depends not just on wealth but also on relative wealth. We treat their analysis as providing a foundation for the induced reduced-form utility functions we consider here which directly incorporate concern for relative position and status.

Another line of economic research in which concern for relative position arises is the literature on rank-order tournaments, or contests, as compensation schemes in economies with imperfect information.² These are compensation

¹See also Abel [1], Cole, Mailath, and Postlewaite [13], and Bagwell and Bernheim [5].

²See Lazear and Rosen [24], Holmstrom [23], Nalebuff and Stiglitz [29], Green and Stokey

schemes in which pay depends on an agent's relative performance rather than only on the absolute level of her input or output. Agents' concern for relative position is thus based solely on economic considerations. Work on rank-order contests is concerned with identification of conditions under which tournaments (contests) provide optimal work incentives, and with derivation of optimal contest structures using monitoring precision and prize spreads as potential choice variables. It is typically assumed that each agent's effort have no direct effect on another agent's output.³

Work on optimal auctions and war of attrition in environments with incomplete information also involve purely economic concern for relative position on the part of participating agents.⁴

The economy we consider in this paper consists of a group of n identical agents who have preferences defined over a private good, a pure public good, and a set of n prizes to be accorded to each agent according to her relative position or status in the group. Each agent starts with an initial endowment $\omega_i \geq 0$ of the private good, the value of which is known only to the agent herself. The single private good can be thought of as composite good or money. Agents then take part in a Bayesian game by simultaneously contributing a nonnegative amount out of their initial endowments toward production of the public good. We assume that the private good consumption financed by the residual wealth, i.e. the amount of initial endowment left after contribution to the public good, is completely observable, and the prize an agent receives depends on her relative position in the distribution of private good consumption levels. The agent with the highest private good consumption receives the highest prize, the agent with the second highest private good consumption receives the second highest prize, and so on.

Prizes that agents care about, and which are distributed according to the rank-order contest described above, can be given an interpretation as in Cole, Mailath, and Postlewaite [12] to be quantities of a nonmarketed good. Concern for ordinal rank and status on the part of agents is not for its own sake, but because rank determines how a nonmarketed good is allocated in the group.

When higher status is associated with higher private good consumption, we demonstrate how in equilibrium agents contribute less toward the public good than the the amount they would have contributed in the absence of concern for relative position. Given the generic inefficiency of the voluntary contribution equilibrium in standard public good environments, this implies that concern for relative position may lead to further inefficiency. When coercive methods, such as taxation, cannot effectively be resorted to for the provision of a public good,

[21], O'Keefe, Viscusi, and Zeckhauser [30].

³An exception is a paper by Drago and Turnbull [16]. They consider a tournament with (positive) team externalities. Group effort is not perfectly separable with respect to individual contributions. Incentives to free-ride increases with the externality, as shirking is less likely to result in loss of the tournament.

⁴See Milgrom and Weber [27], McAfee and McMillan [26], Bulow and Roberts [11] on auctions and bidding, and Bishop et al. [9] and Riley [32] on war of attrition. See also Bliss and Nalebuff [10] for an application of optimal auction literature to the problem of private provision of a discrete public good.

and therefore it is to be provided through voluntary contributions, concern for relative position will exacerbate the free-riding problem.

Arguing that the concern for relative position and status might play a role in allocating certain nonmarketed goods provides an *individual* rationale for why agents will care about them. Arrow [3], on the other hand, argued for the *collective* rationality of norms that involve social rewards, such as prestige and status. Inefficiencies arising from market failures (due to, for example, externalities and public goods), might render it collectively rational for the society to settle on an agreement (a social custom or a norm) to improve the efficiency of the economic system by providing commodities, such as prestige and status, which cannot be bought and sold.⁵

Arrow's interpretation of norms as social agreements to provide certain commodities and to allocate them to curtail inefficiencies brings about the question of whether social rewards, such as prestige and reward, can be used to provide incentives for more efficient provision of public goods. Using the explicitly derived equilibrium behavior of agents in our model, we are able to determine an optimal contest structure by suitably choosing prize spreads in a rank-order contest in which higher private good consumption is punished. A negative prize for higher consumption lead agents in equilibrium to consume less private good, and therefore contribute more toward the public good. Appropriate choice of spread between prizes results in contributions that lead to the ex-post efficient level of public. We can interpret a negative prize as stigma attached to consuming more private good.

The plan of the paper is as follows. In Section 2 we introduce the specific model we use to examine the issues raised above. In Section 3 we derive the equilibrium of the rank-order contest in private good consumption levels, and study its properties. In order to simplify the exposition, the linear equilibrium strategies that arise in our model are first studied in detail for the 2-agent case, followed by the extension to the n -agent case. Section 4 considers a set of prizes that will lead to efficient public good provision. It also includes a discussion of the relation between a contribution contest as a public good provision mechanism and the general expected utility maximizing mechanisms studied in the mechanism design literature. Section 5 includes a discussion of the assumptions deriving the specific forms of our results and some concluding words.

2 Model

Consider a group of n agents, and let $N = \{1, \dots, n\}$ denote the set of agents for a given n , $n \geq 2$. Agents are assumed to have identical preferences over a private good, a pure public good, and a set of n prizes to be accorded to each agent

⁵See Elster [18] for a critique of this view of social norms. He contends that not all norms are Pareto-improvements, which is implicit in Arrow's view of social norms as unanimous agreements to enhance efficiency; that some norms that would make everybody better off are not observed; and, finally, that even if a norm induces a Pareto improvement, this does not by itself explain why it exists.

according to his relative position or status in the group. Each agent i , $i \in N$, starts with an initial endowment $\omega_i \geq 0$ of the private good, the value of which is known only to the agent herself. The single private good can be thought of as composite good or money. The agents then simultaneously contribute a nonnegative amount out of their initial endowments toward production of the public good. We assume that the private good consumption, financed by wealth remaining after the contribution to the public good is made, is completely observable, and the prize an agent receives depends on her relative position in the distribution of private good consumption levels. The agent with the highest final private good consumption level receives the highest prize, the agent with the second highest final private good consumption level receives the second highest prize, and so on. The spread between each prize level, and how much an agent cares about the prize will both have an impact on an agent's voluntary contribution toward the public good.

Let $y_i \in [0, \omega_i]$ denote the contribution of agent i toward public good production. There is no public good initially. The public good is produced through a technology that converts one unit of private good into one unit of public good. With the given technology the total amount of public good produced and consumed will be $Y = \sum_{i \in N} y_i$. Let $x_i = (\omega_i - y_i)$ denote agent i 's private good consumption level. Note that $x_i \in [0, \omega_i]$, for all $i \in N$. Let $\{P_1, P_2, \dots, P_n\}$, $P_j \in \mathfrak{R}$, for all $j \in N$, denote the set of prizes (where \mathfrak{R} represents the real line), and let $R_i \in \{P_1, P_2, \dots, P_n\}$ denote the prize received by agent i , for all $i \in N$. We assume that $P_1 \leq P_2 \leq \dots \leq P_n$, i.e. P_1 is the lowest prize and P_n is the highest prize. The utility functions we consider to represent agents' identical preferences will have the additively separable form $u(x_i, Y) + v(R_i)$, where $u : \mathfrak{R}_+^2 \rightarrow \mathfrak{R}$, and $v : \mathfrak{R} \rightarrow \mathfrak{R}$ is strictly increasing. It will therefore be convenient to express prizes in terms of utility. So let $\{p_1, p_2, \dots, p_n\}$, where $p_j = v(P_j)$, for all $j \in N$, denote the set of prizes in terms of utility, and let $r_i \in \{p_1, p_2, \dots, p_n\}$ denote prize in terms of utility received by an agent i , for all $i \in N$. For simplicity and tractability we make the following assumption on the utility functions of agents.

