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Abstract

Whether fixed factors such as land constrain per-capita income growth depends crucially on two variables: the substitutability of fixed factors in production, and the extent to which innovation is biased towards land-saving technologies. This paper attempts to quantify both. Using the timing of plague epidemics as an instrument for labor supply, I estimate the elasticity of substitution between fixed and non-fixed factors in pre-industrial England to be significantly less than one. In addition, I find evidence that denser populations – and hence higher land scarcity – induced innovation towards land-saving technologies.

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1 Introduction

One of the least understood aspects of economic development is the extent to which fixed factors constrain economic growth.[1] The debate has a long history. Malthus (1798) predicted that humanity, if left unchecked, would breed itself into poverty due to its inability to produce food proportional to a growing population in the face of a fixed amount of arable land. More recently, many have predicted the world running out of oil, coal, clean water, timber, arable land, or other essential natural resources, with disastrous economic consequences. While some scholars such as Ehrlich (1968) and Brown (2004) maintain that the world is heading towards poverty due to the fixedness of natural resources, others such as Simon (1981) and Boserup (1965) argue natural resources pose little if any restrictions on boundless economic growth.[2]

The extent to which fixed factors constrain growth is not only important to our future, but also to understanding our past. Many economists, including Lee (1973), Valor (2005) and Clark (2007) argue that the history of world population before 1750 is well summarized by Malthusian population dynamics. The role of fixed factors in production is a crucial element of this theory. In addition, the nature of fixed factors in these models affects dynamics around the Malthusian steady state. These dynamics are key in understanding the large and active body of literature exploring the escape from the Malthusian steady state and the takeoff to modern growth in Europe during the 19th century (see Lucas (1998), Valor and Weil (1999, 2000), Hansen and Prescott (2002), Doepke (2004), Fernandez-Vilaverde (2005), Voth and Voigtländer (2010)).

Whether fixed factors limit growth depends crucially on two variables. The first is the nature of productivity growth, and the second is the elasticity of

[1] By fixed factors, I mean inputs to production whose quantities are extremely difficult to increase if they become scarce. This includes non-renewable resources which cannot be replaced such as oil and coal, as well as arable land.

[2] A related issue is whether large quantities of natural resources hurt growth. Much research has been devoted to the topic (Sachs and Warner (1995, 1997, 2001), Sala-i-Martin and Subramanian (2003), Leite and Weidmann (1999), Ross (2001), Alexeev and Conrad (2009)), but this is a separate issue from the one described here. This paper focuses on whether fixed factors constrain growth directly via their dilution in the face of increasing population.

substitution between fixed and non-fixed factors.

Productivity growth can help alleviate resource constraints in two ways. First, any type of productivity growth can help overcome the constraining effects of a fixed factor. Therefore, sustained income growth is possible even in the presence of a fixed factor given a high enough rate of productivity growth. Predicting the future rate of productivity growth, however, is an extremely difficult if not impossible task, and I leave it for other work.

Secondly, the *direction* of innovation is important in overcoming resource constraints. If productivity growth is not Hicks-neutral, then whether future innovations are fixed or non-fixed factor saving is important. There are many reasons to think that future productivity growth may take place in the fixed factor sector. Hicks (1932) introduced the concept of "induced innovation", or the tendency for new innovations to save on scarce resources. Many scholars have tied this idea to fixed-factor saving technological progress. Popp et al. (2008) reviews recent literature on induced innovation in the context of energy economics, noting the recent increase in renewable energy R&D as a response to high oil and coal prices. The extent to which technological progress will be fixed-factor saving will influence the degree to which fixed factors will hinder growth.

To what extent non-fixed factors such as labor, physical capital, and human capital can be substituted for fixed factors in production is also important in influencing the extent to which fixed factors impact economic dynamics. A high elasticity of substitution between fixed and non-fixed factors allows the marginal product of non-fixed factors to fall more slowly as these factors accumulate. This can lead to higher levels of income per capita given the same fixed factor endowment.

The value of this elasticity is especially important in two types of models within economics. First, models of the effect of population size on income per capita can have different outcomes depending on how substitutable fixed and non-fixed factors are in production, since this elasticity will affect the intensity of the drag on income caused from congestion of the fixed factor. For example, models which estimate the gain in income via a health intervention which saves lives (Ashram et al (2008)) or fertility reductions (Ashram et al (2010)) depend crucially on this elasticity.

The second type of models for which the elasticity of substitution plays an important role are models of the the demographic transition and takeoff from Malthusian stagnation. The Cobb-Douglas production function, a highly popular description of production technology among economists, implicitly assumes the elasticity to be unity. Virtually every major study explaining the Malthusian regime or the demographic transition has used an elasticity of substitution of one (see Lucas (1998), Valor and Weil (2000), Stokey (2001), Hansen and Prescott (2002), Doepke (2004), Voth and Voigtländer (2010)). The elasticity of substitution can affect a range of outcomes in these models, such as the speed of the demographic transition, the level of pre-industrial income, the speed by which factors shift from the agricultural to the industrial sector, how intensely wages react to population shocks, how quickly wages fall back to the Malthusian steady state after a productivity shock, steady state levels of population density, and predictions of which countries should make the demographic transition first.

However, despite the importance of this parameter, there are few – if any – reliable estimates of this elasticity. A few economists have attempted simple analyses within larger papers on other topics where this elasticity mattered, but no serious attempt has been made at its estimation. A common method used to estimate this elasticity is to regress shares of income paid to factors of production (or alternatively the rental rate paid to factors of production) on factor quantities. However, since these factor quantities are endogenous to their rental rates, these estimates are not well identified. In addition, data on factor shares are notoriously difficult to quantify, leading to imprecise estimates.

The first attempt was by Nordhaus and Tobin (1972), who used data on shares of income paid to land and labor from the U.S. in the first half of the 20th century and find that the elasticity of substitution is about 2. However, they only use 10 observations, and therefore they cannot estimate the elasticity with any certainty[3].

The most cited estimate is from Hayami and Ruttan (1985), who look at a cross section of the agricultural sector in 30 developed countries and test whether the elasticity is unity and fail to reject. However, they do not measure the elasticity of fixed and non-fixed factors, but rather the elasticity of labor and all other factors, then make the assumption that land is the only factor. This is a difficult assumption to make, since capital is heavily used in agriculture in the developed world. Their study also suffers from a small sample size and poor identification.

Given the paucity of good estimates for the elasticity, recently Ashram, Lester, and Weil (2008) use data on the natural resource share in national income from Caselli and Feyrer (2007) for estimation. They find a value of 2.35 and can reject unity. While an improvement over previous studies in regards to sample size, Ashram et al rely heavily on imputed data. First, they use imputed data on land's share of income from Caselli and Feyrer (2007). They also impute the quantity of the fixed factor rather than measuring it directly, due to the fact that there is no obvious unit of measure for "fixed factors". Finally, there is no attempt to overcome the endogeneity issues mentioned previously.

Weil and Wilde (2009) improve upon the work of Ashram et al. by using a set of indicators on natural resource stock per capita, and use instruments for income. Depending on the model specification, they find an elasticity which varies from 1.5 to 5, with most of the estimates being around two.

To estimate this elasticity, all of these studies above measure how the share of national income paid to fixed-factors (or alternatively the rental rate) changes as non-fixed factors accumulate. However, none of the studies accounts for the fact that in an open economy, factor price ratios across countries tend to equalize to the world rate (Ohlin (1933), Stolper and Samuelson (1941), Williamson (1999, 2000a, 2000b)). Therefore, the estimation will be biased upward, since the observed change in factor rent ratios should be zero (or at least smaller than they would be in autarky) as non-fixed factors accumulate

[3]Nordhaus and Tobin fail to reject the elasticity is ∞, but also fail to reject 1.

in an open economy. This implies that the estimation of the elasticity of substitution between fixed and non-fixed factors is better achieved in closed economies.

In addition, the most useful estimates of the elasticity of substitution will come from economies in which fixed factors play an important role – namely developing countries. If we truly want to use an estimate of the elasticity which is useful in explaining the transition from an agricultural economy to an industrial one, or to explain how differing elasticities of substitution may change growth dynamics or the effect of population size on income per capita in the developing world, it would be nice to estimate this elasticity for a set of agrarian economies.

In this paper, I estimate this elasticity of substitution in a closed, agrarian economy – pre-industrial England from 1200-1750. I use changes in labor supply via population size using data on factor rents from Clark (2007) in a simple CES framework. As in the previous literature, my estimation strategy relies on regressing factor rents on factor quantities (specifically I regress the ratio of land rents to wages on the land labor ratio) to uncover the elasticity of substitution. However, I overcome the issue of endogeneity by instrumenting for the land-labor ratio with the timing of plagues in England. As I show below, the timing of plague deaths is likely an exogenous source of variation in population levels, and can be traced back to specific exogenous events such as the arrival of a ship filled with plague-laden rats from an external port.

In addition to my estimation of the elasticity of substitution between fixed and non-fixed factors, my methodology also allows me to estimate the degree of factor-induced productivity growth. If technological progress is induced, that implies the land-labor ratio should not only effect the *level* of the ratio of factor rents via the elasticity of substitution, but also their *growth rate* via a changing ratio of factor-specific productivities. Using a model which incorporates both of these effects, I can control for and quantify both of these phenomena. This study is the first to my knowledge which can put a number on the extent to which induced innovation occurred in pre-industrial England.

Finally, I look at how my results for the elasticity of substitution and induced innovation impact well known economic models. These models fall into two categories. First, I construct a simple model of a Malthusian economy and show that the elasticity of substitution affects the speed by which wages and population converge to their steady state levels after a technology or mortality shock. I then analyze how a small elasticity of substitution would change dynamics in models of the transition from the Malthusian steady state to modern growth such as Valor and Weil (1999, 2000), Hansen and Prescott (2002), Voth and Voigtländer (2010), and Stokey (2001). Second, I analyze how the elasticity of substitution would affect models of the effect of population size on development levels generally.

I find that the elasticity of substitution between land and other factors over this period in England was about 0.6. This implies that the elasticity of substitution is much smaller than previously thought (or at least modeled), since most models addressing the issue of population and the demographic transition implicitly assume an elasticity of 1. This is a novel finding, because the share of income paid to land has been falling since at least the early 1900s (at least in the developed world), which is generally a result of an elasticity of substitution greater than one.

This implies that any short-run deviations of population size and income from their Malthusian steady state were shorter and smaller than previously modeled. In addition, since I estimate the elasticity of substitution over a long period (550 years), this implies that the elasticity I am measuring is a long-run elasticity. In the short run, this elasticity must have been even smaller, implying that land was even more constraining of a factor over shorter periods of time, such as an individual's lifetime.

I also find evidence for factor-biased technological change. Specifically, I find that the difference between the annual growth rates of land- and labor-augmenting productivity was 0.10% higher per million persons – implying higher populations and therefore higher population densities induced innovation towards land-saving technologies. This implies that a doubling of population density in England from its year 1500 level raises the difference in the growth rates of land- and labor-enhancing productivity by 0.22% per year.

Many economic historians have pointed to the fact that technological changes which occurred over this period appeared to be induced, but this is the first study to empirically test and quantify the phenomenon.

I also find that smaller elasticities of substitution slow down the escape from Malthusian stagnation in models which depend on achieving a certain level of population to transition (such as Valor and Weil (1999, 2000)) and in models which rely on shifting production away from the agricultural sector due to biased technological progress (such as Hansen and Prescott (2002)). However, I find that small elasticities of substitution should lead to *faster* transitions in models which rely on achieving a certain level of income to escape from the Malthusian steady state (such as Voth and Voigtländer (2010) and Stokey (2001)).

Finally, I find that smaller elasticities of substitution imply population size has a larger negative effect on income. Interventions in developing countries which affect population size can have widely varying results based on the elasticity of substitution. For example, a health intervention which both saves lives and increases worker productivity could have a positive or negative effect on income depending on the elasticity of substitution. In addition, the elasticity can impact the benefit to an intervention to reduce fertility to have a large or small effect.

The paper continues as follows: Section 2 outlines my basic model and results. Section 3 augments the model to account for biased technological progress and provides results. Section 4 looks at the implications of different elasticities of substitution in economic models. Section 5 concludes. In addition, the appendix details the data on the timing of national plague epidemics in pre-industrial England.

2 Model with Hicks-Neutral Technology

Consider the following constant elasticity of substitution production function introduced by Arrow, Chenery, Minhas, and Solow (1961),

$$Y_t = A_t \left[(\psi X_t)^{\rho} + ((1-\psi)N_t)^{\rho}\right]^{\frac{1}{\rho}}, \tag{1}$$

where X_t is a fixed factor, N_t is a non-fixed factor, Y_t is output, and A_t is total factor productivity, and $\psi \in (0,1)$ is a scale parameter, and t indexes time. Notice that technology in this case is Hicks-neutral. Later in the paper I will relax the assumption of Hicks-neutral technology, which will allow me to estimate the magnitude of induced innovation.

The elasticity of substitution between X_t and N_t is

$$\sigma = \frac{1}{1-\rho}.$$

Taking the ratio of the marginal products of X_t and N_t (denoted r_t^X and r_t^N), and taking logs yields

$$\ln\left(\frac{r_t^X}{r_t^N}\right) = -\frac{1}{\sigma}\ln\left(\frac{\psi}{1-\psi}\right) - \frac{1}{\sigma}\ln\left(\frac{X_t}{N_t}\right). \tag{2}$$

Therefore, in principle I can estimate the elasticity of substitution between fixed and non-fixed factors by regressing the log ratio of fixed factor rent and wages on a constant and the land-labor ratio.

There are several difficulties in estimating this equation directly. First, the quantities of X and N are potentially endogenous to their rental rates, and therefore estimation via OLS can yield biased results. Second, data on the quantity of factors in the economy are difficult to measure, especially in the historical context, and therefore may lead to attenuation bias in estimation. Third, there is no obvious unit of measure to combine different types of natural resources to create a value for the fixed factor stock X_t.[4]

[4] The most logical solution – to convert the value of the stock of each resource into dollar terms to obtain the total value of X – is incorrect since resource values are obtained by merely capitalizing the stream of resource rents. Therefore, the dependent and independent

Measurement issues are especially acute in the context of pre-industrial England, since there is little, if any, data on land use, capital stocks and rents, human capital attainment, or even labor force back to 1200. Therefore, I make two assumptions to simplify the baseline analysis:

1. The total endowment of natural resources, X_t, is constant over the entire sample.

2. The only non-fixed factor in the economy is labor, and labor is a fixed fraction of the population. Put mathematically, $N_t = L_t = \psi P_t$, where P_t and L_t are population and labor supply respectively at time t.

The first assumption side-steps the problem of measuring fixed resources. This allows $X_t = \bar{X}$ to become part of the constant term in my regression equation. The second assumption allows me to use data on factors I can easily acquire for the period 1200-1750 – namely population, wages, and land rents – while forgetting about physical and human capital and their returns. Later in this section I will discuss each assumption and analyze what happens when it is relaxed.

