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Marcus Pivato

Department of Mathematics, Trent University

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# Pyramidal Democracy

Marcus Pivato

Department of Mathematics, Trent University  
1600 West Bank Drive, Peterborough, Ontario, K9J 7B8, Canada  
email: [marcupivato@trentu.ca](mailto:marcupivato@trentu.ca)

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## Abstract

We consider a decentralized, multilayered representative democracy, where citizens participate in deliberative policy formation after self-organizing into a pyramidal hierarchy of small groups. Each group elects a delegate, who expresses the deliberative consensus of that group at the next tier of the pyramid. The pyramid thus acts as a communications network which efficiently aggregates useful information and policy ideas. It is also a powerful meritocratic device, which channels legislative responsibility towards the most committed and competent citizens. This yields a practical implementation of deliberative democracy in a large polity.

‘Deliberative democracy’ emphasizes the importance of widespread dialogue and deliberation amongst the citizenry as an essential part of the democratic process.<sup>1</sup> Such deliberation should have many positive effects. It would yield superior legislation by eliciting and efficiently aggregating the knowledge, creativity, and analytical skills of the entire electorate. It would also encourage compromise and consensus-formation, yielding legislation with broader public support and greater legitimacy. However, there are several practical difficulties in implementing the deliberative ideal:

1. *Scale:* In a polity with tens of millions of citizens, how can we give each citizen an opportunity to meaningfully participate in deliberative policy formation, while still creating a relatively efficient and effective legislative process?
2. *Competency:* Citizens differ hugely in their legislative competency, due to widely varying levels of education (both formal and informal)<sup>2</sup>, intelligence, engagement in the current political discourse, and overall interest in public policy. Unstructured

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<sup>1</sup>See e.g. Fishkin (1991, 1997), Gutmann and Thompson (1996, 2004), Bohman and Rehg (1997), Elster (1998b), Dryzek (2002), Fishkin and Laslett (2003), Amsler (2004), or Bächtiger and Steiner (2005a,b).

<sup>2</sup>See Kuklinski et al. (2000) or Delli-Carpini and Keeter (1996) for studies of voter ignorance or misinformation. See Somin (1998, 2004) or Weinshall (2003) for the ‘public ignorance’ criticism of deliberative democracy.

deliberation in random or self-assembled groups generally gives disproportionate influence to the most vociferous, confident, and charismatic speakers, who are not necessarily the best qualified to formulate public policy. If deliberation is to enhance the quality of legislation, then there must be meritocratic mechanisms which promote the most competent (i.e. intelligent, educated, informed, engaged, ethical, objective, pragmatic, and open-minded) participants, instead of favoring ideologues, extremists, and demagogues.

3. *Time Commitment:* To seriously participate in deliberation, a citizen must acquire and maintain the relevant background knowledge, and then evaluate, critique, and perhaps author policy proposals, while discussing them with fellow citizens. This is a full-time job (at least), and most citizens are too busy with their ordinary lives to participate conscientiously. As a result, they will either participate in a superficial (perhaps counterproductive) manner, or altogether opt out of deliberation.<sup>3</sup>

Some deliberative proposals<sup>4</sup> address these problems by relegating deliberation to a purely ‘educational’ or ‘advisory’ role, adjunct to existing electoral institutions. In these models, deliberation is intended to produce more informed, open-minded, cosmopolitan, rational, and critical citizens, who then vote more intelligently in conventional elections or referenda. However, the alternatives in these elections and referenda would still be determined by conventional political parties or legislatures, so this is still a ‘top-down’ legislative system. Ultimately, citizens affect legislation only by voting, influencing other citizens’ votes, and perhaps sending non-binding policy advice to legislators.

Some proposals [e.g. deliberative polls (Fishkin, 1991, 1997), minipopuli (Dahl, 1989, p.340) or citizen’s juries (Coote and Lenaghan 1997, Jefferson Center 1999)] also address Problem #1 by restricting formal deliberation to relatively small, random samples of citizens, which are intended to be ‘statistically representative’ of the general population (Goodin (2003) calls this ‘ersatz deliberation’). Also, some proposals address Problem #3 by limiting formal deliberation to brief, infrequent, concentrated sessions. For example, Fishkin’s deliberative polls occur over a single weekend, and the aforementioned ‘random sampling’ methodology presumably means that most citizens would only participate in one every few years. Fishkin and Ackerman (2005) suggest limiting formal deliberation to a single national holiday, one week before major elections (although this ‘Deliberation Day’ is also intended to elicit informal deliberation amongst the electorate both before and afterwards).

However, to fully realize the potential of deliberative democracy, we need a deliberative institution which allows the *entire* electorate to *continually* and *substantively* participate in the legislative process, while somehow obviating the three aforementioned practical problems. Pyramidal democracy (PD) is a decentralized, multilayered form of representative democracy which achieves this goal by arranging the electorate in a hierarchical network of small, self-organized, deliberative groups. This allows the whole electorate to meaningfully participate in deliberation and policy formation, but also allows each citizen to voluntarily

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<sup>3</sup>See Warren (1996) for another discussion of Problems #2 and #3.

