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The effect of Trade Liberalization on payroll tax evasion and labor informality

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Abstract

Several developing countries that underwent trade liberalization experienced changes in the share of informal workers in manufacturing industries. This phenomenon deserves careful examination because informal jobs are not only generally viewed as low-quality and low-paying jobs, they also account for more than 30% of the workforce in some countries. In this paper, I develop a theoretical model of the impacts of trade liberalization on labor markets in which domestic and foreign import tariffs affect firms’ payroll tax compliance decisions, which in turn determine the types of jobs (i.e., formal or informal) created. The model is able to replicate several stylized facts such as some small firms hiring formal and informal workers and large firms hiring only formal workers. Moreover, it predicts that a decrease in domestic import tariffs decreases both the share of informal workers and the average formal wage, whereas a decrease in foreign (i.e., trading partner) import tariffs decreases the informality share but increases the average formal wage. The effect on the average informal wage is ambiguous.

Keywords: informal labor markets, trade liberalization, Brazil, payroll tax

JEL classification: F16, F12, H26, O17

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Introduction

The existence of jobs that do not meet legal requirements (hereafter called informal jobs) is a common phenomenon worldwide (Schneider and Enste, 2000), and in developing countries they account for a significant share of the workforce, for instance at least 30% of all jobs in Brazil and Colombia are informal. Although job informality is also a relevant topic for Southern European economies (Schneider, 2012), a significant part of the job informality literature has focused on Latin American countries due to data availability.

Trade liberalization episodes in Latin America were accompanied by almost no change in the industry share of employment as found by Hanson and Harrison (1999) for Mexico and by Pavcnik et al. (2004) for Brazil. Another significant feature was a substantial increase in the share of informal workers in the manufacturing sector (Perry et al., 2007). This last phenomenon is illustrated in Figure 1, which indicates that the share of informal workers in Brazil’s manufacturing sector increased from 12% to 20.4% following the country’s trade liberalization in 1990, whereas the share of informal workers in the services sector remained stable. In light of these facts, labor informality may be a relevant margin of adjustment to trade policy changes and therefore deserves a careful investigation.

The purpose of this paper is to develop a theoretical model in which trade policy changes affect the firm’s decision to offer formal or informal jobs. This means that firms choose whether to comply with labor regulations or not, particularly the payroll tax, which appears to be the key difference between formal and informal labor contracts in Latin America. Thus, government payroll tax revenues represent an important additional channel through which
trade liberalization affects welfare.

To do so, in this paper I present a small open economy version of the Davis and Harrigan (2011) “Good Jobs, Bad Jobs” model, in which the firm’s payroll tax compliance decision is embedded. Next, I conduct comparative statics to evaluate the effect of changes in domestic and trade partner import tariffs on the industry-level share of informal workers, and average formal and informal wages, which are the outcomes examined in the empirical studies such as Goldberg and Pavcnik (2003) and Paz (2012). To briefly summarize the theoretical results, a domestic import tariff reduction lead to a decrease in the share of informal workers and in the average formal wage. A trade partner import tariff reduction also leads to a reduction in the share of informal workers, but it increases the average formal worker wage. The effect of both tariffs on the average informal wage cannot be determined.

This paper is closely related to Goldberg and Pavcnik (2003) and Alemán-Castilla (2006). Goldberg and Pavcnik (2003) developed a representative firm model in which only formal workers have firing costs, whereas informal workers have a higher marginal cost for the firm. As a result, informal workers are employed in order to accommodate demand fluctuations. They found that a reduction in import tariffs results implies a mean decrease in the demand distribution, which leads to a decrease in formal employment and an increase in the share of informal workers.

Alemán-Castilla (2006) assumed that firms have a boost in productivity and access to foreign markets by hiring formal workers, i.e. by complying with the payroll tax. And this is embedded in Melitz’s (2003) heterogeneous firm model. As a result, compliance with payroll tax is not profitable for small firms, and changes in trade policy have an ambiguous effect on
the informality share.

My paper improves on previous studies in several important aspects. First, by assuming wage heterogeneity, my theoretical model is able to replicate several informal labor market stylized facts, including that informal workers are used because they are cheaper than formal workers, some small firms hire formal workers, and in equilibrium firms that hire formal workers will be larger on average. Second, I am able to obtain testable predictions regarding the effect of trade liberalization on the average wages. Third, due to definition of informality used here, my theoretical model is closely linked to the empirical literature such as Goldberg and Pavcnik (2003), Alemán-Castilla (2006), and Paz (2012).

The remainder of the paper is organized as follows. Section 2 presents the theoretical model. In section 3 I conduct the comparative statics exercises to obtain testable predictions in the form of propositions, whose proofs appear in the Appendix. Next, I present the welfare implications of the model, and discuss the model extension to the two large asymmetric economy case. In section 4 the links between the theoretical model testable predictions and the empirical literature findings are discussed. Finally, I offer some conclusions in section 5.

2. The Theoretical model

This section begins with a brief discussion of stylized facts concerning informal labor markets that are taken into account in the theoretical model design. Next, I discuss the core of the theoretical model - the payroll tax and its enforcement framework - which is then introduced into a general equilibrium international trade model.
2.1 Informal labor markets: stylized facts

A large strand of the literature defines informality as being based solely on compliance with regulations. For example, Schneider and Enste (2000) define the informal economy as the “legal value-added creating activities which are not taxed or registered and where the largest part can be classified as clandestine labor, which means that unpaid or ‘pure’ household production, voluntary nonprofit (social) services and criminal activities are excluded”.\(^1\) According to Portes et al. (1989, p.30) “the best-known economic effect of the informalization process is to reduce the costs of labor substantially”. Among these labor costs, the payroll tax is the largest, typically accounting for 15 to 30% of the wage.\(^2\) Furthermore, whether firms comply with the payroll tax is arguably central to the root distinction between formal and informal jobs in Argentina (Pratap and Quentin, 2006) and Brazil (Neri, 2002), for the simple reason that in both cases employers tend to follow other labor regulations. Thus, I define an informal job as an employment relationship in which the employer does not comply with the payroll tax. This definition has two important strengths. First, it can be used both theoretically and empirically, since payroll tax compliance information is available in surveys. Second, this measure is also comparable across countries because payroll taxes are used throughout the world.

Using the above definition of informality, the first stylized fact is concerned with the increase in labor informality in Latin America discussed previously. This increase can be due to changes in the composition of employment across or within manufacturing industries. Goldberg

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\(^1\) In contrast, the International Labor Organization (ILO, 2002) defines informal jobs as those offered by firms with five or less workers. But this definition does not distinguish jobs according to workers’ different welfare levels. 
\(^2\) Schneider and Enste (2000) provide an in-depth discussion of the factors influencing regulation compliance.
and Pavcnik (2003) find that within-industry variation accounted for almost all variation in Colombia (1984-1998) and about 85% of the variation in Brazil (1987-1998). Paz (2012) used a different sample for Brazil (1989-2001) and also found that the within-industry change in informality accounted for 85% of the variation in informality.

The second stylized fact is that informal jobs are more prominent among firms that employ few workers, as reported by Pratap and Quintin (2006) and Neri (2002), for Argentina and Brazil, respectively. From Table 1 we can see that in Brazil although informal workers are more likely to be found in small establishments, some small establishments still employ formal workers, while larger establishments are much more likely to employ formal workers. According to Ellery and Gomes (2007), the large establishments are also responsible for almost all Brazilian manufacturing exports. This strongly suggests that informal workers are not used for producing exported goods.

From Table 2 comes the third stylized fact that in Brazil every manufacturing industry has informal workers, and the share of informal workers is different across industries. Paz (2012) points out that it is commonly highlighted in the literature, e.g. Goldberg and Pavcnik (2003) and Alemán-Castilla (2006), that informal workers are different from formal workers in terms of age, gender, and schooling. This claim is usually supported by evidence consisting of highly aggregated data at the manufacturing level. However, a closer examination of manufacturing data for Brazil disaggregated at the industry level reveals a somewhat different picture (see Table 2). This leads to the fourth stylized fact. Table 2 figures reveal that the average characteristics of formal and informal workers (years of education, age, and gender) are similar in some industries, such as apparel, but slightly different in others, such as
nonmetallic mineral products. So, formal and informal workers are not much different with respect to those observable characteristics.

The idea that formal workers are not very different from informal workers receives additional support from Table 3 makes the fifth and last stylized fact. Table 3 presents evidence that workers in Brazil switched between formal and informal jobs between 1991 and 1996, in particular, approximately one sixth of people employed in the formal market in 1991 switched to the informal market in 1996, while approximately one third of the workers in the informal market in 1991 migrated to the formal labor market in 1996. Tables 2 and 3 suggest that informality is not determined exclusively by either worker or industry-specific characteristics.

2.2 Model set-up

2.2.1 Payroll Tax Mechanism

The model assumes that firms are risk neutral and choose whether or not to comply with payroll taxes based on their expected profits. In either case, firm $i$ has to pay its workers a wage $W_i$. However, if the firm complies with payroll taxes (i.e., establishes a formal labor contract), it pays an *ad valorem* payroll tax ($t$) and a per worker administrative cost ($\Theta$), which encompasses the costs of calculating, preparing and keeping tax records. Therefore, the cost of a formal worker for firm $i$ is $b_i^{\text{for}} = TW_i + \Theta$, with $T = 1 + t$.\(^3\) Thus, the cost of a formal worker for firm $i$ is $b_i^{\text{for}} = TW_i + \Theta$, with $T = 1 + t$.\(^4\)

If a firm is caught employing informal workers, it has to pay an *ad valorem* fine $\eta$, based

\(^3\) Evans (2003) provides evidence about the large private costs of tax compliance. Boisvert et al. (2001) found that in the case of Brazil, these preparation costs can be substantial, ranging from 43 to 86 dollars per worker, or between 15 and 30% of the prevailing minimum monthly wage.

