# Decomposing Time-Frequency Relationship between Interest Rates and Share Prices in India through Wavelets

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***Abstract***

The study analyzed Granger-causality between interest rate (IR) and share prices (SP) for the India by using monthly data covering the period of 1990M1 to 2009M3. The time-frequency relationship between IR and SP was decomposed through continuous wavelet approach for the first time in the study. We found that for the Indian economy the causal and reverse causal relations between SP and IR vary across scale and period viz., *during the late 1993 and early 1994, in 1-4 year scale, IR is lagging with cycle effects from SP*, *whereas during 1998-2001, in 8~12 year scale, IR is leading with cyclical effects on the SP*. Further, results show that *during 2003 to early 2005 (in 1~6 year scale) and again after late 2006 (in 9~14 year scale) IR is lagging and receiving anti-cyclical effects from SP.*

**Keywords:** cyclical effects, anti-cyclical effects, Granger-causality, phase difference, cross wavelets, wavelet coherency.

**JEL Classification**: C10, E32.

## 1. Introduction

Interest rates play a dominant role in affecting other macro-economic variables related, broadly, to the money and capital markets such as share price. For example, volatility of the interest rate affects directly capital reallocation between short-term money markets and long-term capital markets and hence, influences investors' decision to invest in the capital market. In addition to that interest rate is one instrument of monetary policy and it is also affected by money supply. Needless to say, monetary authority of a country i.e., central banks operates concurrently at different time scales with diverse objectives in the short and long run. In addition to that, it is well known fact that in various economic practices several economic agents are involved who have dissimilar term objectives i.e., some economic agents are giving priorities to the daily movements and co-movements and other economic agents are focusing to the longer horizons. However, in the stock markets, situation of share price (which is the focus of the paper) is a little bit more complex vis-à-vis other macroeconomic variable. This is mainly due to the existence of the nonlinearities in stock returns. The major cause of these nonlinearities is the misperceptions of market participants towards the association between risk and return. The above discussion discloses that there are several questions about time series economic data that need to be addressed to understand the behavior of key macroeconomic variables at different frequencies over time. Noteworthy to mention that to discover such information is complicated using pure time-domain or pure frequency-domain methods.

However, it is recently, in order to model the non-coherent financial markets non-parametric or semi-parametric models such as neural networks and wavelets analysis have been used. Gencay and Selcuk (2004) documented that nonlinearity in the stock markets can be captured through wavelets analysis and this has been empirical verified. As Cifter and Ozun (2008a) documented that "the starting point of the wavelets model is that fixed time scales are not sufficient to capture the misperception of risk and returns that maybe present in emerging financial markets. Therefore, forecasting stock returns in emerging markets should be done through a time-adaptive system simultaneously considering all time-scales of the distributions." Norsworty *et al.* (2000) with the application of wavelets analysis examined the relationship between an individual asset return and that of market based on time-scale decomposition in order to test for the existence of discrepancies at different frequencies. The authors found that for the higher frequencies there was higher impact of the market portfolio return on the individual asset.

Further, Ramsey and Lampart (1998) by employing wavelets examined the relationship among consumption, GDP, income and money and showed the time-scale decomposition thereof. The authors conclude that the relationship among the economic variables such as consumption and income change in different time scales. Other works that applied wavelet analyses for causal relationships include Kim and In (2003), Almasri and Shukur (2003), Zhang and Farley (2004), Dalkır (2004), Gencay *et al.* (2002) and Gallegati (2005). Cifter and Ozun (2008a,b) and Cifter (2006) applied wavelet analysis to

Turkish financial market data. Mitra (2006) is one applied wavelet analysis to Indian financial and economic variables data.