Assumption 1: *The preferences of agents are represented by the utility function $U^i(x_i, Y, r_i) = x_i Y + \alpha r_i$, $i \in N$, where $x_i \geq 0$, $Y \geq 0$, $r_i \in \mathfrak{R}$, and $\alpha \geq 0$ is a scalar.*

We assume that all agents have complete information about all aspects of the economy except for the actual level of initial endowments of agents other than themselves. All agents believe it is common knowledge that ω_i are drawn independently from the same continuous and strictly increasing cumulative distribution function $F(\omega)$ defined on $[\underline{\omega}, \bar{\omega}]$, where $0 \leq \underline{\omega} \leq \bar{\omega}$. The corresponding density function $f(\omega)$ is strictly positive on $[\underline{\omega}, \bar{\omega}]$. We will assume that ω is uniformly distributed.

Assumption 2: *The priors of the other agents about the initial endowment ω_i of agent i , for all $i \in N$, is identical and given by the distribution function $F(\omega) = (\omega - \underline{\omega}) / \Delta\omega$, where $\Delta\omega = \bar{\omega} - \underline{\omega}$ is the size of the support.*

The information structure described above induces a Bayesian game of voluntary contributions toward public good in which the initial endowment ω_i is agent i 's type. A pure-strategy for each agent i is a contribution level y_i as a function of her initial endowment ω_i , i.e. $y_i = y_i(\omega_i)$. The payoff to each agent will depend on the amount she contributes, the total amount contributed by the rest of the agents, and on how her private good consumption level $x_i = (\omega_i - y_i)$ ranks among all private good consumption levels, which determines the prize she receives. We will study the symmetric pure-strategy Nash equilibria of the specified game.

3 A Contest in Private Good Consumption Levels

We recall our assumption that private good consumption levels of all agents become perfectly observable after they make their contributions toward the public good. The highest prize p_n is then assigned to the agent with the highest final private good consumption level, the second highest prize p_{n-1} is assigned to the agent with the second highest private good consumption level, and so on.

We will first present the nature of equilibria we are interested in for the case of two agents, i.e. for $n = 2$. This will allow a simpler presentation of the nature of strategic interaction among agents that arises when there is concern for relative position in a public good model of the sort we study here. We will later present the extension of results for $n > 2$.

3.1 The case of two agents

With two agents let p_h and p_l represent the high prize and the low prize, respectively. Since $x_i = (\omega_i - y_i)$, we can equivalently consider each agent i as choosing a private good consumption level. Agent i 's strategy will therefore be a function $x_i : [\underline{\omega}, \bar{\omega}] \rightarrow \mathfrak{R}_+$, specifying for each possible value of ω_i the amount of private good agent i will choose. The agents take their actions simultaneously. If both agents choose the same amount, we assume that each gets the high prize with probability $\frac{1}{2}$. The payoff to agent i if her initial endowment is ω_i , agent j 's strategy is $x_j(\cdot)$, and she chooses a private good consumption level x_i will be

$$V^i(x_i, x_j(\cdot), \omega_i) = \int_{\{\omega_j | x_j(\cdot) < x_i\}} [x_i(\omega_i - x_i + \omega_j - x_j(\cdot)) + \alpha p_h] f(\omega_j) d\omega_j + \int_{\{\omega_j | x_j(\cdot) > x_i\}} [x_i(\omega_i - x_i + \omega_j - x_j(\cdot)) + \alpha p_l] f(\omega_j) d\omega_j \quad (1)$$

The first term is agent i 's expected utility if agent j chooses a private good consumption level less than x_i , and the second term is his expected utility if agent j chooses a level greater than x_i .

Definition 1 : $(\hat{x}_1(\omega_1), \hat{x}_2(\omega_2))$ is a Bayesian-Nash equilibrium for the two person contribution game if for all $i \in \{1, 2\}$, $\omega_i \in [\underline{\omega}, \bar{\omega}]$, and $x_i \in [0, \omega_i]$,

$$V^i(\hat{x}_i(\omega_i), \hat{x}_j(\omega_j), \omega_i) \geq V^i(x_i(\omega_i), \hat{x}_j(\omega_j), \omega_i).$$

We will look for profiles in which each agent's strategy is a strictly increasing and continuous function of his initial endowment. Consider such a function x_i with inverse Φ_i , i.e. $\Phi_i(x_i)$ is the initial endowment of the agent who chooses x_i .⁶ Transforming the variable of integration⁷ in (1) from ω_j to x_j , we differentiate $V^i(x_i, x_j(\cdot), \omega_i)$ with respect to x_i to get

$$\frac{dV^i}{dx_i} = \Phi_i(x_i) - 2x_i + \int_0^{\bar{x}_j} [\Phi_j(x_j) - x_j] f(\Phi_j(x_j)) \Phi_j'(x_j) dx_j, \quad (2)$$

$$+ \alpha(p_h - p_l) f(\Phi_j(x_i)) \Phi_j'(x_i)$$

where, given that $x_j(\cdot)$ is strictly increasing, $\bar{x}_j = x_j(\bar{\omega})$ is the maximum amount of private good that can be consumed by agent $j \neq i$. The first-order condition at an interior optimum is obtained by setting (2) equal to zero, which can also be arrived at by observing that at an equilibrium an agent with initial endowment ω_i and contributing $x_i > 0$ cannot increase her expected utility by choosing $x_i + dx_i$ instead of $x_i \equiv x_i(\omega_i)$. This increase yields a benefit of $\alpha(p_h - p_l)$ if player j chooses in the interval $[x_i, x_i + dx_i]$. This will be the case if ω_j is in the interval $[\Phi_j(x_i), \Phi_j(x_i + dx_i)]$, and this occurs with probability $f(\Phi_j(x_i)) \Phi_j'(x_i) dx_i$. The expected incremental cost associated with an increase of dx_i equals minus the expected value of $\{\partial(x_i(\omega_i - x_i + \omega_j - x_j)) / \partial x_i\} dx_i$. Equating the cost and benefits we obtain the same expression implied by equating (2) equal to zero.⁸ Therefore at a symmetric equilibrium we will have

$$\omega - 2\hat{x}(\omega) + \int_{\Phi(0)}^{\bar{\omega}} [\omega - \hat{x}(\omega)] f(\omega) d\omega + \alpha(p_h - p_l) \frac{f(\omega)}{\hat{x}'(\omega)} = 0, \quad (3)$$

which we obtain from (2) by setting it to zero, dropping the subscripts, substituting $\omega = \Phi(\hat{x})$ throughout, and using the fact that $\Phi' = 1/x'$.⁹

To understand the nature of possible equilibrium strategies for the Bayesian game considered here, it will be helpful to look at the outcome of the complete information version of the voluntary contribution game between two agents when there is no concern for relative position. Let the utilities of two agents i and j be as in (??) with $\alpha = 0$ (or $p_h = p_l$), and let $\omega_i \geq 0$ and $\omega_j \geq 0$ be their mutually known initial endowment levels, respectively. Each agent will therefore choose a contribution level $y_i \in [0, \omega_i]$ to maximize utility, and it can be checked to see

⁶Note that the function $\Phi_i(\cdot)$ is well defined and differentiable (except for a finite number of points), since it is the inverse of a strictly monotonic function, continuous, and bounded.

