Innovations spread more like wildfires than like infections

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Abstract

Conventional theory says that innovations first diffuse slowly, then at faster paces, and finally at asymptotically declining rates. Economists and others explain such behavior with a variety of logistic models. Early models like the contagion model derive their predictive power from reliance on the history of the variables they are trying to predict. New social learning models improve the dynamics of diffusion across heterogeneous populations, while other studies propose various modifications. However, these extensions of the logistic and related models are still too orderly in structure and outcome. In reality one can expect both order from disorder and disorder from order. The argument of this paper is that innovations spread more like wild fire than like systematic epidemics. This analogy is no mere conjecture; some environments are more susceptible to catching fire than others. Just as the rate of the spread of fire is a function of fuel and other factors, so too is the spread of innovations, only that in the latter case the fuel is human population. Human population in general is a necessary fodder for the spread of innovations. The sufficient condition is the quality of the population which can favor or disfavor the spread of innovations, which explains why there are some random chances of finding islands untouched by fire surrounded by a sea of fire devastation.

Keywords: Innovation spread, logistic model, derivative Gompertz, diffusion of innovations

JEL Code: O31, D8, M3, Z00

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Abstract

Conventional theory says that innovations first diffuse slowly, then at faster paces, and finally at asymptotically declining rates. Economists and others explain such behavior with a variety of logistic models. Early models like the contagion model derive their predictive power from reliance on the history of the variables they are trying to predict. New social learning models improve the dynamics of diffusion across heterogeneous populations, while other studies propose various modifications. However, these extensions of the logistic and related models are still too orderly in structure and outcome. In reality one can expect both order from disorder and disorder from order. The argument of this paper is that innovations spread more like wild fire than like systematic epidemics. This analogy is no mere conjecture; some environments are more susceptible to catching fire than others. Just as the rate of the spread of fire is a function of fuel and other factors, so too is the spread of innovations, only that in the latter case the fuel is human population. Human population in general is a necessary fodder for the spread of innovations. The sufficient condition is the quality of the population which can favor or disfavor the spread of innovations, which explains why there are some random chances of finding islands untouched by fire surrounded by a sea of fire devastation.

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1. Introduction

Conventional theory almost always suggests that the spread of innovation follows a logistic distribution. In the beginning there are few adopters; innovations pick up speed as the number of adopters increase; and finally the diffusion rate falls subject to capacity constraints. This theory draws mainly from traditional population growth models interspersed with epidemiological insights.

Gupta, Sharma, and Karisiddappa (1997), for example, examine six conventional diffusion models using publications data on science and technology. While all six models describe an S-curve phenomenon, the authors "... observed that [except for the modified exponential-logistic model] the models were not able to capture the fluctuations in the growth data" (p. 526). Recently H. Peyton Young (2004, 2005, 2007) has injected life in the modeling process by introducing a novel dimension: social learning. In this case diffusion rates depend not only on the conventional rates of change of the Velhurst-Pearl-Reed variety, but also on heterogeneous thresholds associated with payoff matrices that give incentives for, disincentives against, or even discontinuation of, innovations (cf., Rogers,1983). This approach clearly advances understanding of the spread of innovations and helps explain the differential spread of innovations across populations or segments within one population.

A troubling aspect of the logistic model of the spread of innovations, with or without social learning, is that the structural order of the model preordains outcomes. This is rather too artificial a result, because almost all models of innovations identify four elements of how [and why] innovations spread:

the innovation itself, the channels through which the innovation is communicated, the time over which the innovation is communicated, and the social system that surrounds it all (Rogers, 1983). Although less quantifiable than the first three elements, the social system has the ability to constrain or promote both the innovation and the channels of communication, and time. How quickly and successfully adoption occurs and whether a particular innovation takes-off or not, bears directly on the social system, and the dynamic nature of the social system suggests that innovations spread more like wild fire than like systematic contagions, with or without "heterogeneous thresholds".