\(^4\) The tax formula used here is the one used in practice in Brazil, as determined by Laws 8212 and 9876.
on the wage paid to each informal worker. Every firm is audited with probability $\zeta$. The expected cost of an informal worker for firm $i$ is $b_{i}^{\text{inf}} = \lambda W_i$ with $\lambda=1+\eta\zeta$. The payroll tax revenues and fines collected are returned via lump-sum transfer to all unemployed and employed (formal and informal) workers.

The likelihood of a payroll tax audit depends on the government’s ability to detect the existence of firms. An exporting firm can be easily detected because the government monitors the borders, and the trade flow information can then be matched to payroll tax data, uncovering any evasion. Thus exporters are certain to be audited ($\zeta = 1$). This enforcement framework makes it unprofitable for exporting firms to employ informal workers. Furthermore, the government will also know that the exporter serves the domestic market, which rules out the possibility of the firm employing formal workers for export orders and informal workers for domestic orders. However, a firm serving only the domestic market has to be found (i.e., it is harder to detect), so the audit likelihood is smaller ($\zeta = \overline{\zeta} < 1$).\(^{5}\)

Figure 2 shows the formal and informal expected wage bill functions for $\zeta = \overline{\zeta}$. A formal worker is cheaper than an informal worker for firm $i$ if its respective wage is $W_i > \chi \equiv \theta/(\lambda-T)$. Notice that the wage cutoff ($\chi$) is decreasing in $\eta$ and $\zeta$, which means that either an increase in the fine or an increase in the likelihood of an audit will decrease the cutoff. On the other hand, an increase in either one of the payroll taxes leads to an increase in the cutoff. These properties are consistent with findings that both taxation and its enforcement are determinants of informal economy (cf. Schneider and Enste, 2000). When $\zeta = 1$ a formal worker

\(^{5}\) It seems reasonable that the audit probability should increase with firm size. However, this would make calculations more cumbersome, and the results would not be qualitatively different. Moreover, in equilibrium, the probability will differ according to firm size, i.e. it is one for large firms and $\zeta$ for smaller firms.
will always be cheaper, and no firms will offer informal labor contracts.

In order to generate the stylized fact that every industry has a positive share of informal workers, the model’s payroll tax framework requires within-industry wage heterogeneity. And to replicate the fact that small firms are more likely to hire informal workers, but that some small firms do hire formal workers, the wage heterogeneity also has to be present across firms with the same level of employment. This last condition is corroborated by Schmidt and Zimmermann (1991), who find that there is still wage heterogeneity even after controlling for firm size and worker characteristics.⁶

The Melitz (2003) heterogeneous firm model has several extensions that allow for heterogeneous wages. Among them, Kreickemeier and Egger (2009), Helpman, Itskhoki, and Redding (2008), and Helpman and Itskhoki (2010) feature wages that are proportional to firm size. If the payroll tax mechanism proposed here is applied to these models, all small firms will hire only informal workers, which is not supported by the findings discussed earlier.

Fortunately, the Davis and Harrigan (2011) extension of the Melitz model allows firms with the same employment level to pay different wages, which means that among firms with the same level of employment some hire formal workers and others hire informal workers.⁷ This is so because firms differ in terms of productivity and worker monitoring ability (i.e., the employer ability to detect a worker that is shirking her duties). The latter generates wage heterogeneity through an efficiency wage motive (see e.g., Shapiro and Stiglitz 1984).

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⁶ Abowd et al. (1999) used a panel of French firms and workers and found that most of the inter-industry wage differential is due to person effects and not firm effects. Nevertheless, there is still some variation that could not be explained. To the best of my knowledge, there is no similar study conducted in a developing country using formal and informal workers that could discipline the theoretical model design.

⁷ A shortcoming of the Davis and Harrigan model is that the firm-specific wage does not change with the firm profits, contrary to what was found by Amiti and Davis (2011) in Indonesia.
According to Dasgupta and Ray (1986) and Ray (1998), efficiency wages in developing countries are motivated by the fact that workers' physical strength (provided and determined by her nourishment) affects their effort level, so different monitoring abilities could be interpreted as different effort requirements.\(^8\) Hence, a higher wage implies better nourishment and thus the worker is able to exert more effort.

Thus, I embed the payroll tax framework into the Davis and Harrigan (2011) model, which is described next. The world economy consists of two countries Home and Foreign (indicated by "*"). Each country has a one-sector economy where firms produce horizontally-differentiated goods indexed by \(\omega \in \Omega\). It is assumed that homogeneous labor is the only factor of production. Home is a small open economy (SOE) in the sense of Demidova and Rodriguez-Clare (2011), i.e. its policies do not affect Foreign except for the trade flow between the two countries.\(^9\)

2.2.2 Wage Heterogeneity

Workers are risk-neutral, have identical preferences, and their income comes from the wages \((W)\) and the governmental lump-sum transfers \((tr)\). It is assumed that there is no unemployment benefit and workers have no benefit from the payroll taxes.\(^{10}\) This means that workers are ex-ante indifferent between formal and informal jobs. However, the type of jobs in

\(^8\) The effort level is also called labor power, and can include mental strength.
\(^9\) This notion of small open economy is also present in Davies and Paz’s (2011) simplified version of the Melitz (2003) model.
\(^{10}\) In many developing countries, social security is a pay-as-you-go system, in which workers tend to see their contributions as simply a tax because the benefits they receive are not related to the contributions they make. Furthermore, it is not clear that workers are indeed benefiting from social security benefits, since the rate of return of the social security contributions may be smaller than the risk-free asset return.
the economy does matter for the workers’ welfare because of the government transfer received, which is taken as given by the worker. The indirect utility of a worker \((V_i)\) employed at firm \(i\),

\[
V_i(W_i, e, P, tr) = \begin{cases} 
(W_i + tr)^p & \text{if worker shirks} \\
(W_i + tr)/(e + tr)^p & \text{if worker exerts effort} \\
trP^{p-1} P & \text{if worker is unemployed}
\end{cases}
\]  

(1)

where \(e\) is the effort exerted by the worker and is measured in the same units as the wage, and \(P\) is the consumer price index that comes from the standard Dixit-Stiglitz CES aggregate of differentiated goods.

A worker can lose her job due to exogenous firm death, which affects every firm equally with probability \(\delta\), or by being caught shirking her duties with a firm-specific hazard rate \(m_i = (0, \bar{m})\), in which \(\bar{m}\) is the maximum monitoring ability.\(^{11}\)

The derivation of the solution for this efficiency wage problem is similar to Davis and Harrigan (2011), and thus omitted here.\(^{12}\) In the solution of the efficiency wage problem, \(W_1\) is defined to be the efficiency wage paid by the firm with the maximum monitoring ability \((\bar{m})\), as a result, \(W_1\) will be lowest wage paid. The efficiency wages associated with the other monitoring ability levels are given by a monotonically decreasing relationship between \(W_i\) and \(m_i\), as shown by equation (2). So, all wages are proportional to \(W_1\), and when \(W_1\) changes, all wages at Home change proportionately. Moreover, Firm-specific real and nominal wages are in a constant ratio that depends inversely on the firm level relative monitoring abilities as well as common parameters. This can be readily seen by calculating the ratio between equation (2)

\(^{11}\) As in Davis and Harrigan (2011), I also assume the following parameter restriction \(m_i\delta > (e - 1)\). This condition is satisfied as long as the exogenous job loss likelihood and the utility penalty of effort are small.

\(^{12}\) The efficiency wage problem is derived in the Appendix for the referees.
evaluated at two different wage levels.

\[ W_i = \frac{w_i \bar{m} \bar{e}_i}{m_i - \delta(e-1)^l}, \text{ with } \frac{\partial W_i}{\partial m_i} < 0 \quad (2) \]

Equation (2) also allows firms to be fully characterized by the joint distribution of wages and productivity, \( \mathcal{H}(\phi_i, W_i) \). Notice that the support of the wage distribution depends on \( W_i \), but it is just a scale parameter that can be factored out, given that the ratio among wages is constant. The solution to the efficiency wage problem provides one more equation that relates the unemployment level \( u \) to the expected wage distortion, \( E(W_i/m_i) \), which will be discussed in detail in the general equilibrium subsection. This relationship is depicted by equation (3), where \( B_1 \equiv \left[ \frac{\bar{m} \delta(e-1)}{\bar{m} - (e-1)\delta} \right] \) is a constant given \( e \).

\[ u = \left[ \frac{B_1 E\left(\frac{W_i}{m_i}\right)}{W_i + B_1 E\left(\frac{W_i}{m_i}\right)} \right] \quad (3) \]

2.2.3 Firm Profit Maximization Problem

Home has a measure \( M \) of active firms and for Foreign the mass of active firms is given by \( M^* \). The mass of firms is endogenous only for Home given the SOE assumption. When a firm is born, it has to pay a one-time fixed entry cost \( f_e \) units of labor to find out its monitoring ability and productivity \( (m_i, \phi_i) \) and then decide whether or not to enter the market. If it decides to operate and supply only the domestic market, it will incur a per-period fixed cost \( f \) units of labor and a variable cost related to the output level. If the firm decides to serve the foreign market, in addition to the variable cost that corresponds to the units exported, it incurs a per-period fixed export cost \( f_x \) units of labor, and the Foreign country import tariff, with \( \tau^* \equiv 1 + \)
In this heterogeneous firms type of models, the generally-accepted fact that only a small share of firms exports is ensured by assuming the sufficient condition \((r^*)^{\alpha-1} f_x > f\), which is used by Melitz (2003, p. 1709). Another result of this assumption is that every exporting firm will also serve the domestic market.

