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# Asymmetric Information and Inefficient Regulation of Firms Under the Threat of Revolution

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## Abstract

This paper considers the role of asymmetric information in a political agency theory of autocratic economic policy-making. Within the context of a static game, we analyze the strategic interaction between an elite ruling class that sets policy and an imperfectly informed disenfranchised class, who may choose to revolt. We identify the Perfect Bayesian Equilibrium (PBE), which need not be pure strategies. We enrich the basic model in an extension that includes two-sided uncertainty and introduces an additional constraint on the elite's policy decision. The extended model features pure strategies in the PBE, which can include inefficient policy choices and revolution. We characterize the equilibrium strategies in terms of the economy's level of development.

*Keywords:* Political transition, Revolution, Asymmetric information, Perfect Bayesian equilibrium

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# 1 Introduction

This paper considers the political economy of economic policy in a non-democracy. We draw upon the classical theory of democratic political agency to model the strategic interaction between a dictatorial elite class, who sets policy to maximize their own utility, and a disenfranchised majority, whose only political power lies in their ability to revolt. The threat of revolution constrains the extent to which the elite can predate the economy, much like the need to stand for re-election mitigates the extent to which elected politicians can extract rent in the democratic political agency literature (Barro, 1973; Ferejohn, 1986).

The notion of a “revolution constraint” on autocratic leaders is not new (Grossman, 1991; Acemoglu and Robinson, 2001), but the idea that dictators are political agents has only recently been introduced. We follow Dorsch and Maarek (2012) and consider the dictator to be the political agent of the disenfranchised majority.<sup>1</sup> In any agency theory, the presence of information asymmetries is what allows the agent to deviate from serving the interests of the principal and extract rent.<sup>2</sup> As in Dorsch and Maarek (2012), we consider a novel informational asymmetry in analyzing autocratic economic policy-making under the threat of revolution. When there is a bad economic outcome, against which the disenfranchised may wish to revolt, the

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<sup>1</sup>Acemoglu *et al.* (2010) is another example that describes the dictator as a political agent, but their paper considers the strategic interaction between the elite class and the military.

<sup>2</sup>For similar models in a democratic setting, see for example, Coate and Morris (1995), Persson and Tabellini (2000), Besley (2006), and Yared et al (2012).

disenfranchised do not know the extent to which the dictator's economic policy is responsible. The effect is that the economic return to a revolutionary political transition is uncertain. Thus, the dictator can extract rent from the economy in excess of what he could under a perfectly informed revolutionary threat. If there were perfect information and the parameters of the game were stable, revolution would never be an equilibrium outcome because policy would be set to satisfy the revolution constraint. With asymmetric information, however, the elite may rationally violate the constraint to extract excess rents, at the cost of provoking revolution with a strictly positive probability.

We follow a series of papers by Daron Acemoglu in modeling the elite's choice between raising predatory revenue directly through taxation or indirectly through distortionary regulation to benefit elite producers (Acemoglu, 2006a,b, 2010). Along with Dorsch and Maarek (2012), we suppose that the indirect method of manipulating factor prices cannot be observed by the working class, who hold the revolutionary threat. In our model, the distortionary regulation limits the size of non-elite firms, which differentiates our paper from earlier work, which consider barriers to entry on non-elite firms. The limit on firm size reduces labor demand and therefore wages, which allows the elite producers to earn abnormal rents. Modeling distortionary regulation in this way seems to be a more empirically accurate description of economic policy in under-developed economies, where the size of firms in the informal sector is small (Djankov *et al.*, 2002; La Porta and Schliefer, 2008).

Our paper considers the strategic interaction between the elite and the disenfranchised working class within the context of a one-period game with asymmetric information. We suppose that the productive potential of the economy is decided by nature, but that nature's choice is known only to the elites. Should the workers observe a bad economic outcome, they are not certain if it was nature's choice or due to an unobservable distortionary policy. Using a Perfect Bayesian Equilibrium (PBE) solution concept, we characterize when the equilibrium can feature inefficient policy choices and revolution.<sup>3</sup>

As in previous literature, one of the key parameters to shape agents' strategies in our model is the destructive cost of revolution. When the cost of revolution is high enough there is a separating equilibrium in which the elite choose the efficient policy and the workers do not revolt. When the cost of revolt is low, on the other hand, the revolution constraint is tighter and the payoff for the elite from distortionary regulation may be higher than that from the efficient policy. If the cost of revolution is low enough, there may not be a pure strategy equilibrium. In this case, we identify the set of mixed strategy equilibria, which we characterize in terms of when revolutions are more likely to occur.

Due to the difficulty of interpreting mixed strategy equilibria in the context of political institutional change, we extend the basic model to include: (i) a resource cost associated with regulating the size of firms and (ii) uncertainty among the elite over the cost of revolution. The model's extensions

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<sup>3</sup>We find the underlying economic dynamics to be more clear in a single-period game as opposed to the multi-period model found in Dorsch and Maarek (2012).

allow us to identify pure strategy equilibria for the case in which previously equilibrium strategies were mixed. Now, we can more fully characterize the conditions under which choosing inefficient economic institutions and revolutions are pure strategy equilibria. We demonstrate a threshold level of economic development (productive potential of the firms), above which the elite choose the efficient method and below which they choose distortionary regulation.

The nature of the information asymmetry is a compelling feature of our model. Imperfect information has, of course, been previously introduced into rationalist models of revolutionary political transitions (Kuran, 1989; Lohmann, 1994; Ellis and Fender, 2010; Bueno de Mesquita, 2010). This literature typically concentrates on the collective action problem among the disenfranchised class, who are imperfectly informed about one another's "type". In these models, revolutions may be triggered by information shocks which facilitate overcoming the collective action problem. By contrast, we focus on the (non-democratic) political agency problem between the elite and the disenfranchised, so the working class is considered as a single player in a game against the elite class. The nature of the information asymmetry in the kind of game we focus on is, therefore, considerably different from how uncertainty has previously been treated in models of revolutionary transitions.

Our paper is organized in the following way. The next section describes the basic feature of our game-theoretic model, which is presented in its extensive form in the third section. The fourth section demonstrates the

game's Perfect Bayesian equilibria, which may include mixed strategies. The fifth section extends the basic model and characterizes the extended game's pure strategy equilibria. A final section concludes with some suggestions for future extensions.

## 2 Economic Environment

The economic environment is essentially a static version of the model in Dorsch and Maarek (2012). There is a continuum of risk neutral agents consisting of a measure  $L$  of workers, a measure  $\theta^h$  of high-productivity (potential) entrepreneurs, and a measure  $\theta^m$  of elites who control the political institution and can also run firms. It is assumed that  $L > \theta^m + \theta^h$  so that workers are a majority of the population and would set policy if the elite were not in power.

Workers hired by the non-elite entrepreneurs have constant productivity of  $A^h$ , while those in elite firms have productivity  $A^m$ . We assume  $A^h > A^m$ , so that workers are more productive in firms run by entrepreneurs than in firms run by members of the elite.<sup>4</sup> In addition, each worker inelastically supplies one unit of labor as long as the wage is above the reservation wage, which we normalize to zero.

