Social capital, trust, and multiple equilibria in economic performance

Katarzyna Growiec and Jakub Growiec

Warsaw School of Social Sciences and Humanities, Warsaw School of Economics, Poland, National Bank of Poland, Institute for Structural Research, Warsaw, Poland

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Social Capital, Trust, and Multiple Equilibria in Economic Performance

Katarzyna Growiec†  Jakub Growiec‡

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Abstract. We propose a novel mechanism giving rise to poverty traps and multiple equilibria in economic performance. It is a potentially important source of persistent underdevelopment across countries and regions. At the core of this mechanism, bridging social capital and social trust feed back on each other, interdependently affecting individuals’ earnings and subjective well-being. High trust and abundant bridging social capital reinforce each other, leading to a “high” equilibrium where both these variables take persistently high values, and earnings and well-being are high as well, whereas low trust and lacking bridging social capital create a vicious circle, leading to a “low trust trap” where all these variables are persistently low. The workings of our theoretical model are in agreement with a wide range of findings from the contemporary literature in sociology and social psychology.

Keywords: bridging social capital, social trust, multiple equilibria, poverty trap

JEL Classification Numbers: D10, J22, O11
1 Introduction

In a cross-section of countries of the world, observed levels of general social trust\textsuperscript{1} are robustly positively correlated with GDP per capita. Within most countries or regions, though, people’s attitudes such as trust and trustworthiness are puzzlingly persistent across time, reacting to changes in the level of economic development very slowly. Distrust, on the other hand, generally slows growth down, and may even preclude it in some cases. Similar patterns are observed with social capital: in richer countries, people are generally quite willing to form and maintain social ties with people dissimilar to themselves, whereas in poorer areas, people usually restrict their social ties to family members. Again, fast economic growth or relative affluence do not automatically shift people’s attention from kin towards non-kin. On the contrary, family-based closed networks turn out to be very persistent and provide a drag both on social trust and on the pace of economic development.\textsuperscript{2}

This apparent discrepancy between cross-section and time-series evidence on the relationships between GDP per capita, patterns of social ties, and general social trust, makes social capital and trust formation a natural candidate for a mechanism which could give rise to persistent inequality in levels of economic development. To our knowledge, such a mechanism has hitherto never been formalized in the literature. This paper is intended to fill this gap.

The contribution of this paper is thus to propose a novel mechanism able to generate poverty traps and multiple equilibria in economic performance. In our model, high trust and abundant bridging social capital (i.e., plentiful social ties with people in a different socio-economic position, cf. Putnam 2000; Leonard, 2008) reinforce each other leading to a “high” equilibrium where both these variables take persistently high values, and earnings and well-being are high as well, whereas low trust and lacking bridging social capital create a vicious circle leading to a “low trust trap” where all these variables are persistently low.

More specifically, the primary hypothesis of the current study (reflected in the logic of the theoretical model but also in the implications of contemporary literature in sociology and social psychology) is that low levels of bridging social capital go together with low social trust, acting as an impediment for economic catch-up with

\textsuperscript{1}That is, trust towards people whom one does not know, measured, e.g., as a percentage of positive answers to the survey question: “Could most people be trusted?”

\textsuperscript{2}This point has been made perhaps most forcefully for the case of Southern Italy by Putnam, Leonardi, and Nanetti (1993).
wealthier regions and countries. Family-oriented and distrustful societies may become permanently trapped in a low bridging social capital–low trust equilibrium where the formation of social ties with dissimilar people is systematically discouraged by the lack of general trust, and conversely, where low levels of trust are reinforced by the lack of contact with dissimilar others.\textsuperscript{3} Being “trapped” in the currently discussed low equilibrium precludes economic convergence with more developed regions of the world because it imposes substantial transaction costs, slows down the flow of information, prevents implementation of innovative ideas, and limits people’s cooperativeness and thrift (Knack and Keefer, 1997; Zak and Knack, 2001; Inglehart and Baker, 2000; Florida, 2004; Klapwijk and Van Lange, 2009; Algan and Cahuc, 2010). If, on the other hand, bridging social capital is abundant in the society, and individuals are willing to trust strangers (and if there are no other, e.g. structural or institutional, barriers), then we should observe generally high levels of economic development, and fast catch-up of initially backward areas (cf. Beugelsdijk and Smulders, 2003).

