Rational and mechanics of a peak risk variance swap for a property insurance portfolio

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Introduction and motivation

In this technical report we explore the motivation, structuring and actual mechanics of a variance swap contract adapted for a property insurance portfolio. We structure, price and test sensitivities of the swap contract using real historical and modeled loss data. Our key motivation is in proposing an element of financial engineering innovation to insurance portfolio risk management to allow for constructing hedging strategies that may not be possible to achieve with traditional reinsurance treaties and contracts.

Portfolio structure with minimum and required solvency capital reserves

Our exposure at risk is a 10% market share portfolio, spatially distributed in all 90 CRESTA zones of the Netherlands; where in each CRESTA the portfolio insured value is exactly 10% of the insured residential building industry exposure.

The minimum capital reserve $Q_{ep=0.02}$ for the portfolio is defined as the level of stochastic loss equivalent to the 50 year return period in a 10,000 year stochastic simulation, i.e. loss with 2% (0.02) exceedance probability, which we notify as $EP_{0.02}$. The required capital reserve $K_{ep=0.01}$ is defined as equivalent to stochastic loss of the 100 year return period (YRP) in the same 10,000 year stochastic simulation, i.e. loss with 1% $EP_{0.01}$ exceedance probability.

Capital reserve as well as exceedance probability loss is non-additive from policy, line-of-business, or CRESTA to portfolio total level. In our case each CRESTA is a line-of-business and is covered by an individual policy. The non-additive property for stochastic losses at $EP_{0.02}$ and $EP_{0.01}$ is observed:

$$portfolio\ Q_{ep=0.02} > \sum_{cresta\ 1}^{90} policy\ Q_{ep=0.02}$$

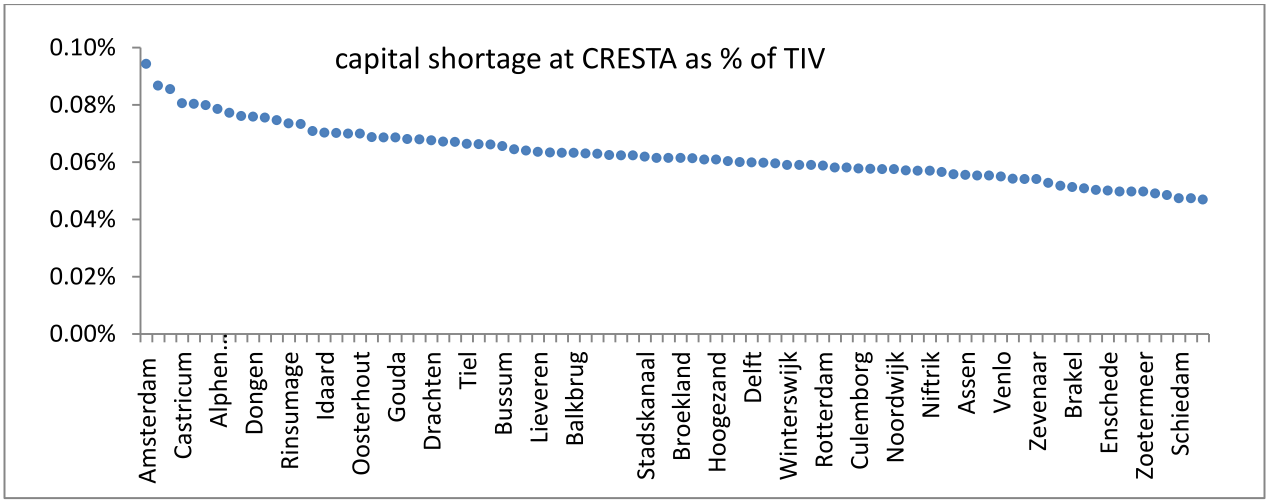
$$portfolio\ K_{ep=0.01} < \sum_{cresta\ 1}^{90} policy\ K_{ep=0.01}$$

The portfolio stochastic loss $portfolio\ Q_{ep=0.02}\ and\ portfolio\ K_{ep=0.01}$ is used to set the level of respectively minimum and required capital reserves. The difference between these two levels of capital reserve is estimated as percentage of total portfolio insured values $portfolio\ TIV$

$$\frac{portfolio\ K_{ep=0.01} - portfolio\ Q_{ep=0.02}}{portfolio\ TIV} = 0.0558\%$$

Table 1 shows the distribution of individual policy level capital reserve differences – shortages, defined as: $\frac{Policy\ K_{ep=0.01} - policy\ Q_{ep=0.02}}{Policy\ TIV}$

Figure 1: Capital shortage by policy - CRESTA as % of total insured values by policy



Capital reserve shortage swap and a reinsurance contract

A financial product or a reinsurance contract that can provide coverage for the solvency capital shortage in case of catastrophe claims exceeding our minimum capital amount of $Q_{ep=0.02}$ up to the required level of capital reserve $K_{ep=0.01}$ will be needed for this portfolio.