However, all above mentioned studies have utilized discrete wavelet approach whereas this paper aims to analyze the effects of changes in interest rates on stock returns in the framework of continuous wavelets (please refer to section two for advances of this approach over the discrete wavelet approach) which is able to detect the possible nonlinearities and complexities in such markets. In this direction, to best of our knowledge, this is the first attempt for an economy. This paper models the relationship between in the Share Prices (SP) (Index Numbers (2005=100): Period Averages) and Interest Rate (IR) (measured by Lending Rate-Percent Per Annum) by using frequency domain analysis. In this respect, detecting interest rates effects on share prices can provide evidence for the absence of market efficiency as well. We found that, for the Indian economy, the causal and reverse causal relations between SP and IR vary across scale and period. Our analysis revealed that *during 1998-2001, in 8~12 year scale, IR was leading with cyclical effects on the SP.*

This paper is organized as follows. Section two presents the data used and a brief introduction about the continuous wavelets analysis methods. Section three presents the empirical findings on the relationship between interest rates and share prices at different time scales. The paper concludes in section four with suggestions for future research in this direction.

## 2. Data Source and Methodology

### *2.1 Data Source*

For the analysis we obtain data of share prices and interest rate form IMF CD ROM (2010) with monthly observations covering the period from 1990M1 to 2009M3.

### *2.2 Motivation and Introduction to Methodology*

In many studies, the analysis is exclusively done in the time-domain and the frequency domain is ignored. However, some appealing relations may exist at different frequencies: interest rate may act like a supply shock at high and medium frequencies (as it is dependent upon short run or medium run monetary targets), therefore, affecting share prices, whereas, in the longer run (i.e., at the lower frequencies) it is the share prices, through a demand effect, that affects interest rate.

There has been a general practice to utilize Fourier analysis to expose relations at different frequencies between share price and interest rate. However, the shortcomings of the use of Fourier transform for analysis has been well established. A big argument against the use of Fourier transform is the total loss of time information and thus making difficult to discriminate ephemeral relations or to identify structural changes which is very much important for time series macro-economic variables for policy purposes. Another strong argument against the use of Fourier transform is the reliability of the results. It is strongly recommended (i.e., it is based on assumptions such as) that this technique is appropriate only when time series is stationary, which is not so usual the case with macroeconomic variables. The time series of these variables are mostly noisy, complex and rarely stationary.

To overcome such situation and have the time dimensions within Fourier transform, Gabor (1946) introduced a specific transformation of Fourier transform. It is known as the short time Fourier transformation. Within the short time Fourier transformation, a time series is broken into smaller sub-samples and then the Fourier transform is applied to each sub-sample. However, the short time Fourier transformation approach was also criticized on the basis of its efficiency as it takes equal frequency resolution across all dissimilar frequencies (see Raihan et al., 2005 for detail).

Hence, as solution to the above mentioned problems wavelet transform took birth. It offers a major advantage in terms of its ability to perform "natural local analysis of a time-series in the sense that the length of wavelets varies endogenously: it stretches into a long wavelet function to measure the low-frequency movements; and it compresses into a short wavelet function to measure the high-frequency movements" Aguiar-Conraria and Soares (2011, p. 646). Wavelet posses interesting features of conduction analysis of a time series variable in spectral framework but as function of time. In other words, it shows the evolution of change in the time series over time and at different periodic components i.e., frequency bands. However, it is worthy to mention that the application of wavelet analysis in the economics and finance is mostly limited to the use of one or other variants of discrete wavelet transformation. There are various things to consider while applying discrete wavelet analysis such as up to what level we should decompose. Further, it is also difficult to understand the discrete wavelet transformation results appropriately and convince economists. The variation in the time series data, what we may get by utilizing any method of discrete wavelet transformation at each scale, can be obtained and more easily with continuous transformation. For example, looking at Fig. 1 in any of quadrants one can immediately conclude the evolution of the variance of the SP or IR at the several time scales along the half-century observation and extract the conclusions with just a single diagram. Even if wavelets posses very interesting features, it has not become much popular among economists because of two important reasons as pointed out by Aguiar-Conraria et al. (2008). Aguiar-Conraria et al. (2008, p. 2865) pointed out that "first, in most economic applications the (discrete) wavelet transform has mainly been used as a low and high pass filter, it being hard to convince an economist that the same could not be learned from the data using the more traditional, in economics, band pass-filtering methods. The second reason is related to the difficulty of analyzing simultaneously two (or more) time series. In economics, these techniques have either been applied to analyze individual time series or used to individually analyze several time series (one each time), whose decompositions are then studied using traditional time-domain methods, such as correlation analysis or Granger causality."