<sup>4</sup>See Ryfe (2002) for a survey of contemporary deliberative organizations.

limit her time commitment by delegating some (or most) of her deliberative responsibilities to an elected representative. The resulting ‘pyramid’ of delegation is a powerful meritocratic mechanism which channels legislative responsibility towards the most committed and competent citizens. The pyramid also acts as a communications network which efficiently aggregates useful information and policy ideas from all citizens, while naturally filtering out fallacy, misinformation and extremism. Formally, pyramidal democracy works as follows:

1. Citizens self-organize into groups (called *nodes*), each containing a minimum number  $B$  of members (we suggest  $B = 7$ , but we expect most nodes will be slightly larger, so as to be more stable). We assume people will form nodes based upon similarity of political views and values. The totality of all such nodes is called *Tier 1* of the pyramid.
2. The members of each node meet regularly to deliberate. Each node selects a single *delegate*, who will represent the consensus positions of that node on various issues.
3. The delegates of all Tier 1 nodes themselves self-organize into nodes, each containing at least  $B$  members. The totality of all such nodes is called *Tier 2* of the pyramid.
4. The members of each Tier 2 node meet regularly to deliberate. Each Tier 2 node chooses a single delegate to represent its consensus positions.
5. These Tier 2 delegates then self-organize into nodes, each containing at least  $B$  members. The totality of these nodes is *Tier 3* of the pyramid.
6. We iterate this process. Clearly, in a society with  $K$  voters, the  $n$ th tier will have at most  $K/B^{n-1}$  nodes. We stop when we reach a tier with less than  $B^2$  members; thus the pyramid will have at most  $\log_B(K)$  tiers. (For example, if  $B = 10$ , then a pyramid with  $K = 100\,000\,000$  voters would have at most seven tiers; the top Tier would have at most 100 members, each indirectly representing a six-tier sub-pyramid with at least 10 000 000 voters.)
7. The top Tier (called the *Parliament*) will contain between  $B$  and  $B^2$  individuals, and will be the legislative branch of the government.

The structure of delegation in the pyramid is fairly elaborate. Two technical remarks are in order:

- (a) Each member of a Tier  $(k+1)$  node  $\mathcal{N}_{k+1}$  is the delegate from some Tier  $k$  node  $\mathcal{N}_k$ , and is tasked with representing the position of  $\mathcal{N}_k$  in  $\mathcal{N}_{k+1}$ . However, she must also be given considerable discretion to modify her views during deliberation in  $\mathcal{N}_{k+1}$ . Presumably, the delegates to Tier  $(k+1)$  will generally be the most serious and politically engaged members of Tier  $k$ ; hence the quality and quantity of deliberation will be much greater in Tier  $(k+1)$  than in Tier  $k$ . Beyond some Tier (say Tier 4), deliberation will become a serious time commitment, and members should be

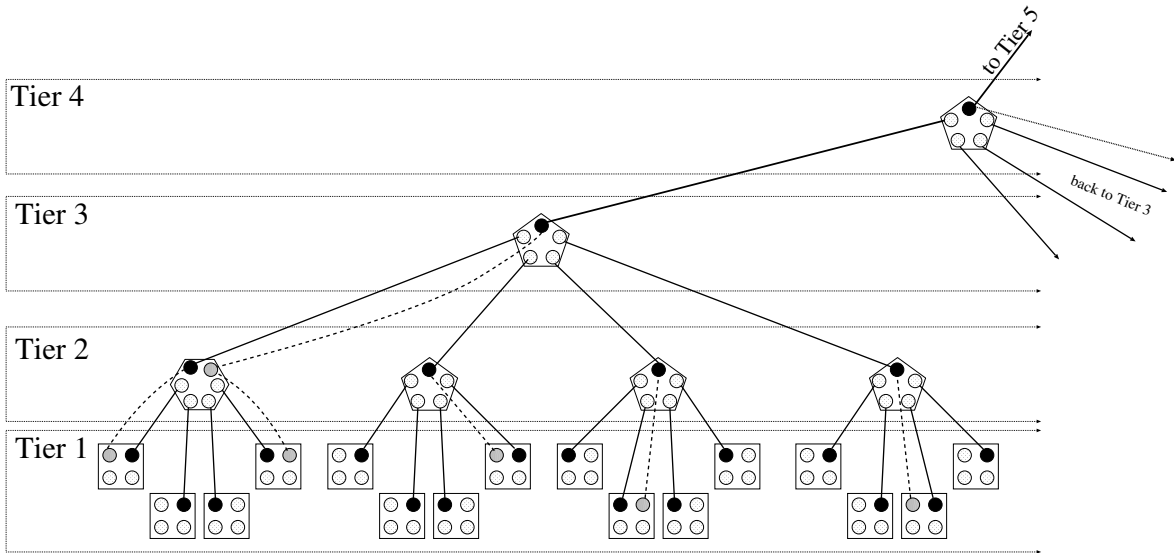


Figure 1: A fragment of a pyramidal democracy. For simplicity we depict nodes with four voting members. Circles are node members, and polygons are nodes. Light grey circles are ordinary members (who can vote). Black circles are delegates (who cannot vote in their home node). Dark grey circles are metadelegates (who can vote). Solid lines represent delegacy, and dashed lines represent metadelegacy.

entitled to part-time release from their ordinary jobs, and commensurate financial compensation (in an electorate of 100 000 000, Tier 4 might contain around 100 000 people). In Parliament and the Tier just below it, deliberation will be a full-time job; members of Parliament and the sub-Parliamentary Tier should be paid as full-time civil servants.

