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Abstract

Disparities in economic outcomes among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative economic standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discriminations in the labor market, seem to distort investment and sectoral decisions and slow down the economic progress of the disadvantaged.

How do these obstacles affect skill investment and sectoral choices of individuals of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of the disadvantaged, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics.

Keywords: ethnic or racial inequality; statistical discrimination; labor market segregation; skill investment.

JEL Classification Numbers: J15, J24, J31, J62, J71, O17

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1 Introduction

Disparities in economic outcomes among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative economic standings of different groups are rather persistent (Borjas, 1994; Darity, Dietrich, and Guilkey, 2001),[1] although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles seem to slow down the economic progress of the disadvantaged. One is costly skill investment: the quality of public schools is not adequate in many countries, thus many people expend on supplementary study materials and tutoring or attend high-quality but costly private schools (Baker et al., 2001; Bray and Kwok, 2003).[2] The other is negative stereotypes or discriminations in the labor market, which compels many from disadvantaged groups to invest less in skill or to choose occupations or sectors where performance is less affected by such handicaps but earnings tend to be lower than those occupied by dominant groups (Telles, 1993; Bayard et al., 1999; van de Walle and Gunewardena, 2001).[3]

How do these obstacles affect skill investment and sectoral choices of individuals of different groups and the dynamics of their economic outcomes and inter-group inequality? Is affirmative action necessary to significantly improve conditions of the disadvantaged, or redistributive policies sufficient? In order to tackle these questions, this paper develops a dynamic model of statistical discrimination and examines how initial economic standings of groups and initial institutionalized discrimination affect subsequent dynamics.

The analysis is based on a discrete-time small-open OLG model. There exist a continuum of two-period-lived individuals who belong to one of two ethnic (racial, religious) groups and are homogeneous in innate ability. In childhood, an individual receives a transfer from her parent and spends it on assets and skill investment needed to become a skilled worker. No

[1] Borjas (1994) shows that a U.S. worker's wage in 1940 and 1980 is significantly related to the average wage of immigrants of the worker's ethnic group (blacks are excluded) in 1910, after individual characteristics are controlled for. Darity, Dietrich, and Guilkey (2001) find that a U.S. worker's occupational status in 1980 and 1990 is significantly related to human capital endowments and the degree of favorable or unfavorable treatment in the labor market in the period between 1880 and 1910 of his/her group.

[2] Baker et al. (2001) find that about 40% of seven and eight graders in a large sample from 41 developed and developing countries participate weekly in private supplementary tutoring, such as tutoring sessions and cram schools, to study mathematics. Further, at the national level, they find that the average participation rate is significantly negatively related to the percentage of public expenditure on education in GNP. Bray and Kwok (2003) briefly review existing studies, which show that the use of private tutoring is extensive even among primary school students in developing countries.

[3] Telles (1993) finds, in Brazilian metropolitan areas, that minorities (except Asians) are overrepresented in low-wage informal-sector jobs in which being minorities has less negative effects on earnings. For the Vietnam economy, van de Walle and Gunewardena (2001) show that, compared to the majority Kinh, returns to education are lower but returns to land are higher for minorities, suggesting that minorities choose to exert more efforts on farming in which performance is less affected by their disadvantaged positions. For the U.S. economy, Bayard et al. (1999) find that greater racial and ethnic wage disparities for men than for women can be explained largely by more severe occupational and industry-level segregation among men.

credit market exists for skill investment, so she cannot invest if the transfer is not enough. Since she can spend wealth on assets too, she invests in skill only if it is affordable *and* profitable. In adulthood, she chooses a sector to work (detailed next), obtains income from assets and work, and spends it on consumption and a transfer to her single child.

The economy is composed of *up to* two production sectors, the primary sector with advanced technology and the secondary sector with backward technology. The primary and the secondary sectors correspond to formal/modern and informal/traditional sectors in developing economies, while in advanced economies, typical secondary-sector jobs would be neighborhood jobs of small businesses. Skilled and unskilled workers are perfectly substitutable in both sectors. In real economy, labor and product markets of the primary sector tend to be ethnically more mixed than the secondary sector. In the integrated primary sector, minorities are prone to face disadvantages in production or suffer greater disutility of work, because prevalent language, customs, taste, code of conduct, and culture are different from theirs, or they are discriminated non-statistically. Hence, skill investment is assumed to raise human capital in the secondary sector equally for both groups, while its effect on human capital in the primary sector, where skilled workers have comparative advantages, *may* be smaller for the minority. Main implications of the model, however, *remain intact without this assumption*: it is imposed for analytical simplicity as well as for reality.

In the primary sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluations are particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above-mentioned inter-group differences.[4] Similarly, qualifications of a job applicant tend to be assessed less precisely when interviewers are from other groups. Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (the average wage) of her group in the sector, and the signal's importance decreases with the share of her group in the sector's skilled workers.[5] In the secondary sector, typically, each worker's contribution is easy to measure or workers in the workplace belong to the same group, thus wage equals human capital.[6]

Wealth in the initial period is unequally distributed over the population, and the in-

[4]Classic models of statistical discrimination by Aigner and Cain (1977) and Lundberg and Startz (1983) are based on a similar idea. See footnotes 16 and 17 for evidence consistent with this and the next claim.

[5]Like other models of statistical discrimination, it is implicitly supposed that education level is not a good signal, which implies that the model is concerned with an economy where the quality of public schools is not adequate or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned at the beginning, is the case in many countries. Skill investment of the model may be interpreted as spendings on these activities.

[6]The wage equations can be derived from profit maximization problems of firms that hire workers *and* physical capital for production (see footnote 20). Further, productivity growth can be incorporated without affecting results qualitatively, as long as the cost of skill investment is assumed to grow proportionately.

equality is transmitted intergenerationally through transfers. Hence, generally, individuals are heterogeneous in accessibility to skill investment, and those without enough wealth do not invest even if it is profitable. Their descendants, however, may become accessible if enough wealth is accumulated. (The opposite is true for offspring of non-poor agents.)

An important property of the model is that skill investment and sectoral choices of different individuals within *and* across groups could be *interrelated*, because a worker's wage in the primary sector depends on her group's average human capital in the sector (also called the group's *reputation*) and the reputation's importance in the wage (also called *the degree of prejudice* toward the group), which decreases with the group's share in the sector's skilled workers. Hence, the dynamics of transfers and economic positions of different people too could be interrelated. The paper examines how the initial distribution of wealth within and across groups affects the dynamics of skill investment, sectoral choices, intra and intergroup disparities, and the steady state outcome.

Main results are summarized as follows. First, sectoral choices and skill investment may not be socially optimal. Even if unskilled workers are less productive in the primary sector,[7] they may choose the sector due to a positive effect from skilled workers through the reputation. Individuals may not carry out productive investment due to the negative effect from unskilled workers. For a similar reason, it is possible that all skilled workers of a group choose the secondary sector and all unskilled workers choose the *primary* sector, even if the *former* have comparative advantages and are more productive in the primary sector.

Second, multiple equilibria could exist regarding skill investment and sectoral choices of skilled workers: both the non-poor of a group invest (skilled workers choose the primary sector) and do not could be equilibria. Within a group, the source of multiplicity is strategic complementarity: to take the investment as an example, as more people invest and get skilled, prejudice toward the group eases, primary-sector wages reflect human capital more closely, and the return to investment rises. Across groups, strategic substitutability is at work: as more people of one group invest, prejudice toward the other group intensifies and their return to investment falls. Hence, if the latter effect is strong for both groups, *either* group invest (choose the primary sector) and the other do not are equilibria; if the former effect too is strong for both, both invest (choose the sector) is also an equilibrium.

Third, the dynamics and long-run outcomes of groups, particularly of the minority, depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since *good* (*bad*) *reputation tends to beget good* (*bad*) *reputation*, a group starting

[7] Individuals, particularly the minority, could be less productive in the primary sector if the quality of formal institutions and thus the sector's productivity are low, if non-statistical discrimination exists, or if the disutility of work is greater in the sector (human capital may be measured *net* of the disutility).

with a good (bad) initial condition, i.e. a high (low) fraction of them can afford skill investment initially, tend to be in a good (bad) position in the long run.

The mechanism is explained based on an economy in which workers always choose the primary sector and offspring of the unskilled majority become accessible to skill investment over time (analyzed in Section 5.1.1). If the minority's initial condition is good and thus a relatively large fraction of them are skilled initially, the wage of the unskilled minority is relatively high because of the group's good reputation. Further, as the majority increasingly become skilled, the reputation becomes more important, which has a positive effect on the wage. As a result, the unskilled minority accumulate relatively large wealth, and some of their offspring come to afford investment at some point. The number of the skilled minority and the reputation start to rise, and the improved reputation stimulates the upward mobility of the unskilled further. In the long run, all workers are skilled. If the initial condition is bad, the similar mechanism affects the skilled minority negatively, and the minority are totally unskilled in the end. Inter-group inequality rises at first, but if the initial condition is good, it is eradicated eventually, while it continues to rise if the condition is bad.[8]

The dynamics of sectoral choices and the degree of labor market segregation too could be affected greatly by initial conditions. The explanation is based on an economy that is similar to the previous one except that the unskilled minority are more productive in the secondary sector and thus choose the sector while the reputation is unimportant (analyzed in Section 5.1.2). As the majority increasingly become skilled and thus prejudice toward the minority intensifies, more and more of the unskilled minority choose the primary sector *inefficiently*, deteriorating their reputation. If the initial condition of the minority is good, however, the reputation remains high enough that the shift to the primary sector continues, and the labor market becomes ethnically integrated eventually (and the dynamics become similar to the previous economy). By contrast, if the initial condition is bad, the downward mobility of the skilled minority starts at some point, which worsens the reputation as well as deepens the prejudice further. Hence, the unskilled minority increasingly choose the *secondary* sector. Eventually, all the minority are unskilled and in the secondary sector, thus *the labor market is segregated completely by ethnicity.* The inefficient sectoral choices make the dynamics sensitive to the initial condition.

Fourth, when multiple equilibria exist regarding skill investment or sectoral choices of skilled workers, which is the case when prejudice is severe or the productivity of human capital investment is low, *given* initial conditions, the *initial selection of equilibrium* could

[8]When the groups are similar in size, the dynamics of the majority too could be affected greatly by initial conditions (Section 5.1.3). If both groups start with bad conditions, *both* could end up without skilled workers: a bad impression each group has about the other group affects the skilled wage negatively, which causes the downward mobility of skilled workers and the impression deteriorates further.

affect the dynamics greatly.[9] When multiple equilibrium choices exist for the minority, it is possible that, if the non-poor minority *happen to* (*not to*) invest [or choose the primary sector] initially, the number of the skilled minority grows (falls) over time and the group are totally skilled (unskilled) eventually (Sections 5.2.1 and 6.2). When multiple equilibria exist for both groups, the long-run outcome of the majority too is sensitive to the initial selection (Section 5.2.2). The majority staring with a *much better* condition than the minority could end up with the *smaller* fraction of skilled workers, if they (the minority) *happen not to* (*to*) invest [choose the primary sector] initially. The result suggests that, if the initial selection is affected by institutionalized discrimination limiting one group's access to skill investment or skilled jobs in the primary sector, the discrimination could have a lasting impact on their well-beings well after its abolishment. Income or wealth redistribution *does little* to change the situation, while affirmative action treating them favorably in investment or primary-sector employment, such as subsidies to tuition or wage, can be very effective.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents and analyzes the model's static part. Section 4 presents the full-fledged model and Section 5 analyzes the dynamics. Section 6 examines a general case by lifting one assumption that excludes situations of severe prejudice and low relative efficacy of skill investment in the primary sector. Section 7 concludes. Appendix contains proofs of lemmas and propositions.

2 Related Literature

This paper is related to the theoretical literature on statistical discrimination that examines the situation where employers cannot observe workers' skills and thus use two kinds of signals, race and a signal imperfectly correlated with individual skill, such as a test and an on-the-job monitoring, to screen workers (see Fang and Moro, 2010, for a survey). The first type of models such as Coate and Loury (1993) explain skill and earnings disparities among groups with equal endowment based on multiple equilibria. Employers assign individuals to two kinds of jobs, jobs requiring skill investment for good performance and those not, based on the signals. Since one's return to investment increases with investments by others of her race, multiple equilibria with different shares of skilled workers could exist. The second type of models, by contrast, assume that the non-race signal is nosier for the disadvantaged group to explain the disparities. Lundberg and Startz (1983), drawing on Phelps (1972) and Aigner and Cain (1977), develop a model where wage equals expected marginal productivity

[9] It is assumed that the initial coordination among individuals of a group continues for subsequent periods: if the group's non-poor happen to invest initially, they continue to invest subsequently. The assumption would be reasonable since children tend to mimic parental behaviors in real society. Kim and Loury (2009) makes a somewhat similar assumption in a dynamic model of statistical discrimination (see footnote 29).

conditional on the signals. The return to investment is lower for the disadvantaged group due to the noisier signal and thus they invest less even if groups' endowment is identical.

Recent major progress in the literature are twofold. One is the extension to a dynamic setting. This is particularly important to the first type of models, where employers' self-confirming beliefs about groups' skill levels select an equilibrium, because a static model does not explain how such beliefs are formed. Kim and Loury (2009) develop a continuous-time OLG model in which employers' beliefs are formed based on objective information on groups' present and future skill levels (reputations) and are updated with changing investments. If the initial reputation of a group is high (low), the group converges to the high (low) reputation steady state, while if it is intermediate, the group could converge to either steady state, i.e. self-confirming expectations determine the final state as in static models.

The other is the consideration of inter-group interactions. In the above models, different groups do not interact and thus behaviors and welfare of one group do not affect those of other groups.[10] Chaudhuri and Sethi (2008) present a static model of the first type in which the investment cost depends on both individual ability and the fraction of skilled peers, which equals a weighted average of the fractions in one's own group and in the overall population and the exogenous weight on own group is interpreted as the degree of segregation. In a special case, they show that, in an economy where inter-group inequality exists under complete segregation, complete integration eliminates inequality and raises (lowers) shares of skilled workers of both groups, if the fraction of the initially disadvantaged group is low (high). Lundberg and Startz (2007) construct a random search model with a second-type element where searchers observe imperfect signals of potential partners' abilities. In a one-sided search model where homogenous white searchers observe more accurate signals of whites than of blacks, there could exist an equilibrium where they trade only with whites with good signal, even if both groups have identical ability distribution. In a two-sided search model where searchers are heterogenous in ability and race (and signals observed by black searchers reveal abilities of both races equally), they numerically show that there could exist an equilibrium of racially segregated transactions where high ability whites (blacks) accept only whites (blacks) with good signal.

This paper shares with the second type of models such as Lundberg and Startz (1983) the feature that the importance of own group's average human capital (reputation) in wage is

[10]Hence, the models cannot provide economic rationales for institutionalized discrimination used to be enforced by dominant groups in many countries. Moro and Norman (2004) construct a static general equilibrium model of the Coate and Loury type, in which productivities of two types of jobs are interrelated. When the two jobs are complementary, an increased share of skilled workers has a negative (positive) effect on the wage of good (bad) jobs, and thus the return to investment of a dominant group is negatively affected by investment of the disadvantaged group, giving dominant groups an incentive for the discrimination.

different among groups (footnote 18). The existing works assume that the importance is *exogenously* greater for a disadvantaged group, while, in this paper, it decreases with the share of own group in primary-sector skilled workers. Unlike these works, the model is dynamic and inter-group disparities could change over time, thus making the reputation's importance endogenous would be crucial. Such formulation yields a different kind of inter-group interactions from works such as Chaudhuri and Sethi (2008) and Lundberg and Startz (2007). Further, the paper models sectoral choice between the primary sector and the secondary sector, where reputation could affect wage only in the former, and the credit constraint in skill investment, both of which are not considered in other works but are important real-economy elements, as stated in the introduction. The credit constraint generates upward and downward mobilities of lineages through intergenerational transmission of wealth and thus the interesting group dynamics described in the introduction, whereas modeling the sectoral choice allows the paper to examine the dynamics of labor market segregation.

Regarding several elements, the paper employs a simpler setting than other works: there is no non-race signal, which implicitly supposes that one's contribution to production cannot be observed initially but is fully revealed later; the investment cost is homogeneous; and the generational structure is simpler than the dynamic model of Kim and Loury (2009). However, because of the simpler setting, it can consider the above-mentioned additional elements and examine how transitional dynamics as well as steady states depend on the initial condition using simple phase diagrams. Further, it can identify conditions under which multiple equilibria exist, the dynamics are different from a "prejudice-free" economy, inter-group disparities are eradicated in the long run, etc.

The paper is also related to works that examine the dynamics of inter-group inequality based on models without statistical discrimination.[11] Lundberg and Startz (1998), based on Loury (1977) and the 'ethnic capital' model of Borjas (1992), examine a dynamic two-group economy in which human capital is the engine of growth and there exist spillovers from coworkers in production and from elder neighbors and, for the minority, from elders of the majority in skill development. Individuals are exogenously segregated by ethnicity both in the workplace and in residence. There are no spillovers from the minority to the majority and inter-group inequality disappears in the long run. Using a version of the model with heterogeneous innate ability and without the third spillover, they examine the effect

[11] Bowles, Loury, and Sethi (2010), building on Loury (1977), construct a discrete-time OLG model with two groups, where the cost of skill investment is modeled as in Chaudhuri and Sethi (2008). They prove that, when the degree of exogeneous segregation is sufficiently high, the long-run group equality cannnot be attained even with very small initial inequality. In a special case, they show a dynamic version of the result of Chaudhuri and Sethi (2008) mentioned earlier. Yuki (2009) examines the dynamics of disparities between educated and uneducated workers in a one-group and one-sector economy where innate ability is heterogeneous and wage is determined as in this paper (education is the signal).

of workplace desegregation, i.e. allowing the minority to move to majority-dominated jobs by paying a mobility cost, on the dynamics. They examine the effect of one-time workplace desegregation, while this paper examines the dynamics of labor market segregation in an economy where workers can freely choose sectors.

