Why one would accept Voting Theory for Democracy and reject the Penrose Square Root Weights

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Summary

Various scientists under the label of "Scientists for a democratic Europe" (SDE) sent a joint "Letter to the governments of the EU member states" (2007) advising the use of the Penrose Square Root Weights (PSRW) for the EU Council of Ministers. When we compare the SDE letter with Colignatus (2001, 2007b) "Voting theory for democracy" (VTFD) then we find that SDE does not fit voting theory for democracy. Inspection of the material upon which the SDE letter is based also shows a moral choice while the rigorous empirical analysis by Gelman, Katz and Bafumi (2007) is actually misrepresented. So the SDE letter can also be rejected on its own grounds. The PSRW approach seems not valid for (indivisible) individuals but may be applicable for divisible shares in shareholder meetings.

Introduction

Various scientists sent a joint letter to the governments of the EU member states, i.e. "Scientists for a democratic Europe" (2007), henceforth called SDE. A key paragraph of SDE is:

> "The basic democratic principle that the vote of any citizen of a Member States ought to be worth as much as for any other Member State is strongly violated both in the voting system of the Treaty of Nice and in the rules given in the draft Constitution. It can be proved rigorously that this principle is fulfilled if the influence of each country in the Council is proportional to the square root of its population. This is known as 'Penrose's Square Root Law'. Such a system may be complemented by a simple majority of states."

This present paper will consider this paragraph and its statements while using the approach in Colignatus (2001, 2007b) "Voting theory for democracy", henceforth VTFD. The conclusion reached is that acceptance of VTFD implies rejection of SDE, or, conversely, that if you accept SDE then you would reject VTFD. Given that science aspires at unanimity on truth, and in this case on what would be optimal, it will be useful to develop the issue.

It may be superfluous to remark that scientists can only offer advice and that our (democratically elected) parliaments take the decisions. Personally, I find it useful to emphasize the distinction by using the name Colignatus for my scientific work while retaining the original birth name for the other realms in society. The SDE initiative can be appreciated, since, when a scientist notices that arguments in a debate are distorted, it is an option to provide unsollicited advice. In the same way, if advice was provided once, but you revise your analysis, then it would be proper to notify the advised body of this revision. The overall issue of the role of scientific advice for economic policy making has been discussed in Colignatus (2005), henceforth DRGTPE, with the suggestion of an Economics Supreme Court, and this is a useful setting for the present discussion as well.

In the following, we will summarize VTFD, review the position of SDE, compare, and finally draw the conclusions. The reason to start this discussion with VTFD is that its approach may be less familiar for proponents of the Penrose Square Root Weights (PSRW).

Summary of VTFD for this discussion

The following summary of VTFD is targetted for the present discussion and will differ a bit from the summary on the book cover text that has an overall and less specific purpose. Points to consider here are:

1. Democracy is rooted in fundamental human characteristics as discussed by Hart (1961, 1967) "The concept of law". To understand democracy we require a rich theoretical setting and a historical understanding of the subject, and it does not suffice to use a simple mathematical model for only one aspect. Voting is seen in the context of an economic system where there are also other instruments such as bargaining and the management of information.
2. The basic notion is the protection of minority rights by veto power (such as property rights), and, for the non-vetoed remainder, the "one man, one vote" approach with proportional representation. The basic decision rule is unanimity to the extent that a person can veto the infringement upon his or her rights. Majority rule can be used when unanimity results into a deadlock (i.e. when more options survive all possible vetos).
3. Key implementation issues concern the multiplicity of parties and options, the possibility of cheating ("strategic voting"), the optimal use of information, the principal-agent problem (does a party what it has been elected for ?), the issue of formation of parties, and bargaining before and after an election. VTFD recognizes this real world situation.
4. Within this general setting, VTFD technically concentrates on solving the "direct single seat election" where voters directly select one item out of a list of items. The voters can be organized in groups that have different weights. In this setting, the principal-agent problem has been defined away. Here, voters select a president discounting the risk that he or she may not keep promisses. The groups are defined by their agreement on a single preference list for the items.

The technical development in VTFD stops here. Thus, VTFD emphasizes the complexity of real world situations but has a technically limited implementation.