Our research relates to at least five complementary strands of literature, all of which will be discussed in more detail in the following section. First and foremost, we dwell on the sociological literature which provides the definition of social capital, and discusses its dimensionality (e.g. the distinction between bridging and bonding social capital), measurement, and implications. Secondly, we relate our results to the literature on the relation between social capital and trust. Thirdly, we justify the assumptions of our model with sociological and psychological literature discussing the observed patterns of social capital formation and its interrelation with individual attitudes and incentives. Fourthly, we relate our work to the literature on the relationship of bridging social capital and trust with individuals’ earnings and (after aggregation) the regional level of economic development. Finally, the workings of our model are presented against the background of selected other dynamic models giving rise to poverty traps and multiple equilibria in economic performance.

The remainder of the article is structured as follows. Section 2 presents background evidence supporting our modeling approach. Section 3 lays out the model and presents its properties, implications, and extensions. Section 4 concludes. Proofs of propositions have been delegated to the appendix. Some evidence for empirical validity of the model predictions in the case of Central and Eastern European countries has been presented in our companion paper, Growiec and Growiec (2011).

\textsuperscript{3}See K. Growiec (2009, 2011), for a sociological rationale as well as empirical evidence for the case of Poland.
2 Background evidence from sociology and social psychology

2.1 Bridging and bonding social capital

The first strand of literature related to the current article includes sociological studies providing the definition of social capital and methods of its measurement. In this respect, we are particularly interested in the network operationalization of social capital (cf. Lin, 2001) and the distinction between bridging social capital (social ties with dissimilar others) and bonding social capital (social ties with similar others), put forward by Putnam (2000). Such an approach is useful for our analytical purposes because it enables us to delineate people’s objective behavior (maintaining social contacts with others) from social norms (trust, reciprocity), and it links social networks people maintain to resources accessed through them (Bourdieu, 1986; Lin, 2001).

Putnam’s (2000) distinction between bridging and bonding social capital is frequently invoked in social capital studies. These two variables are then measured at the individual level with survey questions aimed at capturing the strength and number of appropriate types of social ties. Furthermore, aggregates of such survey-based micro-level measures across communities and societies have become one of the standard ways to proxy stocks of “societal” social capital across populations. The social network perspective on social capital is therefore widely shared in sociology (Lin, 2001; Kadushin, 2002; Li, Pickles, and Savage, 2005; Burt, 2005).

2.2 The relationship between social capital and trust

The second strand of sociological and psychological literature related to the current study deals with general trust. Arguably, modern societies are more then ever based on general trust and social interactions (Simmel, 1971; Giddens, 1991; Sztompka, 1999; Yamagishi, 2002; Glanville and Paxton, 2007; Klapwijk and Van Lange, 2009), whereas without trust societies would effectively disintegrate because trust is a synthetic force within the society (Simmel, 1950; Putnam,Leonardi and Nanetti, 1993). At the same time, general trust turns out to be closely related to bridging social capital while distrust – to bonding social capital, or strong family ties (Alesina and Giuliano, 2010). At the individual level, people whose prevailing form of social capital is the bonding one, or whose social networks are very sparse altogether, are significantly more likely to present general distrust than those with abundant bridging
Individual-level data from Poland provide some preliminary evidence that there might be a universal mutually reinforcing relation between social capital and general trust (K. Growiec, 2009, 2011).

2.3 Why accumulate social capital?

The third strand of literature which we refer to deals with individuals’ motivations to accumulate social capital. Indeed, while forming their social networks, individuals may follow a number of motivations: most importantly, they may seek to satisfy their safety drive or their effectiveness drive (Bowlby, 1969; Greenberg, 1991). Safety is associated with affiliation and the density of networks, while effectiveness – with competition and structural holes (Burt, 2005). These different functions are served by the different forms of social capital which people build: the “motivation for support [provided by bonding social capital] is satisfying basic needs or sustaining status quo. Structural holes [related to bridging social capital] are (...) for creating change and movement” (Kadushin, 2002, p. 86). Furthermore, different psychological predispositions of individuals can have a marked impact on their social networks. Individuals for whom their personal identity is more important than their social identity are more likely to maintain diverse social networks (Kalish and Robins, 2006), i.e. large stocks of bridging social capital. Surprisingly, people who have many structural holes in their network are those who are more neurotic, but reveal also a strong conviction of control over one’s own life (Kalish and Robins, 2006) and are more creative (Burt, 1992). Another important mechanism at work here is due to interpersonal complementarities in social capital formation (Glaeser, Laibson, Sacerdote, 2002): people whose social contacts have more bridging (respectively, bonding) social capital, tend to have more bridging (bonding) social capital themselves.