Given these assumptions, we can rewrite (6) as:

$$\ln\left(\frac{r_t}{w_t}\right) = \alpha + \frac{1}{\sigma}\ln\left(P_t\right), \tag{3}$$

where $\alpha = -\frac{1}{\sigma}\ln(\frac{\psi}{1-\psi}) - \frac{1}{\sigma}\ln(\bar{X})$, w_t is the wage, and r_t is land rent.

2.1 Basic Data and OLS Results

The data on factor rents (wages and land rent) come from Clark (2001, 2002, 2007b)[5]. The unit of observation is the decade. Clark's land rental income series is based on several data sets. For the period 1200-1500, he mainly uses manorial accounts and manor court records which record income from leases for a landholder (see Clark (2001) for more details). This is beneficial

variables would measure the same thing, and the estimates would be biased.

[5]Since land was the main fixed factor in pre-industrial England, I use the rental rate on land as a proxy for the rental rate of fixed factors

since his series is mostly based on actual transactions of land rentals rather than imputed from land prices, although some land price data is used for the period 1200-1320. For 1500-1870, he uses rental values of lands from charitable trusts. Although each of these samples are geographically diverse, they are not nationally representative since more densely populated areas will have disproportionately more data. Therefore, Clark calculates a national rent index by applying weights for regions, plot size, amount of common land, type of land (farmland, meadow, etc.) and population densities (see Clark (2002) for more details).[6]

Nominal wages are determined by similar records on payments to hired farmhands and builders, which are converted to real terms by adjusting by the cost of a bundle of agricultural goods. The data and methods Clark uses to obtain this series are too detailed to be outlined in full here, but can be found in Clark (2007b). The data for population are also from Clark, who in turn used Wrigley et al (1997) for 1540-1870, and Hatcher (1977), Poos (1991), and Hallam (1988) for 1200-1540.

Figure 1 plots the wage and land rent series over time, in addition to the capital rent series from Clark which will be introduced and used later. Figure 2 plots the ratio of land rents to wages from Figure 1 to form a time series of our dependent variable. This is contrasted with population size, our independent variable of interest. It is immediately clear that the two series follow each other quite closely for the period 1200-1750. This is especially evident around 1348, when the Black Death sharply lowered population – the ratio of rental rates immediately followed. In the late 18th century, however, the relationship breaks down as population and factor prices begin to explode with the beginning of the industrial revolution.

It has been widely noted that the Malthusian model of the economy broke down as soon as Malthus wrote it. Not only did wages explode despite population growth, but factor prices began to be driven by forces other than population density, such as trade. O'Rourke and Williamson (2005) note that

[6] Clark graciously provided me with a revised version of his rental rate data from 1200-1800. Therefore, data I use may differ slightly from the published version.

after the beginning of the industrial revolution, factor rents across the globe (but especially in England) tended to equalize to a world rate. With the expansion of overseas trade in the 18th and 19th century, the assumption of a closed economy becomes less palatable. In addition, the fundamental changes in the British economy make it difficult to believe the elasticity of substitution was the same before and after the industrial revolution. For all these reasons, I cut the sample at 1750.

In table 1, column 1, I estimate equation (3) using OLS, and find that the elasticity of substitution is 0.718, and significantly less than one.[7] As mentioned earlier, this is a surprising result, since most contemporary estimates of the elasticity of substitution find that it is greater than one. In column 2, I add a time trend, and the estimate falls slightly to 0.695. Later when I relax the assumption of Hick-neutral technology growth, this time trend will take on economic significance, as will be discussed in section 2.3.

Figure 3 shows a residual plot of the regression in column 2 of Table 1. A residual plot is a useful way of showing the partial correlation between two variables in a multivariate regression, and is useful to see if the regression coefficient on a particular variable is driven by outliers.[8] The relationship between the ratio of land rents to wages and population is positive and very strong. As a result, the estimate of the elasticity of substitution has an extremely small standard error. It is also interesting to note that over a 550-year span, the relationship remained very stable. One might expect that over the 550 year span of the sample something fundamental changed in the English economy regarding the importance of fixed resources and land – so it is remarkable to note that the relationship between factor rents and population was almost exactly the same in 1750 as it was in 1200.

[7]All OLS estimates use Newey-West standard errors with one lag since there is a large degree of autocorrelation in the pure OLS residuals.

[8]Procedure for explaining how to obtain a residual plot will be explained in the note to Figure 3.

2.2 Instrument and 2SLS Results

There are two potential sources of bias in the above analysis. First, there is the issue of measurement error given the historical nature of the data. Estimates of medieval populations in general vary greatly, since they are usually based on a small subset of villages for which data exists and then extrapolated to the country as a whole. More accurate data on population based on family reconstruction are available only after 1541 (see Wrigley et al (1997)).

Even though this paper attempts to correctly quantify the elasticity of substitution between fixed and non-fixed factors, a major contribution of the paper is a qualitative one – that the elasticity of substitution was significantly less than unity. Therefore, if I find a statistical problem in my estimation, determining the direction of bias may strengthen or undermine that conclusion. In the case of measurement error, it strengthens my qualitative results that the elasticity of substitution is less than one. To see this, notice that the elasticity is the inverse of coefficient on log population. Therefore, since measurement error will cause attenuation bias to the coefficient on log population, this implies that the inverse will be biased upward. Therefore, the true elasticity will be even smaller than I estimate.

Secondly, there is concern about the exogeneity of population changes. In the standard Malthusian model, higher wages should lead to higher population growth, and low wages should lead to population decline.

Both statistical problems can be solved by using an instrument. In this paper, I use the timing of plagues in England as an instrument for population. In particular, I use the series of plagues known collectively as the Second Pandemic.

The Second Pandemic began in approximately 1346. It originated in Central Asia and ultimately affected the entire Afro-Eurasian continent. Although accurate estimates are difficult to obtain, it has been estimated that this first wave of the second Pandemic, commonly referred to as the Black Death, killed half of the inhabitants of China, a third of Europe, and an eight of Africa. It was introduced into Europe in 1357 when Mongol armies besieging the Crimean city of Caffa catapulted infected corpses over the city wall in an attempt of biological warfare. The remaining inhabitants of the city fled to Sicily, from which the plague spread into the rest of Europe.

The plague arrived in England via a ship from Gascony in southwestern France to Weymouth shortly before 24 June, 1348. From Weymouth, the disease spread rapidly across the southwest of England, hitting Bristol first and reaching London in the fall. The spread of the disease slowed in the winter, but the next spring spread across all southern England and into the north. It reached York in May, and spread quickly over all of northern England during the rest of the summer, before dying out that winter. Although estimates disagree, the Black Death killed between 25% and 60% of the English population.

The first serious recurrence in England came in the years 1361-62, and then again in 1369, with a death rate of approximately 10-20%. (Gottfried (1983)) Over the following century the plague would return at intervals of five to twelve years, with continuously smaller mortality. There was a resurgence in severity between 1430 to 1480 – the outbreak in 1471 took as much as 10-15% of the population, while the death rate of the plague of 1479-80 could have been as high as 20%.(Gottfried (1983)) From 1480, the outbreaks decreased in frequency in England until the last great plague epidemic, the Great Plague of London in 1665-66. On the European continent, plagues continued to occur well into the 18th century.

There are three types of plague, which differ in the location of infection and vector of transmission. Each type of plague was present during the Black Death, and was caused by the same bacterium, *Yersinia pestis*. First is the bubonic plague, which affects the lymph nodes. It was transmitted via infected flea bites, and was carried by plague-resistant rates. Second, the pneumonic plague infected the lungs, and was transmitted through the air via tiny droplets of infected fluid. This form was especially deadly, since its main symptom was coughing and could be spread human to human. Last was the septicemic plague, which is spread through direct contact with infected tissue or bodily fluids, and infects the blood stream. Mortality from the plague was high, ranging between 30% to 95% of those infected.[9]

[9]Earlier I make that assumption that labor supply is a constant fraction of the population.

In Appendix A, I list every major national plague outbreak in England during the Second Pandemic, starting with the Black Death (1348-1350) and ending with the Great Plague of London (1665-1666). I can use the data in two ways to create an instrument: First, I can use an indicator variable for whether or not there was a national plague epidemic in that decade. Secondly, I can accumulate the plague outbreaks to create the number of plague outbreaks in the last 100 years. My empirical results will be similar using either instrument.

The key identifying assumption is that the timing of plague epidemics were exogenous, and only affect relative factor prices through their effect on labor supply. However, one may worry that the timing of the plagues were endogenous to land scarcity, and therefore to its rental rate – which would imply failure of the exclusion restriction. At the center of this view is the idea that the plagues were a necessary and long overdue Malthusian correction of population, which had reached or even exceeded its sustainable level.

Many historians, however, argue that is is not the case. (see Helleiner (1950), Hallam (1972), Herlihy (1997), Poos (1985), Chavas and Bromley (2005). Hatcher and Bailey (2001) provides a further review.). These scholars argue that the Black Death could not have been caused by a Malthusian crisis for several reasons. First, they argue that the halt in rapid population growth preceding the Black Death was not due to land scarcity, but rather due to weather and climatic shocks. Before the Black Death, the population of England increased rapidly from 2 million in 1000 A.D. to 6 million in 1317, indicative of non-scarcity of land. In 1315-1322, England suffered a period of massive and generalized crop failures due to very extreme and rare weather conditions (Kershaw (1973)), otherwise known as the Great Famine. In addition, in 1319 there was a severe cattle plague which destroyed draught

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One may worry that this may not be the case during the Black Death for two reasons. First, labor supply per person may fall due to morbidity caused by the plague. This does not seem to be the case, as the plague generally killed those infected within a week. Second, the fraction of the population working may change if the Black Death disproportionately affected certain age groups. This also does not seem to be the case – mortality rates for all age groups and social classes were similar. However, mortality rates were particularly high for some occupations, such as monks and priests who directly cared for the infected and dead.

animals and lowered agricultural productivity. This famine led to increases in crime, displacement, and social upheaval, reducing population through the 1330s. These scholars argue that without these weather shocks, population likely would have continued to grow rapidly, implying there was no Malthusian crisis.[10]

Secondly, food prices were quite low on the eve of the Black Death, further indicating land was not scarce. In addition, the ratio of land rents to wages was constant for the 150 years before the Black Death, whereas increasing population pressure would have implied a rising ratio. Third, they argue the plague timing was truly random since it hit all of Europe simultaneously, even the countries whose populations which had much different dynamics and land-labor ratios than England. The Black Death, they argue, would have been just as deadly whether or not there were population pressures, and therefore the fact that population dropped dramatically is not a proof that England was above a sustainable level of population.

Finally, the timing of plagues were not consistent with the theory of when plagues should occur. If Malthusian pressures caused the plagues, then more plagues should occur when population is high, and less when population is low. In reality, we see the exact opposite happening – national plague epidemics were most frequent precisely when population was its lowest. In addition, Herlihy (1997) argues that the Black Death came too late if it were caused by population pressures, since population in 1348 was well off its peak. After the Black Death population remained low for 300 years, but plagues continued to occur.

Since many of the plagues can be tied to specific events, such as a rat-laden ship arriving in a port, it is highly plausible that they were exogenous shocks to population. It may be possible, however, that the *intensity* of plagues when they strike could be affected by land scarcity. A population which faces severe overcrowding and population pressures may be more susceptible to disease. However, I avoid these problems by not using data on plague intensity, but

[10] In fact, Campbell (1984) finds evidence that population did continue to increase in spite of the Great Hunger and its subsequent unrest.

rather plague timing.[11]

In columns 3 and 4 of Table 1 are the results of the 2SLS regression of equation (3) with each instrument. When using the number of plagues in the last 100 years as an instrument, I estimate the elasticity of substitution between fixed and non-fixed factors to be 0.630. This is slightly smaller than the OLS estimate, which we should expect to be the case due to measurement error. Similarly, using the plague indicator as an instrument, I find the elasticity of substitution to be 0.683. The F-test on the first stage instrument coefficient in each regression are 18.34 and 22.04 for plagues in the last 100 years and the plague indicator respectively.

Columns 5 and 6 repeat the analysis with a time trend. The results are similar, with the elasticity of substitution being 0.693 when I use the number of plagues in the last 100 years as the instrument, and 0.695 for the plague indicator. The first stage regressions are stronger in this case, with F-statistics of 126.29 and 33.2. This finding that the elasticity of substitution is less than one has many implications, and will be discussed further in section 4.

2.3 Robustness and Biases

Up to this point, I have used a simple model to estimate the elasticity of substitution. However, one may worry about the validity of several key assumptions. This section will calculate the direction of bias in my estimates when relaxing these assumptions, as well as present robustness checks.

Fixedness of land

I assume that the total endowment of natural resources, X_t, is constant over the entire sample. There are two possible issues with this assumption. First, one may worry that population pressure induced land use to change. However,

[11] One may wonder whether repeated plague outbreaks were in fact the same outbreak, where the bacteria lie dormant in the population, and manifested itself when population pressure became large. However, this possibility is ruled out by the biology of the plague itself as will be discussed later. In short, outbreaks tended to be short and intense, with mortality rates too high, and the duration and incubation period of the plague were too short, to have been dormant in the population.

it is important to realize that Clark's land rent series is not solely based on cropland rents. Rather, it is a national rent index over many different types of land uses, such as arable land, meadow, marsh, pasture, or mixed use. As long as national borders don't change, the overall amount of land available for use in any type of production remains the same. Since I use an aggregate production function, whether natural resources change between uses is irrelevant to the quantity of natural resources in the economy.

Second, the overall amount of land available for use in *any* type of production may be changing due to land reclamation, soil depletion, or erosion. There is a large literature detailing the history of land reclamation, such as the draining of the fen in eastern England or deforestation. If the amount of available land increased over the sample period, this could bias our estimates.

We can determine the direction of bias that would result if the assumption of a constant endowment of fixed factors were violated. Assume that rising population densities increases the amount of land used in the production of goods. This implies a positive correlation between P_t and X_t, meaning the land/labor ratio would change less in response to population growth than if X_t were fixed. Therefore, the coefficient on P_t in regression (3) would be underestimated under the assumption of a fixed X.[12] Since σ is the inverse of the coefficient on P_t, the estimated elasticity would be too large. Therefore, although changing X_t may bias my estimate of σ, it actually strengthens my qualitative argument that the elasticity is less than one.

The opposite would be true if resource depletion were an issue. If resource depletion implied a negative correlation between P_t and X_t, then the estimated elasticity would be too small. My qualitative result depends on which effect was larger, the land reclamation effect, or the land depletion effect. As stated above, the facts suggest that land reclamation was increased the total amount of land available for production, and therefore my estimates for σ are too large – strengthening my argument that the elasticity was less than one.

[12] By underestimate I mean in absolute value.