5. Though VTFD technically does not discuss indirect elections (multiplicity of seats, formation of parties, principal-agent problem and dynamic consistency, separation of powers, two chambers, the role of information), it is fairly straightforward to extrapolate some results in the case of proportional representation, at least conceptually. It is not clear whether it would be so useful to extend the analysis to indirect elections since these would depend very much on the local ideosyncracies. These local conditions are discussed in the applicable literature.
6. VTFD is embedded in the overall analysis of Colignatus (1992b, 2005). This finds that the role of information is undervalued within Montesquieu's Trias Politica model. A conclusion is that it would be better to create a separate constitutional power, namely an Economic Supreme Court that is rooted in science.
7. A major point in VTFD is that economic theory since Arrow (1950, 1951, 1963) has been misdirected towards impossibilities instead of finding the optimal solution for voting for democracy. VTFD presents a better interpretation for Arrow's difficulty and suggests the solution of the Borda Fixed Point method for direct single seat elections. Economic science has been cynical about democracy and at times openly favouring dictatorship. However, with the misinterpretation of Arrow's finding out of the way, the road is open for more democracy (not only political bodies but also other bodies in society).

Scientists can only advise. Taking account of various properties of voting schemes and empirically observable sentiments it would appear to be a good advice to take the Borda Fixed Point method as the optimal scheme for direct single seat elections. With the above in mind, Coligatus (2007h) is a recent application of this analysis to the recent elections in France. That paper summarizes the main finding as follows:

> "What would be a proper democratic voting system ? Given the widely differing opinions of voters and the long lists of candidates, it is generally advisable to use indirect representative democracy and proportional representation. In parliamentary elections, parties advocate their preferences, the popular vote determines party sizes, whereupon the elected party professionals can arrive at the final choice of the executive by using both bargaining and more complex voting schemes, in particular the Borda Fixed Point method. In this manner the information overload is reduced, both for voters who don't have to think about long lists of candidates and for voting

> mechanisms that don't have to calculate with millions of different preferences. Representative democracy also allows for bargaining that allows for optimal compromises."

Jointly, VTFD and DRGTPE also provide some other insights on democracy. Solving unemployment would be a major contribution towards democracy since people under the fear of unemployment might be misguided by that fear. Conversely, a less democratic system would enhance unemployment. Since there is no economic need for mass unemployment, its existence indicates that (national) decision making is insufficiently democratic. Also, in a bicameral system the first chamber (Senate) should focus more on the defence and abuse of minority veto rights by the second chamber (Congress). Another observation is that, in a democracy, it is not fair to exclude minorities from government. Rather than seeing a dichotomy between a ruling coalition and an opposition, we should regard the executive and the legislative as different functions. Rather than forming ruling coalitions and excluding an opposition, we should have an executive that mirrors all parties (of sufficient size), while the legislative provides for the checks. The prime way to achieve this is to enhance voter power and the popular sense of democracy, such that voters can express disapprovement when parties do not respect minority rights and when parties turn co-operative bargaining in factional power politics.

The best way to increase the power of the electorate and to reduce the principal-agent problem is likely by having annual elections. The current political circus at election times arises from the fact that elections are not held frequently enough, and with annual elections the general attitude will become more matter of fact. The recent French phenomenon of having four elections within a few months (presidential and parliamentary, first round and second round) is grossly inefficient and democratically disruptive compared to a single parliamentary election with proportional representation and Borda Fixed Point selection of the executive by parliament. You may note the different approaches. The current approach starts with the doctrine that executive and legislative have to be chosen by the people at different moments in time. The French then noted that this may result into powers of different political sentiment and national paralysis - the French "cohabitation". (One contributing cause is non-proportionality that can enhance such mood swings.) Subsequently, the French sought a remedy by ordering the elections such that there arises a powerful executive and obedient legislative. The VTFD approach starts with the doctrine that there are more objectives to satisfy so that one should not elevate one objective to a starting point. One condition is proportionality. Another condition is the balance of powers. Another condition is that the powers should

work together and not against each other. Another condition is minimal work and maximal use of information. The solution emerges that has been stated in above quote.

