Health insurance as a productive factor

Dizioli, Allan and Pinheiro, Roberto B.

University of Pennsylvania, University of Colorado

29 June 2012

Health Insurance as a Productive Factor*

Allan Dizioli and Roberto Pinheiro†

June 2012

Abstract

In this paper, we present a less-explored channel through which health insurance impacts productivity: by offering health insurance, employers reduce the expected time workers spend out of work in sick days. Using data from the Medical Expenditure Panel Survey (MEPS), we show that a worker with health coverage misses on average 52% fewer workdays than uninsured workers, after controlling for endogeneity. We develop a model that embodies this impact of health coverage in productivity. In our model, health insurance reduces the probability that a healthy worker gets sick, missing workdays, and it increases the probability that a sick worker recovers and returns to work. In our model, firms that offer health insurance are larger and pay higher wages in equilibrium, a pattern observed in the data. We calibrated the model using US data for 2004 and show the impact of increases in health costs, as well as of changes in tax benefits of health insurance expenses, on labor force health coverage and productivity. Finally, we show that a government mandate that forces firms to offer health insurance increases average wages and aggregate productivity while reducing aggregate profits, ultimately having a positive impact on welfare.

Keywords: Health, Health Insurance, Labor Productivity, Labor Markets.

JEL Codes: E20, E24, E25, E62, I10, J32, J63, J78

*We are deeply in debt to Iourii Manovskii for his helpful comments. We would like to thank Bjorn Brugemann, Colin Cameron, Hanming Fang, and Philipp Kircher for their comments, as well as the participants of the Macro Seminar at UC Davis and the West Coast Search and Matching Workshop. Any remaining errors are our responsibility.

†Dizioli: Department of Economics, University of Pennsylvania, dizioli@econ.upenn.edu, Pinheiro: Leeds School of Business, University of Colorado, roberto.pinheiro@colorado.edu

1 Introduction

At the core of the US health system is the role of employers as the main source of insurance for the population at work age (18 to 64 years old). This role generates a peculiar interaction between health care and labor markets. Because health insurance costs outside the workplace are prohibitive to most workers, employers can distinguish themselves by offering health coverage to their employees and obtain a hiring edge over firms that do not offer insurance. On the other hand, as health costs have increased, the labor force's health coverage has become a primary source of variable costs for employers. The increase in health care costs during the last decade was followed by a reduction in the fraction of workers covered by their employers. Consequently, the number of uninsured rose from 36.5 million in 1994 to 45.7 million in 2008, the latter figure representing 17.4% of the non-elderly population. The interaction between the labor market and health insurance in a scenario of rising health care costs is also harmful to labor productivity, since a number of employers hire workers as part-time or contract employees in order to reduce health insurance expenses. Similarly, many workers decide not to move to a job that seems a better match in terms of total productivity but does not offer health insurance. Therefore, a better understanding of the impact of employer-based health insurance on labor market outcomes seems fundamental to estimating the real cost of the US health insurance system.

In this paper, we present a second channel through which health insurance impacts productivity. By offering health insurance, employers reduce employees' expected time out of work in two ways: by reducing the probability a worker gets sick (preventive medicine) and/or increasing the probability a worker recovers from illness (curative medicine). Our empirical results using data from the Medical Expenditure Panel Survey (MEPS) show that a insured worker misses on average 52% fewer workdays per year than an uninsured worker[1], resulting in 2 to 3 more workdays in a year. This reduction in missed workdays implies not only that any given worker becomes a more valuable asset for the firm, but also that fewer sick days reduce the firm's expenses in paid leaves for ill absent workers.

We develop an on-the-job search model that embodies this impact of health coverage in productivity through fewer absences. In our model, employers decide not only which wages to offer, but also wether to offer a health care option to their employees. Offering health insurance has an impact on the probability that a worker gets sick, misses workdays, recovers, and returns to

[1] As usual, we controlled for observables and endogeneity.

work. Through this framework, we match several features empirically observed in the connection between labor market and health insurance coverage. For example, in our model, companies that offer health insurance will be larger in equilibrium as well as offer a higher wage. The reason for higher wages is derived from the productivity boost of health insurance; once employees are working more in expected terms, losing a worker becomes more costly for a firm. In order to avoid workers accepting outside offers, firms offering health insurance pay higher wages. This positive relation between health coverage and wages is also corroborated by our empirical findings with the MEPS. More specifically, according to our empirical results, increases in firm size and wage earned are positively related to the probability of a worker having health insurance coverage. Surprisingly, these labor-related variables are more important predictors of health coverage than health characteristics, such as health habits or addictions.

Once we calibrate the model using US data for 2004, we evaluate the impact of a series of policy changes in the health insurance sector on labor market outcomes. We find that a reduction in health insurance tax subsidies from 35% - as estimated by Gruber (2010) - to 20% generates a reduction in the share of firms providing health insurance from 60% to 47%. Once fewer firms offer insurance, the share of covered workers drops by almost 10%, while the fraction of sick workers goes up by 12.88%. We also show that a 10% increase in health insurance premiums reduces the proportion of workers with health coverage by 4.35%, increasing the number of workers sick in steady state by 5.98%. In addition, we consider a scenario in which the government mandates that all firms provide health insurance. We show that a mandate reduces firms' aggregate profit but increases previously uninsured workers' utility, while the total welfare effect is positive. Finally, we consider the difference in impact of improvements on preventive versus curative care. We compare the case of a governmental investment in medical research that makes preventive methods 10% more efficient to the case in which such an investment is made to improve curative methods (which also become 10% more efficient). Our results show that, although both medical advances have positive impact, choosing to invest in preventive instead of curative care generates a slightly higher gain (0.018%) in labor force's health coverage and consequently a reduction (-0.16%) in the number of sick workers in steady state. Keep in mind that in this exercise we did not take into account potential differences in costs of implementing such advances, that may be considerable.

The next section discusses the related literature. Section 3 describes the data, while Section 4 describes our econometric specifications. Section 5 presents empirical results to motivate the model's main hypothesis, which is the positive effect of holding health insurance on worker productivity. Section 6 describes the model while Section 7 presents comparative statistics and policy experiments. Finally, Section 8 concludes the paper.

2 Related Literature

Many scholars have attempted to explain the predominance of employer provided health insurance in the United States. There are two current leading explanations for this phenomenon. The first explanation has to do with the U.S. tax system, in which firms receive a tax benefit when they provide nondiscriminatory health insurance to their employees. Gruber and Poterba (1996) estimated that the tax-induced reduction in the "price" of employer-provided health insurance is about 27% on average. Woodbury and Huang (1991), Gruber and Poterba (1994), and Gentry and Peress (1994) concluded that taxes are an important factor in the provision of fringe benefits, although, not surprisingly, there is a wide range in the magnitude of the impact of taxes on fringe benefits. The second explanation is the cost advantage that employers gain by reducing adverse selection and lowering administrative expenses through pooling. Together these two factors reduce the cost of providing insurance in large firms relative to small groups. Brown et al. (1990) and Brugemann and Manovskii (2009) hypothesized these factors as the reasons why large firms are much more likely to offer health insurance than smaller ones.

Regarding the effect of health insurance provision on wages, the empirical literature is inconclusive. The conflicting evidence highlights the difficulty associated with isolating the impact of health insurance on labor market outcomes. In principle, we should expect that employees pay for the cost of employer-provided health insurance through lower wages. Similar to general human capital, health remains in possession of the worker as he moves from one job to another, so employers are unable to recover an investment in employees' health. Surprisingly, Monheit et al. (1985) estimated a positive relationship between the two. However, their result does not seem to be robust since Gruber (1994), Gruber and Krueger (1990), and Eberts and Stone (1985), using different datasets and methods,[2] found that most of the cost of the benefit is reflected in lower wages. A problem with these studies, addressed by

[2] Gruber (1994) uses statewide variation in mandated maternity benefits, Gruber and Krueger (1990) employs industry and state variation in the cost of worker's compensation insurance, and Eberts and Stone (1985) relies on school district variation in health insurance costs to estimate the manner in which wages are negatively affected by health insurance provision

subsequent research, is the possible endogenous relationship between health provision and wages. This endogeneity comes from the fact that workers may choose to invest in health through insurance coverage and health habits, knowing that healthier individuals are more productive and obtain higher wages. Several scholars attempted to handle this problem by looking for instrumental variables to obtain a more accurate measure of the health-wage relationship. Leibowitz (1983) used health insurance expenditures as an instrumental variable; she used the RAND Health Insurance Study (RHIS) to estimate the wage/fringe benefit trade-off. The RHIS[3] is considered an "ideal" database to test this trade-off, as it is an individual-level database that includes human capital variables that may be used as controls for ability as well as information about individual health insurance expenses[4]. Using this "ideal" dataset, Leibowitz estimated that employer health insurance expenditures had a positive effect on wages.

In spite of the vast empirical literature on this subject, few theoretical models explain the empirical findings. In the last few years some papers attempted to address this literature gap. Brugemann and Manovskii (2009) developed a quantitative equilibrium model that uses tax-deductibility of employer-provided coverage, non-discriminatory restrictions, and the fixed cost of coverage to understand labor market flows and explain why smaller firms are less likely to provide coverage than large firms. Dey and Flinn (2005) presented an equilibrium model of health insurance provision and wage determination by firms. They investigated the effect of employer-provided health insurance on job mobility rates and economic welfare using an on-the-job search model with Nash-bargaining. They found an equilibrium in which not all employment matches are covered by health insurance and wages at jobs providing health insurance are larger (in a stochastic sense) than those at jobs without health insurance. Moreover, for any given wage rate, workers at jobs with health insurance are less likely to leave those jobs. They also found that the employer-provided health insurance system does not lead to any serious inefficiency in mobility decisions. Finally, Fang and Gavazza (2011) developed a frictional labor market model in which they show that an employment-based health system fails to internalize the entire surplus generated by health investment, which leads to dynamic inefficiencies.

Our paper is different from the previous papers in several ways. Unlike Brugemann and Manovskii (2009), we develop a model of homogeneous firms,

[3]This database is also known as the RAND Health Insurance Experiment (RHIE).

[4]RAND contacted employers to obtain information on employer health insurance expenditures before survey respondents were enrolled in the study.

generating differences in productivity endogenously through firms' health insurance provision decisions. Therefore, our result remains valid even if firms do not have different costs of providing coverage. Our model also delivers the results without the presence of adverse selection, which is fundamental for Brugermann and Manovskii's model even though they found no empirical evidence to support it. Our model differs from Dey and Flinn's in two ways. First, we do not assume that firms that do not offer health coverage necessarily have a larger exogenous job destruction rate. Therefore, our model takes into account not only the productivity impact of large negative health shocks but also the impact of milder ones, which do not necessarily induce job destruction[5]. This approach not only is more general, but also allows us to evaluate the impact of changes or advances in medical treatment - more specifically, investments in curative versus preventive medicine on productivity, as well as the impact of the provision of health insurance on absenteeism and firms' costs in paid leaves. Second, unlike Dey and Flinn, we take into account the impact of taxes on health insurance provision, so we are able to measure the impact of changes in the tax treatment of health insurance expenses on labor market variables.

3 Data and Summary Statistic

The data used for this paper come from the Household Component of the Medical Expenditure Panel Survey (MEPS). The MEPS-HC is a nationally representative survey of the U.S. civilian noninstitutionalized population. The MEPS-HC collects data from a sample of households through an overlapping panel design. Every year a new sample of households is selected to compose a new panel. Five rounds of interviews take place over a two and a half year period to collect the panel data. The purpose of this design is to provide continuous and current estimates of health care expenditures at both the individual and household level for two panels for each calendar year.

The data used in this paper were collected from 2000 to 2007, i.e., we are using information from Panel 5 to Panel 10. A total of 117,994 individuals were interviewed about demographic characteristics, health conditions, health status, access to care, satisfaction with care, health insurance coverage, income, and employment.[6] Our main focus here is estimating the impact of health

[5] In their model, even though diseases imply job destruction, they do not impact workers' future productivity and/or employability. These assumptions seem contradictory, once job destructing diseases or injuries are usually related to chronic or permanent states.

[6] The MEPS sampling frame reflects an oversample of minority groups such as blacks,

insurance on missing workdays for people receving coverage through employment. Therefore, we only consider employed males ages 18 to 64 who do not receive health insurance through other sources than their employers. After we adjust the sample to fit these requirements, 26,731 data points remain.

We use two different variables measuring missing workdays: (a) missing workdays due to illness (DDNWRK) and (b) workdays missed staying in bed (WKINBD), which imply a more serious condition[7]. Since results were similar, we focused on missed workdays due to illness, because there are fewer missing observations for this variable. Mean and standard deviation of this variable are presented in Table 1. Summary statistics for explanatory variables are also presented in Table 1. Health measures include Physical Component Summary (PCS) scores, as well as some objecive measures of health, such as dummies for smoke habits (ADSMOK) and obesity. The PCS score, a self-reported measure of overall health regularly used in health economics, is formed from the answers to the Short-Form 12 questions. We also include a measure of whether the individual currently holds health insurance[8] (INS). Demographic variables include age (AGE), race (WHITE), ethnicity (HISPANICX), marital status (MARRIED), family size (FAMSY) and years of education (EDUCYR). Economic variables include whether or not the individual is part of an union (UNION), real wage at 2000 Dollars (WAGEP), whether paid leave is offered to the individual (SICPAY), the employer's sector of activity (PRIMARY, SECONDARY and TERCIARY), and firm size (NUMEMP). Finally, in order to account for the endogeneity problem of health insurance, we use dummies for region (SOUTH, MIDWEST, WEST) as instrument variables for the probability of holding health insurance coverage. Details about the instrumental variables will be discussed in the next section[9].