The modeling of skill investment and intergenerational transmission of wealth draws on Galor and Zeira (1993) and Yuki (2008), in which, as in this paper, skill investment is constrained by intergenerational transfers motivated by impure altruism.

3 Static Model

This section presents and analyzes the static part of the model. The dynamic part is presented in the next section. Consider a small open economy (interest rate r is exogenous) populated by a continuum of individuals who belong to one of two ethnic (racial, religious) groups and are homogeneous in innate ability. Results in this section can be applied to traits that are not intergenerationally transmitted, such as gender and home province, as well.

Individuals decide whether or not to invest in skill. The cost of skill investment c_h must be self-financed, so they must have enough wealth. Examples of the investment include spending on study materials or private tutoring, going to a high-quality private school instead of a low-quality public school, and spending time on home study, not on helping parents' work.

There exist *up to* two production sectors, the primary sector with advanced technology and the secondary sector with backward technology. The primary and the secondary sectors correspond to formal/modern and informal/traditional sectors in developing economies, while in advanced economies, typical secondary-sector jobs would be neighborhood jobs of small businesses. Skilled and unskilled workers are perfectly substitutable in both sectors. In real economy, labor and product markets of the primary sector tend to be ethnically more mixed than the secondary sector probably because of differences in needed skills, scales of operations, and enforcement of law (Aslund and Skans, 2010).[12]

Two assumptions are made based on the fact. First, skill investment raises human capital from h_u (u is for unskilled) to h_s (s is for skilled) in the secondary sector, while, in the primary sector, it raises human capital of ethnic (racial, religious) group i from $A_{ui}h_u$ to $A_{si}h_s$, where the relative human capital A_{ki} ($k=u,s$) depends on the share of one's own

[12] Primary-sector firms need workers with highly specialized skills and scales of operations tend to be large. Thus, to assign jobs to workers with appropriate skills efficiently, labor markets tend to be anonymous and ethnically integrated. Further, the sector tends to be regulated by laws prohibiting overt employment discrimination. By contrast, in the secondary sector with the contrasting features, employment is largely through personal connections and thus labor markets are more segregated. Also, products of the primary sector are supplied to national markets, while those of the secondary sector, especially services, are mainly for local markets of particular groups. For the Swedish economy, Aslund and Skans (2010) find that the tendency for a minority worker to work with people of his/her group is stronger in smaller establishments.

group in the total population (N_i is the population of group i):

$$A_{ki} = A_k\Big(\frac{N_i}{N_i+N_j}\Big),\ j \neq i,\ A'_k(\cdot) > 0. \tag{1}$$

That is, given skill, human capital in the primary sector is lower for the smaller group.[13] The formulation captures the fact that, in the integrated primary sector, minorities are prone to face disadvantages in production or suffer greater disutility of work (human capital may be measured *net* of the disutility), because prevalent language, customs, taste, code of conduct, and culture are different from theirs.[14] Further, if they are discriminated non-statistically and are not assigned relevant tasks, they end up in lower productivity.[15] Note that $A_{ki} < 1$ is possible if the quality of formal institutions and thus the sector's productivity are low (as shown in footnote 20 below, A_{ki} increases with the sector's relative productivity), if the discrimination exists, or if the disutility of work is greater in the sector.

As is made clear later, the above assumption is imposed for analytical simplicity as well as for reality, and main implications of the model *remain intact* without it. By contrast, the next assumption is crucial to results. In the primary sector, due to complex production processes and organizational structures, evaluating each worker's contribution to output tends to be difficult. Accurate evaluations are particularly difficult at least initially, if a worker and her evaluators belong to different groups due to the above-mentioned inter-group differences (Giuliano, Levine, and Leonard, 2011).[16] Similarly, qualifications of a job applicant tend to be assessed less precisely when interviewers are from other groups (Stoll, Raphael, and Holzer, 2004).[17] Hence, the wage is assumed to depend partly on her human capital and partly on its signal, the average human capital (the average wage) of her group in the sector, and the signal's importance decreases with the share of her group in the sector's skilled workers.[18] Like other models of statistical discrimination, it is implicitly supposed

[13] Skill and human capital are *different*: *skill* is ability and its level is either skilled or unskilled, while *human capital* is the contribution of skill to output (may be net of the cost of public schools) and workers of a given skill level can have different levels of human capital depending on sectoral choices and ethnicity.

[14] When the majority were historically in disadvantaged positions, dominant language, customs, or culture relevant to the primary sector could be different from theirs. In this case, all results remain intact if the argument of $A_k(\cdot)$ is replaced by the group's historical (initial) relative position.

[15] Non-statistical discrimination seems to affect labor market outcomes even in advanced nations. For the U.S., Charles and Guryan (2008) find that white-black wage gaps in a state are related to the degree of bias by whites in the left tail of the bias distribution in the state, consistent with the model of Becker (1971).

[16] Giuliano, Levine, and Leonard (2011) find, for a large U.S. retail firm, that employees generally have better outcomes, particularly in dismissals and promotions, when they are the same race as their supervisor.

[17] Stoll, Raphael, and Holzer (2004) find, for the U.S. economy, that establishments where blacks are in charge of hiring are significantly more likely to employ blacks than those with white hiring agents, and this pattern can be explained largely by the higher application rate of blacks and the higher hiring rate of black job applicants in establishments with black hiring agents.

[18] As mentioned in Section 2, classic works by Aigner and Cain (1977) and Lundberg and Startz (1983) are based on a similar idea. Unlike this model, workers' contributions to output are never revealed and thus

that education level is not a good signal, implying that the model is concerned with an economy where the quality of public schools is not adequate or varies greatly across schools and thus many people expend on supplementary study materials and tutoring or attend private schools, which, as mentioned in the introduction, is the case in many countries.

The wage of an individual with skill level k ($k=u,s$) of group i is given by:

$$(1-s_i)A_{ki}h_k+s_iE[A_ih_i], \tag{2}$$

where $s_i\in[0,1]$ measures the importance of the average human capital, $E[A_ih_i]$, and decreases with the share:[19]

$$s_i=s\left(\frac{p_{si}H_iN_i}{p_{si}H_iN_i+p_{sj}H_jN_j}\right),\ s'(\cdot)<0,\ s(1)=0. \tag{3}$$

H_i is the fraction of skilled workers in group i, and p_{si} is the probability that a skilled worker of group i chooses the primary sector. The size of s_i reflects the degree of the incomplete information and is named *the degree of prejudice* toward the group. If the sector's skilled workers are all from her group, $s(1)=0$ for simplicity. The average human capital, also called the group's *reputation*, equals (p_{ui} is the probability for an unskilled worker):

$$E[A_{ki}h_k]=\frac{p_{si}H_iA_{si}h_s+p_{ui}(1-H_i)A_{ui}h_u}{p_{si}H_i+p_{ui}(1-H_i)}. \tag{4}$$

In the secondary sector, each worker's contribution is easy to measure or workers in the workplace belong to the same group, thus wage equals human capital, h_k ($k=u,s$).

These wage equations can be derived from profit maximization problems of firms that hire workers *and* physical capital for production.[20] Further, productivity growth can be incorporated without affecting results qualitatively, as long as the cost of skill investment c_h is assumed to grow proportionately.

The following assumptions are imposed on A_{ki} ($k=u,s$) and the function $s(\cdot)$.

Assumption 1 (*i*) $A_{si}\geq A_{ui}$ (*ii*) $s(0)\leq\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\Leftrightarrow A_{si}\geq\frac{h_u}{h_s}A_{ui}+\frac{h_s-h_u}{(1-s(0))h_s}$.

The first assumption states that skilled workers have comparative advantages (weakly) in the primary sector, which would be justified from the fact that the sector adopts more advanced technology and thus workers' skills are more important. It also implies that skill investment

a worker's wage equals a weighted average of her group's average human capital and her non-race signal. Further, the importance of the race signal is *exogenously* greater for the disadvantaged group.

[19]An interpretation of s_i is that those who assess her performance cannot recognize her skill during the first s_i fraction of time (and they can identify it after that). Alternatively, $(1-s_i)A_{ki}h_k$ may be construed as the amount of her contribution to output recognized precisely by them.

[20]Suppose that firms with identical CRS technology hire workers and physical capital to produce a final good in each sector. Then, by normalizing the wage rate per unit of human capital (which depends on total factor productivity and the interest rate) of the informal sector to 1, the same wage equations are obtained. The relative human capital in the formal sector A_{ki} ($k=u,s$) increases with the sector's relative TFP.

is (weakly) more productive in the primary sector. The second assumption states that the (private) net return to choosing the primary sector is weakly higher for skilled workers than for unskilled workers even when the degree of prejudice is severest, i.e. $s_i = s(0)$ (the assumption can be expressed as $[(1–s(0))A_{si}–1]h_s \geq [(1–s(0))A_{ui}–1]h_u$). The first assumption is maintained throughout the paper, while the second one is relaxed in Section 6.

3.1 Sectoral choices and skill investment

Since workers are freely mobile between the sectors, they choose the one(s) with higher earnings. The next lemma presents equilibrium values of p_{si} and p_{ui} for given p_{sj} $(j \neq i)$, when $H_i > 0$ and $p_{sj}H_j > 0$, in which case $s_i > 0$ holds from (3).[21] Only equilibria that are stable with respect to small perturbations to equilibrium p_{si} and p_{ui} are considered.[22]

Lemma 1 (Sectoral choices) *Suppose $H_i > 0$ and $p_{sj}H_j > 0$ for $j \neq i$.*

(i) When $A_{si} \geq A_{ui} \geq 1$, $p_{si} = p_{ui} = 1$.

(ii) When $A_{si} > 1 > A_{ui}$, $p_{si} = 1$. $p_{ui} = 0$ for $s_i \leq \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{ui} = \frac{s_iA_{si}h_s + (1-s_i)A_{ui}h_u - h_u}{(1-A_{ui})h_u}\frac{H_i}{1-H_i} \in (0,1)$ for $s_i \in \left(\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}, \frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}\right)$, and $p_{ui} = 1$ for $s_i \geq \frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$.

(iii) When $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$, $p_{si} = p_{ui} = 0$.

When $A_{si} \geq A_{ui} \geq 1$, that is, when both types of workers weakly prefer the primary sector under $s_i = 0$, they do choose the sector. Intuitively, the reason is that, with $s_i > 0$ and $p_{si} > 0$, unskilled workers benefit from the presence of skilled workers in the sector and thus strictly prefer the sector, and the net return from choosing the sector is higher for skilled workers from Assumption 1 (ii). When $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$, that is, when unskilled workers strictly and skilled workers at least weakly prefer the secondary sector under $s_i = 0$, they choose the sector. This is because, the net return to the primary sector for the skilled is lower under $s_i > 0$ due to the negative effect of the unskilled on their wage. When $A_{si} > 1 > A_{ui}$, skilled workers select the primary sector, while choices of unskilled workers depend on s_i: since the positive effect from the skilled increases with s_i, they select the primary (secondary) sector when s_i is large (small), and when s_i is intermediate, they are indifferent between the sectors and $p_{ui} \in (0,1)$ is increasing in s_i.

Taking into account the dependence of wages on sectoral choices, an individual decides on skill investment. As detailed in the next section, she can spend wealth on assets too. Thus, she invests in skill only if it is financially accessible *and* profitable. Let F_i be the

[21] Clearly, when $H_i > 0$ and $p_{sj}H_j = 0$ for $j \neq i$ (thus $s_i = 0$), $p_{ki} = 1 (= 0)$ if $A_{ki} > (<) 1$ and any $p_{ki} \in [0,1]$ if $A_{ki} = 1$ $(k = u, s)$, while when $H_i = 0$, the same result holds for $k = u$.

[22] An equilibrium is defined to be *stable* regarding the small perturbations if there exists a neighborhood of equilibrium p_{si} and p_{ui} such that, from any p_{si} and p_{ui} in the neighborhood, they have tendencies to return to equilibrium values in a simple dynamics in which $p_{ki} (k = s, u)$ increases (decreases) when the net return to choosing the formal sector for type k workers is positive (negative).

proportion of individuals who can afford the investment in group i. H_i cannot exceed F_i but does not necessarily equal F_i. Let p_{hi} be the probability that an individual with enough wealth does invest. To simplify the analysis, the following assumption is imposed on p_{hi}.

Assumption 2 *When individuals are indifferent among multiple values of p_{hi}, the highest value holds in equilibrium.*

For example, when $p_{si}=p_{ui}=0$ and $h_s-(1+r)c_h-h_u=0$, $p_{hi}=1$ holds. The next lemma presents equilibrium $H_i=p_{hi}F_i$ for given H_j and p_{sj} $(j\neq i)$ when $F_i>0$. Only equilibria that are stable with respect to a small perturbation to equilibrium p_{hi} are considered.

Lemma 2 (Skill investment) *Suppose $F_i>0$.*

(i) When $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$.

(ii) When $h_s-(1+r)c_h<h_u$,

(a) If $A_{si}h_s-(1+r)c_h\geq h_u$ (thus $A_{si}>1$) and $A_{ui}\geq 1$, when $p_{sj}H_j=0$ for $j\neq i$, $H_i=F_i\,(=0)$ if $A_{si}h_s-(1+r)c_h\geq(<)A_{ui}h_u$. When for $p_{sj}H_j>0$ (thus $s_i>0$), $H_i=F_i$ if $s(0)\leq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$; otherwise, both $H_i=F_i$ and $H_i=0$ are equilibria ($H_i=0$ is the equilibrium) when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.

(b) If $A_{si}h_s-(1+r)c_h\geq h_u$ and $A_{ui}<1$, $H_i=F_i$ when $p_{sj}H_j=0$ for $j\neq i$. When $p_{sj}H_j>0$, if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ (no stable equilibria exist).

(c) Otherwise, $H_i=0$.

When h_s–$(1+r)c_h\geq h_u$, i.e. the investment is socially productive (and privately profitable) in the secondary sector, every individual with enough wealth invests in skill, because, when choosing the primary sector is more profitable, the (net) private return to the investment is weakly higher than $h_s-(1+r)c_h-h_u$ from Assumption 1 (i) (when $p_{sj}H_j=0$ for $j\neq i$) and (ii) (when $p_{sj}H_j>0$). When $h_s-(1+r)c_h<h_u$ and $A_{si}h_s-(1+r)c_h<h_u$ [(ii)(c) of the lemma], nobody invests because the maximum net return under $s_i=0$, $max[A_{si},1]h_s-(1+r)c_h-h_u$, is negative and thus the net return under $s_i>0$ too is negative. (Note that the skilled [unskilled] wage under $s_i>0$ is lower [higher] than under $s_i=0$.)

By contrast, when $h_s-(1+r)c_h<h_u$ and $A_{si}h_s-(1+r)c_h\geq h_u$ (thus $A_{si}>1$), the decision depends on A_{ui}, s_i, and $s(0)$. When $A_{ui}\geq 1$ and $p_{sj}H_j>0$ for $j\neq i$, since all workers choose the primary sector from Lemma 1 (i), the net return equals $(1-s_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h$ and decreases with s_i. Hence, if the value of s_i at $H_i=F_i$ is small enough that the net return is positive at $H_i=F_i$, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is an equilibrium, while if $s(0)>\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ and thus the return is negative at $H_i=0$, $H_i=0$ is an equilibrium. Since $s(0)>s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)$, *both $H_i=F_i$ and $H_i=0$ are equilibria* when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}<s(0)$ due to *strategic complementarity*: as more people invest and become skilled workers, the degree of prejudice s_i falls and primary-sector wages

reflect human capital more closely, raising the return. The result when $A_{ui}<1$ and $p_{sj}H_j>0$ can be explained similarly. In this case, however, $H_i=0$ is not an equilibrium (since, given $H_i=0$, no unskilled workers choose the primary sector and thus the investment is profitable from $A_{si}h_s-(1+r)c_h\geq h_s$), thus *no stable equilibria exist* if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\geq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, i.e. s_i is too high for the return to be positive at $H_i=F_i$.

By combining Lemmas 1 and 2, skill investment and sectoral choices of group i *given* choices by the other group are summarized in the following proposition.

Proposition 1 (Group i's investment and sectoral choices given choices by group j)

(*i*) *When* $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ *and, if* $p_{sj}H_j>0$ *for* $j\neq i$, $s_i=s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)$.

- (*a*) *When* $p_{sj}H_j>0$ *and* $A_{si}\geq A_{ui}\geq 1$ *or* $A_{si}>1>A_{ui}$, $p_{si}=1$. *If* $A_{si}\geq A_{ui}\geq 1$, $p_{ui}=1$; *and if* $A_{si}>1>A_{ui}$, $p_{ui}=0$ *for* $s_i\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{ui}=\frac{s_iA_{si}h_s+(1-s_i)A_{ui}h_u-h_u}{(1-A_{ui})h_u}\frac{F_i}{1-F_i}\in(0,1)$ *for* $s_i\in\big(\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u},\frac{1}{F_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\big)$, *and* $p_{ui}=1$ *for* $s_i\geq\frac{1}{F_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$.
- (*b*) *When* $p_{sj}H_j>0$ *and* $A_{ui}<A_{si}=1$ *or* $A_{ui}\leq A_{si}<1$, $p_{si}=p_{ui}=0$.
- (*c*) *When* $p_{sj}H_j=0$, $p_{ki}=1(=0)$ *if* $A_{ki}>(<)1$ *and any* $p_{ki}\in[0,1]$ *if* $A_{ki}=1$ $(k=u,s)$.