⁷The density $g(\omega)$ of $x = z(\omega)$ when $z(\cdot)$ is a strictly increasing function is given by the formula $g(x) = f(z^{-1}(x)) (z^{-1})'$, where $f(\omega)$ is the density of ω .

⁸The global second-order conditions are satisfied if the first-order conditions are (see the proof of Proposition 4 below).

⁹ $\Phi(x(\omega)) \equiv \omega \equiv x^{-1}(x(\omega))$, and by the inverse function theorem we have $\Phi' = 1/x'$.

that at the unique Nash equilibrium outcome of this contribution game each agent will contribute an amount equal to

$$\bar{y}_i = \max \left\{ 0, \frac{2\omega_i - \omega_j}{3} \right\}, \quad i \neq j, \quad (4)$$

out of their initial endowments toward the public good and consume an amount equal to

$$\bar{x}_i = \min \left\{ \omega_i, \frac{\omega_i + \omega_j}{3} \right\}, \quad i \neq j, \quad (5)$$

as private good. Expressions (4) and (5) reveal that if $\omega_i \leq \frac{1}{2}\omega_j$, agent i will not contribute anything toward the public good and consume all of her endowment. That is, she will completely free ride on the amount $\frac{1}{2}\omega_j$ contributed by agent j . This property of free riding carries over to the case when there are $n > 2$ agents with identical preferences: there will be a critical initial endowment level ω_0 such that agents with initial endowments above ω_0 will contribute a strictly positive amount, and agents with initial endowments at or below ω_0 will contribute zero.¹⁰

Now assume that agents have incomplete information regarding the initial endowments of all agents other than themselves. With no concern for relative position, i.e. $\alpha = 0$ (or $p_h = p_l$), consider a Bayesian-Nash equilibrium strategy profile $(\hat{x}_1(\omega_1), \hat{x}_2(\omega_2))$ of the contribution game. For each ω_i , $\hat{x}_i(\omega_i)$ must satisfy

$$\hat{x}_i(\omega_i) \in \operatorname{argmax}_{x_i} \left\{ x_i \left(\omega_i - x_i + \int_{\underline{\omega}}^{\bar{\omega}} (\omega_j - \hat{x}_j(\omega_j)) f(\omega_j) d\omega_j \right) \right\}, \quad i \neq j, \quad (6)$$

which implies that either

$$\omega_i - 2\hat{x}_i(\omega_i) + \int_{\underline{\omega}}^{\bar{\omega}} (\omega_j - \hat{x}_j(\omega_j)) f(\omega_j) d\omega_j = 0, \quad (i)$$

or

$$\omega_i - 2\hat{x}_i(\omega_i) + \int_{\underline{\omega}}^{\bar{\omega}} (\omega_j - \hat{x}_j(\omega_j)) f(\omega_j) d\omega_j > 0 \quad \text{and} \quad \hat{x}_i(\omega_i) = \omega_i, \quad (ii)$$

or

$$\omega_i - 2\hat{x}_i(\omega_i) + \int_{\underline{\omega}}^{\bar{\omega}} (\omega_j - \hat{x}_j(\omega_j)) f(\omega_j) d\omega_j < 0 \quad \text{and} \quad \hat{x}_i(\omega_i) = 0. \quad (iii)$$

That the second-order conditions are satisfied whenever the first-order conditions are can easily be checked. We observe that (iii) never holds, so the boundary constraint $x_i \geq 0$ is not binding. Note, however, that (ii) may be binding. That is, an agent may consume all of her endowment as private

¹⁰See Andreoni [2] for an exposition of this property of public good contribution games for identical and heterogenous preferences in a more general setting than ours.

good and contribute none toward the public good. As in the complete information case, agents with initial endowments less than a critical value $\omega_0 = \int_{\underline{\omega}}^{\bar{\omega}} (\omega_j - \hat{x}_j(\omega_j)) f(\omega_j) d\omega_j$ will consume all of their initial endowments.

Now we will exhibit a pair of symmetric equilibrium strategies which will incorporate a cut off initial endowment level $\omega_0 \in [\underline{\omega}, \bar{\omega}]$ and which will be linear in initial endowment when an agent does contribute, i.e.

$$\hat{x}(\omega) = \begin{cases} \omega & \text{if } \underline{\omega} \leq \omega < \omega_0 \\ a\omega + b & \text{if } \omega_0 \leq \omega \leq \bar{\omega} \end{cases} . \quad (7)$$

The continuity of the strategies requires that $a\omega_0 + b = \omega_0$, which implies that $b = (1 - a)\omega_0$. For an agent with $\omega \geq \omega_0$ the first order condition (i) above must be satisfied. Given our assumption that $F(\omega) = \frac{\omega - \underline{\omega}}{\Delta\omega}$, this leads to

$$(1 - 2a)\omega - \left(2 + \frac{\bar{\omega} - \omega_0}{\Delta\omega}\right)b + \frac{1 - a}{2\Delta\omega}(\bar{\omega}^2 - \omega_0^2) = 0. \quad (8)$$

Since this equation must hold for all $\omega \in (\omega_0, \bar{\omega}]$, we can identify that $a = \frac{1}{2}$ by observing that $(1 - 2a)$ must be zero. Similarly, by making use of $b = (1 - 2a)\omega_0 = \frac{1}{2}\omega_0$, we get

$$\omega_0 = (\bar{\omega} + 2\Delta\omega) - 2\sqrt{(\bar{\omega} + \Delta\omega)\Delta\omega}, \quad (9)$$

which also identifies b . Note that $\omega_0 < \bar{\omega}$.¹¹ We must also check that the equilibrium strategy specified in (7) satisfies the first order condition (ii) for $\omega < \omega_0$. That it does can be seen by noting that

$$-\omega + \int_{\omega_0}^{\bar{\omega}} \left[\omega_j - \frac{1}{2}(\omega_j - \omega_0) \right] \frac{1}{\Delta\omega} d\omega > 0 \iff \omega < \omega_0.$$

Figure 1.1 displays the shape of the symmetric equilibrium strategy for $\omega_0 > \underline{\omega}$. The equilibrium strategy $\hat{x}(\omega)$ is continuous and strictly increasing in the initial endowment. The amount contributed toward the public good in equilibrium with initial endowment ω is denoted by $\hat{y}(\omega)$ in the figure, and equals the difference between initial endowment level and the equilibrium choice of private good consumption level. Agents with $\omega \leq \omega_0$ consume all of their initial endowments as private good and contribute zero toward the public good. The slope of the equilibrium private good consumption function for $\omega > \omega_0$ is $\frac{1}{2}$, which is a consequence of the specific functional form $x_i Y$ we employed to represent agents' preferences over the private and the public good.

We noted above that we always have $\omega_0 < \bar{\omega}$. But if $\bar{\omega} < 5\underline{\omega}$, then $\omega_0 < \underline{\omega}$, and (7) no longer applies. We check in this case the first-order condition (i) to see that both agents will contribute a positive amount at a symmetric equilibrium

¹¹The other solution to the quadratic expression involving ω_0 that (8) implies is a value greater than $\bar{\omega}$, and it can be checked from the first order conditions (i)-(iii) above that the suggested strategy will not be an equilibrium in this case.

by setting their private good consumption levels at $\hat{x}(\omega) = \frac{1}{2}\omega + \frac{\bar{\omega} + \omega}{12} < \omega$, for all $\omega \in [\underline{\omega}, \bar{\omega}]$.