- (b) Suppose that a Tier  $(k+1)$  node  $\mathcal{N}_{k+1}$  chooses member  $D$  as its delegate to Tier  $(k+2)$ . Recall that  $D$  is already the delegate to  $\mathcal{N}_{k+1}$  from some Tier  $k$  node  $\mathcal{N}_k$ ; presumably,  $D$  cannot fulfil both delegacy roles simultaneously. Thus, *another* member  $D'$  must be chosen from  $\mathcal{N}_k$  to act as  $\mathcal{N}_k$ 's delegate in  $\mathcal{N}_{k+1}$ . If  $\mathcal{N}_{k+1}$  must vote on some issue, then  $D'$  votes on behalf of  $\mathcal{N}_k$ , and  $D$  casts no vote. (Note that  $D$  is still a voting member of  $\mathcal{N}_k$ —she simply no longer represents  $\mathcal{N}_k$  within  $\mathcal{N}_{k+1}$ ). For terminological clarity, we will say that  $D$  is a ‘metadelegate’ of  $\mathcal{N}_k$ .

Figure 1 portrays a fragment of this structure. Versions of pyramidal democracy have been proposed by Arendt (1965, p.278) and MacPherson (1977; §V(4A), p.108). A four-tier version, called the “communication tree”, was a key feature of MINERVA, an experiment in electronically-mediated democracy in the early 1970’s.<sup>5</sup> For the last decade, a three-tier form of pyramidal democracy (called *Participatory Democracy*) has been a central part of a system of ‘participatory civic planning’ in many cities in Brazil, starting with Porto Alegre.<sup>6</sup> Indeed, the United States Electoral College was originally intended as a three-tier

<sup>5</sup>See Etzioni (1971; §II.2) and Etzioni et al. (1975).

<sup>6</sup>See Santos (1998), Marquetti (2000), Fung and Wright (2001; §I.4), Lieberherr (2003), and Aragonès and Sánchez-Pagés (2005).

pyramidal democracy (the third tier being the President), although in reality it functions nothing like this.

However, our proposal is somewhat different than these earlier models, because it involves more tiers, smaller nodes, and a more fluid structure. In particular:

- The purpose of each node is to share knowledge and ideas, and to build consensus through dialogue. Thus, each node must be small enough that intelligent multilateral dialogue is possible; say around seven to ten people. For this reason we propose setting  $B := 7$ . (In contrast, the earlier pyramid models involved only three or four tiers, with nodes containing hundreds or thousands of people).
- Node membership is entirely voluntary. Citizens choose a node based on ideological affinity, rather than being assigned a node based on geographical proximity. Furthermore, a member can “defect” from a node at any time if she is dissatisfied with the consensus position of that node. (However, until she joins another node, the defector is effectively voiceless in the political process).
- Delegates can be replaced at any time. If they are dissatisfied with her performance, the members of a Tier  $k$  node  $\mathcal{N}_k$  can recall their delegate from Tier  $(k + 1)$  and replace her with a new delegate. (However, if  $D$  is a metadelegate from  $\mathcal{N}_k$  to a Tier  $(j + 1)$  node  $\mathcal{N}_{j+1}$  for some  $j > k$ , then  $\mathcal{N}_k$  *cannot* recall  $D$ , because  $D$  is no longer the official representative of  $\mathcal{N}_k$ . Only the intermediary Tier  $j$  node  $\mathcal{N}_j$  can recall  $D$ , because  $D$  is actually  $\mathcal{N}_j$ ’s delegate).

In §1, we will argue that the pyramid acts as a meritocratic selection device, whereby the most competent individuals will naturally rise to higher tiers and hence acquire greater political influence. In §2, we will examine the stability of the pyramid. Finally, §3 contains some miscellaneous observations about this proposal.

## 1 Pyramidal Meritocracy

In a representative democracy, the delegate chosen to represent each constituency should be the most competent candidate —i.e. the most intelligent, knowledgeable, ethical and dedicated. However, conventional electoral systems often fall far short of this ideal, for several reasons:

- (C1) Electoral success depends on advertising, and advertising requires a lot of money, so conventional elections heavily favor the interests of the wealthy.<sup>7</sup> Political parties have emerged as the most efficient way to obtain and deploy campaign funds, so they now entirely control the nomination process. Thus, candidates are drawn from a small, exclusive clique of party apparatchiks, and are selected not for their competency, but for their charisma, ‘electability’, and partisan loyalty.

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<sup>7</sup>The influence of political advertising (and thus, campaign financing) on elections has been extensively studied; see Morton and Cameron (1992), Austen-Smith (1997), and (Mueller, 2003, §20.2-20.3) for summaries.

- (C2) In a conventional election, a voter is generally presented with very few (e.g. two or three) candidates, who represent widely different ideologies. Of these, she finds an even smaller number (e.g. one or less) ideologically palatable. Thus, after accounting for ideology, she is left with no discretion to optimize on the basis of the candidate's competency.
- (C3) Voters have no opportunity for long-term personal interaction with the candidates. Instead, voters base their opinions on campaign advertising, television soundbites and the facile 'analysis' of media pundits.
- (C4) Even if [despite (C1)-(C3)], voters had a wide variety of candidates to choose from, and easy access to abundant and accurate information about each one, many voters would still make ignorant and irrational decisions [see e.g. Schumpeter (1942 [1976])].
- (C5) Furthermore, voters are usually too apathetic to correct these deficiencies, because of what Downs (1957) calls 'rational ignorance'. Voters have little incentive to spend time and money to become better educated or informed, for the same reason that they have little incentive to vote in the first place: because each voter knows that her vote has only an infinitesimal effect on the outcome of the election, and even less influence on public policy.
- (C6) Even if, despite reasons (C1)-(C5), a conventional election somehow acted as a mechanism to select high-quality candidates, conventional electoral systems involve only one 'iteration' of this mechanism.