Labor as the only Non-fixed Factor

I assume that the only factors of production are land and labor in this economy. What about capital, both human and physical? In this section, I extend the model to account for additional non-fixed factors of production. First, I add just physical capital to the model by allowing N_t in equation (1) become a capital-labor composite factor, $N_t = K_t^{\delta} L_t^{1-\delta}$. From CES production function, I derive the following relationship between L and K:

$$K = \frac{\delta}{1-\delta} L \frac{r_L}{r_K}$$

Substituting this into N_t above results in the following regression equation:

$$\ln\left(r_t^X\right) = \alpha + \frac{1}{\sigma}\ln\left(P_t\right) + \zeta_1 \ln\left(w_t\right) + \zeta_2 \ln\left(r_t^K\right). \tag{4}$$

This is the same as equation (3), without assuming $\zeta_1 = 1$, and controlling for the wage and rental rate on capital in the regression. By extension, I can control for the presence of any non-fixed factor in the composite factor N_t by simply controlling for its rate of return.[13]

Clark (2001, 2002, 2007b) provides data on the rental rate of capital r_K in addition to the data on wages and land rents. He uses documents transferring property by gift or sale to a religious house – or cartularies – which include information on rental payments to capital to calculate capital's rate of return.

Human capital is omitted from the model for two reasons. First, data on human capital over this period are difficult to obtain. Data on any indicator of human capital (education levels, literacy rates, rates of return to education, etc.) are not available back to 1200. However, some data does exist for the latter part of the sample, and they indicate that levels of human capital during the middle ages were quite low. Literacy rates for males in England reached 25% only after 1600, and were even lower for females. Even after the industrial revolution started, educational attainment remained low. For example, the

[13] This statement holds if the composite factor N is of Cobb-Douglas form. If not, then the rates of return would need to be controlled for in a non-linear fashion.

fraction of children 5-14 enrolled in primary eduction did not exceed 10% until after 1850 (Flora et al (1983)).

Secondly, since the existing evidence points to a low level of human capital before the industrial revolution, it likely wasn't an important factor of production. Valor (2005) argues that even during the first phase of the industrial revolution, human capital had a limited role in the production process since factory work did not require literacy.

I estimate equation (4) in table 3. Column 1 reports a simple OLS regression with a time trend but without controlling for capital. Columns 2 and 3 are 2SLS estimates of the same regression using each of the two instruments. The results are similar to the simple OLS regression obtained in table 1.

Columns 4-6 repeat the previous exercise while controlling for capital. The estimated elasticity rises from about 0.6 to 0.8, depending on the specification. While the elasticity is still less than one, I can no longer reject that it is equal to one. This is problematic for my qualitative conclusion in the previous sections that the elasticity of substitution is less than one. However, as we will see in the next section, when I control for factor induced technical change in the next section, the estimated elasticity of substitution will fall back to its previous level of about 0.6, and be significantly less than one (see columns 7-9).

Labor as a Constant Fraction of Population

I assume that labor is a fixed fraction of the population. However, it is possible that labor force participation rates changed as a result of the Black Death. For example, high wages after the Black Death may have induced more labor into the market. Many scholars, including Voigtländer and Voth (2010b) note that female labor force participation increased after the Black Death in response to higher wages, labor scarcity, the change in production from grain to pastoral goods, and increased demand for pastoral products.

Just as before, we can determine the direction of bias if labor supply was not a constant fraction of population. Suppose that labor supply per person increased when population was low and decreased when population was high.

This implies that fluctuations in population overstate the true change in labor supply, and therefore the coefficient on population will be underestimated. This implies σ will be overestimated – further strengthening my finding that $\sigma < 1$.

3 Biased Technological Progress and Induced Innovation

In previous sections I have assumed that technology is Hicks-neutral. I now consider the following CES production function with factor-specific technologies A_X and A_N:

$$Y_t = \left[\left(A_t^X X\right)^{\rho} + \left(A_t^N N\right)^{\rho}\right]^{\frac{1}{\rho}}, \tag{5}$$

This is the same as before, except now productivity is not Hicks-neutral, but rather is factor specific. Following the analysis and making the same assumptions as in Section 2, this results in the following regression equation:

$$\ln\left(\frac{r_t}{w_t}\right) = \alpha - \frac{1}{\sigma}\ln\left(\frac{A_t^X}{A_t^L}\right) + \frac{1}{\sigma}\ln\left(P_t\right). \tag{6}$$

In addition to difficulties estimating the regression model as mentioned in section 2, we now have A_t^X and A_t^L, which will affect the factor rent ratio, but which are unobserved. Since the ratio of factor specific technology $\ln\left(\frac{A_t^X}{A_t^L}\right)$ is unknown, it acts as an omitted variable and therefore will bias the estimation of σ if $\ln\left(\frac{A_t^X}{A_t^L}\right)$ is correlated with the land-labor ratio. The theory of induced innovation implies that the growth *rate* of $\frac{A_t^X}{A_t^L}$ should be higher if the land-labor ratio is low, so it is unclear whether the *level* of $\frac{A_t^X}{A_t^L}$ should be correlated with $\frac{X_t}{L_t}$.

However, I can attempt to control for factor-induced productivity growth by modeling it more explicitly. Assume that the growth rates of A_X and A_L (g_X and g_L respectively) vary depending on the land/labor ratio, such that when land is scare (i.e. when population is high), g_X will be high and g_L will be low. We can rewrite $\ln\left(\frac{A_t^X}{A_t^L}\right)$ as:

$$\ln\left(\frac{A_t^X}{A_t^L}\right) = \ln\left[\frac{A_{t-1}^X\left(1+g_t^X(P_t)\right)}{A_{t-1}^L\left(1+g_t^L(P_t)\right)}\right] = \ln\left[\frac{A_{t-1}^X}{A_{t-1}^L}\left(1+g_t(P_t)\right)\right],$$

where $g_t = \frac{\left(1+g_t^X(P_t)\right)}{\left(1+g_t^L(P_t)\right)} - 1$, $\frac{\partial g_t}{dP_t} > 0$.[14] By iterating this equation and using the approximation that $\ln(1+x) \approx x$, we can express the level of $\ln\left(\frac{A_t^X}{A_t^L}\right)$ into an initial condition and the sum of previous growth rates:

$$\ln\left(\frac{A_t^X}{A_t^L}\right) \approx \ln\left(\frac{A_0^X}{A_0^L}\right) + \sum_{i=1}^{t} g_i(P_i).$$

The true functional form of $g(P_t)$ is unknown. I use a linear specification in my estimation, $g_t = \gamma + \theta P_t$. This implies:

$$\ln\left(\frac{A_t^X}{A_t^L}\right) \approx \ln\left(\frac{A_0^X}{A_0^L}\right) + \gamma t + \theta\sum_{i=1}^{t}(P_i).$$

Plugging back into the regression equation we have:

$$\ln\left(\frac{r_t}{w_t}\right) = \alpha - \frac{1}{\sigma}\gamma t - \frac{1}{\sigma}\theta\sum_{i=1}^{t}(P_i) + \frac{1}{\sigma}\ln\left(P_t\right), \tag{7}$$

where $\alpha = \ln\left(\frac{A_0^X}{A_0^L}\right) - \frac{1}{\sigma}\ln(\bar{X})$. These assumptions lead me to the following regression equation:

$$\ln\left(\frac{r_t}{w_t}\right) = \alpha + \beta_1 t + \beta_2\sum_{i=1}^{t}(P_i) + \beta_3\ln\left(P_t\right), \tag{8}$$

where we can recover the parameters of interest γ, θ, and σ from the βs. Specifically, $\gamma = -\frac{\beta_1}{\beta_3}$, $\theta = -\frac{\beta_2}{\beta_3}$ and $\sigma = \frac{1}{\beta_3}$.

In the previous section, I alluded to the fact that the time trend in the

[14] Note that $g_t \approx g_X - g_L$.

regressions in Table 1 had meaning. In a world with no induced innovation ($\theta = 0$), the time trend will allow us to recover the average difference between land-specific and labor-specific innovation over the sample period. The term $\sum_{i=1}^{t}(P_i)$, or the sum of past levels of population, allows us to determine the amount of induced innovation. This is easier to see if I rewrite equation (8) in differences:

$$\triangle \ln \left(\frac{r_t}{w_t} \right) = \beta_1 + \beta_2 P_t + \beta_3 \triangle \ln \left(P_t \right), \tag{9}$$

Equation (9) states that the change in the ratio of factor rents is affected not only by the change in the land-labor ratio, but also the level of the land-labor ratio itself via its effect on $\frac{A_t^X}{A_t^L}$ through induced innovation.

In the simple regression model without controlling for induced innovation, I used plague deaths as an instrument for population. When I include induced innovation in equation (8), population now enters the equation twice – both in the level of population and in the sum of past levels of population. In order to correctly identify the parameters, I now use two instruments – plagues for the current level of P_t and the sum of past plagues for $\sum_{i=1}^{t} P_t$. In the results that follow, I will therefore report two F-stats for the first stage regression, corresponding to the joint significance of the instruments in each of the first stage regressions.

I estimate both the levels equation (8) and the differences equation (9) in Table 2. Column 1 repeats the simple OLS regression from Table 1 with a time trend and controlling for induced innovation. The elasticity of substitution is about 2/3, almost identical to the estimates in table 1. When using 2SLS with each instrument, the elasticity becomes smaller but qualitatively similar to the OLS regression.

The estimates for implied γ estimate the growth rate of $\frac{A_X}{A_N}$ if population were zero. In the OLS regression in column 1, I estimate that γ is -0.163%, implying that technological progress would be labor saving. This is intuitive – if population is low, labor would be scarce, and innovation would be labor saving. For the 2SLS regression in columns 2 and 3, the estimates are -0.302% and -0.361%. This is about double the size of the coefficient in the OLS case.

The implied θs show the degree of induced innovation – the extent to which the *level* of population influences the growth *rate* of the ratio of factor prices. A positive coefficient implies that higher populations lead to faster growth in $\frac{A_X}{A_N}$. In other words, the difference in the growth rate of land and labor specific productivities, $g_X - g_N$, was larger when population was higher. This implies that innovation was induced to be relatively land saving, since land was the scarce factor when population was high.

In column 1 (the OLS regression), I estimate that θ was 0.051%. This means that for each million people in England, $g_X - g_N$ was 0.051% higher. When I instrument for population in columns 2 and 3, the estimate approximately doubles to about 0.1% per year. Since the overall growth rate in $\frac{A_X}{A_N}$ is equal to $\gamma + \theta P_t$, we can calculate the population at which g_X and g_N grew at the same rate by from $-\frac{\gamma}{\theta}$. In each of the regression is columns 1-3, I calculate this value to be between 3.2-3.4 million inhabitants. Above this value, higher population densities induced innovation to be land saving, and below this value technology was relatively labor saving.

Figures 6 uses these estimates to demonstrate how $\frac{A_X}{A_N}$ evolved from 1200-1750. The OLS estimates imply a smaller rate of induced innovation, and therefore $\frac{A_X}{A_N}$ responds less drastically to changes in population. The level of $\frac{A_X}{A_N}$ is about 25% higher in 1750 than in 1200. The 2SLS estimates should an increase of about 40% over the same time frame, and large changes induced by population.[15] Figure 7 shows how the overall growth rate of $\frac{A_X}{A_N}$ evolved over the period for the two different sets of estimates. For the OLS estimates, it reaches about 0.13% per year in 1300, to a low of about -0.04% in the 1500s, back to about 0.14% in 1700. For comparison, the 2SLS estimates show a growth rate of .21% in 1300, -0.10% in 1500, and .25% in 1750.

In columns 4 and 5 of table 2 I estimate the same regression in differences. The elasticity of substitution is insignificantly greater than 1 in the simple OLS regression. This is surprising, since I obtained an elasticity of substitution of significantly less than 1 in every regression thus far. However, when I

[15] The 2SLS estimates refer to those obtained in column 2, using the number of plagues in the last 100 years as an instrument.

instrument for population the value of σ falls back to its usual levels (although the standard error is still quite high such that I cannot reject an elasticity of 1).

As mentioned above in section 2.3, I re-estimate the augmented model with capital in columns 7-9 of table 3. Column 7 shows the OLS result controlling for technical change, while columns 8-9 show the estimates with each of the two instruments. The elasticity of substitution is slightly lower than before, from 0.583 in the OLS specification to 0.429 in one of the 2SLS regressions. The amount of implied induced innovation is the exact same in the two OLS equations at 0.051% per million people. However, in the 2SLS regression, the degree of induced innovation is slightly smaller when capital is included, down to 0.077% from 0.090%.

4 Implications of the Elasticity of Substitution

In this section, I will explain the significance of the finding that the elasticity of substitution between fixed and non-fixed factors is less than one. Under the assumption of diminishing marginal returns, the marginal product of any factor of production must fall as the quantity of that factor accumulates. The elasticity of substitution between factors influences the degree to which this marginal product falls. If the elasticity is small, then the marginal product falls faster as more of the given factor is added, and vice versa.

So why does this matter? The degree to which factor prices change as their quantities change greatly effect the dynamics of a large number of economic models. For example, the speed by which wages fall as population grows has large implications in models of population's effect on output. In this section, I will show how this elasticity effects two types of models where population matters.

First, many economists have been interested recently in the dynamics of Malthusian economies. Lee (1973) was among the first to estimate a model of dynamics around the Malthusian steady state. More recently, Ashram and Valor (2011) develop a model of the Malthusian economy and test the model's predictions using historical data on income and population size. In addition, there has been a large and active literature recently on understanding the transition from the Malthusian steady state to modern growth. Others have also used Malthusian models to help understand the causes of the Great Divergence.[16]

Models of Malthusian economies rely on the fact that in the presence of a fixed factor, gains in population depress wages. When positive technology shocks occur, the subsequent increase in income per capita will only be fleeting, since future population growth will eventually lower wages back to the Malthusian steady state. The speed by which income falls back to the steady state is a direct reflection of the elasticity of substitution between land and labor. Therefore, the predictions of many models of the takeoff to growth from the Malthusian steady state will be affected by the elasticity of substitution.

Second, the extent to which population levels affect output is a very old question within economics, having been discussed at least since Malthus (1798). While some scholars such as Ehrlich (1968) have argued that continued population growth will lead to economic crises and immiseration, others such as Simon (1981) and Boserup (1965) submit that population growth is not a problem. Both sides of the issue agree that the direct effect of population on income is negative in the presence of a fixed factor. What they disagree on, however, is the degree of substitutability of other factors for the fixed factor, and the ability to innovate away from resource constraints. For example, Boserup maintained that crop intensification would occur in the face of population pressure, leading to proportional increases in food production. This is just another way of saying the elasticity of substitution between fixed and non-fixed factors is low. Simon advocated the idea that people are the "ultimate resource", and that large populations would increase the rate of technological progress.