The position of SDE in favour of the Penrose Square Root Weights

A representative body like the EU Council of Ministers indeed faces the situation that Malta with 400,000 inhabitants sits at the same table with Germany with 82,500,000 inhabitants, i.e. 211 times as large. In the US, one might compare California and Hawaii, and in the UN one might compare China or India with Malta again. The multiple seat problem is not the same as the single seat problem and one can admire Lional Penrose for his wonderful analysis and John F. Banzhaf III for rediscovering it.

For an econometrician, the clearest account of the Penrose Square Root Weights is given by Gelman, Katz and Bafumi (2004) who are critical of its assumptions. Slomczynski and Zyczkowski (2007) refer to Gelman et al. (2004) but do not properly answer the criticism. Kirsch (2007 ?) corroborates the finding by Gelman et al. (2004) but opts for the Square Root Weights anyhow. I checked the Gelman et al. (2004) finding and since Kirsch (2007 ?) states that he corroborates their finding as well I have not checked the particulars of Kirsch (2007 ?). Kirsch approaches the issue from the angle of statistical physics, has the SDE letter on his website, and might be regarded as a key proponent in SDE. An early discussion was already by Leech (2002), which paper does not contain a reference to the issue of the assumptions, which issue was subsequently raised by Gelman et al. (2004). These mentioned papers state that the Penrose result was independently rediscovered by Banzhaf, and that all that Banzhaf presented can also be found in Penrose's publication. Personally, I do not have copies of the original publications by Penrose and Banzhaf but above authors (and other texts not all mentioned) are sufficiently unanimous on the description. The Penrose result is called a "Law" by some but it is better to call it an "idea" and it suffices to use the label PSRW.

The Penrose approach switches focus from "one person, one vote" to another condition, namely to "give every citizen of the EU the same influence on decisions" where the latter influence is also called "being decisive" and can also be called "Penrose (voting) power". Note that VTFD uses "voting power" as a fuzzy notion but mainly for the frequency of elections (yearly or four-yearly). Thus "Penrose voting power" should

rather be abbreviated to "Penrose Power" and not to the fuzzy "voting power". Properties are:

1. The focus is on the coalitions that can be formed with a (qualified) majority.
2. There are two stages: one is the population that provides the (square root) weights, and the other are the representatives (or one with the block weight) who apply that (qualified) majority rule (without principal-agent problems).
3. The problem is considered for one issue, assuming that this holds for all issues without interaction. An issue has only two options (yes / no) and there are not more options that would create the problems known from Kenneth Arrow's impossibility theorem.
4. Under independent random voting, the voters get equal "Penrose power" when populations are represented at the second stage with the weight proportional to the square root of the population size. Alternatively, under assumptions that occur in practice (clusters, traditions), proportional weights are optimal (which is the new result of Gelman et al. (2004) and corroborated by Kirsch (2007 ?)).

This summarizes the position of proponents of the Penrose Square Root Weights. There are many more pages and authors but these don't change this core idea. The SDE preference for PSRW comes not only from the moral choice for decisiveness above "one man, one vote" but also from assuming independent random voting.

There are some additional arguments. One is that the square root would be "transparant" and provide a "general rule", stable under any kind of enlargement of the EU, without the need for additional discussion. This seems to neglect that proportionality is a similar transparant general rule. A new argument is by Slomczynski and Zyczkowski (2007) who argue that a 62% qualified majority (also called "quota" by them) would be a fairly consistent requirement for the EU with square root weights. It would hold when weight and "Penrose voting power" are the same. It is not explained why this condition ought to hold either technically or morally.

Gelman, Katz and Bafumi (2004) are critical of the PSRW. The summary of their article can be fully restated: "Voting power indexes such as that of Banzhaf are derived, explicitly or implicitly, from the assumption that all votes are equally likely (i.e., random voting). That assumption implies that the probability of a vote being decisive in a jurisdiction with n voters is proportional to $1/\sqrt{n}$. In this article the authors show how this hypothesis has been empirically tested and rejected using data from various US and European elections. They find that the probability of a decisive vote is approximately proportional to $1/n$. The random voting model (and, more generally, the square-root rule)

overestimates the probability of close elections in larger jurisdictions. As a result, classical voting power indexes make voters in large jurisdictions appear more powerful then they really are. The most important political implication of their result is that proportionally weighted voting systems (that is, each jurisdiction gets a number of votes proportional to n) are basically fair. This contradicts the claim in the voting power literature that weights should be approximately proportional to $\sqrt{n}$."