Asians and Hispanics. MEPS also oversamples additional policy relevant sub-groups such as low income households.

[7] According to the MEPS' questionnaire, WKINBD is obtained through the following question - "NUMBER OF DAYS MISSED WORK: {NUMBER OF DAYS}. Of those days, how many did (PERSON) stay in bed for a half day or more?". According to MEPS, they ask respondents to "include any time when this occurred because of (PERSON)'s physical illness or injury, or a mental or emotional problem such as stress or depression".

[8] MEPS includes two measures of health insurance coverage: INS and HELD. We will discuss in the paper results with INS. Similar results were obtained with HELD and are available upon request.

[9] We also used as instruments variables derived from questions about how the individual values health insurance, if they believed they did not need Health Insurance (ADINSA), if they thought that health insurance was not worth cost (ADINSB), and if they believed that they could overcome ills without medical help (ADOVER). Since results were qualitatively similar they were ommitted.

Table 1 Summary Statistics

I. Health Variables

	Mean	**SD**
Covered by Health Ins. (%)	67.52	4683
Health (pcs-12 short)	52.16	7.99
Obese (%)	42.88	4949
Smoke (%)	26.40	4408
Workdays Missed	1.752	7.50

II. Demographic Variables

	Mean	**SD**
Age (years)	39.65	10.78
Married (%)	62.45	4703
Family Size	3.292	1.695
Hispanic *(%)*	26.76	4427
Black (%)	10.72	3093

III. Labor Variables

	Mean	**SD**
Real wage ($ per month)	54807	4073
Paid Sick Leave (%)	59.65	4822
Unionized (%)	14.30	3468
Years of Education	12.65	3.229

IV. Regional Variables

	Mean
South (%)	37.90
Northeast (%)	14.49
Midwest *(%)*	20.50
West *(%)*	27.11

Smoke is a dummy that assumes value 1 if the individual is a regular smoker
Obese is a dummy that assumes value 1 if individual's body mass index (BMI) is larger than 30.
Source: MEPS - Authors' calculations

4 Econometric Specification

This section tests the crucial hypothesis implicitly assumed throughout our paper, which is: If a worker holds health coverage, then he will on average miss fewer workdays due to illness than an uninsured worker with similar characteristics. The decision to miss a workday can be treated within the random utility framework used in binary choice models. U_{0i} denotes the utility of not missing a workday while sick, while U_{1i} is the utility of missing a workday. Let $U_0 = x_i'\beta_0 + \varepsilon_{0i}$ and $U_1 = x_i'\beta_1 + \varepsilon_{1i}$ where x_i is a vector of covariates important to explain the number of missed workdays and ε_{ij} are random errors. Thus, If an individual misses a workday, we know that:

$$U_{1i} > U_{0i} \rightarrow \pi_{10} < x_i'(\beta_1 - \beta_0),$$

where $\pi_{10} = \Pr[\varepsilon_{0i} - \varepsilon_{1i}]$. Therefore, the decision to miss a workday can be represented by a Binomial Model. This is the model which motivates the Poisson econometric specification used in this section. Formally, let X be the number of successes in a large number of N independent Bernoulli trials with success probability π_{10} of each trial being small. Then it is a well-known result that as $N \to \infty$ and $\pi_{10} \to 0$, and $N\pi_{10} = \mu > 0$, this Binomial distribution function converges to a Poisson distribution function with parameter μ. The above assertion is an application of a well-known argument used to justify the framework of count data models for the study of medical care utilization based on event counts. Here a missed workday is treated in the same way as a doctor consultation. This model can be generalized in a straightforward manner to allow for unobserved heterogeneity which will imply an overdispersed count model like the negative binomial. We provide empirical evidence suggesting overdispersion of the number of missed workdays due to illness, and for this reason this article also analyzes the negative binomial specification.

Negative Binomial Specification

Let y_i denote the number of workdays missed due to illness, which is obviously a count variable that takes non-negative integer values. The density function for the negative binomial (NB) model is given by:

$$\Pr[Y = m_i|\ \gamma, \lambda) = \frac{\Gamma(m_i + \gamma_i)}{\Gamma(\gamma_i)\Gamma(m_i + 1)} \left(\frac{\gamma_i}{\lambda_i + \gamma_i}\right)^{\phi_i} \left(\frac{\lambda_i}{\lambda_i + \gamma_i}\right)^{y_i}, \tag{1}$$

where

$$\lambda_i = \exp(x_i'\beta)$$

and the precision parameter is given by:

$$\gamma_i = (1/\alpha)\lambda$$

where α is an overdispersion parameter. As a result of this specification, we have:

$$E(y_i|x_i) = \lambda_i$$

and

$$V(y_i|x_i) = \lambda_i(1 + \alpha)$$

This model is called the negative binomial-1 (NB1) model.

4.1 Estimation Procedure

In order to evaluate the impact of health coverage on workdays missed, we must account for the possible endogeneity of health insurance provision, since health insurance may be offered only to healthy people, who naturally miss fewer workdays. To deal with this problem, we follow the two-stage residual inclusion approach (2SRI) suggested by Terza et. al. (2008), which is a version of the control function approach (see details at Navarro (2008)). Our exposition here follows Cameron and Trivedi (2009).

Let m_i denote the number of workdays missed. We are assuming that m_i follows a NB1 distribution. We know that:

$$\mu_i = E(m_i|h_i, x_i, u_i) = exp(\beta_1 h_i + x'_{1i}\beta_2 + u_i) \tag{2}$$

Assume that the error term u_i is correlated with the dummy variable h_i, which is equal to 1 when a worker holds health insurance and 0 otherwise. We also assume that the error term u_i is uncorrelated with x_i,which is a vector of exogenous regressors.

In order to solve this endogeneity problem, we need to find instruments for the health insurance variable h_i. Hence, we specify a probit equation for the dummy variable h_i :

$$h_i = \Phi\left(x'_{2i}\gamma\right) + \varepsilon_i \tag{3}$$

where x_{2i} is a vector which may include variables which affect workdays missed, but x_{2i} also contains variables which affect the probability of holding health insurance while only affecting workdays missed through h_i. Similarly to the linear case, a condition for a robust identification of 2 is that there is at least one valid excluded variable (instrument).

We also assume that there is a common latent factor ε which affects both h_i and m_i and is the only source of dependence between them, after controlling for the influence of the observable variables x_1 and x_2. We can model this assumption as follows:

$$u_i = \rho\varepsilon_i + v_i$$

where v_i is independent of ε_i, v_i is i.i.d., and $E\left[e^{v_i}\right] =$ constant.

Using this additional assumption, it is possible to show that:

$$\mu_i = E(m_i|h_i, x_i, \varepsilon_i) = exp(\beta_1 h_i + x'_{1i}\beta_2 + \rho\varepsilon_i) \tag{4}$$

If ε_i were observable, we could just include it as an additional regressor and this would solve the endogeneity problem. Since we cannot observe it, we replace it with a consistent estimate. Therefore, the first step of our estimation is to estimate 3 and obtain the residuals $\hat{e}_i$. Then we estimate the parameters of the negative binomial given in 4 by replacing ε_i by $\hat{e}_i$.

5 Results

Tables 2 and 3 report the results of our estimation procedure using different models and explanatory variables. To check the consistency of our estimation, we not only estimate the Negative Binomial model but also estimate a Poisson model with a robust standard error estimate[10]. In Table 2, we use an OLS estimator for health insurance in the first step of our procedure. In Table 3, we use a Probit model in the first step, as described above. We use regional dummies as instrumental variables for health insurance. The reason for using regional variables as instruments is that there is a significant difference in health insurance coverage across regions in the US. However, we should not expect that the regional variables would have any impact on the number of workdays a worker misses. In fact, we run a regression using regional variables as explanatory variables in the second step regression and we find that the regional dummies are uncorrelated with missed workdays, confirming the validity of these dummies as instruments for health insurance coverage.

Before we start discussing the empirical results, it is important to notice that the coefficient assigned for residuals is always significant[11], indicating that our data is characterized by endogeneity. The positive coefficient for the residuals indicate that latent factors that increase the probability that an individual will have health coverage also increase the number of missed workdays - an effect consistent with adverse selection. The data also show signs of overdispersion, since the parameter alpha at the negative binomial estimation is always positive and statistically different from zero.

As for the first step regression, we find that the coefficients have the expected sign for explaining health insurance coverage. Workers' characteristic variables indicate that the probability of being covered increases with age, wage, education, family size, and union membership. Being a member of a racial or ethnic minority decreases the probability that a worker is covered. In terms of our regional instrument variables, the Northeast region is the one omitted from our regressions. Thus, we find that people living in the South and West regions have a lower probability of having health coverage when

[10] We use the bootstrap method to control for the first stage estimation of $\widehat{e}_i$, as well as overdispersion.

[11] Following a robust Wald test based on its z-statistic

compared to the Northeast region, while Midwesterners are as likely to be insured as Northeasterners. Finally, health variables have an ambiguous sign. While the self-reported overall health measure (HEALTH) is not statistically significant, being a smoker reduces the probability of having health insurance. We obtain similar qualitatively results for the probit first step in Table 3.

The main paper hypothesis is tested in the second step regression. Thus, we are interested in the *Health Insurance* coefficient, which describes the influence of holding a health insurance plan on the number of workdays an employee misses. Given our specification, if the *Health Insurance* coefficient is negative, then a worker who holds health insurance misses fewer workdays. In all specifications shown below, we find a negative and statistically significant coefficient for *Heath Insurance* at 10%, and at 5% for all but the Negative Binomial regression with a linear regression in the first step. The impact of health insurance on workdays missed is also quantitatively substantial, representing a reduction of between 52.1% and 90.3% in the expected number of workdays missed. Thus, our empirical results support our paper's hypothesis, and workers who hold health insurance are less often absent and consequently more productive.

The other explanatory variables' coefficients have the expected sign or are not statistically significant in the second step count regression. For brevity's sake, we will discuss just a few of them here. The self-reported health status have negative and significant coefficients in all specifications, indicating that a healthier worker misses fewer workdays. Since we control for these health indexes, the dummies indicating if the worker is obese or smoker are not significant for most of our model specifications. Being a member of a union or working for a large company increases the number of workdays missed, the last result corroborating previous research by Barmby and Stephen (2000). Demographic variables such as age and family size have no impact on the number of workdays missed[12]. The *Paid Leave* coefficient is positive in some specifications, an expected sign since paid sick leaves reduce the cost of missing a workday, a result also observed in previous research on absenteeism. We also include other controls, such as dummy variables indicating different economic activity sectors. We find no significant impact of secondary sector on workdays missed. However, coefficients for the primary sector's dummy are positive in all specifications in which they are statistically significant. In order to save space in the tables, the results on activity sectors have been omitted

[12]MEPS also asks if a worker misses a workday because he or she needs to take care of a sick relative. Therefore, it is not surprising that family size has no significant impact on the worker's own sick days.

but are available upon request.

6 Model

There is a continuum of risk neutral workers (measure m). While unemployed, the worker receives a job offer with probability λ_0. When employed, the worker receives a job offer with probability λ_1. Once received, the offer can be accepted or rejected. There is no recall. While unemployed, the worker receives b (unemployment insurance or the utility of leisure) each period. All agents discount future income at rate r.

We assume risk-neutral firms with measure normalized to 1. Firms offer a contract that is comprised of health insurance coverage and an hourly wage. In order to simplify the notation, we use the subscript L for firms that offer health insurance (Low health risk) and the subscript H for firms that do not offer health insurance (High health risk). To offer health coverage, the firm has to pay an up-front cost C. Since the costs of insurance are shared by firm and worker, we allow an employee to decide if he wants coverage or not once it is offered. If yes, he has to pay a flow cost of c_e per period. Otherwise, nothing is paid. We do not assume that health is part of the worker's utility function, but health insurance affects the probability that a worker gets sick (π) (preventive medicine) and/or the expected time he stays sick ($\frac{1}{\rho}$). For instance, a worker who has health insurance has a lower probability of getting sick (π_L) than a worker without coverage (π_H), that is, $\pi_H \geq \pi_L$, as well as a higher probability of healing $(\rho_L \geq \rho_H)$.

The proportion of firms not offering health insurance is γ_H, while the proportion of firms offering it is γ_L, these proportions being pinned down in equilibrium. We assume that the (potentially trivial) distribution of wages offered by firms providing health insurance is given by $F_L(z)$, while the distribution of wages offered by firms which do not provide it is $F_H(z)$.

A sick worker receives only a $\alpha \in (0,1)$ fraction of his wage. This assumption follows from the most recent available data from the Bureau of Labor Statistics' National Compensation Survey (NCS) (covering March 2008), which shows that 39 percent of private-sector workers in the United States have no paid sick days or leave. Whenever paid leaves are available, they cover around 60% of the regular salary a worker receives. Since this value is not taxed, the amount can represent up to 80% of the regular wage. Similarly, a sick employee has a potentially higher job destruction rate (δ_S) than a healthy employee (δ), $\delta_S \geq \delta$. Finally, we assume that sick workers incur additional medical costs χ. Since health insurance covers most costs to its members, we have $\chi_L \leq \chi_H$. A diagram describing the worker's problem is depicted in Figure 1 below.