Figures 1.2 and 1.3 involve cases where $\omega_0 \leq \underline{\omega}$. Figure 1.2 shows the case where $\bar{\omega} = 5\underline{\omega}$ so that $\omega_0 = \underline{\omega}$. In this case type $\underline{\omega}$ agent does not contribute toward the public good while all other types contribute strictly positive amounts. Figure 1.3 is drawn for the case where $\bar{\omega} < 5\underline{\omega}$ so that $\omega_0 < \underline{\omega}$, and all types of agents contribute strictly positive amounts toward the public good in equilibrium.

Proposition 2 below summarizes the symmetric equilibrium strategies we have thus derived for the two-person Bayesian contribution game with uniform distribution of initial endowments.

Proposition 2 : *Let the utilities of agents be as in (??) with $\alpha = 0$ (or $p_h = p_l$), and let their initial endowments be uniformly distributed on $[\underline{\omega}, \bar{\omega}]$. If $\omega_0 < \underline{\omega}$, where ω_0 is as in (9) then private good consumption levels at a symmetric equilibrium of the 2-agent Bayesian contribution game will be given by the function*

$$\hat{x}_*(\omega) = \frac{1}{2}\omega + \frac{\bar{\omega} + \omega}{12}, \quad \forall \omega \in [\underline{\omega}, \bar{\omega}], \quad (10)$$

implying strictly positive contributions toward the public good by all types of agents. If $\omega_0 \geq \underline{\omega}$, then equilibrium private good consumption levels will be given by

$$\hat{x}^*(\omega) = \begin{cases} \omega & \text{if } \underline{\omega} \leq \omega < \omega_0 \\ \frac{1}{2}(\omega + \omega_0) & \text{if } \omega_0 \leq \omega \leq \bar{\omega} \end{cases}, \quad (11)$$

implying that agents with initial endowments less than or equal ω_0 will contribute zero amount toward the public good.

Note that the equilibrium strategies (10) and (11) exhibited in Proposition 2 have the property that $\hat{x}_*(\omega_0) = \hat{x}^*(\omega_0)$. That is, the equilibrium strategy changes continuously with the cut off initial endowment level ω_0 .

Proposition 2 also shows that a Bayesian-Nash equilibrium of the two-person contribution game with uniformly distributed type space may involve strictly positive ex-post contributions toward the public good by both agents, or only one of the agents, or by neither of the agents. When $\bar{\omega} \geq 5\underline{\omega}$ so that $\omega_0 \in [\underline{\omega}, \bar{\omega}]$, if both agents have initial endowments less than or equal to ω_0 , then they will both choose to consume all of their endowments and no public good will be provided, a gross inefficiency in the ex-post sense. If, on the other hand, $\bar{\omega} < 5\underline{\omega}$, i.e. when the support of the type distribution is small, then all types contribute at the equilibrium.

Example 3 : *Let $\omega \in [1, 2]$. Then $\omega_0 = 4 - 2\sqrt{3} \approx 0.54$. Both agents will contribute a positive amount toward the public good and consume $\hat{x}(\omega) = \frac{1}{2}\omega + \frac{1}{4} < \omega$, for all $\omega \in [1, 2]$. If $\omega \in [0, 1]$, then $\omega_0 = 3 - 2\sqrt{2} \approx 0.172$, and agents with $\omega \in [0, 0.172]$ will consume all of their initial endowments as private good, while agents with $\omega \in (0.172, 1]$ will contribute $\hat{y}(\omega) = \omega - \hat{x}(\omega) = \frac{1}{2}(\omega - 0.172)$.*

Returning to the case when relative position does matter, i.e. when $\alpha(p_h - p_l) > 0$, we will now show that linear strategies similar to those we have just described for the case with no concern for relative position will satisfy the first-order conditions given in (3) for an interior optimum. The difference will be that the cut off level of initial endowment $\omega_0 = \Phi(0)$ will change due to the term $\alpha(p_h - p_l) \frac{f(\omega)}{x'(\omega)}$, which reflects the impact of concern for relative position.

Proposition 4 : *Let the utilities of agents be as in (??), and let their initial endowments be uniformly distributed on $[\underline{\omega}, \bar{\omega}]$. If $\alpha(p_h - p_l) < \frac{(4\bar{\omega} - 5\Delta\omega)\Delta\omega}{8}$, then private good consumption levels at a symmetric equilibrium of the 2-agent Bayesian contest will be given by*

$$\hat{x}_*(\omega) = \frac{1}{2}\omega + \frac{\bar{\omega} + \omega}{12} + \frac{2}{3}\alpha \frac{(p_h - p_l)}{\Delta\omega} \quad \forall \omega \in [\underline{\omega}, \bar{\omega}] , \quad (12)$$

implying strictly positive contributions toward the public good by all types of agents. If $\frac{(4\bar{\omega} - 5\Delta\omega)\Delta\omega}{8} \leq \alpha(p_h - p_l) \leq \frac{\bar{\omega}\Delta\omega}{2}$, then equilibrium private good consumption levels will be given by

$$\hat{x}^*(\omega) = \begin{cases} \omega & \text{if } \omega < \omega_0 \\ \frac{1}{2}(\omega + \omega_0) & \text{if } \omega \geq \omega_0 \end{cases} , \quad (13)$$

where

$$\omega_0 = (\bar{\omega} + 2\Delta\omega) - 2\sqrt{(\bar{\omega} + \Delta\omega)\Delta\omega - 2\alpha(p_h - p_l)} , \quad (14)$$

implying that agents with initial endowments less than or equal ω_0 will contribute zero amount toward the public good.

Proof. Assume that $\alpha(p_h - p_l) < \frac{(4\bar{\omega} - 5\Delta\omega)\Delta\omega}{8}$. We check to see that the proposed linear strategy in (12) does satisfy the first-order condition (3) for an interior optimum. To see that the global second order conditions will also be satisfied when the first-order conditions are, note from (2) that $\frac{\partial^2 V^i}{\partial x \partial \omega_i} = 1 > 0$. Suppose that there exists a type ω_i and a strategy x'_i such that $V^i(x'_i, x_j(\cdot), \omega_i) > V^i(x_i, x_j(\cdot), \omega_i)$, where $x_i = \hat{x}_i(\omega_i)$. This implies that

$$\int_{x_i}^{x'_i} \frac{\partial V^i}{\partial x}(x, x_j(\cdot), \omega_i) dx > 0.$$

Using the first-order condition $\frac{\partial V^i}{\partial x}(x, x_j(\cdot), \Phi_i(x)) = 0$, for all x , we have

$$\int_{x_i}^{x'_i} \left(\frac{\partial V^i}{\partial x}(x, x_j(\cdot), \omega_i) - \frac{\partial V^i}{\partial x}(x, x_j(\cdot), \Phi_i(x)) \right) dx > 0.$$

This is equivalent to

$$\int_{x_i}^{x'_i} \int_{\Phi_i(x)}^{\omega_i} \frac{\partial^2 V^i}{\partial x \partial \omega}(x, x_j(\cdot), \omega) d\omega dx > 0.$$

If $x'_i > x_i$, then $\Phi_i(x) > \omega_i$ for all $x \in (x_i, x'_i]$, but in this case the last inequality cannot hold. And similarly for $x'_i < x_i$. So $x_i = x_i(\omega_i)$ is globally optimal for type ω_i .