Larger populations cause resource per capita to shrink. If the elasticity of

[16]See Lucas (1998, 2002), Valor and Weil (1999, 2000), Stokey (2001), Valor Moav (2002), Hansen and Prescott (2002), Lagerlof (2003, 2006), Doepke (2004), Fernandez-Vilaverde (2005), O'Rourke, Rahman, and Taylor (2008), Strulik and Weisdorf (2008), and Voth and Voigtländer (2010), among others.

substitution is low, additional labor does little to increase production, leading to lower average products and smaller incomes per capita. Population size will have a larger negative effect on income levels. Therefore, models which predict the effect of population on output must be effected by this elasticity. In addition, the efficacy of programs intending to raise income per capita which also change population size will depend on this elasticity.

The following section will demonstrate the effect of the elasticity of substitution in these two type of models.

4.1 Malthusian Models

One of the most important ways in which σ affects models of Malthusian economies is via its effect on transition speeds – how fast the model transitions back to steady state after being shocked. This simple observation that σ affects transition speeds has major implications for a large set of models which attempt to explain the takeoff from Malthusian stagnation to growth.

To demonstrate how σ affects transition speeds, I develop a simple Malthusian model based on Lee (1973). Consider the following CES production function with Hicks-neutral technological progress:

$$Y = A_t \left(\alpha L^\rho + (1-\alpha)X^\rho\right)^{\frac{1}{\rho}}$$

where L is labor and can change over time, and X is a fixed factor. This implies a wage of:

$$\frac{\partial Y}{dL} = w = \alpha A_t L^{\rho-1} \left(\alpha L^\rho + (1-\alpha)X^\rho\right)^{\frac{1-\rho}{\rho}} \tag{10}$$

Labor evolves according to the following law of motion:

$$\frac{L_{t+1} - L_t}{L_t} = g_L = f_t - m_t, \tag{11}$$

where f is the fertility rate and m is the mortality rate. Fertility is an function of the wage:

$$f_t = \delta w_t^{\gamma}, \tag{12}$$

where $\gamma > 0$ so that $\frac{\partial f}{dw} > 0$. Combining equations (11) and (12) yields the following difference equation for labor:

$$L_{t+1} = L_t \left(1 + \delta w_t^{\gamma} - m\right) \tag{13}$$

Note that population will be in a steady state when $\delta w_t^{\gamma} - m = 0$, implying

$$w^* = \left(\frac{m}{\delta}\right)^{\frac{1}{\gamma}}.$$

Notice that the wage is independent of the level of technological progress or the amount of land. As in most Malthusian models, any increase in either of these variables will lead to more population, but not increases in the steady state level of population. This can be seen by plugging the steady state wage back into the wage equation to pin down the steady state level of population L^*

$$L_t^* = X_t \left[\left(\frac{\left(\frac{m}{\delta}\right)^{\frac{1}{\gamma}}}{\alpha A_t \left(1-\alpha\right)^{\frac{1-\rho}{\rho}}} \right)^{\frac{\rho}{1-\rho}} - \frac{\alpha}{1-\alpha} \right]^{-\frac{1}{\rho}},$$

where $\frac{\partial L_t^*}{dA_t} > 0$ and $\frac{\partial L_t^*}{dX_t} > 0$.[17]

Using equations (10), (12), and (13) to simulate the system, I can shock technology in the system to see its effect on population and the wage. To calibrate the model, I use $m = \delta = 1$, $\alpha = 0.8$ corresponding to a land share of 20%, and set the pre-shock level of technology to be $A_t = 1$. I also set $\gamma = 0.5$ and $X_t = 1$. I run the model until it reaches its steady state. Then I increase the level of technology by 10% to $A_t = 1.1$. I use a Cobb-Douglas production function initially.

Figure 8 shows how the dynamical system evolves in response to a shock

[17] Notice that we need $\left(\frac{\left(\frac{m}{\delta}\right)^{\frac{1}{\gamma}}}{\alpha A_t} \right)^{\frac{\rho}{1-\rho}} > \alpha$ for this expression to make sense. In the analysis that follows, I choose parameter values such that this is the case.

in technology. The 10% increase in productivity initially raises wages by 10%, leading to higher fertility in the next period. Population grows since now fertility is higher than mortality, and the wage begins to fall. Falling wages leads to falling fertility, and eventually wages fall back to their steady state, while population stabilizes at a new equilibrium level.

From the wage equation (10) we can calculate the elasticity of wages with respect to labor:

$$\frac{\partial w}{dL} \cdot \frac{L}{w} = -\frac{\phi_x}{\sigma},$$

where ϕ_x is the fraction of income paid to land. This elasticity is key in understanding the effect of the σ on transitions speeds. As labor is added, the wage will fall faster if the elasticity of substitution is low. In addition, the wage will also fall faster the more important land is in production as measured by its share of national income.[18]This effect is shown graphically in Figure 9. The three lines represent the dynamics of the model under different elasticities of substitution: 0.5, 1, and 2. The smaller the elasticity of substitution, the faster wages fall.

Since the population growth rate is positively associated with wages, a small σ implies that the population growth rate will fall faster as well. Therefore, the overall population growth caused by a change in technology will be smaller for economies will small elasticities of substitution. This can be seen in Figure 10. When wages fall faster due to smaller σ, this implies population growth will also fall faster. In the case of Cobb-Douglas, a 10% increase in productivity will cause the new steady state level of population to be $\left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{\alpha}} = 1.1^5 \approx 1.61$ times higher than the pre-shock steady state, compared

[18]Vollrath (2010) makes this observation by using a Cobb-Douglas production function with land and labor as inputs to show the effect of α, labor's share of income, on pre-industrial levels of income. He finds that larger α leads to higher income levels before the industrial revolution via a larger fraction of the workforce employed in the agricultural sector. This is also true for σ; the smaller is σ, the larger is the fraction of labor employed in the agricultural sector (analysis available upon request). Although both papers look at the effect of different parameters, the results both run through the speed by which wages fall as labor is added. Therefore, all the results I will describe in this section for small σ will be similar for larger α; and all Vollrath's theoretical results for larger α will be similar for smaller σ.

with 1.78 times higher when $\sigma = 2$ and 1.46 times higher when $\sigma = 0.5$.

Finally, suppose that population suddenly fell in the model. Since labor has declined, the marginal product of labor will rise. However, the degree to which is rises will depend on the elasticity of substitution. In figure 11, I exogenously reduce steady-state population by 0.1% in the model, and trace out the effect on wages over time. When σ is small, the wage rises higher than when σ is large. Afterwards the wage falls faster in the economy with a small σ, as noted previously.

How will this affect current models of the takeoff to growth from Malthusian stagnation? Consider the observations made thus far:

1. Small σ will cause wages to fall faster when labor increases.
2. Small σ will cause wages to rise faster if labor falls.
3. Small σ will cause increases in productivity to translate into smaller steady state population levels.

I group the literature on the takeoff to growth from Malthusian stagnation into 3 categories, and discuss how these observations affect each group.

Models that Depend on Population Size

One set of models explaining the transition to growth from Malthusian stagnation feature a positive relationship between the rate of technological progress and population size. This positive relationship has been used in a number of growth models, beginning with the endogenous growth models of Romer (1990) and Aghion and Howitt (1992). Initially these models focused on explaining cross-country contemporaneous differences in technological progress and economic growth rates. R&D aspects of technological progress were emphasized in these models, and looked at the effect of market size on profitability of innovation, or scale effects in the production of ideas.

Later these endogenous growth models were used to analyze the history of economic development. Kremer (1993) noted that world and regional population density was a surprisingly good predictor of the historical rate of technological progress. Diamond (1997) argued that societies in areas of the world with geographic features compatible with supporting dense populations and trading ideas far surpassed other societies in technological progress by 1500. Jones (2001) and Valor and Weil (1999, 2000) applied this concept to the transition to modern growth from the Malthusian steady state.

One feature of these models is that once technological progress is fast enough, the economy begins to transition away from the Malthusian steady state. Since the rate of technological progress is determined by the size of the population[19], the transition begins once population reaches a threshold level. The main implication of the elasticity of substitution in these models has to do with observation 3 above: improvements in technology will lead to smaller increases in population if the elasticity of substitution is low. Therefore, a small elasticity of substitution slows the transition from Malthusian stagnation to sustained growth. This effect works through two channels. First, the level of technology itself will have to reach a larger threshold to attain the same threshold level of population than economies with a higher elasticity of substitution. Secondly, the growth rate of technological progress will be slower in the economy with a smaller elasticity of substitution, because the level of population will be lower.

Models that Depend on Wage Levels

Another set of models which depend on the elasticity of substitution are those in which the transition occurs after attaining some threshold level of income. An example of a model in this category are Voth and Voigtländer (2006), where the probability of transitioning to growth depends positively on the level of income. Other models depend on rising incomes to spur demand for the manufacturing sector or other luxury goods, thereby igniting the industrial revolution.

Aside from models in which income levels determine when countries will industrialize, divergence in pre-industrial incomes is interesting in its own right.

[19] At least initially – human capital formation plays are role later in many of these unified growth models.

A standard Malthusian prediction is that income levels will return to subsistence in the long run. Therefore, there should be no persistence in differences in cross-country incomes in the Malthusian world, unless there are differences in the subsistence level of income. However, several scholars have argued that incomes in Europe were higher than the rest of the world, even during the Malthusian regime. For example, Voth and Voigtländer (2010b) argue that Western Europe was able to maintain higher levels of income than the rest of the world due to unique fertility patterns from delayed marriage. Diamond (1997) argued that pre-industrial divergence in incomes and technology lead to the great divergence in incomes observed today. Voth and Voigtländer (2010a) explain persistent income differences using a model in which positive wage shocks are reinforced by increases the amount of plagues, wars, and the rate of urbanization, eventually leading to perpetually higher death rates and permanently higher incomes.

The main effect of the elasticity of substitution in this set of models is related to observation 2 above: a small elasticity of substitution will imply that wages rise more for a given decrease in population. All of the Voth and Voigtländer papers point to rising wages after the Black Death as the impetus which lead to plagues, wars, urbanization, and changes in fertility. These in turn kept population low and wages high, eventually leading to a pre-industrial divergence in incomes and the industrial revolution in Europe. If the elasticity of substitution in pre-industrial England was small, then the positive effect of the Black Death on wages would be higher. Therefore, the reinforcing effects of plagues, wars, urbanization, and European marriage patterns would be even stronger, and the eventual divergence in European incomes that much higher. Presumably, this would cause the industrial revolution to happen even faster.

Models that Depend on Sectoral Shift from Biased Technological Progress

Hansen and Prescott (2002) presented a unified growth model in which biased technological progress causes the demographic transition. There are two sectors, the Malthus (agricultural) sector which uses land, labor, and capital as inputs; and the Solow (manufacturing) sector which only uses labor and capital. Each sector produces the same good, and technological progress increases significantly faster in the Solow sector than the Malthus sector. Initially, the Malthus sector is the only sector operative, and is in a Malthusian steady state.

In this setup, the transition is inevitable and will happen rapidly since technological progress in the Solow sector grows over 16 times faster than in the Malthus sector. The transition occurs when technology in the Solow sector grows to the point that it is profitable to produce output. Both sectors are operative, and capital and labor begin to flow from the Malthus sector to the Solow sector. Eventually, virtually all the capital and labor are employed in the Solow sector. Whereas the dynamics of the economy followed the Malthusian economy when only the Malthusian sector was operative, the economy behaves according to the Solow model when the Solow sector employs almost all the capital and labor.

A lower σ in the Malthus sector implies that as labor and capital leave, their marginal products rise faster, inducing more labor and capital to remain. As a result, labor and capital leave the Malthus sector slower, delaying the takeoff to growth. For example, in the first period after the beginning of the transition, there is three times more labor in the Malthus sector when $\sigma = 0.5$ than when $\sigma = 2$, and twice as much than when $\sigma = 1$.[20] This result is similar to the finding of the effect of σ on the Galor-Weil model – low elasticities of substitution delay the demographic transition and takeoff to modern growth.

4.2 Population and Development

Aside from having implications for land share and models of the demographic transition, the elasticity of substitution between land and other factors is integral to understanding the extent to which population size affects output. Surprisingly little concrete empirical research has been devoted to this most simple, yet important question. Most studies of the effect of population on

[20]Due to space constraints, the full analysis and simulation will not be outlined in the paper, but is available upon request.

output deal with effect of the population growth rate, rather than population level. For example, the effects of dependency ratios/demographic structure, accumulable factor dilution, changes in fertility and mortality and their effect on labor force participation, savings, etc. and so forth all work through the growth rate of population rather than its absolute size.

This is even more surprising when one considers that the father of population economics, Malthus, originally framed the question in terms of levels, not rates. He conjectured that in the face of a fixed amount of arable land, food production would not be able to increase fast enough to sustain an exponentially growing population with a shrinking land-labor ratio. This argument contains an implicit assumption about the substitutability of land in food production – that it is small. It is difficult to understand how population size affects production without understanding the substitutability of fixed factors.

Over the last 200 years, the dire predictions of Malthus have not come to fruition. However, this does not imply than large populations do not depress wages. The extent to which this happens is a direct consequence of the elasticity of substitution between land and all other factors. Recently, Weil and Wilde (2009) calculate the effect of a doubling in population size on income per capita. Assuming that each factor of production is paid its marginal product, one can solve for the fraction of national income paid to each factor, and then take the ratio of incomes in two points in time based on differing levels of inputs. Using a CES production function similar to (5), they derive

$$\frac{y_t}{y_0} = \left((1 - \phi_0) + \phi_0 \left(\frac{L_0}{L_t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where y is income per capita, ϕ is the fraction of national income paid to land, and the subscripts index time.

In this simple example, all that is needed to estimate the effect of population size on income per capita is data on ϕ and σ.[21] Table 4 takes several values of each parameter and shows the corresponding change in income per capita.

[21] $L_0/L_t = 0.5$ since we consider a doubling in population.

The larger the elasticity of substitution between land and other factors, the less impact an increase in population has on standards of living.

Another recent paper which depends crucially on the elasticity of land and other factors is Ashram et al (2008). They estimate of the effect of a unspecified health intervention which improves life expectancy from 40 to 60. In addition to improving the productivity of workers via health and education human capital, this reduction in mortality increases population size.

Figure 13 shows the effect of this intervention on income over a 165 year time horizon. With an elasticity of substitution greater than unity, the health intervention improves income in the long run. In this case, the negative effects of population size on income are small relative to the productivity benefits from the health intervention. However, if the elasticity of substitution is less than one, the overall effect becomes negative. Since it is difficult to substitute away from land, shrinking land per person has a large negative effect on production. This negative effect of population on income overwhelms the positive effect of worker productivity. In the case of an elasticity of substitution of 0.5, income per worker decreases by about 25% after 50 years and never recovers. Thus the same health intervention could have opposite results depending on the elasticity of substitution.

Finally, Ashram et al (2010) use a similar model to quantify the effect of a reduction in fertility on income per capita. They model several different channels by which fertility reduction affects income, including what they call the "Malthus" channel, or the effect on income per capita due to fixed factor congestion.