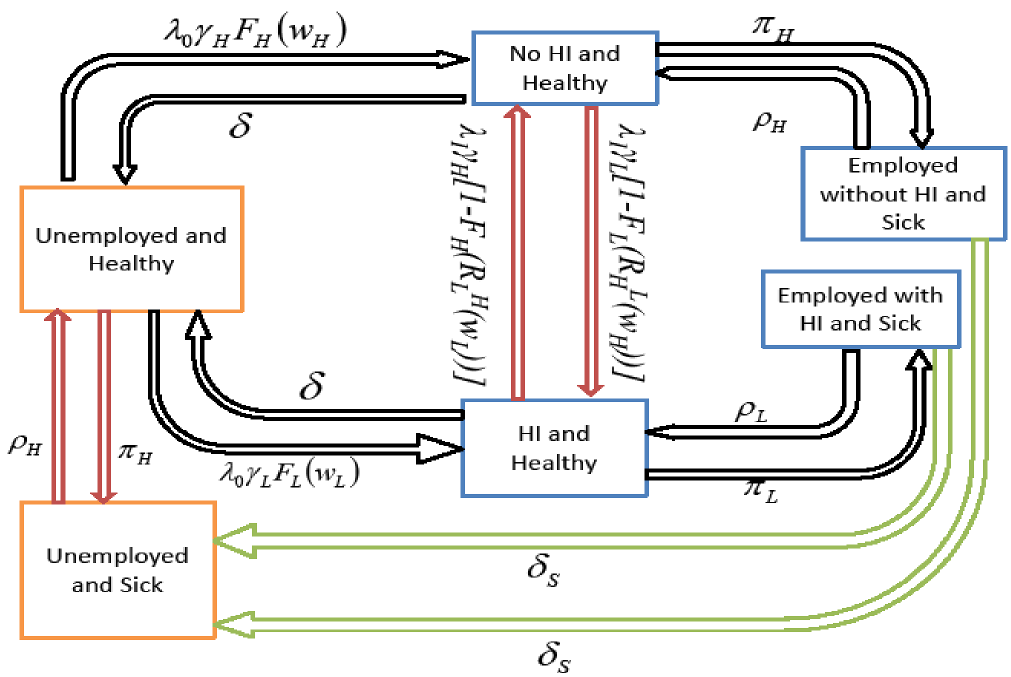

Figure 1

In the next subsection, we will look at the workers' optimal decision. Subsequently, we will look at the firm's optimization problem, and how firms' choice on health insurance coverage and wages will depend on workers' and competitors' behavior. Finally, we will discuss the steady state equilibrium. All proofs and further calculations are in the appendix.

6.1 Worker's Problem

From the framework outlined above, the expected discounted lifetime income when a worker is unemployed and healthy, V_0, can be expressed as the solution of the following equation:

$$rV_0 = b + \lambda_0 \sum_{i=H,L} \gamma_i \int \max\{V_i(z) - V_0, 0\}\, dF_i(z) + \pi_H(D_0 - V_0)$$

where b can be seen as unemployment insurance as well as utility of leisure. A job offer arrives with a probability λ_0. A fraction γ_H of offers comes from firms that do not offer health insurance while the remainder comes from firms offering health coverage. Wages offered are seen by workers as draws from equilibrium distributions $F_i(z)$, where $i \in \{H, L\}$ [13]. D_0 is the value of being an unemployed sick worker. We assume that unemployed workers don't have health insurance and that the only way a worker can obtain health insurance is through his employer. This is a simplifying assumption based on the very low percentage of the working population that has private insurance.[14] It is also without loss of generality in our model, since, as we will see, firms offering wages that would lead workers to buy insurance would optimally offer health insurance. Notice that D_0 is given by:

$$rD_0 = b - \chi_H + \rho_H (V_0 - D_0)$$

where χ_H is an additional cost of being sick without health coverage, while ρ_H is the probability a sick worker without coverage recovers. Rearranging the above expression and substituting it back, we have:

$$rV_0 = b + \lambda_0 \left[\sum_{i=H,L} \gamma_i \int_{R_U^i}^{\infty} (V_i(z) - V_0)\, dF_i(z) \right] + \pi_H \left(\frac{b - \chi_H - rV_0}{r + \rho_H} \right)$$

where R_U^L and R_U^H are the unemployed's reservation wage for working in a health-coverage company and no health-coverage company[15] .

Once a worker is employed at a firm that does not offer health insurance, the value of holding a job with wage w at this company is:

$$rV_H(w) = \left\{ \begin{array}{c} w + \lambda_1 \sum_{i=H,L} \gamma_i \int_{R_H^i(w)} (V_i(z) - V_H(w))\, dF_i(z) \\ +\delta (V_0 - V_H(w)) + \ \pi_H (D_H(w) - V_H(w)) \end{array} \right\}$$

where λ_1 is the probability a job offer arrives. As before, a fraction γ_H of offers comes from firms that don't offer health insurance while the remainder

[13] Using the same approach as Burdett and Mortensen (1998), we initially assume that the distributions of wages, $F_i(z),\ i \in \{H, L\}$ are given and we focus on the optimal workers' decisions given these distributions. We also assume the distributions are well-behaved: continuous, and differentiable (e.g. no mass points). Later, we will derive these distributions and it will be trivial to show that the assumed properties hold.

[14] Most buyers of private health insurance are entrepreneurs/self-employed, a choice that is not allowed in our model.

[15] The fact that the optimal policy is a reservation policy is straightforward and standard. If one accepts wage w_1 as part of an optimal policy, then any wage $w_2 > w_1$ for firms that are otherwise identical, gives more utility and hence should be accepted as well.

comes from firms offering health coverage. Offers above the reservation wage $R_H^i(w) \in \{H, L\}$ are accepted. As expected, reservation wages can differ depending on the company offering health coverage or not. A job match between a firm and a healthy worker is destroyed with probability δ. Finally, a worker without health insurance gets sick with probability π_H and $D_H(w)$ is the value of being sick while holding a job that pays a wage rate of w at a company that does not offer health coverage. Therefore:

$$rD_H(w) = \alpha w - \chi_H + \rho_H (V_H(w) - D_H(w)) + \delta_S (D_0 - D_H(w))$$

where α is the reduction in wages given by the sick leave. We will assume from here on that $\alpha \leq \frac{r+\delta_S}{r+\delta+\lambda_1}$. As mentioned before, a worker without health insurance heals with probability ρ_H and a job match is destroyed with probability $\delta_S \geq \delta$ if the worker is sick.

In the case in which a firm offers health coverage, we need to take into account the worker's decision of accepting the coverage or not. Therefore, the value of holding a job at wage w in a company that offers health coverage is:

$$V_L(w) = \max\{V_L(w, y), V_L(w, n)\}$$

where y and n indicate whether or not the worker accepted the coverage, respectively. But notice that $V_L(w, n) = V_H(w)$. Therefore:

$$rV_L(w) = \max\{V_L(w, y); V_H(w)\}$$

where:

$$rV_L(w, y) = \left\{ \begin{array}{c} w - c_e + \lambda_1 \sum_{i=H,L} \gamma_i \int_{R_L^i(w)} (V_i(z) - V_L(w, y))\, dF_i(z) \\ +\delta (V_0 - V_L(w, y)) + \pi_L (D_L(w) - V_L(w, y)) \end{array} \right\}$$

As mentioned before, in this case the worker pays a flow cost of c_e. We assume that this cost is paid even when the worker is sick, which implies that the value of being a sick worker at this company is given by:

$$rD_L(w) = \alpha w - c_e - \chi_L + \rho_L (V_L(w, y) - D_L(w)) + \delta_S (D_0 - D_L(w))$$

Notice that a firm would only pay the cost C if the worker opted to buy insurance, while a worker would only buy the offered health coverage if at the offered wage $w^{\triangle}$, $V_L\left(w^{\triangle}, y\right) \geq V_H\left(w^{\triangle}\right)$. In Appendix A we show that a worker would buy the coverage offered if the wage received $w^{\triangle}$ were larger than a threshold $\widetilde{w}$, determined implicitly by:

$$\left[1+\frac{\pi_L}{r+\delta_S+\rho_L}\right]c_e=\left\{\begin{array}{c}\left(\frac{\pi_L}{r+\delta_S+\rho_L}-\frac{\pi_H}{r+\delta_S+\rho_H}\right)\left[\alpha\widetilde{\omega}-\left(r+\delta_S\right)V_H\left(\widetilde{\omega}\right)+\delta_S D_0\right]\\+\frac{\pi_H}{r+\delta_S+\rho_H}\chi_H-\frac{\pi_L}{r+\delta_S+\rho_L}\chi_L\end{array}\right\}$$

Finding reservation wages: In principle, we could consider four types of job-to-job transitions (two kinds of transition between companies of different types, two kinds between companies of the same type.). However, it is trivial that the reservation wage for transitions between jobs at firms with the same health coverage is simply the present wage, i.e.

$$R_i^i(w_i)=w_i.$$

When we consider the transition between different types of firms, the following simple result simplifies the problem. Keep in mind that $R_H^L(y)$ is the minimum wage that a health-coverage firm needs to offer to poach a worker employed at a no-health-coverage firm currently earning y. Similarly, $R_L^H(x)$ is the minimum wage that a no-health-coverage firm needs to offer to poach an insured worker currently receiving wage x.

Lemma 1 *Given that $V_i(w)$ is continuous and strictly increasing in w for both $i=L,H$, for a wage x at a health-coverage firm, and a wage y at a no-health-coverage firm, the following should hold*

$$R_H^L(y)=x \iff R_L^H(x)=y.^{16}$$

Hence, we can find a function $\omega^*(\cdot)$ that maps wages at health-coverage firms into wages at no-health-coverage firms, such that for $y=\omega^*(x)$,

$$R_L^H(x)=\omega^*(x), \text{ and } R_H^L(y)=\omega^{*-1}(y)$$

The function ω^* is continuous and strictly increasing.

In Appendix B, we show that $\omega^*(w)>w$, i.e. that the function $\omega^*(\cdot)$ is above the 45 degree line, as well as that $\frac{d\omega^*(w)}{dw}>1$, for every wage above the threshold $\widetilde{w}$. These properties not only imply that all wages can be rescaled into 'health-coverage firm equivalent' wages without loss of generality,[17] but they also show that workers will ask for a wage premium to work in a company that does not offer health coverage ($\omega^*(w)>w$) and this premium is increasing with the current wage level ($\frac{d\omega^*(w)}{dw}>1$).

[16] A particular case of the result above is $R_H^L\left(R_U^L\right)=R_U^H$

[17] Of course, we alternatively could rescale all solid wages into risky firm equivalents.

Since by definition w_H and $w_L = \omega^{*-1}(w_H)$ have the same utility values, we can also replace $V_H(w_H)$ with $V_L(\omega^{*-1}(w_H))$ in the integrals of the value function, and integrate over the cumulative distribution of low-risk-firm-equivalent wages in the economy, $F(z)$ (notice the absence of the subscript!), which we define as follows:

$$F(z) = \gamma_L F_L(z) + (1-\gamma_L) F_H(\omega^*(z))$$

Once we make this adjustment, the only thing that matters for the worker's decision is the wage level in terms of 'health-coverage-firm-equivalent' units.

6.2 Firm's Problem

In this subsection, we take the behavior of workers as given and derive the firms' optimal response. Firms post wages that maximize their profits taking as given the distribution of wages posted by their competitors ($F_i(w),\ i \in \{H, L\}$) and the distribution of wages healthy employed workers are currently earning at other firms, given by distributions $G_i(w)$, $i \in \{H, L\}$. We will assume here that all distributions are stationary and well-behaved. In addition, firms decide about the provision of health insurance. If a firm offers health insurance, then it has to pay an up-front cost of C. Note that firms have to pay taxes t on wages, but they do not pay taxes on health insurance coverage expenditures C.

As we saw previously, a worker's decision depends only on whether an offer is higher in health-coverage-firm-equivalent wages. Therefore, we can construct a cumulative distribution of employed workers' equivalent-wages as follows:

$$G(w) = (1-v_H) G_L(w) + v_H G_H(\omega^*(w))$$

where v_H is the proportion of healthy employed workers in no health-coverage companies.

When a firm is choosing the optimal wage level, it has to take in consideration the amount of active workers it can attract at any given wage. For this reason, before we analyze the firm's wage decision, let's derive the firm's labor force. Since derivations are the same for firms offering and not offering coverage, consider a firm of type $i \in \{H, L\}$. Then the net inflow of workers over time, given an equivalent-wage posted w is:

$$\frac{dl_i(w)}{dt} = \lambda_0 u + \lambda_1 G(w)(m - u - s_e - s_u) + \rho_i d_i(w) - [\delta + \pi_i + \lambda_1(1 - F(w))]\, l_i(w)$$

where $d_i(w)$ is the amount of sick workers the firm keeps in any given period, while u, s_e, s_u is the measure of healthy unemployed workers, sick employed workers and sick unemployed workers in the economy, respectively. Therefore, every period a firm receives an inflow of unemployed workers at rate λ_0, an inflow of currently employed workers at rate $\lambda_1 G(w)$, and an inflow coming from previously sick employees at rate ρ_i. Similarly, every period it loses workers at rate δ to unemployment, at rate $\lambda_1(1-F(w))$ to other firms, and at rate π_i to sickness. Since in steady state we have $\frac{dl_i(w)}{dt}=0$, we have, after substituting $d_i(w)$:

$$l_i(w)=\frac{\lambda_0 u+\lambda_1 G(w)(m-u-s_e-s_u)}{\delta+\lambda_1(1-F(w))+\frac{\delta_S}{\rho_i+\delta_S}\pi_i}$$

Note that the steady state amounts of workers are different, even when equivalent wages are offered, because of different outflows into sickness. Since $\pi_H \geq \pi_L$ and $\rho_L \geq \rho_H$, with at least one inequality strict, $l_L > l_H$ at any 'health-coverage-firm-equivalent' wage. In terms of the total amount of sick workers kept, the result is ambiguous, although we know that companies that offer health coverage keep a smaller fraction of their labor force in sick leave at any period in time. As is standard in on-the-job search models, we focus on the maximization of steady state profits.[18]

Profit Maximization Every wage in distributions F_L, F_H must be optimal in equilibrium; this necessarily means that all wages offered by firms of the same type must yield the same profit. Thus, for a health coverage firm's maximization, the following must be true in equilibrium

$$\mathit{Profit}_L=\max_w\,(p-w(1+t))\,l_L(w)-\alpha w(1+t)d_L(w)-C$$

given $F(w), G(w)$,

$$F_L(w)\subseteq\{w'|w'\in\arg\max\,(p-w(1+t))\,l_L(w)-\alpha(1+t)wd_L(w)-C\}.$$

And, for a firm that does not offer health coverage:

$$\mathit{Profit}_H=\max_w\,(p-\omega^*(w)(1+t))\,l_H(w)-\alpha(1+t)\omega^*(w)\,d_H(w)$$

given $F(w), G(w)$,

$$F_H(\omega^*(w))\subseteq\{\omega^*(w')|w'=\arg\max\,(p-(1+t)\omega^*(w))\,l_H(w)-\alpha(1+t)\omega^*(w)\,d_H(w)\}.$$

[18] See Coles (2001) for a discussion of this focus.

