Economic growth and the optimum size of government in 15 European countries: A threshold panel approach

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Abstract

In the growth literature, there is a nonlinear relationship between economic growth and government size, which is similar to an inverted U-shaped curve. This curve can be used to determine the optimum share of government expenditures. This paper, using threshold panel approach, attempts to investigate this nonlinear effect for 15 European countries, empirically. For the size of government, four measures are considered as follows: (i) total expenditures to gross domestic product, (ii) final consumption expenditures to gross domestic product, (iii) current expenditures other than final consumption to gross domestic product and (iv) government gross fixed capital formation to gross domestic product. Estimation results show that the inverted U-shaped curve is approved for four measures. The estimated optimum shares are 41.7\%, 15.8\%, 19.4\% and 2.5\%, respectively.

Keywords: Economic growth; Government expenditures; Government size; Threshold panel; Bootstrap procedure; European countries

\textbf{JEL:} C23; H50; H54; O40; O52

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1. Introduction

Do governments have to increase or decrease their costs for higher economic growth? Does changing the government cost structure affects economic growth? In this paper, these two questions are addressed. Some activities are due to government and it can improve other economic activities classified in the following two categories: (i) Government must have a role in fields like legislation, security of property rights and providing proper environment for investment and production through decreasing transaction costs. (ii) Government must interfere in other fields sometimes belong to the private sector for playing roles and/or giving services. Infrastructures, human force enhancement, public health, and education are some examples. One reason is that the private sector cannot carry out such duties sufficiently and/or for the total society.

In general, government can ameliorate economic activity and growth through entering sectors in which the market mechanism fails or when it has comparative advantage in comparison with the private sector. However, these interferences have costs as below: (i) Government does not have a "magic source" of money and has to afford its costs through borrowing and taxes. Receiving taxes from economic agents makes them discouraged. Borrowing also increases financial costs of investments, crowds out private investment and tax increase in future (Mitchell, 2005; Gwartney et al., 1998; Ram, 1986). (ii) Market forces bring economy to optimality through effective allocation of resources but political forces will prevail over market forces in allocating public and private resources thus efficiency decreases and less products are produced. Some government actions impose heavy costs on productive sectors because of having decision making and interference power. Government subsidy payments distort resource allocation as well (Mitchell, 2005; Ram, 1986). (iii) Centralization and bureaucratic activities decrease creativity in both public and private sectors. In the market mechanism, rewards and penalties of decisions are subject to wise choices directly, because it will appear in his or her wealth and properties very soon. In this mechanism, entrepreneurs are searching for new techniques and technologies and are spending their capital on goods and services with the utmost value, which causes wealth creation and economic growth. Thus the economic growth is a discovery process that is advanced by market mechanism. Government interference lessens market mechanism and private decision making which decreases economic growth. For example, the consequences of non-optimal investments of governmental do not appear rapidly or if they appear, decision makers are not punished, so the discovery process of growth is disturbed or made slow (Mitchell, 2005; Gwartney et al., 1998; Ram, 1986).
Therefore, Activities of government have both positive and negative effects on economic growth. On one hand, it increases economic growth by providing a proper environment for private activities, legislation regarding private possession and its guarantee, building infrastructures and public goods. On the other hand, it decreases economic growth through borrowing and taxation policies, decreasing creativity and increasing inefficiency, in which the intensity of negative effects depends on the amount and how to spend this money (types of expenditures). Thus, the final effect of government expenditures depends on the kind of government expenditures (protection of property rights, subsidies, infrastructures, etc.) and its positive or negative effects on economic growth.

Ghali (1998), Loizides and Vamvoukas (2005) and Wu et al. (2010) show that the government size is the Granger cause of economic growth. Guseh (1997), Gwartney et al. (1998), Fölster and Henrekson (2001) and Dar and Amirkhalkhali (2002) show that the effect of the government size on economic growth is negative. Against, theoretical (Barro, 1990; Mourmouras and Lee, 1999; Kosempel, 2004) and empirical studies (Ram, 1986; Barro, 1991; Vedder and Gallaway, 1998; Chiou-Wei and et al., 2010) show that in the lower levels of government activities, the effect of government expenditures is positive but it is reversed if it increases, which is shown with an inverted U-shaped curve. This curve can be used to determine the optimum share of government expenditures.

In the mentioned empirical studies, simple linear methods in the form of quadratic function are used to test non-linear relationship between economic growth and the government size. In this paper, threshold panel approach was used to detect linear and non-linear effects of the government size, which leads to more reliable results. In addition, the present study considered four measures of government expenditures rather than one. They are total government expenditures, final consumption expenditures, current expenditures other than final consumption and fixed investment expenditures. Then the non-linear effect for all of the four measures was considered.