From this literature we infer that social ties with others should be considered a source of individuals’ utility (or subjective well-being) which they maximize, separate from consumption or leisure. We also infer that the ease of forming bridging social capital, satisfying the effectiveness but not the safety drive, should be related to the individuals’ levels of social openness and – importantly for the setup of the model

\[4\]Apart from social capital, general trust is also related to risk taking and coping with uncertainty (Dasgupta, 1988; Molm, Takahashi and Peterson, 2000; Cook, Yamagishi, Cheshire, Cooper, Matsuda, and Mashima, 2005). Low-trust societies which primarily avoid risk taking, put themselves at a competitive disadvantage in global markets by doing so, as they can’t build complex social institutions (Fukuyama, 1995).
2.4 Social capital, trust, and economic performance

The fourth strand of related literature deals with the impact of social capital and trust on economic performance at the level of individuals, communities, regions, and whole countries. Given the aforementioned findings, one should naturally expect large differences between the impacts of bridging and bonding social capital here. And indeed, sociological literature argues that bridging social capital, but not bonding social capital, goes together with civil liberties and the support for gender and racial equality, and strengthens the functioning of democracy by reducing corruption (Putnam et al., 1993; Putnam, 2000). On the other hand, “bonding social capital (as distinct from bridging social capital) has negative effects for society as a whole, but may have positive effects for the members belonging to this closed social group or network” (Beugelsdijk and Smulders, 2003). Beugelsdijk and Smulders (2003) proceed to show that bridging social capital is empirically good for economic growth at the level of European regions, whereas bonding social capital is bad for growth.

Bridging social capital is also found to be individually beneficial for those who possess it. Granovetter’s (1973) most prominent discovery is that weak ties (i.e., ties between dissimilar people) facilitate better job finding than strong ties (between similar people). Friendship ties have also been shown to be positively related to individuals’ wages and upward mobility in the workplace (Podolny and Baron 1997; Słomczyński and Tomescu-Dubrow 2005). Most strongly perhaps, Burt (2005) claims that bridging social capital, as opposed to bonding social capital, is positively related to individuals’ economic performance, creativity, social trust, and happiness. The question whether sophisticated social networks indeed improve the individuals’ earnings potential remains unsettled, though: recent research from Franzen and Hangartner (2006) indicates that using social networks might not necessarily increase the monetary payoff but improve the nonpecuniary characteristics of the job like better career perspectives instead.

Despite Burt’s (2005) clear suggestions that bridging social capital should be positively related to individuals’ happiness, the issue of whether social networks influence subjective well-being (SWB) has not been fully settled either. Even more worryingly, earnings and SWB are directly interrelated as well, complicating the matter even further (Helliwell, 2003), e.g., people with higher relative incomes have been found to show significantly higher measures of subjective well-being (Diener, Suh, Lucas, ...
and Smith, 1999). It could also be true that these ambiguous results were due to a non-linear relation between SWB and income: “Theory and some previous research suggest that the effects of individual and national incomes may be non-linear in nature, with smaller well-being effects attached to increases in income beyond levels set by each individual’s or society’s expectations and habits” (Helliwell, 2003, p. 344).

Another relevant empirical finding is that social capital levels are generally positively correlated with human capital levels, both at the individual and aggregate level. This finding is strongly supported, for example, in Putnam’s (2000) investigations based on US state-level data. The literature has also discussed a few causal links between these two forms of capital. Goldin and Katz (1999) have identified community-level social capital to be the primary determinant of the timing of introduction and the current extent of public education provision across the US. In individual data, the positive empirical relation between social and human capital has been confirmed, among others, by Glaeser, Laibson and Sacerdote (2002) and Helliwell and Putnam (2007). Hence, empirical studies preoccupied with identification of the impact of social capital on economic performance, such as our companion paper (Growiec and Growiec, 2011), control for human capital levels in all the relevant regressions. Social capital accumulation is also correlated with occupational categories. In particular, according to Burt (2005), most bridging social capital is accumulated by managers; K. Growiec (2011), based on Polish data, finds this for supervisors and entrepreneurs. A few specific jobs have also been identified as the ones where social capital has particularly high returns, e.g., physicians, clergymen, and policemen (Glaeser, Laibson and Sacerdote, 2002).