In the long run, the elasticity of substitution plays a large role in determining the magnitude of the effect on income per capita. Figure 12 shows the effect of this intervention on income over a 165 year time horizon. If $\sigma = 2$, the Malthus effect is small since land is easily substitutable. Income 150 years after the fertility intervention is about 45% higher. However, if $\sigma = 0.5$, income rises to about 130% higher than baseline over the same time horizon – about three times higher than the previous case. The Malthus channel becomes the most important channel by which fertility decline affects income.

These studies show that understanding the value of σ is essential for accurately assessing the impact of policies which affect population. The extent to which fixed factors can be substituted for non-fixed factors in production, especially in agriculture, is central to understanding population's affect on living standards. Considering the large investment of resources devoted to health interventions and promoting fertility control across the developing world, the fact that almost no research until now has been devoted to the estimation of σ is quite surprising.

5 Conclusion

The elasticity of substitution between fixed and non-fixed factors is an extremely important parameter in a large number of economic models. It governs dynamics around the steady state in Malthusian models of the economy. It also is essential for understanding the movement away from the Malthusian steady state and takeoff to modern growth which has occurred in most areas of the world during the last 200 years. It predicts speed of the demographic transition and the order in which countries will industrialize.

In addition, its has important applications to the question of how population size affects economic development. The extent to which fixed factors can be substituted in production has implications for how strong the negative effect of population on growth is. The efficacy of interventions which affect population size, such as family planning programs or health interventions which affect mortality, will be affected by this substitutability.

Despite the importance of this parameter, no credible estimates of this elasticity are available. In this paper, I estimated this elasticity in pre-industrial England. I found that the elasticity of substitution was about 0.6 – significantly lower than most models assume. In addition, I estimate the direction of biased technological progress and the degree of induced innovation. I find that technology was scarce factor saving. After the Black Death when land was plentiful and labor scarce, productivity growth saved on labor. Afterwards when population grew again and land became scarce, productivity was land saving. An additional 1 million people (an increase of about 1/3 over population's median level over the period 1200-1750) increased the differential growth rate between land- and labor-augmenting productivity growth by 0.1% per year.

There are several directions future research could take. One interesting implication of my work is that the elasticity of substitution should predict the order in which countries begin the demographic transition. Estimating this elasticity in a set of countries would allow us to determine if this correlation exists. Since different models predict different effects of the elasticity of substitution on transition speeds, this analysis could be a method of testing the veracity of these models.

In addition, obtaining a well-identified estimate of the elasticity of substitution for a modern developing country would be extremely important. As mentioned previously, it is extremely difficult to estimate this elasticity well due to a number of econometric problems. But since this parameter affects the efficacy of any number of health interventions currently being pursued in the developing world, having a good idea of the value of this elasticity should be of utmost importance to policy makers.

Finally, the implications of induced innovation for the dynamics around the Malthusian steady state could be explored, as well as its effect of the transition from this steady state to modern growth.

Appendix: Plague Epidemics

List of national epidemics of the plague. List compiled by Brian Williams, 1996.

1348-1349 The Black Death.

1361 Pestis secunda or Pestis puerorum [Gottfried (1983) pg. 130]; [Shrewsbury (1970) pg. 23].

1369 Pestis tertia [Shrewsbury (1970) pg. 23].

1375 Pestis quarta [Shrewsbury (1970) pg. 23, 135-136].

1379-1383 Pestis quinta; 'In 1379 there was a great plague in the Northern parts...under the year 1382, a very pestilential fever in many parts of the country' [Creighton (1965) pg. 218]; London was afflicted in 1382, with Kent and others parts in 1383 [219].

1390-1391 A prolonged outbreak through 1390-1391 'described not without some detail' [1:219]; [Gottfried (1983) pg. 131]; '"a grete pestilence in England"' [Shrewsbury (1970) pg. 137].

1399-1400 [Gottfried (1983) pg. 131].

1405-1406 [Gottfried (1983) pg. 131]; [Shrewsbury (1970) pg. 141].

1411-1412 'another national epidemic' [Gottfried (1983) pg. 131].

1420-1423 Norfolk, 'but the Rolls of Parliament bear undoubted witness to a very severe prevalence of plague in the North about the same time' [Creighton (1965) pg. 221]; 1420 and 1423 [Gottfried (1978) pg. 36]; 1423 [Gottfried (1983) pg. 131].

1426-1429 [Gottfried (1978) pg. 36]; 'a London visitation in 1426, and a national epidemic in 1428-29' [Gottfried (1983) pg. 132].

1433-1435 'Here then, early in 1434, is the first distinct suggestion in the period 1430-1480 of something more that a local or regional epidemic' [Gottfried (1978) pg. 37]; 'a national epidemic that lasted from 1433 to 1435' [Gottfried (1983) pg. 132]; 1433 or 1434 [Shrewsbury (1970) pg. 143].

1438-1439 'a national epidemic' [Gottfried (1983) pg. 132]; [Shrewsbury (1970) pg. 144].

1447-1454 'Parts of England experienced plague epidemics in 11 out of the 18 years between 1442 and 1459' [Gottfried (1983) pg. 132]; 1447-1454 [Shrewsbury (1970) pg. 145].

1463-1465 [Creighton (1965) pg. 229]; 'From 1463 to 1465, another severe epidemic hit the entire kingdom' [Gottfried (1983) pg. 132]; 1463 '"a greate pestilence...all England over'" [Shrewsbury (1970) pg. 146].

1467 'In 1467 another epidemic swept through parts of England, and was possibly national in scope. If the Rolls of Parliament are to be believed, it was unquestionably an epidemic of plague' [Gottfried (1978) pg. 42]; [Gottfried (1983) pg. 132].

1471 'evidence indicates that this epidemic was one of plague' [Gottfried (1978) pg. 44]; 'in 1471, all of England was overwhelmed' [Gottfried (1983) pg. 132].

1479-1480 'This year [1479] saw great mortality and death in London and many other parts of this realm' [Creighton (1965) pg. 231-232], 'the great epidemic of 1479 in London and elsewhere' [286]; 'The most virulent epidemic of the fifteenth century was the plague of 1479-1480' [Gottfried (1978) pg. 14]; 'From autumn to autumn, a combined epidemic of bubonic and pneumonic plague devastated all of Britain' [Gottfried (1983) pg. 133].

1499-1500 'the great epidemic of 1499-1500, in London and apparently also in the country' [Creighton (1965) pg. 287]; [Gottfried (1978) pg. 14]; [Gottfried (1983) pg. 156]; the sixteenth century opened with 'a great pestilence throughout all England' [Shrewsbury (1970) pg. 159].

1509-1510 [Gottfried (1978) pg. 156]; 1509, a 'great plague' that afflicted various parts of England [Shrewsbury (1970) pg. 160].

1516-1517 [Gottfried (1983) pg. 156].

1523 [Shrewsbury (1970) pg. 163].

1527-1530 [Gottfried (1983) pg. 156].

1532 'There is supporting evidence that the disease was widespread' [Shrewsbury (1970) pg. 168].

1544-1546 1544 'several scattered, localized outbreaks of plague in England' [Shrewsbury (1970) pg. 178], 1545 north-east [pg. 180], south coast [pg. 181], 1546 westwards [pg. 182].

1563 'probably the worst of the great metropolitan epidemics' [Shrewsbury (1970) pg. 176], 'and then extended as a major national outbreak of it' [Shrewsbury (1970) pg. 189].

1585-1587 'bubonic plague was busy in numerous places in England in the years from 1585 to 1587 inclusively' [Shrewsbury (1970) pg. 237].

1593 the 'great metropolitan and national epidemic of 1593' [Shrewsbury (1970) pgs. 176, 222].

1603-1604 [Shrewsbury (1970) pg. 264].

1609-1610 'The next two years, 1609 and 1610, witnessed several severe outbreaks of bubonic plague in English towns' [Shrewsbury (1970) pg. 299].

1625 'the great outburst of 1625' [Shrewsbury (1970) pg. 313].

1637 'widely distributed in 1637 and a number of places experienced more or less severe visitations of it' [Shrewsbury (1970) pg. 389].

1645 'The year 1645 was one of severe plague in several towns at the same time' [Creighton (1965) pg. 557].

1665 The Great Plague, affecting London in the main.

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Figure 1: Land Rents, Wages, and Capital Rental Rates -- England 1200-1750