In parts 2 and 3, theoretical and empirical studies are expressed, respectively. Data and variables are presented in part 4. In part 5, linear estimations of the effect of government size on economic growth are discussed. In parts 6 and 7, the non-linear method of threshold and its estimation results are presented, respectively. In the final part, conclusion and recommendations for future research are offered.
2. Theoretical literature

In the growth literature, Barro (1990, 1991) enters the public sector in a simple model of endogenous growth. He assumes that government tax incomes spent on providing public services, so that all producers have the same share, and there is no congest effects. Considering that government spending enter the representative firm production function, Barro’s production function is given by following AK form:

\[ Y(t) = AK(t)^\alpha G(t)^{1-\alpha}, \quad 0 < \alpha < 1, \]  

(1)

and utility function takes the constant intertemporal elasticity of substitution (CIES) form:

\[ u[c(t)] = \begin{cases} 
\frac{c(t)^{1-\sigma}}{1-\sigma}, & \sigma > 0, \sigma \neq 1, \\
\ln c(t), & \sigma = 1.
\end{cases} \]  

(2)

Equilibrium condition is \( Y(t) \equiv C(t) + I(t) + G(t) \). Based on the solution of infinite horizon Ramsey model, steady state growth rate is determined by:

\[ \frac{\dot{Y}(t)}{Y(t)} = \sigma^{-1} \left[ \alpha (1-\tau_G) A^{1/\alpha} \tau_G^{(1-\alpha)/\alpha} - \delta - \rho \right]. \]  

(3)

Therefore, government size has positive and negative bi-effects on growth rate. \( \alpha \) is between zero and one, then by increasing the size of government, its positive effect on growth rate is decreased. Barro (1988) concludes that "The economy's growth rate and saving rate initially rise with the ratio of productive government expenditures to GNP, \( g/y \), but each rate eventually reaches a peak and subsequently declines" (Fig. 1). Its graph became known as Barro curve, and it is a basis for government expenditures optimum determination. Mourmouras and Lee (1999) expanded Barro model by combining Barro production function and Blanchard overlapping generations model (Barro, 1990, 1991; Blanchard, 1985). They considered consumer with limited life horizon and logarithmic utility function, \( U(i,t) = \int_t^\infty \text{LnC}(i,t) e^{-\rho + \lambda} \text{d}v \), and in a more general model concluded that Barro’s curve is confirmed.
Kosempel (2004) expanded Mourmouras and Lee’s model by assuming the case in which government spends its expenditures in two ways: (i) Government allocates part of its expenditures for providing free services (x) like health, parks, museums, and art exhibitions that directly enter consumer’s utility function. Utility function takes the following CIES form:

\[
\begin{align*}
\dot{u}[c(i, t), x(i, t)] &= \begin{cases} 
  (1 - \beta) c(i, t)^{1-\sigma} + \beta x(i, t) - 1, & \sigma > 0, \sigma \neq 1, \\
  (1 - \beta) \ln c(i, t) + \beta \ln x(i, t), & \sigma = 1.
\end{cases}
\end{align*}
\]  

(ii) The other part is spent on providing free services to producers like road construction, airports, railways, harbors, R&D, and human forces enhancement services that enter production function like Barro and Mourmouras and Lee models. He assumes that government incomes are derived from proportional taxes, and so \( X(t) + G(t) = \tau Y(t) \) or \( \tau X + \tau G = \tau \).

Finally, based on solving the optimization problems of consumer with finite horizon \( (U(i, t) = \int_t^\infty u[c(i, t), x(i, t)] e^{-(\rho + \lambda)(\nu - t)} dv) \) and firm with infinite horizon \( (\varphi(t) = \int_0^\infty [(1 - \tau) Y(t) - I(t) - W(t)] e^{-\int_0^t \tau(u) du} dv) \), the following equations are derived

\[
\frac{\dot{C}(t)}{C(t)} = \sigma^{-1} \left( \alpha(1 - \tau) A^{1/\alpha} \tau_G^{(1-\alpha)/\alpha} - \delta - \rho \right) - \\
\left( \lambda + \rho + (\sigma - 1)(\alpha(1 - \tau) A^{1/\alpha} \tau_G^{(1-\alpha)/\alpha} - \delta + \lambda) \lambda \bar{C}(t)^{-1} \right) \sigma^{-1},
\]

\[
\frac{\dot{K}(t)}{K(t)} = \frac{1}{\alpha} \left( \alpha(1 - \tau) A^{1/\alpha} \tau_G^{(1-\alpha)/\alpha} - \delta \right) - \delta - \bar{C}(t),
\]

where \( \bar{C}(t) \) is consumption-capital ratio and
\[
\frac{d[K(t)]}{d\tau_x} < 0, \quad \frac{d[C(t)]}{d\tau_x} < 0, \quad \frac{d[K(t)]}{d\tau_g} \geq 0, \quad \frac{d[C(t)]}{d\tau_g} \geq 0.
\]

The first two derivatives are negative, but the other ones are ambiguous. Kosempel (2004) indicated that the steady state growth rate is determined according to the following equation

\[
\frac{\ddot{Y}(t)}{Y(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{\ddot{K}(t)}{K(t)}
\]

(7)

Now the existence of Barro’s curve can be investigated in two cases: a) Assume that \(\tau_x\) and thereby \(\tau\) is increased. In this condition, according to Eq. (7), steady state growth rate is decreased. b) But assume that \(\tau_g\) and so \(\tau\) is increased. In this condition, Eqs. (5) to (7) imply that the steady state growth rate depends on \((1 - \alpha)(1 - \tau_x) - \tau_g\). Steady state growth rate is increased if this expression is positive, and is decreased if it is negative. By increasing \(\tau_g\), the expression \((1 - \alpha)(1 - \tau_x) - \tau_g\) will be negative if \(0 < \alpha < 1\). The latter expenditures follow Barro’s curve, while the former expenditures always have negative effects on economic growth.

3. Empirical studies

There are many empirical studies concerning the government size or the government expenditures share of total production. Ram (1986) using cross-sectional data and separate time series for 115 developing (those with market economy) and developed countries concluded that the finding of time series are consistent with cross-sectional results and the overall effect of government size on economic growth is positive in almost all cases. Guseh (1997) using fixed effects method for 59 middle-incomes developing countries (1984 classification of World Bank), concluded that the effect of government size on economic growth is negative. He took the share of total government spending in GDP as an index for the government size.