Condorcet's (1785) Jury Theorem<sup>8</sup> is sometimes invoked to argue that majority vote will pick the best candidate. However, the Jury Theorem is inapplicable for three reasons:

- The Jury Theorem assumes that each voter has a probability greater than 50% to pick the best candidate. This is far from clear, especially in light of reasons (C2)-(C5) above.
- The Jury Theorem models the voters' choices as independent random variables —i.e. the voters make their decisions entirely independently of one another. This is clearly false: voters discuss politics extensively with one another prior to voting, and they are all inundated by the same torrent of propaganda and punditry, so their choices are highly correlated.
- Even if its hypotheses were satisfied, the Jury Theorem implies that a majority vote will choose the best option *from a given ballot of alternatives*. In conventional representative democracy, this ballot is assembled by political parties, so in light of (C1), the candidates on the ballot are not necessarily of high competency.

In contrast, pyramidal democracy should do a much better job of selecting the most competent candidates for Parliament, for the following six reasons:

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<sup>8</sup>See §6.1 of Mueller (2003).

- (P1) PD is economically egalitarian: everyone has equal opportunity to rise to higher tiers of the pyramid, regardless of her wealth or political connections. There is no longer any role for campaign financing, because there are no longer any election campaigns. Instead of being indispensable sources of campaign funds, parties will revert to their original role as mere debating clubs or advocacy groups, with no real political clout.
- (P2) Nodes are ideologically homogeneous (except perhaps at the very top tiers), because they self-organize on the basis of ideological affinity. Thus, the delegacy candidates in each node are ideologically similar, so that the choice amongst them will be made primarily based on competency, not ideology.
- (P3) Each delegacy candidate is extremely well-known to her ‘constituents’ (i.e. fellow node members), because they have discussed policy and personally interacted with her over a long period. Hence, presumably, these constituents can make a well-informed choice, and choose the most competent candidate as their delegate.
- (P4) Because of reasons (P1)-(P3), the delegates who appear in Tier 2 will generally be the most competent members of the Tier 1 population. Likewise, those in Tier 3 will generally be the most competent members of Tier 2. Inductively, each successive Tier will generally contain the most competent members of the previous Tier. The more competent the members of a Tier become, the more willing and able they will be to choose the most competent candidates amongst themselves to ascend to the next Tier.
- (P5) Furthermore, each voter has a strong incentive to educate herself and vote intelligently, because her vote has a clear and significant influence on the decisions made by her node. It is no longer ‘rational’ to be ignorant.
- (P6) To the extent that delegate election is a mechanism which selects high-competency candidates, pyramidal democracy involves several iterations of this mechanism, and each iteration further winnows the pool of candidates. Suppose we disregard reasons (P1)-(P5) and suppose that nodes are no better at choosing their delegates than conventional elections are at choosing legislators; even then, multiple iterations of this process will still improve the outcome.

To illustrate these arguments with a simple mathematical model, we make the following simplifying assumptions:

- (A1) Each candidate’s intelligence, education, creativity, integrity, etc. is aggregated into a single real number, which we call the *competency* of that candidate. Candidates with higher competency are preferable to those with lower competency.
- (A2) All voters have the same notion of competency —i.e. all voters place the same relative importance on personal traits such as intelligence, integrity, etc.
- (A3) The competency levels over the population are distributed according to a probability distribution with probability density function  $g : \mathbb{R} \rightarrow \mathbb{R}$ .



(A4) Each constituency tries to select the most competent candidate; thus candidates of higher competency are more likely to be chosen than those of lower competency. However, because of imperfect information, there is always nonzero probability that a low-competency candidate will be chosen instead of a high-competency candidate. We assume there is some nondecreasing function  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x)$  is the conditional probability density that a candidate will be elected, given that her competency level is  $x$ .

The function  $f$  describes the ‘efficacy’ of the electoral process in choosing a high-competency candidate. If  $f(x) = 1$  for all  $x$ , then the electoral process is totally blind to competency. If  $f$  is increasing, then the process favors higher-competency candidates. The more rapidly  $f$  increases, the more effective the process becomes at choosing the most competent candidate.

Let  $k := \int_{-\infty}^{\infty} f(x)g(x) dx$ . Then given assumptions (A1)-(A4), the competency of the elected candidate will be a random variable with density function  $h(x) := f(x)g(x)/k$ . For example, suppose that  $g(x)$  is a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ . Let  $\alpha > 0$ , and suppose that  $f(x) = e^{\alpha x}$ . (Thus, a larger value of  $\alpha$  represents an electoral system which is better at picking high-competency candidates.) Then  $f(x)g(x) = k h(x)$ , where  $k := e^{\alpha(\mu + \sigma^2/2)}$  and where  $h(x)$  is a Gaussian distribution with variance  $\sigma^2$  and mean  $\mu + \sigma^2\alpha$ . In other words, the mean competency of elected delegates is  $\sigma^2\alpha$  higher than the mean competency of the general population.

Our choice of a Gaussian for  $g$  is quite natural, but our choice for  $f$  is somewhat artificial. Nevertheless, this example illustrates a general principle: the more rapidly the function  $f$  increases, the higher will be the average competency of the elected candidates.

Let  $f_0 : \mathbb{R} \rightarrow \mathbb{R}$  describe the ‘efficacy’ of conventional elections. Then arguments (C1)-(C5) above suggest that  $f$  will be, at best, slowly increasing. Let  $f_1 : \mathbb{R} \rightarrow \mathbb{R}$  describe the efficacy of the delegate elections in the first Tier of the pyramid. We have insufficient information to posit a specific expression for  $f_0$  or  $f_1$ ,<sup>9</sup> but arguments (P1)-(P5) suggest that  $f_1$  will be more rapidly increasing than  $f_0$ . Now, for all  $k > 1$ , let  $f_k$  describe the efficacy of the delegate selection process at the  $k$ th Tier. Then argument (P4) suggests that  $f_2$  will increase more rapidly than  $f_1$ , because the mean level of competency in Tier 2 will be higher than that in Tier 1, and hence, Tier 2 voters will be even more capable of selecting the most competent candidates amongst themselves. Likewise,  $f_3$  will increase even more rapidly than  $f_2$ , and so on.