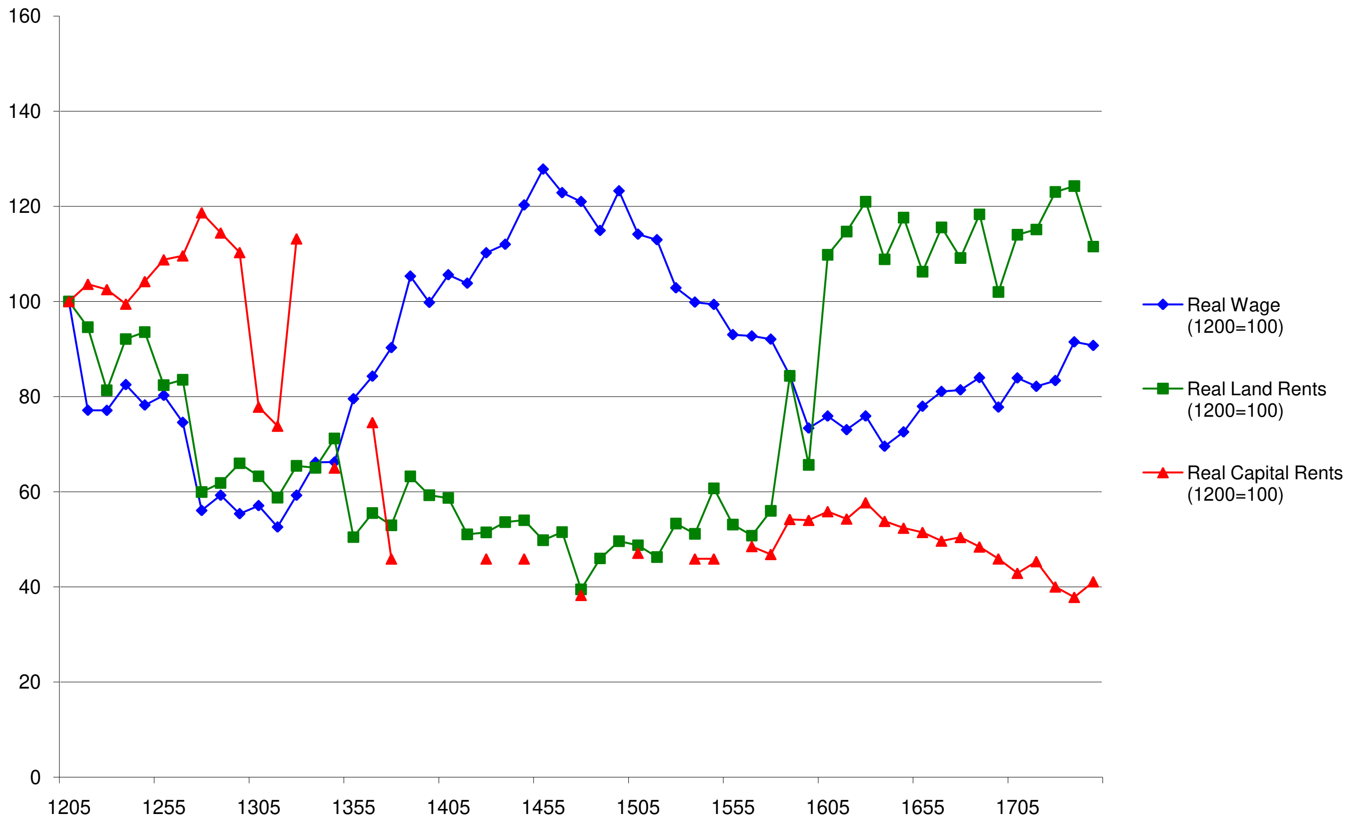


Figure 2: Land Rents over Wages and Population in England, 1200 - 1750

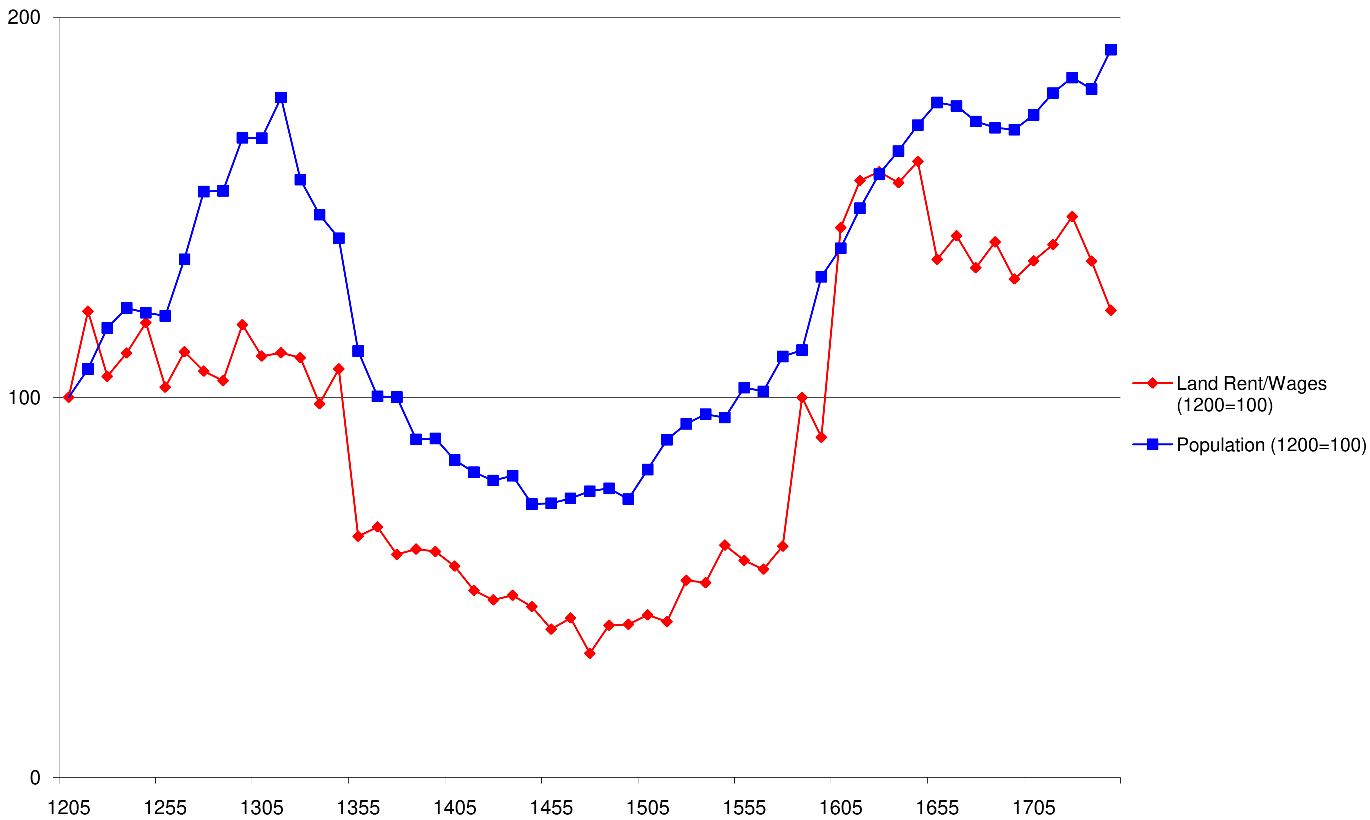
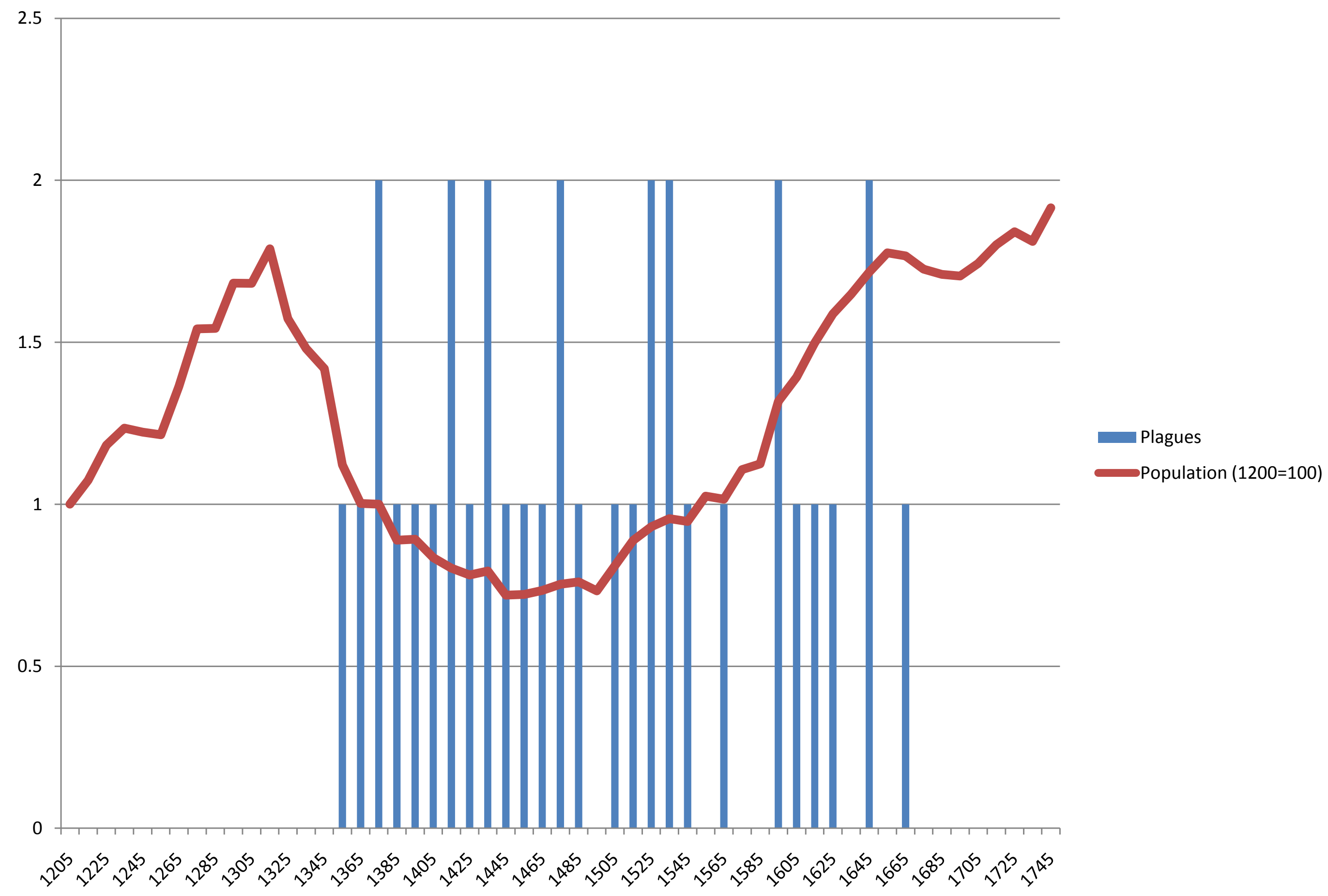
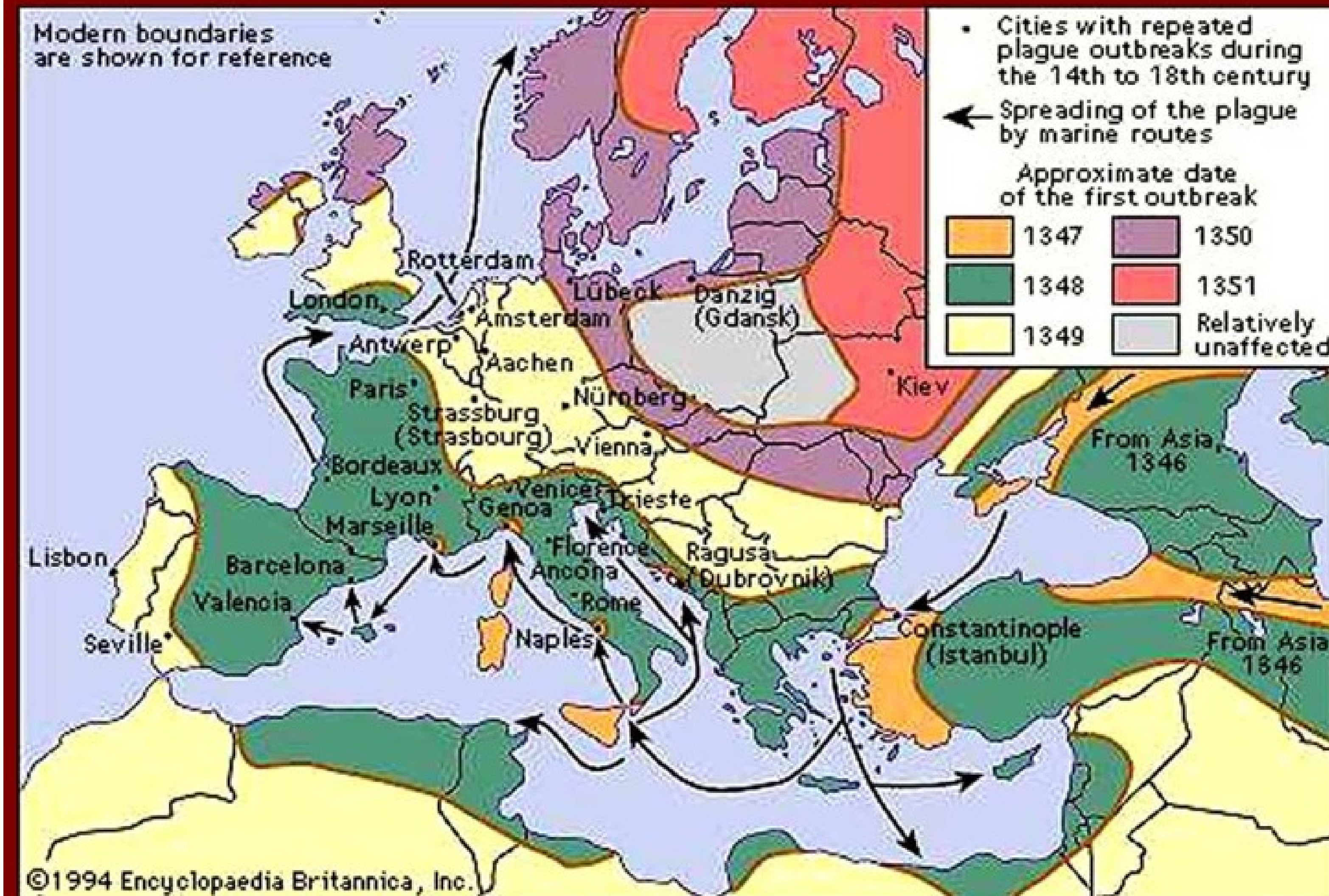


Figure 3: English Population and Plagues: 1200-1750





Pestis Prima -- the first wave of plague -- moved very quickly along sea and land trade routes.

Table 1: Results with Hicks-Neutral Technology

	(1)	(2)	(3)	(4)	(5)	(6)
Method	OLS	OLS	2SLS	2SLS	2SLS	2SLS
Instrument			Plagues 100	Plague Current	Plagues 100	Plague Current
Dep. Var.	ln(r/w)	ln(r/w)	ln(r/w)	ln(r/w)	ln(r/w)	ln(r/w)
ln(Pop)	1.392***	1.438***	1.587***	1.464***	1.443***	1.438***
	-0.0778	-0.094	-0.172	-0.133	-0.0758	-0.115
Year		-0.000290			-0.000293	-0.000290
		(0.000254)			(0.000202)	(0.000223)
Obs.	55	55	55	55	55	55
R-squared			0.834	0.848	0.858	0.858
Implied σ	0.718***	0.695***	0.630***	0.683***	0.693***	0.695***
	(0.040)	(0.045)	(0.068)	(0.062)	(0.036)	(0.056)
1st Stage F-			18.34	22.04	126.29	33.2

*** $p<0.01$, ** $p<0.05$, * $p<0.1$. Standard errors in parentheses. OLS regressions use Newey-West errors with one lag. The significance levels on ln(Pop), Δln(Pop), and Implied σ test whether the coefficient is different from one. All other significance levels test whether they are different from zero.

Table 2: Results with Biased Technology Change

	Levels				Differences	
	(1)	(2)	(3)		(4)	(5)
Method	OLS	2SLS	2SLS		OLS	2SLS
Instrument		Plagues 100	Plague Current			ΔPlagues 100
Dep. Var.	ln(r/w)	ln(r/w)	ln(r/w)		Δln(r/w)	Δln(r/w)
ln(Pop)	1.501***	1.685***	1.572***	Δln(Pop)	0.753	1.92
	(0.103)	(0.115)	(0.139)		(0.359)	(1.220)
Year	0.00244	0.00508***	0.00567***	Const	0.0248	0.0329
	(0.00147)	(0.00183)	(0.00179)		(0.0448)	(0.1230)
ΣPop	-0.00763*	-0.0152***	-0.0166***	Pop	-0.00751	-0.0131
	(0.00403)	(0.00523)	(0.00514)		(0.00987)	(0.02970)
Obs	55	55	55		54	54
Implied σ	0.666***	0.594***	0.636***		1.327	0.521
	(0.046)	(0.040)	(0.056)		(0.632)	(0.331)
Implied γ	-0.163%*	-0.302%***	-0.361%***		-0.329%	-0.171%
	(0.096)	(0.097)	(0.104)		(0.615)	(0.639)
Implied θ	0.051%*	0.090%***	0.106%***		0.100%	0.068%
	(0.003)	(0.003)	(0.029)		(0.132)	(0.152)
1st Stage		82.94	16.7			1.04
F-tests		66.47	36.7			75.35

*** p<0.01, ** p<0.05, * p<0.1. Standard errors in parentheses. OLS regressions use Newey-West errors with one lag. The significance levels on ln(Pop), Δln(Pop), and Implied σ test whether the coefficient is different from one. All other significance levels test whether they are different from zero. Since two instruments are used in the 2SLS regressions, the two 1st stage F-test values report the joint significance of the two instruments in each of the two 1st stage regressions.

Table 3: Robustness

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Method	OLS	2SLS	2SLS	OLS	2SLS	2SLS	OLS	2SLS	2SLS
Instrument		Plagues 100	Plague Cur.		Plagues 100	Plague Cur.		Plagues 100	Plague Cur.
Dep. Var	ln(r)	ln(r)	ln(r)	ln(r)	ln(r)	ln(r)	ln(r)	ln(r)	ln(r)
ln(Pop)	1.649***	1.573***	1.539*	1.272	1.239	1.064	1.715***	2.332***	1.829**
	(0.269)	(0.303)	(0.458)	(0.245)	(0.311)	(0.427)	(0.242)	(0.480)	(0.608)
ln(wage)	1.312***	1.215***	1.173*	1.445***	1.414***	1.249**	1.716***	2.194***	1.742***
	(0.418)	(0.441)	(0.625)	(0.304)	(0.362)	(0.464)	(0.264)	(0.398)	(0.540)
ln(cap. Rent)				0.710***	0.721***	0.779***	0.452**	0.167	0.353
				(0.178)	(0.183)	(0.220)	(0.210)	(0.279)	(0.312)
Year	-0.000508	-0.000434	-0.000401	0.000907*	0.000956*	0.001210	0.00394**	0.00568**	0.00581***
	(0.000416)	(0.000399)	(0.000556)	(0.000471)	(0.000547)	(0.000737)	(0.00158)	(0.00230)	(0.00200)
ΣPop							-0.0104**	-0.0179**	-0.0162**
							(0.0047)	(0.0079)	(0.0072)
Obs.	55	55	55	41	41	41	41	41	41
R-squared		0.726	0.726	0.745	0.745	0.739	0.768	0.745	0.758
Implied σ	0.606***	0.636***	0.650*	0.786	0.807	0.939	0.583***	0.429***	0.547**
	(0.099)	(0.123)	(0.193)	(0.152)	(0.203)	(0.377)	(0.082)	(0.088)	(0.182)
Implied γ							-0.230%**	-.244%***	-0.318%***
							(0.088520)	(0.073880)	(0.105220)
Implied θ							0.061%**	0.077%***	0.088%***
							(0.024)	(0.023)	(0.025)
1st Stage		99.49	11.76		163.99	14.93		98.01	8.2
F-tests								247.68	60.78

*** p<0.01, ** p<0.05, * p<0.1. Standard errors in parentheses. OLS regressions use Newey-West errors with one lag. The significance levels on ln(Pop), Δln(Pop), and Implied σ test whether the coefficient is different from one. All other significance levels test whether they are different from zero. When two instruments are used in the 2SLS regressions, the two 1st stage F-test values report the joint significance of the two instruments in each of the two 1st stage regressions.