2.5 Dynamic mechanisms generating multiple equilibria

The fifth strand of literature which we relate to deals with dynamic models with non-convexities. Such models are able to generate multiple equilibria and/or poverty traps in economic performance, and thus to imply persistent earnings inequality. The list of potential driving forces behind such non-convexities is very long and includes, among others: threshold externalities in physical capital accumulation or productivity (Azariadis and Drazen, 1990); social externalities in human capital accumulation, due to a persistent and nonlinear wedge between social and private returns to education (Bénabou, 1996; Tamura, 2001; Belzil and Hansen, 2002; Davies, 2003; Rangazas, 5 See also the discussion in Galor (1996) as well as Acemoglu (2009), Chapter 21.
2005); borrowing constraints binding for low-income individuals but not high-income individuals (Aghion and Bolton, 1997; Piketty, 1997; Matsuyama, 2000); borrowing constraints precluding low-income individuals from getting education (Galor and Zeira, 1993; Moav, 2002); discrete choices of school and occupation (Cardak, 2004; Fall, 2005); the adoption of consecutive technological vintages in discrete steps (Chari and Hopenhayn, 1991; Jovanovic, 1998); and a discrete switch from the “traditional” agriculture-based technology of stagnant economies to the “modern” technology allowing for perpetual growth (cf. Galor, 2010 and references therein).

To our knowledge, the potential of social capital and social trust to generate multiple equilibria has not been studied in the literature yet.

3 The model

The model presented below is a thoroughly reworked version of the models analyzed in Beugelsdijk and Smulders (2003) and Growiec and Growiec (2010). There are several crucial differences between those two earlier setups and the current one. The most important novelty of the current paper is the assumption that the ease of forming new interpersonal contacts (i.e., bridging social capital) is proportional to the pool of contacts one already has and the pool of people with whom one is not yet acquainted but might consider being. The size of this pool is in turn determined by the total number of people in the society and, most importantly, by the level of social trust. In Beugelsdijk and Smulders (2003), bridging social capital was treated as a flow and not a stock, and thus that model completely neglected the dynamics of social capital formation. In Growiec and Growiec (2010), we treated bridging social capital as a stock, but we assumed that the ease of forming new interpersonal ties was related only to the amount of acquaintances one already had, thus neglecting one important source of nonlinearity. We also completely abstracted from the relationship between social capital and social trust.

In result, both earlier models were purely neoclassical (convex) in nature, which made them unable to capture the possibility that bridging social capital and social trust, interacting with one another, could give rise to multiple equilibria. This property is, on the other hand, central to the model presented below.

The role of the current model is to describe a mechanism which could show how bridging social capital and trust may generate multiple equilibria due to a mutually reinforcing relationship between them (both a “vicious” and a “virtuous” circle, giving
rise to a low and high equilibrium, respectively), and to relate them to individuals’
earnings and well-being. Such a transmission mechanism can potentially explain the
persistence of differences in the discussed social variables, and show why they are
capable of forming a serious obstacle in economic development.\textsuperscript{6}

3.1 Setup of the model

Our model economy is populated by individuals who maximize their lifelong sum
of subjective well-being (SWB). Following Helliwell (2003) as well as O’Brien and
Quimby (2006), we presuppose that SWB is composed of (i) consumption, (ii) satis-
faction from social life outside the family, and (iii) other characteristics such as the
evaluation of one’s health, satisfaction from family life, and general conditions and
circumstances of life.\textsuperscript{7} The last component (iii) we consider exogenous to the model
and set aside hereafter (though in reality, it will be correlated with earnings). We are
thus taking a markedly broader view of the maximized objective function to what is
customary in economics – in the discussed framework, individuals derive utility also
from other variables than just consumption. Mathematically, this means that the
instantaneous flow of well-being is given by

\[ \text{SWB} = H c^{\gamma} v^{\theta}, \]  

where $H$ is the constant exogenous constituent factor of SWB, $c$ is consumption, and
$v$ is the stock of bridging social capital. $\gamma \in (0, 1)$ and $\theta \in (0, 1)$ are the exogenous
partial elasticities of SWB with regard to consumption and bridging social capital,
respectively.