It can also be easily checked that for $\hat{x}_*(\omega) < \omega$ for $\alpha(p_h - p_l) < \frac{(4\bar{\omega} - 5\Delta\omega)\Delta\omega}{8}$, i.e. all agents will contribute positive amounts toward the public good, as specified by the equilibrium strategy.

Now assume that $\frac{(4\bar{\omega} - 5\Delta\omega)\Delta\omega}{8} \leq \alpha(p_h - p_l) \leq \frac{\bar{\omega}\Delta\omega}{2}$. We check 14 to see that in this case ω_0 is real and belongs to the interval $[\underline{\omega}, \bar{\omega}]$. For $\omega \in [\omega_0, \bar{\omega}]$, where $\omega_0 = \Phi(0)$, the proposed strategies in (13) do satisfy the first-order conditions in (3) for an interior optimum, and the second-order conditions can be similarly checked to hold as above. That the boundary constraint $x(\cdot) \leq \omega$ is binding for $\omega \in [\underline{\omega}, \omega_0]$ is a consequence of the fact that $\omega - 2\omega + \int_{\omega_0}^{\bar{\omega}} [\frac{1}{2}(\omega - \omega_0)] f(\omega) d\omega + \frac{2\alpha(p_h - p_l)}{\Delta\omega} > 0 \iff \omega < \omega_0$. ■

Proposition 4 shows how the concern for relative position effects contributions toward the public good. The higher the magnitude of the concern for relative position in terms of observable private good consumption levels the more agents will reserve resources for their private good consumption, as to be expected. Comparing (14) and (9) we observe that for a given (uniform) initial endowment distribution, concern for relative position leads to more expected free riding when private good consumption levels determine relative position. The cut off level of initial endowment below which agents do not contribute toward the public good is higher, leading to less expected total contributions.

Proposition 4 also shows that, for a given α , with a prize difference $(p_h - p_l)$ low enough all possible types of agents will contribute. This brings out the possibility that if one considers the problem at hand as one of designing a reward system based on relative position in terms private good consumption, the designer can choose a negative $(p_h - p_l)$, i.e. impose a tax on higher private good consumption, to bring about desired levels voluntarily provided public good. We will later turn to the issue of designing a reward structure based on relative position to induce efficient levels voluntary contribution.

Example 5 : Let $\omega \in [1, 2]$. We know from Example 3 that all possible types of agents will contribute toward the public good when $\alpha(p_h - p_l) = 0$. With $\alpha(p_h - p_l) = \frac{1}{2}$, only agents with initial endowments in the interval $(1.172, 2]$ will contribute. If $\omega \in [0, 1]$, then only with $\alpha(p_h - p_l) \leq \frac{-1}{8}$ will all types of agents contribute at the equilibrium.

3.2 An n -agent rank order contest

With n agents and n prizes $p_1 < p_2 < \dots < p_n$, the agent with the highest private good consumption level receives the highest prize p_n , the agent with the second highest private good consumption level receives p_{n-1} , and so on. This is an n -person rank order contest in which rewards depend on the rank order of agents in terms of private good consumption levels. In an n -agent rank order contest of this kind, agent i will receive prize p_k if and only if x_i is the k th-order statistic of (x_1, \dots, x_n) .

The definition of the Bayesian-Nash equilibrium of the contribution game for the n -agent case is a straightforward extension of that for the 2-agent case, and the derivation of equilibrium strategies is also a straightforward generalization. Let the vector of strategies for agents other than agent i be denoted by $x_{-i}(\cdot) = (x_1(\cdot), \dots, x_{i-1}(\cdot), x_{i+1}(\cdot), \dots, x_n(\cdot))$. If $m > 1$ agents choose the same amount, we assume that each gets the associated prize with probability $\frac{1}{m}$. The payoff to agent i of type ω_i when $(n-1)$ other agents choose $x_{-i}(\cdot)$ and he chooses x_i will be

$$V^i(x_i, x_{-i}(\cdot), \omega_i) = \sum_{k=1}^n \phi_{k,n-1} \int_{\Psi_i(x_i)} \cdots \int_{\Psi_i(x_i)} \left[x_i \left(\omega_i - x_i + \sum_{j \neq i} (\omega_j - x_j(\cdot)) \right) + \alpha p k \right] \prod_{j \neq i} f(\omega_j) d\omega_j, \quad (15)$$

where, $\Psi_i(x_i) = \{\omega_i \mid x_i \text{ is the } k\text{th-order statistic of } (x_i, x_{-i}(\cdot))\}$, and $\phi_{k,n-1} = \frac{(n-1)!}{(n-k)!(k-1)!}$.¹² As in the 2-agent case, we will look for profiles in which each agent's strategy is strictly increasing and continuous in type. After transforming the variable of integration in (15) and rearranging, we have that for every ω_i an equilibrium strategy $\hat{x}_i(\omega_i)$ for agent i must satisfy

$$\hat{x}_i(\omega_i) \in \operatorname{argmax} \left\{ \int_0^{\bar{x}_j} \cdots \int_0^{\bar{x}_j} \left[\left(\sum_{j \neq i} \Phi_j(x_j) - x_j \right) \right] \prod_{j \neq i} f(\Phi_j(x_j)) \Phi'_j(x_j) dx_j \right. \\ \left. + \alpha \sum_{k=1}^n \phi_{k,n-1} p_k F^{k-1}(\Phi_j(x_i)) [1 - F(\Phi_j(x_i))]^{n-k} \right\}, \quad (16)$$

where, as in the 2-agent case, Φ_i is the inverse of x_i , and \bar{x}_j is the maximum amount of private good that can be consumed by an agent $j \neq i$. Therefore, at an interior optimum with symmetric strategies we must have

$$0 = \Phi(\hat{x}) - 2\hat{x} + \int_0^{\bar{x}} \cdots \int_0^{\bar{x}} \left[\sum_{j=1}^{n-1} (\Phi(x) - x) \right] \prod_{j=1}^{n-1} f(\Phi(x)) \Phi'(x) dx \\ + \alpha \sum_{k=2}^n \phi_{k,n-1} (p_k - p_{k-1}) F^{k-2}(\Phi(\hat{x})) [1 - F(\Phi(\hat{x}))]^{n-k} f(\Phi(\hat{x})) \quad (17)$$

Substituting $\omega = \Phi(x)$ throughout, and using $\Phi' = 1/x'$, we can express (17)

¹²We recall that the density function for the k th-order statistic in a sample of size n drawn from a distribution $G(x)$ with density $g(x)$ is given by $\phi_{k,n} = \frac{n!}{(n-k)!(k-1)!} g(x) G^{k-1}(x) [1 - G(x)]^{n-k}$.

as

$$\begin{aligned}
0 = \omega - 2\hat{x}(\omega) + \int_{\Phi(0)}^{\bar{\omega}} \cdots \int_{\Phi(0)}^{\bar{\omega}} \left[\sum_{j=1}^{n-1} (\omega_j - \hat{x}(\omega_j)) \right] \prod_{j=1}^{n-1} f(\omega_j) d\omega_j \\
+ \alpha \frac{f(\omega)}{x'(\omega)} \sum_{k=2}^n \phi_{k,n-1} (p_k - p_{k-1}) F^{k-2}(\omega) [1 - F(\omega)]^{n-k}
\end{aligned} \quad (18)$$

Proposition 6 below shows that for the case of evenly spaced prizes, i.e. $p_k - p_{k-1} = \Delta p$, for all $k = 2, \dots, n$, the qualitative features of the linear symmetric equilibrium strategy we derived for the 2-agent case generalizes to the n -agent case.