However, we do not know yet what the distributions $F(w), F_L(w_L)$, and $F_H(w_H)$ look like. All we know at this stage is that in the equilibrium, every equivalent-wage in the support of $F(w)$ must be offered by a firm either offering or not offering health coverage.

To construct the distributions of wages offered in equilibrium, we will need to show some additional properties of wages posted by firms that offer and do not offer health coverage. Theorem 1 will allow us not only to put additional structure on the support of wages offered by each firm type, but also to say that in this environment firms do not pay compensating differentials for higher health risks.

But before that, let us present formally the result previously mentioned that no firm that offer health-coverage will offer a wage below $\widetilde{w}$.

Lemma 2 *Any firm that pays the up-front cost C will offer a wage that induces workers to join the health insurance plan.*

Now we are ready to present the result that allow us to pin down the wage distributions:

Theorem 1 *Suppose that w_L and w_H are profit-maximizing equivalent-wages offered in equilibrium by a firm providing health insurance and by a firm not providing insurance, respectively. For these wages it holds that*

$$\begin{aligned} w_L &\in \arg\max_w \{(p - w(1+t))\, l_L(w) - \alpha w(1+t) d_L(w) - C\}; \\ w_h &\in \arg\max_w \{(p - \omega^*(w)(1+t))\, l_H(w) \quad - \alpha(1+t)\omega^*(w)\, d_H(w)\} \end{aligned}$$

Then, we must have $w_L \geq w_H$. Moreover, the sets of equivalent-wages offered by health-coverage firms, and likewise by no-health-coverage firms, are connected sets.

This proposition shows that the compensating wage differentials demanded by the worker for an increase in health risk are not 'supplied' by the other side of the market. In the labor market equilibrium, firms that decide not to provide health insurance cannot profitably compete in wages with firms providing it, especially when the required compensating differential becomes large. The reason for this is that paying a high wage for a worker who will be uninsured is not profit maximizing, since the worker will be less productive due to sick leaves, while still receiving a fraction α of that high wage while sick. As a result, firms that do not offer insurance prefer to make more profit per worker and to keep this worker for a shorter period than to pay higher wage rates and risk keeping an unproductive worker for a long period of time due to sickness. Firms offering health insurance, on the other hand, pay higher wages to attract and keep the workers for a longer period since they have already invested in health insurance to keep them healthy and therefore more productive.[19] The differences in average size among firms offering and not offering health coverage are also an immediate consequence of Theorem 1. As firms with health coverage offer more attractive wages than firms with no coverage, they have a higher expected inflow of workers and a lower expected outflow, leading to a higher steady state labor force.

An important last remark is that since all firms are identical at the beginning of each period, they all must have the same profit, otherwise either all firms will invest in health insurance or no firm will invest in it. Therefore, the fraction of firms not investing in health insurance (γ_H) is endogenously determined by the following equal profit condition. For any wages w_L and w_H offered in equilibrium by firms offering and not offering health coverage, respectively, we have:

$$\begin{aligned} Profit_L &= \left(p - \left[1 + \frac{\alpha\pi_L}{\rho_L + \delta_S}\right] w_L(1+t)\right) l_L\left(w_L\right) - C = \\ &= \left(p - \left[1 + \frac{\alpha\pi_H}{\rho_H + \delta_S}\right] w_H(1+t)\right) l_H\left(w_H\right) = Profit_H \end{aligned}$$

Clearly, depending on the parameters, we may have three possible outcomes:

1.) All firms offer health insurance;
2.) No firm offers health insurance;
3.) A fraction $(1-\gamma_H) \in (0,1)$ offers health insurance.

As expected, in the next section, our discussion will focus in the third case.

We are now ready to define the steady state equilibrium formally:

Definition 1 *A steady state equilibrium in the labor market is a tuple* $\{R_U^H, \omega^*(\cdot), F_L(\cdot), F_H(\cdot), G_L(\cdot), G_H(\cdot), u, s_e, s_u, \gamma_H\}$*, such that*

[19] This result is similar to the one found in Fu (2011) in a different context. Fu (2011) have proved that in an environment with search friction, firms will pay part of the investment in General Human Capital. In the introduction we argued that we can interpret health as a kind of General Human Capital since it is valued by employers and employees take it with them from job to job.

1. *Given* $\{F_L, F_H\}$*,* R_U^H, ω^* *follow from worker's optimization*

2. *Given* $\{F_L, F_H, G_L, G_H, u, s_e, s_u, R_U^H, \omega^*\}$ *firms maximize;*

3. G_L, G_H *are stationary distributions,* u *is stationary unemployment for healthy workers,* s_e *is stationary measure of sick employees,* s_u *is stationary measure of sick unemployed workers, given the optimal decisions of workers in (1), and firms in (2);*

The first two items have been covered in the last two sections. Using these previously presented results, we can show 3 constructing the stationary distributions G_L and G_H as well as the measures of unemployed and sick workers mentioned.

In Appendix C, we explicitly characterize these equilibrium distributions and outline the existence of a steady state equilibrium by construction.

7 Discussion and Policy Analysis

The benefit of an equilibrium analysis is that it allows us to analyze the impact of changes in policy, while taking into account the overall effects and potential externalities of such measures. In this section, we present some policy exercises in order to evaluate the impact of changes in health costs and health treatments (preventive vs. curative) on relevant endogenous variables, such as the measure of firms offering health coverage, the measure of workers with health coverage, the measure of sick workers in steady state, and unemployment.

We calibrate the parameters in our model according to the data for the American economy in 2004. The unit of time considered is 1 month. First of all, the labor product p is obtained from the output per worker provided by the Bureau of Economic Analysis (BEA) through the Survey of Current Business for 2004[20]. Unemployed benefits b are set to 36% of monthly average wage, which is the national average according to the National Employment Law Center. The measure of workers relative to the number of firms, m, is obtained from the 2004 Census by dividing the total number of employer firms by the number of establishments. For the labor-market arrival rates, λ, we use the estimates by Jolivet et. al. (2002) based on data from the Panel Study of Income Dynamics (PSID) for 1994-1997. The probabilities of getting sick and healing, π and ρ respectively, are derived from our estimates of the number of days lost using the MEPS dataset described in a previous

[20] We also calculated p as the GDP per employed worker for 2004, which gave us similar qualitative results.

section. The cost of health insurance, C, is determined according to the 2004 average premium of an individual health insurance plan reported by the Kaiser Family Foundation. The exogenous termination rates are computed from the MEPS data set to match the unemployment rate of the healthy workers, δ, and the unemployment rate of sick workers, δ_S[21]. Finally, the disutility of getting sick without health insurance, χ_H, is determined by the average cost of health services in the MEPS data set. Following Gruber (2010), we consider here a tax price of 0.65, i.e., a dollar of health insurance costs 35 cents less than a dollar of other goods purchased with after-tax wages[22]. The calibrated parameters are presented in the table below:

Table 4: Calibration

p	b	$H.I.\ cost$	m	χ_H	χ_L	α	λ_0	
4934.30	804	3695	19.96	4400	0	0.81	0.143	
λ_1	π_H	π_L	ρ_H	ρ_L	δ	δ_s	r	tax
0.0112	0.088	0.086	3.33	12.05	0.00477	0.01 275	0.001	0.35

The model performance can be evaluated in Table 5. The model does a reasonably good job matching the measure of firms offering health insurance and the measure of workers with health insurance. The model underestimates the percentage of workers unemployed in equilibrium, although this is probably related to a problem with the PSID as presented by Brown and Duncan (1996). The model underestimates the wage for covered workers while overestimating the wage for uninsured workers, which is understandable given the significant presence of heterogeneity among workers in the data. One potential issue here is that the PSID has been criticized for having noisy and often inconsistent measures of job turnover, which result from questions about job tenure that are somewhat ambiguous.[23] In order to overcome that criticism, we also calibrate the model in which the labor-market arrival rates are derived from the NLSY

[21] We also used the estimates from Dey and Flinn (2005) for δ and δ_s with similar results.

[22] Therefore, we multiply firm's cost of providing health insurance C and worker's flow cost c_e by $(1 - 0.35)$.

[23] Brown and Light (1992) show that the coefficients from probit estimation using PSID turnover measures as the dependent variable are quitesensitive, both in sign and magnitude, to how one cleans the data.

by Bowlus et. al. (1995) with results that are qualitatively similar.

Table 5: Model vs. Actual Data			
Variables	Benchmark	Actual data	% Variation
Share of firms offering HI	60%	59%	1.69%
Unemployment	3.72%	5.41%	−31.24%
Avg. wage of insured	4,694.39	5,342.14	−12.13%
Avg. wage of uninsured	4,381.07	3,251.01	34.76%
STDV of insured's wage	84.92	2,063	−95.88%
STDV of uninsured's wage	108.98	4,472	−97.56%

Let us now consider the impact of rising health insurance costs on the measure of firms that offer coverage in equilibrium. This is initially tricky since the cost is divided among firms and employees. According to Buchmueller and Monheit (2009), the share of premiums paid directly by employees has remained constant over the past decade at around 15 percent for single coverage and 25 percent for family coverage. Therefore, we will assume that the worker pays 19 percent of the cost while the company pays the rest of it. The graph below summarizes our results:

As we can see, an increase in health insurance costs steeply reduces the fraction of firms offering health coverage in equilibrium. Since firms offering health coverage tend to be larger in equilibrium, the reduction in health coverage among workers is not as pronounced, but it is still significant. As expected, the measure of sick workers in steady state goes up. The measure of unemployed workers goes slightly up.[24]

Considering an increase in 10% of the health insurance premium, while keeping the share paid by employee and firm constant, we find the following result, in which the first column represents the values of the current calibration:

Table 6: Effect of Higher Health Insurance Premium			
	Benchmark	Higher Premium	% Variation
Share of firms offering insurance	59.95%	54.08%	-9.79%
Share of workers insured	83.19%	79.57%	-4.35%
Fraction of sick workers	1.087%	1.152%	5.98%
Unemployment rate	3.278%	3.282%	0.122%

Therefore, an increase in 10% in the price of health insurance generates a reduction of 9.79% in the fraction of firms offering coverage and a reduction of 4.35% in the measure of workers covered. This reduction in health insurance coverage generates an increase in the measure of sick workers in steady state by 5.98%.

Now, let us suppose that the U.S. government decides to reduce the tax benefits on health insurance expenses. Following Gruber (2010), we consider here a tax price of 0.65, i.e., a dollar of health insurance costs 35 cents less than a dollar of other goods purchased with after-tax wages. We then simulate the model with a reduction of tax benefits such that the tax price becomes 0.8, with results presented in Table 7. The main result is the large reduction in the fraction of firms providing health insurance, which drops from 60% in the benchmark model to 47% with the reduction in tax benefits. Similarly, the share of workers covered by health insurance is reduced by almost 10%. It is not surprising that the fraction of sick workers in steady state goes up, given the reduction in the fraction of the labor force covered. Finally, even though average wages go up in each group - with and without health insurance - since there is a shift of a fraction of the labor force from firms with health coverage to firms without it, the overall average wage goes down, from $\$4,641.70$ to $\$4,630.60$.

[24] This results are qualitatively independent from $\delta_S > \delta$.

Table 7: Model with Lower Tax Benefits vs Benchmark			
Variables	Benchmark	Low Tax Benefits[1]	% Variation
Share of firms offering insurance	60%	47%	−21.67%
Share of workers insured	83.19%	75.18%	−9.63%
Fraction of sick workers	1.087%	1.227%	12.88%
Unemployment	3.72%	3.74%	0.54%
Average wage of insured	4,694.39	4,700.15	0.12%
Avg. wage of uninsured	4,381.07	4,419.75	0.88%

[1]Tax Benefits reduced from 35 to 20%

Using our calibrated model, we can also analyze the effects of a policy change in which the U.S. government mandates that all firms provide health insurance. The main outcomes predicted by the model in this hypothetical situation are described in Table 8 below. As a result of this new policy, firms are worse off, as aggregate profits are reduced, even though the monthly aggregate product increases as a result of more productive workers - notice the reduction of sick workers in steady state. This result is not surprising, since the mandate reduces firms' choices and the decision not to offer health insurance by some firms was a profit-maximizing choice. The results for workers are a bit more ambiguous. Workers that have been previously uninsured are better off, their average wage goes up and their expected time sick goes down. However, workers previously insured are slightly worse off, because their average wage goes down. To pin down the net effect, we calculate the utilitarian social welfare function for this economy and we see that welfare goes up, showing that the extra utility gained by new covered workers more than fully compensated by the reduction in firms' profit or utility losses by previously insured workers.