To keep things as simple as possible, we neglect the possibility of savings and
physical capital accumulation. Thus, all earnings $w$ are always immediately spent
on consumption and output is not stored across time.\textsuperscript{8} The production function is

\textsuperscript{6}Jovanovic (1998) divides the explanations of persistent income inequality into three groups: the
ones driven by (i) initial conditions, (ii) random factors, and (iii) compensating differentials. The
mechanism proposed in this paper falls into the first group: if the model economy begins with a
one-point distribution of social capital and trust, it will converge to a unique steady state. Also,
for simplicity and clarity of the obtained results, the current investigation unlike the previous ones
concentrates on bridging social capital only, and bonding social capital is disregarded here.

\textsuperscript{7}By general conditions and circumstances of life, we mean housing conditions, congestion in the
place of residence, frequency of problems with neighbors, etc.

\textsuperscript{8}One could argue that social capital accumulation is a form of \textit{indirect} savings here, however,
because it implies investing in a stock that would yield productive gains in the future, and thus may
linear in labor (which is the only production factor here), and further augmented by a positive spillover from bridging social capital. Ignoring also the possibility of human capital augmentation of labor, we write:

\[ w = c = A^\sigma \ell_Y v^\phi, \tag{2} \]

with

\[ \ell_v + \ell_Y = 1, \tag{3} \]

\( A^\sigma \) being the constant “total factor productivity”, \( \ell_Y \) denoting the fraction of the total time endowment spent effectively at work, \( \ell_v \) denoting the fraction of time spent on socializing with people outside of the family, and the parameter \( \phi > 0 \) measuring the strength of the spillover from bridging social capital to production. The individual’s total time endowment at each instant of time is normalized to unity.

The spillover \( \phi \) is included in the production function since it is argued (cf. Dasgupta, 2002) that social capital – and in particular bridging social capital (Burt, 2005) – facilitates the matching of workers and firms, speeds up information transmission, and reduces transaction costs and deadweight losses in economic activity. Please note that this spillover effect is fully internalized by the decision-making individuals: they treat their social ties with friends and acquaintances both as ends (direct increases in SWB, with an elasticity \( \theta \)) and (instrumentally) as means for raising the level of consumption (with an elasticity \( \phi \)).

The assumption that \( w \) increases with \( v \), at least upon aggregation, is essential to the workings of the current model. There exist, however, quite a few alternatives to the production function assumed in equation (2) which also seem plausible. First, one may doubt whether the spillover from bridging social capital to production is fully internalized by individuals. Thus, one could replace (2) with \( w = A^\sigma \ell_Y \bar{v}^\phi \), where \( \bar{v} \) is the average level of bridging social capital in the economy, external to the individuals’ decisions. This would not change our results qualitatively. Second, one could doubt whether there exists a true spillover from bridging social capital to productivity. In fact, its apparent presence in individual-level data could also be an artifact of the so-called “fallacy of composition” (see the discussion in Durlauf and Fafchamps, 2005): bridging social capital may improve the earnings of some individuals only at the expense of others without having an impact on aggregate productivity. In such case, we would write \( w = A^\sigma \ell_Y (v/\bar{v})^\phi \) and thus \( w = A^\sigma \ell_Y \) in the symmetric equilibrium. A final possibility is a generalization of our previous ideas, a function \( w = A^\sigma \ell_Y v^\mu \bar{v}^\nu \) that help smooth consumption intertemporally.
includes both internal and external effects of bridging social capital on productivity. Quantitatively, the outcomes of the model will clearly differ depending on the values of \( \mu \) and \( \nu \). Qualitatively, however, as we shall see shortly, these differences do not overturn the main predictions and characteristics of the model, as long as \( \mu + \nu \neq 0 \), i.e. there exists a non-negligible spillover from aggregate bridging social capital to aggregate productivity. In the light of the empirical findings of Granovetter (1973), Burt (2005), among others, we view this as a very plausible assumption. One natural interpretation of this assumption is that bridging social capital constitutes a part of “social infrastructure”, a component of total factor productivity (Hall and Jones, 1999): it reduces transaction costs, improves the flow of information, facilitates cooperation and implementation of innovative ideas, and thus increases TFP by reducing the extent of input misallocation (Hsieh and Klenow, 2009).