Only workers and the elite are considered to be strategic players in the game. The elite act as a group to maximize their total payoff. The game described below will give the payoffs to individual workers. However, since

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<sup>4</sup>This assumption is common in the literature; however, the model can be easily solved when the productivities are equal.

all workers are identical, they will make the same choices. Therefore, we will also think of workers as a single player in the game, acting to maximize their total payoff. Notice that this means that we are not considering any coordination problems facing the workers: if one worker revolts then all workers revolt and the revolution will be successful.<sup>5</sup> The entrepreneurs will start businesses if possible and hire the profit-maximizing amount of labor.

In the previous literature there is usually an exogenously given maximum number of workers per entrepreneur. In this paper, we make this the choice variable for the elite and also the subject of the asymmetric information. Let  $\lambda^e$  be the number of workers that can be employed per elite firm. We will normalize units of labor so that  $\lambda^e = 1$ . The elite also choose the maximum number of workers per entrepreneur,  $\lambda$ , which we interpret as an institutional choice. Rauch (1991), for example, shows that regulation may cause entrepreneurs to move to the informal sector where firms are constrained to be smaller. Smaller firms in the informal sector could also be due to lack of access to resources or public goods. There could also be institutions that create a cost of entry for middle-class entrepreneurs, or even bar entry explicitly as in Acemoglu (2010) and Dorsch and Maarek (2012). Whatever the case, we consider the limit on firm size as a proxy for these kinds of elite regulation of the private sector.

We assume that the workers have incomplete information about the possible values of  $\lambda$  that the elite could choose. Specifically, we assume that there is a maximum possible value of this parameter,  $\bar{\lambda}$ , that is known by the

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<sup>5</sup>Considering an exogenous probability of success, as in Acemoglu *et al.* (2010) and Ellis and Fender (2010) among others, would not change the equilibrium properties.

elite, but not by the workers. This  $\bar{\lambda}$  can be thought of as describing those features of an economy that affect the capacity constraint of the non-elite firms. Loosely speaking, we think of  $\bar{\lambda}$  as summarizing the level of development of the economy, which determines the maximum size of non-elite firms. The elite can impose additional institutional or regulatory constraints on firm size by choosing  $\lambda < \bar{\lambda}$ . The workers are able to infer  $\lambda$  from the equilibrium wage, but are uncertain about whether firms are small because the elite have restricted their size or the economic situation cannot support larger firms.

The payoff to the elite comes from two potential sources. First, they receive profits from the firms they run. This total profit to the elite is  $(A^m - w)$  times the amount of labor hired times the measure of the elite producing, where  $w$  is the wage rate. Elite producers cannot earn a positive profit if  $w \geq A^m$ . If this is the case, then the elite earn a payoff from their second source, which is their ability to tax wage income. When  $w \geq A^m$ , the elite choose a tax rate  $0 \leq \tau \leq 1$  and their payoff includes the tax revenue, which is  $\tau w$  times the measure of workers receiving wage  $w$ . In principle the elite could receive a payoff from both sources. However, given our assumptions, only one of these will be nonzero in a specific situation.

The payoff to a worker when the elite are in power will be their after-tax wage,  $(1 - \tau)w$ . The workers can also choose to revolt at a cost of  $\mu$  per worker. If workers choose to revolt then, since they are a majority, they will choose not to limit the size of firms and not to tax wage income. Therefore, a worker's payoff is  $w^* - \mu$  if there is a revolution, where  $w^*$  is the wage that

would result when  $\bar{\lambda}$  is the maximum size of firms. This post-revolution wage is not known to the workers. If the workers revolt then the elite's payoff is zero.

Given the assumption of constant marginal products of labor, there are essentially three different states of the economy, which are characterized by different values of the equilibrium wage. These states are defined by the values of  $\bar{\lambda}$ ,  $\theta^m$ , and  $\theta^h$ . One can think of the following characterization of the states of the economy as describing what will happen in a democracy with no constraints of firm size.

The first possibility is that the maximum number of workers that can be hired by the entrepreneurs (both elite and middle-class) is less than the number of workers. In other words,

$$\bar{\lambda} \leq \frac{L - \theta^m}{\theta^h} \equiv \lambda_l. \quad (1)$$

If this is true then the equilibrium wage is zero. Dorsch and Maarek call this a “tragedy of development”. Notice that in this case, any choice of  $\lambda$  by the elite will result in  $w = 0$  and, therefore, there will be no tax revenue from a wage tax. This will also be the labor market outcome if there is a revolution and the workers take power. We call this a situation with “low  $\bar{\lambda}$ ” and assume that it occurs with probability  $\rho_l$ .<sup>6</sup>

The second possibility is that the marginal employer is an elite en-

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<sup>6</sup>Alternatively,  $\rho_l$  can be thought to represent the workers' prior probability of this case, i.e. their prior that there is a development tragedy. This state features unemployment even though it does not appear explicitly in the model because wages are zero.

trepreneur, i.e. when

$$\lambda_l \equiv \frac{L - \theta^m}{\theta^h} < \bar{\lambda} \leq \frac{L}{\theta^h} \equiv \lambda_m. \quad (2)$$

In this case the equilibrium wage will be  $A^m$  when there are no additional constraints. Notice that it is possible that only a fraction of the elite are in business. All elite firms producing will earn zero profits. So, in this case, the elite's payoff comes from a tax on wage income unless they choose to limit the size of middle-class firms so that the equilibrium wage falls to zero. We call this a situation with “mid  $\bar{\lambda}$ ” and assume that it occurs with probability  $\rho_m$  (since the equilibrium wage will be  $A^m$  in this case).

The final possibility is that all workers can be hired by high-productivity middle-class entrepreneurs. This will be true if

$$\lambda_m \equiv \frac{L}{\theta^h} < \bar{\lambda}. \quad (3)$$

In this situation, the equilibrium wage will be  $A^h$  and there will be no elite producers. We call this a situation with “high  $\bar{\lambda}$ ” and assume that it occurs with probability  $\rho_h$ .

It will be convenient to consider inequalities (1), (2) and (3) with  $\bar{\lambda}$  replaced by  $\lambda$ . We will call the resulting values of  $\lambda$  low, mid or high, respectively. These result in the different possible equilibrium wages as a function of the choice of  $\lambda$  by the elite. So that if the elite choose low  $\lambda$  then the equilibrium wage is zero, if they choose mid  $\lambda$  then the wage is  $A^m$  and if high  $\lambda$  is chosen then the wage is  $A^h$ .

All of the above, which is assumed to be common knowledge to workers and the elite, defines an extensive form game with incomplete information.

This extensive form is described in the next section along with the notation describing the behavioral strategies of the players.