Table 8: Effects of Government Mandate			
Variables	Benchmark	Mandate	% Variation
Firms' Aggregate Profit	3,721.58	2,479.47	−33.38%
Aggregate Product	93,785.63	94,103.06	0.3385%
Share of firms offering HI	60%	100%	66.67%
Share of workers insured	83.19%	100%	20.21%
Fraction of sick workers	1.087%	0.781%	−28.15%
Avg. wage of insured	4,694.39	4,677.27	−0.3647%
Avg. wage of uninsured	4,381.07	4,677.27	6.761%
Unemployment	3.73%	3.70%	−0.804%
Social Welfare	88,440,807.83	89,462,704.01	1.156%

We also wanted to explore which would be the better health insurance coverage, one that reduces the probability that a worker gets sick (preventive medicine - reduction in π) or one that reduces the time that a worker stays sick (curative medicine - reduction in ρ). In order to further investigate the impact of investments in preventive versus curative medicine, we consider the following exercise. Assume that the government has as its main goals to reduce the number of sick workers and to increase the number of workers with health insurance. In order to achieve such goals the government can invest a given amount in scientific advances for preventive or curative medicine. This investment can reduce the probability a worker gets sick or increase the probability that he or she recovers once sick by 10%. Considering that only workers with health insurance could benefit from the medical advance, which choice would be the best? The following table compares the results of both cases to the benchmark calibrated model:

Table 9: Preventive vs Curative Methods

	Benchmark	Preventive	Curative
Share of firms offering health insurance	59.946%	60.475%	60.449%
Share of workers covered by insurance	83%	83.511%	83.496%
Fraction of sick workers	1.087%	1.053%	1.055%
Unemployment rate	3.728%	3.725%	3.726%

As expected, even though both investments have a positive impact, the preventive medicine has a slightly greater impact than the curative one, even though differences are small. Clearly, this is just a first step in this topic. Natural extensions of this exercise need to consider differences in the cost of investments, as well as differences in the cost of treatments in both cases, as well as a deeper discussion of social welfare.

8 Concluding Remarks

In this paper, we show that health coverage has a positive impact on labor productivity by reducing the number of sick days a worker needs to take. Our empirical results using data from the Medical Expenditure Panel Survey (MEPS) show that a worker with health coverage misses on average 52% fewer workdays per year than workers without health coverage. We introduce this productivity edge to an on-the-job search model in which employers not only post wages, but also decide whether or not to offer health coverage. In equilibrium, firms offering health coverage are bigger and offer higher wages on average. These results are also corroborated by our empirical findings with the MEPS. According to our empirical results, increases in firm size and wage earned are positively related to the probability of a worker having health insurance coverage. Surprisingly, these labor-related variables are more important predictors of health coverage than health characteristics, such as health habits or addictions.

Once we calibrate the model using US data for 2004, we evaluate the impact of a series of policy changes in the health insurance sector on labor market outcomes. We show that an increase of 10% in health insurance premiums reduces the proportion of workers with health coverage by 4.35%, increasing the number of sick workers in steady state by 5.98%. We also find that a reduction in health insurance tax benefits from 35% to 20% generates a reduction in the share of firms providing health insurance from 60% to 47%. We also consider a scenario in which the government mandates that all firms provide health insurance. We show that a mandate reduces firms' aggregate profit but increases previously uninsured workers' utility, while the total welfare effect is positive. Finally, we consider the difference in impact of improvements on preventive versus curative care. We compare the case of a governmental investment in medical research that makes preventive methods 10% more efficient to the case in which such an investment is made to improve curative methods (which also become 10% more efficient). Our results show that, although both medical advances have positive impact, choosing to invest in preventive instead of curative care generates a slightly higher gain (0.018%) in labor force's health coverage and consequently a reduction (-0.16%) in the number of sick workers in steady state.

References

[1] Barmby, T., and G. Stephen, (2000). "Worker Absenteeism: Why firm size may matter," *The Manchester School* 68 (5), 568-577.

[2] Bowlus, A., N. Kiefer, and G. Neumann, (1995). "Estimation of equilibrium wage distributions with heterogeneity," *Journal of Applied Econometrics* 10, S119-S131.

[3] Brown, C., and G. Duncan, (1996). "Data Watch: The Panel Study of Income Dynamics," *Journal of Economics Perspectives*, Vol. 10 (2) ,155-168.

[4] Brown, C., and J. Hamilton and J. Meadoff, (1990). *Employers Large and Small*, Harvard University Press, Cambridge, MA.

[5] Brown, J., and A. Light, (1992). "Interpreting Panel Data on Job Tenure," *Journal of Labor Economics*, X , 219-257.

[6] Brugemann, B., and I. Manovskii, (2009). "Fragility: A quantitative Analysis of the US Health Insurance System," Working Paper, University Of Pennsylvania, mimeo.

[7] Buchmueller, T., and A. Monheit, (2009). "Employer-Sponsored Health Insurance and the Promise of Health Insurance Reform," Working Paper 14839, National Bureau of Economic Research.

[8] Burdett, K., and D. Mortensen, (1980). "Search, Layoffs, and Market Equilibrium," *Journal of Political Economy*, 88 , 652-672.

[9] Burdett, K., and D. Mortensen, (1989). "Wage Differentials, Employer Size," MEDS Discussion Paper no. 860, Northwestern University.

[10] Burdett, K., and D. Mortensen, (1998). "Wage Differentials, Employer Size and Unemployment," *International Economic Review* 39, 257-273.

[11] Cameron, A. C., and P. Trivedi, (2009). *Microeconometrics Using STATA*, STATA Press.

[12] Coles, M., (2001). "Equilibrium Wage Dispersion, Firm Size, and Growth," *Review of Economic Dynamics* 1, 159-187.

[13] Currie, J., and B. Madrian, (1999). "Health, Health Insurance, and the Labor Market," in: Ashenfelter, O., and D. Card (eds.), *Handbook of Labor Economics* Vol. 3C, 3309-3416.

[14] Cutler, D., (2003). "Employee Cost and the Decline in Health Insurance Coverage," Working Paper 9036, National Bureau of Economic Research.

[15] Cutler, D., (1994). "Market Failure in Small Group Health Insurance," Working Paper 4879, National Bureau of Economic Research.

[16] Dey, M, and C. Flinn, (2005). "An equilibrium Model of Health Insurance Provision and Wage Determination," *Econometrica,* 73 (2), 571-627.

[17] Eberts, R.W. and J.A. Stone (1985), "Wages, fringe benefits and working conditions: an analysis of compensating differentials," *Southern Economic Journal* 52, 274-280.

[18] Fang, H., and A. Gavazza, (2011). "Dynamic Inefficiencies in an Employment-Based Health-Insurance System: Theory and Evidence," *American Economic Review*, 101 (7), 3047-77.

[19] Fu, C., (2011). "Training, Search and Wage Dynamics," *Review of Economic Dynamics* 14, 650-666.

[20] Gentry, W. and E. Peress, (1994). "Taxes and fringe benefits offered by employers," Working paper 4764, National Bureau of Economic Research.

[21] Gruber, J., (1994). "The incidence of mandated maternity benefits," *American Economic Review,* 84, 622-641.

[22] Gruber, J., (2010). "The tax exclusion for employer-sponsored health insurance," Working paper 15766, National Bureau of Economic Research.

[23] Gruber, J., and A. Krueger, (1990). "The Incidence of Mandated Employer-Provided Insurance: Lessons from Workers' Compensation Insurance," Princeton Industrial Relations Section Working Paper 279.

[24] Gruber, J., and J. Poterba, (1994). "Tax incentives and the decision to purchase health insurance: evidence from the self employed," *Quarterly Journal of Economics*, 109, 701-733.

[25] Gruber, J., and J. Poterba, (1996). "Tax subsidies to employer-provided health insurance," University of Chicago Press, Chicago, IL, 135-164.

[26] Jolivet, G., F. Postel-Vinay, and J.-M. Robin, (2006). "The Empirical Content of the Job Search Model: Labor Mobility and Wage Dispersion in Europe and the U.S.," *European Economic Review*, 50(4), 877-907.

[27] Leibowitz, A., (1983). "Fringe benefits in employee compensation," in: Triplett, J. (ed.): *The measurement of labor cost*, The University of Chicago Press, 371-389.

[28] Levy, H., and D. Meltzer, (2001). "What Do We Really Know about Whether Health Insurance Affects Health?," Economic Research Initiative on the Uninsured Working Paper 6, University of Michigan.

[29] Monheit, A., M. Hagan, M. Berk, and P. Farley, (1985). "The employed uninsured and the role of public policy," *Inquiry*, 22, 348-364.

[30] Navarro, S., (2008). "Control Functions," in: Durlauf, S. and L. Blume (eds.), *The New Palgrave Dictionary of Economics*, 2nd. edition, Palgrave Macmillan Press, London.

[31] Pinheiro, R., and Visschers, L., (2009). "Job Security and Wage Differentials," University of Pennsylvania, mimeo.

[32] Terza, J., A. Basu, and P. Rathouz, (2008). "Two-stage residual inclusion estimation: Addressing endogeneity in health econometric modeling," *Journal of Health Economics*, 27, 531-543.

[33] Woodbury, S. and W. Huang, (1991). "The tax treatment of fringe benefits," Working Paper, W.E. Upjotm Institute for Employment Research, Kalamazoo, MI.

Table 2 Effects of health insurance on productivity (OLS)

First step regression: Linear Regression

	Health	*South*	Midwest	West	*Hispanic*
Held insurance	$-.0005414$ (−0.92)	$-.033103^{**}$ (−2.49)	.01064 (0.74)	$-.02999^{**}$ (−2.16)	$-.193276^{**}$ (−16.12)
	Real Wage[1]	*Firm Size*[2]	*Union*	*Smoke*	*Black*
	0.0117^{**} (11.86)	0.3567^{**} (15.23)	$.14335^{**}$ (11.74)	$-.06313^{**}$ (−6.20)	$-.061258^{**}$ (−4.59)
	Age	*Education*	*Family Size*	*White Collar*	
	$.004893^{**}$ (11.63)	$.022993^{**}$ (12.85)	$.00809^{**}$ (3.00)	$.03225^{**}$ (2.95)	

** Significant at 5%

t-stats between parenthesis

[1]wages in thousands of dollars

[2]in thousands of workers

Dummies for sector and year included

Second Step: Bootstrapped Errors

	NB	**Poisson**
Health Insurance	-2.0323^{*} (−1.92)	-2.333^{**} (−2.18)
Health	$-.0806^{**}$ (−12.90)	$-.07920^{**}$ (−13.38)
Age	.00991 (1.38)	.00951 (1.21)
Union	$.78571^{**}$ (3.63)	$.73123^{**}$ (3.16)
Family Size	−.03787 (−1.24)	−.03306 (−0.93)
Pay Leave	.08684 (0.64)	$.30911^{**}$ (2.08)
Obese	.01792 (0.18)	$-.20993^{*}$ (−1.68)
Smoke	.1309 (0.94)	−.1388 (0.386)
Firm Size[1]	1.0761^{**} (2.38)	1.4665^{**} (3.37)
Residuals	2.3028^{**} (2.12)	2.4774^{**} (2.28)

** Significant at 5%, * Significant at 10%

t-stats between parenthesis

[1]in thousands of workers

Dummies for sector and year included

% Change in E(Workdays Missed)

	NB	**Poisson**
Health Insurance	$-86.9\%^{*}$ (−1.92)	$-90.3\%^{**}$ (−2.18)
Health	$-7.7\%^{**}$ (−12.90)	$-7.61\%^{**}$ (−13.38)
Age	1.0% (1.38)	.96% (1.21)
Union	$119.4\%^{**}$ (3.63)	$107.8\%^{**}$ (3.16)
Family Size	−3.7% (−1.24)	−3.3% (−0.93)
Pay Leave	9.1% (0.64)	$36.2\%^{**}$ (2.08)
Obese	1.8% (0.18)	$18.9\%^{*}$ (−1.68)
Smoke	14.0% (0.94)	−13.0% (0.386)
Firm Size[1]	$193.3\%^{**}$ (2.38)	$333.4\%^{**}$ (3.37)
Residuals	$900.2\%^{**}$ (2.12)	$1091\%^{**}$ (2.28)

** Significant at 5%, * Significant at 10%

t-stats between parenthesis

[1]in thousands of workers

Dummies for sector and year included

Table 3 Effects of health insurance on productivity (Probit)

First step regression: Probit

	Health	*South*	Midwest	West	*Hispanic*
Held insurance	.00273 (−1.00)	−.1448** (−2.17)	.03777 (0.51)	−.1436** (−2.04)	−.6264** (−12.28)
	Real Wage[1]	*Firm Size*[2]	*Union*	*Smoke*	*Black*
	.0135** (16.87)	2.0514** (15.02)	.8452** (10.11)	−. 2606** (−5.87)	−.2815** (−4.66)
	Age	*Education*	*Family Size*	*White Collar*	
	.0201** (10.13)	.0707** (9.07)	.0403** (3.27)	.2441** (4.29)	

** Significant at 5%, * Significant at 10%
t-stats between parenthesis
[1]wages in thousands of dollars
[2]in thousands of workers
Dummies for sector and year included