Bridging social capital \( v \) is modeled as a stock and not as a flow as in Beugelsdijk and Smulders (2003). In line with intuition, we assume that bridging social capital might be accumulated through purposeful investments of time – i.e. time spent socializing with friends and acquaintances – and that it depreciates gradually over time if not enough effort is made to maintain the social ties. We write

\[
\dot{v} = \xi \ell_v^\mu v(zn - v) - \delta v, \tag{4}
\]

where \( \mu > 0 \) is the returns-to-scale parameter in bridging social capital accumulation and \( \delta > 0 \) is the depreciation rate of bridging social capital (the rate of natural decay of social ties). The variable \( n > 0 \) captures the total number of people in the population with whom it is possible to establish social ties if trust permits, and \( z \in [0, 1] \) is the individual’s level of social trust; \( zn - v \) is thus the total number of “eligible” people whom one trusts but with whom she has not established a tie yet.

Hence, the current bridging social capital accumulation function parallels the assumptions of the logistic diffusion model (cf. Benhabib and Spiegel, 2005): the increments to bridging social capital are proportional both to the current stock of this variable and the remaining pool of trusted people \( zn - v \). An important feature of this model is that it offers a positive long-run equilibrium (steady state) only if time investments \( \ell_v \) and the total pool of trusted people \( zn \) are large enough. Otherwise, it predicts gradual decay of social ties.

It is important to note here that \( n \) does not just capture the raw number of people in the population. What actually matters for each particular individual is the number of people who could potentially establish social ties with her, or total
population scaled by an appropriate societal measure of social distance. This definition corresponds directly to the classical Bogardus (1926) Social Distance Scale. The social distance measured by the Bogardus Scale captures individuals’ willingness to participate in social contacts with members of diverse social groups, such as other racial and ethnic groups, sex offenders, and homosexuals. Alternative scales of social distance which could be used in an empirical operationalization of \( n \) include, among others, sociometric measures of frequency of contact/interaction with members of appropriately defined outgroups. The result on such a scale tells how open a society is, or equivalently, how strong the prejudice toward out-groupers is there. Other issues which could potentially affect the value of \( n \) are: mobility, diversity of the society (measured partially by ethno-linguistic fractionalization), and the general cultural background in the society, governing the average individual’s “exogenous” willingness to form social ties with others, irrespective of their characteristics, such as e.g. the tradition of hospitality.

Assuming a constant discount rate \( \rho > 0 \) and using (1) and (2), the individual’s subjective well-being (SWB) maximization problem is the following:

\[
\max_{\{\ell_v(t)\}_{t=0}^{\infty}} \int_0^{+\infty} HA^{-\sigma}(1 - \ell_v)^{\gamma\phi} e^{-\rho t} \, dt \quad \text{s.t.} \quad \dot{v} = \xi \ell_v^\mu (zn - v) - \delta v. \tag{5}
\]

In the following analysis, we will assume social trust \( z \) to be constant and exogenous. Later on, we will relax this assumption and discuss the consequences.

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9. The scale asks people to point the degree of closeness they wish to maintain with members of a given diverse social group. The degrees of closeness are: marrying one’s daughter/son, being one’s personal friend, being one’s neighbor on the same street, being a co-worker in the same occupation, being a citizen of one’s country, being only a visitor in one’s country, and wishing to exclude this person from one’s country. The Bogardus Scale implies that the possible attitudes toward the out-groupers span a continuum, from exclusion from the country, to close relations via marriage. It also implies that if you accept an out-grouper as a spouse of your child, you must also accept out-groupers as neighbors, co-workers, etc.

10. Under an alternative interpretation, \( n \) can also be viewed as the Dunbar’s (1993) constant. This author has argued that due to the construction of human brain, the maximum number of stable, coherent social ties that a person can effectively maintain is approximately 150.

