A NLIP Model on Wage Dispersion and Team Performance

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Abstract

Using a Non-Linear Integer Programming (NLIP) model, I examine if wage differences between Super talents and Normal players improve the performance of four teams which participate in a tournament, such as in the UEFA Champions League (UCL) group matches. With ad-hoc wage differences, the optimal solutions of the model show that higher wage equality seems to improve the performance of all teams, irrespectively if the elasticity of substitution between Super- and Normal- players is high or low. In addition to that, a U-type performance exists in two teams with the highest and the second high elasticity of substitution. With team data from the 2011-12 UCL group matches and from the Italian Serie A over 2010-12 seasons, the wage dispersion has no effect on team performances.

Keywords: Players, Teams, Wages, Performance, Tournament

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1. Introduction

The optimal wage structure of teams has been frequently discussed over the last decades. The main hypothesis is whether the compressed or the dispersed wages among a team's players have a stronger impact on the performance of the team. As is very often in economics, there are at least two schools of thought. Lazear & Rosen (1981) argued that the team performance increases if the best talents are paid higher wages than the normal players. Milgrom (1988) and Lazear (1989) on the other hand, stressed the possibility of bad field cooperation between players, and consequently the performance could be inferior, if the under-paid players feel discriminated. Levine (1991) took an extreme position and favoured the egalitarian wages. Fehr and Schmidt (1999) tried to balance these two effects and argued that, as a whole, the team losses more if wages are more unequal than equal.

Franck & Nüesch (2007) reviewed the empirical studies from sport teams, in baseball, hockey, basket and football. Some studies seem to support the compressed wages hypothesis. They argue that these findings can partly be explained because the majority of empirical studies assume a linear relationship between wage dispersion and team performance. In their own study, based on 5281 individual salary proxies from German soccer players between 1995-07, they allowed for squares of the Gini coefficient and the coefficient of variation of the wage distribution at the beginning of the season. They found a U-formed sportive success, i.e. teams perform better by either an egalitarian pay structure or a steep one[1]. On the other hand, Pokorny (2004) found an inverted U-formed success, i.e. the performance is higher with intermediate wage differences. Avrutin & Sommers (2007), using baseball data from 2001-05, found no effect. Torgler et al (2008), using also data from the German Bundesliga (and the NBA), found that players care more about the salary distribution within the team and not just about their own salary. Generally, players prefer a reduced inequality and in that case their performance improves. In addition, a detailed investigation of the basketball data shows also that when a player moves from a relative income advantage to a relative disadvantage, his performance decreases in a statistically significant way. On the other hand, moving from relative income disadvantage to relative advantage has no effects. Wiseman & Chatterjee (2003), using baseball data from 1980-02, found a negative effect of wage dispersion. In a similar study recently, i.e. with baseball data from 1985-10, Breunig et al (2012) found also a negative effect of wage dispersion. On the other hand, Simmons & Berri (2011), using basket statistics, supported the Lazear & Rosen (1981) hypothesis that higher wage dispersion increases team performance.

[1] Also they found evidence that teams with dispersed wages entertain the public better since the number of seasonal dribbling and runs increases significantly!

In this paper I formulate and solve a NLIP model to examine mainly the effects of wage differences between Super (S) - and Normal (N) - players in the performance of four teams, which participate in a tournament like the UEFA Champions League (UCL) group matches. Using ad-hoc wage differences and various “team production functions”, it is of interest to see if the optimal solutions favour the compressed or the dispersed wages hypotheses for all, or some teams. In addition, two small data sets (UCL group matches from 2011-12 and Italian Serie A from 2010-12 seasons) are used to test the above hypotheses.

2.1 The model

There are four football teams which play three home and three away matches in the group stage of the UCL tournament. As a whole there are 12 matches and the maximum number of points is 36. The two teams which collect most points are qualified for the next round and the other two are eliminated[2].

The four participating teams have different qualities and consequently different ranking. To differentiate the teams, four different “team production functions” are assumed. I follow Kesenne (2007) and assume that the formation of the teams consists of a certain number of S- and N-players. The S- and N-players are considered as being from almost complements, to almost substitutes. All S-players are equally “Super” and all N-players are equally “Normal”[3]. Teams use their S- and N- players in order to “produce” points. The supply of S- and N-players is unlimited.

Since the value of the marginal product of players can’t be observed or measured, teams have a certain wage structure, from very dispersed to very compressed. The wages is the policy variable of the model. It is assumed that teams have no other fixed costs (like managers or other facilities); their only variable costs consist of the wages they pay to their players. It is also assumed that all teams receive similar revenues, either directly from UEFA, and/or from their public, TV-rights and sponsors and the wages they pay can’t be higher than their revenues. All teams play a “Cournot” type game and maximize simultaneously their points.