### 3 Extensive Form of the Game

First, nature chooses the value of  $\bar{\lambda}$ , which is observed by the elite but not the workers, with a probability density given by  $\rho(\bar{\lambda})$ . So the probabilities,  $\rho_l, \rho_m$ , and  $\rho_h$ , defined in the previous section are given by

$$\begin{aligned}\rho_l &= \int_0^{\lambda_l} \rho(\lambda) d\lambda, \\ \rho_m &= \int_{\lambda_l}^{\lambda_m} \rho(\lambda) d\lambda, \text{ and} \\ \rho_h &= \int_{\lambda_m}^{\infty} \rho(\lambda) d\lambda.\end{aligned}\tag{4}$$

Then we let  $E(\bar{\lambda})$  denote the elite's information set when  $\bar{\lambda}$  is revealed. At each such information set the elite choose functions  $\lambda$  and  $\tau$ , where for each  $\bar{\lambda}$ ,  $\lambda(\bar{\lambda}) \in [0, \bar{\lambda}]$  and  $\tau(\bar{\lambda}) \in [0, 1]$ . Note that if the equilibrium wage is zero, i.e. a low  $\lambda$  is chosen, the latter tax rate could be irrelevant. The values  $\lambda(\bar{\lambda})$  and  $\tau(\bar{\lambda})$  are the choices of an elite who observe the state of the economy  $\bar{\lambda}$ , i.e. the behavioral strategy of an elite of type  $\bar{\lambda}$ . We usually suppress the function notation since it will be clear that we are considering a particular  $\bar{\lambda}$ .

However, we can simplify things since from a payoff and strategic perspective, the only thing that matters is if  $\bar{\lambda}$  is either low, mid or high as defined by inequalities (1), (2) or (3), respectively, and whether the elite choose a  $\lambda$  that is low, mid, or high as defined by the appropriately modified versions of these three inequalities. Therefore, we will use the following

notation to describe the three different types of information sets the elite can have.

$$\begin{aligned}
E_l &= E(\bar{\lambda}) \text{ for all } 0 \leq \bar{\lambda} \leq \lambda_l, \text{ i.e. } \bar{\lambda} \text{ is low} \\
E_m &= E(\bar{\lambda}) \text{ for all } \lambda_l \leq \bar{\lambda} \leq \lambda_m, \text{ i.e. } \bar{\lambda} \text{ is mid, and} \\
E_h &= E(\bar{\lambda}) \text{ for all } \lambda_m \leq \bar{\lambda}, \text{ i.e. } \bar{\lambda} \text{ is high.}
\end{aligned} \tag{5}$$

At  $E_l$ , the elite's only possible choice is to set a low  $\lambda$ . At  $E_m$ , the elite can choose either a low  $\lambda$  or a mid  $\lambda$  with a tax rate  $\tau$ . At  $E_h$ , the elite can choose any of the three types of  $\lambda$  with tax rate for mid  $\lambda$  and high  $\lambda$ . Of course, a strategy choice by the elite can also consist of a mixed strategy.

The workers do not know  $\bar{\lambda}$  and do not observe the elite's choice of  $\lambda$ , although they can deduce the latter from the equilibrium wage. What they observe is the equilibrium wage in the economy and the tax rate chosen by the elite. These two things define the workers' information sets, which can be denoted by  $W(w, \tau)$ . If  $w = 0$  then the tax rate is irrelevant and we would have  $W(0, \tau) \equiv W(0, \tau')$  for all  $\tau$  and  $\tau'$ . At each information set, the workers select a probability of revolting,  $r(w, \tau) \in [0, 1]$ . Since there are only three different possible equilibrium wages, 0,  $A^m$ , or  $A^h$ , we can again simplify possible workers' corresponding information sets. Associated with each of the three possible wages we denote the workers' corresponding information set as  $W_l$ ,  $W_m$  and  $W_h$ , respectively. The last two of these information sets should also be indexed by the observed tax rate,  $\tau$ . However, we will find the only tax rates that are consistent with PBE below and so will not introduce specific notation showing that the workers' information sets really also depend on the tax rate. At each information set, workers

choose the probability of revolting, which will be denoted  $r_l$ ,  $r_m$ , and  $r_h$  at the corresponding information sets. Again, technically these probabilities should also depend on the tax rate. We will show how they must do so at a PBE below.

The payoffs of workers and the elite are given by the following. If workers revolt ( $r = 1$ ) then the elite receives zero and workers receive the equilibrium wage that would result with  $\bar{\lambda}$  minus the cost of revolting  $\mu$ . This depends on whether nature has chosen  $\bar{\lambda}$  to be low, mid or high and is a random variable from the workers' point of view. In these three possible cases, a worker's payoff when revolting is  $-\mu$ ,  $A^m - \mu$ , or  $A^h - \mu$ , respectively. When the workers do not revolt, they receive the after-tax wage, which now depends on the  $\lambda$  (and tax rate  $\tau$ ) chosen by the elite. This results in payoffs of 0,  $(1 - \tau)A^m$ , or  $(1 - \tau)A^h$ , depending on whether the elite have chosen  $\lambda$  to be low, mid, or high, respectively. When  $\lambda$  is low, the elite's payoff is their profit from running firms,  $A^m\theta^m$  (remember that  $\lambda^e \equiv 1$ ), since there can be no wage tax revenue. If  $\lambda$  is mid or high then the elite's payoff is  $\tau A^m L$  or  $\tau A^h L$ , respectively, where  $\tau$  is the chosen tax rate which can be different in these two cases. Also, notice that when  $\lambda$  is mid then some elite can be producing. However, any elite producing earns zero profits in this case since  $w = A^m$ . So all of the elite's payoff comes from taxing labor in this case.

The following summarizes the timing of the game, with payoff matrices given for the penultimate period.

1. Nature chooses  $\bar{\lambda}$  with probability density  $\rho$ .
2. Elite observe  $\bar{\lambda}$  with general information sets denoted by  $E(\bar{\lambda})$  or just

$E_l$ ,  $E_m$ , or  $E_h$  when  $\bar{\lambda}$  is low, mid, or high, respectively.

3. At each information set, elite choose a maximum firm size  $0 \leq \lambda \leq \bar{\lambda}$  and tax rate  $0 \leq \tau \leq 1$  with the tax rate irrelevant if  $\bar{\lambda}$  is low.
4. Given the elite's choice of  $\lambda$  and  $\tau$ , the equilibrium wage,  $w$ , is determined and all entrepreneurs demand a profit-maximizing number of workers less than or equal to the maximum ( $\lambda$  for non-elite and 1 for elite).
5. Workers observe the equilibrium wage,  $w$ , and any tax rate,  $\tau$  with general information sets denoted by  $W(w, \tau)$  or just  $W_l$ ,  $W_m(\tau)$ , or  $W_h(\tau)$  for the three possible equilibrium wages, 0,  $A^m$ , or  $A^h$ , respectively.
6. At each information set, workers choose a probability of revolt,  $0 \leq r \leq 1$ .
7. The following summarizes the payoffs in the three possible states of the economy, i.e.  $\bar{\lambda}$  is low, mid, or high, in terms of the pure strategies of the elite and workers. Note that the first expression in each cell is the elite's payoff and the second is the payoff to a worker. Remember that the workers do not know  $\bar{\lambda}$  and so their expected payoff depends on their beliefs about  $\bar{\lambda}$  and the strategy choice of the elite.