Figure 5: Partial Correlation Residual Plot --
Ln(Land Rent/Wages) on Ln(Population)

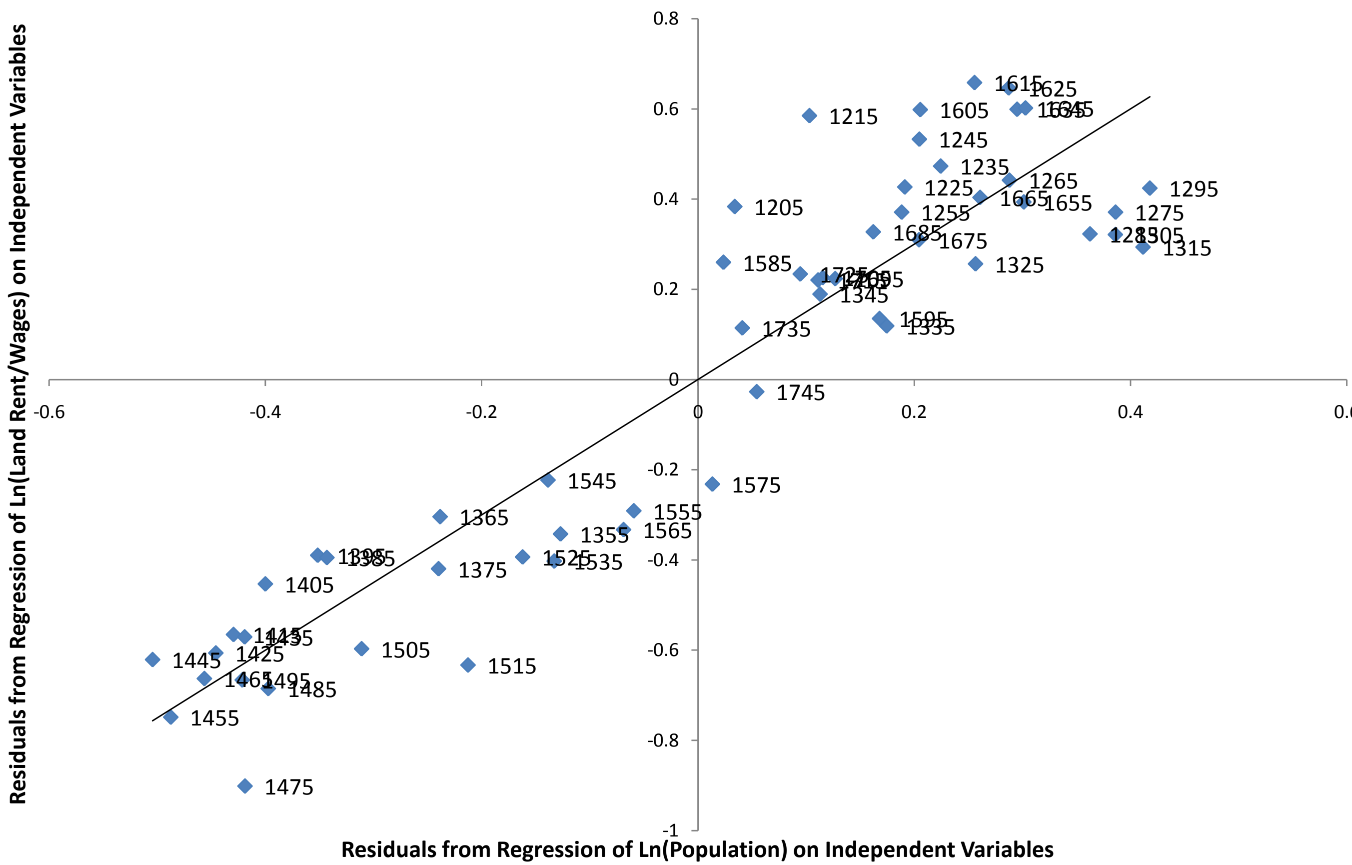


Figure 6: Levels of Population and Estimated A_X/A_N (1200=1)

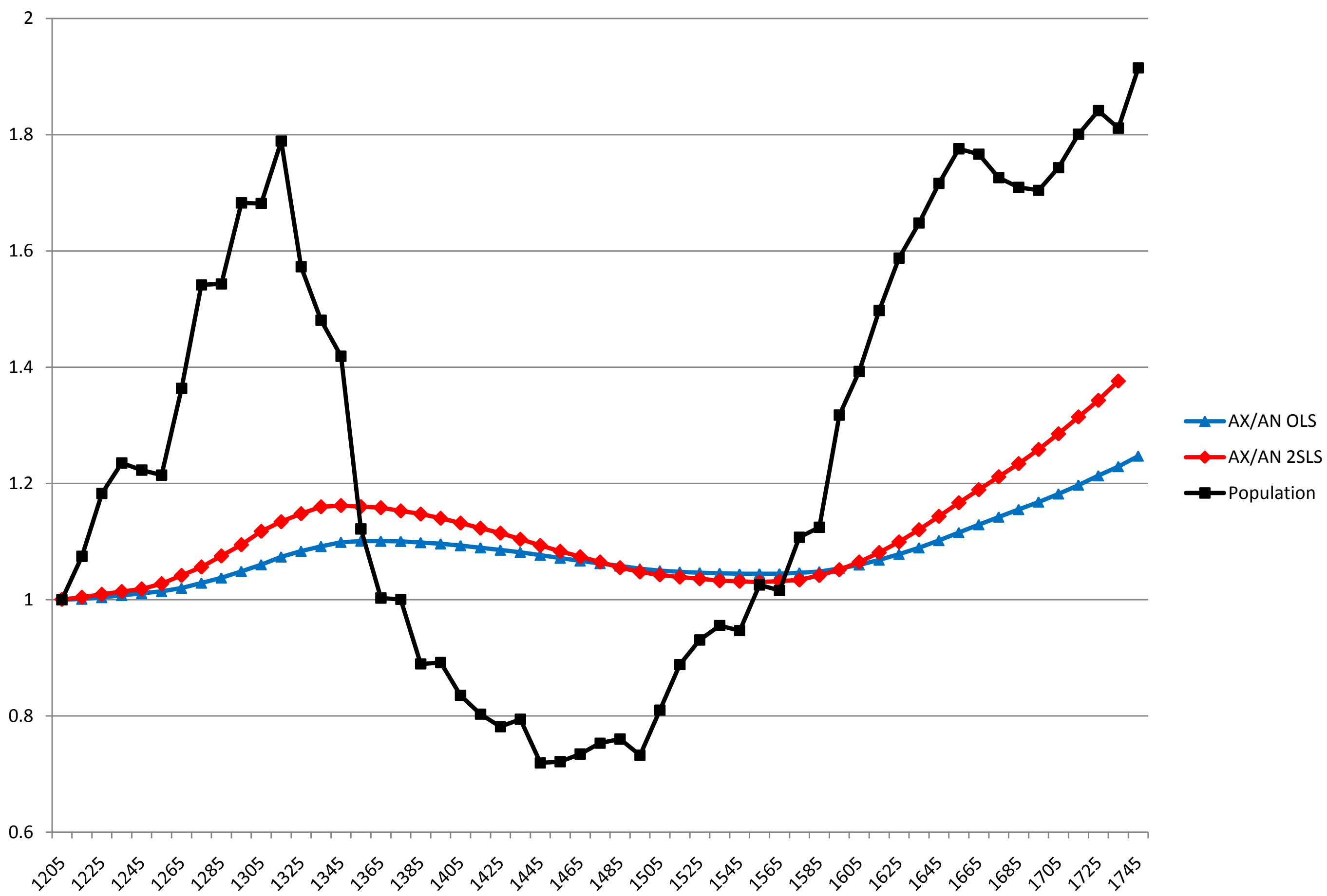


Figure 7: Estimates of Growth Rates of A_X/A_N

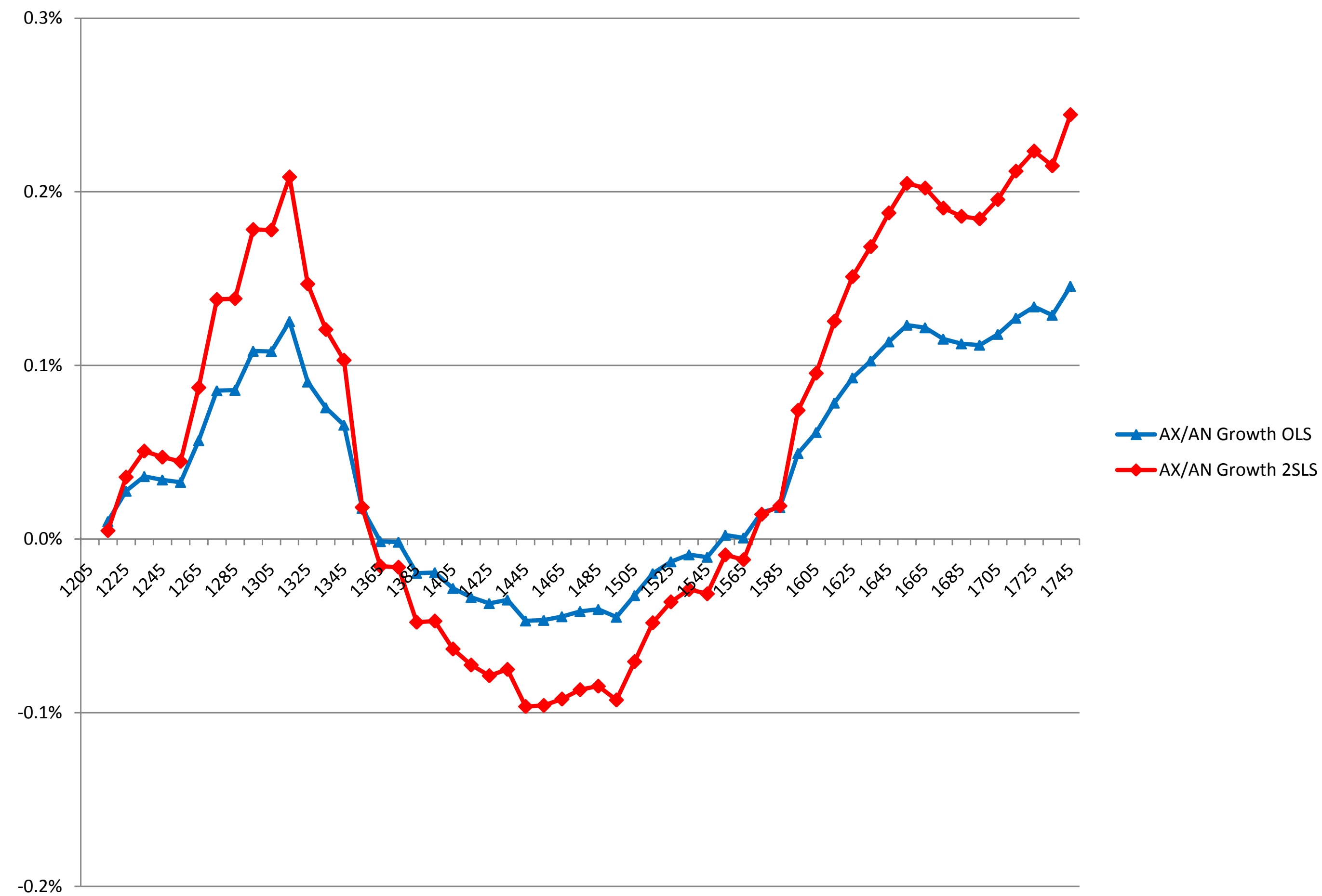


Figure 8: Technology Shocks in Simple Malthusian Model

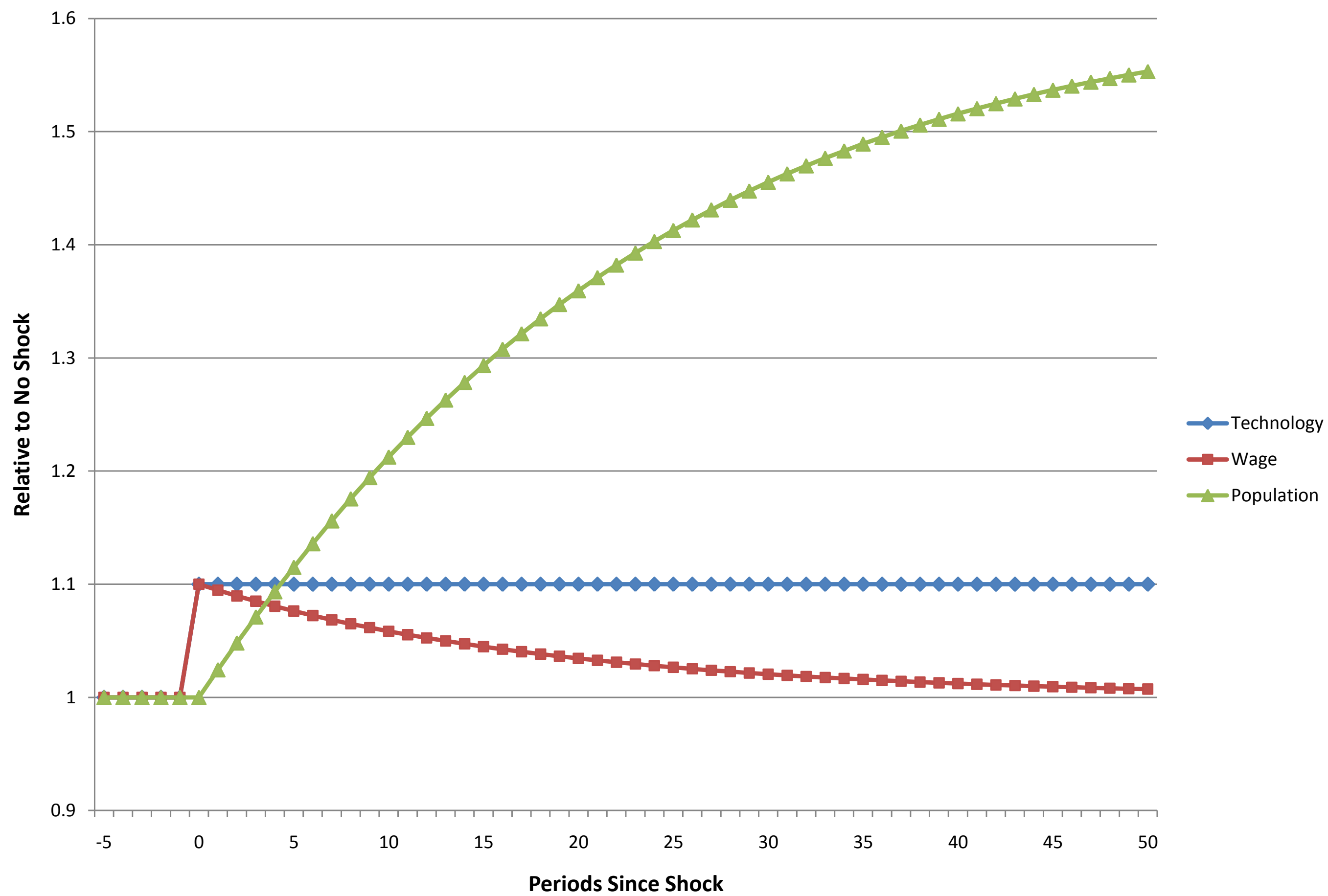
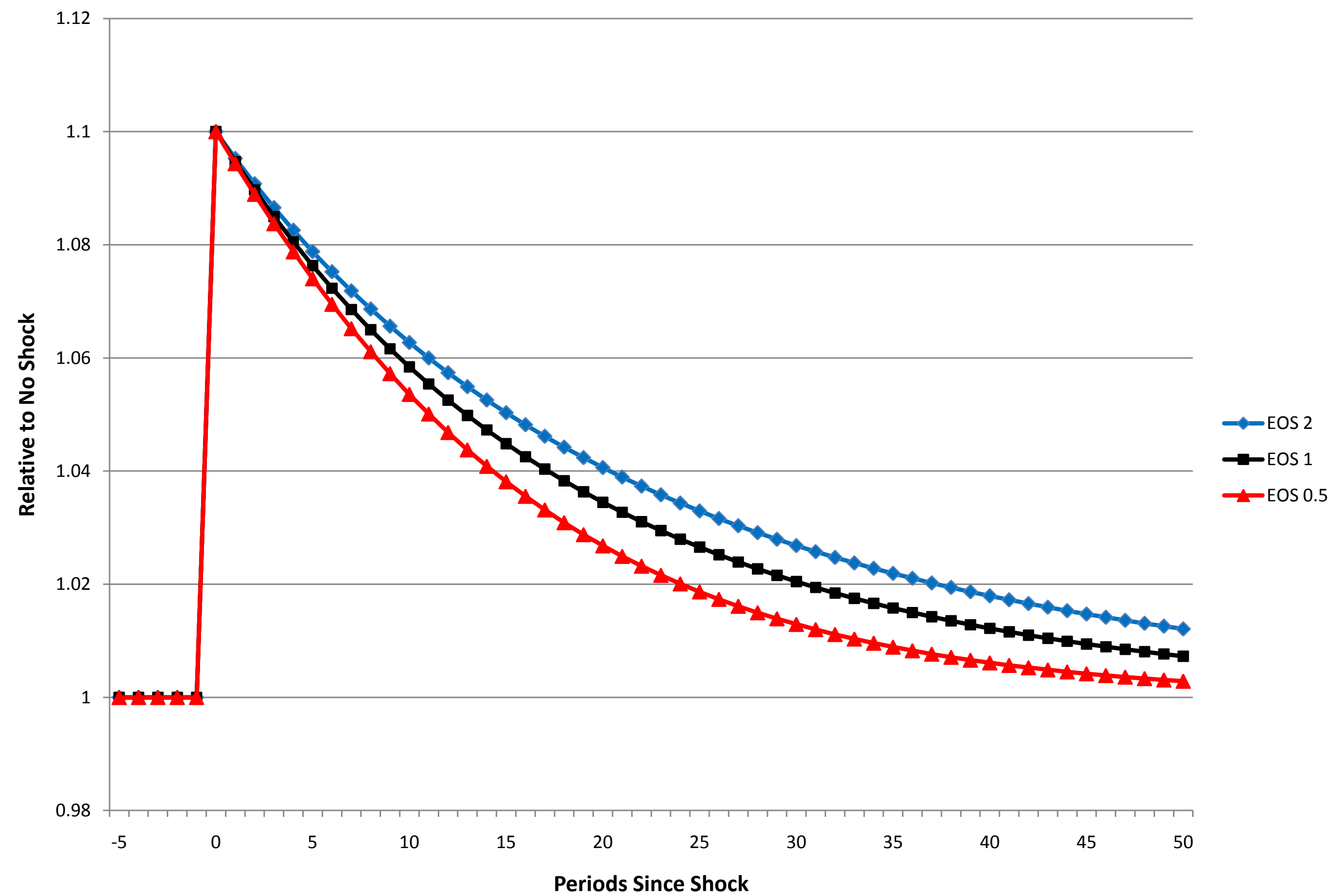