The competency distribution of conventional election winners is given by the density  $h(x) := f(x)g(x)/k$ , as described above. In contrast, the competency distribution of the Parliament in an eight-tier pyramidal democracy will be described by the density  $H(x) := f_7(x)f_6(x) \cdots f_2(x)f_1(x)g(x)/K$ , where  $K := \int_{-\infty}^{\infty} f_7(x)f_6(x) \cdots f_2(x)f_1(x)g(x) dx$ .

For example, suppose  $f_k(x) = e^{\alpha_k x}$  for all  $k$ . Then the previous argument implies that  $\alpha_7 \geq \alpha_6 \geq \cdots \geq \alpha_2 \geq \alpha_1 > \alpha_0$ . Assuming an initially Gaussian distribution with mean 0 and variance  $\sigma^2$ , the mean competency of a conventional election winner is  $\sigma^2\alpha_0$ . However,

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<sup>9</sup>However, see Huckfeldt (2001) for an interesting empirical study of subjective estimates of ‘political competency’ in deliberative settings.

the mean competency of the pyramidal Parliament is  $\sigma^2(\alpha_1 + \alpha_2 + \dots + \alpha_7)$ , which is more than seven times as large.

## 2 Stability

Pyramidal democracy is extremely fluid and responsive to the electorate. At any time, a dissatisfied node can replace its delegate. A dissatisfied voter can defect from a node, and too many defections can force the node to dissolve. This fluidity and responsiveness is an asset, but it can also lead to excessive political instability: it is possible for a ‘cascade’ of delegate replacements or defections/dissolutions to propagate up the hierarchy of the pyramid. We will now construct a simple mathematical model to study this problem.

**Delegate Replacement.** We first distinguish between two kinds of delegate replacement. An *endogenous* replacement occurs when the majority of the members of a node become dissatisfied with their delegate and replace her. In a node  $\mathcal{N}$  in Tier 2 or higher, there can also be *exogenous* replacements: this occurs when one or more members of  $\mathcal{N}$  (each being a delegate from a lower tier) are themselves replaced, and this changes the balance of power in  $\mathcal{N}$  so as to precipitate an immediate delegate replacement in  $\mathcal{N}$ . We make the following assumptions:

- (a) In any single node  $n$ , the endogenous replacement of delegates is a continuous-time Poisson process with some rate  $v_n > 0$ , where  $v_n$  measures the endogenous political ‘volatility’ of node  $n$ . (Nodes with higher volatilities replace their delegates more frequently.) The endogenous replacements in distinct nodes are independent processes.
- (b) The volatilities of all nodes are independent random variables. The volatilities of all nodes in Tier  $t$  have the same distribution, with mean  $\bar{v}_t$ .
- (c) Whenever a node at Tier  $t$  replaces its delegate, there is a probability  $\alpha_t > 0$  that this triggers an exogenous replacement event in the next higher tier.

If  $\mathcal{N}_2$  is a Tier 2 node, then assumptions (a)-(c) imply that the delegate replacements (both endogenous and exogenous) in  $\mathcal{N}$  obey a Poisson process with rate

$$v_{\mathcal{N}}^* := v_{\mathcal{N}} + \alpha_1 \sum_{n \in \mathcal{N}} v_n. \quad (1)$$

Inductively, suppose  $\mathcal{N}$  is a Tier  $T$  node. For all  $t \in \{1, \dots, T-1\}$ , let  $\mathcal{N}_t$  be the set of all Tier  $t$  nodes below  $\mathcal{N}$ . Then the replacements in  $\mathcal{N}$  obey a Poisson process with rate  $v_{\mathcal{N}}^*$ , where  $v_{\mathcal{N}}^*$  is defined inductively by:

$$\begin{aligned} v_{\mathcal{N}}^* &= v_{\mathcal{N}} + \alpha_{t-1} \sum_{n \in \mathcal{N}_{t-1}} v_n^* = \dots\dots\dots \\ &= v_{\mathcal{N}} + \alpha_{t-1} \sum_{n \in \mathcal{N}_{t-1}} v_n + (\alpha_{t-1}\alpha_{t-2}) \sum_{n \in \mathcal{N}_{t-2}} v_n + \dots + (\alpha_{t-1}\dots\alpha_2\alpha_1) \sum_{n \in \mathcal{N}_1} v_n. \end{aligned} \quad (2)$$

Let  $N$  be the average node size, suppose the pyramid has  $T$  tiers below the Parliament, and suppose  $\alpha_1 \approx \alpha_2 \approx \dots \approx \alpha_{T-1} \approx \alpha$  for some constant  $\alpha$ . If  $\mathcal{N}$  is a Tier  $T$  node (i.e. its delegate is in Parliament), then the Law of Large Numbers says we can approximate eqn.(2) by

$$v_{\mathcal{N}}^* \approx v_{\mathcal{N}} + \alpha N \bar{v}_{T-1} + \alpha^2 N^2 \bar{v}_{T-2} + \dots + \alpha^{T-1} N^{T-1} \bar{v}_1. \quad (3)$$

For example, suppose  $\bar{v}_1 \approx \bar{v}_2 \approx \dots \approx \bar{v}_T \approx \bar{v}$  for some constant  $\bar{v}$ . In equation (3), if  $\alpha \ll 1/N$ , then  $v_{\mathcal{N}}^* \approx \bar{v}$  for any node  $\mathcal{N}$ —all nodes about equally volatile. If  $\alpha = 1/N$ , then  $v_{\mathcal{N}}^* \approx T\bar{v}$ . However, if  $\alpha > 1/N$ , then  $v_{\mathcal{N}}^* \approx (\alpha C)^{T-1} \bar{v}_1$ , so Tier  $T$  nodes are exponentially more volatile than Tier 1 nodes.