(a)  $\bar{\lambda}$  is low:

		<b>Workers</b>	
		$r = 0$	$r = 1$
<b>Elite</b>	low $\lambda$	$A^m \theta^m, 0$	$0, -\mu$

(b)  $\bar{\lambda}$  is mid:

		<b>Workers</b>	
		$r = 0$	$r = 1$
<b>Elite</b>	low $\lambda$	$A^m \theta^m, 0$	$0, A^m - \mu$
	mid $\lambda, \tau'$	$\tau' A^m L, (1 - \tau') A^m$	$0, A^m - \mu$

(c)  $\bar{\lambda}$  is high:

		<b>Workers</b>	
		$r = 0$	$r = 1$
<b>Elite</b>	low $\lambda$	$A^m \theta^m, 0$	$0, A^h - \mu$
	mid $\lambda, \tau_m$	$\tau_m A^m L, (1 - \tau_m) A^m$	$0, A^h - \mu$
	high $\lambda, \tau_h$	$\tau_h A^h L, (1 - \tau_h) A^h$	$0, A^h - \mu$

A Perfect Bayesian Equilibrium (PBE) of this game also requires specifying the players' beliefs about where they are in each information set. In general, the workers' beliefs would be given by  $p(\bar{\lambda}|w, \tau)$ , i.e. a probability density of the workers' beliefs about  $\bar{\lambda}$  conditional on  $w$  and  $\tau$ . This would give the workers' probability density over nodes at their information set  $W(w, \tau)$ . However, the only payoff-relevant properties of this density are the probabilities assigned to  $\bar{\lambda}$  being low, mid and high, which can depend on the observed wage of either 0,  $A^m$ , or  $A^h$ . So workers' beliefs can be represented by three triples of probabilities,  $p_j^i$ , where  $p_j^i$  is the probability that workers believe that  $\bar{\lambda}$  is low ( $j = l$ ), mid ( $j = m$ ) or high ( $j = h$ ) when they observe that the wage is 0 ( $i = l$ ),  $A^m$  ( $i = m$ ), or  $A^h$  ( $i = h$ ). In addition, for all  $i$  we must have  $p_l^i + p_m^i + p_h^i = 1$  with each  $p_j^i \geq 0$ . When workers observe  $w = A^h$  they know that  $\bar{\lambda}$  must be high and the elite must have chosen a high  $\lambda$ . Therefore, beliefs must have  $p_h^h = 1$  with  $p_l^h = p_m^h = 0$ .

When workers observe  $w = A^m$  then they know that  $\bar{\lambda}$  cannot be low, i.e.  $p_l^m = 0$  and  $p_m^m + p_h^m = 1$ . When workers observe  $w = 0$  then they know nothing about  $\bar{\lambda}$  and so there are no additional restrictions of the probability vector  $p^l \equiv (p_l^l, p_m^l, p_h^l)$ . Technically, these beliefs should also depend on the tax rate when relevant.

**Definition 1** *A PBE for this game gives a choice of (actually a probability distribution over)  $\lambda(\bar{\lambda})$  and a tax rate  $\tau(\bar{\lambda})$  (if the chosen  $\lambda$  is not low) at each elite's information set,  $E(\bar{\lambda})$ , a choice by the workers of a probability of revolting,  $r(w, \tau)$ , at each of their information sets,  $W(w, \tau)$ , and workers' beliefs,  $p(\bar{\lambda}|w, \tau)$  at each of their information sets, such that*

- *at every information set, the given behavioral strategy is a best reply for that player given that player's beliefs at that information set and the strategies of the other player; and*
- *the beliefs of a player are consistent with Bayes' rule given the strategies and prior probabilities  $\rho$  whenever possible.*

## 4 Perfect Bayesian Equilibria

In this section, PBE are found for all possible combinations of values of the parameters. First we show several properties that a PBE must have.

We start by describing the only possible  $\tau$  chosen at  $E_h$  along with a high  $\lambda$  that is consistent with a PBE. To do this we first describe the workers' best replies at an information  $W(A^h, \tau)$ . If workers observe  $w = A^h$  then they know that  $\bar{\lambda}$  must be high and that the elite have chosen a high  $\lambda$ .

Therefore,  $p_h^h = 1$  and the workers know that if they revolt they can get a wage of  $A^h$  with no taxes. So the workers' best reply will be given by:

$$\left\{ \begin{array}{l} r(A^h, \tau) = 0 \\ r(A^h, \tau) \in [0, 1] \\ r(A^h, \tau) = 1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} (1 - \tau)A^h > A^h - \mu \\ (1 - \tau)A^h = A^h - \mu \\ (1 - \tau)A^h < A^h - \mu \end{array} \right\} \quad (6)$$

In other words, when workers observe the high wage they will revolt with positive probability only if the tax rate  $\tau$  is greater or equal to  $\frac{\mu}{A^h}$ . Note that if  $\mu > A^h$  then the workers will never revolt.

This implies that in a PBE if the elite choose a high  $\lambda$  they will also choose a tax rate of  $\tau_h \equiv \frac{\mu}{A^h}$  since a higher tax results in a revolt that yields zero to the elite and lower rates will yield lower revenue. This means that the only relevant choice in  $E_h$  associated with choosing a high  $\lambda$  is to also choose  $\tau_h$ . This yields payoffs to the elite of either 0 if there is a revolt or  $\tau_h A^h L = \mu L$  if there is no revolt. Worker's receive a payoff of  $A^h - \mu$  whether or not they revolt.

Actually, the workers will never revolt if the elite choose (high  $\lambda, \tau_h$ ) at a PBE. If workers were to revolt with some positive probability (which is a weakly best reply for them) then the payoff to the elite would be less than  $\mu L$  when they choose (high  $\lambda, \tau_h$ ) at  $E_h$ . However, the elite could then achieve a payoff arbitrarily close to  $\mu L$  by choosing a tax rate slightly less than  $\tau_h$ , which would cause all workers not to revolt. Therefore, there cannot be a PBE where the elite choose (high  $\lambda, \tau_h$ ) at  $E_h$  and the workers revolt with a strictly positive probability at  $W(A^h, \tau_h)$ . This proves the following.

**Proposition 1** *At a PBE, if the elite choose high  $\lambda$  at  $E_h$  then they also*

choose the tax rate  $\tau_h \equiv \frac{\mu}{A^h}$ . Furthermore, at such a PBE, when workers observe a wage equal to  $A^h$  they will not revolt, i.e.  $r_h = 0$ .

Next, we show that the elite will never choose a mid  $\lambda$  when  $\bar{\lambda}$  is high at a PBE. Suppose to the contrary that the elite choose mid  $\lambda$  with tax rate  $\tau$  at  $E_h$ . At  $W(A^m, \tau)$ , workers will revolt with positive probability if and only if  $(1 - \tau)A^m \leq p_m^m A^m + p_h^m A^h - \mu$  or  $\tau \geq \frac{\mu - (A^h - A^m)p_h^m}{A^m}$  since  $p_h^m = 1 - p_m^m$  in this case. This last inequality gives the highest tax rate at which there will be no revolt. Also note that  $p_h^m$  by Bayes' Rule depends on the  $\rho_i$  and the strategies the elite have chosen at  $E_h$  and  $E_m$ . Therefore, the elite's highest (conditional) payoff possible when mid  $\lambda$  is selected at  $E_h$  is to also choose  $\tau = \frac{\mu - (A^h - A^m)p_h^m}{A^m}$ . The elite's payoff by doing this will be at most  $\frac{\mu - (A^h - A^m)p_h^m}{A^m} A^m L = \mu L - (A^h - A^m)p_h^m L$ . Since the elite can attain a payoff of  $\mu L$  by selecting (high  $\lambda, \tau_h$ ) at  $E_h$ , they will never select a mid  $\lambda$  at  $E_h$ , which would imply that  $p_h^m > 0$  and result in a payoff less than  $\mu L$ .