We divide the total portfolio capital shortage into five tranches using the stochastic exceedance probability curve in the interval: $[EP_{1.00\%} - EP_{2.00\%}]$, where the length of each interval is equal to $EP_{0.20\%}$. For each tranche we estimate loss cost, capital shortage and expected tranche loss as % of tranche volume - $E_{A,..,E}(X)$. We use standard notification as *attachment point (loss)* and *exhaustion point (loss)* to indicate the tranche loss boundaries.

$$Loss\ Cost = \frac{Exhaustion\ loss}{TIV}$$

$$Capital\ shortage = \frac{Exhaustion\ loss - Minumum\ reserve}{TIV}$$

$$E_{A,\dots,E}(X)\% = \frac{Expected\ loss\ for\ tranches(A, \dots, E)}{Exhaustion\ Loss(A, \dots, E) - Attachment\ Loss(A, \dots, E)}$$

$$E_A(X) = \sum_{ep=1.0\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep} - \sum_{ep=1.2\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep}$$

And expected loss for the full tranche $E(X)$

$$E(X) = \sum_{ep=1.0\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep} - \sum_{ep=2.0\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep}$$

Table 1; Capital reserve shortage by tranche parameters

E.P.	Loss cost	Capital shortage	Tranche	Expected loss as % of tranche
1.00%	0.1527%	0.0558%	Tranche A	1.5025%
1.20%	0.1342%	0.0373%	Tranche B	2.7602%
1.40%	0.1248%	0.0278%	Tranche C	2.3673%
1.60%	0.1152%	0.0182%	Tranche D	1.8048%
1.80%	0.1036%	0.0066%	Tranche E	2.8961%
2.00%	0.0970%	*reserve*	**full Tranche**	**2.0933%**

And expected loss for the full tranche $E(X)$

$$E(X) = \sum_{ep=1.0\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep} - \sum_{ep=2.0\%}^{ep=99.9999\%} ep_{0.0001} * X_{ep}$$

The expected loss of each tranche and the expected loss of the full tranche give us the expected premiums for each individual tranche – layer and for the full tranche. These premiums can be used to structure both reinsurance and capital markets contracts.

Computing the value of the contract with a notional cover equal to the difference between the minimum reserve capital $Q_{ep=0.02} = X_{ep=2.0\%}$ and the required reserve capital $K_{ep=0.01} = X_{ep=1.0\%}$ using traditional swap net present value expression allows:

$$Swap_{NPV} = \sum_{i=1}^{n} (X_{ep=1.0\%} - X_{ep=2.0\%}) * EP_{2.0\%} - \sum_{i=1}^{n} E(X) * (1 - EP_{2.0\%})$$

Where:

$(X_{ep=1.0\%} - X_{ep=2.0\%}) -$

$is\ the\ size\ of\ the\ full\ tranche\ equivalent\ to\ a\ comparative\ reinsurance\ contract\ notional\ cover$

- We assume full recovery and full payment transactions of both swap and reinsurance contracts, hence this factor is unity.

$E(X) - expected\ value\ of\ loss\ of\ the\ tranche, as\ \%\ of\ the\ tranche\ notional$

$EP_{2.0\%}$
$- probability\ of\ triggering\ the\ swap\ transaction\ or\ an\ equivalent\ reinsurance\ contract$
$(1 - EP_{2.0\%})$
$- probability\ of\ survival, i.e. of\ the\ swap\ or\ reinsurance\ contracts\ not\ being\ triggered$
Given that the net present value of the swap contract at time of inception is zero, for a one year contract and excluding the impact of risk free interest rates, the swap rate as a percentage of the tranche or contract notional cover becomes:

$$swap_{r,T+1} = 1 - \frac{E(X) * (1 - EP_{2.0\%})}{(X_{ep=1.0\%} - X_{ep=2.0\%}) * EP_{2.0\%}} = 2.5064\%$$