The model is rather general and can explain, not only the own performance, but the effect to the other teams as well, even if they keep their wages unchanged. It can also

[2] The third team continues in the UEFA Europa League.
[3] Obviously, there are players who, objectively, belong to one category or the other. Perhaps, most players are neither that excellent to be classified in the first category, nor that “Normal” to be classified in the second category. In the football world, it is very common that the supporters of a team tend to overvalue their own players and undervalue the competitor’s players. This is an empirical issue, left to supporters, to managers, to journalists or even to those who do research in efficiency analysis.

show whether the tournament remains balanced and whether teams who have more S-players can collect more points.

Let P_1, P_2, P_3, P_4, be the points collected by the teams; let S_1, S_2, S_3, S_4, be the number of S-players, and N_1, N_2, N_3, N_4 the number of N-players the teams use in these matches. Let w_1, w_2, w_3, w_4, be the victories of the teams, d_1, d_2, d_3, d_4, the draw matches and l_1, l_2, l_3, l_4, the losses (defeats) of the teams. All these 24 variables are positive integers.

Since all firms aim at maximizing points (simultaneously), their objective function is given by (1), under the following constraints.

The key constraints that differentiate teams are the teams' "production" functions, (2a – 2d). All functions are of Constant Elasticity of Substitution (CES) type of degree one, with different elasticity. The function of team 1 is closed to Leontief, i.e. very low elasticity of substitution between its S- and N-players[4]. Team 4 on the other hand, has a closed to Cobb-Douglas "production" function, i.e. almost excellent elasticity of substitution, team 2 is close to team 1, while team 3 is close to team 4.

The use of S- and N-players for each team will be endogenously determined from the optimal solution. When the team formation has been determined, it is assumed that the same team will be used for all six matches, unless the wage structure has changed. Teams of course change the composition of their players for tactical reasons, such as if they play away against a stronger team or if they play at home against a weaker team, or because some of their players might be injured or punished and are not available for a particular match. The model neglects such possibilities. The model also assumes that players who are not used (because the roster of teams consists of more than 11 players), receive zero wages. Notice that, due to the integer constraint of players, the composition of the team can remain unchanged, even if the wages change. t_i is the efficiency parameter and a_i is the distribution parameter.

Constraints (3a – 3d) restrict the number of team players to 11. In reality, even for top and wealthy teams, the number of S- is often lower than the number of N-players, which is given by constraints (4a – 4d). Despite the fact, that such condition is not necessary, it is stated explicitly in order to speed up the solution of this complex model.

As is well known, victories are worth 3 points; draws are worth 1 point and losses, zero points. Thus, the number of points collected to every team is given by constraints (5a – 5d).

[4] In reality, an S-player can substitute an N-player more easily than the reverse. I assume that the elasticity of substitution is unchanged, irrespectively if the S- substitutes the N-player, or if the N-substitutes the S-player.

$$\max \sum_{i=1}^{4} P_i \qquad (1)$$

$s.t.$

$$P_1 = t_1\left(\alpha_1 S_1^{-100} + (1-\alpha_1) N_1^{-100}\right)^{-\frac{1}{100}}$$

$$P_2 = t_2\left(\alpha_2 S_2^{-10} + (1-\alpha_2) N_2^{-10}\right)^{-\frac{1}{10}}$$

$$P_3 = t_3\left(\alpha_3 S_3^{-0.5} + (1-\alpha_3) N_3^{-0.5}\right)^{-\frac{1}{0.5}} \qquad (2a-2d)$$

$$P_4 = t_4\left(\alpha_4 S_4^{-0.1} + (1-\alpha_4) N_4^{-0.1}\right)^{-\frac{1}{0.1}}$$