(*ii*) *When* $h_s-(1+r)c_h<h_u$,

- (*a*) *If* $A_{si}h_s-(1+r)c_h\geq h_u$ *and* $A_{ui}\geq 1$, *when* $p_{sj}H_j=0$, $H_i=F_i(=0)$ *if* $A_{si}h_s-(1+r)c_h\geq(<)A_{ui}h_u$. *When* $p_{sj}H_j>0$, $H_i=F_i$ *if* $s(0)\leq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, *otherwise, both* $H_i=F_i$ *and* $H_i=0$ *are equilibria* ($H_i=0$ *is the equilibrium*) *when* $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$.
- (*b*) *If* $A_{si}h_s-(1+r)c_h\geq h_u$ *and* $A_{ui}<1$, $H_i=F_i$ *when* $p_{sj}H_j=0$. *When* $p_{sj}H_j>0$, *if* $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<(\geq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ (*no stable equilibria exist*).
- (*c*) *If* $A_{si}h_s-(1+r)c_h<h_u$, $H_i=0$.
- (*d*) *When* $H_i=F_i$, *if* $p_{sj}H_j>(=)0$, p_{si} *and* p_{ui} *are determined as in* (*i*)(*a*) [(*i*)(*c*)], *while when* $H_i=0$, $p_{ui}=1(=0)$ *if* $A_{ui}>(<)1$ *and any* $p_{ui}\in[0,1]$ *if* $A_{ui}=1$.

Based on Proposition 1 (i), Figure 1 illustrates sectoral choices when $h_s-(1+r)c_h\geq h_u$ (thus $H_i=F_i$) and $p_{sj}H_j>0$ for $j\neq i$, i.e. $s_i>0$, on the (A_{ui},A_{si}) plane. A_{ui} and A_{si} must satisfy Assumption 1 (i) and (ii), thus only the upper left region of the two bold solid lines is feasible. As for skilled workers, $p_{si}=1$ when $A_{si}>1$ and $p_{si}=0$ when $A_{si}\leq 1$. Choices of unskilled workers, by contrast, are determined by the two bold broken lines, and $p_{ui}=0$ $(=1)$ in the region at the left (right) side of the left (right) broken line and $p_{ui}\in(0,1)$ in the region between the two lines. (When $p_{ui}\in(0,1)$, p_{ui} increases with A_{ui} and A_{si}.) $p_{ui}>0$ is possible with $A_{ui}<1$ because of the positive effect from skilled workers in the primary sector. Positions of the bold lines and the value of p_{ui} depend on F_i through s_i.

Figure 2 illustrates skill investment and sectoral choices when $h_s-(1+r)c_h<h_u$ and $p_{sj}H_j>0$, based on Proposition 1 (ii). When $A_{ui}\geq 1$, $p_{ui}=1$ and $p_{si}=1$ (when $H_i>0$)

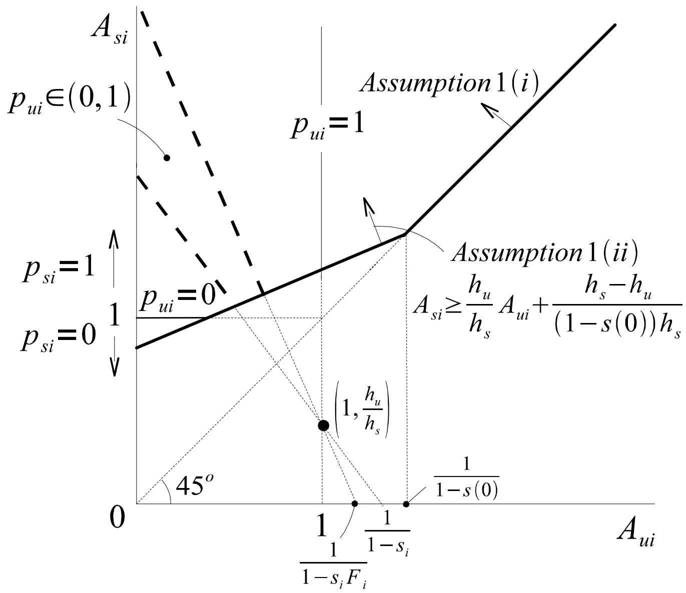

Figure 1: Sectoral choices when $h_s - (1+r)c_h \geq h_u$ and $p_{sj}H_j > 0$ for $j \neq i$

always hold. As for skill investment, $H_i = F_i$ ($H_i = 0$) is the only equilibrium in the region on or above $A_{si} = \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-s(0))h_s}$ (on or below $A_{si} = \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-s_i)h_s}$), while *both* $H_i = F_i$ *and* $H_i = 0$ *are equilibria* between the two lines, the area with slanting lines. When $A_{ui} < 1$, $H_i = 0$ holds in the region below $A_{si} = \frac{h_u + (1+r)c_h}{h_s}$, while in the region on or above the line, $H_i = F_i$ holds above $A_{si} = \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-s_i)h_s}$ and no equilibria exist on or below it (the area with vertical lines). Sectoral choices when $A_{ui} < 1$ and $H_i = F_i$ are as in Figure 1, thus $p_{si} = 1$ always ($H_i = F_i$ only when $A_{si} \geq \frac{h_u + (1+r)c_h}{h_s} > 1$). Positions of $A_{si} = \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-s_i)h_s}$ and of the two broken lines and thus their choices depend on F_i through s_i.

Sectoral choices and skill investment may *not* be socially optimal when $s_i > 0$. When $A_{ui} < 1$, because of the positive effect from skilled workers, some or all of unskilled workers choose the less productive primary sector at the right side of the left broken line of Figures 1 and 2. As for skill investment, if workers are optimally allocated to sectors, the investment is socially productive on or above the dotted line below the area with slanting lines when $A_{ui} \geq 1$ (where $A_{si}h_s - (1+r)c_h \geq A_{ui}h_u$ is satisfied), and on or above $A_{si} = \frac{h_u + (1+r)c_h}{h_s}$ when $A_{ui} < 1$ in Figure 2. However, when $A_{ui} \geq 1$, an individual may not carry out the productive investment in the region between $A_{si} = \frac{h_u}{h_s}A_{ui} + \frac{(1+r)c_h}{(1-s(0))h_s}$ and the dotted line due to the negative effect of unskilled workers on the return to investment.

Skill investment and sectoral choices in a general equilibrium are determined by applying the proposition to the two groups simultaneously. Cases in which the investment is always profitable for both groups can be easily identified from the proposition (see Figure 2 too).

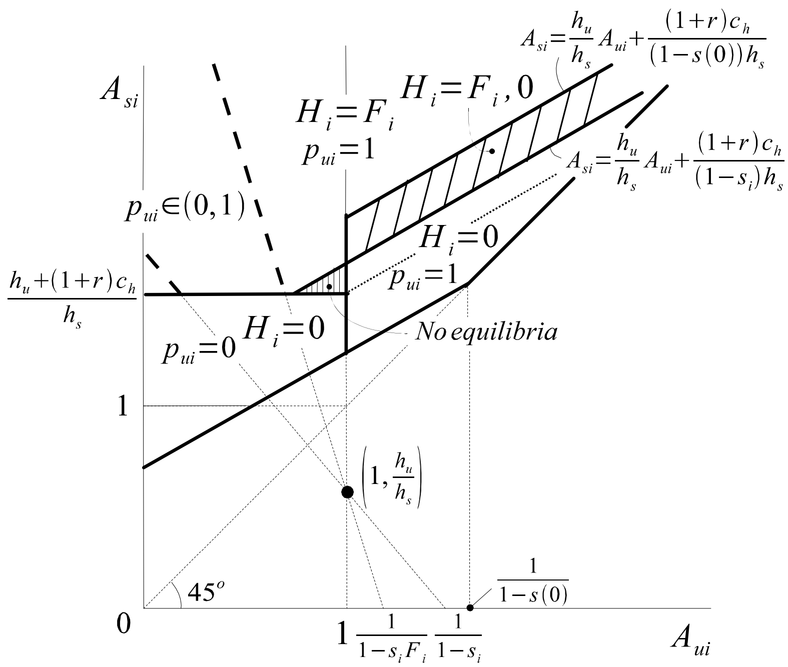

Figure 2: Skill investment and sectoral choices when $h_s-(1+r)c_h<h_u$ and $p_{sj}H_j>0$

Corollary 1 (Cases in which the investment is always profitable for both groups) *$H_i=F_i$ holds for any i and F_i, when $h_s-(1+r)c_h\geq h_u$ or when $h_s-(1+r)c_h<h_u$, $A_{si}h_s-(1+r)c_h\geq h_u$ (thus $A_{si}>1$), and $(1-s(0))[A_{si}h_s-A_{ui}h_u]\geq(1+r)c_h$. For group i with $A_{si}\geq 1$, $s_i=s\left(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\right)$, where $p_{sj}=1(=0)$ when $A_{sj}\geq(<)1$, and for one with $A_{si}<1$, $s_i=s(0)$.*

In other cases, the determination of H_i is not simple, which is examined in Proposition 2 of Section 5 with an additional assumption.

3.2 Wages

Wage levels depend on skill investment and sectoral choices. Denote the unskilled wage of group i by w_{ui} and the skilled wage *net of the investment cost* by w_{si}. Then, the wages when $H_i=F_i$ and $p_{sj}H_j>0$ for $j\neq i$, i.e. $s_i>0$, are:

if $p_{si}=p_{ui}=1$,
$$w_{ui}=(1-s_i)A_{ui}h_u+s_i[F_iA_{si}h_s+(1-F_i)A_{ui}h_u]$$
$$=A_{ui}h_u+s_iF_i(A_{si}h_s-A_{ui}h_u), \quad (5)$$
$$w_{si}=(1-s_i)A_{si}h_s+s_i[F_iA_{si}h_s+(1-F_i)A_{ui}h_u]-(1+r)c_h$$
$$=A_{si}h_s-s_i(1-F_i)(A_{si}h_s-A_{ui}h_u)-(1+r)c_h; \quad (6)$$

if $p_{si}=1$ and $p_{ui}\in(0,1)$,
$$w_{ui}=(1-s_i)A_{ui}h_u+s_i\frac{F_iA_{si}h_s+p_{ui}(1-F_i)A_{ui}h_u}{F_i+p_{ui}(1-F_i)}=h_u, \quad (7)$$
$$w_{si}=(1-s_i)A_{si}h_s+s_i\frac{F_iA_{si}h_s+p_{ui}(1-F_i)A_{ui}h_u}{F_i+p_{ui}(1-F_i)}-(1+r)c_h$$

$$= h_u + (1-s_i)(A_{si}h_s - A_{ui}h_u) - (1+r)c_h; \tag{8}$$

and if $p_{si} = 1 (= 0)$ and $p_{ui} = 0$, $w_{ui} = h_u$ and $w_{si} = A_{si}h_s - (1+r)c_h (= h_s - (1+r)c_h)$. When $H_i = 0$ or when $s_i = 0$, $w_{ui} = \max\{A_{ui}h_u, h_u\}$ and $w_{si} = \max\{A_{si}h_s, h_s\} - (1+r)c_h$ (if $H_i > 0$).

4 Dynamic model

Based on the results in the previous section, this section presents the dynamic part of the model. Consider an OLG economy composed of a continuum of two-period-lived individuals. The distribution of wealth over the initial generation of each group is given, while wealth distributions of subsequent generations are determined endogenously.

4.1 Lifetime of an individual

Childhood: In childhood, an individual receives a transfer from her parent (if she belongs to the initial generation, it is given) and spends it on two options, assets (yields interest rate r) and skill investment (costs c_h), to maximize future income. Consider an individual born into a lineage of group i in period $t-1$ (generation t) who receives b_{it} units of transfer and allocates it between asset a_{it} and skill investment v_{it}. As shown in the previous section, when an equilibrium exists, $H_{it} = F_{it}$ or $H_{it} = 0$ (i.e. $p_{hit} = 1$ or $p_{hit} = 0$), depending on F_{it} and exogenous variables such as A_{si} and A_{ui} (and, through s_{it}, corresponding variables of other groups). When $H_{it} = F_{it}$, the allocation is determined by b_{it}:

$$a_{it} = b_{it}, \qquad v_{it} = 0, \quad \text{if} \quad b_{it} < c_h, \tag{9}$$

$$a_{it} = b_{it} - c_h, \quad v_{it} = c_h, \quad \text{if} \quad b_{it} \geq c_h. \tag{10}$$

By contrast, when $H_{it} = 0$, $a_{it} = b_{it}$ and $v_{it} = 0$.

Adulthood: In adulthood, she chooses a sector based on the skill investment, obtains income from assets and work, and spends it on consumption c_{it} and a transfer to her single child b_{it+1}. Her utility maximization problem is:

$$\max u_{it} = (c_{it})^{1-\gamma_b}(b_{it+1})^{\gamma_b}, \quad s.t. \quad c_{it} + b_{it+1} = w_{it} + (1+r)a_{it}, \tag{11}$$

where $\gamma_b \in (0,1)$ and w_{it} is her gross wage, which depends on her human capital, F_{it}, and exogenous variables (and corresponding variables of other groups). By solving the maximization problem, her consumption and transfer rules equal

$$c_{it} = (1-\gamma_b)\{w_{it} + (1+r)a_{it}\}, \tag{12}$$

$$b_{it+1} = \gamma_b\{w_{it} + (1+r)a_{it}\}. \tag{13}$$

Generational change: At the beginning of period $t+1$, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the total (adult) population of the group is time-invariant and equals N_i.

4.2 Dynamics of individual transfers

The dynamic equation linking the received transfer b_{it} to the transfer given to the next generation b_{it+1} is derived from the transfer rule (13). For a current unskilled worker, it is obtained by substituting $w_{it}=w_{uit}$ and $a_{it}=b_{it}$ into (13):

$$b_{it+1} = \gamma_b\{w_{uit}+(1+r)b_{it}\}. \tag{14}$$

The assumption $\gamma_b(1+r)<1$ is made so that the fixed point of the equation for given w_{uit}, $b^*(w_{uit})\equiv\frac{\gamma_b}{1-\gamma_b(1+r)}w_{uit}$, exists. The fixed point becomes crucial in later analyses.

For a current skilled worker, who exists only when $H_{it}=F_{it}$, the dynamic equation is

$$b_{it+1} = \gamma_b\{w_{sit}+(1+r)b_{it}\}, \tag{15}$$

which is obtained by substituting $w_{it}=w_{sit}+(1+r)c_h$ and $a_{it}=b_{it}-c_h$ into (13).

The equations show that the dynamics of transfers within a lineage depend on those of wages and H_{it}, which in turn are determined by the time evolution of F_{it} and F_{jt} $(j\neq i)$.

4.3 Aggregate dynamics

The time evolution of F_{it} (the fraction of group i individuals who can afford skill investment) is determined by the dynamics of individual transfers. That is, the individual and aggregate dynamics are interrelated.

More specifically, when $H_{it}=F_{it}$, if offspring of some unskilled workers become accessible to the investment through wealth accumulation, $F_{it+1}>F_{it}$, while, if some of present skilled workers cannot leave enough transfers to cover the investment cost, $F_{it+1}<F_{it}$.

The former takes places iff there exist lineages satisfying $b_{it}<c_h$ and $b_{it+1}\geq c_h$. From (14), the following condition must hold for such lineages to exist:

$$b^*(w_{uit}) \equiv \tfrac{\gamma_b}{1-\gamma_b(1+r)}w_{uit} > c_h. \tag{16}$$

By contrast, the latter occurs iff lineages satisfying $b_{it}\geq c_h$ and $b_{it+1}<c_h$ exist. From (15), the necessary condition is

$$b^*(w_{sit}) \equiv \tfrac{\gamma_b}{1-\gamma_b(1+r)}w_{sit} < c_h. \tag{17}$$

Since $b^*(w_{sit})\geq b^*(w_{uit})$, the above equations do not hold simultaneously. If (16) holds, $F_{it+1}\geq F_{it}$, while if (17) is true, $F_{it+1}\leq F_{it}$: $F_{it+1}=F_{it}$ is possible depending on the distribution of transfers over the group, but, if the condition continues to hold, F_{it} does change at some point. When neither equations are satisfied, $F_{it+1} = F_{it}$. The dynamics when $H_{it}=0$ depend on the relative value of $b^*(w_{uit})$ to c_h only.

Regarding the value of $b^*(w_{uit})$, the following is assumed.

Assumption 3 $h_u \leq \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$

This implies that $b^*(w_{uit}) \leq c_h$ when $p_{ui,t} < 1$, that is, offspring of unskilled workers can never afford the investment if the unskilled wage stays at the lowest level, h_u. The assumption is imposed to rule out the trivial case in which $h_u > \frac{1-\gamma_b(1+r)}{\gamma_b} c_h$ and thus F_{it} always increases.

Since the dynamics of individual transfers depend on the evolution of F_{jt} $(j \neq i)$ through skill investment and wages, the dynamics of F_{it} and F_{jt} are interrelated. The next section analyzes the joint dynamics of the variables and those of related variables of interest.