Kirsch (2007 ?) states: "It does not come as a surprise that we obtain a square-root law for a model with independent voters (...) However as the coupling between voters exceeds a certain threshold, the fairest representation in the council is no longer given by votes proportional to $\sqrt{N_i}$ but rather by votes proportional to N_i. (...) " As said, I checked the results of Gelman et al. (2004), as likely did the referees of the BJPS, and Kirsch (2007 ?) corroborates it.

We should note how the proponents of the Square Root Weights react to the criticism by Gelman et al. (2004).

Slomczynski and Zyczkowski (2007:2) state:

> "Note that this [i.e. their / TC] approach is purely normative, not descriptive: we are interested in the a priori voting power arising from the voting procedure itself. The actual voting power depends on the polarisation of opinion in the voting body and changes from voting to voting (...)".

This reaction is not adequate. Consider an opinion poll on views on climate change. S & Z say that they interviewed 1000 trees and that those were 69.4% against climate change and 5.1% don't knows. You say: "But these are *trees* ! You don't apply a method intended for people to non-people !" S & Z react: "So what, our approach is purely normative, we don't have to check whether our respondents were trees or whatever." Perhaps this example is exaggerated but it indicates the point that one shouldn't confuse the issue of applicability with the issue of morality. A safe conclusion is that these authors don't really respond to the Gelman et al. (2004) criticism. It may be observed that Gelman et al. (2004) also state that proponents of the Square Root Weights are being normative and not empirical, but it would not be correct to reduce their criticism to only that. The point is that a theory also should be adequate before you start applying norms.

Kirsch (2007 ?:14) states:

> "To design a nonhomogeneous voting system for a constitution in the light of our results is a question of different nature. Even knowing the correlation structure of the countries in question exactly would be of limited value to design a constitution. Constitutions are meant for a long term period, correlation structures of countries on the other hand are changing even on the scale of a few years. One might argue that modern societies have a tendency to decrease the correlation between their members. In all modern states, at least in the West, the influence of churches, parties, and unions is constantly declining. In addition to this it seems more important to protect small countries against a domination of the big ones than the other way round. This motivates us to choose a square-root law in these long term cases."

The latter position is more sophisticated than the earlier but the few last lines still are a giant leap from a theoretically small probability model (that takes the body of the paper) towards conclusions on constitutional law for ages to come. Gelman et al. (2004) provide empirical evidence that deviations from the random voting model are the key property of practical elections. Yet, Kirsch (2007 ?) holds *ex cathedra* that any empirical result still would be random (where the qualification "in the long run" is without substantiation and where it is not said that Gelman et al. (2004) would not have constitutional law in mind as well).

A point to observe is that this literature seems to lose the distinction between the two chambers of parliament. The SDE letter concentrates on the EU Council of Ministers but its logic might also apply the the EU parliament. PM. Some papers mention the use of block voting, i.e. that all weight goes for the single vote and cannot be distributed. It would seem that this is not a requirement for the original model, as presented by Gelman, Katz and Bafumi (2004). However, it would hold naturally for the EU Council of Ministers where a Member State has only one voice with a weight.

Another point to observe is that the Penrose model, as restated by Gelman et al. (2004), contains some elements that don't always apply. It uses only two parties and has "winner takes all" district voting. One might want to see this adapted to more parties and proportional representation. Likely this would not change much about the point that parliament could form a majority coalition, but it blends in with the observation that some coalitions may come about more naturally than others. Also, their regression where they corroborate the case of the Square Root Weights under the assumption of random voting still shows a large dispersion so that the square root result also comes about from a specification that has few parameters and few explanatory variables.

Comparison of the two approaches

We discussed the Penrose idea while already referring to alternative points of view. These alternative points were merely provided to clarify the Penrose idea. Let us now consider the critical issues.