$$S_i + N_i = 11 \qquad (3a-3d)$$

$$S_i \le N_i \qquad (4a-4d)$$

$$P_i = 3w_i + d_i \qquad (5a-5d)$$

$$w_{i,j}^h + w_{j,i}^a + d_{i,j}^h = 1,\ pairwise, \qquad (6a-6l)$$

$$w_{i,j}^h + w_{j,i}^a + d_{j,i}^a = 1,\ pairwise, \qquad (7a-7l)$$

$$d_{i,j}^h + d_{i,j}^a = d_i, \qquad (8a-8d)$$

$$w_{i,j}^h + w_{i,j}^a = w_i, \qquad (9a-9d)$$

$$w_i + d_i + l_i = 6, \qquad (10a-10d)$$

$$P_i \le 18, \qquad (11a-11d)$$

$$\sum_{i=1}^{4} P_i \le 36, \qquad (12)$$

$$\pi_i = 12P_i - 0.5P_i^2 - w_{S_i} S_i - w_{N_i} N_i \ge 0, \qquad (13a-13d)$$

$$(P_i,\ S_i,\ N_i, w_i\ d_i,\ l_i) \in Integer$$

$$(w_{i,j}^h,\ w_{j,i}^a,\ d_{i,j}^h,\ d_{j,i}^a) \in Binary$$

$$i, j = 1,..,4,\ i \ne j$$

$$t_i \succ 0$$

$$0 \prec \alpha_i \prec 1$$

$$w_{S_i},\ w_{N_i} = wage\ \ parameters$$

Each one of the 12 matches in the group of four teams, ends either with a home team victory, $w_{i,j}^h$, with an away team victory, $w_{j,i}^a$, or with a home team draw, $d_{i,j}^h$, which is obviously equal to the away team draw, $d_{j,i}^a$. But, in order to identify the correct pair of teams which play draw at home and/ or draw away, we must separate the home team draw from the away team draw, i.e. we need 24 additional constraints (6a – 7l). The first 12 constraints (6a – 6l) relate each pair of teams to home team draw and the remaining 12, (7a – 7l), relate to the away team draw. If for instance $d_{1,2}^h = 1$, (and consequently $w_{1,2}^h = w_{2,1}^a = 0$), from constraints (6a) and (7a) we are ensured that $d_{2,1}^a = 1$, as well. If that match ends with a home or away victory, it implies that $d_{1,2}^h = d_{2,1}^a = 0$. Moreover, these constraints do not exclude impossible (non-binary) match results, such as $d^h_{1,2} = d^a_{2,1} = 0.5$ and $w^h_{1,2} = w^a_{2,1} = 0.25$. Therefore we require that all possible match results are binary as well.

Obviously, the draws (and the victories) for each team is the sum of all possible draws (and victories) against all other teams. Constraints (8a – 8d) ensure that the number of draws in constraints is always an integer (and actually even) number, or zero. For instance, when team 1 plays only one match draw, it is $d_1 = 1$.If the draw match was a home match against team 2, it must be $d^h_{1,2} = 1$, (from 8a), and $d^a_{2,1} = 1$, (from 8b), as well. In that case, if team 2 does not play another match draw, it must be $d_2 = 1$as well. Of course, if team 2 plays three draw matches, i.e. $d_2 = 3$, it implies that team 2 must have played draw against the other teams as well. A similar interpretation applies for the victory constraints (9a – 9d).

Since each team play six matches, there are 6 possible results from its games, i.e. constraints (10a – 10d) are required. Constraints (11a – 11d) show that no team can collect more than 18 points. In addition to that, constraint (12) shows that the maximum number of points from all matches is 36, composed with 12 victories (for two teams) and 12 defeats for the other two teams.

Finally, constraints (13a – 13d) ensure non-negative profits. The revenue function is quadratic in the points collected. All teams pay the same, higher wages to their S-players, and the same, lower wages to their N-players. The initial parameters are: $w_{S_i} = 8$ and $w_{N_i} = 4.8$, i.e. the N-players receive only 60% of the S-players' wages. Keeping $w_{S_i} = 8$, the model is repeated and solved for higher and lower values of the parameter w_{N_i}. Obviously the wage dispersion or compression does not influence the performance of the own team, but the performances of the other teams too. The non-negative profits constraints are satisfied if each team collects at least 6 points. With less than 6 points there is no integer value of N- and S-players to ensure non-negative profits.

The model is now complete and was solved in Lingo, using Global Solver.

2.2 The Solution

I solved the model 124 times, i.e. 31 times per team, using 31 different w_{N_i}, starting from as low as 3.4 up to 6.4, increased at a range of 0.1. When one team changed its w_{N_i}, all other teams keep their own w_{N_i} unchanged. When $w_{N_i} > 6.4$, there is no feasible (integer) solution, because the non-negative profits constraint(s) are violated.

Table 1 depicts the own effects[5] on performance and team composition, where the policy variable is w_{N_i}. Some points from the Table 1 are worth mentioned.

The initial solution, with $w_{S_i} = 8$ and $w_{N_i} = 4.8$, shows that the tournament is completely balanced, because each team wins its three home matches and collects 9 points. Despite the differences in their "production" functions, all four teams have a similar team composition, consisting of 1 S-player and 10 N-players and each team makes a profit of 11.5.