Ghali (1998) with government expenditures share of GDP for 10 countries of OECD concluded that the government size influences an economic growth directly or indirectly (via investment and trade). Vedder and Gallaway (1998) calculated optimum government expenditures for USA, Canada, Britain, Italy, Sweden and Denmark based on Armey curve (1995). They estimated a quadratic function for each country with independent variables of the government size and square government size and dependent variable of economic growth. According
to their results, the optimum expenditures are 17.45%, 21.37%, 20.97%, 22.23%, 19.43% and 26.14% for these countries respectively. Gwartney et al. (1998) took the total government expenditures to GDP proportion as an index for the government size. They studied the US, 23 countries of OECD and 60 countries of less developed with high income levels and concluded that the government size effect is negative in all three cases.

Fölster and Henrekson (2001) considered two indices for the government size: total tax share of GDP and total government expenditures share of GDP. They took two groups of countries and concluded that this negative effect is certified by the second index only, but in non-OECD by both two indices. In this research, the panel extreme bounds analysis (EBA) is used. Dar and Amirkhalkali (2002) with the government expenditures to GDP index for 19 countries of OECD concluded that the government size has negative effect on economic growth. Loizides and Vamvoukas (2005) with the share of total expenditure in GNP as index of government size for Greece, UK and Ireland concluded that in all three countries government size is the Granger cause of economic growth in short term (all three countries) and long term (Ireland and UK). Mitchell (2005) studied the government expenditures and economic growth trend in the US and 15 European countries. The results show that the expansion of government expenditures does not necessarily improve economic activities.

Romero-Ávila and Strauch (2008) using various indices concluded that government consumption expenditures and direct tax have negative effect but the public investment share has positive effect on economic growth. This research was done for 15 European countries with cointegration panel method. Chiou-Wei et al. (2010) investigated the nonlinear effect of government size on economic growth in South Korea, Singapore, Taiwan, Thailand and Malaysia. They used the share of government expenditures in GDP as an index for government size. The optimum size was estimated by Solow growth model and a dynamic STAR model for South Korea, Singapore, Taiwan and Thailand (nonlinear effect did not approved for Malaysia) as 10.8%, 11%, 15.9% and 10.8%, respectively (of course, nonlinear relationship was estimated as U-shape for Singapore). Wu et al. (2010), using the panel Granger causality for 182 countries, concluded that there is a bilateral relation between economic growth and government size. They considered all countries, OECD countries, non-OECD countries, high-income countries, middle-income countries, low-income countries, high-corruption and low-corruption countries. They concluded that bi-directional causality is confirmed in all groups except low-income countries.

There are researches about the government size on the basis of non-economic criteria. De Witte and Moesen (2009) determined factors affecting the
government size like country size, economic growth, Wagner’s law, urbanization and family size and estimated optimum government size for 23 OEDC countries using data envelopment analysis (DEA). Davies (2009) considered government consumption expenditures as a share of GDP and government investment expenditures as a share of GDP and used human development index to determine the optimum amount of them. He selected 154 high and low-income countries and determined the optimum expenditures for 5 years (1975, 1980, 1985, 1990, 1995, 2000, 2002). A second order function with dependent variable of HDI (instead of economic growth rate) is estimated by generalized method of moments (GMM) and then it was used to estimate the optimum value of government size. This research has two results: first, the optimum share of consumption and investment expenditures are 17% and 13% in high-income countries, respectively. Second, in low-income countries, consumption expenditures always have positive effect while investment expenditures have negative effect (-40%). In these countries, a clear optimum level is not defined yet.

4. Data

Total government expenditures are divided to current and capital expenditures. Current expenditures are divided to final consumption expenditures and other current expenditures. The government expenditures that are spent for providing goods and services are called the final consumption expenditures. The second part of current spending contains social transfers other than social transfers provided to households via market producers, subsides, etc. Capital expenditures contain two parts: gross government fixed capital formation (tangible properties like infrastructures and intangible properties like computer software) and other capital expenditures (like the change in inventories of government sector, precious stones and metals, etc.).

Four measures for government expenditures are selected here: (i) Total government expenditures to product, (ii) Final consumption expenditures to product, (iii) Current expenditures other than final consumption expenditures to product, (iv) Fixed investment expenditures to product. In the majority of empirical studies, as mentioned in section 3, the second measure and sometimes the forth one has been used. The present study has investigated the Barro curve for all four measures mentioned above.

15 European countries of OECD and members of European Union to 1995 include Austria (1995), Belgium (1952), Denmark (1973), Finland (1995), France (1952), Germany (1952), Greece (1981), Ireland (1973), Italy (1952), Luxembourg (1952), Netherlands (1952), Portugal (1986), Spain (1986), Sweden
(1995) and United Kingdom (1973). The data are from the WDI and AMECO. The four measures (at current prices in Local Currency Unit) are as follows:

(i) TE: total current and investment expenditures proportion to GDP.
(ii) FCE: final consumption expenditures proportion to GDP.
(iii) OCE: current expenditures other than final consumption proportion to GDP.
(iv) GFCF: gross fixed capital formation of government proportion to GDP.

5. Linear Estimations results

In this study, panel approach is used for estimation. Economic (resource endowment such as labor force and natural, physical and human capitals, financial system and economic freedom), social and political structures may be different in countries under the study. Individual-specific effects method is used to take into consideration these differences. In addition, the estimation results of panel data compare to cross-sectional or time-series data are more reliable (Baltagi, 2008).