Let us first recall the key paragraph of SDE:

> "The basic democratic principle that the vote of any citizen of a Member States ought to be worth as much as for any other Member State is strongly violated both in the voting system of the Treaty of Nice and in the rules given in the draft Constitution. It can be proved rigorously that this principle is fulfilled if the influence of each country in the Council is proportional to the square root of its population. This is known as 'Penrose's Square Root Law'. Such a system may be complemented by a simple majority of states."

With respect to this key paragraph of SDE:

1. It is *not* proven rigorously that equal Penrose power is served by the Square Root Weights. Gelman, Katz and Bafumi (2004), checked by me, likely checked by the referees and editors of BJPS, and corroborated by Kirsch (2007 ?) show that for *practical* situations the equal Penrose power is served by proportionality. Thus, precisely the opposite has been proven rigorously. It may also be observed that key papers of the SDE group, Slomczynski and Zyczkowski (2007) and Kirsch (2007 ?) take positions in morality and propound issues ex cathedra, which is fine in terms of developing hypotheses for futher research but which cannot be presented as results already attained.
2. The basic democratic principle is "protection of minority rights, and for the remainder: one person, one vote". Thus, the basic democratic principle is not "the vote of any citizen of a Member States ought to be worth as much as for any other Member State". It is OK that these authors propose that the EU adopts another principle but it is not OK that they suggest that this already has been adopted. When Penrose wrote his article in 1946 it was an illuminating contribution since it provided another angle on the issue of voting power but it is incorrect to suggest that he changed the principle of "one man, one vote". Indeed, Penrose didn't change much, Banzhaf had to rediscover the issue, and it still is not commonly accepted. Authors on the Penrose Square Root Weights

should stop their rhetoric on what "the" basic democratic principle is and should adapt their statements to its common definition.

3. There is no 'Penrose's Square Root Law'. There is only a mathematical theorem that if you have various assumptions then square root weights would satisfy some criterion. How that criterion and these assumptions relate to democracy is a complex issue, witness this paper.
4. The statement "Such a system may be complemented by a simple majority of states." is confusing and may have been included without proper thought. It is confusing since a state is a unit with a weight of 1. The standard Penrose derivation uses 50% + 1 of the votes and not of the states. The papers considered here don't seem to do this differently (with Slomczynski and Zyczkowski (2007) making it 62% but still using votes and not states).

Next there is the overall relation to the analysis in VTFD.

The SDE misrepresent the issue of the protection of minority rights. These rights are essentially protected by veto power (e.g. property rights) and it would be wrong to forget about veto power and replace this by some weights and a qualified majority rule. For example, take a country with 100 districts with (predominantly) whites and 4 districts with (predominantly) blacks, and assume that there is a problem with discrimination. Then SDE suggest to solve this problem by assigning 10 votes to the whites and 2 votes to the blacks. One wonders why these intelligent people think that this solves anything.

A hypothesis is that the SDE participants mainly come from countries that have a "winner take all" way of thinking and a mentality that emphasizes majority coalition power. To some extent that is merely realistic, since, for example, assigning 100% of the votes to 1% of the population definitely expresses a shift in power. But, properly seen, the SDE approach focusses on a phenomenon that is a distortion of true democracy. They try to meddle with that distortion but actually still allow for it, since in their approach, the reweighted EU Council of Ministers would still be allowed to fully elbow out reweighted minorities. As such behaviour is a distortion of democracy, the best remedy is to refocus on democracy instead of trying to meddle with that distortion.

To understand this, it helps to consider the question why Germany with 82.5 million people would want to sit at one table with Malta with 0.4 million people. There are basic economic issues, like the ease of travel and the prevention of cross-border crime, capital flight, tax evasion, infectious diseases and the like. It is also a derived issue, in that Germany is linked to Italy and Italy is linked to Malta. And there is a sense of common history and destiny. Thus, crucially, Malta has bargaining power, and the weight that it

exerts at the table reflects that bargaining power. From this it follows that once the basic rights of Malta and Germany have been respected, that there is no obvious need to elaborate on the bargaining power that already exists.