Following Davis and Harrigan (2011), I assume that the fixed costs \(f_o\), \(f\), and \(f_x\) consist of activities performed by workers receiving a wage of \(W_1\), and the per worker cost of accounting \(\Theta\) consists of \(\theta\) units of labor that also receive wage \(W_1\). The wage paid to these workers do not suffer payroll tax incidence, whereas the workers related to the variable cost activities receive the efficiency wage, \(W_i\), and are hired using formal or informal labor contracts. In order to have formal and informal jobs coexisting in the same industry, I further assume that \(\theta > \lambda T > 0\), which guarantees the existence of an interval in which the expected wage bill of an informal worker is smaller than the expected wage bill of a formal worker.

The last element of the firm profit maximization problem is the output demand that comes from the consumer expenditure minimization problem,

\[
q_{i,d} = \left[\frac{p_{i,d}}{P}\right]^{-\sigma} Q
\]

where \(q_{i,d}\) is the quantity of firm \(i\)'s output that is demanded domestically at price \(p_{i,d}\). As in Dixit and Stiglitz (1977), \(Q\) is the aggregate production level of the economy, \(\rho \in (0,1)\) is the parameter of the workers’ utility CES aggregator, and \(\sigma \equiv (1-\rho)^{-1}\) is the elasticity of substitution across goods. Firm \(i\)'s labor requirement for the variable cost activity is given by \(q_{i,d}/\phi_i\) and its domestic market profit when employing formal workers \((\pi_{i,d}^{for})\) is

\(r^*\) can also be interpreted as an iceberg type of variable export cost (e.g. transportation and insurance costs).
The profit maximization problem for a firm that employs informal workers is analogous to the case for formal workers. If the firm decides to export, it always finds cheaper to employ only formal workers. The foreign demand for a Home made variety is given by

\[ q_{i,x} = (p_{i,x})^{-\sigma} A \]

where \( A \equiv Q^*(P^*)^{-\sigma} \) and is constant due to the SOE assumption. The profit from exporting is given by equation (6).

\[ \pi_{i,x} = \max_{p_{i,x}} p_{i,x} q_{i,x} - W_1 f_x - \tau_i b_{i,for}^x \]

Consequently, firms have to choose between staying out of the market due to negative profits, producing only for domestic consumers using informal workers, producing only for domestic consumers using formal workers, or producing for domestic and foreign consumers using formal workers. Firm \( i \)'s ex post profit is given by \( \pi_i = \max \{ 0, \pi_{i,d}^{\text{inf}}, \pi_{i,d}^{\text{for}}, \pi_{i,d}^{\text{for}} + \pi_{i,x}^{\text{for}} \} \).

2.2.4 General equilibrium

In this subsection, I present and discuss the equations needed to solve for the general equilibrium of the model. First, let's define the inverse of firm \( i \)'s marginal cost as \( s_i = \varphi_i/b_i(W_i, \varphi) \).

The entry cutoff \( (s_{\text{open}}) \) is the inverse of the marginal cost at which firms make zero profit, so firms with \( s_i < s_{\text{open}} \) do not operate. Similarly, the inverse of the marginal cost that provides zero profit from exporting is defined as \( s_{s,for} = \inf \{ s_i : \pi_{i,d}^{\text{for}} + \pi_{i,x}^{\text{for}} > \max \{ \pi_{i,d}^{\text{for}}, \pi_{i,d}^{\text{inf}} \} \} \), which takes into account not only whether formal firms decide to serve foreign markets, but also the export decision of informal firms that have to switch to formal workers in order to supply foreign
markets.

Figure 3 shows the cutoffs in the wage-productivity space. The diagonal lines represent $s$ and its slope is the marginal cost $(b/\phi)$. Notice that the active firms will be on the right side of the $s_{\text{open}}$ curve, and the firms on the left exit the market as soon as they learn their pair $(m_i, \phi)$. The exporters, which are larger in size and productivity, are located to the right of $s_{x,\text{for}}$. The informal firms are located inside the polygon ABCD. Since the difference between the informal and formal wage bills decreases in $W_i$, the segment AB belongs to the line $TW_i + \Theta = \phi/s_{x,\text{for}}$. The employment level is constant along the $s$ line. This means that small firms, the ones with $s$ close to $s_{\text{open}}$, can either hire formal or informal workers depending on the wage paid. This matches the second stylized fact discussed earlier.

Let $G(s) \equiv \Pr[S \leq s]$ be the cumulative distribution function of $s$, with its density given by $g(s)$, and, following Melitz (2003), the $\sigma - 1$ uncentered moment of $g(s)$ is assumed to be finite. So, the probability of a firm being an exporter, i.e. its inverse marginal cost being larger than the export cutoff, is given by $\Pr(s > s_{x,\text{for}}) = 1 - G(s_{x,\text{for}})$. Let’s define $\tilde{s}(s)$ as a measure of aggregate inverse marginal costs for firms with $s_i \geq s$,

$$\tilde{s}(s) \equiv \frac{1}{1 - G(s)} \left[ \int_{s}^{\infty} s^{\sigma - 1} G(s) ds \right]^{\frac{1}{\sigma - 1}}$$  \hspace{1cm} (7)

The endogenous variables that need to be solved for in general equilibrium are $s_{\text{open}}$, $s_{x,\text{for}}$, $s_{x,\text{for}}^*$, $W_1$, $M$, $P$, $Q$, $E(W_i/m_i)$, $u$, and tr. The first equation needed for the general equilibrium is equation (8) that Melitz (2003) called the free entry condition (FE), which states that a sufficient mass of ex ante identical firms enters the market so that the expected profit from entry equals the fixed cost of entry.
with $f(s) \equiv [1 - G(s)][(\tilde{s}(s)/s)^{σ−1} − 1]$.

Equation (9) is the zero profit cutoff condition for entry. It requires the marginal entrant to have zero profits, i.e. the variable profit ($π_v$) must be equal to the per period fixed cost of production.

$$π_v^*(s_{open}) = σ^{−1}(ρPs_{open})^{σ−1}Q = W_1^*f$$  \hspace{1cm} (9)

Equation (10) is the zero cutoff profit for exporting, and it means that for the marginal entrant in the Foreign market the variable profit from exporting has to equal the fixed cost of exporting.

$$π_x^*(s_{for}) = σ^{−1}(ρs_{for}/τ^*)^{σ−1}A = W_1^*f_x$$  \hspace{1cm} (10)

Similarly for the Foreign exporter, the zero profit cutoff for entering the Home market must equal the fixed cost of exporting, as depicted in equation (11). Notice that I set $W_1^* = 1$ to be the numeraire.

$$π_x^*(s_{for}^*) = σ^{−1}(ρPs_{for}/τ)^{σ−1}Q = W_1^*f_x$$  \hspace{1cm} (11)

From the ratio between equation (9) and equation (11), a relationship emerges between the Home entry cutoff and the Foreign export cutoff as shown by equation (12).

$$s_{for}^* = τs_{open}(f_x/W_1^*)^{1/(σ−1)}$$  \hspace{1cm} (12)

Next, the static nature of the model requires balanced trade flows. Thus, sales from Home to Foreign must be equal to the sales from Foreign to Home, as depicted by equation (13).

$$\frac{M}{1−G(s_{for})} \int_{s_{for}}^{∞} (ρs/τ^*)^{σ−1}AdG(s) = \frac{M^*}{1−G^*(s_{for}^*)} \int_{s_{for}^*}^{∞} (ρPs/τ)^{σ−1}QdG^*(s)$$  \hspace{1cm} (13)
where the price index faced by the Home consumer is given by

\[ p^{1-\sigma} = \frac{M}{1-G(s_{open})} \int_{s_{open}}^{\infty} (ps)^{\sigma-1} dG(s) + \frac{M^*}{1-G^*(s_{x,for})} \int_{s_{x,for}}^{\infty} (ps/\tau)^{\sigma-1} dG^*(s) \]  

(14)

The labor force \((L)\) is divided into unemployed workers \((U)\), workers in the fixed entry cost sector \((L^e)\), workers in per-period fixed cost activities \((L^f)\), workers in per-period export fixed cost activities \((L^x)\), formal workers in variable cost activities \((L^{v,for})\), and informal workers in variable cost activities \((L^{v,inf})\).

In the steady state, the mass of active firms does not change, so the mass of entrants should be enough to replace the firms that experienced a negative shock and exited the market, so \(L^e = \delta M_f\). The level of employment in the per-period fixed cost activity is \(L^f = M_f\), and the level of employment in the per-period export fixed cost activity is \(L^x = [1-G(s_{x,for})]M_f\). The labor market clearing condition is given by:

\[ (1-u)L = \frac{M\delta_f}{1-G(s_{open})} + Mf + M[1-G(s_{x,for})]f_x + (1+\theta)L^{v,for} + L^{v,inf} \]  

(15)

Where

\[ L^{v,for} = \int_{s_{open}(T\chi+\Theta)}^{\infty} \int_{\chi} \frac{q_{i,d}}{\phi} \psi(\phi, W) dW d\phi + \int_{s_{x,for}(T\chi+\Theta)}^{\infty} \int_{\chi} \frac{q_{i,x}}{\phi} \psi(\phi, W) dW d\phi \]

\[ + \int_{s_{x,for}(TW_1+\Theta)}^{\infty} \int_{W_1} \frac{\lambda_x}{\phi} \psi(\phi, W) dW d\phi \]

\[ + \int_{s_{x,for}(T\chi+\Theta)}^{\infty} \int_{W_1} \frac{\lambda_x}{\phi} \psi(\phi, W) dW d\phi \]
\[
L^{\text{inf}} = \int_{s_{\text{open}}(TW_1 + \Theta)}^\infty \frac{\varphi}{\lambda x_{\text{open}}} \psi(\varphi, W) dW d\varphi + \int_{(TW_1 + \Theta)s_{x,\text{for}}}^\Theta \frac{\chi}{\lambda x_{\text{open}}} \psi(\varphi, W) dW d\varphi
\]