Based on the theoretical (Solow, 1956; Barro, 1990; Mourmouras and Lee, 1999; Kosempel, 2004) and empirical growth literature (Solow, 1957; Barro, 1991; Ram, 1986; Guseh, 1997; Romero-Ávila and Strauch, 2008), the following growth regression is specified for estimation:

\[
GGDP_{it} = \alpha_0 + \alpha_1 LAB_{it} + \alpha_2 INV_{it} + \alpha_3 EXP_{it} + \alpha_4 IMP_{it} + \alpha_5 GOV_{it} + u_{it},
\]

where GGDP is growth rate of real gross domestic product (in constant 2000 US dollar), LAB is labor growth rate, INV is gross fixed investment proportion to GDP (in constant 2000 US dollar), EXP and IMP are growth rate of exports and imports of goods and services, respectively (in constant 2000 US dollar), and finally, GOV is government size that can be one of the four TE, FCE, OCE or GFCF. So, four measures can be considered (as shown in Table 1). Model (1) and (4) face up to the problem of multicollinearity. In model (1), TE contains fixed government investment, and therefore there is multicollinearity between TE and INV. In model (4), GFCF is government investment itself, and then there is multicollinearity between GFCF and INV. In models (1) and (4), due to prevent multicollinearity, INV (gross fixed investment to GDP) is replaced with private fixed investment to GDP (in other words, in these models, INV is private gross fixed capital formation to GDP).
Table 1
Different models of government size

<table>
<thead>
<tr>
<th>Measure</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOV</td>
<td>TE</td>
<td>FCE</td>
<td>OCE</td>
<td>GFCF</td>
</tr>
<tr>
<td>Min-Max</td>
<td>31.58</td>
<td>64.82</td>
<td>12.9</td>
<td>29.9</td>
</tr>
<tr>
<td>Countries</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Observations</td>
<td>165</td>
<td>375</td>
<td>165</td>
<td>165</td>
</tr>
</tbody>
</table>

Source: a AMECO and b WDI

At first, the panel unit root tests of LLC (Levin, Lin and Chu, 2002) and IPS (Im, Pesaran and Shin, 2003) are performed for two relatively stable periods of 1995-2005 and 1981-2005. The results show that all variables are stationary (Table 2).

Table 2
Panel unit root tests

<table>
<thead>
<tr>
<th>Measure</th>
<th>(1), (3), (4) [1995-2005]</th>
<th>(2) [1981-2005]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LLC</td>
<td>IPS</td>
</tr>
<tr>
<td>GGDP</td>
<td>-4.4***</td>
<td>-5.9***</td>
</tr>
<tr>
<td>LAB</td>
<td>-11***</td>
<td>-13***</td>
</tr>
<tr>
<td>INV</td>
<td>-2.3**</td>
<td>-6.5***</td>
</tr>
<tr>
<td>INVP*</td>
<td>-2.1***</td>
<td>-4.5***</td>
</tr>
<tr>
<td>EXP</td>
<td>-7.5***</td>
<td>-9.1***</td>
</tr>
<tr>
<td>IMP</td>
<td>-6.7***</td>
<td>-9.0***</td>
</tr>
<tr>
<td>TE</td>
<td>-6.0***</td>
<td>-2.7***</td>
</tr>
<tr>
<td>FCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCE</td>
<td>-7.7***</td>
<td>-1.9**</td>
</tr>
<tr>
<td>GFCF</td>
<td>-6.8***</td>
<td>-6.8***</td>
</tr>
</tbody>
</table>

Note: *** denotes significance at 1%, ** at 5% and * at 10% levels. ^: INVP is private gross fixed capital formation to GDP.

In order to select fixed or random individual effects, Hausman test based on linear regression (8) is done. The null hypothesis is that there is no dependency between explanatory variables and disturbances. With model of fixed individual effects, the estimators under the null and alternative hypotheses are consistent but estimators of random effects will be consistent under the null one. So if null hypothesis is rejected, fixed effects must be used (Baltagi, 2008). The Hausman test result shows that the null hypothesis is rejected with 99% confidence in all models and so fixed effects must be used. According to linear regression
estimations with fixed individual effects, total expenditures (TE), the current expenditures other than final expenditures (OCE) and government gross fixed capital formation expenditures (GFCF) do not have effects on economic growth, and only the effect of final expenditures share (FCE) is significant and negative (Table 3).

Table 3
Linear regression results

<table>
<thead>
<tr>
<th>Regression</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_0)</td>
<td>(a_1)</td>
<td>(a_2)</td>
<td>(a_3)</td>
</tr>
<tr>
<td></td>
<td>-5.383</td>
<td>0.084</td>
<td>0.278</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(2.747)*</td>
<td>(0.085)***</td>
<td>(0.071)***</td>
<td>(0.032)***</td>
</tr>
<tr>
<td></td>
<td>5.161</td>
<td>0.224</td>
<td>0.079</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(1.669)***</td>
<td>(0.073)***</td>
<td>(0.037)***</td>
<td>(0.019)</td>
</tr>
<tr>
<td></td>
<td>-5.634</td>
<td>0.086</td>
<td>0.276</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td>(2.386)***</td>
<td>(0.085)</td>
<td>(0.071)***</td>
<td>(0.032)***</td>
</tr>
<tr>
<td></td>
<td>-3.010</td>
<td>-0.086</td>
<td>0.240</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.275)***</td>
</tr>
<tr>
<td></td>
<td>Hausman test</td>
<td></td>
<td></td>
<td>(0.64)***</td>
</tr>
</tbody>
</table>

Note: *** denotes significance at 1%, ** at 5% and * at 10% levels.