The objective of this note is to model that hypothesis by incorporating into the logistic model the wild fire analogy. Section 2 describes the conventional logistic model, and its Young extension. The first subsection of Section 3 shows the applicability of a basic fire spread model to innovation spread models. The second subsection of the same modifies the logistic model in a manner consistent with the fire spread model. It turns out that the fire spread component of the model is just a human capital dimension in the innovation spread model. Section 4 describes the (least squares) estimation of the modified logistic model, while Section 5 outlines the distributions that are consistent with the logistic model. Without denying either the relevance of conventional logistic models or the importance of social learning, the paper concludes that social learning makes the spread of innovations less predictable. The reason is that innovations spread more like wild fire than like systematic contagions, with or without "heterogeneous thresholds", a situation that calls for more realistic modifications of the logistic model.

2. Background

What is the rate of spread of an innovation X, such as a new technology or product? How and why do diffusion rates vary across populations and even within one population? Conventional theory suggests that the spread of innovations follows a logistic model with time as one, if not the key, determinant. According to this model an innovation appears from some exogenous source. A few members of a population adopt the innovation. First the innovation catches on slowly, then faster and faster as more and more people convert to it. In time diminishing returns of a sort set in and the diffusion rate slows as it approaches its asymptote, leaving behind an S-shaped trace.

Formally: Assume we have a population with a maximum absorptive capacity of K(Z) at any time. Let these people be potential adopters of an innovation. Out of K(Z), N(Z) people are actual adopters of an innovation X at any time. These N(Z) people spread X across at least some K(Z). Assume that each Nth spreader spreads X to λ(Z) individuals, and some of λ(Z) become spreaders themselves. Over time the total number of spreaders (A(Z)) is

$$A(Z) = \lambda(Z) N(Z), \tag{1}$$

where Z is a multidimensional matrix of variables, including time, *t*, that determine the spread of an innovation. Given (1) the proportion of K(Z) that is not N(Z) is $[K(Z) - N(Z)] / K(Z)$. The number of new adopters (A^*(Z) is

$$A^*(Z) \equiv dN/dZ = \lambda N(K-N) = \theta NK - \theta N^2, \quad \theta = \lambda/K. \tag{2}$$

From (2) $dA^* / dN = \Theta K - 2\Theta N = 0 \Rightarrow K/2.$ Since $dN / dZ = \Theta NK - \Theta N^2$, $dN = [\Theta NK - \Theta N^2]dZ$ such that

$$N(Z) = \int A^*(Z)dZ = \frac{K}{\alpha + \beta e^{-\gamma Z}} = \frac{1}{1+\beta e^{-\gamma Z}}, \tag{3}$$

for K =1, α = 1 and $\beta = -1 + 1/N(Z=0)$. Thus, (3) is a simple contagion version of the conventional Velhurst-Pearl-Reed logistic model (seeYoung, 2004, 2005, 2007). It assumes that all λ(Z) individuals who learned about X for N(Z) convert at an instantaneous rate of γ. The Bass model on the other hand generalizes (3) by assuming that conversion rates differ depending upon whether learning is from a familiar (internal) source or it is from an unfamiliar (external) source. Following Young let $\gamma_1 > 0$ be the conversion rate when learning is internal and $\gamma_2 > 0$ when learning is from an external source. Then (3) can be restated as

$$N(Z) = \frac{[1 - \phi\gamma_2 e^{-(\gamma_1+\gamma_2)Z}]}{[1 + \phi\gamma_2 e^{-(\gamma_1+\gamma_2)Z}]}, \quad \gamma_i > 0, \quad \phi > 0. \tag{4}$$

Young then shows that for N(Z=0), $\phi = 1/\gamma_2$ and (4) turns into

$$N(Z) = \frac{1-e^{-\gamma * Z}}{\pi e^{-\gamma * Z}}, \quad \gamma * = \gamma_1 + \gamma_2, \quad \pi = (\gamma_2 + \gamma_1\gamma_2)/\gamma_2. \tag{5}$$