Table 1: Points maximization, subject to non-negative profits

w_{N_i}	S_1	N_1	P_1	α_1	S_2	N_2	P_2	α_2	S_3	N_3	P_3	α_3	S_4	N_4	P_4	α_4
3.4	1	10	**6**	0.55	1	10	**6**	0.43	3	8	**6**	0.5	1	10	**9**	0.78
3.5	1	10	**6**	0.5	1	10	**9**	0.5	1	10	**9**	0.89	1	10	**9**	0.74
3.6	1	10	**6**	0.34	1	10	**6**	0.5	1	10	**9**	0.65	3	8	**9**	0.01
3.7	1	10	**9**	0.5	1	10	**6**	0.01	3	8	**6**	0.65	1	10	**9**	0.28
3.8	1	10	**9**	0.66	1	10	**6**	0.29	2	9	**6**	0.5	2	9	**6**	0.5
3.9	1	10	**9**	0.66	2	9	**6**	0.48	1	10	**9**	0.48	2	9	**6**	0.65
4.0	1	10	**9**	0.5	2	9	**6**	0.01	3	8	**9**	0.65	1	10	**6**	0.01
4.1	1	10	**6**	0.37	4	7	**9**	0.44	5	6	**9**	0.5	1	10	**9**	0.45
4.2	1	10	**9**	0.37	1	10	**6**	0.30	1	10	**6**	0.03	1	10	**6**	0.4
4.3	1	10	**9**	0.5	1	10	**6**	0.66	1	10	**6**	0.72	1	10	**6**	0.5
4.4	1	10	**9**	0.5	1	10	**9**	0.37	1	10	**6**	0.01	1	10	**6**	0.61
4.5	1	10	**9**	0.02	1	10	**6**	0.44	5	6	**9**	0.65	5	6	**9**	0.5
4.6	1	10	**9**	0.5	1	10	**6**	0.44	1	10	**6**	0.5	1	10	**6**	0.5
4.7	1	10	**9**	0.66	1	10	**9**	0.5	4	7	**9**	0.5	4	7	**9**	0.65
4.8	1	10	**9**	0.34	1	10	**9**	0.36	1	10	**9**	0.57	1	10	**9**	0.67
4.9	1	10	**9**	0.66	4	7	**9**	0.5	1	10	**9**	0.89	1	10	**9**	0.74
5.0	1	10	**9**	0.5	1	10	**9**	0.44	1	10	**9**	0.5	4	7	**9**	0.65
5.1	1	10	**9**	0.34	1	10	**9**	0.01	1	10	**9**	0.99	3	8	**9**	0.8
5.2	1	10	**9**	0.66	1	10	**9**	0.34	2	9	**9**	0.02	1	10	**9**	0.03
5.3	1	10	**9**	0.5	1	10	**9**	0.37	1	10	**9**	0.01	1	10	**9**	0.62
5.4	1	10	**9**	0.5	1	10	**9**	0.5	2	9	**9**	0.5	1	10	**9**	0.01
5.5	1	10	**9**	0.5	1	10	**9**	0.65	1	10	**9**	0.43	1	10	**9**	0.5
5.6	1	10	**9**	0.5	2	9	**9**	0.76	1	10	**9**	0.65	1	10	**9**	0.78
5.7	1	10	**9**	0.66	1	10	**9**	0.5	2	9	**9**	0.5	1	10	**9**	0.03
5.8	1	10	**9**	0.5	1	10	**9**	0.65	1	10	**9**	0.5	1	10	**9**	0.68
5.9	1	10	**9**	0.5	1	10	**9**	0.65	1	10	**9**	0.63	1	10	**9**	0.02
6.0	1	10	**12**	0.5	2	9	**10**	0.65	1	10	**10**	0.62	1	10	**12**	0.5
6.1	1	10	**10**	0.21	2	9	**12**	0.44	1	10	**10**	0.99	1	10	**12**	0.99
6.2	1	10	**10**	0.55	1	10	**10**	0.5	2	9	**12**	0.71	1	10	**10**	0.72
6.3	1	10	**12**	0.44	1	10	**12**	0.5	1	10	**12**	0.65	1	10	**12**	0.21
6.4	1	10	**12**	0.11	1	10	**12**	0.2	1	10	**12**	0.5	1	10	**12**	0.5

[5] To save space, I have excluded the cross effects, i.e. effects on other teams. The four additional Tables can be provided by the author at a request.

As expected, team 1 (closed to Leontief), never changes its team composition and uses 1 S- and 10 N-players. On the other hand, it is team 3 that changes its team composition more frequent (in 11 out of 31 times) and not team 4, (closed to Cobb-Douglas).