The next step is to calculate the expected wage distortion. To do so, let \( \psi(i/s^*) \) be the probability distribution function of active firms, in which the firm type is given by \( i(\varphi, m) \) and this density also depended on the entry and export cutoffs. The employment weighted average wage distortion per unit of mass of active firms is

\[
E \left( \frac{W_i}{m_i} \right) = \left( \frac{\delta f_e}{1 - G(s_{\text{open}})} + f + [1 - G(s_{x,\text{for}})]f_x + \theta L^{\text{for}} \right) \frac{W_1}{m} + \int \frac{q_{d,i}}{\varphi m_i} \psi(i|s_{\text{open}}) d\varphi
\]

\[
+ \int \frac{q_{i}}{\varphi m_i} \psi(i|s_{x,\text{for}}) d\varphi
\]

The per capita government transfer is given by the sum of collected payroll taxes and the expected value of the fines, where \( \bar{W}_{\text{for}} \) is the average formal wage and \( \bar{W}_{\text{inf}} \) is the average informal wage,

\[
tr = \frac{tw_{\text{for}}L^{\text{for}} + \eta \bar{w}_{\text{inf}}L^{\text{inf}}}{L} \]

The last equation needed comes from the efficiency wage problem and relates the unemployment rate to the expected wage distortion and \( W_1 \), equation (3).

Similarly to Demidova and Rodriguez-Clare (2011), by recursive substitution the general equilibrium ends up depending upon \( W_1 \) and \( s_{x,\text{for}} \) only. This is so, because all the other endogenous variables can be written as a function of both of them. So the general equilibrium
problem collapses to equation (10) and equation (18) that is a modified version of equation (13).

\[
\frac{M(W_1s_{x,for})}{1-G(s_{x,for})} \int_{s_{x,for}}^{\infty} \left( \frac{\rho s}{\tau} \right)^{\sigma-1} AdG(s) = \\
\frac{M^*}{1-G^*[s_{x,for}(W_1s_{x,for})]} \int_{s_{x,for}(W_1s_{x,for})}^{\infty} \left[ \frac{\rho P(W_1s_{x,for})s}{\tau} \right]^{\sigma-1} Q(W_1, s_{x,for}) dG^*(s) \tag{18}
\]

**Proposition 1:** There exists a solution to the system of equations (10) and (18) and it is unique.\(^{14}\)

3 Comparative statics

The analysis conducted here is a comparative steady state analysis, in which I assess the effects of changes in Home and Foreign import tariffs on the share of informal workers and on the average formal and informal wages. The effects of these changes on the ex ante workers’ welfare are discussed next. Finally, I show how the results are altered when Home and Foreign become two large and asymmetric economies.

The share of informal workers is defined as $share \equiv L^{V,inf}/(L^{V,inf} + L^{V,for})$. This is the most common measure used in the literature because given the usual belief that workers prefer formal to informal jobs, its increase would mean a reduction in workers’ welfare. The average formal and informal wages not only allow for evaluation of changes in wage inequality

\(^{14}\) The proof of this proposition is similar to Demidova and Rodriguez-Clare (2011), hence it is omitted here.
between average formal and informal workers, but are also an important and easily measurable component of worker welfare. The share of informal workers and the average formal and informal wages are very convenient measures because they are expressed as ratios. And this simplifies considerably the derivations since the general equilibrium effects on $P$, $Q$, and $M$ do not matter for the sign of the derivatives.

3.1 Effects of a reduction in Home import tariffs

A reduction in Home import tariffs ($\tau$) affects only the trade balance condition, equation (18). From Lemma 1 in Demidova and Rodriguez-Clare (2011), for a fixed export cutoff ($s_{x,for}$) the fall in Home import tariff forces a reduction in Home wages, $\frac{dW_1}{d\tau} > 0$. Remember that the ratio between wages ($W_i$) and $W_1$ is always constant, so in this case, a decrease in $W_1$ makes every Home wage at Home to decrease proportionally, and the formal-informal cutoff ($\chi$) also changes proportionately. The reduction in $W_1$ in equation (10) leads to a decrease in $s_{x,for}$ ($\frac{\partial s_{x,for}}{\partial \tau} > 0$), which is equivalent to an increase in the marginal cost export cutoff, the $s_{x,for}$ line rotates to the left in Figure 3.

The free entry condition, equation (8), implies that $s_{open}$ moves in the opposite direction of $s_{x,for}$, so a decrease in $\tau$ leads to an increase in $s_{open}$ ($\frac{\partial s_{open}}{\partial \tau} < 0$), i.e. the $s_{open}$ line rotates to the right in Figure 3. These high cost and high price firms exit the market and that leads to a decrease in the price index ($\frac{dP}{d\tau} > 0$).

The effect of a change in Home import tariffs on the informal share is derived in equation (19). The decrease in the marginal cost entry cutoff induced by a reduction in Home
import tariff leads to a loss of both formal and informal jobs for Home firms, so the sign of the first term on the right-hand side of equation (19) is undetermined. The second term in the right-hand side of equation (19) reflects the increase in the marginal cost export cutoff due to the decrease in $\tau$. The Home firms in the $s_{x,for}$ locus with $W_i < \chi$ switch from informal to formal workers in order to gain access to foreign markets, and firms with $W_i > \chi$ hire more formal workers to engage in exporting. As a result, the second term of equation (19) is positive, since formal jobs are created and informal jobs are destroyed, thus the share of informal workers decreases. But the sign of $\frac{d \text{share}}{d \tau}$ is ambiguous, since informal jobs are destroyed and formal jobs are created and destroyed.

\[
\frac{d \text{share}}{d \tau} = \frac{\partial \text{share}}{\partial s_{open}} \frac{\partial s_{open}}{\partial \tau} + \frac{\partial \text{share}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{\partial \tau} \tag{19}
\]

Although the model intuition is simple, its derivation quickly becomes cumbersome when the ambiguity concerning the sign of the $\partial \text{share}/\partial s_{open}$ term is addressed. One way to obtain testable predictions is to develop (sufficient) assumptions about $\Psi(\varphi_i, W_i)$ and the model parameters. This leads to Proposition 2:

Proposition 2. A reduction in the Home import tariff ($\tau$) decreases the share of informal workers in total industry employment if the sufficient conditions (S1)-(S4) hold,

(S1)$\sigma > 3$; (S2)$f_x = f$; (S3)$0 > \Psi(\varphi, W)^{-1} \frac{\partial \Psi(\varphi, W)}{\partial \varphi} > \frac{1-\sigma}{\varphi}$; (S4)$\frac{\partial \Psi(\varphi, W)}{\partial W} < 0$. \(^{15}\)

Condition (S1) is a consequence of the earlier assumption about the finiteness of $g(s)$

\(^{15}\) The proof for Proposition 2 appears in Appendix A.1.
moments, and for instance it was also assumed by Helpman et al. (2004). Moreover, this assumption does not seem to be restrictive because the empirical estimates of $\sigma$ by Hummels (2001) and Romalis (2007) are above 3. Using data for 15 developing countries (some in Latin America) Broda et al. (2008) estimated the elasticity of substitution and found that across HS-4 level varieties it has a median above 3.

One way to solve the ambiguity is to guarantee that the number of formal jobs along the $s_{x,for}$ line (in Figure 3) is larger than the number of formal jobs along the $s_{open}$ line. For this to happen, we need $|\frac{\partial s_{x,for}}{\partial \tau}| > |\frac{\partial s_{open}}{\partial \tau}|$ and $|\frac{\partial \text{share}}{\partial s_{x,for}}| > |\frac{\partial \text{L}_{v,for}}{\partial s_{open}}|$. Condition (S2) guarantees that the former inequality holds, and this leads to Lemma 1.

**Lemma 1.** A sufficient condition for $|\frac{\partial s_{x,for}}{\partial \tau}| > |\frac{\partial s_{open}}{\partial \tau}|$ is $f_x = f$.\(^{16}\)

The sufficient conditions (S3) and (S4) are needed to ensure that the latter inequality holds. Both conditions are met if, for example, the marginal distributions are Pareto. Condition (S3) means that the change in probability as $\varphi$ is smaller in absolute value than the rate of employment growth as $\varphi$ increases. The reason of this assumption is that the formal employment of the firms lying on $S_{x,for}$ ray is larger than the formal employment of the firms lying on the $s_{open}$ ray, which solves the ambiguity.

The effects of a reduction in Home import tariff on the average formal wage is presented in equation (20) and the average informal wage is presented in equation (21).