Eq. (5) = (4) = (3) if $\gamma2 = 0$, $\gamma_1 = \gamma > 0$, K = 1, and α = 1. When $\gamma_1 = \gamma = 0$, and $\gamma_1 = \gamma_2 > 0$, $N(Z) = 1 - \exp(-\gamma Z)$, suggesting that everyone who could learn had learned, but the adoption rate has slowed down a lot, or it is taking place with a significant time lag. At this point Young introduces “social learning with heterogenous thresholds”, so that the rate of the spread of an innovation is subject to some constraints like resistance, friction, and uncertainty. The introduction of thresholds specifically challenges the fundamental assumption of the simple contagion model “that people adopt an innovation simply because they heard about its existence” (p.9). Such barriers are lower the higher the penalty for not converting, or similarly the higher the payoff for converting. Thus, the rate of spread is either reduced or accelerated by “heterogeneous thresholds”, which is Young’s Equation (14, p. 13), and can be restated as

$$N(Z)=\frac{\nu F(N(Z))-N(Z+1)}{\nu+1}. \qquad (6)$$

Eq. (6) approximates the number of individuals, given today's payoff, tat can be expected to overcome their resistance and convert tomorrow. This is a simplification of Young, of course.

3. Innovations spread like wild fire

This section consists of two parts. The first part next below shows the applicability of a basic fire spread model to innovation spread models. The second subsection modifies the logistic model in a manner consistent with the fire spread model.

3.1. Fire versus innovations - an analogy

Young's formulation shows that N(Z) depends on both the conventional rate γ and some resistance factor. The formulation is understandable because innovations are human creations, and as such they depend on human population whose numbers and behaviors are difficult to predict. While an innovation may be governed by logistic rules in some stages of its spread, other stages may be dynamic, perhaps even chaotic. A close view of Young's and Gupta, Sharma, and Kirisiddappa's graphical fits of diffusion models shows that they miss the data in the middle of the S-curve. Therefore, it is not unreasonable to claim that innovations spread like wild fire. In fact, incorporating the wild fire phenomenon into the logistic models is like incorporating uncertainty, instead of risk, in Young's model. As Robert Frank (2007) explains in *Black Swans*, uncertain matrices almost always have relevant, albeit unknown and unpredictable, information content. They are like wildfires.

In its basic form a fire spread model relates physical and behavioral properties of the fire to its environment, pre-ignition and during the burning process. Although there are many and increasingly sophisticated models of fire spread, almost all of them are based on Rothermel (1972). The Rothermel model states the fire spread (R) as the energy intensity of the reaction of fire (Ω), adjusted for factors that favor or hinder the spread of fire such as wind and slope (ψ), to fuel (U), i.e.,

$$R=\Omega(1+\psi)U^{-1}. \qquad (7)$$

Eq.(8) is basically a fire-over-fuel model. The question then becomes: How do we make fire properties consistent with economic properties of the spread of an innovation? The answer turns out surprisingly simple. With respect to innovations R is equivalent to the reaction intensity of a population to an innovation as described by Rogers (1983). Since a population's reaction to an innovation depends on the expected benefit/cost ratio, R acts like a productivity shifter. It is the expected intensity of use (X/N), where X is a measure of the innovation output like a product that embodies an innovation X. As such it reflects socio-economic factors that facilitate or hinder the adoption and spread of X: culture, politics, religion, and so on (Bulliet, 2002). Just like in the fire case, the spread of an innovation over a population (fuel) is an energy conservation problem.