Combining this with the earlier result and the fact that a wage of  $A^m$  will not occur when  $\bar{\lambda}$  is high gives

**Proposition 2** *At a PBE, the probability that the elite choose a (mid  $\lambda, \tau$ ) at  $E_h$  is zero. Furthermore, this implies that  $p_h^m = 0$  and therefore  $p_m^m = 1$  at all PBE, i.e. when workers observe a wage of  $A^m$  they must believe that  $\bar{\lambda}$  is mid.*

This implies that, at  $W(A^m, \tau)$ , workers will not revolt if  $(1 - \tau)A^m > A^m - \mu$  or  $\tau < \frac{\mu}{A^m}$ . Define  $\tau_m \equiv \frac{\mu}{A^m}$ , which will be the tax rate if mid  $\lambda$  is chosen by the elite at  $E_m$  at a PBE. So we have

**Proposition 3** *At a PBE, if the elite choose a mid  $\lambda$  at  $E_m$  then they will also choose a tax rate  $\tau_m \equiv \frac{\mu}{A^m}$ . Furthermore, as above, the workers will not revolt at  $W(A^m, \tau_m)$ .*

Finally, we describe what consistency with Bayes' Rule implies about the workers' beliefs,  $p^l$ , at  $W_l$ . To do this let  $q_j^i$ , where  $i, j \in \{l, m, h\}$  denote the probability that the elite choose a  $\lambda$  that is {low, mid, high} [ $j$  is  $l, m$ , or  $h$ ] when they know  $\bar{\lambda}$  is {low, mid, high} [ $i$  is  $l, m$ , or  $h$ ]. The triple  $q^i$  describes the elite's mixed strategy at its information set  $E_i$ . Recall that  $q_l^l = 1$  since when  $\bar{\lambda}$  is low the elite can only choose a low  $\lambda$ . Then we have

$$\begin{aligned} p_l^l &= \frac{\rho_l}{\rho_l + \rho_m q_l^m + \rho_h q_l^h}, \\ p_m^l &= \frac{\rho_m q_l^m}{\rho_l + \rho_m q_l^m + \rho_h q_l^h}, \text{ and} \\ p_h^l &= \frac{\rho_h q_l^h}{\rho_l + \rho_m q_l^m + \rho_h q_l^h}. \end{aligned} \tag{7}$$

The following gives the PBE for various possible values of the parameters. There are 3 cases that describe all of the possible sets of parameters (up to a set of measure zero given by a set of equalities). We also describe the additional possible PBE that arise at the boundary of each case.

**Case 1:**  $\theta^m A^m < \mu L$

At  $E_l$ , low  $\lambda$  is the only possible choice for the elite. At  $E_m$ , choosing (mid  $\lambda, \tau_h$ ) yields a payoff of  $\mu L$  to the elite since the workers know  $\bar{\lambda}$  is mid if they observe a wage of  $A^m$  and will therefore not revolt if the tax rate is  $\tau_h$ . At  $E_h$ , the elite choose (high  $\lambda, \tau_h$ ) as shown above. Since the elite are choosing different policies in each state, we have a separating equilibrium

and the workers know the state by observing the wage. The given policies make not revolting a best reply in each of the workers' information sets. So these policy choices by the elite and the workers not revolting with beliefs given by  $p_l^l = p_m^m = p_h^h = 1$  is a pure strategy PBE in this case.

Note that this is also a PBE when  $\theta^m A^m = \mu L$ . However, in that case, there could also be mixed strategy PBE where the elite could choose a low  $\lambda$  with positive probability at any information set since their payoff is the same. This would require that the workers' beliefs be different since if the wage was zero the workers would not know the state. The beliefs at  $W_l$ , which are given by (7) at a PBE would be determined by the mixed strategy used by the elite and Bayes' Rule. [It would still be the case that  $p_m^m = p_h^H = 1$ .] Any mixed strategy that resulted in beliefs for which not revolting at  $W_l$  is a best reply would be part of a PBE.

**Case 2:**  $\theta^m A^m > \mu L$  and  $\mu > \rho_m A^m + \rho_h A^h$

The second inequality in this case says that the cost of revolting is greater than the expected wage using the prior probabilities of the different states, i.e. the probabilities of whether  $\bar{\lambda}$  is low, mid or high. Note that if the expected wage was high enough, i.e. greater than  $\theta^m A^m$ , then this case would be empty.

With  $\theta^m A^m > \mu L$ , the elite will do better by choosing low  $\lambda$  at  $E_m$  and  $E_h$  if the workers do not revolt at  $W_l$ . If the elite choose low  $\lambda$  at every information set then the workers will always observe a wage of zero. Therefore, the workers must believe that the probability of a given state is

just the probability that nature chooses that state, i.e. for all  $i, p_i^l = \rho_i$ .

Given the second inequality involving  $\mu$ , the workers will therefore never revolt at  $W_l$ . Therefore, we have a PBE where the elite are always choosing low  $\lambda$ , the workers always observe a wage of zero and never revolt. The beliefs must be  $p^l = \rho$  with  $p^m$  and  $p^h$  arbitrary since all information sets where the workers observe a nonzero wage are off the equilibrium path. Actions by workers at these other information sets must be optimal given the beliefs and the first constraint on  $\mu$  guarantees that the elite will not want to deviate from choosing a low  $\lambda$ . These features define the pure strategy PBE in this case.

Notice that this is also a PBE when  $\mu = \rho_m A^m + \rho_h A^h$ . However, in this case, there would also be mixed strategy PBE of the form given in the next case.

**Case 3:  $\theta^m A^m > \mu L$  and  $\mu < \rho_m A^m + \rho_h A^h$**

In this case there is no pure strategy PBE as in the previous two cases. To see this suppose that the workers do not revolt at  $W_l$ . Then the elite will want to choose low  $\lambda$  at every information set. Then the workers' beliefs must be  $p^l = \rho$  and the second constraint on  $\mu$  implies that the workers will revolt at  $W_l$  so that the elite would receive a payoff of zero. If the workers are revolting at  $W_l$  then the elite can do better by choosing (mid  $\lambda, \tau_m$ ) at  $E_m$  and (high  $\lambda, \tau_h$ ) at  $E_h$ , which both yield a payoff of  $\mu L$  to the elite. If this is the elite's strategy then the workers will know that  $\bar{\lambda}$  is low when the wage is zero and will therefore not want to revolt at  $W_l$ . Therefore, there is

no pure strategy choice at  $W_l$  that is part of a Nash equilibrium.

So there must be a PBE with the workers choosing a mixed strategy at  $W_l$ . For this to be true, we must have the expected payoff of revolting being equal to the expected payoff of not revolting when the workers observe a wage of zero. In other words, the workers' beliefs at  $W_l$  must be such that

$$\mu = p_m^l A^m + p_h^l A^h.$$

Using (7) gives

$$\mu = \frac{\rho_m q_l^m A^m + \rho_h q_l^h A^h}{\rho_l + \rho_m q_l^m + \rho_h q_l^h}$$

or

$$(A^h - \mu)\rho_h q_l^h = \mu\rho_l - (A^m - \mu)\rho_m q_l^m. \quad (8)$$

Note that  $A^m > \mu$  by the first inequality constraint in this case and the assumption that  $\theta^m < L$ .