Thus, the membership of Parliament will be reasonably stable as long as  $\alpha_1, \dots, \alpha_{T-1}$  and  $\bar{v}_1, \dots, \bar{v}_T$  are small enough. To ensure this, we suggest the following policies:

- A Tier  $t$  node must wait  $M_t$  days before replacing its delegate. (This introduces a ‘cooling period’, during which time a reconciliation might occur). Increasing  $M_t$  will decrease  $\bar{v}_t$ .
- When a Tier  $t$  node replaces its delegate, there is an  $N_t$  day ‘initiation period’ during which the new delegate cannot vote in the higher tier. (Thus, the node is ‘penalized’ for replacing its delegate, by being disenfranchised for  $N_t$  days; this discourages capricious replacement of delegates). Increasing  $N_t$  will decrease both  $\bar{v}_t$  and  $\alpha_t$ .

$M_1, \dots, M_T$  and  $N_1, \dots, N_T$  are control parameters with which to ‘tune’ the stability of the pyramid. If the pyramid exhibits too many delegate replacements in Tier  $t$ , then we can decrease  $\bar{v}_t$  and/or  $\alpha_t$  by increasing  $M_t$  and/or  $N_t$ . (However, it is important that  $M_t$  and  $N_t$  still be kept as small as possible, to maximize the accountability of delegates to their constituents).

**Defection and Dissolution.** If (through defection) a node ever drops below the minimum size  $B$ , then it has a very short ‘grace period’ (e.g. 30 days) to replace the defector(s) and satisfy the minimum size requirement—otherwise the node is dissolved, and its members must join other nodes or be disenfranchised. To create a safety cushion against such a ‘membership crisis’, most nodes will probably choose to have an excess of members over the minimum  $B$  (so that no single member can extort concessions by threatening to defect). For example, if  $B = 7$ , then most nodes would probably have nine or ten members. There is no maximum size to nodes. However, increasing the size of a node dilutes the effective political power of each member, so members have an interest in keeping nodes small. Thus, the size of the node represents a trade-off between greater influence for each member, versus greater stability for the node as a whole.

If a Tier 1 node dissolves due to a membership crisis, then its delegate immediately leaves the corresponding Tier 2 node; this could compromise the viability of the Tier 2 node, leading to *its* dissolution, and so forth. We call this scenario a *dissolution cascade*. Clearly, a high frequency of dissolution cascades could compromise the stability of the pyramid.

A node can dissolve in two ways. An *endogenous* dissolution occurs when some member defects, reducing the node's population below  $B$ , and the node is unable to replace the defector within the grace period. A node  $\mathcal{N}$  in Tier 2 or higher can also suffer *exogenous* dissolution: this occurs when some member  $D$  of  $\mathcal{N}$  is a delegate from a lower-tier node  $n$ , and the node  $n$  dissolves, so that  $D$  must immediately leave  $\mathcal{N}$ , reducing  $\mathcal{N}$ 's population below  $B$  and indirectly causing the dissolution of  $\mathcal{N}$  as well.

Suppose the population of each node fluctuates according to a stationary, continuous time Markov process. Then the dissolution of each node occurs according to a Poisson process. We make the following assumptions:

- (a) The endogenous dissolution of node  $n$  occurs according to a continuous-time Poisson process with some rate  $v_n > 0$ , where  $v_n$  measures the endogenous political ‘volatility’ of node  $n$ . (Nodes with higher volatilities are more likely to dissolve.) The endogenous dissolutions of distinct nodes are independent processes.<sup>10</sup>
- (b) The volatilities of all nodes are independent random variables. The volatilities of all nodes in Tier  $t$  have the same distribution, with mean  $\bar{v}_t$ .
- (c) Whenever a node at Tier  $t$  dissolves, there is a probability  $\alpha_t > 0$  that this triggers an exogenous dissolution in the next higher tier.

Clearly, this yields a mathematical model of which is formally identical to the previous model of delegate replacement. The conclusion is the same: as long as the values  $\alpha_1, \dots, \alpha_{T-1}$  and  $\bar{v}_1, \dots, \bar{v}_T$  are small enough, dissolution cascades will be rare events, and the pyramid will be relatively stable. To make these values small enough, we suggest the following policies:

- As suggested above, if the population of a Tier  $t$  node drops below  $B$ , then this node has a ‘grace period’ of  $L_t$  days to recruit new members before it is dissolved. Increasing  $L_t$  decreases  $\bar{v}_t$ ; if  $t \geq 2$ , then it also decreases  $\alpha_{t-1}$ .
- A Tier  $t$  node member must wait  $M_t$  days before defecting. (This introduces a ‘cooling period’, during which time a reconciliation might occur; it also gives the node time to find a replacement for the defector). Increasing  $M_t$  decreases  $\bar{v}_t$ .
- If a Tier  $t$  node member defects, there is an  $N_t$  day period before the defector can join a new node. (Thus, the defector is ‘penalized’ for defecting, by being disenfranchised for  $N_t$  days; this discourages capricious defections). Increasing  $N_t$  decreases  $\bar{v}_t$ .