3.2. Modified logistic model of the spread of innovations

Taking (7) into account we can adjust (5) such that

$$N^{*}(Z)=R\cdot N(Z)=[\frac{\Omega(1+\psi)}{U}][\frac{1-e^{\gamma^{*}Z}}{\pi e^{-\gamma^{*}Z}}] \tag{8.1}$$

Equivalently,

$$N^{*}(Z)=[\frac{\Omega(1+\psi)}{U}][\frac{1}{1+\beta e^{-\gamma Z}}]=[\frac{\Omega(1+\psi)}{U}][\frac{K}{\alpha+\beta e^{-\gamma Z}}] \tag{8.2}$$

Hence, $N^{*}(Z) \Leftrightarrow N(Z)\ \forall R= 1$. But in the standard model such as (3) K/α is the upper limit and $K/(\alpha+\beta) = N(Z=0)$. According to (8) the upper limit is $RK/(\alpha+\beta) = N(Z=0)$, and $RK/(\alpha+\beta) < K/\alpha$ for $0 < R < 1$.

Clearly R simply refines N(Z) for quality, and is consistent with T.W. Schultz's (1981) concept of "quality population" or human capital (cf. Becker, 1993) . From Jones (1997), among many, human capital (H(Z)) is

$$R\equiv H(Z)=e^{\eta q}L(Z), \tag{9}$$

where η is the human capitalization rate (appreciation/depreciation rate), and q include factors such as years of schooling, literacy rate, or even health indicators like life expectancy. However, although a reasonable proxy for human capital in production, L(Z) is too narrow a basis for H(Z) in characterizing the spread of an innovation. Children can spread innovations to other children and to their own parents. In this case both the past and the present depend on the future instead of the usual case in which the past determines the future; advertisers and marketing people understand this fact too well. Those involved in the production of a new technology may not necessarily be its first adopters. Many scientists and engineers who built the first atomic bomb had in fact opposed its adoption long before and after it was used against Japan in WWII. Thus, a broader basis of H(Z) is needed and we call that "quality (as in grade) population" (G(Z)) so that

$$H(Z)=e^{\eta q}G(Z). \tag{10}$$

Hence,

$$N^{**}(Z)=[H(Z)N^{*}(Z)][\frac{1-e^{-\gamma^{*}Z}}{\pi e^{\gamma^{*}Z}}]. \tag{11}$$

Eq.(11) is better understood in terms of (3) or (4) by which

$$N^{**}(Z)=[e^{\eta q}G(Z)][\frac{K}{\alpha+\beta e^{-\gamma Z}}]=\frac{H(Z)K}{\alpha+\beta e^{-\gamma Z}}. \tag{12}$$

4. Estimating an H-modified logistic model of the spread of innovation

Let $K^{*} \equiv H(Z)K$, and restate (12) as

$$N^{**}(Z)=K^{*}(\alpha+\beta e^{-\gamma Z})^{-1}. \tag{12'}$$

Then,

$$dN^{**}(Z)/dZ=-K^{*}(\alpha+\beta e^{-\gamma Z})^{-2}(-\gamma\beta e^{-\gamma Z}) \tag{13}$$

Simplification of (13) shows that $-K^{*}(\alpha+\beta e^{-\gamma Z})^{-2}=-(N^{**}(Z))^{2}/K^{*}$. Then from (12)

$$K^{*}=\alpha N^{**}(Z)+\beta N^{**}(Z)e^{-\gamma Z}\Rightarrow\beta e^{-\gamma Z}=K^{*}/N^{**}(Z)-\alpha. \tag{13'}$$

Eq. (13') suggests that

$$-\gamma\beta e^{-\gamma}=-\gamma(\frac{K^{*}}{N^{**}(Z)}-\alpha). \tag{14}$$

Plugging $-(N^{**}(Z))^{2}/K^{*}$ and $-\gamma(K^{*}/N^{**}(Z)-\alpha)$ into (14) and simplifying leads to

$$dN^{**}(Z)/dZ = \frac{\gamma N^{**}(Z)K^{*} - \alpha\gamma(N^{**}(Z))^{2}}{K^{*}}. \tag{15}$$

Given K^* in (14) $\alpha = (K^* - \beta N^{**}(Z)e^{-\gamma Z}/N^{**}(Z)$ and $\beta = (K^* - \alpha N^{**}(Z))/N^{**}(Z)e^{-\gamma Z}$.