This equation gives the constraint on the elite's mixed strategies at  $E_m$  and  $E_h$ , which are defined by the the probabilities of choosing low  $\lambda$ ,  $q_l^m$  and  $q_l^h$ , at these information sets, that make the workers indifferent between revolting and not revolting at  $W_l$ . These probabilities must be such that the workers' expected wage under democracy conditional on observing a zero wage is equal to the cost of revolting. In general, (8) implies that the elite are choosing mixed strategies when they know  $\bar{\lambda}$  is mid and when they know it is high. However, it is also possible that the elite only mix at one of these information sets.

For example, (8) is satisfied by

$$q_l^m = 0 \text{ and } q_l^h = \frac{\mu\rho_l}{(A^h - \mu)\rho_h}, \quad (9)$$

which says that the elite choose a mid  $\lambda$  at  $E_m$  and mix between a low  $\lambda$  with probability  $q_l^h$  and a high  $\lambda$  (with tax rate  $\tau_h$ ) at  $E_h$ .

Alternatively, (8) is also satisfied by

$$q_l^h = 0 \text{ and } q_l^m = \frac{\mu\rho_l}{(A^m - \mu)\rho_m}, \quad (10)$$

which says that the elite choose a mid  $\lambda$  at  $E_h$  and mix between a low  $\lambda$  with probability  $q_l^m$  and a mid  $\lambda$  (with tax rate  $\tau_m$ ) at  $E_m$ .

In order for the above mixed strategies to be optimal at  $E_m$  and  $E_h$  it must be that a low  $\lambda$  yields the same expected payoff as (mid  $\lambda, \tau_m$ ) at  $E_m$  and the same expected payoff as (high  $\lambda, \tau_h$ ) at  $E_h$ . In other words, we must have

$$(1 - r_l)\theta^m A^m = \tau_m A^m L = \mu L \text{ and } (1 - r_l)\theta^m A^m = \tau_h A^h L = \mu L. \quad (11)$$

Both of these constraints give

$$r_l = \frac{\theta^m A^m - \mu L}{\theta^m A^m} \quad (12)$$

as the probability the workers will revolt when they observe a wage of zero. Notice that, perhaps surprisingly, this probability of a revolt does not depend on the prior probabilities of the various states,  $\rho_i$ . However, these probabilities do affect whether this case or the previous case, in which there is no probability of a revolt, occurs. For example, higher  $\rho_m$  or higher  $\rho_h$ , other things being equal, make this case more likely than the previous case and therefore can cause a jump in the equilibrium probability of a revolt from zero to some positive value.

Equations (8) and (12) define mixed strategies for the elite and workers that are part of a PBE in this case. Substituting a solution to these equations

into (7) gives the workers' beliefs at  $W_l$ . When the workers observe a mid  $\lambda$  they know that  $\bar{\lambda}$  is mid and will not revolt if and only if  $\tau \leq \tau_h$ . When workers observe a high  $\lambda$  they know that  $\bar{\lambda}$  is high and do not revolt if and only if  $\tau \leq \tau_h$ . Since the elite choose tax rates  $\tau_h$  when they choose a mid  $\lambda$  and  $\tau_h$  when they choose a high  $\lambda$ , there will be no revolt when the wage is above zero. These features define a PBE in this case.

The following summarizes the three possible PBE of the game.

1. When the cost of revolt and the economic return of taxation relative to elite productivity is high, i.e.  $\theta^m A^m < \mu L$ , there is a pure strategy PBE in which there is no revolution and all elite types choose taxation if possible.
2. When elite productivity is high relative to the revenue from taxation (i.e.  $\theta^m A^m > \mu L$ ) and the cost of revolt is sufficiently high (i.e.  $\mu > \rho_m A^m + \rho_h A^h$ ), there is a pure strategy PBE in which there is no revolution and all elite choose factor price manipulation.
3. When elite productivity is high relative to the revenue from taxation (i.e.  $\theta^m A^m > \mu L$ ) and the cost of revolt is sufficiently low (i.e.  $\mu < \rho_m A^m + \rho_h A^h$ ), there are only mixed strategy PBE in which workers choose to revolt with positive probability of revolution and some elites choose factor price manipulation with positive probability.

Mixed strategy equilibria are a bit difficult to interpret since it is unclear what exactly it means for the elite to mix between different limits on firm size. Another interpretation of the  $q_l^i$  given by (8) is that it represents the

fraction of the elites in  $E_i$  choosing a low  $\lambda$ . So, all elites choose a pure strategy; however, not all of them observing the same type of information choose the same strategy. For example,  $q_t^h$  is the fraction of all elites observing a high  $\bar{\lambda}$  who choose a low  $\lambda$ . The other  $1 - q_t^h$  of elites observing a high  $\bar{\lambda}$  would choose not to restrict firm size. This is also not completely satisfactory since it is unclear what determines whether an elite observing a high  $\bar{\lambda}$  chooses to restrict firm size or not. Similarly, the workers' probability of revolt given by (12) is difficult to interpret.

In the next section we add incomplete information about the value of the cost of revolting,  $\mu$ , and a cost of regulation. These additional features will result in pure strategy PBE existing in all cases.

## 5 Incomplete Information about $\mu$

In this section, we make the cost of revolution,  $\mu$ , unknown to the elite. This implies that there is always a probability of revolt, unlike in the previous model in which the elite can choose a tax rate that makes the probability of revolt zero. It is also more realistic that the elite do not know the workers' cost of revolt. Things like the cost and ease of coordination or the extent of the workers' distrust of the elite, which can be included in the cost of revolt, are clearly better known to the workers than to the elite.

Formally, we assume that the elite only know the probability distribution function,

$$F(x) \equiv \int_0^x f(\mu) d\mu \tag{13}$$

giving the probability that  $\mu \leq x$ . We will assume that  $F(0) = 0$  so that

$\mu > 0$  with probability one. In addition, we will assume that  $F(A^m) = 1$ , which guarantees that elite with mid or high  $\bar{\lambda}$  will choose a tax rate less than 1 when they choose not to restrict firm size.

Workers know the value of  $\mu$ . So, we now have to index the workers' information sets by  $\mu$ . Therefore,  $W(w, \tau, \mu)$  now represents an information set of the workers. At such an information set, the workers will revolt only if  $p_m A^m + p_h A^h - \mu \geq (1 - \tau)w$ , where  $p_m$  and  $p_h$  give the conditional probability given  $w$  that the workers believe that the equilibrium wage will be  $A^m$  or  $A^h$ , respectively, after a revolt.

Additionally, we now introduce a cost for the elite of restricting firm size. Limiting firms to a size below their potential requires more regulation and costly monitoring. We suppose that the cost is strictly increasing in the difference between the productive capacity of non-elite firms and the size of firms that is desired by the elite. That is, the regulatory cost is higher the more the elite restrict firm size below  $\bar{\lambda}$ . We capture this regulatory cost of limiting firms to  $\lambda$  when the economy is characterized by  $\bar{\lambda}$  by a strictly increasing continuous function  $c(\bar{\lambda} - \lambda)$ . Assume that  $c(0) = 0$  so that if the elite impose no restriction on firms there is no regulatory cost.

