Social security family finance and demography

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Abstract: In this paper we analyzed a model of endogenous fertility in presence of financial market assets and social security pensions. Given the children externality and in the absence of corrective policy, the fertility rate chosen in market economy is too low. Indeed, in his optimal choice of family size, the representative household does not take into account of this children externality which leads to a sub optimal demography. We have shown that an optimal demographic allocation exists and can be implemented through a subvention taxation policy if it is available.
Introduction

In the literature of population growth, two main dominant views of reproductive motivations are presented (Dasgupta, 1993). In the first one, children are considered as durable consumer goods. In the second one, and particularly, in developing countries, children are mainly viewed as investment goods, hence the parents reproductive motivation is thus associated to the so-called old-age security hypothesis (Leibenstein, 1957). According this hypothesis (Jellal, 2000; Jellal and Wolff, 2002), in a socio-economic environment where parents are unsure about their potential ability to support themselves during old age, they may rear children in the expectation of receiving support from their children in their old age (Neher, 1971; Willis, 1980; Nerlove et al. 1987). The underlying transfer mechanism is one of direct reciprocity driven by evolutionary cultural transmission, where parents give first and children give later, without any intergenerational conflict, children are implicitly considered as passive agents under the control of parents, who honor the contract loan by reimbursing their parents during their old-age. This transfer mechanism is self enforcing. Indeed, in the literature of the motives underlying the inter vivos transfers within the family, Jellal and Wolff (2002) have proposed an evolutionary cultural model of filial altruism formation. Moving away from the altruism and exchange hypotheses, they study the endogenous altruism approach in which private transfers are rooted in a purposeful shaping of preferences within the family. The purpose of their paper has been to study the motives underlying upstream inter vivos transfers between the generations by departing from the traditional approach suggested by Becker (1991) with exogenous altruism and the exchange reciprocity hypothesis. Instead, these authors have presented a model of endogenous altruism, where private transfers are aimed at shaping preferences within the family. Rather than relying on the so-called demonstration effect theory, they have presented a model of cultural
transmission of filial altruism in which individuals can be of two types, altruistic or non-altruistic with respect to their parents, and the probability of a child being altruistic is positively influenced by the amount of upstream care shown by parents. Mainly, they have shown that only the altruistic type of preferences survives evolutionary selection, and hence the cultural system converges in the long run to a population having kin obligations. Further the authors present an empirical analysis of a French tri-generational survey which indicates that only the predictions of the cultural model of filial altruism are borne out by the data, while alternatives approaches (exogenous altruism, exchange) are rejected. In particular, helping one’s elders is an effective way to receive services in old age from one’s children. Further, the old-age security hypothesis has been strongly supported by many empirical studies in developing countries, either in Africa or in Asia (Nugent, 1985). For instance, in the case of Malaysia, Raut (1996) has shown that the probability that parents will rely on the support of their children when they are old is significantly lower for households having high income, with better access to private pensions and other financial assets. Therefore, we infer that, when parents consider children as investment goods, the development of social security policy or financial market are expected to lower fertility and leading to a demographic transition. Indeed, Nugent and Gillaspy (1983) and Entwisle and Winegarden (1984) has shown that the demand for children tends to decrease with the public pension programs development,

Hence it seems that the presence of social security tends to substitute public for private transfers as a support for old age and thus provided motivations to have fewer children. However, having few children per household create a large negative externality on the social security pension. Indeed, when deciding to choose optimally the optimal size of his family, the representative household does not take into account this externality; this fact
may be harmful for the social security system viability in a decentralized economy.

The main purpose of the present paper is to shed further light about the links between the desire of fertility, financial market, private transfers within the family and social security. Since social security pension induce a demographic externality, we show that the optimal family size is suboptimal. Therefore, in order to correct this distortion, we propose a mechanism of taxation which restores an optimal demography.

Our paper is organized as follows: Section 2 presents a simple model, Section 3 analyses suboptimal fertility choice under market economy, the implementation of optimal centralized fertility allocation is presented in section 4 and Section 5 concludes.