6. Threshold non-linear method

As discussed earlier, the government effect on economic growth may be nonlinear (like Barro curve). To test and estimate nonlinearity effects, the threshold panel method is used. Hansen (1999) suggests this method with fixed individual effects that is used here. On the one hand, using panel data approach, can lead to more reliable results than cross-sectional and time-series data. On the other hand, using a threshold approach, the existence and significance of the threshold can be tested.

First, existence of the threshold(s), or breakdown(s), is tested. If \(I(.)\) is an index function and parameter(s) of \(\gamma\) shows threshold(s), its regressions are shown below.

Single threshold regression:
\[ GGDP_{it} = \alpha_1 LAB_{it} + \alpha_2 INV_{it} + \alpha_3 EXP_{it} + \alpha_4 IMP_{it} + (\delta_1 + \beta_1 GOV_{it}) I(GOV_{it} \leq \gamma_1) + (\delta_2 + \beta_2 GOV_{it}) I(GOV_{it} > \gamma_1) + u_{it}. \]  

(9)

Double threshold regression:

\[ GGDP_{it} = \alpha_1 LAB_{it} + \alpha_2 INV_{it} + \alpha_3 EXP_{it} + \alpha_4 IMP_{it} + (\delta_1 + \beta_1 GOV_{it}) I(GOV_{it} \leq \gamma_1) + (\delta_2 + \beta_2 GOV_{it}) I(\gamma_1 < GOV_{it} \leq \gamma_2) + (\delta_3 + \beta_3 GOV_{it}) I(GOV_{it} > \gamma_2) + u_{it}. \]  

(10)

In general, regression with J thresholds is as follows:

\[ GGDP_{it} = \alpha_1 LAB_{it} + \alpha_2 INV_{it} + \alpha_3 EXP_{it} + \alpha_4 IMP_{it} + \sum_{j=2}^{J-1} (\delta_j + \beta_j GOV_{it}) I(\gamma_{j-1} < GOV_{it} \leq \gamma_j) + (\delta_J + \beta_J GOV_{it}) I(GOV_{it} > \gamma_J) + u_{it}. \]  

(11)

Threshold approach suggests minimizing sum of squared residuals that are obtained from a consistent estimation (Chan, 1993). For simplicity, assume that thresholds are estimated from smallest to largest:

\[ \hat{\gamma}_1 = arg\min S_1(\gamma_1), \]

\[ \hat{\gamma}_2 = arg\min S_2(\gamma_2 | \hat{\gamma}_1), \]

\[ \vdots \]

\[ \hat{\gamma}_J = arg\min S_J(\gamma_J | \hat{\gamma}_1, \ldots, \hat{\gamma}_{J-1}). \]  

(12)

According to Eqs. (9)-(11) the hypothesis testing for presence of threshold(s) (for each of the cases above) can be shown as below:

\[ \gamma_1: H_0^1: \beta_1 = \beta_2, \quad H_1^1: \beta_1 \neq \beta_2, \]

\[ \gamma_2: H_0^2: \beta_2 = \beta_3, \quad H_1^2: \beta_2 \neq \beta_3, \]

\[ \vdots \]

\[ \gamma_J: H_0^J: \beta_{J-1} = \beta_J, \quad H_1^J: \beta_{J-1} \neq \beta_J. \]  

(13)

These hypotheses must be tested while the null hypothesis is not rejected. If the \( i \)-th null hypothesis is not rejected, the \( i \)-th threshold, \( \gamma_i \), is not significant and
the model has only threshold of \( i-1 \). The \( F_i \) test for \( i-th \) null hypothesis, \( i-th \) threshold, is done:

\[
F_1 = \frac{S_0 - S_1(\hat{y}_1)}{\hat{\sigma}_1^2},
\]

\[
F_2 = \frac{S_1(\hat{y}_1) - S_2(\hat{y}_2 | \hat{y}_1)}{\hat{\sigma}_2^2},
\]

\[\vdots\]

\[
F_j = \frac{S_{j-1}(\hat{y}_{j-1} | \hat{y}_1, \ldots, \hat{y}_{j-2}) - S_j(\hat{y}_j | \hat{y}_1, \ldots, \hat{y}_{j-1})}{\hat{\sigma}_j^2}. \tag{14}
\]

\( S_0 \) is regression residuals sum of squares and variances are defined as follows:

\[
\hat{\sigma}_1^2 = \frac{1}{N(T-1)} S_1(\hat{y}_1),
\]

\[
\hat{\sigma}_2^2 = \frac{1}{N(T-1)} S_2(\hat{y}_2 | \hat{y}_1),
\]

\[\vdots\]

\[
\hat{\sigma}_j^2 = \frac{1}{N(T-1)} S_j(\hat{y}_j | \hat{y}_1, \ldots, \hat{y}_{j-1}). \tag{15}
\]

\( F_i \) distribution is non-standard and depends on the moments of sample, and so the critical values cannot be tables (Hansen, 1996). So following the suggestion of Hansen (1997, 1999, 2000), the bootstrap procedure is used: (i) First, through minimizing the sum of squares of sample residuals, coefficients are estimated. (ii) Based on sample residuals distribution, a new sample under null hypothesis is produced. The independent variables are non-random and do not vary, and just the dependent variable is reproduced. (iii) With the new sample, the coefficients are estimated (under the null and alternative hypotheses) and the simulated \( F_i \) statistic is obtained. (iv) The process is repeated and using the simulated statistics, the critical values are calculated. The p-value based on the number of simulated statistics exceeds actual estimation of \( F \) is calculated.