5 Analyses

This section analyzes the time evolution of F_{it}, skill composition, sectoral choices, wages, and intergroup inequality by relying on phase diagrams. Depending on values of exogenous variables such as A_{si} and A_{ui}, many qualitatively different dynamics arise, hence analyses are restricted to cases that are representative and yield clear-cut results.

For simplicity, the elasticity of s_i with respect to F_i (in absolute value) is assumed to be less than 1 in dynamic analyses, although all of main results hold without the assumption.

Assumption 4 $s(x)+s'(x)x(1-x) > 0$ *for any* $x \in [0,1) \Leftrightarrow \frac{\partial(s_i F_i)}{\partial F_i} > 0$ *always.*

5.1 When skill investment is always profitable

First, consider the case in which $H_i = F_i$ always holds for any group i. From Corollary 1, this is true when $h_s-(1+r)c_h \geq h_u$, or when $h_s-(1+r)c_h < h_u$, $A_{si}h_s-(1+r)c_h \geq h_u$ (thus $A_{si}>1$), and $(1-s(0))[A_{si}h_s - A_{ui}h_u] \geq (1+r)c_h$ for any i (see Figure 2). That is, it must be that skill investment is socially productive and individually profitable, as long as workers are optimally assigned to sectors.

5.1.1 A majority and a large minority

Consider an economy where the majority (*group 1*) and the minority (*group 2*), i.e. $N_1 > N_2$, exist. Suppose that the relative size of the minority, $\frac{N_2}{N_1}$, or the relative productivity of the primary sector (footnote 20) is high enough that $A_{ki} > 1$ $(k=u,s;\ i=1,2)$, i.e. all workers are more productive in the sector. Then, $p_{si} = p_{ui} = 1$ from Proposition 1 (i)(a), (i)(c), and

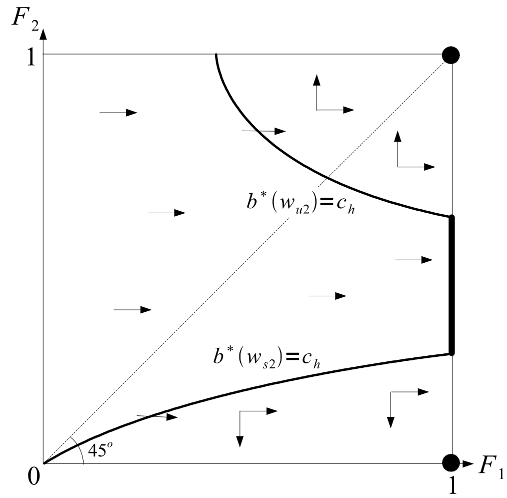

Figure 3: Dynamics when $H_{it}=F_{it}$ always and the relative size of the minority is large

(ii)(d), i.e. all workers choose the sector. Skill investment of those with enough wealth and sectoral choices are socially optimal from $A_{ki}>1$ and the conditions of Corollary 1.

As for the dynamics of F_{1t}, assume $A_{u1}h_u > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, that is, even with the lowest wage, descendants of the unskilled majority can afford the investment eventually. Then, F_{1t} increases over time and $H_1^*=F_1^*=1$ in the long run (superscript $*$ indicates the steady state value). In contrast, A_{u2} and A_{s2} are lower and the following is assumed: $A_{u2}h_u < \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the lowest wage, descendants of the unskilled minority remain unskilled; $\left[1-s\left(\frac{N_2}{N_1+N_2}\right)\right]A_{u2}h_u+s\left(\frac{N_2}{N_1+N_2}\right)A_{s2}h_s > \frac{1-\gamma_b(1+r)}{\gamma_b}c_h$, i.e. with the highest wage (at $(F_1,F_2)=(1,1)$ from Assumption 4), they can afford the investment eventually; $A_{s2}h_s > \frac{c_h}{\gamma_b}$ and $(1-s(0))A_{s2}h_s+s(0)A_{u2}h_u < \frac{c_h}{\gamma_b}$, i.e. with the highest (lowest) wage, descendants of the skilled minority can (cannot) stay skilled. Then, $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ exist and equal:

$$A_{u2}h_u+s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)F_2(A_{s2}h_s-A_{u2}h_u)=\frac{1-\gamma_b(1+r)}{\gamma_b}c_h, \tag{18}$$

$$A_{s2}h_s-s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)(1-F_2)(A_{s2}h_s-A_{u2}h_u)=\frac{c_h}{\gamma_b}, \tag{19}$$

which are obtained by plugging (5) and (6) with $s_2=s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)$ into (16) (with $>$ replaced by $=$) and (17) (with $<$ replaced by $=$), respectively.[23]

The dynamics of F_{1t} and F_{2t} can be analyzed graphically by placing $b^*(w_{u2}) = c_h$ and $b^*(w_{s2})=c_h$ on the (F_1,F_2) plane (Figure 3).[24] As F_1 rises, F_2 satisfying $b^*(w_{u2})=c_h$ ($b^*(w_{s2})=$

[23]Because w_{u2} for given $\frac{F_2}{F_1}$ increases linearly with F_2 and $b^*(w_{u2})>(<)c_h$ at $(F_1,F_2)=(1,1)$ (as $F_2\to 0$ on $\frac{F_2}{F_1}=1$) from the two assumptions on the wage (see eq. 18), there exists $F_2\in(0,1)$ on $\frac{F_2}{F_1}=1$ satisfying $b^*(w_{u2})=c_h$. $b^*(w_{s2})=c_h$ exists for any $F_1\in(0,1]$ (at $F_1=0$, $b^*(w_{s2})>c_h$ always) since, for any such F_1, w_{s2} increases with F_2 and $b^*(w_{s2})<(>)c_h$ at $F_2=0$ $(=1)$ from the two assumptions on the wage (see eq. 19).

[24]$b^*(w_{u2})=c_h$ intersects with $F_1=1$ at $F_2\in(0,1)$ and with $F_2=1$ at $F_1\in(0,1)$ from the two assumptions

c_h) falls (rises) from (18) and (19), since s_2 increases (decreases) with F_1 (F_2) and s_2F_2 increases with F_2. The direction of motion of F_{2t} (F_{1t}) is represented by vertical (horizontal) arrows. Since w_{s2} decreases and w_{u2} increases with F_1, in the region at the right (left) side of $b^*(w_{s2})=c_h$, $b^*(w_{s2})<(>)c_h$ and F_{2t} decreases (non-decreases) over time, while in the region at the right (left) side of $b^*(w_{u2})=c_h$, $b^*(w_{u2})>(<)c_h$ and F_{2t} increases (non-increases).

Unlike the economy in which reputation does not affect wages, i.e. $s_{it}=0$ always, where F_{1t} increases and F_{2t} is constant over time, the long-run fate of the minority could be very different depending on the level of F_2 in the initial period, F_{20}. When the initial distribution of wealth is such that a sufficiently large portion of them can afford the investment, to be more accurate, when $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{20})$, $H_2^*=F_2^*=1$ as well as $H_1^*=F_1^*=1$ in the long run. As an illustration, suppose that F_{10} is not so high that $b^*(w_{u20})<c_h$ holds. Then, as $H_{1t}=F_{1t}$ increases over time, the influence of the majority in wage determination becomes stronger and wages of the minority are affected more by their reputation as a group, i.e. s_{2t} increases. As a result, the unskilled (skilled) wage of the minority rises (falls) over time. Since $H_{2t}=F_{20}$ is not low and thus their reputation (average human capital) is not bad, the wage of the skilled minority stays high enough for their descendants to remain skilled, while the unskilled wage grows to the point that the investment becomes affordable to some of their offspring at some point, i.e. $b^*(w_{u2t})>c_h$. $H_{2t}=F_{2t}$ and the reputation start to rise, and the improved reputation further stimulates the upward mobility of unskilled workers. In the long run, everyone becomes a skilled worker.

By contrast, when F_{20} is small enough that $b^*(w_{s2})<c_h$ at $(F_1,F_2)=(1,F_{20})$, $(H_1^*,H_2^*)=(1,0)$ in the long run. Since their initial reputation is low and its effect on the wages increases over time, the skilled wage falls to the point that offspring of skilled workers become unable to afford the investment at some point. F_{2t} start to decrease and the deteriorated reputation spurs the downward mobility of skilled workers. In the long run, all of the majority (minority) are skilled (unskilled). (When F_{20} is in the intermediate range, $(H_1^*,H_2^*)=(1,F_{20})$.)

As long as (F_{10},F_{20}) is located at the left side of the two loci, the minority's average skill and wage levels relative to the majority fall at first. However, if F_{20} is sufficiently high, they start to rise at some point and both groups are totally skilled in the long run, otherwise, the relative levels continue to fall and, in particular, if F_{20} is low, the two groups are totally segregated by skill levels in the long run. The initial condition affects the long-run fate of the minority through their reputation: *good (bad) reputation begets good (bad) reputation.*

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on w_{u2}. $b^*(w_{s2})=c_h$ intersects with $F_1=1$ at $F_2\in(0,1)$ from the assumptions on w_{s2}, does not intersect with $F_2=0$ from $(1-s(0))A_{s2}h_s+s(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$, and not with $F_1=0$ from $A_{s2}h_s>\frac{c_h}{\gamma_b}$. (Thus, it approaches $(F_1,F_2)=(0,0)$.) $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ do not intersect from $w_{s2}>w_{u2}$ for $F_2>0$. In the figure, $b^*(w_{s2})=c_s$ is always below the 45^0 line, but if $\left[1-s\left(\frac{N_2}{N_1+N_2}\right)\right]A_{s2}h_s+s\left(\frac{N_2}{N_1+N_2}\right)A_{u2}h_u<\frac{c_s}{\gamma_b}$, it crosses the line.

5.1.2 A majority and a small minority

Next consider an economy where $\frac{N_2}{N_1}$ or the relative productivity of the primary sector is low enough that $A_{s2}>1>A_{u2}$ and $A_{s1}\geq A_{u1}>1$ hold, i.e. unskilled workers of the minority are less productive (*net* of the disutility of work) in the sector. Then, from Proposition 1 (i)(a), (i)(c), and (ii)(d), $p_{si}=p_{u1}=1$ $(i=1,2)$, while $p_{u2}=0$ for $s_2\leq\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}$, $p_{u2}=\frac{s_2A_{s2}h_s+(1-s_2)A_{u2}h_u-h_u}{(1-A_{u2})h_u}\frac{F_2}{1-F_2}\in(0,1)$ for $s_2\in\left(\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u},\frac{1}{F_2}\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}\right)$, and $p_{u2}=1$ for $s_2\geq\frac{1}{F_2}\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}$. Unlike the previous economy, sectoral choices of the unskilled minority are not socially optimal when $p_{u2}>0$. The dividing line between $p_{u2}=0$ and $p_{u2}\in(0,1)$ and the one between $p_{u2}\in(0,1)$ and $p_{u2}=1$ are given respectively by:

$$s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)=\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}, \tag{20}$$

$$s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)F_2=\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}. \tag{21}$$

Assumptions related to the dynamics of F_{1t} and F_{2t} are same as the previous case except that $(1-s(0))A_{s2}h_s+s(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$ is strengthened to $h_u+(1-s(0))(A_{s2}h_s-A_{u2}h_u)<\frac{c_h}{\gamma_b}$ (and $A_{u2}h_u<\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$ now follows from Assumption 3). Thus, $b^*(w_{u2})=c_h$ and $b^*(w_{s2})=c_h$ when $p_{u2}=1$ are given by (18) and (19), respectively. When $p_{u2}\in(0,1)$, $b^*(w_{s2})=c_h$ equals

$$h_u+\left[1-s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)\right](A_{s2}h_s-A_{u2}h_u)=\frac{c_h}{\gamma_b}, \tag{22}$$

which is obtained by substituting (8) into (17) (with $<$ replaced by $=$).[25]

Figure 4 illustrates the dynamics of F_{1t}, F_{2t}, and p_{u2t} graphically. On the (F_1,F_2) plane, the dividing line between $p_{u2}=0$ and $p_{u2}\in(0,1)$ is a positively-sloped straight line that is located above the 45^0 line and approaches the origin ($p_{u2}=0$ at $F_2=0$). The dividing line between $p_{u2}\in(0,1)$ and $p_{u2}=1$ is a negatively-sloped curve, because the LHS of (21) depends positively on s_2F_2, like the LHS of the equation for $b^*(w_{u2})=c_h$, (18).[26] The two lines are located at the left side of $b^*(w_{u2})=c_h$ (from Assumption 3) and intersect at $F_2=1$. $b^*(w_{s2})=c_h$ when $p_{u2}\in(0,1)$ is a positively-sloped straight line approaching the origin.[27]

The dynamics of F_{1t} and F_{2t} are as in the previous case: when F_{20} is large [small] enough that $b^*(w_{u2})>c_h[b^*(w_{s2})<c_h]$ at $(F_1,F_2)=(1,F_{20})$, F_{2t} starts to increase [decrease] eventually and $(F_1^*,F_2^*)=(H_1^*,H_2^*)=(1,1)[=(1,0)]$ in the long run.

[25] $b^*(w_{s2})=c_h$ when $p_{u2}\in(0,1)$ exists since the LHS of (22) is strictly higher (lower) than the RHS at lowest (highest) s_2, i.e. at s_2 satisfying (20) ($s_2=s(0)$), from $A_{s2}h_s>\frac{c_h}{\gamma_b}$ and $h_u+(1-s(0))(A_{s2}h_s-A_{u2}h_u)<\frac{c_h}{\gamma_b}$.

[26] Since $s\left(\frac{N_2}{N_1+N_2}\right)>\frac{(1-A_{u2})h_u}{A_{s2}h_s-A_{u2}h_u}$, i.e. $p_{u2}>0$ on $\frac{F_2}{F_1}=1$ and $p_{u2}=1$ at $(F_1,F_2)=(1,1)$, from $\left[1-s\left(\frac{N_2}{N_1+N_2}\right)\right]A_{u2}h_u+s\left(\frac{N_2}{N_1+N_2}\right)A_{s2}h_s>\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$ and Assumption 3, the dividing line between $p_{u2}=0$ and $p_{u2}\in(0,1)$ is above the 45^0 line and the one between $p_{u2}\in(0,1)$ and $p_{u2}=1$ exists and intersects with the 45^0 line (and with $F_2=1$).

[27] Unlike the figure, $b^*(w_{s2})=c_h$ when $p_{u2}\in(0,1)$ may be located above the 45^0 line or it may not intersect with the dividing line between $p_{u2}\in(0,1)$ and $p_{u2}=1$, although main results are not affected.

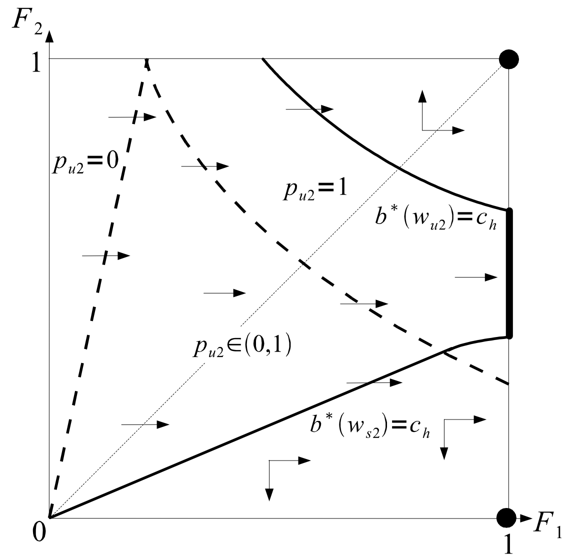

Figure 4: Dynamics when $H_{it} = F_{it}$ always and the relative size of the minority is small

What is new is that sectoral choices of the unskilled minority change over time. Suppose F_{10} is small enough that $p_{u20} = 0$, i.e. they choose the secondary sector initially. Then, as long as $p_{u2t} = 0$ is satisfied, wages of the minority equal human capital levels and are constant. After F_{1t} and thus s_{2t} become high enough that $p_{u2t} \in (0,1)$ holds, induced by the growth of s_{2t}, more and more of the unskilled minority choose the primary sector over time despite such choice is *inefficient*, i.e. $A_{u2} < 1$. This deteriorates their reputation and, together with the increasing importance of reputation (an increase in s_{2t}), lowers the wage of the skilled minority, while that of the unskilled minority remains constant at h_u. That is, average earnings of the minority *fall* (note $A_{u2} < 1$).

After that, the dynamics of p_{u2t} and the wages differ greatly depending on the initial condition. When F_{20} is sufficiently high, $p_{u2t} = 1$ is realized at some point and wage dynamics become qualitatively same as the previous economy. The labor market is integrated in the long run in the sense that all individuals work in the primary sector. By contrast, when F_{20} is small, the wage of the skilled minority falls to the point that $b^*(w_{s2t}) < c_h$ and F_{2t} starts to decrease at some point. The fall of F_{2t}, like the growth of F_{1t}, raises s_{2t}, but it also worsens the minority's reputation directly. While the positive effect on s_{2t} is stronger, p_{u2t} rises as before, but eventually the negative effect dominates and p_{u2t} starts to *fall*. In the long run, all of the minority are unskilled and in the secondary sector, thus *the labor market is segregated completely by ethnicity*. Inefficient sectoral choices of the unskilled minority make the outcome sensitive to the initial condition and quite different from a "prejudice-free" economy: if their choices are optimal, i.e. $p_{u2t} = 0$, F_{2t} is constant as under $s_{2t} = 0$.