An actuarial quote in the same terms as percentage of contract notional cover, also known as rate-on-line is given as:

$$RE_{r,T} = \frac{E(X)}{X_{ep=1.0\%} - X_{ep=2.0\%}} = 2.0933\%$$

Swap contracts are settled at maturity and reinsurance contracts are settled at inception. For an annual risk free interest rate of 2.1%, the reinsurance contract rate at maturity or expiration is:

$$RE_{r,T+1} = RE_{r,T} * e^{0.021} = 2.1377\%$$

In summary the annual swap rate and reinsurance rate-on-line is:

contract type	at time	*inception*	*expiration / maturity*
reinsurance		2.0514%	2.5064%
swap		2.0933%	2.1377%

Visibly in this case the rates for both contracts are comparable, which leads us to explore new hedging techniques and structures for swap contracts that cannot be easily replicated with traditional reinsurance treaties.

Adapting a financial variance swap to a property insurance portfolio

The traditional financial variance swap is defined at settlement as:

$$Settlement = Notional * (Relized\ variance - Stike\ variance)$$

The building block of a variance swap is the estimation of realized and strike variances. The realized annual variance is computed over the length of the contract: $\{S_t, \dots, S_T\}$ from the log – returns of price or rate observations of the underlying security, index, or basket of securities.

$$VAR_T = \frac{252}{N-1}\sum_{t=1}^{N}\left(ln\left(\frac{S_t}{S_{t-1}}\right)\right)^2$$

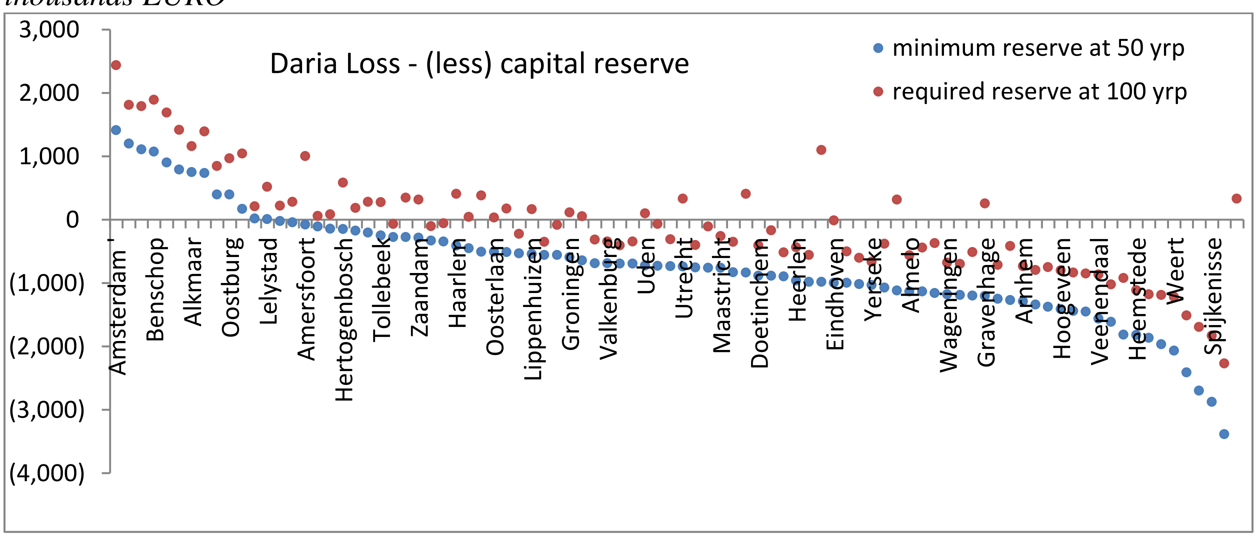
The strike variance is estimated through a stochastic variance model which may contain additional benchmarking methodology to an implied variance from forward market contracts.

The usability of a variance swap contract for a property insurance portfolio will be in hedging the variance of realized CAT event losses to actual required reserve capital amounts, the latter being estimated from stochastic loss at the 100 YRP - $EP_{0.01}$ $i.e.$ our required capital of $K_{ep=0.01}$.