The most striking result is that more S-players do not always increase the performance of teams! This is due to two reasons. First, other teams might also play with more S-players too. Second the efficiency[6] of the teams changes too. For instance, when $w_{N4} = 4.7,$ both team 4 and team 2 use 7 N-players and all teams collect 9 points and their respective efficiencies are almost similar, $t_4 = 43.6,\ t_2 = 38.6.$ When $w_{N4} = 6.0,$ both teams use 10 N-players, team 4 collects 12 points, while team 2 collects 7 points, because their new efficiencies are $t_4 = 35.5,\ t_2 = 7.5.$ Obviously, when the performance does not increase, the profits of the teams are reduced with more S-players. And the points collected per team are not related to their distribution parameter of players' a_i.

Finally, the tournament appears to be frequently balanced. For instance, it is completed balanced in 23/ 31 cases when team 1 selects wages; it is also completely balanced in 16/ 31 cases when team 2 selects wages, and in 19/ 31 cases when team 3 or team 4 select wages. And while in most cases the total number of points selected equals to 36, there are some draw matches with a total number of points equal to 35 (always when $w_{N_i} \geq 6.0$).

2.2.1 The regressions from the optimal solutions

(i) The own effects

In order to find out how the wage dispersion influences the performance of the teams, I run the following two regression equations, based on all 124 optimal solutions.

$$P_i = \alpha + \beta_1 (8 - w_{N_i})$$
$$P_i = \alpha + \beta_1 (8 - w_{N_i}) + \beta_2 (8 - w_{N_i})^2 \cdot$$

The right hand side variable is the wage difference between the respective fixed $w_{S_i} = 8$ and w_{N_i}, while the dependant variable is points collected. Negative (positive) β_1-estimates imply that the performance of teams improves if the wage equality (inequality) between the S- and N-players increases. Similarly, negative β_1-estimates

[6] Due to space constraints, the efficiency parameters are not presented.

and positive β_2-estimates imply a U-type success, i.e. either highly unequal or highly equal wages will improve performance.

Table 2a summarizes the regression estimates for every team. Based on the average number of points, team 1 and 4 are qualified. It is clear that in the linear model, all teams improve their performances when the wage dispersion decreases (all β_1-estimates are strongly negative). Consequently, the findings support the Milgrom (1988) and Lazear (1989) hypothesis.

Table 2a: OLS own estimates

	Team 1 $\overline{P} = 8.97$ $\sigma = 1.47$	**Team 2** $\overline{P} = 8.39$ $\sigma = 1.89$	**Team 3** $\overline{P} = 8.68$ $\sigma = 1.72$	**Team 4** $\overline{P} = 8.74$ $\sigma = 1.80$
α	*12.65*** (19.24)*	*13.6*** (18.52)*	*12.75*** (15.64)*	*12.77*** (14.16)*
β_1	*-1.19*** (-5.84)*	*-1.68*** (-7.39)*	*-1.32*** (-5.20)*	*-1.30*** (-4.65)*
$\overline{R^2}$	*0.52*	*0.64*	*0.46*	*0.41*
α	*12.97*** (5.44)*	*17.25*** (6.73)*	*18.36*** (6.70)*	*21.7*** (7.88)*
β_1	*-1.41 (-0.87)*	*-4.25* (-2.43)*	*-5.26*** (-2.82)*	*-7.58*** (-4.04)*
β_2	*0.036 (0.14)*	*0.414 (1.48)*	*0.637* (2.13)*	*1.01*** (3.38)*
$\overline{R^2}$	*0.51*	*0.66*	*0.52*	*0.56*

** significant at 0.01 level; * significant at 0.05 level; t-statistics is in parentheses

Regarding the quadratic function, the U-type success for teams 3 and 4 is not rejected, over the relevant range $w_{N_i} = (3.4,...,6.4)$. For instance, team 3 will minimize its performance and collect about 7.5 points if it pays $w_{N_3} \approx 3.9$, (i.e. if its N-players receive slightly less than 50% of the S-players wages), it will collect about 8.5 points if it pays the lowest wage $w_{N_3} = 3.4$ and it will collect about 11.5 points if it pays the upper-limit wage $w_{N_3} = 6.4$. Similarly, team 4 will minimize its selected points (7.5), if it pays $w_{N_4} \approx 4.25$.

(ii) The cross effects

In order to find out the performance effects to the other teams, I run the following two regression equations:

$$P_i = \alpha + \beta_1(8 - w_{N_j})$$

$$P_i = \alpha + \beta_1(8 - w_{N_j}) + \beta_2(8 - w_{N_j})^2, \quad i \neq j$$

The estimates are given in Table 2b. In the linear model, the estimates are completely consistent with those in Table 2a. A decrease of wage dispersion in team *i* reduces the performance of team *j* (all β_1-estimates are strongly positive).