If the pyramid exhibits too many dissolutions in Tier  $t$ , then we can decrease  $\bar{v}_t$  and/or  $\alpha_t$  by increasing  $L_t$ ,  $M_t$  and/or  $N_t$ . (However, it is important that these parameters be kept as small as possible, to grant citizens maximal mobility to migrate between nodes).

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<sup>10</sup>This is somewhat unrealistic: defections and dissolutions may occur in response to a polity-wide controversy or crisis, which would simultaneously impact many nodes.

### 3 Concluding remarks

**Suffrage.** It is generally acknowledged that children and mentally infirm adults should not vote. Presumably, they should not participate in other deliberative institutions either. However, any legal distinction between ‘child’ and ‘adult’ or between ‘firm’ and ‘infirm’ is arbitrary and leads to intellectually indefensible inconsistencies. For example, in many western democracies, the voting age is eighteen; yet there certainly exist thoughtful, well-informed, politically engaged youths who may be more competent to vote than many ignorant, politically apathetic adults. Presumably, the age of eighteen is chosen to roughly coincide with the completion of high school. However, an adult who quit school in grade nine can still vote; indeed, even functionally illiterate and innumerate adults have the franchise, and any proposal to restrict suffrage to people with some minimum educational level is regarded as highly undemocratic. The distinction between mentally ‘firm’ and ‘infirm’ (as decided by a psychiatrist) is even more subjective and ambiguous, and transfers troubling political power to the psychiatric profession.

Pyramidal democracy provides a natural solution to this problem. We can allow all citizens—even children and the mentally infirm—to participate in Tier 1 of the pyramid (perhaps in familial nodes). The meritocratic mechanisms discussed in §1 will naturally identify incompetent voters and curtail their political influence. Presumably children and the mentally infirm will *not* be chosen as delegates to Tier 2. However, by participating in Tier 1, these citizens can still communicate their needs and wishes through their delegate, and thus we ensure that society takes these needs into account in legislation.

**Bargaining and Consensus.** In a conventional majority/plurality vote, it is possible for a large minority (or even a majority) to be strongly dissatisfied with the outcome. This undermines the legitimacy of the decision and may lead to problems with implementation and compliance. Representative democracies using plurality vote are also vulnerable to many ‘voting paradoxes’, such as Anscombe’s (1976) and Ostrogorski’s (1902) Paradoxes (Nurmi, 1998, 1999). To address these problems, a variety of elaborate voting systems have been proposed (e.g. Borda count, transferable votes, approval voting). However, any voting system, no matter how elaborate, is vulnerable to inconsistent and pathological outcomes (by Arrow’s Impossibility Theorem) and is manipulable by ‘strategic voters’ (by the Gibbard-Satterthwaite theorem).<sup>11</sup> Voting systems are also prone to ‘Condorcet cycles’, creating political instability and possible manipulation through agenda control.<sup>12</sup>

In the sub-parliamentary nodes of the pyramid, however, we imagine that most political decisions (e.g. the choice of delegate, the endorsement of certain policy proposals) will *not* be made by voting. Instead, they will be made through a process of deliberation and bargaining leading to unanimous or near-unanimous consensus. This is for two reasons:

1. Each node is very small (e.g. ten people) so multilateral discussion and negotiation is entirely feasible. In a small group, it is possible to continuously deploy new pro-

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<sup>11</sup>See (Riker, 1982, Ch.5-6) or (Mueller, 2003, Ch.24) for a summary of these results.

<sup>12</sup>See (Riker, 1982, Ch.7), (Austen-Smith and Banks, 1999, Ch.6), or (Mueller, 2003, §5.12.1) for a summary.

posals which creatively compromise between conflicting positions, until a mutually acceptable arrangement is found.

2. Node membership is voluntary; a dissatisfied member can ‘defect’ at any time.

Reason #2 means that a node member will not be satisfied if she is out-voted on an important issue; she will defect to some other node more congenial to her views. Thus, on every issue, each node must struggle to achieve a consensus which is at least tolerable to all its members, or the node will cease to exist. Fortunately, Reason #1 means that this struggle can usually succeed.

Of course, unanimous consensus is sometimes impossible, because the individual positions are too divergent. This becomes more likely at higher tiers of the pyramid, where nodes must bring together delegates with increasingly different views. Furthermore, each delegate is constrained to represent the consensus position of her constituency, and may only have limited discretion to compromise. Consensus is also less likely for urgent decisions which leave no time for negotiations. Finally, on some issues, there is an exogenous, finite ballot of alternatives, and it is not possible to introduce new alternatives through ‘creative compromise’. Nevertheless, on most decisions in sub-parliamentary nodes, we expect to see supermajoritarian support, often approaching unanimity. Supermajoritarian decisions are far less likely to exhibit voting paradoxes<sup>13</sup>, and are less vulnerable to Condorcet cycles<sup>14</sup>.