Then the confidence intervals based on likelihood ratio test are made. Consistent estimations are obtained by minimizing the residuals sum of squares but the asymptotic distributions of estimators are non-standard. Hansen (1999)
suggests using the likelihood ratio test. The likelihood ratio is estimated for different threshold values under the null hypothesis. For single threshold:

\[ H_0^1: \gamma_1 = \gamma_1^*, \]

\[ LR_1(\gamma_1) = \frac{(S_1(\gamma_1) - S_1(\hat{\gamma}_1))}{\hat{\sigma}_1^2}. \] (16)

For two thresholds:

\[ H_0^2: \gamma_2 = \gamma_2^*, \quad H_0^1: \gamma_1 = \gamma_1^*, \]

\[ LR_2(\gamma) = \frac{(S_2(\gamma_2|\hat{\gamma}_1) - S_2(\hat{\gamma}_2|\hat{\gamma}_1))}{\hat{\sigma}_2^2}, \quad LR_1(\gamma) = \frac{(S_1(\gamma_1|\hat{\gamma}_2) - S_1(\hat{\gamma}_1|\hat{\gamma}_2))}{\hat{\sigma}_2^2}. \] (17)

The same process can be done for more than two thresholds. Hansen (1999, 2000) shows that the likelihood statistics tend to converge to a random variable in distribution with the following reverse distribution function:

\[ LR(\gamma) \overset{d}{\rightarrow} \xi, \quad P(\xi \leq x) = \left(1 - e^{-x/\tau}\right)^2, \] (18)

\[ c(\alpha) = -2 \log\left(1 - \sqrt{1 - \alpha}\right). \] (19)

By replacing the percentage of significance level (for example, 5%) in Eq. (19), the critical value of likelihood statistic is calculated. Likelihood ratio for each of actual thresholds under the null hypothesis is calculated and then based on the inequality \( LR_1 \leq c(\alpha) \), a% confidence interval is calculated. It should be noted that the hypothesis \( H_0: \beta_1 = \beta_2 \) is different from the hypothesis \( H_0: \gamma = \gamma_0 \). The \( F_1 \) statistic is for testing the presence of the threshold, while the \( LR_1 \) statistic is used for constructing the confidence interval for the present threshold.

### 7. Non-linear estimations results

The results are shown in Table 4. The bootstrap was repeated for 10000 times for each model and then the critical values and p-values were calculated. The existence of one threshold for each of the four measures is confirmed. The second threshold in regressions (2) and (4) is not confirmed at 10% level, but in regressions (1) and (3), it is certified. In regression (3), the third threshold is even significant, but the direction does not change and just the intensity changes slightly, which is insignificant. In 1-th threshold, smallest one, the effect changed from positive to negative so these thresholds are regarded as optimum amounts.
Albeit for the exact estimation of all coefficients, the second threshold in regressions (1) and (3) are considered (change in direction and intensity in Table 4 is quite obvious).

Table 4
Threshold test

<table>
<thead>
<tr>
<th>Measure</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>20.363???</td>
<td>29.693???</td>
<td>23.953???</td>
<td>7.081*</td>
</tr>
<tr>
<td>Critical values</td>
<td>8.4, 11.0, 17.2</td>
<td>7.9, 10.3, 16.1</td>
<td>8.2, 10.9, 17.0</td>
<td>6.1, 8.1, 12.6</td>
</tr>
<tr>
<td>Bootstrap p-value</td>
<td>0.005</td>
<td>0.001</td>
<td>0.002</td>
<td>0.071</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>10.468???</td>
<td>8.824</td>
<td>6.301**</td>
<td>3.832</td>
</tr>
<tr>
<td>Critical values</td>
<td>4.2, 5.5, 8.8</td>
<td>9.4, 12.0, 18.3</td>
<td>4.3, 5.7, 9.0</td>
<td>7.4, 9.7, 15.4</td>
</tr>
<tr>
<td>Bootstrap p-value</td>
<td>0.005</td>
<td>0.113</td>
<td>0.037</td>
<td>0.286</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>5.512</td>
<td>8.701**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical values</td>
<td>9.7, 12.1, 17.9</td>
<td>5.0, 6.7, 10.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bootstrap p-value</td>
<td>0.277</td>
<td></td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>( F_4 )</td>
<td></td>
<td>2.965</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical values</td>
<td></td>
<td>3.4, 4.4, 6.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bootstrap p-value</td>
<td></td>
<td>0.134</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *** denotes significance at 1%, ** at 5% and * at 10% levels.