11. Regarding empirical tests of the current model, one has to keep in mind that in reality, social trust might also change for reasons unrelated to social capital. Knack and Keefer (1997) document that social trust has been declining in the US throughout 1950–1990. In the same period, however, the US society has been growing in number and increasing its mobility, both these factors being captured by an increasing \( n \).
3.2 The dynamic equations

To solve this intertemporal optimization problem, one should apply the standard optimal control approach. The associated Hamiltonian reads:

$$H = HA^{\gamma}(1 - \ell_v)\gamma v^{\gamma\phi + \theta} e^{-\rho t} + \lambda[\xi \ell_v^\mu v(zn - v) - \delta v].$$ (6)

From this, and with the assumptions that $H$, $A$ and $z$ are constant over time, the dynamic equation for $\ell_v$, i.e. the evolution of optimal time investment in bridging social capital over time, is derived as:

$$\dot{\ell}_v = \ell_v \left( \frac{\rho + \delta(\gamma\phi + \theta + \frac{v}{zn - v}) - \xi \ell_v^\mu (zn - v)(\gamma\phi + \theta) \left(1 + \frac{v}{\gamma}(1 - \ell_v)\frac{1-\ell_v}{1-\ell_v}\right)}{1 - \mu + (1 - \gamma)\ell_v}\right).$$ (7)

The transversality condition $\lim_{t \to \infty} \lambda(t)v(t) = 0$ is automatically satisfied because the dynamics of $\lambda(t)$ are dominated by the vanishing term $e^{-\rho t}$ and the stock of bridging social capital $v(t)$ is bounded. This, coupled with the equation of motion of the stock of bridging social capital

$$\dot{v} = \xi \ell_v^\mu v(zn - v) - \delta v,$$ (8)

completes the description of the dynamics. Let us now proceed to the description of the steady state.

3.3 Interior steady state: conditions for existence and uniqueness

The model meets the usual concavity requirements for an interior maximum (see Appendix A.2) and it possesses a unique interior steady state such that $\dot{\ell}_v = \dot{v} = 0$ with $v^* \in (0, zn)$, provided that the unit efficiency of social capital formation and/or the pool of trusted people is large enough. More precisely, it is the case if the steady state level of time investment in social capital $\ell_v^*$ exceeds a certain pivotal value:

$$\ell_v^* > \left(\frac{\delta}{\xi zn}\right)^{1/\mu} \equiv \ell_v^{piv}.$$ (9)

The condition above is equivalent to the condition $\frac{\partial \ell_v}{\partial v}\bigg|_{v=0} > 0$, i.e., that if the individual has zero social capital, it is beneficial for her to accumulate it.
Provided that the condition (9) holds, one can insert the steady-state relationship \( v = zn - \frac{\delta}{\xi \ell_v} \) into (7) with the restriction \( \dot{\ell}_v = 0 \). The steady state is then computed as the implicit solution \( \ell_v^\star \) to the equation:

\[
\varphi(\ell_v) = \xi zn \ell_v^{\mu+1} + \left( \frac{\delta \mu}{\gamma} (\gamma \phi + \theta) + \rho - \delta \right) \ell_v - \frac{\delta \mu}{\gamma} (\gamma \phi + \theta) = 0.
\] (10)

As stated in the following propositions, a solution to \( \varphi(\ell_v) = 0 \) is guaranteed to exist only in a certain parameter range:

**Proposition 1** Equation (10) has a unique solution \( \ell_v^\star \in (0, 1) \) provided that \( \xi zn + \rho > \delta \) and that the inequality (9) holds.

**Proposition 2** The condition (9) is equivalent to the inequality

\[
\ell_v^{\text{min}} = \left( \frac{\delta}{\xi zn} \right)^{1/\mu} < \frac{\delta \mu}{\gamma} (\gamma \phi + \theta) \cdot \frac{\rho + \delta \mu}{\gamma} (\gamma \phi + \theta).
\] (11)

In words, this means that for an interior solution to exist – so that there could be a positive stock of bridging social capital in the long run – forming social ties must be “sufficiently easy”. This in turn requires that either the total pool of people with whom social ties could be potentially formed \( n \) is large enough, or so is the “efficiency” of social capital formation \( \xi \), or so is social trust \( z \). The last option is most relevant to our study and will be exploited more in the following paragraphs.

### 3.4 Comparative statics of the steady state

In the above section, we have presented the conditions under which an interior steady state exists and is unique. Unfortunately, due to the statement of the problem, we are unable to derive a closed-form solution for \( \ell_v^\star \) and \( v^\star \). It is however straightforward to compute the relevant comparative statics using the implicit function theorem.