To overcome the problems and accommodate the analysis of time frequency dependencies between two time series Hudgins et al. (1993) and Torrence and Compo (1998) developed approaches of the cross-wavelet power, the cross-wavelet coherency, and the phase difference. We can directly study the interactions between two time series at different frequencies and how they evolve over time with the help of the cross-wavelet tools. Whereas, (single) wavelet power spectrum help us understand the evolution of the variance of a time series at the different frequencies, with periods of large variance associated with periods of large power at the different scales. In brief, the cross-wavelet power of two time series illustrates the confined covariance between the time series. The wavelet coherency can be interpreted as correlation coefficient in the time–frequency space. The term "phase" implies the position in the pseudo-cycle of the series as a function of frequency. Consequently, the phase difference gives us information "on the delay, or synchronization, between oscillations of the two time series" (Aguiar-Conraria et al., 2008, p. 2867).

*2.2.1 The Continuous Wavelet Transform (CWT)*[1]

A wavelet is a function with zero mean and that is localized in both frequency and time. We can characterize a wavelet by how localized it is in time ($\Delta t$) and frequency ($\Delta\omega$ or the bandwidth). The classical version of the Heisenberg uncertainty principle tells us that there is always a tradeoff between localization in time and frequency. Without properly defining $\Delta t$ and $\Delta\omega$, we will note that there is a limit to how small the uncertainty product $\Delta t \cdot \Delta\omega$ can be. One particular wavelet, the Morlet, is defined as

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}. \tag{1}$$

where $\omega_0$ is dimensionless frequency and $\eta$ is dimensionless time. When using wavelets for feature extraction purposes the Morlet wavelet (with $\omega_0$=6) is a good choice, since it provides a good balance between time and frequency localization. We therefore restrict our further treatment to this wavelet. The idea behind the CWT is to apply the wavelet as a band pass filter to the time series. The wavelet is stretched in time by varying its scale (s), so that $\eta = s \cdot t$ and normalizing it to have unit energy. For the Morlet wavelet (with $\omega_0$=6) the Fourier period ($\lambda_{wt}$) is almost equal to the scale ($\lambda_{wt} = 1.03$s). The CWT of a time series ($x_n, n = 1,...,N$) with uniform time steps $\delta t$, is defined as the convolution of $x_n$ with the scaled and normalized wavelet. We write

$$W_n^X(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^{N} x_{n'} \psi_0\left[(n'-n)\frac{\delta t}{s}\right]. \tag{2}$$

[1] The description of CWT, XWT and WTC is heavily drawn from Grinsted et al. (2004). I am grateful to Grinsted and coauthors for making codes available at: http://www.pol.ac.uk/home/research/waveletcoherence/, which was utilized in the present study.

We define the wavelet power as $\left|W_n^X(s)\right|^2$. The complex argument of $W_n^X(s)$ can be interpreted as the local phase. The CWT has edge artifacts because the wavelet is not completely localized in time. It is therefore useful to introduce a Cone of Influence (COI) in which edge effects can not be ignored. Here we take the COI as the area in which the wavelet power caused by a discontinuity at the edge has dropped to $e^{-2}$ of the value at the edge. The statistical significance of wavelet power can be assessed relative to the null hypotheses that the signal is generated by a stationary process with a given background power spectrum ($P_k$).