In Table 5, the optimum amount of the four measures and their confidence intervals are shown. The optimum share of total expenditures (TE), the optimum share of final consumption expenditures (FCE), the optimum share of current expenditures other than final consumptions (OCE) and the optimum share of government fixed investment (GFCF) are estimated 41.7%, 15.8%, 19.4% and 2.5%, respectively. The sum of the optimum share of current expenditures (19.4%+15.8%) with the optimum share of fixed investment expenditures (2.5%) will be 37.7%, and the optimum share of total expenditures is 41.7%. Therefore, there is 3% different between 41.7% and 37.7%. Total expenditures comprise current and capital expenditures. The capital expenditures comprise fixed investment expenditures and non-fixed capital expenditures (or other capital expenditures). The non-fixed capital expenditures comprise change in inventories of government sector, precious stones and metals, etc., having a 1% to 3% share of GDP. Adding the share of non-fixed capital expenditures (with 37.7%), the share of total expenditures will be almost 41.7%. Therefore, there is no remarkable difference and the results are consistent.
Table 5
The optimum size of government

<table>
<thead>
<tr>
<th>Regression</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOV</td>
<td>TE</td>
<td>FCE</td>
<td>OCE</td>
<td>GFCF</td>
</tr>
<tr>
<td>Optimum Size</td>
<td>41.7%</td>
<td>15.8%</td>
<td>19.4%</td>
<td>2.5%</td>
</tr>
<tr>
<td>(1-th threshold)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confidence Intervals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99%</td>
<td>[41.7 41.7]</td>
<td>[15.8 15.8]</td>
<td>[16.3 20.0]</td>
<td>[1.2 2.6]</td>
</tr>
<tr>
<td>95%</td>
<td>[36.0 43.6]</td>
<td>[15.8 16.0]</td>
<td>[16.3 20.1]</td>
<td>[1.2 3.6]</td>
</tr>
<tr>
<td>90%</td>
<td>[36.0 44.0]</td>
<td>[14.0 16.0]</td>
<td>[16.3 21.4]</td>
<td>[1.2 3.6]</td>
</tr>
<tr>
<td>2-th threshold</td>
<td>51.9%</td>
<td></td>
<td>25.1%</td>
<td></td>
</tr>
<tr>
<td>3-th threshold</td>
<td></td>
<td></td>
<td>22.8%</td>
<td></td>
</tr>
</tbody>
</table>

Regressions (1) and (3) with Eq. (10) and regressions (2) and (4) with Eq. (9) are estimated (Table 6). Considering the threshold(s), the significance of coefficients improves, and especially the coefficients of government expenditures share are significant before and after the threshold. The optimum share of total expenditures was estimated 41.7%. If total expenditures share is below the optimum, an increase in TE will have a meaningful positive effect on economic growth but the effect is negative if it is above the optimum. After the 51.9% level, the second threshold, TE has no meaningful effect. Based on these estimations, 1% increase in TE causes 0.4% increase in economic growth but after the optimum level 1%, increase in TE causes 2% decrease in economic growth averagely. The word averagely is important because change in the share of total expenditures depends on whether these changes have happened on which kind of expenditures and their combination ($\beta_1$, $\beta_2$ and $\beta_3$ in regression (1)).

The optimum share of final consumption expenditures is estimated 15.8%. Both of the coefficients before and after the threshold are significant at 1% level. Before the threshold, 1% increase in FCE causes 0.9% increase in economic growth but after that 1% increase causes 0.4% increase in economic growth ($\beta_1$ and $\beta_2$ in regression (2)). The optimum share of current expenditures other than final consumptions is estimated 19.4%, for which the direction of effect is reversed after the threshold. Before the optimum level, 1% increase in OCE causes 0.7% increase and after it 0.3% decrease in economic growth. Of course, after the second threshold, 25.1%, coefficient of OCE is insignificant ($\beta_1$, $\beta_2$ and $\beta_3$ in regression (3)). Government fixed investment optimum share is estimated 2.5%, then before the optimum level with 1% increase in GFCF, causes 1% increase and after it with no effect on economic growth ($\beta_1$ and $\beta_2$ in regression (4)).
### Table 6
Threshold regression results

<table>
<thead>
<tr>
<th>Regression</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.085</td>
<td>0.239</td>
<td>0.070</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.070)**</td>
<td>(0.079)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>α&lt;sub&gt;2&lt;/sub&gt;</td>
<td>0.296</td>
<td>0.071</td>
<td>0.373</td>
<td>0.247</td>
</tr>
<tr>
<td></td>
<td>(0.070)**</td>
<td>(0.036)**</td>
<td>(0.073)**</td>
<td>(0.063)**</td>
</tr>
<tr>
<td>α&lt;sub&gt;3&lt;/sub&gt;</td>
<td>0.110</td>
<td>0.032</td>
<td>0.123</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>(0.031)**</td>
<td>(0.019)*</td>
<td>(0.030)***</td>
<td>(0.032)**</td>
</tr>
<tr>
<td>α&lt;sub&gt;4&lt;/sub&gt;</td>
<td>0.093</td>
<td>0.198</td>
<td>0.072</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.031)**</td>
<td>(0.017)**</td>
<td>(0.031)**</td>
<td>(0.032)**</td>
</tr>
<tr>
<td>δ&lt;sub&gt;1&lt;/sub&gt;</td>
<td>-17.657</td>
<td>-11.345</td>
<td>-17.625</td>
<td>-4.384</td>
</tr>
<tr>
<td></td>
<td>(4.729)**</td>
<td>(3.460)**</td>
<td>(3.857)**</td>
<td>(1.478)**</td>
</tr>
<tr>
<td>δ&lt;sub&gt;2&lt;/sub&gt;</td>
<td>4.247</td>
<td>8.279</td>
<td>-0.077</td>
<td>-3.449</td>
</tr>
<tr>
<td></td>
<td>(4.138)</td>
<td>(1.937)**</td>
<td>(4.022)</td>
<td>(1.598)**</td>
</tr>
<tr>
<td>δ&lt;sub&gt;3&lt;/sub&gt;</td>
<td>-7.226</td>
<td>-7.742</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.409)**</td>
<td>(2.552)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.379</td>
<td>0.891</td>
<td>0.666</td>
<td>1.005</td>
</tr>
<tr>
<td></td>
<td>(0.099)**</td>
<td>(0.226)**</td>
<td>(0.149)***</td>
<td>(0.484)**</td>
</tr>
<tr>
<td>β&lt;sub&gt;2&lt;/sub&gt;</td>
<td>-0.171</td>
<td>-0.421</td>
<td>-0.265</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>(0.077)**</td>
<td>(0.079)**</td>
<td>(0.146)*</td>
<td>(0.371)</td>
</tr>
<tr>
<td>β&lt;sub&gt;3&lt;/sub&gt;</td>
<td>0.066</td>
<td>0.064</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.70)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *** denotes significance at 1%, ** at 5% and * at 10% levels.