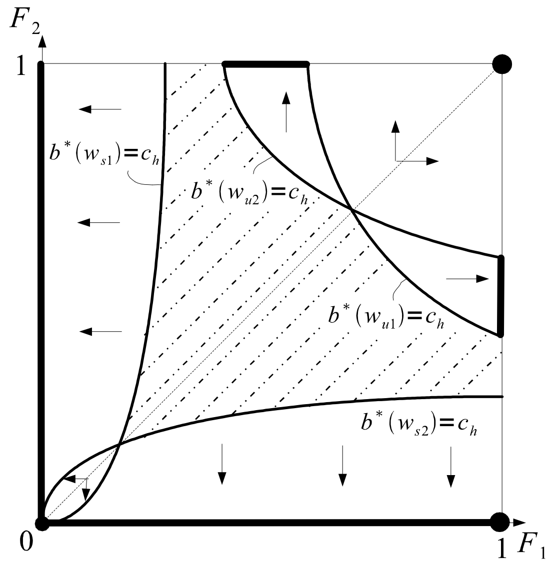

Figure 5: Dynamics when $H_{it} = F_{it}$ always and two groups are equal in size

5.1.3 Two equally sized groups

Finally, consider an economy composed of two equally sized groups, as an approximation of an economy composed of the groups with similar sizes. Thus, $A_{k1} = A_{k2} = A_k(\frac{1}{2})$ $(k = u, s)$ and denote it by A_k and assume $A_k > 1$ for simplicity. Then, $p_{si} = p_{ui} = 1$ $(i = 1, 2)$ from Proposition 1 (i)(a), (i)(c), and (ii)(d). As for the dynamics of F_{it}, assume that A_k is not very high and thus the same assumptions as the ones for the minority in the first economy (Figure 3) hold. Then, $b^*(w_{ui}) = c_h$ and $b^*(w_{si}) = c_h$ exist and are given by (18) and (19) with $N_1 = N_2$ and $A_{k1} = A_{k2} = A_k$, respectively.

Figure 5 illustrates the dynamics of F_{1t} and F_{2t}. Since $b^*(w_{k1}) = c_h$ and $b^*(w_{k2}) = c_h$ $(k = u, s)$ are symmetric with respect to the 45^0 line, directions of motion of F_{1t} and F_{2t} too are symmetric regarding the line. Unlike Figures 3 and 4, $\left[1 - s\left(\frac{1}{2}\right)\right] A_s h_s + s\left(\frac{1}{2}\right) A_u h_u < \frac{c_s}{\gamma_b}$ is assumed and thus $b^*(w_{si}) = c_s$ intersects with the 45^0 line (see footnote 24).[28]

Now, the long-run fate of the groups depends on both F_{10} and F_{20}. When (F_{10}, F_{20}) is above $b^*(w_{u1}) = c_h$ (at the right side of $b^*(w_{u2}) = c_h$), F_{1t} (F_{2t}) increases over time and $H_1^* = 1$ $(H_2^* = 1)$ in the long run. Particularly, when both F_{10} and F_{20} are high, i.e. when $b^*(w_{ui}) > c_h$ at $(F_1, F_2) = (F_{10}, F_{20})$ for at least one group i and, for $j \neq i$, $b^*(w_{uj}) > c_h$ at $F_i = 1$ and $F_j = F_{j0}$, everyone is skilled in the long run. By contrast, when (F_{10}, F_{20}) is at the left side of $b^*(w_{s1}) = c_h$ (below $b^*(w_{s2}) = c_h$), F_{1t} (F_{2t}) decreases over time and $H_1^* = 0$ $(H_2^* = 0)$ in

[28] A minor assumption, $b^*(w_{si}) = c_h$ and $b^*(w_{uj}) = c_h$ $(i \neq j)$ do not intersect, too is imposed. For $\frac{F_2}{F_1} > (<) 1$, $b^*(w_{u2}) = c_h$ is located at the left (right) side of $b^*(w_{u1}) = c_h$ since $w_{u2} > (<) w_{u1}$ from Assumption 4.

the long run. In particular, when (F_{10},F_{20}) satisfies both $b^*(w_{s1})<c_h$ and $b^*(w_{s2})<c_h$, it is possible that *nobody is skilled* in the long run: a bad impression that each group has about the other group affects the skilled wage negatively, thus F_{it} decreases and the impression deteriorates further. (In the area with chained lines, both F_{1t} and F_{2t} are constant.) Like the previous economies, the effect of initial conditions on long-run outcomes tends to be more critical than a "prejudice-free" economy: $H_i^*=F_{i0}$ when $s_{it}=0$ always.

5.1.4 Summary and discussions

Analyses have shown that the dynamics and long-run outcomes of groups, particularly of the minority, depend greatly on groups' initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition, i.e. a high (low) fraction of them can afford skill investment initially, tend to be in a good (bad) condition in the long run. In the first economy, if the initial condition of the minority is good (bad), all of them are skilled (unskilled) in the long run. In the second economy, if the condition is good (bad), not only all of them are skilled (unskilled) but also are in the primary (secondary) sector, hence the labor market becomes ethnically integrated (segregated) eventually. The third economy shows that, when the two groups are similar in size, the dynamics of the majority too could be affected greatly by initial conditions.

The strong dependence on initial conditions arises because, unlike a "prejudice-free" economy in which the dynamics of an individual lineage are affected only by the initial condition of the lineage, they are affected by initial conditions of groups too owing to the dependence of primary-sector wages on group-level variables, reputation and the degree of prejudice. Empirical findings support the positive and strong intergenerational association at the group level; for example, Borjas (1994) finds that the intergenerational correlation of mean log wages of ethnic groups (about 0.4−0.5) is much higher than that of within-group individual log wages (about 0.25) for the U.S. economy.

Note that main implications derived from the first two economies *remain intact* even if $A_{k1}=A_{k2}$ $(k=u,s)$ holds *irrespective of* the relative population size. $A_{k1}>A_{k2}$ is assumed for analytical simplicity as well as for reality: when $A_{k1}=A_{k2}$, as in Figure 5, critical loci exist for the majority too and thus analyses become complicated without affecting the implications. That is, the main implications hold even if the minority do not face disadvantages in production, do not suffer greater disutility of work, and do not face non-statistical discrimination in the primary sector. This applies to later analyses as well.

5.2 When skill investment is not always profitable

Section 5.1 has examined the case in which $H_i = F_i$ always holds. Now consider the case in which $H_i = 0$ holds at least for one group (and equilibria are stable). From Proposition 1 (ii) (see Figure 2 too), this is true when $h_s - (1+r)c_h < h_u$ and, for such group i, either $A_{ui} \geq 1$ and $(1-s(0))[A_{si}h_s - A_{ui}h_u] < (1+r)c_h$ or $A_{ui} < 1$ and $A_{si}h_s - (1+r)c_h < h_u$ are satisfied. That is, given optimal assignments to sectors, at least for such group, skill investment is unproductive or unprofitable at least when the degree of prejudice s_i is high.

Investment decisions of the two groups are interrelated, thus, depending on A_{si} and A_{ui}, equilibrium combinations of H_1 and H_2 are varied and multiple equilibria are possible. To limit possible combinations, the following is assumed.

Assumption 5 $A_s'\left(\frac{N_i}{N_i+N_j}\right)h_s - A_u'\left(\frac{N_i}{N_i+N_j}\right)h_u \geq 0.$

It states that the social return to investment in the primary sector is weakly higher for a larger group, which is reasonable because the minority tend to have greater disadvantages in jobs requiring high interpersonal skills usually occupied by skilled workers (e.g. management jobs). The next proposition presents equilibrium (H_1, H_2) based on Proposition 1.

Proposition 2 (Equilibrium $(\mathbf{H}_1, \mathbf{H}_2)$ when the investment is not always profitable) *Assume* $h_s - (1+r)c_h < h_u$ *and* $N_1 \geq N_2$.

(i) *When* $A_{u2} \geq 1$ (*thus* $A_{u1} \geq 1$ *and* $A_{s1} \geq A_{s2} > 1$), $(1-s(0))[A_{s2}h_s - A_{u2}h_u] < (1+r)c_h$, *and* $A_{s2}h_s - (1+r)c_h \geq A_{u2}h_u$,

(a) *If* $(1-s(0))[A_{s1}h_s - A_{u1}h_u] \geq (1+r)c_h$, $H_1 = F_1$ *and both* $H_2 = F_2$ *and* $H_2 = 0$ *are equilibria* ($H_2 = 0$ *is the equilibrium*) *when* $s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right) < (\geq)\ \frac{A_{s2}h_s - A_{u2}h_u - (1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$.

(b) *Otherwise,* $(H_1, H_2) = (0, F_2), (F_1, 0)$, *and, when* $s\left(\frac{F_iN_i}{F_iN_i+F_jN_j}\right) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ *for any* $i, j = 1, 2$ $(j \neq i)$, $(H_1, H_2) = (F_1, F_2)$ *as well.*

(ii) *When* $A_{s2}h_s - (1+r)c_h < \max\{A_{u2}, 1\}h_u$, *if* $A_{s1}h_s - (1+r)c_h < \max\{A_{u1}, 1\}h_u$, $(H_1, H_2) = (0, 0)$, *otherwise,* $(H_1, H_2) = (F_1, 0)$.

Given optimal sectoral allocations, when the investment is not productive for the minority (Proposition 2 (ii)), i.e., when (A_{u2}, A_{s2}) is below the dotted line for $A_{u2} \geq 1$ and below $A_{s2} = \frac{h_u + (1+r)c_h}{h_s}$ for $A_{u2} < 1$ in Figure 2, they do not invest since the individual return is lower than the social one under $s_2 > 0$. Then, the majority's individual and social returns coincide and $H_1 = F_1$ ($H_1 = 0$) when investment is productive (unproductive). Wages equal human capital levels and the result is same as when reputation does not matter.

By contrast, when $A_{u2} \geq 1$ (thus $A_{u1} \geq 1$ and $A_{s1} \geq A_{s2} > 1$) and (given optimal sectoral allocations) the investment is productive but is not profitable for the minority with highest s_2 (Proposition 2 (i)), i.e. when (A_{u2}, A_{s2}) is in the region between $A_{s2} = \frac{h_u}{h_s}A_{u2} + \frac{(1+r)c_h}{(1-s(0))h_s}$ and the dotted line in Figure 2, multiple equilibria are possible, which are examined next.

5.2.1 When investment is always profitable for the majority

If the investment is always (weakly) profitable for the majority, i.e. (A_{u1},A_{s1}) is in the region on or above $A_{s1}=\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-s(0))h_s}$ (and $A_{u1}\geq 1$) in Figure 2 (Proposition 2 (i)(a)), $H_1=F_1$ is always true, while both $H_2=F_2$ and $H_2=0$ are equilibria ($H_2=0$ is the equilibrium) when $\frac{F_2N_2}{F_2N_2+F_1N_1}$ is strictly greater (smaller) than the value satisfying

$$s\Big(\frac{F_2N_2}{F_1N_1+F_2N_2}\Big)=\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}. \tag{23}$$

As explained after Lemma 2, multiple equilibria arise due to strategic complementarity within the minority: as more of them invest in skill and become skilled workers, s_2 decreases and the investment becomes more profitable. As for sectoral choices, since $A_{s1}\geq A_{s2}>1$ and $A_{u1}\geq A_{u2}\geq 1$, $p_{si}=1$ (when $H_i>0$) and $p_{ui}=1$ ($i=1,2$) from Proposition 1 (ii)(d).

Suppose that the relative population $\frac{N_2}{N_1}$ or the relative productivity of the primary sector is not so large that assumptions related to the dynamics of F_{1t} and F_{2t} are same as the first economy in Section 5.1 (thus F_{1t} always increases), except that $(1-s(0))A_{s2}h_s+s(0)A_{u2}h_u<\frac{c_h}{\gamma_b}$ now follows from $A_{u2}h_u<\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$. When multiple equilibria exist, assume that the initial coordination among the minority continues for subsequent periods: for example, if $H_{20}=F_{20}$ happens to hold, then $H_{2t}=F_{2t}$ for any $t>0$. This assumption would be reasonable considering that children tend to mimic parental behaviors in real society.[29]

Then, if $s\Big(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\Big)<\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$ and $(H_{10},H_{20})=(F_{10},0)$ *happens to* be an initial equilibrium, *the minority never make productive investment*, F_{1t} rises and F_{2t} falls (since $H_{2t}=0$ and $A_{u2}h_u<\frac{1-\gamma_b(1+r)}{\gamma_b}c_h$) over time, and $H_1^*=F_1^*=1$ and $H_2^*=F_2^*=0$.

Otherwise (thus $(H_{10},H_{20})=(F_{10},F_{20})$ if $s\Big(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\Big)<\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$), the dynamics are as illustrated in Figure 6. The dividing line between $H_2=F_2$ and $H_2=0$ (eq. 23) is a positively-sloped straight line approaching the origin, and $H_2=0$ holds below the line. (The line is located below $b^*(w_{s2})=c_h$, if $b^*(w_{s2})=c_h$ and $b^*(w_{u2})=c_h$ do not intersect.) The dynamics of F_{2t} when $H_{2t}=F_{2t}$ are qualitatively same as the first economy of Section 5.1 (Figure 3), while when $H_{2t}=0$, F_{2t} decreases over time.

Hence, if F_{20} is not so small that $b^*(w_{s2})\geq c_h$ at $(F_1,F_2)=(1,F_{20})$, *given the initial condition*, the long-run fate of the minority is drastically different depending on which equilibrium *happens to* be realized initially: if $H_{20}=F_{20}$, $H_2^*=1$ (if $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{20})$) or $H_2^*=F_{20}$ (otherwise), whereas if $H_{20}=0$, $H_2^*=F_2^*=0$.[30] The initial selection of good (bad)

[29] Relatedly, in a dynamic model of statistical discrimination, Kim and Loury (2009) assume that, when there exist equilibrium paths to both good and bad steady states, an initial consensus on the final state shared by group members picks one path and the consensus is maintained over generations.

[30] When $A_{u2}<1<A_{s2}$, $(1-s(0))[A_{s2}h_s-A_{u2}h_u]<(1+r)c_h$, and $A_{s2}h_s-(1+r)c_h\geq h_u$ (the case not considered in the proposition or Corollary 1, see Figure 2), the dynamics are illustrated by a figure similar to Figure

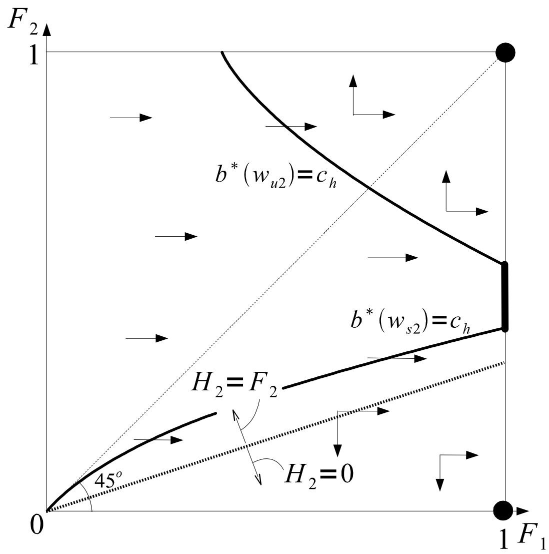

Figure 6: Dynamics when $H_{2t}=F_{2t}$ is selected in the region where both $H_{2t}=0$ and $H_{2t}=F_{2t}$ are equilibria

equilibrium brings the better (worse) long-run outcome than under $s_i=0$.

5.2.2 When investment is not always profitable for both groups

If the investment is not always profitable for the majority too, i.e. (A_{u1},A_{s1}) is in the region between $A_{s1}=\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-s(0))h_s}$ and the dotted line in Figure 2 (Proposition 2 (i)(b)), equilibria are $(H_1,H_2)=(0,F_2)$, $(F_1,0)$, and, when $s\left(\frac{F_iN_i}{F_iN_i+F_jN_j}\right)<\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any $i,j=1,2$, $(H_1,H_2)=(F_1,F_2)$ too. ($p_{si}=p_{ui}=1$ as before.) $(H_1,H_2)=(0,F_2),(F_1,0)$ are equilibria because strategic substitutability is at work between the groups: as more individuals of one group invest, prejudice toward the other group increases and their return to investment falls. As in the previous economy, assumptions related to the dynamics of F_{it} are same as the first economy in Section 5.1, and the initial coordination continues for subsequent periods when multiple equilibria exist.

Then, if only the minority (majority) happen to make productive investment initially, i.e. $H_{10}=0$ and $H_{20}=F_{20}$ ($H_{10}=F_{10}$ and $H_{20}=0$), F_{2t} is constant (falls) and F_{1t} rises and $H_1^*=0(=1)$ and $H_2^*=F_{20}(=0)$. Since this type of equilibria exist for *any* F_{10} and F_{20}, it is possible that the majority with a *much better* initial condition than the minority,

6. Differences are that no stable equilibria exist, not $H_2=0$, in the region on or below $s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)=\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, and in the region above the line, $p_{u2}<1$ is possible depending on F_1 and F_2 like Figure 4. Hence, if $s\left(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\right)<\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$ and $s\left(\frac{F_{20}N_2}{N_1+F_{20}N_2}\right)\geq\frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$, F_{1t} rises and $H_{it}=F_{it}$ $(i=1,2)$ at first, but after the economy crosses the line, the stable equilibrium fails to exist.

i.e. $F_{10} >> F_{20}$, end up with the *smaller* fraction of skilled workers, i.e. $H_1^* = 0 < H_2^* = F_{20}$ ($F_1^* = 1 > F_2^* = F_{20}$, though). If $s\big(\frac{F_{i0}N_i}{F_{i0}N_i+F_{j0}N_j}\big) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ for any $i, j = 1, 2$ and $(H_{10}, H_{20}) = (F_{10}, F_{20})$ happens to hold, the dynamics are similar to those illustrated in Figure 6.[31] Unlike the previous economy, the long-run outcome of *the majority too* is sensitive to the initial selection of equilibrium given the initial condition.