\(^{16}\) The proof for Lemma 1 appears in Appendix A.1.
\[
\frac{d\bar{w}_{\text{for}}}{dt} = \left( \frac{\partial \bar{w}_{\text{for}}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{dt} + \frac{\partial \bar{w}_{\text{for}}}{\partial s_{\text{for}}} \frac{\partial s_{\text{for}}}{dt} \right) \frac{dW}{dt} (20)
\]

\[
\frac{d\bar{w}_{\text{inf}}}{dt} = \left( \frac{\partial \bar{w}_{\text{inf}}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{dt} + \frac{\partial \bar{w}_{\text{inf}}}{\partial s_{\text{for}}} \frac{\partial s_{\text{for}}}{dt} \right) \frac{dW}{dt} (21)
\]

The reduction in all wages caused by the domestic import tariff leads to a decrease in the average formal and informal wages. Given that \( \frac{dW}{dt} > 0 \), the sign of the derivatives will be determined by the sign of the expression inside the parentheses, which is in principle undetermined. This expression consists of the changes on entry and export cut-offs that lead to changes in the job composition, since the wages of the destroyed jobs are different from the wages of the created jobs, as shown below.

\[
\text{sgn}\left\{ \frac{d\bar{w}_{\text{for}}}{dt} \right\} = \text{sgn}\left\{ \frac{\partial \bar{w}_{\text{for}}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{dt} - \bar{w}_{\text{for}} \frac{\partial L^{Y_{\text{for}}}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{dt} + \frac{\partial \bar{w}_{\text{for}}}{\partial s_{\text{for}}} \frac{\partial s_{\text{for}}}{dt} - \bar{w}_{\text{for}} \frac{\partial L^{Y_{\text{for}}}}{\partial s_{\text{for}}} \frac{\partial s_{\text{for}}}{dt} \right\}
\]

The sign of the above expression can be determined by comparing the average formal wage with the average wage of the formal firms lying on the curve in which the derivative is calculated, in this case \( s_{\text{open}} \) and \( s_{\text{for}} \). For the sign to be positive, we need the average formal wage along \( s_{\text{open}} \) to be larger than \( \bar{w}_{\text{for}} \) and the average formal wage along \( s_{\text{for}} \) to be smaller than \( \bar{w}_{\text{for}} \). This is ensured by conditions (S1)-(S3) and (S5). This leads to the third proposition.

Proposition 3. A reduction in the Home import tariff leads to a decrease in the industry-level average formal wage if the sufficient conditions (S1)-(S3) and the additional sufficient condition (S5) hold,

\[
(S5) 0 > -\sigma \rho^{\sigma} \left( \frac{\phi}{TW + \Theta} \right)^{\sigma - 1} > \frac{\partial \Psi(\phi, W)}{\partial W}. \quad 17
\]

\[17\] The proof for Proposition 3 is presented in Appendix A.1.
Condition (S5) is more restrictive than condition (S4), and it is needed to guarantee that the average formal wage along \( s_{open} \) is larger than the average wage along \( s_{x,for} \). Unfortunately, the previous assumptions are not sufficient to prevent sign ambiguity concerning the effects of Home tariff changes on the average informal wage. So, unless the model is fully parameterized a prediction cannot be derived.

3.2 Effects of a reduction in Foreign import tariffs

Next the effect of a reduction in Foreign import tariffs on informality share and average formal and informal wage is investigated. From Demidova and Rodriguez-Clare (2011) Lemma 1 a reduction in Foreign import tariffs \((\tau^*)\) affects equation (10), and for a fixed export cutoff \((s_{x,for})\) it implies that \( W_1 \) has to increase, \( \frac{dW_1}{d\tau^*} < 0 \). Remember that the wage ratios are always constant, so in this case, an increase in \( W_1 \) makes every wage at Home to increase proportionally. The increase in \( W_1 \) leads to a decrease in \( s_{x,for} \) \((\frac{ds_{x,for}}{d\tau^*} > 0)\) through equation (19), which means an increase in the marginal cost export cutoff. Finally, equation (8) implies that \( s_{open} \) moves in the opposite direction of \( s_{x,for} \), so a decrease in \( \tau \) leads to an increase in \( s_{open} \) \((\frac{ds_{open}}{d\tau^*} < 0)\), or a decrease in the marginal cost entry cut-off, which as in the previous case leads to a decrease in the price index \((\frac{dp}{d\tau^*} > 0)\). The effect of Foreign import tariff on the Home share of informal workers is calculated in a similar fashion to the effect of Home import tariffs, but now as shown in equation (22), the derivative of the informality share with respect to \( \tau^* \), has an additional term that accounts for the change in the share due to the change in the formal
employment of current exporters.

\[
\frac{d{\text{share}}}{d\tau^*} = \frac{\partial{\text{share}}}{\partial{s_{\text{open}}}} \frac{\partial{s_{\text{open}}}}{d\tau^*} + \frac{\partial{\text{share}}}{\partial{s_{\text{for}}}} \frac{\partial{s_{\text{for}}}}{d\tau^*} + \frac{\partial{\text{share}}}{\partial{\tau^*}}
\] (22)

So, a decrease in the foreign import tariff affects the share of informal workers by destroying informal jobs and creating and destroying formal jobs due to changes in the entry and export cut-offs (as discussed before), and now by creating more formal jobs as the result of the increase in the volume exported, which raises formal employment by current exporting firms, as depicted by the last term in the right hand side of equation (22), which is positive. Thus, the sign of \(\frac{d{\text{share}}}{d\tau^*}\) is ambiguous. But this ambiguity can be solved using a strategy similar to the one used in Proposition 2. This leads to Proposition 4.

Proposition 4. A reduction in the Foreign import tariff (\(\tau^*\)) decreases the share of informal workers in total industry employment if the sufficient conditions (S1)-(S4) hold.\(^{18}\)

Next, the effects of a reduction in Foreign import tariff on the Home average formal and informal wage are investigated, which also suffers from sign ambiguity, as shown in equation (23), for the effect of \(\tau^*\) on the average formal wage.

\[
\frac{d\bar{w}_{\text{for}}}{d\tau^*} = \left(\frac{\partial\bar{w}_{\text{for}}}{\partial\tau^*} + \frac{\partial\bar{w}_{\text{for}}}{\partial{s_{\text{open}}}} \frac{\partial{s_{\text{open}}}}{d\tau^*} + \frac{\partial\bar{w}_{\text{for}}}{\partial{s_{\text{for}}}} \frac{\partial{s_{\text{for}}}}{d\tau^*} + \frac{\partial\bar{w}_{\text{for}}}{\partial{\tau^*}}\right) d\bar{w}_{\text{for}}
\] (23)

Like in Proposition 3, under conditions (S1)-(S3) and (S5) we obtain a positive sign for the last four terms in equation (24).

\(^{18}\) The proof for Proposition 2 appears in Appendix A.1.
Now, we also need to have a positive sign for the first two terms, so the sign of the term in parentheses in equation (23) is positive. Notice that \( \frac{\partial \text{twp}_{for}}{\partial \tau^*} < 0 \) and \( \frac{\partial L_{for}}{\partial \tau^*} < 0 \). Furthermore, conditions (S3) and (S5) ensure that \( \bar{w}_{for} > 0 \). Thus, \( \bar{w}_{for} \), i.e. \( \bar{w}_{for} \), increases with a decrease in \( \tau^* \). This leads to Proposition 5:

**Proposition 5:** A reduction in the Foreign import tariff leads to an increase in the industry-level average formal wage if the conditions (S1)-(S3) and (S5) hold.\(^{19}\)

As before, the previous assumptions are not sufficient to prevent sign ambiguity concerning the effects of tariff changes on the average informal wage.

3.3 Welfare Analysis

The welfare analysis conducted here consists of contrasting the workers’ ex ante welfare at two different steady states. Welfare changes during the transition between steady states cannot be evaluated because the adjustment path is not contemplated by the theoretical model.

The workers’ ex ante welfare is affected by the price index, the government transfers,  

\(^{19}\) The proof for Proposition 4 is presented in Appendix A.1.
and the expected wage, which depends on the distribution of active firms and unemployment. All these three factors are affected by changes in Home and Foreign import tariffs.

A decrease in the Home import tariff leads to a decrease in the price index, which improves the workers’ welfare. This change in tariff also reduces the informality share, which affects workers’ welfare through the government transfers, in the sense that higher informality leads to lower government transfers, all else equal. But the effect on unemployment and on the overall average wage is undetermined. As a result, it is not clear whether workers’ ex ante welfare increases or decreases. A similar analysis is applicable to a decrease in Foreign import tariff, but again, the total effect on the ex ante welfare cannot be determined.

3.4 Two Large and Asymmetric Economies case

The theoretical model can be extended to the case of two large asymmetric economies. Now, the mass of firms on Foreign becomes endogenous, and the Foreign demand for Home products will depend on the now endogenous $P^*$ and $Q^*$. This means that changes in Home economy will affect the Foreign economy. As discussed in Demidova and Rodriguez-Clare (2011), the comparative statics of this version of the model gets extremely cumbersome, to the point that it is not even possible to figure out what happens with $s_{x,for}$ when tariffs change. I present in Appendix A.2 the equations needed to solve for the general equilibrium in the case of two large asymmetric economies.

4. Discussion
In this section I contrast the propositions from theoretical model with the findings from the empirical studies about the effect of domestic and trade partner import tariff reductions on the industry-level share of informal workers and average formal and informal wages.

The empirical evidence of the effects of domestic import tariff reduction is mixed. Alemán-Castilla (2006) found no effect in Mexico, and Goldberg and Pavcnik (2003) also found no effect for Colombia and Brazil. On the other hand, Paz (2012) found that a decrease in domestic tariffs increased informality share in Brazil. This result was also found by Acosta and Montes-Rojas (2010) using Argentinean data.\textsuperscript{20}

Both results contradict Proposition 2. But this fact does not rule against the theoretical model because it has ambiguous predictions at first. And it is not clear which of conditions (S1)-(S4) used to derive proposition 2 is the culprit for such failure. Intuitively, both the lack of effect and the increase in informality share can be generated whenever the net formal job loss is equal or exceeds the informal job destruction. One possibility is to have very few firms exporting due to a very high fixed exporting cost, which could be the case in these countries aforementioned, since their trade exposure is small. Another possibility is to have the marginal density of productivity to decrease very fast as productivity increases.\textsuperscript{21}

Alemán-Castilla (2006) and Paz (2012) are the only studies that assessed the effects of a trade partner import tariff reduction. They found that it decreased the informality share as predicted by Proposition 4. Remember that conditions (S1)-(S4) are sufficient conditions, and in particular, in the proof of Proposition 4 the fact that a decrease trade partner import tariff

\textsuperscript{20} Notice that only Paz (2012) empirical strategy takes into account the potential endogeneity between of tariffs and informality share due to the effects of both policies on government revenues.