### 8. Conclusions

This research has four main results:

(i) In all of the 15 European countries, the effects of government expenditures on economic growth are nonlinear which is confirmed based on all measures of TE (total expenditures to GDP), FCE (final consumption expenditures to GDP), OCE (current expenditures other than final consumption to GDP) and finally GFCF (gross fixed capital formation of governmental to GDP). In the previous empirical studies, the second measure and sometimes the forth one has been used. Increasing government expenditures causes economic growth up to a certain level but the effect will be negative beyond it. Therefore, the Barro curve is confirmed based on all four measures. Then the optimum expenditures level can be determined by this curve, which helps countries with their budgetary policies. The optimum share of final consumption expenditures and other current expenditures (FCE, OCE) are calculated 15.8% and 19.4%, respectively. FCE is calculated in this study is close to previous research. According to results of Vedder and Gallaway (1998), the optimum final consumption expenditures are 17.45%, 21.37%, 20.97%, 22.23%, 19.43% and 26.14% for USA, Canada, Britain, Italy,
Sweden and Denmark, respectively. Also based on the HDI, Davies (2009) calculated optimum share of expenditures in high-income countries that approximately is 17%. Government fixed capital formation expenditures share is calculated 2.5% and with the share of 1-3% for non-fixed capital expenditures, the sum of these four expenditures mentioned will be almost 41.7%.

(ii) Before the optimum level, the fixed investment share has the most positive effect on economic growth with a 1 to 1 relation. After the optimum level, the share of final consumption expenditures has the most negative effect with a 2 to 1 relation. As different expenditures have different effects, change in the budget combination can increase or lessen economic growth even if the share of total expenditures is constant. As in none of the 15 European countries, the consumption expenditures share is not less than the optimum level, so the share of final consumption expenditures increase together with other expenditure decrease (other current expenditures and/or investment expenditures) causes increase in economic growth.

(iii) In figure 2, the four measures are shown for 15 European countries during 2001-2005 in which for eight countries (Austria, Belgium, Denmark, Finland, France, Germany, Italy, Sweden), the total expenditures share is almost 10% more than 41.7%. In these countries, FCE and OCE are 5% more than the optimum level. In four countries (Austria, Belgium, Denmark, Germany), the share of investment expenditures is almost 1% less than the optimum share, 2.5%, while in the other four countries (Finland, France, Italy, Sweden), this share is at maximum 1% more than 2.5%. Therefore, these eight countries can reach higher economic growth by decreasing the current expenditures and increasing investment expenditures. In 6 countries (Austria, Belgium, Denmark, Germany), the total expenditures share is equal to the optimum but in some countries the combination is not optimum. In 3 countries (Finland, France, Italy, Sweden), the final consumption expenditures is 5% more than the optimum level. In six countries (Greece, Luxembourg, Netherlands, Portugal, Spain, United Kingdom), the share of investment expenditures is 3% to 4% (except in the UK which is almost 1% less than the optimum level). Only in Ireland, the total expenditures are less than the optimum share remarkably (which is 8% less than optimum share) which pertains to current expenses other than final consumption. It means that the share of current expenditures other than final consumptions is 7% less than the optimum level.
(iv) According to Kosempel model (in section 2), theoretically, the Barro curve is only approved for expenditures in providing free services to producers. Meanwhile expenditures for providing free services to consumers always have a negative effect on economic growth. However, in section 7, the non-linear effects for both current and capital expenditures were confirmed. Are these results contradictory? First, government activities have both positive and negative effects (they are described in section 1). These effects are for all of the kinds of expenditures (expenditures for providing free services to producers/consumers and current/capital expenditures) although with different intensities. Therefore, the final effect of government expenditures depends on the kinds and amount of government expenditures. As these expenditures increase, the negative effects are
expected to increase that may create a non-linear relation between government and economic growth. Second, budget classification in Kosempel model (the kind of user groups: consumers and producers) is different from the common budget classification (current and capital). In Kosempel model, expenditures for providing free services to producers were regarded as effective expenditures, and enter the production function and affect the economic growth directly, while expenditures to construct parks and museums are consumer services expenditures and enter consumer utility function. But in budget classification, non-capital expenditures like education or human forces enhancement expenditure, are considered in current budget, and capital expenditures whether for roads and harbors or parks and museums are considered as capital expenditures. Thus, budget classification that was used in this research and many others is not an accurate classification based on the user groups (although it can be said that current expenditures mainly pertain to consumers and capital expenditures mainly pertain to producers). Such questions can be discussed in future researches.

References


[17] Hansen, B.E. (1996) Inference when a nuisance parameter is not identified under the null hypothesis, Econometrica, 64, pp. 413-430.