Using historical losses by CRESTA zone from windstorm Daria (January 25'th and 26'th 1990), one of the highest loss events on record for the Netherlands and for Western Europe in general, we develop and test the mechanics of a variance swap contract for an insurance portfolio. For this contract case study, our portfolio required capital reserve $K_{P,\ ep=0.01}$ becomes the aggregated additive sum of the policies individual capital reserves, estimated by CRESTA as stochastic loss equivalent to 100 YRP - $E(X)_{ep=0.01} = policy\ K_{ep=0.01}$

$$K_{P,\ ep=0.01} = \sum_{cresta\ 1}^{90} policy\ K_{ep=0.01}$$

Figure 2: Daria (1990) loss by CRESTA less minimum and required capital reserves by policy in thousands EURO



Realized event loss variance is computed from reported losses by administrative – GEO unit or by policy and LOB - L_{cresta} to required reserve capital amounts by policy and CRESTA - $K_{ep=0.01}$ i.e. by same unit of aggregation. Our portfolio contains ninety policies each by individual single CRESTA zone. The portfolio realized variance - VAR_R for historical Daria (1990) losses is expressed as.

$$VAR_R = \frac{1}{N} \sum_{cresta\ 1}^{90} \left(ln \left(\frac{L_{cresta}}{K_{ep=0.01}} \right) \right)^2 = 59.5577\%$$

At inception of the contract, we estimate a strike variance using as a strike benchmark the 200 YRP stochastic losses - $E(X)_{ep=0.005}$by CRESTA and portfolio reserve capital amounts by policy - $E(X)_{ep=0.01} = K_{ep=0.01}$, always using the same unit of aggregation for losses, capital amounts and variance quantities.

$$VAR_K = \frac{1}{N} \sum_{cresta\ 1}^{90} \left(ln \left(\frac{E(X)_{ep=0.005}}{K_{ep=0.01}\left[E(X)_{ep=0.01}\right]} \right) \right)^2 = 60.4998\%$$

At settlement the contract pay-out is cleared between the counterparties as:

$$Settlement = Notional * (VAR_R - \ VAR_K)_+ = Notional\ *0.9421\%$$

In this contract settlement the insurance firm will pay 0.9421% of the agreed notional to the swap counterparty. Had the realized CAT event – Daria (1990) variance exceeded the strike variance of 60.49%; the insurance firm will have received payments from the swap counterparty.

The latter scenario can be derived from a less ‘conservative’ estimation of the strike variance at inception and negotiation of the contract. As a general observation, the methodology for determining a strike variance for insurance portfolios swap contracts is less well established than for securities portfolios, where historical and market data on futures contracts in prices, variance and volatility is standardized and available to all parties. An example of a typical linear method for defining a strike variance, used in equity variance futures follows:

$$VAR_K = \frac{(T-1) * VAR_{futures} - (t-1) * VAR_{realized}}{T-t}$$

Where:

$T - observation\ period\ to\ be\ used\ as\ length\ of\ the\ swap\ contract$

$t - number\ of\ trading\ days\ in\ which\ a\ futures\ variance\ was\ quoted\ during\ T$

$VAR_{futures} - quoted\ future'\ variance\ at\ T$

$VAR_{realized} - estimated\ realized, historical\ variance\ at\ time\ of\ contract\ inception$

In context of an insurance portfolio hedging scheme we remove the time dimension, and assume that $VAR_{futures}$ is equivalent to the variance of 200 YRP losses $\rightarrow VAR[E(X)_{ep=0.05}]$ and $VAR_{realized}$ is equivalent to the variance of expected value losses$\rightarrow$ $VAR[E(x)]$; both quantities are by policy i.e. equivalently by CRESTA zone in our portfolio case study. In order to adapt this methodology to stochastic CAT losses we test a strike variance VAR_K' for our portfolio case study:

$$VAR'_K = \frac{1}{N}\sum_{cresta\ 1}^{90}\left(ln\left(\frac{E(X)_{ep=0.005} - E(X)}{K_{ep=0.01}}\right)\right)^2$$

$$VAR'_K = 56.2825\%$$

We find this expression to produce a reasonable strike variance. In this second case our swap contract will transfer 3.3% of agreed notional from the swap counterparty to the insurance firm.

$$Settlement = Notional * (VAR_R - VAR_K)_+ = Notional *3.2752\%$$

We summarize both swap contracts realized and strike variances and payouts in the table below:

realized VAR		59.5577%
strike VAR	*VAR [X ep=0.02]*	60.4998%
	VAR [X ep=0.02 - EV(x)]	56.2825%
pay-outs as % of Notional	*VAR [X ep=0.02]*	-0.9421%
	VAR [X ep=0.02 - EV(x)]	3.2752%

Since at inception the net present value of the swap contract is expected to be zero then:

$$NPV\ swap = e^{-r.\Delta t}\{Notional * (VAR_R - VAR_K)_+\}$$

We can conclude that for a strike variance estimated from stochastic losses closely below a 200 YRP stochastic loss, i.e.$E(X)_{ep=0.005}$, the NPV of the swap contract will begin to converge to zero.