The SDE focus on one issue, which is perhaps the best that one can achieve in science with authors who fly out in all kinds of directions, but singling out this issue (and distorting it) is misrepresenting the multiplicity, sizes and order of the real problems in current representative democracy. These rather are: (a) distortion of information (the need for an Economic Supreme Court), (b) the principal-agent problem (frequency of elections), (c) non-proportionality (use of districts) in major countries like France and the UK, (d) the mentality of "winner take all" coalition formation (wrong education on the meaning of Arrow's impossibility theorem), (e) co-ordination of the national parliaments with various international bodies (neglect of Jan Tinbergen's analysis of the optimal order).

The SDE letter also contains these statements:

> "The experts on voting theory agree that the Treaty of Nice gives too much power to a number of countries while others obtain less power than appropriate."
>
> "We urge our politicians to take into consideration the contribution of the scientific community to this issue."

These statements suggest not only that voting theory has reached a unanimous point of view but also that there is a level playing field so that all voices have been heard and all relevant questions answered. Both suggestions are misplaced. The capacity of the scientific community to integrate different approaches is severely limited. Scientific research groups are fragmented, like small fiefdoms with their own agendas and hobbies, in the best case targetted towards publication in journals, but still limited by those conventions and hobbies, and that type of scientist cannot be assumed to be aware of what it takes to run a continent. In fact, this episode again provides an example for the parliaments in the world that it is advisable to consider a constitutional amendment on installing an Economic Supreme Court, to warrant adequate scientific advice.

Speaking as the scientist Colignatus, I can observe that citizen Cool thinks that the issue of national sovereignty is misstated in the EU debate. This particular citizen thinks that it would be wiser to dress down the EU ambitions, have countries first improve their democracy at home (see (a) to (d) above), and have the EU concentrate on creating the economic space with enlargement with Russia and Turkey. For scientist Colignatus these are interesting hypotheses for research and much more important than the PSRW. For

example, when we use the model of a proportionally representative parliament who elects an executive, and apply this to an EU that is mainly an economic space and not a federation, then this issue of "voting by country" does not arise since votes in the economic space would be along party lines and not countries. The crux would be to create such an economic space such that this would be possible.

The square root idea can be used from the angle of information management. If we apply proportional numbers of seats in a parliament then Malta would have 1 seat, Germany 211, and the EU parliament perhaps becomes too large for the sheer size of numbers. We might allow Germany 15 seats with still the total weight of 211, creating manageable numbers of delegates. This might be combined with a shift attention to the Europe of the regions, fostering the reduction of national sentiments and separation along national lines. For purposes like these, the square root is a function that has advantages in transparancy compared to other functions, see Colignatus (2003).

A final point to note is that the Penrose Square Root Weights can also be applied to shareholder meetings in companies. For voting theory we have a sound moral base in individual integrity (excluding animals and plants) and arrive at "one person, one vote". For shareholders we might wonder about the divisibility of money. Shares not only express the shares in profits and losses but may also express voting shares. In this economic realm it might make more sense to use the Penrose voting power concept, and the aspiration of equal decisiveness. It would seem to have economic value to have shareholder voting shares that reflect the Penrose voting power. Holding shares while knowing that you will have no influence due to some other coalition, would be less attractive, unless you are only piggy-backing on the business sense of that other coalition (and then belong to it). Perhaps one can derive the result from a homogeneity condition that voting with a coalition must be like shifting the weights. Thus, the rejection of the PSRW is only limited to representative democracy and not necessarily for other applications.

Conclusions

We considered the SDE "Scientists for a democratic Europe" (2007), "Letter to the governments of the EU member states", and compared this with my book VTFD "Voting theory for democracy" (2001, 2007b).

This comparison shows that SDE does not fit voting theory for democracy.

When we consider the material upon which the SDE letter is based then we find moral choices and a misrepresentation of rigorous empirical analysis. Thus, SDE can also be rejected just for itself.

The comparison with VTFD clarifies that there is little scope to repair the SDE approach and points into directions for useful research.

Literature

Thomas Colignatus is the preferred name of Thomas Cool in science.

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