The elite's payoff is now reduced by the regulatory cost of achieving their chosen  $\lambda$ . This means that the elite will choose the  $\lambda$  that results in their desired outcome and has the lowest regulatory cost. Therefore, if  $\bar{\lambda}$  is already of the desired type (i.e. low, mid or high) then the elite will chose  $\lambda = \bar{\lambda}$  and incur no cost. If the elite want to limit firm size then they will pick the highest  $\lambda$  consistent with the desired equilibrium wage. In other

words, if the elite want to restrict firm size to a low  $\lambda$  they will pick  $\lambda_l$  and if they want to restrict to a mid  $\lambda$  they will pick  $\lambda_m$ .

When the elite are uncertain about the value of  $\mu$ , Propositions 1-3 do not directly apply. It will no longer be true, as in Propositions 1 and 3, that elite with high  $\bar{\lambda}$  and mid  $\bar{\lambda}$  can prevent revolution by choosing a conciliatory tax rate. However, we demonstrate below that one implication of these propositions continues to hold. Namely, elites at  $E_e$  and  $E_H$  who choose not to restrict firm size have the same expected payoff. We also show that Proposition 2 still holds when the elite are uncertain about the value of  $\mu$  and there is a regulatory cost associated with restricting firm size.

First, consider an elite at an  $E_h$ . Remember that when the wage is  $A^h$  the workers know that  $\bar{\lambda}$  is high and therefore  $p_h = 1$  at any  $W(A^h, \tau, \mu)$ . This implies that workers revolt at such information sets whenever  $\mu \leq \tau A^h$  and therefore the elite choosing a high  $\lambda$  will face a probability of a revolt of  $F(\tau A^h)$ . So at an  $E(\bar{\lambda})$  with  $\bar{\lambda}$  high, the elite's expected payoff of choosing  $(\bar{\lambda}, \tau)$  is  $[1 - F(\tau A^h)]\tau A^h$ . This means the elite's maximum expected payoff when they do not restrict firm size and choose a tax rate of  $\tau$  is given by

$$\max_{\tau} [1 - F(\tau A^h)]\tau A^h L. \quad (14)$$

Let  $\tau_h$  be the tax rate that solves this problem.

Now consider an elite at an  $E_m$  and assume that when workers observe a wage of  $A^m$  they believe that  $\bar{\lambda}$  is mid, i.e.  $p_m^e = 1$ . [We will show that this is in fact true.] Therefore, workers will revolt if  $\mu \leq \tau A^m$  and the probability of revolt facing an elite is  $F(\tau A^m)$ . So such an elite's maximum expected payoff when they do not restrict firm size and choose a tax rate of  $\tau$  is given

by

$$\max_{\tau} [1 - F(\tau A^m)] \tau A^m L. \quad (15)$$

Let  $\tau_m$  be the tax rate that solves this problem.

Note that problems (14) and (15) are equivalent to the following problem.

$$\Pi^* \equiv L \max_x [1 - F(x)] x. \quad (16)$$

This is true because we have assumed that  $F(A^m) = 1$ . Note that the function being maximized in (16) is continuous and is zero at both  $x = 0$  and  $x = A^m$ . This means that the solution,  $x^*$ , to (16) occurs at  $0 < x^* < A^m$ . Therefore, the solution to (15) will be  $\tau_m \equiv \frac{x^*}{A^m}$  and the solution to (14) will be  $\tau_h \equiv \frac{x^*}{A^h}$ .

Therefore, if  $p_m^m = 1$  then elite at an  $E_m$  and  $E_h$  will have the same expected payoff,  $\Pi^*$ , if they choose not to restrict firm size. The condition  $p_m^m = 1$  will be satisfied if elite with a high  $\bar{\lambda}$  never select a mid  $\lambda$ . We next show that this is true as in Proposition 2.

Suppose the elite at  $E_h$  choose a mid  $\lambda$  with tax rate  $\tau$ . In this case, workers observing a wage of  $A^m$  will revolt if  $(1 - \tau)A^m < p_m^m A^m + p_h^m A^h - \mu$  or, equivalently if  $\mu < \tau A^m + (A^h - A^m)p_h^m$ , since  $p_m^m + p_h^m = 1$ . Therefore such an elite's expected payoff is  $[1 - F(\tau A^m + (A^h - A^m)p_h^m)] \tau A^m L$  and their maximum payoff associated with a mid  $\lambda$  is

$$\max_{\tau} [1 - F(\tau A^m + (A^h - A^m)p_h^m)] \tau A^m L. \quad (17)$$

Note that this has the form

$$L \max_x [1 - F(x + a)] x, \quad (18)$$

where  $a > 0$ . Since  $F$  is increasing, the maximum value of (18) will be less than the solution to (16). This means that if a positive measure of elite with a high  $\bar{\lambda}$  choose a mid  $\lambda$ , so that  $p_h^m > 0$ , then the payoff of an elite at  $E_h$  choosing not to restrict firm size will have a higher expected payoff than an elite at  $E_h$  who chooses a mid  $\lambda$ . So, at a PBE, no elite will choose a mid  $\lambda$  at an  $E_h$ .

Therefore, Proposition 2 holds in our extended model. This also implies that the previous argument showing that elites with both mid and high  $\bar{\lambda}$  have the same payoff,  $\Pi^*$ , applies.

One feature of Propositions 1 and 3 that no longer holds is that there is always a positive probability of revolt in the extended model. Since  $\mu$  was common knowledge in the baseline model, the elite could prevent revolution by choosing a conciliatory tax rate that satisfies the revolution constraint. When  $\mu$  is uncertain for the elites, that is not possible. In the extended model, the payoff-maximizing tax rates are associated with a strictly positive probability of revolt.

The value  $\Pi^*$  will be now used to characterize the different types of PBE. At each PBE each type of worker and elite will choose a pure strategy, unlike in the previous section where there are only mixed strategy PBE for certain parameter values. Workers will choose to revolt at  $W(w, \tau, \mu)$  iff  $\mu < \text{expected wage after a revolt minus } (1 - \tau)w$ , where the expected wage depends on the workers' conditional beliefs about the true value of  $\bar{\lambda}$ . These conditional beliefs are given by Bayes' Rule and the elite's strategy in the PBE. The elite with a low  $\bar{\lambda}$  will choose to make no change. The elite at

$E(\bar{\lambda})$  with  $\bar{\lambda} > \lambda_l$  will choose either not to restrict firm size by selecting  $\lambda = \bar{\lambda}$  with tax rates  $\tau_m$  if  $\bar{\lambda}$  is mid or  $\tau_h$  if  $\bar{\lambda}$  is high, or choose to restrict firm size by selecting  $\lambda = \lambda_l$ . The following describes when each of these strategies is chosen.

Our characterization will also depend on the following function. Define

$$\Pi(\ell) \equiv \left[ 1 - F \left( \int_{\lambda_l}^{\min\{\ell, \lambda_m\}} \rho(\lambda) A^m d\lambda + \int_{\lambda_m}^{\max\{\ell, \lambda_m\}} \rho(\lambda) A^h d\lambda \right) \right] \theta^m A^m - c(\ell - \lambda_l). \quad (19)$$

This function gives the expected payoff of an elite at  $E(\ell)$  choosing to restrict firms size by selecting  $\lambda_l$  assuming that all types of elite with  $\bar{\lambda} \leq \ell$  also choose  $\lambda_l$  and all types of elite with  $\bar{\lambda} > \ell$  choose not to restrict firm size. The argument of the distribution  $F$  is the expected wage given the equilibrium beliefs of workers observing a zero wage when all types of elite with  $\bar{\lambda} \leq \ell$  choose a low  $\lambda$ . Note that  $\Pi(\lambda_l) = \theta^m A^m$  and  $\Pi$  is a decreasing continuous function.