Figure 9: Wages in Simple Malthusian Model



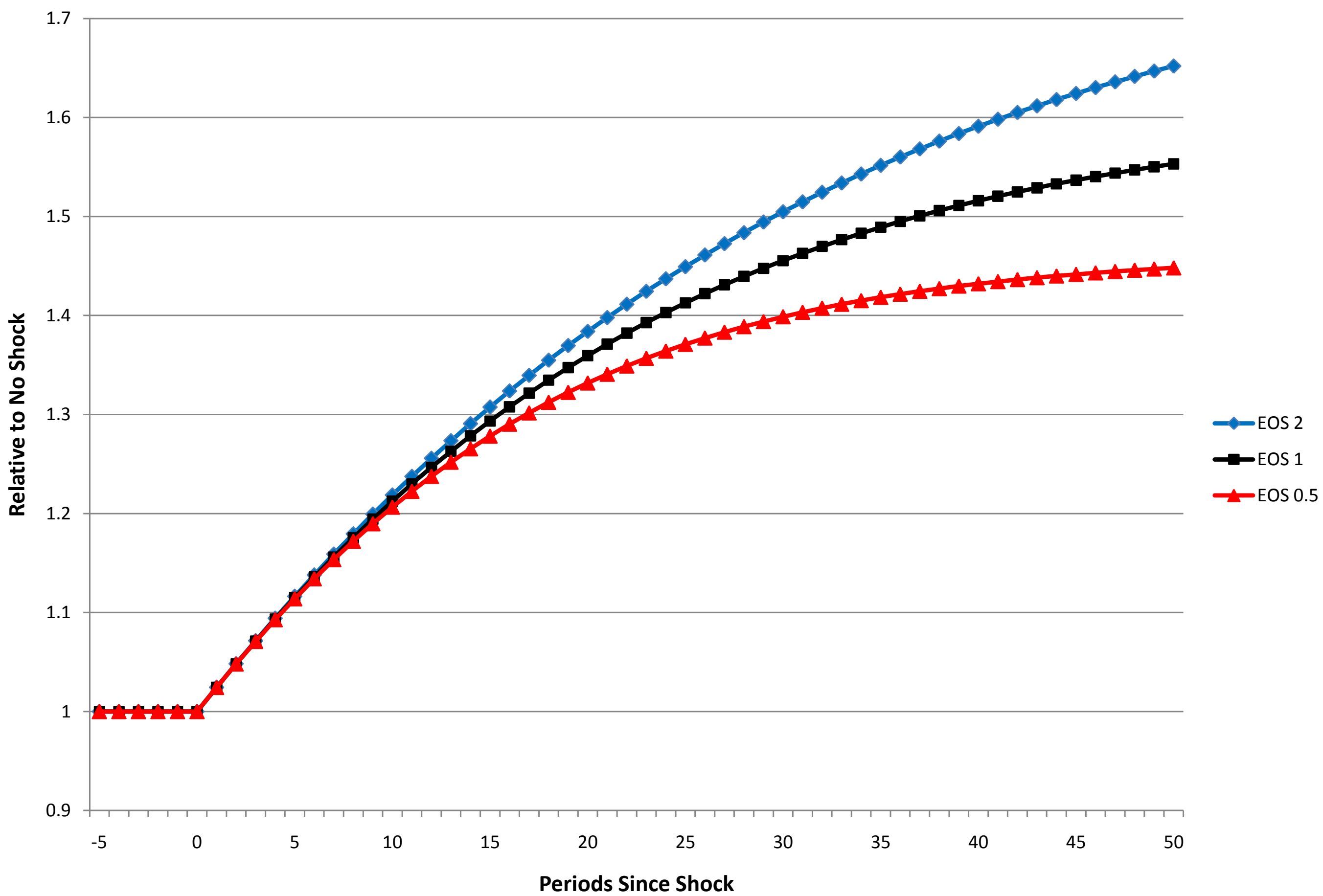


Figure 10: Population in Simple Malthusian Model

Figure 11: Effect of 0.1% Reduction in Population on Wages

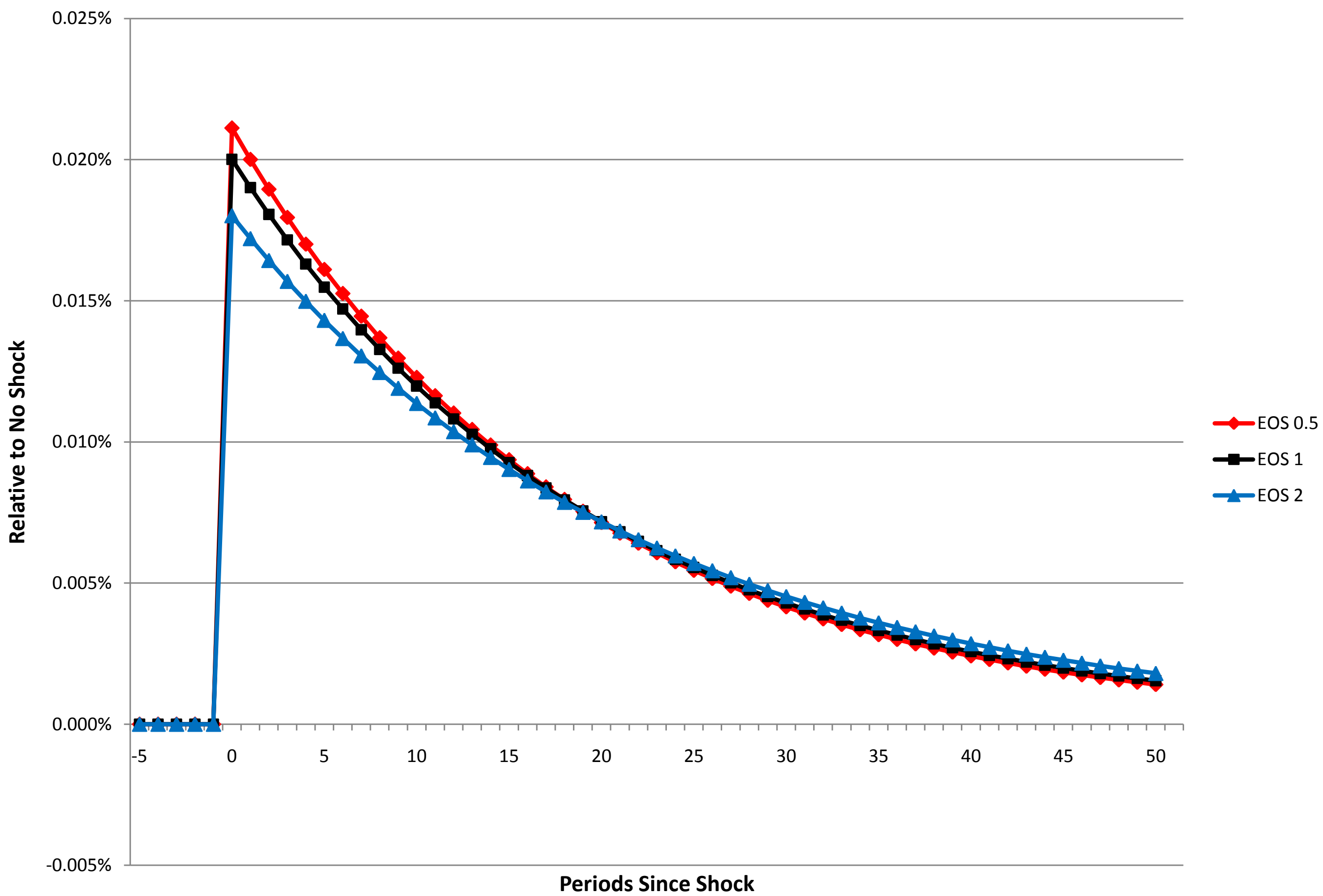


Table 4: Change in Income Per Capita if Population Were Doubled

		Elasticity of Substitution				
		0.5	1	2	3	5
Current	0.1	-9.1%	-6.7%	-5.8%	-5.5%	-5.3%
Natural	0.2	-16.7%	-12.9%	-11.4%	-10.9%	-10.5%
Resource	0.3	-23.1%	-18.8%	-16.8%	-16.2%	-15.7%
Share in	0.4	-28.6%	-24.2%	-22.1%	-21.4%	-20.8%
Income	0.5	-33.3%	-29.3%	-27.1%	-26.4%	-25.9%

Figure 12: Ashraf, Weil, and Wilde
Effect of Decrease in TFR of 1.00 on Income per Capita

Land Share of Income = 30%

No Labor Force Participation or Schooling Effects

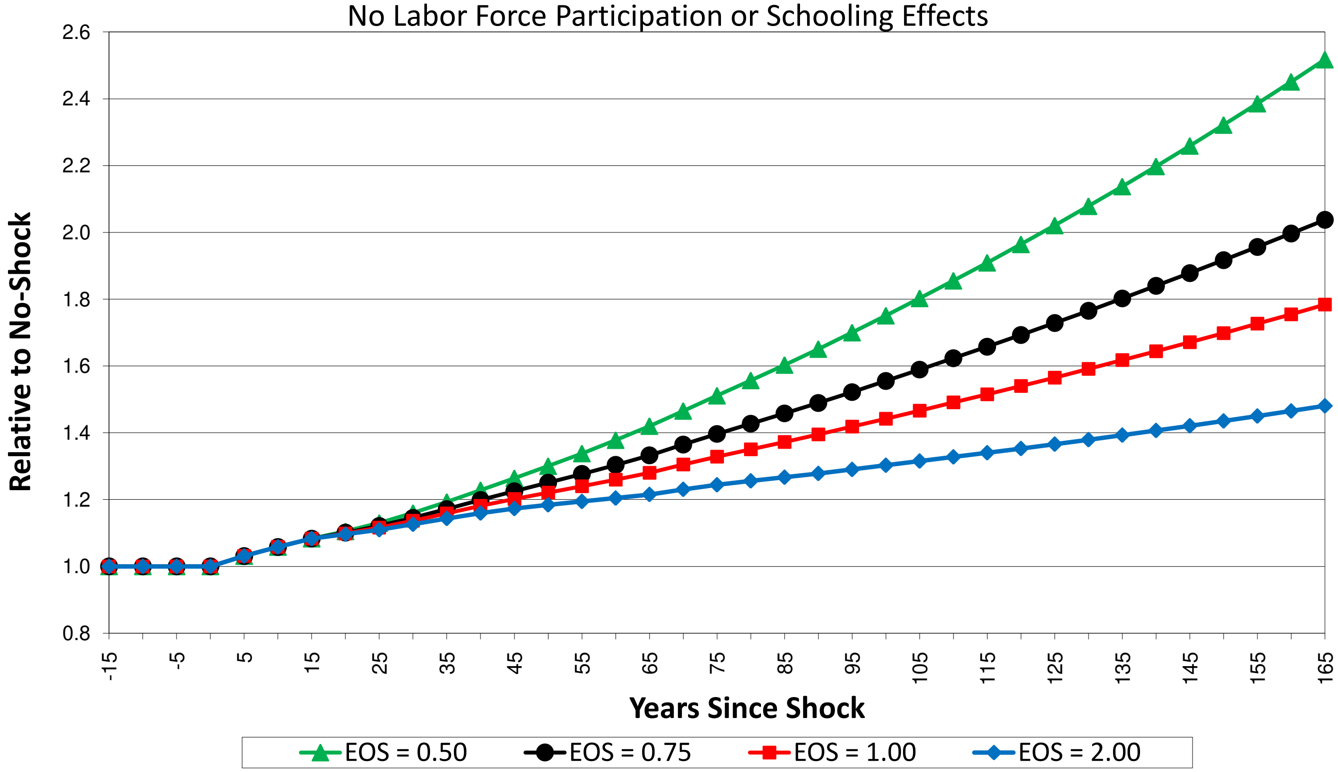


Figure 13: Ashraf, Lester, and Weil (2008)
Effect of Health Intervention on Income per Capita