Proposition 6 : *Let the utilities of agents be as in (??), and let their initial endowments be uniformly distributed on $[\underline{\omega}, \bar{\omega}]$. Assume that $p_k - p_{k-1} = \Delta p$, for all $k = 2, \dots, n$. If*

$$\alpha \Delta p < \frac{\bar{\omega} \Delta \omega}{2(n-1)} - \frac{(n+1)^2 - 4}{8(n-1)^2} (\Delta \omega)^2,$$

then private good consumption levels at a symmetric equilibrium of the n -agent Bayesian contest will be given by

$$\hat{x}_*(\omega) = \frac{1}{2}\omega + \frac{(n-1)(\bar{\omega} + \underline{\omega})}{4(n+1)} + \frac{2(n-1)}{n+1} \alpha \frac{\Delta p}{\Delta \omega}, \quad \forall \omega \in [\bar{\omega}, \underline{\omega}], \quad (19)$$

implying strictly positive contributions toward the public good by all types of agents. If

$$\frac{\bar{\omega} \Delta \omega}{2(n-1)} - \frac{(n+1)^2 - 4}{8(n-1)^2} (\Delta \omega)^2 \leq \alpha \Delta p \leq \frac{\bar{\omega} \Delta \omega}{2(n-1)},$$

then equilibrium private good consumption levels will be given by

$$\hat{x}^*(\omega) = \begin{cases} \omega & \text{if } \omega < \omega_0 \\ \frac{1}{2}(\omega + \omega_0) & \text{if } \omega \geq \omega_0 \end{cases},$$

where

$$\omega_0 = \left(\bar{\omega} + \frac{2}{n-1} \Delta \omega \right) - 2 \sqrt{\left(\bar{\omega} + \frac{\Delta \omega}{n-1} \right) \frac{\Delta \omega}{n-1} - 2\alpha \Delta p}, \quad (20)$$

implying that agents with initial endowments less than or equal ω_0 will contribute zero amount toward the public good.

Proof. We will only show that the assumption $p_k - p_{k-1} = \Delta p$, for all $k = 2, \dots, n$, will allow linear equilibrium strategies of the sort we described in Proposition 4. Parameters of the proposed strategy are then calculated as in

Proposition 4. We note that with initial endowments uniformly distributed on $[\underline{\omega}, \bar{\omega}]$, and for a given Δp , the summation term in (18) that involves Δp becomes

$$\alpha \frac{(n-1) \Delta p}{x'(\omega) \Delta w} \sum_{k=0}^n \frac{(n-2)!}{(n-2-k)!k!} F^k(\omega) [1 - F(\omega)]^{(n-2)-k}. \quad (21)$$

By the binomial formula the summation term in (21) equals $(F(\omega) + 1 - F(\omega))^{n-2} = 1$, so (18) becomes

$$0 = \omega - 2\hat{x}(\omega) + \frac{n-1}{\Delta w} \int_{\Phi(0)}^{\bar{\omega}} [\omega - \hat{x}(\omega)] d\omega + \alpha \frac{(n-1) \Delta p}{\hat{x}'(\omega) \Delta w},$$

This is of the same form that allowed linear equilibrium strategies in Proposition 4 for the 2-agent case. ■

It can be observed from the expression in (18) and the proof of Proposition 6 that when n prizes are not evenly spaced there will not in general be an equilibrium in linear strategies, even with the uniform distribution.

4 Efficient Set of Prizes

Adopting a designer's point of view in a public good environment a natural question to ask is whether Pareto efficient outcomes can be reached through a mechanism under consideration. Following the suggestion by Arrow [3] to view social rewards such as prestige and status as a mechanism to cope with the inefficiencies arising from externalities, we can use the model we have developed so far to consider whether a set of prizes can be designed to provide incentives for more efficient provision of public goods. In certain cases using prestige and status to discourage private good consumption among a group of agents may be an inexpensive way of promoting voluntary supply of a public good at more efficient levels.

With incomplete information as in the present model, the Pareto efficiency of a mechanism can be considered in both the ex-ante sense, where the expected utility of no type of agent can be increased without decreasing the expected utility of another type, or in the ex-post sense, where when types of all agents are revealed there can be no outcome that dominates the outcome under the mechanism in the usual Pareto sense.

Assume that preferences of an agent i over a private good and a public good are represented by the utility function $U^i(x_i, Y) = x_i Y$, where $x_i \geq 0$ is his private good consumption level, and Y is the total public good level, $\forall i \in N$.

Definition 7 : A public good provision mechanism is a function $\rho : \prod_{i=1}^n [\underline{\omega}, \bar{\omega}] \rightarrow \mathfrak{R}_+^n \times \mathfrak{R}_+$ that assigns, for each possible profile of agents' initial endowments $\Omega = (\omega_1, \dots, \omega_n)$, a vector $(\rho_1^x(\Omega), \dots, \rho_n^x(\Omega))$ indicating the private good consumption level for each agent i , and a public good level $\rho^Y(\Omega)$ such that $\sum_{i=1}^n \rho_i^x(\Omega) + \rho^Y(\Omega) \leq \sum_{i=1}^n \omega_i$.

Definition 8 : A public good provision mechanism $\rho : \prod_{i=1}^n [\underline{\omega}, \bar{\omega}] \rightarrow \mathfrak{R}_+^n \times \mathfrak{R}_+$ is ex-post efficient if for no profile $\Omega = (\omega_1, \dots, \omega_n)$ is there an $((x_1, \dots, x_n), Y) \in \mathfrak{R}_+^n \times \mathfrak{R}_+$, satisfying $\sum_{i=1}^n x_i + Y \leq \sum_{i=1}^n \omega_i$, such that $U^i(x_i, Y) \geq U^i(\rho_i^x(\Omega), \rho^Y(\Omega))$ for every i , and $U^i(x_i, Y) > U^i(\rho_i^x(\Omega), \rho^Y(\Omega))$ for some i .

It can be checked that the unique ex-post efficient level of public good when $U^i(x_i, Y) = x_i Y, \forall i \in N$, will be

$$\frac{1}{2} \sum_{i=1}^n \omega_i. \quad (22)$$

4.1 Efficient prizes for the contest in private good consumption levels

The results of Section 3 show that while a rank-order contest in private good consumption levels in which higher consumption is rewarded exacerbates the underprovision of the public good, a contest in which higher consumption is punished will ameliorate the level of public good provided through voluntary contributions. Proposition 9 below demonstrates how the ex-post efficient level of the public good can be achieved in a rank-order contest.

Proposition 9 : Assume that the preferences of agents over a private good, a public good, and a set of n prizes $\{p_1, p_2, \dots, p_n\}$ are as in (??), and let their initial endowments be uniformly distributed on $[\underline{\omega}, \bar{\omega}]$. In an n -agent rank-order contest in private good consumption levels, the unique ex-post efficient level of public good (22) can be attained by setting the difference between each prize level equally at

$$\Delta p = p_k - p_{k-1} = -\frac{E(\omega)\Delta\omega}{4\alpha}, \quad \forall k = 2, \dots, n, \quad (23)$$

where $E(\omega)$ denotes the expected value of initial endowments, and $\Delta\omega = \bar{\omega} - \underline{\omega}$.