5.2.3 Summary and discussions

To summarize, when workers are more productive in the primary sector and (given optimal sectoral allocations) skill investment is productive but is not profitable with highest s_i at least for the minority, multiple equilibria could exist regarding skill investment, and *given* the initial distribution of wealth, the *initial selection of equilibrium* could affect the dynamics greatly. When the investment is profitable for the majority, it can be the case that, if the minority with enough wealth *happen to* (*not to*) invest initially, F_{2t} increases (decreases) over time and all of the minority are skilled (unskilled) in the long run. When the investment is not profitable with high s_i for the majority too, given the initial condition, the long-run outcome of *the majority too* is sensitive to the initial selection of equilibrium. The majority with a *much better* initial condition than the minority could end up with the *smaller* fraction of skilled workers, if the majority (minority) happen not to (to) invest initially.

The results suggest that, in an economy where prejudice is severe ($s(0)$ is high) or the productivity of skill investment is low, if the initial selection is affected by institutionalized discrimination against one group that limits their access to investment opportunities, such discrimination could have a lasting impact on their well-beings well after its abolishment. Income or wealth redistribution raising F_i *does little* to change the situation, while affirmative action that treats them favorably in investment, such as tuition subsidy, *can be very effective*. To be successful, their c_h must be lowered so that, for any group, $(1-s(0))[A_{si}h_s - A_{ui}h_u] \geq (1+r)c_h$ holds and thus $H_i = F_i$ becomes the unique equilibrium (Corollary 1). Redistribution becomes effective only after such policy is implemented. As in Section 5.1, the main implications of the analysis remain intact even when $A_{k1} = A_{k2}$ ($k = u, s$) always holds.

[31] Differences are that another positively-sloped straight line, $s\big(\frac{F_1N_1}{F_1N_1+F_2N_2}\big) = \frac{A_{s1}h_s - A_{u1}h_u - (1+r)c_h}{A_{s1}h_s - A_{u1}h_u}$, exists above the 45^0 line, and the equilibrium $(H_1, H_2) = (F_1, F_2)$ exists only in the region between this line and $s\big(\frac{F_2N_2}{F_1N_1+F_2N_2}\big) = \frac{A_{s2}h_s - A_{u2}h_u - (1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$ (the dotted bold line). The economy is in this region and F_{1t} increases at first. If $s\big(\frac{F_{20}N_2}{N_1+F_{20}N_2}\big) \geq \frac{A_{s2}h_s - A_{u2}h_u - (1+r)c_h}{A_{s2}h_s - A_{u2}h_u}$, the economy crosses the dotted bold line at some point, after which the equilibrium switches to $(H_{1t}, H_{2t}) = (0, F_{2t})$ *or* $(F_{1t}, 0)$. If a group with smaller F_{it} is assumed to switch to $H_{it} = 0$, as in the figure, F_{2t} continues to decline and $H_2^* = F_2^* = 0$ and $H_1^* = 1$ in the long run.

6 General case

Analyses so far are performed under Assumption 1 (ii), $A_{si} \geq \frac{h_u}{h_s}A_{ui} + \frac{h_s-h_u}{(1-s(0))h_s} \Leftrightarrow A_{si}h_s - A_{ui}h_u \geq \frac{h_s-h_u}{(1-s(0))}$. When $s(0)$ is high or when the relative productivity of skill investment in the primary sector, $\frac{A_{si}h_s-A_{ui}h_u}{h_s-h_u}$, is low, however, the assumption does not hold. Hence, it is dropped now. The assumption implies that the net return to choosing the primary sector is weakly higher for skilled workers even when the degree of prejudice is severest ($s_i = s(0)$) and thus $p_{si} \geq p_{ui}$ always holds. Without it, it is possible that $p_{si} = 0$ and $p_{ui} = 1$ hold, i.e. all skilled workers choose the secondary sector and all unskilled workers choose the *primary* sector, even if the *former* have comparative advantages and are more productive in the primary sector. Further, multiple equilibria could exist regarding *sectoral choices of skilled workers* as well as skill investment. Hence, the initial selection of equilibrium on sectoral choices too could have lasting impacts on the dynamics.

6.1 Sectoral choices and skill investment

To analyze the model without Assumption 1 (ii), this subsection examines sectoral choices and skill investment when the assumption does *not* hold. Assumption 1 (ii) is replaced by:

Assumption 6 $s(0) > \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u} \Leftrightarrow A_{si} < \frac{h_u}{h_s}A_{ui} + \frac{h_s-h_u}{(1-s(0))h_s}$.

As in Section 3.1, Assumptions 3 through 5 are not imposed in this subsection.

The following lemma on sectoral choices is parallel to Lemma 1 under the old assumption.

Lemma 3 (Sectoral choices under Assumption 6) *Suppose $H_i > 0$ and $p_{sj}H_j > 0$, $j \neq i$.*

(i) When $A_{si} \geq A_{ui} \geq 1$, $p_{ui} = 1$. $p_{si} = 1$ if $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$; both $p_{si} = 1$ and $p_{si} = 0$ if $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$; otherwise, $p_{si} = 0$.

(ii) When $A_{si} > 1 > A_{ui}$,

(a) If $(1-s_i)A_{si}h_s - h_s > (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1 \Leftrightarrow s_i = s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, $p_{si} = 1$ and p_{ui} is determined as in Lemma 1 (ii).

(b) Otherwise, $p_{si} = p_{ui} = 1$ if $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, or else, no stable equilibrium exists.

(iii) When $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$, as in Lemma 1 (iii), $p_{si} = p_{ui} = 0$.

When $A_{si} \geq A_{ui} \geq 1$, unskilled workers always choose the primary sector as before, while choices of skilled workers now depend on the net return to the primary sector: if it is positive with $p_{si} = 1$, i.e. $s\left(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, $p_{si} = 1$ as before, whereas if it is negative with $p_{si} = 0$, i.e. $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, $p_{si} = 0$.[32] That is, when $s(0)$ or $\frac{h_s}{h_u}$ is sufficiently high, all skilled workers choose the secondary sector and all unskilled workers choose the *primary*

[32]An equilibrium with $p_{si} \in (0,1)$ is not stable because the net return for the skilled increases with p_{si}.

sector, even if *skilled* workers have comparative advantages and are more productive in the primary sector. Since $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big) < s(0)$, *both* $p_{si}=1$ *and* $p_{si}=0$ *are equilibria* for some combinations of A_{si} and A_{ui} due to strategic complementarity among skilled workers (their net return increases with p_{si}). When $A_{si}>1>A_{ui}$ and the net return with $p_{si}=1$ is weakly lower for skilled workers, i.e. $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big) \geq \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, if the return for the skilled (with $p_{si}=p_{ui}=1$) is positive, $p_{si}=p_{ui}=1$ holds, otherwise (thus $p_{si}<1$), *no stable equilibrium exists*: $p_{si}=0$ cannot be an equilibrium from $A_{si}>1>A_{ui}$, while an equilibrium with $p_{si}\in(0,1)$ is not stable due to strategic complementarity. Choices are same as the corresponding cases of Lemma 1, when $A_{si}>1>A_{ui}$ and the net return with $p_{si}=1$ is higher for skilled workers, and when $A_{ui}<A_{si}=1$ or $A_{ui}\leq A_{si}<1$.

Based on Lemma 3, the next lemma presents equilibrium values of H_i, which corresponds to Lemma 2 under the original assumption.

Lemma 4 (Skill investment under Assumption 6) *Suppose* $F_i>0$.

(*i*) *When* $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ *if* $p_{sj}H_j=0$ *for* $j\neq i$. *If* $p_{sj}H_j>0$,

(*a*) *When* $A_{si}\geq A_{ui}\geq 1$,

1. *If* $h_s-(1+r)c_h\geq A_{ui}h_u$, $H_i=F_i$.
2. *If* $h_s-(1+r)c_h<A_{ui}h_u$, $H_i=F_i$ *when* $s(0)\leq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$; *both* $H_i=F_i$ *and* $H_i=0$ *are equilibria when* $s(0)>\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}>s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)$; *or else,* $H_i=0$.

(*b*) *When* $A_{si}>1>A_{ui}$, *if* $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<\min\big\{\max\big[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)},\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\big],\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\big\}$, $H_i=F_i$, *otherwise, no stable equilibrium exists.*

(*c*) *If* $A_{ui}<A_{si}=1$ *or* $A_{ui}\leq A_{si}<1$, $H_i=F_i$.

(*ii*) *When* $h_s-(1+r)c_h<h_u$, *Lemma 2* (*ii*) *applies* (*no case* $s(0)\leq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, *however*).

When $h_s-(1+r)c_h<h_u$, H_i is same as before except that the case $s(0)\leq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ of Lemma 2 (*ii*)(*a*) does not arise now. By contrast, when $h_s-(1+r)c_h\geq h_u$, $H_i=F_i$ always under the old assumption, while *inefficient* $H_i=0$ *can be an equilibrium and stable equilibria may not exist* under the new assumption. When $A_{si}\geq A_{ui}\geq 1$, $p_{si}=0$ or 1 and $p_{ui}=1$ from Lemma 3 (i). Hence, if the net return to investment is non-negative even under $p_{si}=0$ and $p_{ui}=1$ (the return is lower than under $p_{si}=p_{ui}=1$), i.e. $h_s-(1+r)c_h\geq A_{ui}h_u$, $H_i=F_i$ holds; otherwise, when the net return with $H_i=0$ is negative even under $p_{si}=p_{ui}=1$, $H_i=0$ holds, while when the net return with $H_i=F_i$ is positive under $p_{si}=p_{ui}=1$, $H_i=F_i$ holds (and both $H_i=0$ and $H_i=F_i$ are equilibria when both conditions hold due to strategic complementarity). When $A_{si}>1>A_{ui}$, no stable equilibria exist if stable p_{si} and p_{ui} do not exist, which is when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\geq\max\big[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)},\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}\big]$ from Lemma 3 (ii)(b), or if stable H_i does not exist, which is when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\geq\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ (otherwise, $H_i=F_i$). Stable H_i fails to exist in such case since the net return under $H_i=0$ is *higher* than under $H_i>0$ after the dependence of p_{ui} on H_i is taken into account ($p_{si}=1$

always from Lemma 3 (ii)): given $H_i = 0$, $p_{ui} = 0$ from $A_{ui} < 1$ and thus $H_i = 0$ is not an equilibrium from $A_{si}h_s–(1+r)c_h > h_u$, whereas, given $H_i \in (0, F_i]$, the net return is non-positive under $p_{ui} > 0$ and thus $H_i \in (0, F_i]$ is not a stable equilibrium.[33]

Finally, investment and sectoral choices of group i for given choices by the other group under Assumption 6 are summarized as follows.

Proposition 3 (Investment and sectoral choices of group i under Assumption 6)

(i) When $h_s - (1+r)c_h \geq h_u$, if $p_{sj}H_j = 0$ for $j \neq i$, $H_i = F_i$ and Proposition 1 (i)(c) applies for p_{si} and p_{ui}. If $p_{sj}H_j > 0$ (thus $s_i > 0$),

(a) When $A_{si} \geq A_{ui} \geq 1$, $p_{ui} = 1$.

1. *When $h_s - (1+r)c_h \geq A_{ui}h_u$, $H_i = F_i$. $p_{si} = 1$ if $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$; otherwise, both $p_{si} = 1$ and $p_{si} = 0$ are equilibria if $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}$, or else $p_{si} = 0$.*
2. *When $h_s - (1+r)c_h < A_{ui}h_u$ (thus $A_{ui} > 1$), if $s(0) \leq \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$, $H_i = F_i$ and $p_{si} = 1$; otherwise, both $H_i = 0$ and $H_i = F_i$, $p_{si} = 1$ are equilibria if $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$, or else $H_i = 0$.*

(b) When $A_{si} > 1 > A_{ui}$,

1. *If $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $H_i = F_i$, $p_{si} = 1$, and Proposition 1 (i)(a) applies for p_{ui}.*
2. *Otherwise, if $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \min\left\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}, \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}\right\}$, $H_i = F_i$ and $p_{si} = p_{ui} = 1$; or else, no stable equilibrium exists.*

(c) When $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$, as in Proposition 1 (i)(b), $H_i = F_i$ and $p_{si} = p_{ui} = 0$.

(ii) When $h_s - (1+r)c_h < h_u$, Proposition 1 (ii) applies (no case $s(0) \leq \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$).

If $h_s–(1+r)c_h \geq h_u$, investment and sectoral choices when $p_{sj}H_j > 0$ ($j \neq i$) are as illustrated in Figure 7. When $A_{ui} \geq 1$, if $h_s - (1+r)c_h \geq A_{ui}h_u \Leftrightarrow A_{ui} \leq \frac{h_s-(1+r)c_h}{h_u}$, $H_i = F_i$, $p_{ui} = 1$, and, depending on A_{si} and A_{ui}, $p_{si} = 0$, both $p_{si} = 0$ and $p_{si} = 1$, or $p_{si} = 1$ (sectoral choices of the skilled are inefficient when $p_{si} = 0$), while if $A_{ui} > \frac{h_s-(1+r)c_h}{h_u}$, the choices are same as the case of $h_s - (1+r)c_h < h_u$ under Assumption 1 (ii) (see Figure 2) and $H_i = 0$ is possible (then, investment is inefficient). When $A_{ui} < 1$ and stable equilibria exist, they are determined as in the corresponding case under the old assumption (see Figure 1). (The difference is that stable equilibria do not exist for some combinations of $A_{si} > 1$ and $A_{ui} < 1$.)

If $h_s–(1+r)c_h < h_u$, the choices are very similar to the corresponding case under Assumption 1 (ii), thus they are mostly as illustrated in Figure 2.[34] (The only difference is that, when $A_{ui} \geq 1$, the region in which $H_i = F_i$ is the unique equilibrium does not arise now.)

[33]Since $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) \geq \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u} > \frac{(1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{ui} = 0$ cannot be true from Lemma 3 (ii) and Lemma 1 (ii).

[34]Note that $H_i = F_i$ is possible under the new assumption as well: when $h_s - (1-s(0))(1+r)c_s < h_u$, the dotted line below the area with slanting lines in Figure 2 is located below the bold solid line dividing the regions satisfying Assumption 1 (ii) and Assumption 6.

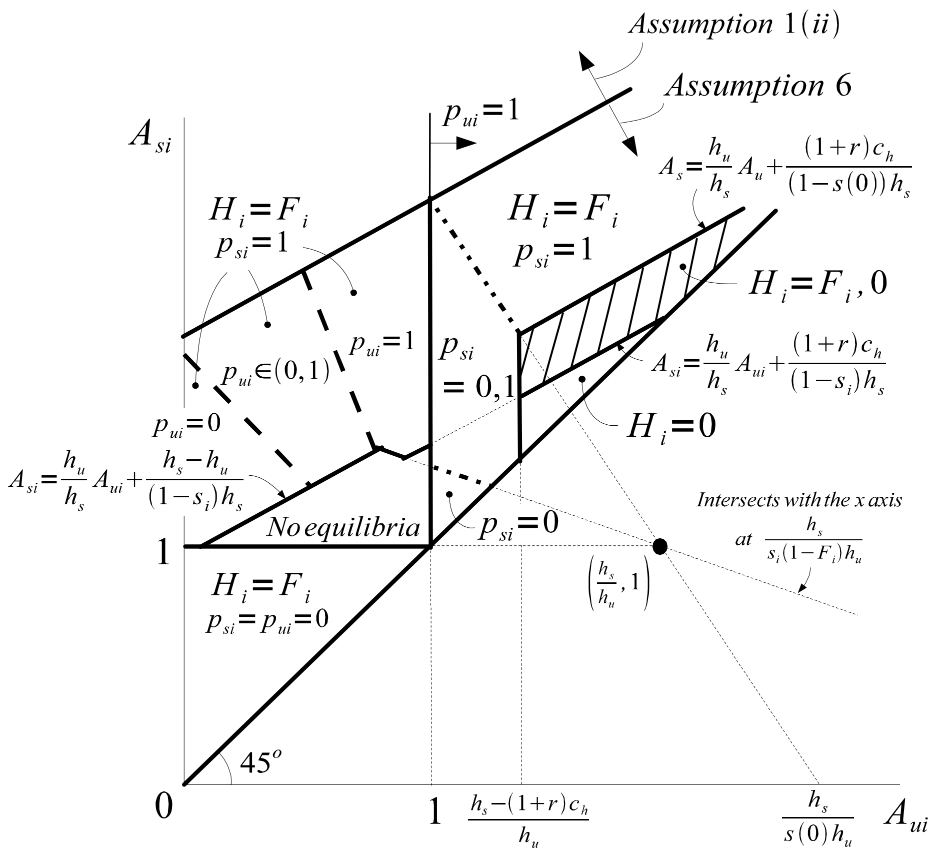

Figure 7: Investment and sectoral choices under Assumption 6 when $h_s-(1+r)c_h \geq h_u$ and $p_{sj}H_j>0$ $(j \neq i)$

6.2 Analyses

The dynamics are examined without imposing Assumption 1 (ii), based on results of the previous subsection and Section 3.1. Qualitatively new dynamics arise when $A_{si} \geq 1$, $A_{ui} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$ (thus, $h_s-(1+r)c_h \geq h_u$), $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, and Assumption 6 hold for at least one group i (the region below the upper dashed double-dotted line in Figure 7), and $A_{uj} \geq 1$ or Assumption 1 (ii) holds for the other group j.[35]

For example, consider an economy in which $A_{s2}>1$, $A_{u2} \in \left[1, \frac{h_s-(1+r)c_h}{h_u}\right]$, $s(0) > \frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}$, and Assumption 6 hold for the minority (group 2), and either $A_{u1} > \frac{h_s-(1+r)c_h}{h_u}$ and $A_{s1} \geq$

[35]The dynamics are similar to Section 5 in other cases. When $h_s-(1+r)c_h < h_u$, analyses of the corresponding case in Section 5 go through from Proposition 3 (ii). When $h_s-(1+r)c_h \geq h_u$, if $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$ for some i, as before, $p_{ui}=0$ and either $p_{si}=0$ or $p_{sj}H_j=0$ for $j \neq i$ from Propositions 3 (i) and 1 (i), thus either $s_j=0$, $s_i=0$, or $p_{si}=p_{sj}=0$ and analyses are simple. Otherwise, when either $A_{ui} \geq 1$ and $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ (the region on or above the upper dashed double-dotted line in Figure 7) or $A_{ui} > \frac{h_s-(1+r)c_h}{h_u}$ is true for any i, analyses in Section 5 apply, while when Assumption 6 and $A_{si}>1>A_{ui}$ hold for some i and thus stable equilibria fail to exist depending on F_i, analyses in footnote 30 of Section 5.2 apply.