Table 2b: OLS cross estimates

	Linear			*Quadratic*			
	α	β_1	$\overline{R}^2$	α	β_1	β_2	$\overline{R}^2$
	Effect on Team 2						
	*7.19***	*0.64***	*0.27*	*9.08***	*-0.69*	*0.22*	*0.27*
	(12.2)	*(3.48)*		*(4.3)*	*(-0.5)*	*(1.14)*	
	Effect on Team 3						
Team 1	*8.20***	*0.22**	*0.13*	*5.87***	*1.85***	*-0.26**	*0.26*
	(27.1)	*(2.32)*		*(5.89)*	*(2.74)*	*(-2.43)*	
	Effect on Team 4						
	*6.99***	*0.59***	*0.34*	*4.57***	*2.3**	*-0.27*	*0.37*
	(14.8)	*(4.1)*		*(2.78)*	*(2.05)*	*(-1.53)*	
	Effect on Team 1						
	*6.95***	*0.66***	*0.34*	*7.5***	*0.28*	*0.06*	*0.32*
	(13.2)	*(4.05)*		*(3.93)*	*(0.2)*	*(0.3)*	
	Effect on Team 3						
Team 2	*6.87***	*0.66***	*0.30*	*2.61*	*3.66***	*-0.48**	*0.39*
	(11.9)	*(3.73)*		*(1.37)*	*(2.81)*	*(-2.32)*	
	Effect on Team 4						
	*7.60***	*0.62**	*0.16*	*5.14*	*2.35*	*-0.28*	*0.16*
	(9.95)	*(2.60)*		*(1.88)*	*(1.27)*	*(-0.94)*	
	Effect on Team 1						
	*6.83***	*0.85***	*0.30*	*5.88**	*1.51*	*-0.11*	*0.27*
	(9.21)	*(3.69)*		*(2.20)*	*(0.83)*	*(-0.37)*	
	Effect on Team 2						
Team 3	*8.15***	*0.23**	*0.15*	*5.24***	*2.28***	*-0.33***	*0.37*
	(27.3)	*(2.53)*		*(5.72)*	*(3.66)*	*(-3.3)*	
	Effect on Team 4						
	*7.3***	*0.49***	*0.23*	*3.0*	*3.52***	*-0.49**	*0.37*
	(14.3)	*(3.13)*		*(1.82)*	*(3.13)*	*(-2.7)*	
	Effect on Team 1						
	*7.91***	*0.34**	*0.14*	*6.59***	*1.27*	*-0.15*	*0.14*
	(17.5)	*(2.45)*		*(4.08)*	*(1.16)*	*(-0.8)*	
	Effect on Team 2						
Team 4	*7.57***	*0.49**	*0.13*	*1.37*	*4.86***	*-0.7***	*0.31*
	(11.0)	*(2.30)*		*(0.63)*	*(3.28)*	*(-2.97)*	
	Effect on Team 3						
	*6.78***	*0.73***	*0.28*	*2.84*	*3.5**	*-0.45*	*0.33*
	(10.3)	*(3.56)*		*(1.26)*	*(2.28)*	*(-1.82)*	

In the quadratic model, some estimates are consistent with those in Table 2a, because a reversed U-type success appears. For instance, when team 3 pays $w_{N_3} = 6.4$, while team 2 keeps its initial wages at $w_{N_2} = 4.8$, team 2 collects about 8 points, while team 3 would collect 11.5 points. The pair of teams (first entry denotes wage dispersion of team i and second entry denotes the performance of team j) with a reversed U-type performance are: (1, 3), (2, 3), (3, 2), (3, 4) and (4, 3), while in the remaining seven pairs, there is no effect. Notice that team 3, which improves its own performance by low or high wage dispersion (Table 2a), is affected negatively by all other teams too.

To summarize the estimates, we conclude that team 1 (with the almost Leontief type), which collects the highest average number of points (8.97), improves its own performance linearly with lower wage dispersion. And despite the fact that the U-formed wage dispersion does not improve its own performance, it is the only team that is not influenced from low or high wage dispersions of other teams.

3. Empirical study

Obviously, to test the same hypotheses with real data is almost impossible, because there are many problems with the observed players' wages and of course the team formation. It will require a huge amount of time to refine all available data set by observing each individual player, to examine if he was injured or punished and unavailable for some matches, or if he played only a small part in a match. And even if such information were relatively easy to collect, the wage dispersion of the team would vary depending upon the various weights one uses for each player participation in the matches. To ad hoc exclude some players (often the youngsters with low wages) might be erroneous too, because sometimes managers do not play the "expensive" players and deliberately use "cheap" players in some matches.