Although Torrence and Compo (1998) have shown how the statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by an *AR (0)* or *AR (1)* stationary process with a certain background power spectrum ($P_k$), for more general processes one has to rely on Monte-Carlo simulations. Torrence and Compo (1998) computed the white noise and red noise wavelet power spectra, from which they derived, under the null, the corresponding distribution for the local wavelet power spectrum at each time *n* and scale *s* as follows:

$$D\left(\frac{\left|W_n^X(s)\right|^2}{\sigma_X^2} < p\right) = \frac{1}{2} P_k \chi_v^2(p), \tag{3}$$

where $v$ is equal to 1 for real and 2 for complex wavelets.

*2.2.2 The Cross Wavelet Transform*

The cross wavelet transform (XWT) of two time series $x_n$ and $y_n$ is defined as $W^{XY} = W^X W^{Y*}$, where $W^X$ and $W^Y$ are the wavelet transforms of $x$ and $y$, respectively, * denotes complex conjugation. We further define the cross wavelet power as $\left|W^{XY}\right|$. The complex argument arg($W^{xy}$) can be interpreted as the local relative phase between $x_n$ and $y_n$ in time frequency space. The theoretical distribution of the cross wavelet power of two time series with background power spectra $P_k^X$ and $P_k^Y$ is given in Torrence and Compo (1998) as

$$D\left(\frac{\left|W_n^X(s) W_n^{Y*}(s)\right|}{\sigma_X \sigma_Y} < p\right) = \frac{Z_v(p)}{v} \sqrt{P_k^X P_k^Y}, \tag{4}$$

where $Z_v(p)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two $\chi^2$ distributions.

*2.2.3 Wavelet Coherence (WTC)*

As in the Fourier spectral approaches, Wavelet Coherency (WTC) can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local correlation, both in time and frequency, between two time series. While the Wavelet power spectrum depicts the variance of a time-series, with times of large variance showing large power, the Cross Wavelet power of two time-series depicts the covariance between these time-series at each scale or frequency. Aguiar-Conraria et al. (2008, p. 2872) defines Wavelet Coherency as "the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local (both in time and frequency) correlation between two time-series".

Following Torrence and Webster (1999) we define the Wavelet Coherence of two time series as

$$R_n^2(s) = \frac{\left| S\left(s^{-1} W_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|W_n^X(s)\right|^2\right) \cdot S\left(s^{-1}\left|W_n^Y(s)\right|^2\right)}, \tag{5}$$

where $S$ is a smoothing operator. Notice that this definition closely resembles that of a traditional correlation coefficient, and it is useful to think of the Wavelet Coherence as a localized correlation coefficient in time frequency space.

However, following Aguiar-Conraria and Soares (2011) we will focus on the Wavelet Coherency, instead of the Wavelet Cross Spectrum. Aguiar-Conraria and Soares (2011, p. 649) gives two arguments for this: "(1) the wavelet coherency has the advantage of being normalized by the power spectrum of the two time-series, and (2) that the wavelets cross spectrum can show strong peaks even for the realization of independent processes suggesting the possibility of spurious significance tests".

*2.2.4 Cross Wavelet Phase Angle*

As we are interested in the phase difference between the components of the two time series we need to estimate the mean and confidence interval of the phase difference. We use the circular mean of the phase over regions with higher than 5% statistical significance that are outside the COI to quantify the phase relationship. This is a useful and general method for calculating the mean phase. The circular mean of a set of angles ($a_i, i = 1,...,n$) is defined as

$$a_m = \arg(X, Y) \text{ with } X = \sum_{i=1}^{n} \cos(a_i) \text{ and } Y = \sum_{i=1}^{n} \sin(a_i) \tag{6}$$

It is difficult to calculate the confidence interval of the mean angle reliably since the phase angles are not independent. The number of angles used in the calculation can be set arbitrarily high simply by increasing the scale resolution. However, it is interesting to know the scatter of angles around the mean. For this we define the circular standard deviation as

$$s = \sqrt{-2\ln(R/n)}, \quad (7)$$

where $R = \sqrt{(X^2 + Y^2)}$. The circular standard deviation is analogous to the linear standard deviation in that it varies from zero to infinity. It gives similar results to the linear standard deviation when the angles are distributed closely around the mean angle. In some cases there might be reasons for calculating the mean phase angle for each scale, and then the phase angle can be quantified as a number of years.