\textsuperscript{21} But the calculations to determine such speed are very cumbersome and require the specification of a functional form for the marginal density of productivity.
increase formal employment of the current exporters was not used. This means that the sufficient conditions can be relaxed to some extent and the result will remain.

Paz (2012) investigated the effects of tariff changes on the average formal and informal wages in Brazil. His estimates imply that a decrease in domestic import tariffs decreases the average formal wage, as predicted by Proposition 3. A similar change in the trade partner’s import tariff increases the average formal wage by 0.32%, which supports the result from Proposition 5. For the average informal wage, he found that it increases with a reduction in import tariff and decreases with a reduction in the trade partner’s import tariffs. In sum, of the four testable predictions generated by the theoretical model, the empirical evidence supports three of them.

5. Conclusions

Recent trade liberalization episodes in developing countries were accompanied by changes in the share of informal jobs, which are commonly seen as low wage and quality jobs, and also cause payroll tax evasion.

To shed some light on this important issue, this paper presented a novel theoretical model of a small open economy in which trade policy affects firms’ payroll tax compliance, which is the major determinant of whether jobs are formal or informal. In particular, I focus on the effects of trade policy changes on the share of informal jobs but also the average formal and informal wages.

The theoretical model predicts that a decrease in import tariffs reduces the share of
informal workers, and a decrease in the trade partner’s import tariff also reduces the informality share. Additionally, an import tariff reduction decreases the average formal wage, whereas a decrease in the trade partner’s import tariff increases the average formal wage. The last three results received strong support from empirical studies such as Alemán-Castilla (2006) and Paz (2012). Nevertheless, the effect on the average informal wage is ambiguous.

One interesting direction for future research would use the theoretical model to investigate the effect of a government revenue-neutral switch from payroll tax to consumption tax on the informality outcomes studied in this paper.

References


Perry, G., W. Maloney, O. Arias, P. Fajnzylber, A. Mason, and J. Saavedra-Chanduvi


Table 1 – Employment shares in Brazil, by firm employment level and formality status.

<table>
<thead>
<tr>
<th>Number of employees</th>
<th>1989 Formal share</th>
<th>1989 Informal share</th>
<th>1999 Formal share</th>
<th>1999 Informal share</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.83</td>
<td>0.45</td>
<td>1.50</td>
</tr>
<tr>
<td>3 to 5</td>
<td>2.27</td>
<td>3.41</td>
<td>3.33</td>
<td>5.82</td>
</tr>
<tr>
<td>6 to 10</td>
<td>3.21</td>
<td>2.05</td>
<td>6.17</td>
<td>4.19</td>
</tr>
<tr>
<td>11 or more</td>
<td>82.05</td>
<td>5.97</td>
<td>69.50</td>
<td>9.04</td>
</tr>
<tr>
<td>Total</td>
<td>87.74</td>
<td>12.26</td>
<td>79.45</td>
<td>20.55</td>
</tr>
</tbody>
</table>

Source: Brazilian 1989 and 1999 PNAD data.

Notes: An informal worker is one whose employer does not comply with payroll taxes.

The data is presented at the most disaggregated level available.

The question about firm employment level is included in the PNAD survey only in selected years.
Table 2 – Descriptive statistics of worker characteristics in Brazil, by industry

<table>
<thead>
<tr>
<th>Industry</th>
<th>Share of informal workers</th>
<th>Average informal worker characteristics</th>
<th>Average formal worker characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Std. dev.</td>
<td>Min</td>
</tr>
<tr>
<td>Nonmetallic Mineral Product</td>
<td>0.288</td>
<td>0.027</td>
<td>0.242</td>
</tr>
<tr>
<td>Metals Production and Processing Machinery, Equipment and Commercial Installation Electrical and Electronic Equipment</td>
<td>0.070</td>
<td>0.037</td>
<td>0.027</td>
</tr>
<tr>
<td>Automobile, Truck and Bus</td>
<td>0.056</td>
<td>0.023</td>
<td>0.032</td>
</tr>
<tr>
<td>Wood Sawing and Wood Products</td>
<td>0.325</td>
<td>0.063</td>
<td>0.239</td>
</tr>
<tr>
<td>Pulp and Paper Production, Paper Products, Printing and Publishing</td>
<td>0.134</td>
<td>0.048</td>
<td>0.062</td>
</tr>
<tr>
<td>Rubber Products</td>
<td>0.071</td>
<td>0.037</td>
<td>0.012</td>
</tr>
<tr>
<td>Non-petrochemical Chemical and Fertilizer</td>
<td>0.072</td>
<td>0.016</td>
<td>0.048</td>
</tr>
<tr>
<td>Pharmaceutical, Perfumes, Detergents and Candles</td>
<td>0.103</td>
<td>0.028</td>
<td>0.063</td>
</tr>
<tr>
<td>Plastics Products</td>
<td>0.094</td>
<td>0.028</td>
<td>0.047</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.104</td>
<td>0.030</td>
<td>0.055</td>
</tr>
<tr>
<td>Apparel</td>
<td>0.260</td>
<td>0.040</td>
<td>0.186</td>
</tr>
<tr>
<td>Footwear and leather products</td>
<td>0.181</td>
<td>0.030</td>
<td>0.115</td>
</tr>
<tr>
<td>Coffee, Food, Beverage, Animal</td>
<td>0.181</td>
<td>0.029</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Source: Brazilian PNAD data.

Note: An informal worker is one whose employer does not comply with payroll taxes.
Table 3 - Formal-informal job transitions in Brazil between 1991 and 1996

<table>
<thead>
<tr>
<th>Employment status in May 1991</th>
<th>Formal job</th>
<th>Informal job</th>
<th>Self-employed</th>
<th>Employers</th>
<th>No-wage employment</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formal job</td>
<td>41.4%</td>
<td>10.1%</td>
<td>8.1%</td>
<td>1.4%</td>
<td>0.2%</td>
<td>61.2%</td>
</tr>
<tr>
<td>Informal job</td>
<td>4.9%</td>
<td>8.5%</td>
<td>2.9%</td>
<td>0.4%</td>
<td>0.1%</td>
<td>16.9%</td>
</tr>
<tr>
<td>Self-employed</td>
<td>2.1%</td>
<td>2.4%</td>
<td>11.2%</td>
<td>1.8%</td>
<td>0.3%</td>
<td>17.8%</td>
</tr>
<tr>
<td>Employers</td>
<td>0.3%</td>
<td>0.2%</td>
<td>0.9%</td>
<td>1.9%</td>
<td>0.1%</td>
<td>3.5%</td>
</tr>
<tr>
<td>No-wage employment</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Non-declared</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Totals</td>
<td>48.9%</td>
<td>21.4%</td>
<td>23.4%</td>
<td>5.5%</td>
<td>0.8%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: Data from the May 1996 special supplement of the PME (Pesquisa Mensal de Emprego - Brazilian Monthly Employment Survey), in which every worker interviewed reported his/her formality status in 1991 and in 1996.

Note: An informal worker is one whose employer does not comply with payroll taxes.
Figure 1 – Informality share in services and manufacturing in Brazil, 1984-2001

Source: PNAD data.

Note: Dashed vertical line indicates the start of Brazil’s trade liberalization
Figure 2 – Wage ($w_i$) and expected wage bill ($b_i$) for formal and informal labor contracts

\begin{align*}
TW_i + \Theta &= \lambda \chi \\
\lambda w_i \text{ (informal contract)} \\
TW_i + \Theta \text{ (formal contract)} \\
\lambda w_i &= \Theta / (\lambda - T)
\end{align*}
Figure 3—Productivity ($\phi_i$) and wage ($w_i$) space and firms’ choice of labor contracts and markets served in an open economy.
Appendix

A.1 Comparative Statics

**Lemma 1.** A sufficient condition for \( \left| \frac{\partial s_{x,for}}{\partial \tau} \right| > \left| \frac{\partial s_{open}}{\partial \tau} \right| \) is \( f_x = f \).

**Proof**

By differentiating equation (7) with respect to \( \tau \), I obtain

\[
\frac{\partial f(s_{open})}{\partial s_{open}} \frac{\partial s_{open}}{\partial \tau} f + \frac{\partial f(s_{x,for})}{\partial s_{x,for}} \frac{\partial s_{x,for}}{\partial \tau} f_x = 0
\]

Notice that

\[
\frac{\partial f(s_{open})}{\partial s_{open}} = 1 - \sigma \int_{s_{open}}^{\infty} \left( \frac{s}{s_{open}} \right)^{\sigma-1} \sigma \, dG(s)
\]

Then,

\[
\frac{\partial s_{x,for}}{\partial \tau} = -\frac{\partial f(s_{x,for})}{\partial s_{x,for}} \frac{\partial s_{open}}{\partial \tau} f_x \left( \frac{\partial f(s_{x,for})}{\partial s_{x,for}} \right)^{-1}
\]

\[
\frac{\partial s_{x,for}}{\partial \tau} = -\frac{\partial s_{open}}{\partial \tau} f_x \left( \frac{s_{x,for}}{s_{open}} \right)^{\sigma-1} \sigma \int_{s_{x,for}}^{\infty} \frac{s}{s_{open}}^{\sigma-1} \sigma \, dG(s)
\]

Given that \( s_{x,for} > s_{open}, \frac{\partial s_{open}}{\partial \tau} < 0 \), and under the assumption that \( f = f_x \), the term multiplying \( \frac{\partial s_{open}}{\partial \tau} \) is positive and larger than one, I conclude that \( \left| \frac{\partial s_{x,for}}{\partial \tau} \right| > \left| \frac{\partial s_{open}}{\partial \tau} \right| \).

**Lemma 2.** A sufficient condition for \( \left| \frac{\partial s_{x,for}}{\partial \tau^*} \right| > \left| \frac{\partial s_{open}}{\partial \tau^*} \right| \) is \( f_x = f \).