Proof. Observe from (19) in Proposition 6 that with differences in prizes equally set as in (23), each agent i will contribute $\frac{1}{2}\omega_i$, leading to the ex-post efficient total public good level $\frac{1}{2}\sum_{i=1}^n \omega_i$. ■

Proposition 9 shows that punishing higher private good consumption, or, equivalently, rewarding lower private good consumption with suitable prize differences will lead to efficiency. Note also that Δp in (23) is independent of n .

The assumption that allows us to obtain ex-post efficiency in Propostion 9 is that the utility functions in (??) are additively separable in prizes. This renders the prize in our model a third good, distinct from the private and the public goods. In fact, we interpreted the prizes, following Cole, Mailath, and Postlewaite [12], as a nonmarketed good allocated to the agents through a rank-order contest. Additive separability in this third good is a certain special case.

To understand the role of additive separability of the utility functions in this third good, consider instead the case where the prizes are denominated in terms of the private good. Specifically, consider for the 2-agent case that the

agent with the higher private good consumption pays an amount T in private good, taken as the numeraire good, to the agent with lower private good consumption. That is, consider a two-good model in which there is a lump-sum tax on higher consumption. Such a model is similar to the models treated in Bayesian mechanism design literature, the relevant class in our case being the expected externality mechanism due to d'Aspremont and Gérard-Varet [15] and Arrow [4]. Note that the taxing scheme we impose will be ex-post balanced, since $\sum_i T_i = 0$. In parallel to the model used so far, let the utility of agent i be given by

$$U^i(x_i, Y, T) = \begin{cases} (x_i - T)Y & \text{if } x_i > x_j \\ x_i Y & \text{if } x_i = x_j \\ (x_i + T)Y & \text{if } x_i < x_j \end{cases}, j \neq i.$$

Following the same procedure as in Section 3.1 we obtain symmetric strategies linear in type ω given by

$$\hat{x}(\omega) = \begin{cases} \omega & \text{if } \underline{\omega} \leq \omega < \omega_0 \\ \left(\frac{1}{2} - \frac{3T}{\Delta\omega}\right)\omega + b & \text{if } \omega_0 \leq \omega \leq \bar{\omega} \end{cases}, \quad (24)$$

which are of the form similar to the case with additive separability and involve a cut off initial endowment level below which types of agents do not contribute toward the public good. It can immediately be observed from (24) that, as opposed to the case with additive separability, the slope of the strategy for the contributing agents involves the parameter T , and the cut off point ω_o can not be reduced to $\underline{\omega}$ to have all agents with $\omega > 0$ contribute on the way to achieving the ex-post efficient level of public good.

The expected externality mechanism due to d'Aspremont and Gérard-Varet [15] mentioned above considers an environment in which agents' utility functions are quasi-linear, which is not the case in our model, and their types are statistically independent, as in our model.¹³ They constructed a mechanism involving transfers among agents for the quasi-linear environment which they demonstrate to be ex-post efficient. We do not have ex-post efficiency in the two-good version of our model of contribution contest, as we have just shown. However, since our utility functions are not quasi-linear we are not considering the same environment as in their model. Another problem with the expected externality mechanism of d'Aspremont and Gérard-Varet is that it may lead to outcomes which are not individually rational, i.e. some agents may lack incentives to participate in the mechanism when reservation utility constraints are taken into consideration. The outcome in a contest of the sort we examine here will always satisfy individual rationality, since agents have the option of not contributing to the public good.

¹³See Mas-Colell, Whinston, and Green [25], Section 23D, for a discussion of the properties of the mechanism due to d'Aspremont and Gérard-Varet.

4.2 Possibility of a contest in contributions toward the public good

The discussion above on the relation between our model and the Bayesian mechanism design literature suggests the possibility of having agents compete for prizes in terms of amount contributed toward the public good, instead of the competition in private good consumption levels as we have done here. That is, we could equivalently consider a rank-order contest in terms of contributions toward the public good.

The rank-order contest in terms of contributions toward the public good does not, however, have an equilibrium in pure strategies in our model. To see this, consider a contest that rewards higher contributions to the public good. First note that the equilibrium in this case cannot involve a symmetric strategy with a cut off initial endowment level ω_0 below which types do not contribute. Given that types below ω_0 do not contribute, an agent with $\omega < \omega_0$ would have a finite expected gain for an arbitrarily small increase in the amount contributed, hence he is better off not following the strategy which prescribes a zero contribution. A pair of asymmetric strategies, one strictly increasing in initial endowment and the other involving a cut off level ω_0 , cannot be an equilibrium, either. Facing a pure strategy with a cut off level ω_0 , for agents with types $\omega < \omega_0$ any nonzero choice of contribution will be dominated by a smaller contribution. Equilibrium cannot involve a strictly increasing symmetric pure strategy either. The type $\underline{\omega}$ agent has no incentive to distort his contribution from the case when the prize does not matter, since the probability that he wins the contest is zero. He will continue to contribute zero. Given that type $\underline{\omega}$ does not change his contribution, a type infinitesimally higher $\underline{\omega}$ will also have no incentive to distort his contribution from the level with no prize. And similarly for all types $\omega < \omega_0$ that contribute zero in the no prize situation which will also continue not to contribute. But we have just argued that a pure strategy equilibrium cannot involve any types not contributing.

That even simple bidding games may have no equilibria, pure or otherwise, is well known.¹⁴ The difference between the contest in terms of private good consumption levels and contest in contributions to the public good in our model is that the payoff function is not continuous in contributions around zero for all types of agents.¹⁵

5 Discussion and Concluding Words

We studied a public good environment with incomplete information in which agents' decisions to contribute to public good were further complicated by agents' concern for relative position in the group. Depending on their ordinal rank in the distribution of private good consumption levels, financed by

¹⁴See Milgrom and Weber [28].

¹⁵On the existence of Nash equilibrium in games with discontinuities see Dasgupta and Maskin [14], Simon [34], and Reny [31].

their residual wealth after the contribution, agents were assigned prizes. We derived and studied the properties of a symmetric equilibrium for the simple model considered. We showed that this symmetric equilibrium involves strategies linear in initial endowment, and demonstrated how, depending on whether higher private good consumption is awarded or punished, it leads either to lower or higher contributions toward the public good, respectively. We also showed that a proper choice of prize levels in our model will result in the ex-post efficient level of public good.

The utility function we consider as describing the identical preferences of individuals is additively separable in the prize that is distributed through a rank-order contest. This is admittedly a special case, and it partly derives the linearity of the equilibrium strategies that we obtain. We adopted a simple form for preferences for simplicity and tractability in examining the questions we found of interest in the Introduction. We believe that equilibrium strategies in public good environments incorporating concern for relative position can be shown to exist for more general preferences, and is likely to exhibit the qualitative aspects of some of the results derived here. We will investigate more general classes of preferences in future research.

The main force deriving the linearity of strategies in our model is the fact that each agent's maximization problem involves an expression which is quadratic in the choice variable.¹⁶ A particular feature of the equilibrium in our model is that the concern for relative position has only a level effect on agents' strategies. Concern for relative position does not affect the slope of the symmetric equilibrium strategy in Section 3. This property is a consequence jointly of the additive separability and the quadratic nature of each agent's maximization problem.