Reason #2 ceases to be applicable in Parliament. Here, defection is not an option, so some decisions may ultimately be decided by voting, possibly irritating of a large minority. However, even in this context, Reason #1 still applies. Furthermore, deliberation may make Arrowian pathologies less likely, by encouraging the formation of ‘single-peaked’ preferences.<sup>15</sup>

**Public vs. private deliberation.** Public deliberation (i.e. in front of an audience) tends to be superficial, adversarial and demagogic. It also makes compromise difficult, because it is embarrassing for speakers to admit mistakes or modify their position; see e.g. Stasavage (2007). Private deliberation allows serious, honest discussion without rhetorical theatrics; however it also creates the possibility for corruption, collusion, and cooption; see e.g. (Gutmann and Thompson, 1996, Ch.3). The best deliberative institution would require both private and public deliberation. As observed by Elster (1998a),

The process ought to contain elements of both secrecy (committee discussion) and publicity (plenary assembly discussions). With total secrecy, partisan interests and logrolling come to the forefront, whereas full publicity encourages

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<sup>13</sup>See Nurmi and Uusi-Heikkilä (1985), Wagner (1983, 1984) and Deb and Kelsey (1987). See (Nurmi, 1998, §3.2) or (Nurmi, 1999, §7.6) for summaries.

<sup>14</sup>See Greenberg (1979), McKelvey and Schofield (1986), Caplin and Nalebuff (1988, 1991), Weber (1993), Banks (1995), and Saari (1997). See (Mueller, 2003, §5.8.2-5.8.3) for a summary.

<sup>15</sup>See e.g. Knight and Johnson (1994), Miller (1992, 2003), or Dryzek and List (2004).

grandstanding and rhetorical overbidding. Conversely, secrecy allows for serious discussion, whereas publicity ensures that any deals struck are capable of withstanding the light of day. (p.117)

Pyramidal democracy implements this private/public dichotomy. The members of a Tier  $t$  node can discuss an issue in private, until they reach a consensus position. The node's delegate can then articulate this position within the more public context of a Tier  $(t + 1)$  node, where she must defend it against other delegates from other Tier  $t$  nodes, and perhaps compromise until the Tier  $(t + 1)$  tier node reaches some consensus. This process iterates all the way up the pyramid. Deliberation within the Parliament will involve both private discussions and public debates. Furthermore, each Parliament member must justify her decisions in face-to-face dialogue with her constituents (who must then justify these decisions to *their* constituents, and so on), so it will be difficult for her to betray the public interest for personal gain.

**Group Polarization.** Self-assembled deliberative nodes are vulnerable to a phenomenon Sunstein (2003) calls 'group polarization': ideologically similar individuals form an 'enclave' where they reinforce one another's beliefs, causing the whole enclave to evolve towards a more ideologically extreme position. A preponderance of divergent, extremist enclaves can undermine the stability of the polity. Sunstein notes that this danger is greatest when the enclaves evolve in isolation from one another. However, he observes that enclaves also provide a space for marginalized political communities to articulate their views: "A special advantage of... 'enclave deliberation' is that it promotes the development of positions that would otherwise be invisible, silenced, or squelched by general debate. ...[m]any desirable social movements have been made possible through this route" (p.94). He concludes: "It is desirable to create spaces for enclave deliberation without insulating enclave members from those with opposing views, and without insulating those outside of the enclave from the views of those within it" (p.98).

Pyramidal democracy instantiates Sunstein's suggestion: citizens can aggregate into ideologically homogeneous nodes ('enclaves'), where marginalized (perhaps extremist) ideologies can flourish, but each nodes must send a delegate, who must deliberate and ultimately compromise with the representatives of opposing views.

**Implementation.** Radical and utopian political schemes either founder upon the resistance of vested interests, or unleash dangerously unpredictable consequences. For this reason, most deliberative-democratic proposals are firmly embedded in the conventional political framework. Pyramidal democracy is considerably more radical than many of these proposals, and it would be both reckless and politically impossible to implement it immediately in a large-scale polity. Instead, we propose to first test pyramidal democracy in small and informal democratic settings, such as the governance of student groups, private clubs, and professional associations. These 'micropolities' generally deal with rather trivial matters, and there are ample exit opportunities for dissatisfied members if pyramidal democracy spectacularly fails as a form of governance. (These polities also generally

have high levels of voter apathy, but pyramidal democracy automatically adjusts to this: apathetic voters simply remain in the lower tiers of the pyramid.)

If pyramidal democracy succeeds in these micropolities, the next target would be the governance of publicly traded corporations. As first argued by Berle and Means (1932 [1968]), the widely dispersed shareholders of a large public firm actually have very little real oversight or control, even though they exercise formal democratic discipline over the management. We propose to replace the existing shareholder democracy with a pyramidal democracy, where each shareholder receives a weight proportional to her share ownership. The Parliament of this pyramid would act as the Board of Directors; it would appoint the Officers of the firm, and would be consulted on major policy decisions.

For example, a firm having  $7^5 = 16807$  shares in circulation could form a pyramid with five tiers, with each seven-member node at Tier  $k$  representing  $7^{k-1}$  shares. A person owning one share would begin in Tier 1, but a person owning more than 7 shares would constitute a ‘node’ at Tier 1, and so could act as a delegate to Tier 2. Likewise, the owner of more than 49 shares would automatically ascend to Tier 3, and the owner of more than 343 shares would ascend to Tier 4. The owner of more than 2401 shares (i.e. more than one seventh of the entire firm) would automatically be the Tier 5 Parliament (which would contain at most seven members).

Corporate governance provides a natural laboratory to tinker and experiment with pyramidal democracy, and limits the fallout from catastrophic failure: dissatisfied shareholders can simply disinvest in a badly governed firm. If pyramidal democracy succeeds in this setting, the next target would be municipal governments. Once pyramidal democracy has been tested and perfected in these small polities, it could be introduced into larger polities (e.g. regional or federal governments), first perhaps in an advisory role, but with gradually increasing levels of control, until it entirely eclipsed conventional legislative institutions.

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