Two very small data sets have been collected, assuming simply that all players, for whom wage observations exist, are available to play, and all players' wages have the same weights, irrespectively if they played or not. The team spirit or the envy of players exists for the entire team roster and not necessarily for those players who are fielded. The first data set consists of 32 European teams (496 players) from 48 UCL group matches, played in 2011. The aggregate statistics are shown in Appendix (Table A). As is shown, for some teams there are accurate observations for at most 10-13 players. The second data set consists of 40 Italian teams from the Serie A (about 1015 players), over two seasons 2010-11/ 2011-12, obtained from the following sites: http://www.football-marketing.com/2010/09/07/italian-serie-a-wage-list/, http://www.xtratime.org/forum/showthread.php?t=261972. Notice that 34 of these Italian teams appear in both seasons, while the remaining 6 teams appear only in one season (3 of them relegated in the first season and 3 of them advanced in the second season). Since the team roster of these 34 teams change and/or some wage contracts are re-negotiated, the wage dispersion within the same team varies from year to year.

Table 3 summarizes the OLS estimates for these data sets. In UCL, the wage dispersion is measured by Coefficient of Variation (CV) and by Mean Absolute Deviation (MAD) as well; as control variables, two different proxies are used, the team value (V) and the respective UEFA ranking (R); in Serie A, the wage dispersion is also measured by CV and by Coefficient of Dispersion (CD) while as a control variable the average wage (W) per team is used.

In case (i), the number of points does not automatically reflect qualification or elimination from the tournament, because it depends on the group. For instance in group D, the eliminated third team's points (Ajax) are identical to the second team's (Lyon), but Lyon had a better goal difference against Ajax. Similarly, while Manchester City eliminated with 10 points, four other teams in other groups qualified with less points. Consequently, in case (ii), a dummy variable was introduced with the following values: 1 for the two qualified teams per group; 0.5 for the third eliminated team and 0 for the fourth team.

Table 3: OLS estimates for UCL and Serie A

UCL: (i) Points							
α	γ_1	γ_2	$\overline{R}^2$	α	γ_1	γ_2	$\overline{R}^2$
	MAD	*V*			*CV*	*V*	
*4.205*** (4.50)*	*1.775 (0.78)*	*0.014 (1.89)*	*0.45*	*4.66*** (2.61)*	*-0.45 (-0.09)*	*0.02*** (4.91)*	*0.44*
	MAD	*R*			*CV*	*R*	
*3.066*** (2.66)*	*3.056 (1.93)*	*0.044** (2.16)*	*0.47*	*2.14 (1.12)*	*2.865 (0.6)*	*0.07*** (4.60)*	*0.41*
UCL: (ii) (Dummy)*(Points)							
	MAD	*V*			*CV*	*V*	
2.24 (1.79)	*4.37 (1.45)*	*0.008 (0.75)*	*0.35*	*1.72 (0.74)*	*2.76 (0.42)*	*0.02*** (3.61)*	*0.30*
	MAD	*R*			*CV*	*R*	
0.64 (0.43)	*3.52 (1.73)*	*0.051 (1.95)*	*0.41*	*-1.13 (-0.47)*	*5.55 (0.92)*	*0.08*** (4.17)*	*0.37*
Italian Serie A: Points							
	CD	*W*			*CV*	*W*	
*42.73*** (6.81)*	*-0.08 (-0.68)*	*14.34*** (5.99)*	*0.50*	*43.49*** (6.49)*	*-0.08 (-0.62)*	*14.44*** (6.01)*	*0.50*

** significant at 0.01 level; * significant at 0.05 level; t-statistics is in parentheses.

Both data sets show similar estimates, with rather low explanatory power. None of the dispersion variables is statistically significant from zero. On the other hand, as expected, stronger or richer teams perform better. Moreover, since both samples are very small and the players' statistics are not refined, it is difficult to draw certain conclusions.

4. Conclusions

The purpose of this paper was to develop a kind of "general equilibrium" model to investigate if teams perform better or worse when they pay rather compressed or more dispersed wages to their S- and N- quality players. In the model, four different football teams compete, in a tournament like the UCL group matches, to maximize their points and qualify in the next ground. Some key features of the model are the non-linearity of "production" functions (which are of CES type with different elasticity of substitution among players) and a number of various integer variables (like players, points, victories and draws). Assuming a high wage level for the implicitly solved S-quality players in every team and a rather large range of lower wages for the implicitly solved N-quality players, global optimal solutions were obtained.

In most cases, the team formation is composed by 1 S- and 10 N-players. Moreover, as it often happens in football, teams which field more S- players do not always perform better than teams with just 1 S-player! This is mainly due to differences in absolute and/ or relative efficiency the teams.

Despite the fact that all four teams perform almost equally well and the wage parameters lead to a rather balanced tournament, team 1, with the lowest elasticity of substitution between S- and N- players, collects on average, slightly more points than all others (followed by team 4, with the highest elasticity of substitution).