The role played by the uniformity of the distribution of initial endowments for obtaining linear equilibrium strategies is also apparent from derivation of the equilibrium in Section 3. There will be equilibrium strategies in our model linear in initial endowment for probability distributions other than the uniform distribution.¹⁷

The symmetric linear strategy we exhibited for the n -agent case also involve a specific assumption on the prize structure, namely that prizes are equally spread. That there may be no equilibrium strategy, symmetric or otherwise, for arbitrary prize structures is apparent. An interesting question to ask in our model is to compare an n -agent rank order contest with n prizes and an n -agent rank order contest with only 2 prizes. There will not be an equilibrium linear in initial endowments in Section 3.2 when we consider only 2 prizes, even with uniformly distributed initial endowments. The flexibility offered by the existence of many levels of prizes is partially demonstrated in Proposition 6,

¹⁶The existence and uniqueness of equilibrium strategies in a quadratic decision problem with uncertainty has been extensively studied in optimal control literature. The linearity of the equilibrium strategies for certain probability distributions for the random variable is also well established. On both points see Başar [7]. Also see Başar and Ho [8].

¹⁷See Başar and Ho [8] for existence of linear equilibrium strategies in environments similar to ours. For example, we can consider the form $f(\omega) = \beta\omega + \varepsilon$ for the density function of initial endowments. There will be a symmetric equilibrium for certain values of the parameters β and ε . Note that the uniform distribution is the special case of this form, with $\beta = 0$.

which exhibits a simple linear strategy with evenly spaced prize levels.

In Section 4.2 we discussed the possibility of having a rank-order contest in the amount contributed to the public good. Unlike the rank-order contest in private good consumption levels, we argued that there is no pure strategy equilibrium in our model when the contest is in the amount contributed to the public good. In future research we will address the issue of existence of mixed strategy equilibria in such cases.

The contribution games we examined are one-period static games. Alternatively, in an environment similar to ours one can consider a multi-period game in which agents make incremental contributions to the public good each period until they find it optimal to stop and receive a prize according to the ordinal rank of their contribution.

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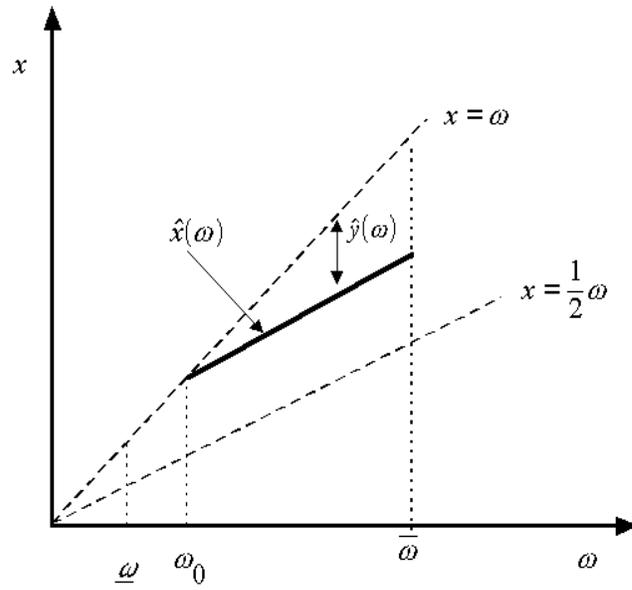


Figure 1:

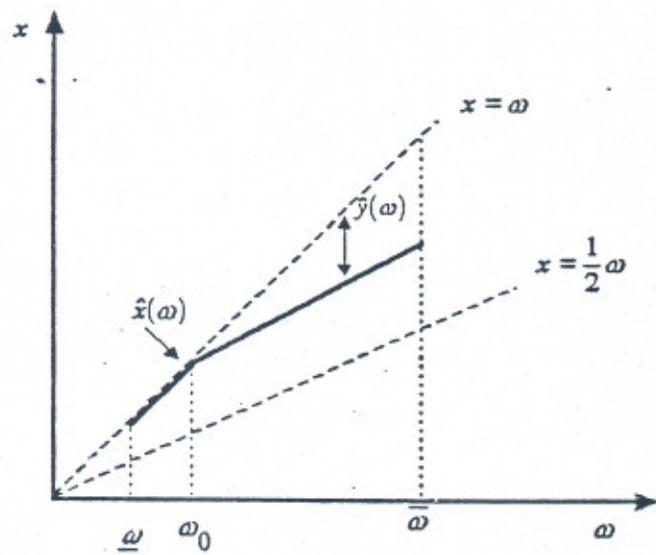


Figure 1: The symmetric equilibrium strategy when $\omega_0 > \underline{\omega}$.

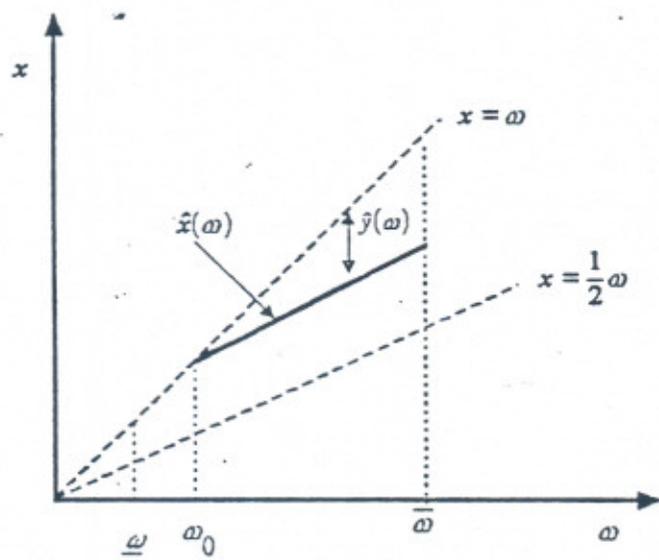


Figure 2: The symmetric equilibrium strategy when $\omega_0 = \underline{\omega}$.

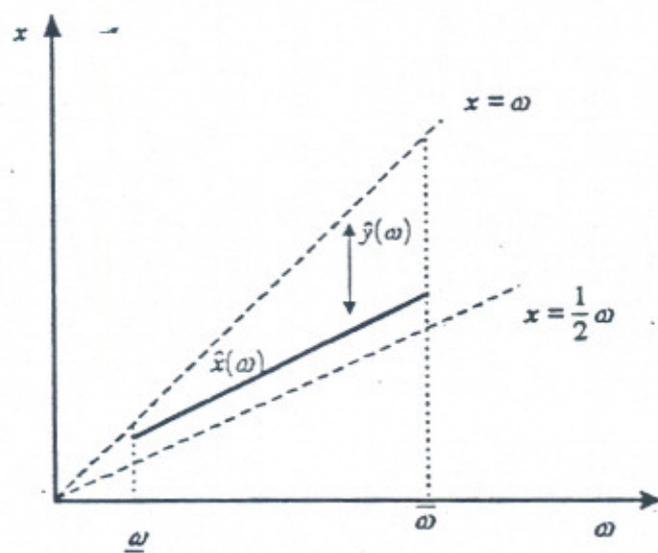


Figure 3: The symmetric equilibrium strategy when $\omega_0 < \underline{\omega}$.