$\frac{h_u}{h_s}A_{u1}+\frac{(1+r)c_h}{(1-s(0))h_s}$ or $A_{u1}\geq 1$ and Assumption 1 (ii) hold for the majority (group 1). This is the case in which skill investment is productive even in the secondary sector, all workers are more productive in the primary sector but A_{s2} and A_{l2} are low, and $s(0)$ is high. From Figures 7 and 1, $H_i = F_i$, $p_{ui}=1$ $(i=1,2)$, and $p_{s1}=1$, while p_{s2} is 0 or 1. From Proposition 3 (i)(a)1., the dividing line between the region $p_{s2}=0,1$ and the region $p_{s2}=0$ is:

$$s\left(\frac{F_2N_2}{F_1N_1+F_2N_2}\right)(1-F_2)=\frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}. \tag{24}$$

Suppose that $\frac{N_2}{N_1}$ or the relative productivity of the primary sector is not very high so that assumptions on the dynamics are same as the first economy in Section 5.1, implying that F_{1t} increases and, when $p_{s2}=1$, $b^*(w_{s2})=c_h$ (eq. 19) and $b^*(w_{u2})=c_h$ (eq. 18) exist. To make results more interesting, assume $h_s<\frac{c_h}{\gamma_b}$ and thus F_{2t} falls when $p_{s2t}=0$. When multiple equilibria exist, initial coordination continues for subsequent periods as in Section 5.2.

Then, if $s\left(\frac{F_{20}N_2}{F_{10}N_1+F_{20}N_2}\right)(1-F_{20})<\frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}$ holds, i.e. both $p_{s20}=0$ and $p_{s20}=1$ are equilibria, and $p_{s20}=0$ *happens to* be realized initially, F_{1t} rises and F_{2t} falls over time and $H_1^*=1$ and $H_2^*=0$ in the long run. Although the skilled minority are more productive in the *primary* sector, they choose the secondary sector to avoid the negative effect from the unskilled minority. The sector's wage, however, is not high enough for their descendants to remain skilled and the minority are totally unskilled in the long run.

Instead, if $p_{s20}=1$ *happens to* be realized under the *same* situation, the dynamics of F_{1t} and F_{2t} are as illustrated in Figure 8.[36]The skilled minority efficiently choose the primary sector and earn the higher wage than the previous case. In particular, if F_{20} is high enough that $b^*(w_{u2})>c_h$ at $(F_1,F_2)=(1,F_{20})$, F_{2t} starts to increase at some point and $H_1^*=H_2^*=1$ in the long run. The unskilled minority benefit from the presence of the skilled minority in the primary sector, which enables the upward mobility of their descendants.

Given the initial condition, the long-run fate of the minority is very different depending on the *initial selection of* p_{s20}. The result suggests that initial institutionalized discrimination against the group limiting their access to skilled jobs in the primary sector could have a lasting negative impact on their well-beings well after its abolishment. Affirmative action treating them favorably in the sector, such as wage subsidy making $p_{s2}=1$ the unique equilibrium, can be very effective to change the situation.

[36]From (24), the dividing line (the dashed line) is positively sloped and approaches the origin (note $s(0)>\frac{(A_{s2}-1)h_s}{A_{s2}h_s-A_{u2}h_u}$). (F_{10},F_{20}) is above the dividing line since $p_{s2}=1$ is possible only in the region above the line. Shapes of $b^*(w_{s2})=c_h$ and $b^*(w_{u2})=c_h$ are as explained in Section 5.

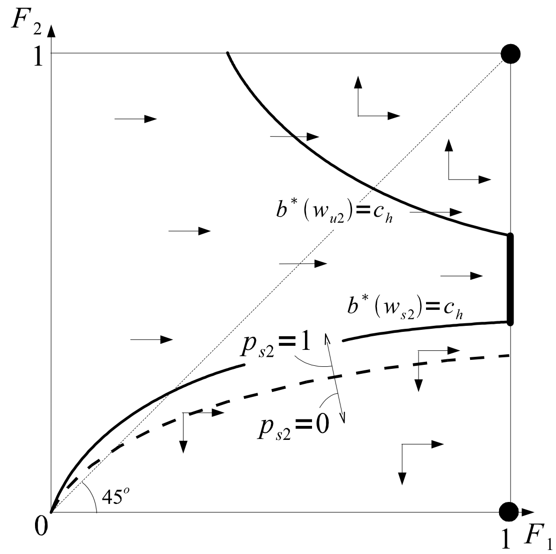

Figure 8: Dynamics when $p_{s2t} = 1$ is selected in the region where both $p_{s2t} = 0$ and $p_{s2t} = 1$ are equilibria

7 Conclusions

Disparities in economic outcomes among different ethnic, racial, or religious groups continue to be serious concerns in most economies. Relative economic standings of different groups are rather persistent, although some groups initially in disadvantaged positions successfully caught up with then-advantaged groups. Two obstacles, costly skill investment and negative stereotypes or discriminations in the labor market, seem to distort investment and sectoral decisions and slow down the economic progress of the disadvantaged.

This paper has developed a dynamic model of statistical discrimination in which these obstacles affect skill investment and sectoral choices of individuals of two groups and examined how initial economic standings of the groups and initial institutionalized discrimination affect subsequent dynamics. The model economy has (up to) two sectors, the primary sector that is ethnically mixed and group reputation affects wages due to statistical discrimination, and the secondary sector with the contrasting features.

Main results are summarized as follows. First, sectoral choices and skill investment may not be socially optimal because choices of different individuals within and across groups could be interrelated. Second, multiple equilibria could exist regarding skill investment and sectoral choices of skilled workers: both the non-poor of a group invest (skilled workers choose the primary sector) and do not could be equilibria. Third, the dynamics and long-run outcomes of groups, particularly of the minority, depend greatly on initial conditions and could be quite different from a "prejudice-free" economy. Since good (bad) reputation tends to beget good (bad) reputation, a group starting with a good (bad) initial condition tend to be in a good (bad) position in the long run. Fourth, when multiple equilibria exist, which is the case when the effect of stereotypes is strong or the productivity of human capital investment is low, given initial conditions, the initial selection of equilibrium could affect the dynamics greatly. The result suggests that, if the initial selection is affected by institutionalized discrimination limiting one group's access to skill investment or skilled jobs in the primary sector, the discrimination could have a lasting impact on their well-beings well after its abolishment. Income or wealth redistribution does little to change the situation, while affirmative action could affect the dynamics greatly.

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Appendix Proofs of lemmas and propositions

Proof of Lemma 1. (i) If $p_{si}>0$, $p_{si}=p_{ui}=1$ is the only stable equilibrium because

$$\begin{aligned}(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s &\geq (1-s(0))A_{si}h_s+s(0)E[A_{ki}h_k]-h_s \quad (25)\\ &\geq (1-s(0))A_{ui}h_u+s(0)E[A_{ki}h_k]-h_u \quad (26)\\ &\geq (1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u \quad (27)\\ &> [(1-s_i)A_{ui}-1]h_u+s_iA_{ui}h_u=(A_{ui}-1)h_u\geq 0, \quad (28)\end{aligned}$$

where the second inequality is from Assumption 1(ii) and the fourth inequality is from $p_{si}>0$. If $p_{si}=0$, $p_{ui}=0$ must hold from $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s\geq(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. However, $p_{si}=p_{ui}=0$ is not an equilibrium when $A_{si}>1$ (skilled workers deviate), and it is not stable when $A_{si}=1(=A_{ui})$ (the unskilled deviate whenever p_{si} increases).

(ii) As shown in (i), $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s\geq(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. Thus, if $p_{si}=0$, $p_{ui}=0$ must hold, which, as shown in (i), is not an equilibrium from $A_{si}>1$. If $p_{si}\in(0,1)$ and thus $s_i<s(0)$, (27) holds with strict inequality and $p_{ui}=0$ must hold, which, however, is not an equilibrium from $A_{si}>1$. Thus, if an equilibrium exists, $p_{si}=1$ and the net return to the primary sector for the unskilled is $(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u=[(1-s_i)A_{ui}-1]h_u+s_i\frac{H_iA_{si}h_s+p_{ui}(1-H_i)A_{ui}h_u}{H_i+p_{ui}(1-H_i)}$, which is decreasing in p_{ui}. Hence, $p_{ui}=0$ when the return is non-positive with $p_{ui}=0$, i.e. $[(1-s_i)A_{ui}-1]h_u+s_iA_{si}h_s\leq 0\Leftrightarrow s_i\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; $p_{ui}=1$ when it is non-negative with $p_{ui}=1$, i.e. $[(1-s_i)A_{ui}-1]h_u+s_i[H_iA_{si}h_s+(1-H_i)A_{ui}h_u]\geq 0\Leftrightarrow s_i\geq\frac{1}{H_i}\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; otherwise, $p_{ui}\in(0,1)$ and the value of p_{ui} is obtained by solving the zero return condition. Such p_{ui} and $p_{si}=1$ is an equilibrium when $p_{ui}=0$ from $A_{si}>1$ (and the above condition for the return) and when $p_{ui}>0$ from $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s>(1-s_i)A_{ui}h_u+s_iE[A_{ki}h_k]-h_u$. When $p_{ui}=0$ ($p_{ui}=1$) and the return for the unskilled is negative (positive), such equilibrium is stable since the return for the skilled is positive. In other cases, it is stable because the return for the skilled is positive and the return for the unskilled decreases with p_{ui}.

(iii) When $A_{si}=1$, $p_{si}>0$ only if $p_{ui}=0$, which, however, is not stable regarding a small increase in p_{ui}. $p_{si}=0$ and $p_{ui}>0$ is not an equilibrium from $A_{ui}<1$. Thus, $p_{si}=p_{ui}=0$ is the equilibrium when $A_{si}=1$, which is stable because the return for the skilled becomes negative whenever p_{ui} increases. When $A_{si}<1$, $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-h_s\leq[(1-s_i)A_{si}-1]h_s+s_iA_{si}h_s=(A_{si}-1)h_s<0$ and thus $p_{si}=p_{ui}=0$, which is clearly stable. ■

Proof of Lemma 2. (Proof when $p_{sj}H_j=0$ for $j\neq i$) $H_i=F_i(=0)$ iff $max[A_{si},1]h_s-(1+r)c_h-max[A_{ui},1]h_u\geq(<)0$ from Assumption 2. (i) $max[A_{si},1]h_s-(1+r)c_h-max[A_{ui},1]h_u\geq h_s-(1+r)c_h-h_u\geq 0$, where the first inequality is from Assumption 1 (i). (ii)(a)/(b) Since $A_{si}>1$, $max[A_{si},1]h_s-(1+r)c_h-max[A_{ui},1]h_u=A_{si}h_s-(1+r)c_h-max[A_{ui},1]h_u$, which equals $A_{si}h_s-(1+r)c_h-h_u\geq 0$ when $A_{ui}<1$. When $A_{ui}\geq 1$ it equals $A_{si}h_s-(1+r)c_h-A_{ui}h_u$. (c) $max[A_{si},1]h_s-(1+r)c_h-max[A_{ui},1]h_u\leq max[A_{si},1]h_s-(1+r)c_h-h_u<0$.

(Existence/nonexistence of $H_i>0$ when $p_{sj}H_j>0$) (i) Given $H_i>0$, if the condition of Lemma 1 (iii) holds and thus $p_{si}=p_{ui}=0$, the net return to investment is non-negative, i.e. $h_s-(1+r)c_h-h_u\geq 0$, and thus $p_{hi}=1$ from Assumption 2. If the condition of (i) or (ii) of the lemma holds, $p_{si}=1$ given $H_i>0$. Thus, when $p_{ui}>0$, $(1-s_i)[A_{si}h_s-A_{ui}h_u]-(1+r)c_h>(1-s(0))[A_{si}h_s-A_{ui}h_u]-(1+r)c_h\geq h_s-h_u-(1+r)c_h\geq 0$ from Assumption 1 (ii) (thus $p_{hi}=1$). When $p_{ui}=0$, since $A_{si}>1$ from the lemma, $(1-s_i)A_{si}h_s+s_iE[A_{ki}h_k]-(1+r)c_h=A_{si}h_s-(1+r)c_h>h_s-(1+r)c_h\geq h_u$. Hence, when $h_s-(1+r)c_h\geq h_u$, $H_i=p_{hi}F_i=F_i$ is the only equilibrium with $H_i>0$, which exists for any A_{si} and A_{ui} and is clearly stable.

(ii) (a)/(b) Since $A_{si}>1$, given $H_i>0$, $p_{si}=1$ from Lemma 1 (i) and (ii) and thus s_i is independent of p_{si} and equals $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)$. When $p_{ui}=0$, i.e. $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from Lemma 1 (ii) (occurs only in (b)), $A_{si}h_s-(1+r)c_h\geq h_u$ and thus $p_{hi}=1$ from Assumption 2. When $p_{ui}>0$, i.e. $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)>\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the net return equals $\big[1-s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s-A_{ui}h_u)-(1+r)c_h$ and thus, if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big)\leq(>)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $p_{hi}=1(=0)$ from Assumption 2. Note that $\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\geq\frac{(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ from $A_{si}h_s-(1+r)c_h\geq h_u$. Hence, when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)>\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, the net return is negative for any $H_i\in(0,F_i]$ and thus an equilibrium with $H_i>0$ does not exist, while when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)<\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only stable equilibrium with $H_i>0$ because the net return is increasing in H_i (when $s(0)>\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, another equilibrium with $H_i\in(0,F_i)$ exists but is not stable). Similarly, when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)=\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i=F_i$ is the only equilibrium with $H_i>0$ but is not stable.

(c) Since the skilled (unskilled) wage is lower (higher) under $s_i>0$ than under $s_i=0$, the net return to the investment cannot exceed the maximum return under $s_i=0$, $max[A_{si},1]h_s-(1+r)c_h-h_u$, which is negative. Hence, $H_i>0$ is not an equilibrium.

(Existence/nonexistence of $H_i=0$ when $p_{sj}H_j>0$) Given $H_i=0$, if $A_{ui}<1$ (thus $p_{ui}=0$), the net return is $max[A_{si},1]h_s-(1+r)c_h-h_u$, which is ≥ 0 in (i) and (ii)(b) and < 0 in (ii)(c). Hence, $H_i=0$ is not an equilibrium in (i) and (ii)(b) (when the net return is 0, $p_{hi}=1$ from Assumption 2), while it is an equilibrium in (ii)(c). Given $H_i=0$, if $A_{ui}\geq 1$ (thus $A_{si}\geq 1$ from Assumption 1 (i)), the net return is $max[(1-s_i)A_{si}h_s+s_iA_{ui}h_u,h_s]-(1+r)c_h-A_{ui}h_u=max\{(1-s(0))[A_{si}h_s-A_{ui}h_u],h_s-A_{ui}h_u\}-(1+r)c_h=(1-s(0))[A_{si}h_s-A_{ui}h_u]-(1+r)c_h\geq h_s-h_u-(1+r)c_h$ from Assumption 1 (ii) and $A_{ui}\geq 1$. Thus, in (i), $H_i=0$ is not an equilibrium when $A_{ui}\geq 1$ too (note Assumption 2). In (ii), the net return is negative (non-negative) when $s(0)>(\leq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. Hence, $H_i=0$ is (is not) an equilibrium if $s(0)>(\leq)\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ in (ii)(a) and is an equilibrium in (ii)(c). In (ii)(c), equilibrium $H_i=0$ is stable since, as shown above, the net return when $H_i>0$ is always negative. It is stable in (ii)(a) too since $s(0)>\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ implies that the net return is negative with small $H_i>0$ (since $A_{ui}\geq 1$, given $H_i>0$, $p_{si}=p_{ui}=1$ from Lemma 1 (i)). ■

Proof of Proposition 1. (i) $H_i=F_i$ from Lemma 2 (i), thus Lemma 1 applies with $H_i=F_i$. (ii)(a)/(b) The value of H_i is from Lemma 2 (ii)(a)/(b). (c) $H_i=0$ is from Lemma 2 (ii)(b). (d) Lemma 1(iii) does not apply since $H_i=F_i$ only if $A_{si}h_s-(1+r)c_h\geq h_u$ (thus $A_{si}>1$). ■

Proof of Proposition 2. When $(1-s(0))[A_{s2}h_s-A_{u2}h_u]\geq(1+r)c_h$ and $A_{s2}h_s-(1+r)c_h\geq h_u$, the same condition holds for the majority (group 1) from Assumption 5, which is the case covered in Section 5.1. When $A_{u2}<1$, $(1-s(0))[A_{s2}h_s-A_{u2}h_u]<(1+r)c_h$, and $A_{s2}h_s-(1+r)c_h\geq h_u$ (thus $A_{s2}>1$), given $p_{s1}H_1>0$, stable H_2 does not exist for some F_2 from Proposition 1 (ii)(b). Given $p_{s1}H_1=H_1=0$ ($p_{s1}=1$ when $H_1=F_1$ from $A_{s1}\geq A_{s2}>1$), $H_2=F_2$ from the proposition, but $(H_1,H_2)=(0,F_2)$ is an equilibrium for any F_1 and F_2 only when $A_{u1}\geq 1$, $(1-s(0))[A_{s1}h_s-A_{u1}h_u]<(1+r)c$, and $A_{s1}h_s-(1+r)c_h\geq A_{u1}h_{uh}$ (see Figure 2), *and* $H_1=0$ *happens to* hold ($H_1=F_1$ too could hold depending on F_1 and F_2) from Assumption 5 and Proposition 1 (ii)(a). Hence, equilibria that are stable for any F_1 and F_2 may not exist and thus this case is not considered in the proposition (briefly discussed in footnote 30).