The statistical significance level of the wavelet coherence is estimated using Monte Carlo methods. We generate a large ensemble of surrogate data set pairs with the same AR1 coefficients as the input datasets. For each pair we calculate the Wavelet Coherence. We then estimate the significance level for each scale using only values outside the COI.

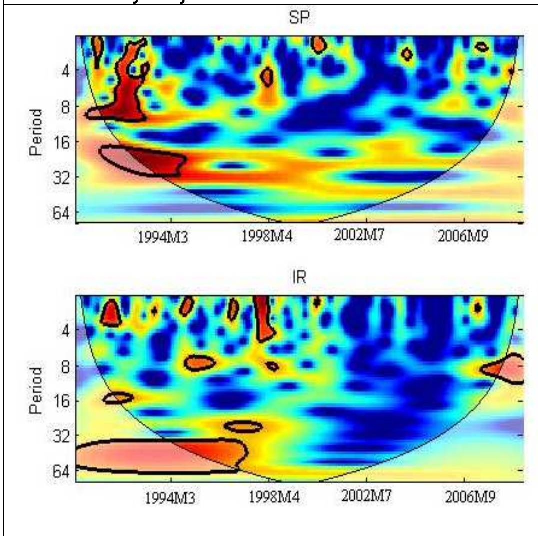
### 3. Data Analysis and Empirical Findings

First of all descriptive statistics of variables has been analyzed to see the sample property of both variables. Descriptive statistics show that IR is log non-normal while SP is (see Table-1 in appendix). Further, from the correlation analysis we found that correlation is marginal and its value is 0.56. In the next step stationary property of the data series of all test variables has been tested through ADF and PP test.[2] We find that both variables are non-stationary in the log level form while they are stationary at their first differenced form. Then for further analysis we adopted two approaches. In the first case we adjusted data for seasonality and in the second case we presented results for the data without seasonal transformation of log level form of data. In both cases first difference form of the data is utilized.

Firstly, in Fig.1 we present results of continuous wavelet power spectrum of both SP (in the top) and IR (in the bottom) for seasonally adjusted and non-seasonally adjusted data.

[2] Time series plot and descriptive statistics of the variables are presented in Figure 1 and Table 1 respectively, in appendix. ADF and PP unit root test are not presented to save space, however, can be obtained from the author upon request.

| Seasonally adjusted data | Non-seasonally adjusted data |
|---|---|



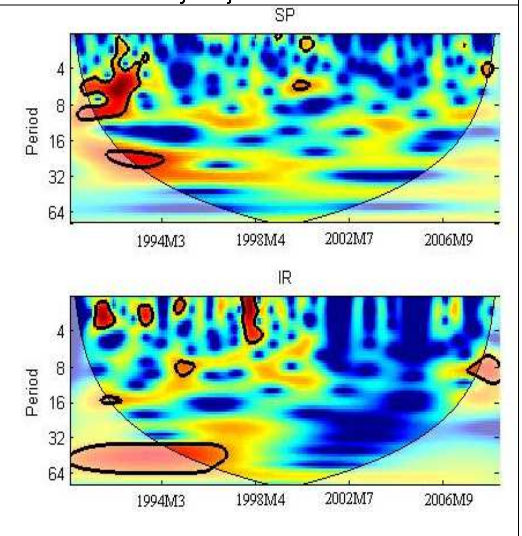
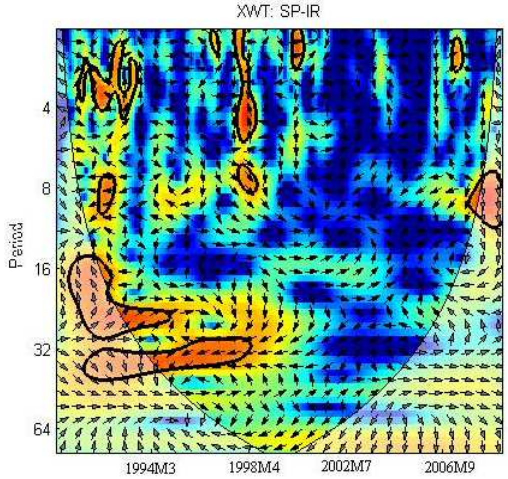