Second Step: Bootstrapped Errors

	NB	**Poisson**
Health Insurance	−.7364** (−2.16)	−1.0058** (−2.39)
Health	−.08078** (−12.9)	−.07804** (−13.52)
Age	.00411 (0.81)	.00279 (0.52)
Union	.1514** (4.00)	.50298** (3.23)
Family Size	−.04411 (−1.41)	−.04504 (−1.37)
Pay Leave	.11063 (0.82)	.3326** (2.16)
Obese	.03726 (0.38)	−.1860 (−1.51)
Smoke	.2100* (1.77)	−.0429 (−.32)
Firm Size[1]	.5293** (2.04)	.9628** (3.01)
Residuals	.62630** (3.33)	.7008** (3.01)

** Significant at 5%, * Significant at 10%
t-stats between parenthesis
[1]in thousands of workers
Dummies for sector and year included

% Change in E(Workdays Missed)

	NB	**Poisson**
Health Insurance	−52.1%** (−2.16)	−63.4%** (−2.39)
Health	−7.8% (−12.9)	−7.5%** (−13.52)
Age	0.4% (0.81)	0.3% (0.52)
Union	83.2%** (4.00)	65.4%** (3.23)
Family Size	−4.3% (−1.41)	−4.4% (1.37)
Pay Leave	11.7% (0.82)	39.5%** (2.16)
Obese	3.8% (0.38)	−17.0% (−1.51)
Smoke	23.4%* (1.77)	−4.2% (−.32)
Firm Size[1]	69.77% ** (2.04)	161.9%** (3.01)
Residuals	87.1%** (3.33)	101.5%** (3.01)

** Significant at 5%, * Significant at 10%
t-stats between parenthesis
[1]in thousands of workers
Dummies for sector and year included

9 Appendix A

In this appendix, we look at $V_L(w) = \max\{V_L(w,y); V_H(w)\}$. A firm would only pay the cost C if the worker opts to buy insurance. Therefore, we can continue with $V_L(w,y)$ and $V_H(w)$ and at the end check that for any wage $w^{\triangle}$ offered by a company that pays C, $V_L\left(w^{\triangle},y\right) \geq V_H\left(w^{\triangle}\right)$.Given this, assuming that the value functions are increasing in w (which we are going to check later), we may have a cut off (that could be below zero) $\widetilde{w}$, such that for $w > \widetilde{w}$, $V_L(w,y) > V_H(w)$ (this is only true if we have a single crossing condition - i.e., we will need log concavity. We can show by obtaining $\frac{dV_L(w,y)}{dw} > \frac{dV_H(w)}{dw}$). So, first of all, let's look at the conditions for the cut off.

First of all, let's obtain $\frac{dV_H(w)}{dw}$. Manipulating the integrals and using the result that, by definition $V_L\left(R_H^L(w), y\right) = V_H(w)$, we have that:

$$\frac{dV_H(w)}{dw} = \frac{\left[1 + \frac{\alpha\pi_H}{r+\rho_H+\delta_S}\right]}{r + \delta + \frac{\pi_H(r+\delta_S)}{r+\rho_H+\delta_S} + \lambda_1\left[1 - F\left(R_H^L(w)\right)\right]}$$

where $F(\cdot)$ is defined as follows:

$$F(z) = \gamma_H F_H\left(R_L^H(z)\right) + (1-\gamma_H) F_L(z)$$

Notice that if $w > \widetilde{w}$, we must have $R_L^H(w) > w$ and $R_H^L(w) < w$:

$$V_H\left(R_L^H(w)\right) = V_L(w)$$

and

$$V_H(w) = V_L\left(R_H^L(w)\right)$$

Therefore, if $R_L^H(w) > w$, for monotonicity of the value functions, we must have $R_H^L(w) < w$. This implies that:

$$\lambda_1\left[1 - F\left(R_H^L(w)\right)\right] > \lambda_1\left[1 - F(w)\right], \text{ for any } w > \widetilde{w}$$

Now, looking at the derivative for $V_L(w,y)$, we obtain:

$$\frac{dV_L(w,y)}{dw} = \frac{\left(1 + \frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r + \delta + \frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L} + \lambda_1\left(1 - F(w)\right)}$$

We already know that the last term in the denominator is smaller for $\frac{dV_L(w,y)}{dw}$. Now, notice that $\frac{\pi_H}{r+\rho_H+\delta_S} > \frac{\pi_L}{r+\rho_L+\delta_S}$, to consider the impact of the increase in this value, let's assume that $x = \frac{\pi}{r+\rho+\delta_S}$ and to simplify consider the last term in the denominator equals to $\lambda_1 (1 - F(\widetilde{w}))$ (this actually helps $\frac{dV_H(w)}{dw}$). Then, we have that:

$$\frac{d\left(\frac{1+\alpha x}{r+\delta+(r+\delta_S)x+\lambda_1(1-F(\widetilde{w}))}\right)}{dx} = \frac{\alpha\left(r+\delta+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)\right)-\left(r+\delta_S\right)}{\left(r+\delta+\left(r+\delta_S\right)x+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)\right)^2}$$

and this is negative if:

$$\alpha\left(r+\delta\right)-\left(r+\delta_S\right)<0 \Rightarrow \alpha < \frac{\left(r+\delta_S\right)}{\left(r+\delta+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)\right)}$$

Since we assume that $\alpha \leq \frac{r+\delta_S}{r+\delta+\lambda_1}$ this is always satisfied and we have the single-crossing property that we need.

Therefore, whenever $\delta_S > \delta$, for any $w > \widetilde{w}$, $\frac{dV_L(w,y)}{dw} > \frac{dV_H(w)}{dw}$. Since $V_L(\widetilde{w}, y) = V_H(\widetilde{w}) \Rightarrow V_L(w) = V_L(w, y)$, for $w > \widetilde{w}$. This also implies that for $w > \widetilde{w}$, $R_L^H(w) > w$, as we show in the Lemma B.1.

Now, let's find an implicit expression for $\widetilde{w}$. From $V_L(\widetilde{w}, y) = V_H(\widetilde{w})$, we obtain:

$$c_e = \pi_L\left(D_L\left(\widetilde{w}\right)-V_L\left(\widetilde{w},y\right)\right)-\pi_H\left(D_H\left(\widetilde{w}\right)-V_H\left(\widetilde{w}\right)\right)$$

Since:

$$\pi_H\left(D_H\left(\widetilde{\omega}\right)-V_H\left(\widetilde{\omega}\right)\right)=\frac{\pi_H}{r+\delta_S+\rho_H}\left\{\alpha\widetilde{\omega}-\chi_H-\left(r+\delta_S\right)V_H\left(\widetilde{\omega}\right)+\delta_S D_0\right\}$$

and

$$\pi_L\left(D_L\left(\widetilde{\omega}\right)-V_L\left(\widetilde{\omega},y\right)\right)=\frac{\pi_L}{r+\delta_S+\rho_L}\left\{\alpha\widetilde{\omega}-c_e-\chi_L-\left(r+\delta_S\right)V_L\left(\widetilde{\omega},y\right)+\delta_S D_0\right\}$$

Substituting the terms inside parenthesis, we have:

$$\left[1+\frac{\pi_L}{r+\delta_S+\rho_L}\right]c_e=\left\{\begin{array}{c}\left(\frac{\pi_L}{r+\delta_S+\rho_L}-\frac{\pi_H}{r+\delta_S+\rho_H}\right)\left[\alpha\widetilde{\omega}-\left(r+\delta_S\right)V_H\left(\widetilde{\omega}\right)+\delta_S D_0\right]\\+\frac{\pi_H}{r+\delta_S+\rho_H}\chi_H-\frac{\pi_L}{r+\delta_S+\rho_L}\chi_L\end{array}\right\}$$

10 Appendix B

Proof of Lemma 1:

Proof. At the reservation wage y of a move from a solid firm with wage x to a risky firm (i.e. we suppose that $y = R_L^H(x)$), and the reservation wage $R_H^L(y)$ of the reverse transition, it must be the case that

$$V_L(x) = V_H(R_L^H(x)) = V_H(y) = V_L(R_H^L(y)).$$

But then it follows that $R_H^L(y) = x$. Similarly, it follows if $R_H^L(y) = x$, then $R_L^H(x) = y$. By the strict monotonicity of the value functions the mapping $R_i^j(y) = x$ is unique. It is straightforward to see that the resulting function must be continuous and increasing, if the value functions are increasing and continuous. ■

Lemma B.1 $\omega^*(w) > w$.

Proof. From the previous result, we obtain through manipulations that:

from $V_H\left(R_L^H(w)\right) = V_L(w)$, we obtain:

$$\omega^*(w) = \frac{1}{\left[1+\frac{\alpha\pi_H}{r+\delta_S+\rho_H}\right]}\left\{\begin{array}{c} \left[1+\frac{\alpha\pi_L}{r+\delta_S+\rho_L}\right] w - c_e\left[1+\frac{\pi_L}{r+\delta_S+\rho_L}\right] \\ +\left(\frac{\pi_L}{r+\delta_S+\rho_L}-\frac{\pi_H}{r+\delta_S+\rho_H}\right)\left[\begin{array}{c} +\frac{\delta_S b}{r+\rho_H}+\frac{\delta_S\rho_H}{r+\rho_H}V_0 \\ -(r+\delta_S)V_L(w,y) \end{array}\right] \\ +\left[\begin{array}{c} \frac{\pi_H}{r+\rho_H} \\ -\frac{\delta_S}{r+\rho_H}\frac{\pi_L}{r+\delta_S+\rho_L} \end{array}\right]\chi_H - \frac{\pi_L}{r+\delta_S+\rho_L}\chi_L \end{array}\right\}$$

Rearranging the expression obtained for $\omega^*(w)$, we have:

$$\omega^*(w) = w + \frac{\left(\frac{\pi_H}{r+\delta_S+\rho_H}-\frac{\pi_L}{r+\delta_S+\rho_L}\right)}{\left[1+\frac{\alpha\pi_H}{r+\delta_S+\rho_H}\right]}\left[\begin{array}{c} \alpha\widetilde{\omega}-\alpha\omega \\ +(r+\delta_S)\left(V_L(w,y)-V_H(\widetilde{\omega})\right) \end{array}\right]$$

Rearranging the expressions for $V_L(w,y)$ and $V_H(\widetilde{w})$, we obtain:

$$\begin{aligned} V_L(w,y)-V_H(\widetilde{w}) &= \left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)\int_{\widetilde{w}}^{w}\frac{1}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1(1-F(z))}dz \\ &> \left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)\int_{\widetilde{w}}^{w}\frac{1}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1(1-F(\widetilde{w}))}dz \\ &= \frac{\left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1(1-F(\widetilde{w}))}(w-\widetilde{w}) \end{aligned}$$

Therefore, we have:

$$\omega^*(w) > w + \frac{\left(\frac{\pi_H}{r+\rho_H+\delta_S} - \frac{\pi_L}{r+\rho_L+\delta_S}\right)}{\left[1+\frac{\alpha\pi_H}{r+\rho_H+\delta_S}\right]} \left\{ \left[\frac{(r+\delta_S)\left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)} - \alpha \right] (w-\widetilde{w}) \right\}$$

Therefore, if:

$$\frac{(r+\delta_S)\left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)} > \alpha$$

the second term in the RHS is positive. Rearranging the above inequality, we have:

$$\alpha < \frac{(r+\delta_S)}{r+\delta+\lambda_1\left(1-F\left(\widetilde{w}\right)\right)}$$

which is satisfied by α, once $\alpha \leq \frac{r+\delta_S}{r+\delta+\lambda_1}$. ■

Lemma B.2 $\forall w > \widetilde{w}$, $\frac{d\omega^*(w)}{dw} > 1$.

Proof.

$$\omega^*(w) = w + \frac{\left(\frac{\pi_H}{r+\rho_H+\delta_S} - \frac{\pi_L}{r+\rho_L+\delta_S}\right)}{\left[1+\frac{\alpha\pi_H}{r+\rho_H+\delta_S}\right]} \left\{ \begin{array}{c} \alpha\widetilde{w} - \alpha w \\ +(r+\delta_S)\left(V_L(w,y) - V_H\left(\widetilde{w}\right)\right) \end{array} \right\}$$

$$V_L(w,y) - V_H\left(\widetilde{w}\right) = \left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)\int_{\widetilde{w}}^{w} \frac{1}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1\left(1-F(z)\right)}dz$$

Taking the integral, we have:

$$\frac{d\left(V_L(w,y) - V_H\left(\widetilde{w}\right)\right)}{dw} = \frac{\left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1\left(1-F(w)\right)}$$

Then:

$$\frac{d\omega^*(w)}{dw} = 1 + \frac{\left(\frac{\pi_H}{r+\rho_H+\delta_S} - \frac{\pi_L}{r+\rho_L+\delta_S}\right)}{\left[1+\frac{\alpha\pi_H}{r+\rho_H+\delta_S}\right]} \left\{ \left(\frac{(r+\delta_S)\left(1+\frac{\alpha\pi_L}{r+\rho_L+\delta_S}\right)}{r+\delta+\frac{\pi_L(r+\delta_S)}{r+\delta_S+\rho_L}+\lambda_1\left(1-F(w)\right)} - \alpha \right) \right\}$$

The second term is positive if:

$$\alpha < \frac{(r+\delta_S)}{r+\delta+\lambda_1 (1-F(w))}$$

Since the RHS of the inequality above is decreasing in w, we have that it is satisfied for any $w > \widetilde{w}$ if:

$$\alpha < \frac{(r+\delta_S)}{r+\delta+\lambda_1 (1-F(\widetilde{w}))}$$

■

Proof of Lemma 2:

Proof. Suppose that a firm that pays the up-front C and offers a wage lower than $\widetilde{w}$. As we saw, the worker will not differentiate it from a firm that does not pay the up-front cost, therefore $\omega^*(w) = w$. Therefore, at the end the number of workers this firm keeps in steady state $l_H(w)$. Therefore:

$$\mathit{Profit}_L(w) = [p - w(1+t)]\, l_H(w) - \alpha w (1+t)\, d_H(w) - C = \mathit{Profit}_H(w) - C,\ \forall w < \widetilde{w}$$

Therefore, this firm would have a profitable deviation, which would be not pay the up-front cost C and become a H firm. ■

Proof of Theorem 1:

Proof. Suppose there exists w_B, w_A, such that $w_B > w_A$, and w_B is offered by a risky firm while w_A by a low-risk firm. Then it must be that

$$\begin{aligned} & \left(p - \left[1 + \frac{\alpha \pi_H}{\rho_H + \delta_S}\right] \omega^*(w_B)(1+t)\right) l_H(w_B) \\ \geq \ & \left(p - \left[1 + \frac{\alpha \pi_H}{\rho_H + \delta_S}\right] \omega^*(w_A)(1+t)\right) l_H(w_A) \end{aligned} \tag{5}$$

Now, note that:

$$\frac{l_L(w)}{l_H(w)} = \frac{\delta + \lambda_1 (1-F(w)) + \frac{\delta_S \pi_H}{\rho_H + \delta_S}}{\delta + \lambda_1 (1-F(w)) + \frac{\delta_S \pi_L}{\rho_L + \delta_S}} > 1$$

Since $\pi_H > \pi_L$ and $\rho_L > \rho_H$.