Using the collected points from the respective optimal solutions, linear and quadratic regressions were run to examine the wage dispersion on (i) the own effects and (ii) the cross effects. All four teams improve their own performance if the wage dispersion decreases. In most cases, the decrease in wage dispersion deteriorates the performance of other teams as well. In addition, some evidence on the U-type own success and a reverse U-type cross success appears. Consequently, while highly depressed wages improve performances linearly, intermediary wage dispersion is worse than a highly dispersed one.

On the other hand, using two small data sets, from the UCL and the Italian Serie A, the observed wage dispersion from all players of the teams was not statistically significant from zero, when other controlled variables (like the ranking or the value of teams) were included in the regressions.

The model can be easily extended to catch other important aspects. For instance, instead of maximizing points, subject to non-negative profits, teams can maximize profits. One can also increase the "price" parameter of points from 12 to 15, so that teams can collect more than 12 points that some global solutions provided. Another interesting extension can be to allow different teams compositions in different matches, or combine the "production" functions per match, to find out if the "Leontief team" beats or is defeated by the "Cobb-Douglas team", at home or away, and try to collect appropriate match data to test the derived hypothesis. Finally, since the selected CES functions do not let teams use various tactical dispositions of players, it would be desirable to stress the field positions of the S- and N-players. That can be similar in spirit to Hirotsu & Wright (2006), who applied a Nash-Cournot game to figure out the win probabilities of the 4-4-2 strategy over the 4-5-1 one.

Table A: MAD, CV, UEFA Ranking, Team value in m. of € and Points

Teams per group	Observed players	MAD (1)	CV (2)	Rank (3)	Value (4)	Points (5)
FC Bayern	24	1.8196	0.5517	122.507	359.95	13
Napoli	24	0.4203	0.6453	39.853	194.2	11
Man. City	16	1.3656	0.3051	61.507	467.0	10
Villarreal	16	0.2875	0.3647	78.551	142.6	0
Inter	26	1.3231	0.6479	102.853	246.85	10
CSKA	16	0.2625	0.2153	80.566	137.3	8
Trabzonspor	13	0.2793	0.3130	20.115	87.8	7
Lille	13	0.3562	0.2580	38.802	119.75	6
Benfica	16	0.2672	0.2189	86.835	168.2	12
Basel	13	0.2414	0.2704	53.360	48.3	11
Man. United	16	0.9422	0.3329	141.507	415.0	9
Otelui Galati	13	0.1402	0.2970	7.764	18.15	0
Real Madrid	16	1.5078	0.4148	110.551	542.0	18
Lyon	15	0.9173	0.3394	94.802	152.2	8
Ajax	13	0.4911	0.3211	57.943	97.35	8
Dinamo Zagreb	13	0.2734	0.2972	24.774	44.8	0
Chelsea	16	1.3500	0.3127	122.507	381.0	11
Bayer Leverkussen	13	0.5609	0.2907	59.403	137.0	10
Valencia	13	0.3030	0.1755	83.551	180.0	8
Genk	13	0.2367	0.2882	12.480	47.9	3
Arsenal	15	0.8089	0.3250	113.507	299.25	11
Marseille	13	0.5882	0.2520	84.802	140.65	10
Olympiacos	15	0.3067	0.3118	61.420	81.4	9
Borussia Dortmund	16	0.6031	0.4752	30.403	179.75	4
Apoel	13	0.2604	0.3221	33.599	14.95	9
Zenit	13	0.3172	0.2421	79.066	155.2	9
Porto	13	0.3976	0.2730	97.835	210.0	8
Shakhtar	13	0.4391	0.4539	83.894	143.45	5
Barcelona	16	1.6711	0.3680	151.551	579.0	16
AC Milan	25	1.2758	0.6483	88.853	266.3	9
Plzen	13	0.2166	0.3445	14.070	163.0	5
Bate Borisov	13	0.1112	0.2403	29.641	17.05	2

Notes: (1) & (2): For the three Italian teams, the annual wages are found in: http://www.xtratime.org/forum/showthread.php?t=261972 ; for all other teams, searching extensively various sport sites, forums and the teams' official sites; most values for the following teams are uncertain and for the non- Euro teams, estimated with various exchanges rates into €: Trabzonspor, Otelui Galati, Dinamo Zagreb, Genk, Apoel, Plzen, Bate Borisov.

(3): http://kassiesa.home.xs4all.nl/bert/uefa/data/method4/trank2012.html

(4): http://www.transfermarkt.co.uk/

(5): http://www.uefa.com/uefachampionsleague/season=2012/matches/round=2000263/index.html

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