Fig. 1. The continuous wavelet power spectrum of both SP (in the top) and IR (in the bottom) series are shown here. The thick black contour designates the 5% significance level against red noise and the cone of influence (COI) where edge effects might distort the picture is shown as a lighter shade. The color code for power ranges from blue (low power) to red (high power). X-axis measures frequencies or scale and y-axis represent the time period studied.

It is evident from Fig.1 that seasonal transformation of the data has improved the wavelet power (i.e., red color, within the thick black contour, is darker in the seasonally adjusted data vis-à-vis non-seasonally adjusted data). So, our focus will be on seasonally adjusted data only. Now if we see the common features in the wavelet power of these two time series i.e., SP and IR we find that there are some common island. In particular, the common features in the wavelet power of the two time series are evident in 1~3 year scale that belongs to 1990s, 4~5 year scale that belongs to 1998s, one year scale that belongs to 2000s and 2006s. In these different year scales both series have the power above to the 5% significance level as marked by thick black contour. However, the similarity between the portrayed patterns in these periods is not very much clear and it is therefore hard to tell if it is merely a coincidence. The cross wavelet transform helps in this regard. We further, analyzed the nature of data through cross wavelet and presented results in Fig.2 for both seasonally adjusted and non-seasonally adjusted data for comparison purposes. However, as we indicated above, our focus and discussion is only on the seasonally adjusted data.

Seasonally adjusted data




Non-seasonally adjusted data

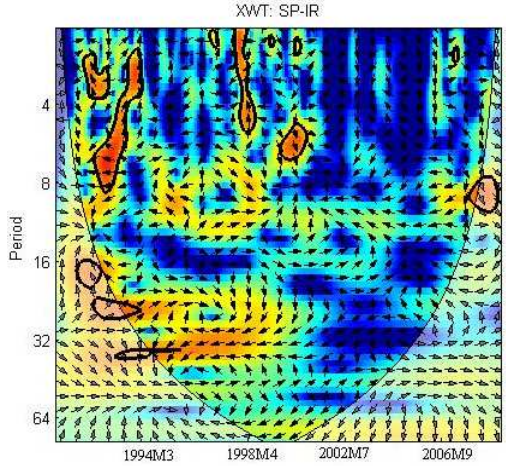



Fig. 2. Cross wavelet transform of the SP and IR time series. The thick black contour designates the 5% significance level against red noise which estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from blue (low power) to red (high power). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with IR is lagging. To the right and down, with IR is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with IR is leading. To the left and down, with IR is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having ant-cyclical effect on each other.