By taking derivatives, we have:

$$\frac{d\left(\frac{l_L(w)}{l_H(w)}\right)}{dw} = \frac{\lambda_1 F'(w)\,\delta_S\left[\frac{\pi_H}{\rho_H+\delta_S} - \frac{\pi_L}{\rho_L+\delta_S}\right]}{\left\{\delta + \lambda_1\left(1-F(w)\right) + \frac{\delta_S \pi_L}{\rho_L+\delta_S}\right\}^2} > 0$$

Therefore, this ratio is larger than 1 and increasing, In particular, it follows that:

$$\frac{l_L(w_B)}{l_L(w_A)} > \frac{l_H(w_B)}{l_H(w_A)}. \tag{6}$$

To study the instantaneous profit per worker, notice:

$$\frac{d\left(\frac{p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w(1+t)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)}\right)}{dw} = \frac{\left\{\begin{array}{c} -\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)(1+t)\left(p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)\right) \\ +\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)(1+t)\frac{d\omega^*(w)}{dw}\left[p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w(1+t)\right]\end{array}\right\}}{\left\{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)\right\}^2}$$

Now, for any $w > \widetilde{w}$, $\frac{d\omega^*(w)}{dw} > 1$, which implies that:

$$\begin{aligned}\frac{d\left(\frac{p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w(1+t)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)}\right)}{dw} &> \frac{\left\{\begin{array}{c} -\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)(1+t)\left(p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)\right) \\ +\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)(1+t)\left[p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w(1+t)\right]\end{array}\right\}}{\left\{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)\right\}^2} \\ &= \frac{\left\{\begin{array}{c} \left[\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)\right](1+t)\,p \\ +\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)(1+t)\left[\omega^*(w)-w\right]\end{array}\right\}}{\left\{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)\right\}^2} > 0\end{aligned}$$

since by Lemma 3, no firm that offers health insurance would offer a wage lower than $\widetilde{w}$, there is no loss of generality.

Therefore:

$$\frac{d\left(\frac{p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w(1+t)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w)(1+t)}\right)}{dw} > 0$$

But this means that:

$$\frac{p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w_B(1+t)}{p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w_A(1+t)} > \frac{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w_B)(1+t)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*(w_A)(1+t)}. \tag{7}$$

Now, putting (6) and (7) together, it follows that (5) implies

$$
\begin{aligned}
& \left(p-\left[1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right]w_B\left(1+t\right)\right)l_L\left(w_B\right)-C \\
\geq \quad & \left(p-\left[1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right]w_A\left(1+t\right)\right)l_L\left(w_A\right)-C,
\end{aligned}
$$

which contradicts that w_A was the profit maximizing choice of the solid firm. The connectedness follows from the fact that any 'holes' will give an opportunity for a profitable deviation by the next (higher) firm, it can increase instantaneous profit per worker, without losing workers faster, or gaining slower. For details in this argument, please see Burdett and Mortensen (1998). ■

Corollary B.1 *The minimum wage posted by a firm that do not offer health insurance is* R_U^H*, while the minimum wage posted by a firm that offers health insurance is* $\widetilde{w}$*.*

Corollary B.2 There is no mass point in the distribution of offered wages.

11 Appendix C

Using the stationary offer distributions $F_L(w_L), F_H(w_H)$, and the optimal decisions of workers, we can derive the stationary distribution of workers of wages. Employing that all equivalent-wages offered by solid firms are higher, we can derive the stationary risky firm distribution. First, looking at the more general case to, to use derivatives to find the change in the wage distribution G_L over time:

$$\begin{aligned}\frac{dG_L(w,t)}{dt} &= \lambda_0(1-\gamma_H)F_L(w,t)u(t) + \lambda_1(1-\gamma_H)F_L(w,t)\upsilon_H(m-u-s_e-s_u) \\ &+ \left\{\begin{array}{c} +\rho_L S_L(w,t)(1-\mathfrak{s}_H)s_e \\ -\left[\begin{array}{c}\lambda_1(1-\gamma_H)(1-F_L(w,t)) \\ +\delta+\pi_L\end{array}\right]G_L(w,t)(1-\upsilon_H)(m-s_e-s_u-u)\end{array}\right\}\end{aligned}$$

in steady state:

$$G_L(w) = \frac{\left\{\begin{array}{c}[\lambda_0 u + \lambda_1\upsilon_H(m-u-s_e-s_u)](1-\gamma_H)F_L(w) \\ +\rho_L S_L(w)(1-\mathfrak{s}_H)s_e\end{array}\right\}}{\left[\begin{array}{c}\lambda_1(1-\gamma_H)(1-F_L(w)) \\ +\delta+\pi_L\end{array}\right](1-\upsilon_H)(m-s_e-s_u-u)}$$

while the proportion of sick employees at health-coverage firms working at wage $\leq w$

$$\frac{dS_L(w,t)}{dt} = \pi_L G_L(w,t)(1-\upsilon_H)(m-s_e-s_u-u) - (\delta_S+\rho_L)S_L(w,t)(1-\mathfrak{s}_H)s_e$$

in steady state:

$$S_L(w) = \frac{\pi_L G_L(w)(1-\upsilon_H)(m-s_e-s_u-u)}{(\delta_S+\rho_L)(1-\mathfrak{s}_H)s_e}$$

Similarly:

$$\begin{aligned}\frac{dG_H(w,t)}{dt} &= \lambda_0\gamma_H F_H(w,t)u(t) + \rho_H S_H(w,t)\mathfrak{s}_H s_e \\ &\quad -\left[\begin{array}{c}\lambda_1\gamma_H(1-F_H(w,t)) \\ +\lambda_1(1-\gamma_H)+\delta+\pi_H\end{array}\right]G_H(w,t)\upsilon_H(m-s_e-s_u-u)\end{aligned}$$

Since in steady state $\frac{dG_H(w,t)}{dt} = 0$, we have:

$$G_H(w) = \frac{\lambda_0 \gamma_H F_H(w) u + \rho_H S_H(w) \mathfrak{s}_H s_e}{\begin{bmatrix} \lambda_1 \gamma_H (1 - F_H(w)) \\ +\lambda_1 (1-\gamma_H) + \delta + \pi_H \end{bmatrix} \upsilon_H (m - s_e - s_u - u)}$$

while:

$$\frac{dS_H(w,t)}{dt} = \pi_H G_H(w,t) \upsilon_H (m - s_e - s_u - u) - (\delta_S + \rho_H) S_H(w,t) \mathfrak{s}_H s_e$$

in steady-state, we have:

$$S_H(w) = \frac{\pi_H G_H(w) \upsilon_H (m - s_e - s_u - u)}{(\delta_S + \rho_H) \mathfrak{s}_H s_e}$$

To obtain $F_L(\cdot)$, we use the profit equality condition for all wages offered by companies that supply health insurance:

$$\begin{aligned} & \left(p - \left[1 + \frac{\alpha \pi_L}{\rho_L + \delta_S}\right] w_0^L (1+t)\right) \frac{\lambda_0 u + \lambda_1 G\left(w_0^L\right)(m - u - s_e - s_u)}{\delta + \lambda_1 \left(1 - F\left(w_0^L\right)\right) + \frac{\delta_S}{\rho_i + \delta_S} \pi_L} - C \\ = \; & \left(p - \left[1 + \frac{\alpha \pi_L}{\rho_L + \delta_S}\right] w_L (1+t)\right) \frac{\lambda_0 u + \lambda_1 G(w_L)(m - u - s_e - s_u)}{\delta + \lambda_1 (1 - F(w_L)) + \frac{\delta_S}{\rho_i + \delta_S} \pi_L} - C \end{aligned}$$

where $F(z) = \gamma_H F_H(\omega^*(z)) + (1-\gamma_H) F_L(z)$ and $G(z) = \upsilon_H G_H(\omega^*(z)) + (1-\upsilon_H) G_L(z)$. Then, since w_0^L is the minimum wage offered by a company with health insurance and therefore it must be the highest wage offered by a company that does not offer health insurance from Theorem 1, we have that $F\left(w_0^L\right) = \gamma_H$. Similarly, $G\left(w_0^L\right) = \upsilon_H$. Therefore,introducing this values and the expression obtained previously to $G(\cdot)$, we obtain:

$$F_L(w) = \frac{\left[\delta + \lambda_1(1-\gamma_H) + \frac{\delta_S}{\rho_L + \delta_S}\pi_L\right]}{\lambda_1(1-\gamma_H)} \left\{ 1 - \left(\frac{p - \left(1 + \frac{\alpha\pi_L}{\rho_L + \delta_S}\right) w_L(1+t)}{p - \left(1 + \frac{\alpha\pi_L}{\rho_L + \delta_S}\right) w_0^L (1+t)} \right)^{\frac{1}{2}} \right\}$$

Using this expression we can obtain $G_L(w)$ and $S_L(w)$.

Now let's look at $F_H(\cdot)$. From previous results, we know that the minimum wage offered by a firm not offering health insurance is R_U^H. Then, we have that:

$$\left(p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)R_U^L\left(1+t\right)\right)\frac{\lambda_0 u}{\delta+\lambda_1+\frac{\delta_S}{\rho_H+\delta_S}\pi_H}$$

$$=\left(p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*\left(w_H\right)\left(1+t\right)\right)\frac{\lambda_0 u+\lambda_1 \upsilon_H G_H\left(\omega^*\left(w_H\right)\right)\left(m-u-s_e-s_u\right)}{\delta+\lambda_1\left(1-\gamma_H F_H\left(\omega^*\left(w_H\right)\right)\right)+\frac{\delta_S}{\rho_H+\delta_S}\pi_H}$$

Substituting $G\left(\cdot\right)$ and rearranging, we have:

$$F_H\left(\omega^*\left(w_H\right)\right)=\frac{\left[\delta+\lambda_1+\frac{\delta_S}{\rho_H+\delta_S}\pi_H\right]}{\lambda_1\gamma_H}\left\{1-\left(\frac{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\omega^*\left(w_H\right)\left(1+t\right)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)R_U^L\left(1+t\right)}\right)^{\frac{1}{2}}\right\}$$

with few manipulations, we have:

$$F_H\left(w_H\right)=\frac{\left[\delta+\lambda_1+\frac{\delta_S}{\rho_H+\delta_S}\pi_H\right]}{\lambda_1\gamma_H}\left\{1-\left(\frac{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)w_H\left(1+t\right)}{p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)R_U^H\left(1+t\right)}\right)^{\frac{1}{2}}\right\}$$

Using the wage distributions obtained above, we are able to fully characterize the wage offered in equilibrium. From Theorem 1 we know that in equivalent-wage terms, no health-coverage firms pay lower wages than firms that offer health coverage. This means that the very lowest nominal wages (R_U^H) are always offered by the risky firms.[25]

In this case, to fully characterize the wages offered in equilibrium we need to find the reservation wage, R_U^H, and the maximum wage paid by firm which does not provide health insurance, $\bar{w}_H$. With these information we are able to determine the high risk firms' offered wage range, $[R_{0,}\bar{w}_H]$, and the low risk firms' wage range, $[w_0^L,\bar{w}_L]$.

First let's find the highest wage offered by a firm which doesn't offer health insurance (to find it, put $F_H(w_H)=1$, and solve,

$$\overline{w}_H=\frac{1}{\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)\left(1+t\right)}\left\{p-\left[\begin{array}{c}\left(\frac{\delta+\lambda_1(1-\gamma_H)+\frac{\delta_S}{\rho_H+\delta_S}\pi_H}{\delta+\lambda_1+\frac{\delta_S}{\rho_H+\delta_S}\pi_H}\right)^2\times\\ \times\left(p-\left(1+\frac{\alpha\pi_H}{\rho_H+\delta_S}\right)R_U^H\left(1+t\right)\right)\end{array}\right]\right\},$$

Note that the value of $\overline{w}_H$ depends crucially on γ_L. Indeed, as $\gamma_H\rightarrow 0$, $\overline{w}_H\rightarrow R_0$. Since the dispersion of wages offered by risky companies on the

[25] Provided there is a positive mass of risky firms.

interval $[R_0, \overline{w}_H]$ are generated by the competition between no health-coverage companies, as the measure of no health-coverage companies reduces, this dispersion shrink to 0.