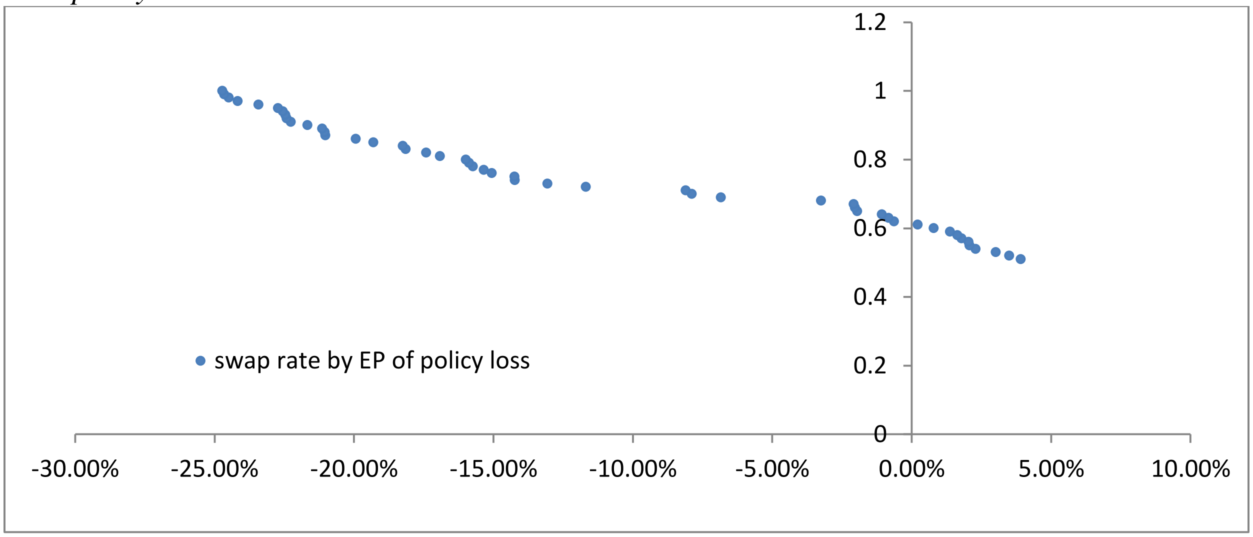
To study the sensitivity of the swap rate to modeled strike variance changes, we price the swap contract for the full interval of stochastic policy losses $\left[E(X)_{ep=1\%}{\sim}E(X)_{ep=0.5\%}\right]$ in $[EP_{0.01}{\sim}EP_{0.005}]$ interval for each individual policy, with our second formula:

$$VAR'_{K,\ ep=1\%\sim0.5\%} = \frac{1}{N}\sum_{cresta\ 1}^{90}\left(ln\left(\frac{E(X)_{ep=0.01\sim0.005} - E(X)}{K_{ep=0.01}}\right)\right)^2$$

And keeping required portfolio capital reserve $K_{ep=0.01}$ and expected value of the full tranche loss $E(X)$ as constant. This sensitivity study indicates that with strike variance derived from policy losses in the interval $\left[E(X)_{ep=0.61\%}{\sim}E(X)_{ep=0.62\%}\right]$ - $[EP_{0.0061}{\sim}EP_{0.0062}]$, the net present value of the swap contract approached zero.

For swap rates less than zero in exceedance probability interval$[EP_{0.0062}{\sim}EP_{0.01}]$, the insurance firm will transfer the amount of $swap\ rate * notional$ to the swap counterparty. And for swap rates above zero in exceedance probability interval$[EP_{0.005}{\sim}EP_{0.0061}]$, the swap counterparty will transfer the amount of $swap\ rate * notional$ to the insurance firm.

Figure 3: swap rate derived from policy stochastic losses in the interval $[EP_{0.005} \sim EP_{0.01}]$ *for each policy*



Continuing work

Given an understanding of the rational and mechanics of a variance swap next steps in developing this financial instrument will be in defining a coherent model for strike variance and a market accepted methodology for computing of realized variance. Practical hedging strategies, with impact on real risk profile and P&L of a re/insurance firm will need to be studied in detail with a simulation cases studies. Variations in definitions of realized and strike variance and adding a swap trigger mechanism will be explored.

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