**Proof**
By differentiating equation (7) with respect to $\tau^*$, I obtain

$$
\frac{\partial f(s_{\text{open}})}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{\partial \tau^*} f + \frac{\partial f(s_{x,\text{for}})}{\partial s_{x,\text{for}}} \frac{\partial s_{x,\text{for}}}{\partial \tau^*} f_x = 0
$$

Notice that

$$
\frac{\partial f(s_{\text{open}})}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{\partial \tau^*} = 1 - \sigma \int_{s_{\text{open}}}^{\infty} \left( \frac{s}{s_{\text{open}}} \right)^{\sigma-1} dG(s)
$$

Then,

$$
\frac{\partial s_{x,\text{for}}}{\partial \tau^*} = -\frac{\partial f(s_{\text{open}})}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{\partial \tau^*} f x \left( \frac{\partial f(s_{x,\text{for}})}{\partial s_{x,\text{for}}} \right)^{-1}
$$

$$
\frac{\partial s_{x,\text{for}}}{\partial s_{\text{open}}} = -\frac{\partial f(s_{\text{open}})}{\partial \tau^*} f_s \frac{s_{x,\text{for}}}{s_{\text{open}}} \int_{s_{x,\text{for}}}^{\infty} \left( \frac{s_{\text{open}}}{s_{x,\text{for}}} \right)^{\sigma-1} dG(s)
$$

Given that $s_{x,\text{for}} > s_{\text{open}}$, $\frac{\partial s_{\text{open}}}{\partial \tau^*} < 0$, and under the assumption that $f = f_s$, the term multiplying

$\frac{\partial s_{\text{open}}}{\partial \tau^*}$ is positive and larger than one, I conclude that $|\frac{\partial s_{x,\text{for}}}{\partial \tau^*}| > |\frac{\partial s_{\text{open}}}{\partial \tau^*}|$.

**Proposition 2**: A reduction in the Home import tariff decreases the share of informal workers in total industry employment if the sufficient conditions (S1)-(S4) hold,

$$
(S1) \sigma > 3; (S2) f_s = f; (S3) 0 > \Psi(\phi, W)^{-1} \frac{\partial \Psi(\phi, W)}{\partial \phi} > \frac{1-\sigma}{\phi}; (S4) \frac{\partial \Psi(\phi, W)}{\partial W} < 0.
$$

**Proof** I need to prove that $\frac{d\text{share}}{d\tau} = \frac{\partial \text{share}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{\partial \tau} + \frac{\partial \text{share}}{\partial s_{x,\text{for}}} \frac{\partial s_{x,\text{for}}}{\partial \tau} > 0$. As discussed earlier, the sign of the last term is positive, but the sign of the first term is ambiguous. Let’s examine the sign of the first term in detail, since $\frac{\partial s_{\text{open}}}{\partial \tau} < 0$. 

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\[
\frac{\partial \text{share}}{\partial s_{\text{open}}} = (L^{\text{v,inf}} + L^{\text{v,for}}) \left\{ \frac{\partial L^{\text{v,inf}}}{\partial s_{\text{open}}} - \frac{\partial L^{\text{v,for}}}{\partial s_{\text{open}}} \right\} \]

Notice that the first term inside the curly brackets is positive, while \( \frac{\partial L^{\text{v,for}}}{\partial s_{\text{open}}} > 0 \). So the resulting sign of \( \frac{\partial \text{share}}{\partial s_{\text{open}}} \) is ambiguous. The proposition result requires \( \frac{\partial \text{share}}{\partial s_{\text{open}}} < 0 \), so the strategy adopted here is to find a term that can dominate \( \frac{\partial L^{\text{v,inf}}}{\partial s_{\text{open}}} L^{\text{v,for}} \). This is done by looking at \( \frac{\partial \text{share}}{\partial s_{\text{for}}} \), where

\[
\frac{\partial \text{share}}{\partial s_{\text{for}}} = (L^{\text{v,inf}} + L^{\text{v,for}}) \left\{ \frac{\partial L^{\text{v,inf}}}{\partial s_{\text{for}}} - \frac{\partial L^{\text{v,for}}}{\partial s_{\text{for}}} \right\}
\]

The first term inside the curly bracket is positive since \( \frac{\partial L^{\text{v,inf}}}{\partial s_{\text{for}}} > 0 \). The second term is also positive because \( \frac{\partial L^{\text{v,for}}}{\partial s_{\text{for}}} < 0 \). So, \( \frac{\partial \text{share}}{\partial s_{\text{for}}} > 0 \).

The ambiguity can be solved if

\[
\left| \frac{\partial s_{\text{for}}}{\partial \tau} \frac{\partial L^{\text{v,for}}}{\partial s_{\text{for}}} L^{\text{v,inf}} \right| > \left| \frac{\partial s_{\text{open}}}{\partial \tau} \frac{\partial L^{\text{v,for}}}{\partial s_{\text{open}}} L^{\text{v,inf}} \right|.
\]

This means that the number of formal jobs along the \( s_{\text{for}} \) line is larger than the number of formal jobs along the \( s_{\text{open}} \) line. For this to happen, we need \( \left| \frac{\partial s_{\text{for}}}{\partial \tau} \right| > \left| \frac{\partial s_{\text{open}}}{\partial \tau} \right| \) and \( \left| \frac{\partial L^{\text{v,for}}}{\partial s_{\text{for}}} \right| > \left| \frac{\partial L^{\text{v,for}}}{\partial s_{\text{open}}} \right| \). Condition (S2) with Lemma 1 guarantees that that the former inequality holds. The latter inequality is met if conditions (S3) and (S4) hold. The intuition is the following. For a given wage, as productivity increases two things happen. First, the firm employment level increases at the \( (\sigma-1)\varphi^{-1} \) rate. So, as long as the marginal density of productivity decreases at a slower rate, i.e. \( 0 > \Psi(\varphi,W)^{-1} \frac{\partial \Psi(\varphi,W)}{\partial \varphi} > \frac{1-\sigma}{\varphi} \), there will be more formal workers employed on firms lying on \( s_{\text{for}} \) than on \( s_{\text{open}} \). 

**Proposition 3.** A reduction in the Home import tariff leads to a decrease in the industry-level
average formal wage if the sufficient conditions (S1)-(S3) and the additional sufficient condition (S5) hold.

\[(S5)0 > -\sigma \rho^\sigma \left( \frac{\varphi}{TW + \Theta} \right)^{\sigma - 1} \frac{\partial \Psi(\varphi, W)}{\partial W} \]

**Proof** Notice that the derivative of the average formal wage with respect to Home import tariff is given by

\[
\frac{d\bar{W}_{for}}{d\tau} = \left( \frac{\partial \bar{W}_{for}}{\partial s_{open}} \frac{\partial s_{open}}{d\tau} + \frac{\partial \bar{W}_{for}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{d\tau} \right) dW_1
\]

The sign of \(\frac{d\bar{W}_{for}}{d\tau}\) is given by the sign of the term inside the parentheses because \(\frac{dW_1}{d\tau} > 0\).

\[
sgn\left(\frac{d\bar{W}_{for}}{d\tau}\right) = sgn\left(\frac{\partial \bar{w}_{for}}{\partial s_{open}} \frac{\partial s_{open}}{d\tau} - \bar{W}_{for} \frac{\partial \nu_{for}}{\partial s_{open}} \frac{\partial s_{open}}{d\tau} + \frac{\partial \bar{w}_{for}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{d\tau} - \bar{W}_{for} \frac{\partial \nu_{for}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{d\tau}\right)
\]

The sign of the above expression can be determined by comparing the average formal wage with the average wage of the formal firms lying on the curve in which the derivative is calculated, in this case \(s_{open}\) and \(s_{x,for}\). For the sign of curly bracket term to be positive, we need the average formal wage along \(s_{open}\) to be larger than \(\bar{W}_{for}\) and the average formal wage along \(s_{x,for}\) to be smaller than \(\bar{W}_{for}\). This is ensured by conditions (S1)-(S3) and (S5). Notice that condition (S5) is more restrictive than condition (S4). The intuition for conditions (S2) and (S3) is the same from Proposition 2. Condition (S5) means that for a fixed productivity level the decrease rate of the marginal density of the wage is faster than the rate of decrease in employment level due to higher wages. As a result, the average formal wage along \(s_{open}\) is larger than the average formal wage, which by its turn is larger than the average formal wage along \(s_{x,for}\).
**Proposition 4.** A reduction in the Foreign import tariff ($\tau^*$) decreases the share of informal workers in total industry employment if the sufficient conditions (S1)-(S4) hold.

**Proof.** In comparison with Proposition 2, the derivative of the share of informal workers has an additional term which accounts for the change in the formal employment by the current exporters.

$$\frac{d\text{share}}{d\tau^*} = \frac{\partial \text{share}}{\partial s_{open}} \frac{\partial s_{open}}{d\tau^*} + \frac{\partial \text{share}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{d\tau^*} + \frac{\partial \text{share}}{d\tau^*}$$

A decrease in the Foreign import tariff leads to an expansion of the formal employment of the current exporters, hence $\frac{\partial \text{share}}{d\tau^*} > 0$. Given that $\frac{\partial s_{open}}{d\tau^*} < 0$ and $\frac{\partial s_{x,for}}{d\tau^*} > 0$, the ambiguity is similar to the one discussed in Proposition 2. So, given condition (S2), Lemma 2 can be applied to arrive at $|\frac{\partial s_{x,for}}{d\tau^*}| > |\frac{\partial s_{open}}{d\tau^*}|$. Then, the rest of the proof is similar to the one from Proposition 2. ■

**Proposition 5:** A reduction in the Foreign import tariff leads to an increase in the industry-level average formal wage if the conditions (S1)-(S3) and (S5) hold.