**Case 1:**  $\theta^m A^m \leq \Pi^*$

At all  $E(\bar{\lambda})$  with  $\bar{\lambda} \leq \lambda_l$ , low  $\lambda$  is the only possible choice for the elite. For the other elite types, this case says that  $\Pi^*$  is larger than what such elite would obtain by choosing a low  $\lambda$ . Therefore, at  $E(\bar{\lambda})$  with  $\lambda_l < \bar{\lambda} \leq \lambda_m$ , the elite choose  $(\bar{\lambda}, \tau_h)$  and at  $E(\bar{\lambda})$  with  $\bar{\lambda} > \lambda_m$ , the elite choose  $(\bar{\lambda}, \tau_h)$ .

This gives essentially the same kind of PBE as in the case where  $\theta^m A^m < \mu L$ . All elite choose not to restrict firm size and the workers know the state by observing the state. Here all elite choose  $\lambda = \bar{\lambda}$ , which need not occur

in the previous section since all  $\lambda$  of the same type are payoff equivalent. Here, the regulatory cost leads to the elite keeping  $\bar{\lambda}$  unchanged. Another difference is that in the previous section there was a zero probability of revolt. Here, except when  $w = 0$ , there is a probability of revolt. Furthermore, elite with mid and high  $\bar{\lambda}$  get the same expected payoff,  $\Pi^*$ , and face the same probability of revolt,  $F(x^*)$ . However, as in the previous section, elite with mid  $\bar{\lambda}$  will have a higher tax rate,  $\tau_m = x^*/A^m$ , than elite with high  $\bar{\lambda}$ , who will select a tax rate of  $\tau_h = x^*/A^h$ .

**Case 2:**  $\Pi^* < \theta^m A^m$  and  $\Pi^* < \Pi(\ell)$  for all  $\ell > \lambda_l$

The second condition implies that all elite with  $\bar{\lambda} > \lambda_l$  have a higher payoff if they restrict firm size to  $\lambda_l$  than if they choose no restrictions on firm size with the payoff-maximizing tax rate. This is analogous to the second case in the previous section, where all elite also choose a low  $\lambda$ . However, in the previous section workers never revolt while here there is a positive probability of revolt.

**Case 3:**  $\Pi^* < \theta^m A^m$  and  $\Pi^* \geq \Pi(\ell)$  for some  $\ell > \lambda_l$

Since  $\Pi$  is a decreasing continuous function with  $\theta^m A^m = \Pi(\lambda_l) > \Pi^* \geq \Pi(\ell)$  for some  $\ell$ , there exists a  $\lambda^* > \lambda_l$  such that  $\Pi(\lambda^*) = \Pi^*$ . For all types of elite with  $\bar{\lambda} < \lambda^*$  their payoff when choosing  $\lambda_l$  is greater than their payoff from not restricting firm size, which is  $\Pi^*$ . All types of elite with  $\bar{\lambda} > \lambda^*$  have a higher payoff when not restricting firm size than if they choose  $\lambda_l$ , which yields less than  $\Pi(\lambda^*)$  since the regulatory costs of such elites is larger than for the type with  $\bar{\lambda} = \lambda^*$ .

This case corresponds to the case in the previous section that had no pure strategy PBE. However, here we have all players choosing pure strategies at every information set. All types of elite in economies below some level of development (i.e.  $\bar{\lambda} < \lambda^*$ ) choose to restrict firm size, while types with more developed economies choose not to restrict firm size. In effect, from the point of view of the workers this is like selecting a particular type of mixed strategy satisfying the analogue of the constraint (8). The  $\lambda^*$  defines a fraction of elite with mid  $\bar{\lambda}$  that restrict firm size. If this fraction is less than one then all types of elite with high  $\bar{\lambda}$  do not restrict firm size. The fraction of types of elite with high  $\bar{\lambda}$  that restrict firm size is strictly positive only if the fraction of types of elite with mid  $\bar{\lambda}$  that restrict firm size is one. These fractions of these two types of elite restricting firm size determines the workers' expected post-revolution wage when they observe a wage of zero. In equilibrium this expected wage must be such that the probability of revolt faced by the elite makes the type of elite with  $\lambda^*$  indifferent between restricting firm size and not restricting firm size.

## 6 Conclusion

As in Dorsch and Maarek (2012) we have introduced incomplete information into a political economy model of economic policy in a non-democracy. However, our model differs from that paper and the other previous literature along several dimensions. Our model is a static two-player game with incomplete information rather than a repeated game. The uncertainty is about the potential size of firms (which we interpret as representing the

level of development or productive potential of the economy) rather than about the size of an entrepreneurial middle class. In addition, we consider an extension in which the elite are also uncertain about the cost of revolt to the workers. Our equilibrium concept is perfect Bayesian equilibrium rather than Markov perfect equilibrium. In our basic model, we show that it is possible that there are no pure strategy equilibrium in certain situations. Our extended model with two-sided uncertainty also includes a regulatory cost of restricting the size of firms. This cost is increasing in the amount by which firm size is reduced. This enables us to show that a pure strategy PBE always exists in the extended model. These equilibria have the property that there exists a level of development such that if the level observed by the elite is less than this amount then the elite choose distortionary regulation while if they know the level of development is higher then they choose the efficient policy.

Since we are thinking of the parameter  $\bar{\lambda}$  as representing the level of development of an economy it would be useful to consider a repeated game where the parameter evolved over time and/or was endogenously determined. This would allow one to examine how the institution choice of the elite evolved over time. One can get a rough sense of what might happen in this situation by examining how the equilibrium changes in our model as the expectation of the probability distribution  $\rho$  increases. This would place more weight on states of the economy with higher equilibrium wages, which would increase the expected wage of the workers after a revolution and thus make revolts more likely. This would make the first case in each of our mod-

els less likely, other things being equal. Therefore, one would expect to see more equilibria where the elite choose the efficient policy as the productive potential of the economy increases. This is similar to our interpretation of the equilibrium with the threshold level in our extended model. However, it would be useful to have a more explicit dynamic model with such a results.

Another possible extension that might be interesting is to consider a game where the middle-class entrepreneurs are also strategic players. This would make the model similar to Acemoglu (2010) where he has two competing groups of elites, but no workers. One could ask questions like when and/or whether the middle-class will support the elite against the workers or support the workers in a revolt against the elite. In the basic model in this paper, the middle-class would prefer to have restrictions that limit the wage to  $A^m$  (or less) since they make no profits when  $\lambda$  is high and the wage is  $A^h$ . So they would seem likely to support the elite to prevent a revolution when  $\bar{\lambda}$  is high. However, when  $\bar{\lambda}$  is low (or mid) the middle-class would do better under democracy than under the elite with a restriction on firm size. So, in such cases, it appears that the middle-class might support the workers against the elite. It would be interesting examine this issue of coalition formation more precisely in our model.

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