2. The Model

We consider a simple two-period model economy. Economic agents maximize an inter-temporal utility of their current and future consumption. Young agents work, earn labor wage and by filial obligation and cultural transmission, provide an exogenous private transfer to their old parent. Following the old-age hypothesis, we suppose that the young individuals rear children as investment goods. Although their fertility is costly in terms of forgone labor income, when they become old, they receive a transfer from their Childs. We suppose that the government subsidies their rearing cost by balancing his budget through a flat tax. Further, to be eligible for social security pension, the young agents pay a proportional tax on their labor income, which is used by the government to
finance a public pension system and save on the financial market to smooth their consumption profile. 

The representative young agent maximizes the following inter temporal utility function:

\[ V = U(c_t) + \beta U(c_{t+1}) \]  

(1)

where \( c_t \) and \( c_{t+1} \) represent respectively the consumption profile in youth and in old age for an individual born at time \( t \) and \( \beta \) is the standard discount factor. 

This representative agent maximizes his utility taking into consideration the following budget constraints:

\[
    c_t + s_t + T_t = w_t (1 - \theta - m - (1 - \tau) \psi(n_t)) \\
    c_{t+1} = w_{t+1} n_t m + (1 + r_{t+1}) s_t + p_{t+1}
\]

Where \( \theta \) is the pension contribution fixed rate paid by all young individuals, \( m w_t \) is the transfer to the parents, while \( \psi(n_t) \) denotes the fertility convex function cost or the cost of raising \( n_t \) children, which is supposed subsided at fixed rate given by \( \tau \), \( s_t \) is saving amount and \( T_t \) is a flat tax collected by the government to finance the granted subvention. In their second period of her life, the representative agent receives a gross interest factor \((1 + r_{t+1})\) on her optimal saving. Further, according to the self enforcing old age norm, the agent receives also a fixed transfer from each of her child \( w_{t+1} n_t m \) and a public pension of amount \( p_{t+1} \).

The government budget constraints are presented as follows:

\[
    T_t = \tau w_t \psi(n_t) \\
    p_{t+1} = \theta w_{t+1} \overline{n_t}
\]

Where \( \overline{n_t} \) is the ex post optimal average fertility rate in the total population, this is this average fertility which represent an intra generation externality.
3. Fertility under Decentralized Market Economy

By taking factor prices, and the average number of children as given, the individual representative of generation $t$ chooses material consumption over the life cycle and fertility to maximize under budget constraints her lifetime utility function by solving the following problem:

\[ \text{Max } V = U(c_t) + \beta U(c_{t+1}) \]  \hspace{1cm} (2)

\[ c_t = w_t(1 - \theta - m - (1 - \tau)\psi(n_t)) - s_t - T_t \]

\[ c_{t+1} = w_{t+1}n_t m + (1 + r_{t+1})s_t + p_{t+1} \]

Ex post \hspace{1cm} n_t = \bar{n}_t

The first order conditions for an interior solution for savings and fertility rate are given respectively by:

\[ U'(c_t) = \beta (1 + r_{t+1})U'(c_{t+1}) \]  \hspace{1cm} (3)

And

\[ w_t(1 - \tau)\psi'(n_t)U'(c_t) = \beta (w_{t+1}m)U'(c_{t+1}) \]  \hspace{1cm} (4)

Of these standard equilibrium conditions we obtain the following first result.

**Proposition 1**

The optimal number of children of the representative household is given by:

\[ \psi'(n_t) = \frac{w_{t+1}}{w_t} \frac{m}{(1 + r_{t+1})(1 - \tau)} \]

Proof

It is obtained by substitution from the equations (3) and (4):

\[ U'(c_t) = \beta (1 + r_{t+1})U'(c_{t+1}) \]

\[ w_t(1 - \tau)\psi'(n_t)U'(c_t) = \beta (w_{t+1}m)U'(c_{t+1}) = \beta (w_{t+1}m) \frac{U'(c_t)}{\beta (1 + r_{t+1})} \]

Thus \[ \psi'(n_t) = \frac{w_{t+1}}{w_t} \frac{m}{(1 + r_{t+1})(1 - \tau)} \]
Since the representative household considers children as investment goods, then according to this first theoretical finding, his optimal family size depends positively on the return from investing in children, and thus on the private transfer $m$ dictated by filial obligation (Jellal and Wolff, 2002), and on the growth of wage, but negatively on the return from his optimal saving option. Further, since the cost of raising children is subsided, this incentive to procreate encourages the representative household to have more children.

To conclude, depending on the extent of the taxation subvention scheme, the optimal number of children may be too low since the household do not take into account the children externality given by the average fertility. The availability of an appropriate taxation policy may lead to an optimal demography which internalizes the population impact on sustainability of social security pension.