It is very interesting to see that in Fig.2, the direction of arrows at different periods (i.e., frequency bands) over the time period studied is not same. In 1990s itself, pointing direction of arrows is not same i.e., variables appear to have within the phase and also they are out of phase. For example, in the 1~4 year scale, arrows appears to be right and up, indicating variables are in phase and IR is lagging. That is IR is accommodating cyclical effect from SP. And in the same frequency band (i.e., year scale) arrows appears to be left and down indicating variables are out of phase and IR is lagging, which indicates that IR is accommodating anti-cyclical effects from SP. Further, in the same year scale we have arrows pointing to the left and up indicating that variables are out of phase and IR is leading. Further, in 1990s, in the 8~10 year scale, arrows are pointing to the left and down indicating that variables are out of phase and IR is lagging. Further, during 1993-1994, IR is lagging (whether variables are in the phase or out of the phase) because in 25~30 year scale arrows are right up, and in 33~40 year scale, arrows are left down. We have some significant area in higher frequencies but it is affected by edge effects, therefore we ignored that area in the discussion. In 1998s, in the 1~6 and 7~8 year scale, again we find that arrows are pointing to the left and right and up and down and thus giving mixed results. In 2006 and 2007 we observe similar situation. Further, outside the areas with significant power, the phase relationship is also not very clear. Even if, now, we do not have very clear results but this type of results one analyst would have not got if he/she would have utilized either time series or frequency analysis methods. Overall we, therefore, speculate that there is a stronger link between IR and SP than that implied by the cross wavelet power. Finally, we relied on Cross-wavelet coherency for above stated reasons (those are stated in section 2) and presented results of Cross-wavelet coherency in Fig.3.

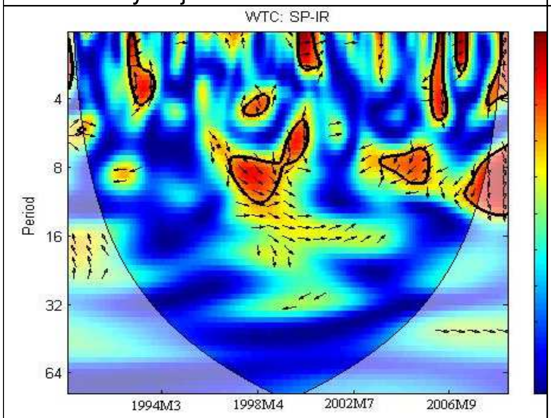
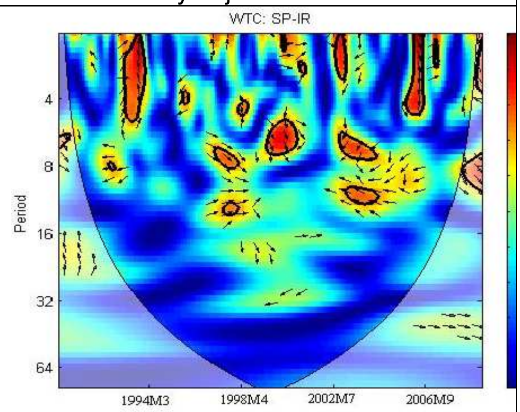
| Seasonally adjusted data | Non-seasonally adjusted data |
|---|---|
|  |  |





Fig. 3. Cross-wavelet coherency or Squared wavelet coherence. The thick black contour designates the 5% significance level against red noise which is estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is also shown with a light black line. The color code for coherency ranges from blue (low coherency-close to zero) to red (high coherency-close to one). The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, with IR is lagging. To the right and down, with IR is leading. Arrows pointing to the left mean that the variables are out of phase. To the left and up, with IR is leading. To the left and down, with IR is lagging. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having ant-cyclical effect on each other.