(i) Since $A_{s1}\geq A_{s2}>1$, from Proposition 1 (ii)(d), $p_{si}=1$ when $H_i>0$ $(i=1,2)$. Then, from Proposition 1 (ii)(a), given $p_{s1}H_1=H_1=0$, $H_2=F_2$, and given $H_1>0$, both $H_2=F_2$ and $H_2=0$ ($H_2=0$) when $s\left(\frac{F_2N_2}{F_2N_2+H_1N_1}\right)<(\geq)\ \frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$. (a) In this case, from Proposition 1 (ii)(a), $H_1=F_1$ always. Hence, $H_1=F_1$ and both $H_2=F_2$ and $H_2=0$ are equilibria ($H_2=0$ is the equilibrium) when $s\left(\frac{F_2N_2}{F_2N_2+F_1N_1}\right)<(\geq)\ \frac{A_{s2}h_s-A_{u2}h_u-(1+r)c_h}{A_{s2}h_s-A_{u2}h_u}$. (b) In this case, given H_2, the value of H_1 is determined in the same way as H_2. Hence, $(H_1,H_2)=(0,F_2)$, $(F_1,0)$, and, when $s\left(\frac{F_iN_i}{F_iN_i+F_jN_j}\right)<\frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$ for any $i,j=1,2$ $(j\neq i)$, $(H_1,H_2)=(F_1,F_2)$ as well. (ii) From Proposition 1 (ii)(a) and (c), $H_2=0$ always. Then, from Proposition 1 (ii), given $H_2=0$, $H_1=0$ either when $A_{u1}\geq 1$ and $A_{s1}h_s-(1+r)c_h<A_{u1}h_u$ or when $A_{u1}<1$ and $A_{s1}h_s-(1+r)c_h<h_u$, and otherwise, $H_1=F_1$. ■

Proof of Lemma 3. (iii) The proof of Lemma 1 (iii) does not rely on Assumption 1(ii) and thus the same result as the lemma holds with Assumption 6.

(i)/(ii) An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 1$, if exists, is not stable, because the return to the primary sector for the skilled becomes positive whenever p_{si} increases. An equilibrium with $p_{si} \in (0,1)$ and $p_{ui} = 0$, which can occur only when $A_{si} = 1$, is not stable since the return for the skilled becomes negative whenever p_{ui} increases. An equilibrium with $p_{si}, p_{ui} \in (0,1)$, which satisfies $(1-s_i)A_{si}h_s + s_i E[A_{ki}h_k] - h_s = (1-s_i)A_{ui}h_u + s_i E[A_{ki}h_k] - h_u = 0$, is not stable because, whenever p_{si} increases and p_{ui} non-increases, the return for the skilled becomes positive and p_{si} does not have a tendency to return to the original value: the effect of p_{si} on the return for the skilled is greater than for the unskilled from $A_{si}h_s > A_{ui}h_u$ and the effect of p_{ui} on the return is same for both types of workers.

Thus, if a stable equilibrium exists, $p_{si} = 0$ or 1. As shown in the proof of Lemma 1 (i), $p_{si} = p_{ui} = 0$ cannot be a stable equilibrium. $p_{si} = 0$ and $p_{ui} \in (0,1)$ is not an equilibrium when $A_{ui} \neq 1$, while it is not stable when $A_{ui} = 1$ (since the return for the unskilled becomes positive whenever p_{si} increases). $p_{si} = 0$ and $p_{ui} = 1$ is not an equilibrium when $A_{ui} < 1$. When $A_{ui} \geq 1$, it is a stable equilibrium if the return for the skilled is negative, i.e. $(1-s(0))A_{si}h_s + s(0)A_{ui}h_u - h_s < 0 \Leftrightarrow s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$, because the return for the unskilled is positive (when $A_{ui} > 1$) or it increases whenever p_{si} increases (when $A_{ui} = 1$). (When $s(0) = \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$, it is not stable since the returns for the skilled increases with p_{si}.)

As for possible equilibria with $p_{si} = 1$, if $(1-s_i)A_{si}h_s - h_s > (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1 \Leftrightarrow s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, the proof of Lemma 1 (i) and (ii) can be applied with a slight modification, thus the result of the lemma holds. If $(1-s_i)A_{si}h_s - h_s \leq (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1 \Leftrightarrow s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) \geq \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{si} \leq p_{ui}$ and thus $p_{ui} = 1$ must hold. $p_{si} = p_{ui} = 1$ is a stable equilibrium when $(1-s_i)A_{si}h_s + s_i E[A_{ki}h_k] - h_s = (1-s_i)A_{si}h_s + s_i[H_iA_{si}h_s + (1-H_i)A_{ui}h_u] - h_s > 0$ with $p_{si} = 1 \Leftrightarrow s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)}$, since the returns for both types are positive. When $s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) = \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)}$, it is not stable because the return for the skilled falls with a decrease in p_{si}. (When $(1-s_i)A_{si}h_s - h_s = (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1$, the additional reason is that the effect of p_{si} on the return for the skilled is greater than for the unskilled and the effect of p_{ui} on the returns are same.)

To summarize, when $A_{si} \geq A_{ui} \geq 1$, since $\frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)} \geq \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $p_{si} = p_{ui} = 1$ if $s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)}$ and $p_{si} = 0$ and $p_{ui} = 1$ if $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$. Hence, because $s(0) > \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$ (from Assumption 6), $\frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u} \leq \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)}$, and $s(0) > s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right)$ hold, the stable equilibrium(a) is $p_{si} = p_{ui} = 1$ when $s(0) \leq \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$; both $p_{si} = p_{ui} = 1$ and $p_{si} = 0, p_{ui} = 1$ when $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ and $s\left(\frac{H_iN_i}{H_iN_i + p_{sj}H_jN_j}\right) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s - A_{ui}h_u)}$; and $p_{si} = 0$ and $p_{ui} = 1$ otherwise.

When $A_{si} > 1 > A_{ui}$, if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big) < \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the result of Lemma 1 (ii) applies, while if $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big) \geq \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$, the stable equilibrium is $p_{si} = p_{ui} = 1$ when $s\big(\frac{H_iN_i}{H_iN_i+p_{sj}H_jN_j}\big) < \frac{(A_{si}-1)h_s}{(1-H_i)(A_{si}h_s-A_{ui}h_u)}$, otherwise, no stable equilibrium exists. ■

Proof of Lemma 4. (Proof when $p_{sj}H_j = 0$ for $j \neq i$) The proof of Lemma 2 applies.

(Existence/nonexistence of $H_i > 0$ when $p_{sj}H_j > 0$)

(i)(a)/(b) When $A_{si} \geq A_{ui} \geq 1$ or $A_{si} > 1 > A_{ui}$, if $(1-s_i)A_{si}h_s - h_s > (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$, and $p_{si} = 1$ for given $H_i = F_i$ hold, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i) and (ii)), the corresponding part of Lemma 2 (i) applies and thus $H_i = F_i$ is the only stable equilibrium with $H_i > 0$.[37] Instead, if $(1-s_i)A_{si}h_s - h_s \leq (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$ and $p_{si} = 1$ for given $H_i = F_i$ hold, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) \in \big[\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}, \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}\big)$ (from Lemma 3 (i) and (ii)), $p_{si} = p_{ui} = 1$ from the lemma and the net return to investment with $H_i = F_i$ equals $\big[1-s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big)\big](A_{si}h_s - A_{ui}h_u) - (1+r)c_h$. Hence, if the return is positive, i.e. if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$, $H_i = F_i$ is the only stable equilibrium with $H_i > 0$, otherwise, no stable equilibrium with $H_i > 0$ exists. (If the return is positive, an equilibrium with $H_i \in (0, F_i)$ too may exist, and if it is zero, $H_i = F_i$ is the only equilibrium with $H_i > 0$, both of which are not stable.) Finally, when $A_{si} \geq A_{ui} \geq 1$, if $p_{si} = 0$ for $H_i = F_i$ holds, i.e. $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ (from Lemma 3 (i)), the net return is $h_s - (1+r)c_h - A_{ui}h_u$, thus, if it is non-negative, $H_i = F_i$ is the only stable equilibrium with $H_i > 0$ (note Assumption 2), otherwise, no stable equilibrium with $H_i > 0$ exists.

To summarize, when $A_{si} \geq A_{ui} \geq 1$, $H_i = F_i$ if $h_s - (1+r)c_h \geq A_{ui}h_u$ and $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ or if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \min\big\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\big\}$. Note that, when $h_s - (1+r)c_h \geq A_{ui}h_u$, $\min\big\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\big\} \geq \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$ and thus $H_i = F_i$ always. Hence, when $A_{si} \geq A_{ui} \geq 1$, $H_i = F_i$ if $h_s - (1+r)c_h \geq A_{ui}h_u$ or if $h_s - (1+r)c_h < A_{ui}h_u$ and $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}$. When $A_{si} > 1 > A_{ui}$, if $\frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u} \geq \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}$, $H_i = F_i$ when $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$; otherwise, $H_i = F_i$ if $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \min\big\{\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s-A_{ui}h_u)}, \frac{A_{si}h_s-A_{ui}h_u-(1+r)c_h}{A_{si}h_s-A_{ui}h_u}\big\}$.

(i)(c) When $A_{ui} < A_{si} = 1$ or $A_{ui} \leq A_{si} < 1$, $p_{si} = p_{ui} = 0$ from Lemma 3 (iii), thus the net return equals $h_s - (1+r)c_h - h_u \geq 0$ and $H_i = F_i$ is the only stable equilibrium with $H_i > 0$.

(ii) When $A_{si} \geq A_{ui} \geq 1$ or $A_{si} > 1 > A_{ui}$, if $(1-s_i)A_{si}h_s - h_s > (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$ and $p_{si} = 1$ for $H_i = F_i$ hold, i.e. $s\big(\frac{F_iN_i}{F_iN_i+p_{sj}H_jN_j}\big) < \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{A_{si}h_s-A_{ui}h_u}$ holds (see the proof of (i)(a)/(b)), the corresponding part of the proof of Lemma 2 (ii) applies.

[37] To be exact, if $s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s-A_{ui}h_u}$, for given F_i, there exists $\widetilde{H}_i \in (0, F_i)$ such that $s\big(\frac{\widetilde{H}_iN_i}{\widetilde{H}_iN_i+p_{sj}H_jN_j}\big) = \frac{(A_{si}-1)h_s+(1-A_{ui})h_u}{(1-\widetilde{H}_i)(A_{si}h_s-A_{ui}h_u)}$, and when $A_{si} \geq A_{ui} \geq 1$, $p_{si} = 0$ and $p_{ui} = 1$ is the only equilibrium for $H_i \in (0, \widetilde{H}_i]$. However, such H_i ($p_{hi} \in (0,1)$) is not an equilibrium because the net return equals $h_s - (1+r)c_h - A_{ui}h_u$ (note Assumption 2). The same reasoning applies to the next case and the corresponding cases of (ii) too.

In particular, since $\frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u} < \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $H_i = F_i$ is a stable equilibrium in the same cases as Lemma 2 (ii) (a)/(b), except that now $s(0) \le \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ is not possible from Assumption 6. No equilibrium with $H_i > 0$ exists in the remaining cases: when $A_{si} \ge A_{ui} \ge 1$ or $A_{si} > 1 > A_{ui}$, if $(1-s_i)A_{si}h_s - h_s \le (1-s_i)A_{ui}h_u - h_u$ with $p_{si} = 1$ and $H_i = F_i$ and $p_{si} = 1$ hold, $p_{ui} = 1$ and the net return is $(1-s_i)[A_{si}h_s - A_{ui}h_u] - (1+r)c_h \le h_s - (1+r)c_h - h_u < 0$; when $A_{si} \ge A_{ui} \ge 1$, if $p_{si} = 0$, $p_{ui} = 1$ and the net return is $h_s - (1+r)c_h - A_{ui}h_u \le h_s - (1+r)c_h - h_u < 0$; and when $A_{ui} < A_{si} = 1$ or $A_{ui} \le A_{si} < 1$, $p_{si} = p_{ui} = 0$.

(Existence/nonexistence of $H_i = 0$ when $p_{sj}H_j > 0$) Given $H_i = 0$, if $A_{ui} < 1$ (thus $p_{ui} = 0$), the corresponding part of the proof of Lemma 2 applies and thus $H_i = 0$ is not an equilibrium in (i)(b) and (c) and when $A_{ui} < 1$ in (ii). Given $H_i = 0$, if $A_{ui} \ge 1$ (thus $A_{si} \ge 1$), the net return is $max\{(1-s_i)A_{si}h_s + s_i A_{ui}h_u, h_s\} - (1+r)c_h - A_{ui}h_u = max\{(1-s(0))[A_{si}h_s - A_{ui}h_u], h_s - A_{ui}h_u\} - (1+r)c_h < h_s - (1+r)c_h - h_u$. Thus, $H_i = 0$ is always an equilibrium when $A_{ui} \ge 1$ in (ii), while it is (is not) an equilibrium if $max\{(1-s(0))[A_{si}h_s - A_{ui}h_u], h_s - A_{ui}h_u\} - (1+r)c_h < (\ge) 0$ in (i)(a). That is, $H_i = 0$ if $(1-s(0))[A_{si}h_s - A_{ui}h_u] < h_s - A_{ui}h_u \Leftrightarrow s(0) > \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ and $h_s - (1+r)c_h < A_{ui}h_u$, or if $s(0) \le \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ and $s(0) > \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$. Since $\frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u} > \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u} \Leftrightarrow h_s - (1+r)c_h < A_{ui}h_u$, $H_i = 0$ if $h_s - (1+r)c_h < A_{ui}h_u$ and $s(0) > \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ in (i)(a). ■

Proof of Proposition 3. (When $p_{sj}H_j = 0$ for $j \ne i$) Since H_i, p_{si}, and p_{ui} are determined independent of $s(\cdot)$, the corresponding result of Proposition 1 applies.

(When $p_{sj}H_j > 0$) (i)(a)1 From Lemmas 4 (i)(a)1 and 3 (i). (a)2 From Lemmas 4 (i)(a)2 and 3 (i). Note that $p_{si} = 1$ when $H_i = F_i$, since, if $p_{si} = 0$, $H_i = 0$ from $h_s - (1+r)c_h - A_{ui}h_u < 0$. (b)1 From Lemmas 4 (i)(b) and 3 (ii)(a) and Proposition 1 (i)(a). (b)2 From Lemmas 4 (i)(b) and 3 (ii)(b). (Since $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) \ge \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$, $\max\left[\frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}, \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}\right] = \frac{(A_{si}-1)h_s}{(1-F_i)(A_{si}h_s - A_{ui}h_u)}$.)(c)From Lemmas 4 (i)(c) and 3 (iii).

(ii) The determination of H_i is from Lemma 4 (ii). When $H_i = 0$, p_{si} and p_{ui} are determined independent of $s(\cdot)$ and the corresponding result of Proposition 1 applies. When $H_i = F_i$, from Lemma 4 (ii) and thus Lemma 2 (ii), $A_{si}h_s - (1+r)c_h \ge h_u$ (thus $A_{si} > 1$) and $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u}$ must hold. Hence, when $A_{ui} \ge 1$, $p_{si} = p_{ui} = 1$ from Lemma 3 (i), since $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u} < \frac{(A_{si}-1)h_s}{A_{si}h_s - A_{ui}h_u}$ and $H_i = 0$ holds if $p_{si} = 0$. When $A_{ui} < 1$, from Lemma 3 (ii)(a), p_{si} and p_{ui} are determined as in Lemma 1 (ii) and thus Proposition 1 (ii), since $s\left(\frac{F_iN_i}{F_iN_i + p_{sj}H_jN_j}\right) < \frac{A_{si}h_s - A_{ui}h_u - (1+r)c_h}{A_{si}h_s - A_{ui}h_u} < \frac{(A_{si}-1)h_s + (1-A_{ui})h_u}{A_{si}h_s - A_{ui}h_u}$. ■