But $K^* \equiv H(Z)\,K$, implying that

$$K^{*} = \frac{[N^{**}(Z)(\alpha + \beta e^{-\gamma Z})]}{e^{\eta q G(Z)}}. \tag{16}$$

Thus, in (12)

$$N^{**}(Z) = \frac{N^{**}(Z)(\alpha - \beta e^{-\gamma Z}}{\alpha + \beta e^{-\gamma Z}} \approx (12) \tag{17}$$

As one useful digression, we can also set H(Z) = α ≡ *I*, for "*I*" called "initial conditions" (Masterton-Gibbons, 1995). Then it becomes clear that

$$N^{**}(Z) = \frac{IK}{I + (K - I)e^{-\gamma Z}},\; K - I = \beta, \tag{18}$$

which is just a restatement of (12). Either (12) or (18) is a Markov property of the spread of innovation. It says that the spread of innovations depend only on the current activity, and not on history - a "memoryless" process, to use Masterton-Gibbons words.

5. The distribution of the H-modified logistic model of the spread of innovation

Our discussion so far has assumed a logistic distribution without being specific. This is not maintainable because while N(Z) may follow a logistic distribution, H(Z) may follow a different distribution such that the product of the two (H(Z)N(Z)), let us call that n(z), may follow yet another distribution. Assume n(z) is lognormally distributed such that

$$n(z)=\frac{K^*}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\ln z)^2+\frac{\mu}{\sigma^2}\ln z-\frac{\mu^2}{2\sigma^2}} \tag{19}$$

For simplicity set $\frac{K^*}{\sigma\sqrt{2\pi}e^{(\mu/2\sigma)^2}}=a_0,\quad 1/2\sigma^2=a_1,\quad \mu^2/2\sigma^2=a_2$. Then

$$\ln n(z)=a_0+a_1(lnz)^2+a_2\ln z\Rightarrow dlnn(z)/dlnz=a_1lnz+a_2. \tag{19'}$$

In addition, $K^* = KH$, so that lognormally

$$n(z)=e^{\eta q}G(Z)\frac{K}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\ln z)^2+\frac{\mu}{\sigma^2}\ln z-\frac{\mu^2}{2\sigma^2}} \tag{20}$$

Again for $\frac{K}{\sigma\sqrt{2\pi}e^{(\mu/2\sigma)^2}}=a_0^*,\quad 1/2\sigma^2=a_1^*\quad \mu^2/2\sigma^2=a_2^*$,

$$\ln n^*(z)=a_0^*+a_1^*(lnz)^2+a_2^*\ln z+\eta qG(z)\Rightarrow dlnn^*(z)/dlnz=a_1^*lnz+a_2^*+d/d\ln(z)(\eta qG(z)+\eta qG'(z)). \tag{20'}$$

One reason why Young (ib.) stresses a free-distribution of the F(.) of his model is that a logistic spread of innovation is similar to the Gompertz (Harris 1992, Thompson, 1998). A Gompertz is a model of form

$$n^*(z)=e^{\alpha-\beta\gamma^Z}. \tag{21}$$

Consequently, the derivative of (21) is

$$dn^*(z)/d\ln z=-\beta\ln\gamma\gamma^z e^{\alpha-\beta\gamma^z}=-\beta\ln\gamma\gamma^z n^*(z) \tag{21'}$$