The squared WTC of SP and IR is shown in Fig.3 for both seasonally adjusted and non-seasonally adjusted data. However, as previously discussed our focus will be on seasonally adjusted data. If we compare results of WTC and XWT i.e., if we compare Fig.2 and Fig.3 we find three main differences. First, power of the wavelet has increased in Fig.3 vis-à-vis Fig.2 as indicated by dark red color within the thick black contours. Second, in comparison with the XWT a larger section stands out as being significant and all these areas show a clear picture of phase relationship between SP and IR. Worthy to note that the area of a time frequency plot above the 5% significance level (i.e., the area which is outside the thick black contour) is not a reliable indication of causality. Therefore, we will focus on the arrows appears within the thick black contour. During the late 1993 and early 1994 there is significant area which corresponds to 1~4 year scale. In this area arrows are right and up suggesting that IR is lagging with cycle effect on SP (i.e., variables are in phase). However, during 1998-2001, in 8~12 year scale, arrows are downwards and to the right suggesting that IR is leading with cyclical effects on the SP. The most interesting part which comes now in existence (which did not appear in XWT analysis) is that during 2003 to early 2005 (in 1~6 year scale) and again after late 2006 (in 9~14 year scale) arrows are pointing downwards and to the left suggesting that IR is lagging variable, and receiving anti-cyclical effects from SP. Now with the application of WTC analysis we have very clear evidence on lead-lag relationship between IR and SP. Further, we also come to know whether one variable affects or affected by the other through anti-cyclical or cyclical nature. Definitely these results would have not been drawn through the application of time series or Fourier transformation analysis if one could have tried.

## 4. Conclusions

The study analyzed Granger-causality between IR and SP for the India by using monthly data covering the period of 1990M1 to 2009M3. To analyze the issue in depth, study decomposes the time-frequency relationship between IR and SP through continuous wavelet approach. To the best of our knowledge this is first ever study in this direction with the present approach to any economy. Our testing of stationarity property of the data revealed that both variables were nonstationary in log level form and stationary in log first difference form. We found from the continuous power spectrum figure that the common features in the wavelet power of IR and SP are evident in 1~3 year scale that belongs to 1990s, 4~5 year scale that belongs to 1998s, one year scale that belongs to 2000s and 2006s. Results of cross Wavelet Transform, which indicate the covariance between IR and SP, are unable to give clear-cut results but indicate that both variables have been in phase and out phase (i.e., they are anti-cyclical and cyclical in nature) in some or other durations. However, our results of Cross-Wavelet Coherency or Squared Wavelet Coherence, which can be interpreted as correlation, reveal that *during the late 1993 and early 1994, in 1~4 year scale, IR is lagging with cycle effect from SP*. However, *during 1998-2001, in 8~12 year scale, IR is leading with cyclical effects on the SP*. Further, results show that *during 2003 to early 2005 (in 1~6 year scale) and again after late 2006 (in 9~14 year scale) IR is lagging and receiving anti-cyclical effects from SP.*

Our results show, for the Indian economy, that causal and reverse causal relations between SP and IR vary across scale and period. There are evidence of both cyclical and anti-cyclical relationship between IR and SP. We found that SP Granger-cause IR at short scales of 1~4 year where IR receives cyclical effect from SP and in 1~6 year scale and in 9~14 year scale IR receives anti-cyclical effects from SP. Further, in 8~12 year scale we found that IR is leading (i.e., IR Granger-causes SP) with cyclical effects on the SP. The unique contribution of the present study lies in decomposing the causality on the basis of time horizons and in terms of frequency.

The present study can be extended by analyzing different interest rates over the Indian yield curve to see if similar results are observed using different frequency of interest rates. Another possibility to extend the work is to analyze the effect of volatility in exchange rates on both on interest rates and stock returns, either in bivarate or trivariate framework through continuous wavelet analysis as theoretically all the three variables are expected to be highly correlated with each other.

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## Appendix

**Table 1: Summary statistics of the variables**

| | Ln(IR) | Ln(SP) |
|---|---|---|
| Mean | 2.610263 | 3.820943 |
| Median | 2.583998 | 3.707210 |
| Maximum | 2.995732 | 5.404433 |
| Minimum | 2.374906 | 2.002830 |
| Std. Dev. | 0.168959 | 0.699305 |
| Skewness | 0.455160 | 0.132108 |
| Kurtosis | 2.221489 | 3.351788 |
| Jarque-Bera | 13.80960 | 1.863062 |
| Probability | 0.001003 | 0.393950 |
| Source: Author's compilation | | |

**Figure 1: Time series plot of the variables**