Similarly, we can derive the maximum wage paid by a solid company,

$$\overline{w} = \frac{1}{\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)(1+t)} \left\{ \begin{array}{c} p - \left(\frac{\delta+\frac{\delta_S\pi_L}{\rho_L+\delta_S}}{\delta+\lambda_1(1-\gamma_H)+\frac{\delta_S\pi_L}{\rho_L+\delta_S}}\right)^2 \times \\ \times\left(p-\left(1+\frac{\alpha\pi_L}{\rho_L+\delta_S}\right)w_0^L(1+t)\right) \end{array} \right\}$$

Again, as $\gamma_H \to 1$, we have that $\overline{w}_L \to w_0^L$.

To obtain w_0^L, we need to no compare the minimum wage asked by employees to accept health coverage once offered, $\widetilde{w}$ and the optimal wage w_0^{L*} obtained calculating: $\omega^*\left(w_0^{L*}\right) = \overline{w}_H$, since once the constraint $\widetilde{w}$ is not binding, we can easily show that the wage set must be connected. Then:

$$w_0^L = \max\left\{w_0^{L*}, \widetilde{w}\right\}$$

Finally, we can obtain an expression for R_U^H, as we substitute the results obtained previously.

To close the model, we use the profit equality condition to pin down γ_H. We can show that the equilibrium is unique.

$\to u$: unemployed healthy:
inflow: $\delta(m-u-s_e-s_u)+\rho_H s_u$;
outflow: $\pi_H u + \lambda_0 u$

Rearranging:

$$u = \frac{\delta(m-s_e-s_u)+\rho_H s_u}{\pi_H+\lambda_0+\delta}$$

$\to s_u$: sick out of job workers:
inflow: $\delta_S s_e + \pi_H u$;
outflow: $\rho_H s_u$

Rearranging:

$$s_u = \frac{\delta_S s_e + \pi_H u}{\rho_H}$$

Substituting it back in the expression for u :

$$u=\frac{\delta m+\left(\delta_S-\delta\left(1+\frac{\delta_S}{\rho_H}\right)\right)s_e}{\lambda_0+\delta+\frac{\delta\pi_H}{\rho_H}}$$

$\rightarrow s_e$: sick and employed workers:
inflow: $\upsilon_H\left(m-u-s_e-s_u\right)\pi_H+\left(1-\upsilon_H\right)\left(m-s_e-s_u-u\right)\pi_L$;
outflow: $\mathfrak{s}_H s_e\rho_H+\left(1-\mathfrak{s}_H\right)s_e\rho_L+\delta_S s_e$

Rearranging:

$$s_e=\frac{\left(\upsilon_H\pi_H+\left(1-\upsilon_H\right)\pi_L\right)\left(m-u-s_u\right)}{\left[\begin{array}{c}\mathfrak{s}_H\rho_H+\left(1-\mathfrak{s}_H\right)\rho_L+\\ +\delta_S+\upsilon_H\pi_H+\left(1-\upsilon_H\right)\pi_L\end{array}\right]}$$

Now, equations to obtain the proportions : $\upsilon_H,\mathfrak{s}_H$.

$\rightarrow \upsilon_H$:
inflow: $\lambda_0\gamma_H u+\rho_H\mathfrak{s}_H s_e$;
outflow: $\left\{\begin{array}{c}\delta\upsilon_H\left(m-s_e-s_u-u\right)+\pi_H\upsilon_H\left(m-s_e-s_u-u\right)\\ +\lambda_1\left(1-\gamma_H\right)\upsilon_H\left(m-s_e-s_u-u\right)\end{array}\right\}$

Rearranging:

$$\upsilon_H=\frac{\lambda_0\gamma_H u+\rho_H\mathfrak{s}_H s_e}{\left[\delta+\pi_H+\lambda_1\left(1-\gamma_H\right)\right]\left(m-s_e-s_u-u\right)}$$

$\rightarrow \mathfrak{s}_H$:
inflow: $\pi_H\upsilon_H\left(m-s_e-s_u-u\right)$;
outflow: $\delta_S\mathfrak{s}_H s_e+\rho_H\mathfrak{s}_H s_e$.

$$\mathfrak{s}_H=\frac{\pi_H\upsilon_H\left(m-u-s_e-s_u\right)}{\left(\delta_S+\rho_H\right)s_e}$$

Now, we are going to solve this system with 5 equations and 5 unknowns ($\upsilon_H,\ s_e,\ \mathfrak{s}_H,\ u,\ s_u$). Rearranging the above expressions, we have:

$$\left\{\begin{array}{ll}\left(m-u-s_e-s_u\right)=\frac{\left(\delta_S+\rho_H\right)s_e\mathfrak{s}_H}{\pi_H\upsilon_H} & \left(\mathfrak{s}_H\right)\\ \left(m-u-s_e-s_u\right)=\frac{\left[\mathfrak{s}_H\rho_H+\left(1-\mathfrak{s}_H\right)\rho_L+\delta_S\right]s_e}{\upsilon_H\pi_H+\left(1-\upsilon_H\right)\pi_L} & \left(s_e\right)\\ \left(m-u-s_e-s_u\right)=\frac{\lambda_0\gamma_H u+\rho_H\mathfrak{s}_H s_e}{\left[\delta+\pi_H+\lambda_1\left(1-\gamma_H\right)\right]\upsilon_H} & \left(\upsilon_H\right)\\ \delta\left(m-u-s_e-s_u\right)+\rho_H s_u=\pi_H u+\lambda_0 u & (u)\\ \delta_S s_e+\pi_H u=\rho_H s_u & \left(s_u\right)\end{array}\right.$$

Then, substituting (s_u) into (u), we have:

$$\begin{cases} (m-u-s_e-s_u) = \frac{(\delta_S+\rho_H)s_e\mathfrak{s}_H}{\pi_H\rho_H} & (\mathfrak{s}_H) \\ (m-u-s_e-s_u) = \frac{[\mathfrak{s}_H\rho_H+(1-\mathfrak{s}_H)\rho_L+\delta_S]s_e}{\upsilon_H\pi_H+(1-\upsilon_H)\pi_L} & (s_e) \\ (m-u-s_e-s_u) = \frac{\lambda_0\gamma_H u+\rho_H\mathfrak{s}_H s_e}{[\delta+\pi_H+\lambda_1(1-\gamma_H)]\upsilon_H} & (\upsilon_H) \\ \delta(m-u-s_e-s_u)+\delta_S s_e = \lambda_0 u & (u) \end{cases}$$

Then multiplying (u) by γ_H and substituting into (υ_H), we have:

$$\begin{cases} (m-u-s_e-s_u) = \frac{(\delta_S+\rho_H)s_e\mathfrak{s}_H}{\pi_H\upsilon_H} & (\mathfrak{s}_H) \\ (m-u-s_e-s_u) = \frac{[\mathfrak{s}_H\rho_H+(1-\mathfrak{s}_H)\rho_L+\delta_S]s_e}{\upsilon_H\pi_H+(1-\upsilon_H)\pi_L} & (s_e) \\ (m-u-s_e-s_u) = \frac{(\delta_S\gamma_H+\rho_H\mathfrak{s}_H)s_e}{[(\delta+\pi_H+\lambda_1(1-\gamma_H))\upsilon_H-\delta\gamma_H]} & (\upsilon_H) \end{cases}$$

Then, equalizing $(\mathfrak{s}_H)$ and (s_e), we obtain:

$$\frac{(\delta_S+\rho_H)\,\mathfrak{s}_H}{\pi_H\upsilon_H} = \frac{[\mathfrak{s}_H\rho_H+(1-\mathfrak{s}_H)\,\rho_L+\delta_S]}{\upsilon_H\pi_H+(1-\upsilon_H)\,\pi_L}$$

$$\mathfrak{s}_H = \frac{\pi_H\upsilon_H\,(\rho_L+\delta_S)}{\pi_H\upsilon_H\,(\rho_L+\delta_S)+(1-\upsilon_H)\,\pi_L\,(\rho_H+\delta_S)}$$

Similarly, equalizing $(\mathfrak{s}_H)$ and (υ_H), we have:

$$\frac{(\delta_S+\rho_H)\,\mathfrak{s}_H}{\pi_H\upsilon_H} = \frac{(\delta_S\gamma_H+\rho_H\mathfrak{s}_H)}{[(\delta+\pi_H+\lambda_1\,(1-\gamma_H))\,\upsilon_H-\delta\gamma_H]}$$

$$\mathfrak{s}_H = \frac{\pi_H\upsilon_H\delta_S\gamma_H}{\{[(\delta+\pi_H+\lambda_1\,(1-\gamma_H))\,\upsilon_H-\delta\gamma_H]\,(\delta_S+\rho_H)-\rho_H\pi_H\upsilon_H\}}$$

Then, we have:

$$\begin{aligned} & (\rho_L+\delta_S)\,\{[(\delta+\pi_H+\lambda_1\,(1-\gamma_H))\,\upsilon_H-\delta\gamma_H]\,(\delta_S+\rho_H)-\rho_H\pi_H\upsilon_H\} \\ = \; & \delta_S\gamma_H\,\{(\upsilon_H\pi_H+(1-\upsilon_H)\,\pi_L)\,(\delta_S+\rho_H)+(\rho_L-\rho_H)\,\pi_H\upsilon_H\} \end{aligned}$$

From this expression we can obtain υ_H. Rearranging it, we have:

$$\upsilon_H = \frac{(\delta_S+\rho_H)\,\{\delta\,(\delta_S+\rho_L)+\delta_S\pi_L\}}{(\delta_S+\rho_H)\,\pi_L\delta_S\gamma_H+(\delta_S+\rho_H)\,(\delta_S+\rho_L)\,(\delta+\lambda_1\,(1-\gamma_H))+\rho_L\delta_S\pi_H\,(1-\gamma_H)}$$

Substituting this into $\mathfrak{s}_H$, we have:

$$
\mathfrak{s}_H = \frac{\pi_H\left(\delta_S+\rho_H\right)\left(\delta_S+\rho_L\right)\gamma_H\left\{\delta\left(\delta_S+\rho_L\right)+\delta_S\pi_L\right\}}{\left\{\begin{array}{c}\pi_H\left(\delta_S+\rho_H\right)\left(\delta_S+\rho_L\right)\gamma_H\left[\delta\left(\delta_S+\rho_L\right)+\delta_S\pi_L\right]\\+\pi_L\left(1-\gamma_H\right)\left(\delta_S+\rho_L\right)\left[\left(\delta_S+\rho_H\right)\left(\delta_S+\rho_L\right)\left(\delta+\lambda_1\right)+\rho_L\delta_S\pi_H\right]\end{array}\right\}}
$$

Few more calculations, we obtain:

$$
s_e = \frac{\lambda_0\rho_H\left(\upsilon_H\pi_H+\left(1-\upsilon_H\right)\pi_L\right)m}{\left\{\begin{array}{c}\left[\delta_S+\mathfrak{s}_H\rho_H+\left(1-\mathfrak{s}_H\right)\rho_L\right]\left(\lambda_0\rho_H+\delta\left(\pi_H+\rho_H\right)\right)\\+\left[\upsilon_H\pi_H+\left(1-\upsilon_H\right)\pi_L\right]\left(\lambda_0\left(\rho_H+\delta_S\right)+\left(\pi_H+\rho_H\right)\delta_S\right)\end{array}\right\}}
$$

Substituting $\mathfrak{s}_H$ and υ_H, we have:

$$
s_e = \frac{\lambda_0\rho_H\left(\frac{(\delta_S+\rho_H)\{\delta(\delta_S+\rho_L)+\delta_S\pi_L\}}{(\delta_S+\rho_H)\pi_L\delta_S\gamma_H+(\delta_S+\rho_H)(\delta_S+\rho_L)(\delta+\lambda_1(1-\gamma_H))+\rho_L\delta_S\pi_H(1-\gamma_H)}\left(\pi_H-\pi_L\right)+\pi_L\right)m}{\left\{\begin{array}{c}\left[\begin{array}{c}\delta_S+\\\frac{\pi_H(\delta_S+\rho_H)(\delta_S+\rho_L)\gamma_H\{\delta(\delta_S+\rho_L)+\delta_S\pi_L\}}{\left\{\begin{array}{c}\pi_H\left(\delta_S+\rho_H\right)\left(\delta_S+\rho_L\right)*\\*\gamma_H\left[\begin{array}{c}\delta\left(\delta_S+\rho_L\right)\\+\delta_S\pi_L\end{array}\right]\\+\pi_L\left(1-\gamma_H\right)\left(\delta_S+\rho_L\right)\left[\begin{array}{c}\left(\delta_S+\rho_H\right)*\\*\left(\delta_S+\rho_L\right)\\*\left(\delta+\lambda_1\right)+\rho_L\delta_S\pi_H\end{array}\right]\end{array}\right\}}\left(\rho_H-\rho_L\right)\\+\rho_L\end{array}\right]*\\\left(\begin{array}{c}\lambda_0\rho_H\\+\delta\left(\pi_H+\rho_H\right)\end{array}\right)+\left[\begin{array}{c}\upsilon_H\left(\pi_H-\pi_L\right)\\+\pi_L\end{array}\right]\left(\begin{array}{c}\lambda_0\left(\rho_H+\delta_S\right)\\+\left(\pi_H+\rho_H\right)\delta_S\end{array}\right)\end{array}\right\}}
$$