The logistic and Gompertz models belong to the family of modified exponential model (Harris, 1992, Young, 2004, 2005, 2007). For example, if Y is any raw data, then the following data transformation shows the relationship between the Gompertz and logistic models, and the association of both to the modified exponential model (Harris, 1992, Barnerjee, 200x, Koch and Lind, 1971, Gupta, Sharma, Kirisiddappa, 1997), i.e.,

$$n^{**}(z)=K^{*}/Y=\frac{K^{*}}{\alpha-\beta\gamma^{z}};\quad lnY=\ln n^{*}(z)=\alpha-\beta\gamma^{z}. \tag{22}$$

Eq.(22) implies that

$$n^{**}(z)=\frac{K^{*}}{\ln Y}=\frac{K^{*}}{\ln(e^{\alpha-b\gamma^{z}})}. \tag{23}$$

For H(Z)K ≡ K*, the logistic model in terms of a Gompertz is

$$n^{**}(z)=\frac{e^{\eta q}G(z)K}{\ln(e^{\alpha-b\gamma^{z}})}. \tag{23'}$$

Since $\ln n^{**}(z)=C+\eta q-\ln(\alpha-\beta\gamma^{z}),\quad C=G(z)-\ln K,$ one can show that

$$n^{**}(z)=-\ln(\alpha-\beta\gamma^{z})=\frac{1}{\alpha-\beta\gamma^{z}}\Rightarrow n^{**}(z)/dz=(16),\ K=1. \tag{23''}$$

6. Functional form of H(Z)

The discussion so far assumes that H(Z) is linear in G(Z) so that η is a constant. There is no reason not to think that H(Z) can be raised to some power, say ρ. Below are four possible forms:

$$\begin{aligned} &n^{**}(z)=H^{\rho}K/[\ln(e^{\alpha-b\gamma^{z}})],\quad n^{**}(z)=K/[\ln(H^{\rho}e^{\alpha-b\gamma^{z}})],\\ &n^{**}(z)=K/[H^{\rho}\ln(e^{\alpha-b\gamma^{z}})],\quad n^{**}(z)=K/[\ln(e^{\alpha-b\gamma^{(z+H)}})]\bullet \end{aligned} \tag{24}$$

Unfortunately pursuing these does not add any more to clarity than leaving them unattended.

7. Concluding Remark

Appropriately modified logistic models continue to be useful tools for describing the spread of innovations. Generally logistic regression techniques such as the Logit technique, are nearly indispensable in assessing the loglikelihood of some economic event. Where repeated sampling is possible and learning processes can be modeled, logistic models have few competitors in predicting probable behaviors of economic variables. Their flexible lognormal distributions give them an additional benefit that captures varying rates of innovation spreads. Moreover, a logistic model can be transformed into a Gompertz (Harris, 1992), and into a modified exponential model of which both are family members.

However, the benefit of flexibility is often purchased with too strong a currency, so to speak, in terms of the assumption about the tight structure that permits diffusion to proceed first at a slow rate, then at a fast rate, and eventually at an asymptotically declining rate. Diffusion has no end?

In reality the spread of an innovation is more complex than a simple logistic model predicts. A lot of institutional constraints and prospects are involved. Characterizing the constraints and prospects are underlying individual behavioral interactions of many infrastructural and superstructural elements. The interactions determine the spread of innovations in dynamic, perhaps even chaotic, ways. In this paper the argument is that innovations spread more like wild fire than like systematic epidemics. This is no wild analogy; some environments are just more susceptible to catching fire than others. Just as the rate of the spread of fire is a function of fuel and other factors, so too is the spread of innovation, only that in the latter case the fuel is human population. Human population is a necessary fodder for the spread of innovation. The sufficient condition is the quality of the population which can favor or disfavor the spread of innovation, which explains why there are some random chances of finding islands untouched by fire surrounded by a sea of fire devastation.

Social learning can speed up or slow down the spread of innovations depending on how the population perceives Young's benefit/cost (payoff) matrix. However, social learning makes the spread of innovations less predictable. This calls for more realistic modifications to the logistic model. The rest of this papere attempts to do just that. Obviously without its empirical component, a lot remains to be done.

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