**Proof.** In comparison with Proposition 3, the derivative of the average formal wage has an additional term which accounts for the change in the average formal wage due to changes in the employment level of the current exporters.

$$\frac{d\bar{w}_{for}}{d\tau^*} = \left(\frac{d\bar{w}_{for}}{d\tau^*} + \frac{\partial \bar{w}_{for}}{\partial s_{open}} \frac{\partial s_{open}}{d\tau^*} + \frac{\partial \bar{w}_{for}}{\partial s_{x,for}} \frac{\partial s_{x,for}}{d\tau^*}\right) \frac{dW_1}{d\tau^*}$$

The sign of $\frac{d\bar{w}_{for}}{d\tau^*}$ is given by the opposite sign of the term inside the curly brackets since
Thus the term inside the curly bracket has to be positive.

\[
\text{sgn}\left\{ \frac{d \bar{w}_{\text{for}}}{d \tau^*} \right\} = -\text{sgn}\left\{ \frac{\partial \text{tpwp}_{\text{for}}}{\partial \tau^*} \bar{w}_{\text{for}} + \frac{\partial \text{L}^V_{\text{for}}}{\partial \tau^*} \frac{\partial s_{\text{open}}}{\partial \tau^*} - \bar{w}_{\text{for}} \frac{\partial \text{L}^V_{\text{for}}}{\partial s_{\text{open}}} \frac{\partial s_{\text{open}}}{\partial \tau^*} \right\}
\]

\[+ \frac{\partial \text{tpwp}_{\text{for}}}{\partial s_{x,\text{for}}} \frac{\partial s_{x,\text{for}}}{\partial \tau^*} - \bar{w}_{\text{for}} \frac{\partial \text{L}^V_{\text{for}}}{\partial s_{x,\text{for}}} \frac{\partial s_{x,\text{for}}}{\partial \tau} \}
\]

The reasoning behind the proof here is similar to the reasoning used in Proposition 3. But now, condition (S2) allows the use of Lemma 2. As in Proposition 3, conditions (S3) and (S5) and Lemma 2 ensure that the sign of the last four terms of the curly bracket is positive. Now, we also need to have a positive sign for the first two terms, so the sign of the term in parentheses in equation (23) is positive. Notice that \( \frac{\partial \text{tpwp}_{\text{for}}}{\partial \tau^*} < 0 \) and \( \frac{\partial \text{L}^V_{\text{for}}}{\partial \tau^*} < 0 \), consequently the first two terms will be positive if the average formal wage of the exporters were smaller than the overall average formal wage. And this happens whenever conditions (S3) and (S5) hold. Thus, \( \frac{d \bar{w}_{\text{for}}}{d \tau^*} < 0 \), i.e. \( \bar{w}_{\text{for}} \) increases with a decrease in \( \tau^* \).

A2. Case of Two Large asymmetric economies

Remember that Foreign is different from Home only in terms of the population. That said, I present first the Foreign economy equations, and then I discuss the general equilibrium equations presented before that need to be changed.

The demand faced by Foreign producers when they supply Foreign market is given by equation (25), and the demand faced by Foreign producers at Home market is given by equation (26).

\[
q_{l,d}^* = \left[ \frac{p_{l,d}^*}{p^*} \right]^{-\sigma} Q^*
\]

(25)
\[ q^*_i = \left[ \frac{p^*_i}{p} \right]^{-\sigma} Q \]  

(26)

The Foreign firm zero cutoff profit (ZCP) for entry is shown in equation (27). Notice that from the ratio between equation (27) and equation (10), we obtain equation (28).

\[ \pi^v_{d}(s^*_{open}) = \sigma^{-1}(\rho P^* s^*_{open})^{\sigma^{-1}} Q^* = W_1 f \]  

(27)

\[ s^*_{x,for} = \tau^* s^*_{open}(W_1 f_x/W_1 f)^{1/\sigma^{-1}} \]  

(28)

The free-entry condition for Foreign is depicted by equation (29).

\[ J(s^*_{open})f + J(s^*_{x,for})f_x = \delta f_e \]  

(29)

The labor market clearing condition, the price index for Foreign, the unemployment equation, the expected wage distortion, and the government transfer equations are analogous to the ones presented earlier for the Home economy. The equations that need to be changed are the Home firms zero cutoff profit for exporting, and the Trade Balance equation, which are equations (30) and (31) respectively.

\[ \pi^v_x(s^*_{x,for}) = \sigma^{-1}(\rho P^* s^*_{x,for}/\tau^*)^{\sigma^{-1}} Q^* = W_1 f_x \]  

(30)

\[ \frac{M(W_1 s^*_{x,for})}{1-G(s^*_{x,for})} \int s^*_{x,for} \left( \frac{\rho P^* (W_1 s^*_{x,for}) s^*}{\tau^*} \right)^{\sigma^{-1}} Q^*(W_1, s^*_{x,for}) dG(s) = \]  

(31)

\[ \frac{M^*(W_1 s^*_{x,for})}{1-G^*(s^*_{x,for}(W_1 s^*_{x,for}))} \int s^*_{x,for} \left[ \frac{\rho P(W_1 s^*_{x,for}) s^*}{\tau^*} \right]^{\sigma^{-1}} Q(W_1, s^*_{x,for}) dG^*(s) \]

By recursive substitution, the general equilibrium depends only on \( W_1 \) and \( s_{x,for} \), so the two equations needed are (9) and (31). And from Demidova and Rodriguez-Clare (2011) Lemma 1, there exists one equilibrium and it is unique.
A.3 Appendix for the Referees

A.3.1 Efficiency wage problem with government transfers

The flow value of a non-shirker worker employed at firm is given by equation (32), where \( w_i \equiv \frac{W_i}{p} \).

\[
rV^N_{E_i} = \frac{w_i}{e} + \frac{tr}{p} + \delta(V_u - V^N_{E_i})
\]

(32)

And the flow value of a shirker worker employed at firm \( i \) is

\[
rV^s_{E_i} = w_i + \frac{tr}{p} + (\delta + m_i)(V_u - V^s_{E_i})
\]

Let’s impose the non-shirking condition, \( V^N_{E_i} = V^s_{E_i} = V_{E_i} \).

\[
\frac{1}{r + \delta} \left[ \frac{w_i}{e} + \frac{tr}{p} + \delta V_u \right] = \frac{1}{r + \delta + m_i} \left[ w_i + \frac{tr}{p} + (\delta + m_i)V_u \right]
\]

Next, I solve for \( rV_U \), and I obtain

\[
rV_U = \frac{w_i}{m_i} \left[ \frac{(1 - e)(r + \delta) + m_i}{e} \right] + \frac{tr}{p}
\]

Let \( \bar{m}_i \equiv \frac{m_i - e(1+r+\delta)}{e} \). Isolating \( w_i \) in the previous equation leads to

\[
w_i = \frac{rV_U}{\bar{m}_i} - \frac{tr}{\bar{m}_i p}
\]

(33)

Now, let’s calculate \( V_{E_i} - V_U \),

\[
(r + \delta + m_i)[V_{E_i} - V_U] = -rV_U + \frac{W_i}{p} + \frac{tr}{p}
\]

Plugging \( rV_U \) in the above equation, I obtain

\[
V_{E_i} - V_U = \frac{W_i}{m_i p} \left[ \frac{e - 1}{e} \right]
\]

For any firms with different \( i \)’s, using equation (33) I construct the wage ratios as shown below.

\[
\frac{w_A}{w_B} = \frac{\bar{m}_B}{\bar{m}_A} = \frac{W_A}{W_B}
\]

Let’s the wage paid by the firm with monitoring ability \( \bar{m} \) be \( W \), then
\[ W_i = W_1 \left( \frac{m_i}{\bar{m}} \right) > W_1, \text{ and } \frac{\partial W_i}{\partial m_i} = -\frac{(e-1)(r+\delta)}{m - (e-1)(r+\delta)} < 0. \]

It is now time to aggregate firm level variables in order to solve for the unemployment level. Given the wage density is common knowledge, we can calculate

\[ V_E = E[V_{E_i}] = \left[ \frac{e-1}{e} \right] p^{-1} E \left[ \frac{W_i}{m_i} \right] + V_U \]

The flow value of being unemployed is given by

\[ rV_U = b(V_E - V_U) + \frac{tr}{p} \]

\[ rV_U = b \left( \frac{e-1}{e} p^{-1} E \left[ \frac{W_i}{m_i} \right] \right) + \frac{tr}{p} \]

In equilibrium, unemployment inflow equal unemployment outflow, then

\[ bU = \delta(L - U), u \equiv U/L \]

\[ b = \delta \left( \frac{1-u}{u} \right) \]

Where \( b \) is the unemployment escape rate.

\[ rV_U = \delta \left( \frac{1-u}{u} \right) \left( \frac{e-1}{e} p^{-1} E \left[ \frac{W_i}{m_i} \right] \right) + \frac{tr}{p} \]

Plugging \( rV_U \) into the wage equation, we obtain for firm \( i \),

\[ W_i = \frac{\delta}{\bar{m}} \left( \frac{1-u}{u} \right) \left( \frac{e-1}{e} \right) E \left[ \frac{W_i}{m_i} \right] \]

Then for the firm with monitoring ability \( \bar{m} \), we have

\[ u = \left[ \frac{B_1 E \left[ \frac{W_i}{m_i} \right]}{W_1 + B_1 E \left[ \frac{W_i}{m_i} \right]} \right], \text{ where } B_1 = \frac{\delta(e-1)}{em_1} \text{ and } \bar{m}_1 = \frac{m - (e-1)(r+\delta)}{em} \]

Notice that Davis and Harrigan (2011) let \( r \) go to zero, as in Melitz (2003).