4. Fertility under Centralized Economy

In order to examine the sub optimality of demography in presence of market economy, we consider the fertility problem which is implemented by a benevolent planner. The main role of later is to recognize the population externality impact on the viability of social security system. This recognition leads the planner to internalize the positive children externality and choose the optimal fertility rate. Indeed, a benevolent planner is supposed to maximize the representative agent’s utility, and thus solves the following program:

$$\begin{align*}
\text{Max} \ V &= U(c_t) + \beta U(c_{t+1}) \\
c_t &= w_t(1 - \theta - m - (1 - \tau)\psi(n_t)) - s_t - T_t \\
c_{t+1} &= w_{t+1}n_t m + (1 + r_{t+1})s_t + p_{t+1} \\
T_t &= \tau w_t \psi(\bar{n}_t) \\
p_{t+1} &= \theta w_{t+1} \bar{n}_t \\
\text{Ex ante} \quad n_t = \bar{n}_t
\end{align*}$$
The first order optimal conditions for an interior solution for savings and fertility rate are given respectively by:

\[ U'(c_t) = \beta (1 + r_{t+1}) U'(c_{t+1}) \]  \hspace{1cm} (6)

And

\[ w_t \psi'(n_t) U'(c_t) = \beta (w_{t+1}.(m + \theta)) U'(c_{t+1}) \]  \hspace{1cm} (7)

From these optimal equilibrium conditions we obtain the optimal fertility rate given by the following result.

**Proposition 2**

The planner implements the optimal number of children of the representative household which is given by:

\[ \psi'(n_t^*) = \frac{w_{t+1}}{w_t} \cdot \frac{(m + \theta)}{(1 + r_{t+1})} \]

Proof:

It is directly obtained by substitution from:

\[ U'(c_t) = \beta (1 + r_{t+1}) U'(c_{t+1}) \]

and

\[ w_t \psi'(n_t) U'(c_t) = \beta \cdot w_{t+1} \cdot (m + \theta) U'(c_{t+1}) \]

We observe immediately that the fertility rate choice in a centralized economy differs from that chosen in a decentralized economy by the fact that the planner takes into account the positive children externality. Therefore, now, the optimal number of children depends on the fixed rate of pension contribution given by \( \theta \). Hence, higher is this contribution, higher is number of children.

Given this first best allocation, we now examine how the government can use the available taxation policy to make possible the implementation of centralized allocation market economy.
Proposition 3
There exists an optimal tax-subvention scheme which leads to a fertility rate optimal in market economy and this tax is given by the following rate:

$$
\tau^* = \frac{\theta}{\theta + m}
$$

Proof:

The implemented optimal number $n_t^*$ of children of the representative household which is given by:

$$
\psi'(n_t^*) = \frac{w_{t+1}}{w_t} \cdot \frac{(m + \theta)}{(1 + r_{t+1})}
$$

While the number $n_t$ of children chosen in a market economy is given by:

$$
\psi'(n_t) = \frac{w_{t+1}}{w_t} \cdot \frac{m}{(1 + r_{t+1})(1 - \tau)}
$$

Therefore we have $n_t = n_t^*$ if and only if:

$$
\frac{w_{t+1}}{w_t} \cdot \frac{(m + \theta)}{(1 + r_{t+1})} = \frac{w_{t+1}}{w_t} \cdot \frac{m}{(1 + r_{t+1})(1 - \tau)}
$$

This equality holds if and only if we have: $\tau = \tau^* = \frac{\theta}{\theta + m}$.

We observe that optimal tax subvention rate depends only on the parameters of the transfers, those within the family as well as those provided by social security pension. Further, the optimal tax depends positively on the social security contribution rate and negatively on size of private transfers within the family. These results are intuitive. Then, we showed how we can implement an optimal demography size in a market economy if an appropriate taxation subvention policy is available and feasible.
5. Conclusion

In this paper we analyzed a simple model of endogenous fertility in presence of the availability of financial assets and social security pensions. Given the presence of a children externality and in the absence of corrective policy, the fertility rate chosen in market economy is too low. Indeed, in his optimal choice of family size, the representative household does not take into account of this children externality which leads to a sub optimal demography. We have shown that an optimal demographic allocation exists and can be implemented through a subvention taxation policy if it is available.

References