Two parameters of crucial importance for the properties of the model are: the amount of social trust \( z \) and the spillover parameter \( \phi \) measuring the pecuniary benefits of individuals’ bridging social capital.\(^{12}\) We obtain the following results regarding these two parameters.

\(^{12}\)Obviously, one could also compute comparative statics with regard to other parameters of the model, such as \( n, A, \delta, \) or \( \xi \). These results are however not directly conducive to the workings of our model, so we do not present them here. They are available from the authors upon request.
Proposition 3 The steady state level of bridging social capital $v^*$ increases with social trust $z$, but the share of time devoted to social capital accumulation $\ell_v^*$ decreases with $z$.

The interpretation of this result is the following: social trust gives rise to both substitution and wealth effects – if one finds others more trustworthy than before, it becomes easier and quicker for her both (1) to establish more social ties (wealth effect) and (2) to maintain a fixed level of social capital with less effort, leaving more time for productive work (substitution effect). In our current setup, the substitution effect outweighs the wealth effect, thereby implying a negative relationship between the intensity of social capital accumulation and social trust.

As far as the equilibrium level of individuals’ bridging social capital $v^*$ is concerned, the above two effects are augmented with one more effect, a direct positive one: more social trust provides direct increases to the pool of people with whom one could get acquainted. This effect dominates the two indirect effects (i.e., via the equilibrium level of time investment in social capital, $\ell_v^*$), and hence the total impact of social trust on equilibrium level of bridging social capital is unambiguously positive.\(^{13}\)

Since the impact of social trust $z$ on social capital $v^*$ is positive at the steady state, its impact on average earnings $w^* = A^\sigma (1 - \ell_v^*)^{(v^*)^\phi}$ and subjective well-being $SWB = HA^\gamma (1 - \ell_v^*)^{(v^*)^\phi + \theta}$ is positive as well:

$$\frac{\partial w}{\partial z} \bigg|_{v=v^*} = w^* \left( -\frac{1}{1 - \ell_v^*} \frac{\partial \ell_v}{\partial z} \bigg|_{\ell_v = \ell_v^*} + \frac{\phi}{v^*} \frac{\partial v}{\partial z} \bigg|_{v=v^*} \right) > 0;$$

(12)

$$\frac{\partial SWB}{\partial z} \bigg|_{v=v^*} = SWB^* \left( -\frac{\gamma}{1 - \ell_v^*} \frac{\partial \ell_v}{\partial z} \bigg|_{\ell_v = \ell_v^*} + \frac{\phi + \theta}{v^*} \frac{\partial v}{\partial z} \bigg|_{v=v^*} \right) > 0.$$

(13)

The dependence of the steady state $(\ell_v^*, v^*)$ on the spillover parameter $\phi$ can be summarized as follows.

Proposition 4 The spillover parameter $\phi$ relates positively both to the equilibrium time investment in social capital formation $\ell_v^*$ and to the social capital level $v^*$.

The interpretation of this result is the following. The larger the spillover parameter $\phi$, i.e., the larger is the positive impact of one’s stock of bridging social capital on

\(^{13}\)Testing this prediction empirically is possible (e.g. Growiec and Growiec, 2011) but requires one to distinguish between empirical measures of social capital (a characteristic of social networks) and social trust (a concept of social norms). Unfortunately, these two distinct ideas are often convoluted in the economic literature on social capital and economic performance.
her earnings, the more social capital will be accumulated, and the more time will be
devoted to its accumulation. As opposed to the relationship between trust and social
capital investment, there is no substitution effect at work here, inducing people to
shift their time allocations towards work. Trust makes it easier to establish social
ties; the spillover $\phi$ makes it more profitable. This leads to different impacts on their
incentives in each of the two cases.

3.5 Two equilibria

As noted above, the existence of an interior steady state with a positive level of
bridging social capital hinges on the crucial assumption (9) (or equivalently, (11))
which states that the level of social trust should be sufficiently high:

$$
\left( \frac{\delta}{\varepsilon zn} \right)^{1/\mu} < \frac{\delta \mu (\gamma \phi + \theta)}{\rho + \delta \mu (\gamma \phi + \theta)} \Leftrightarrow z > \left( \frac{\rho + \delta \mu (\gamma \phi + \theta)}{\delta \mu (\gamma \phi + \theta)} \right)^\mu \frac{\delta}{\varepsilon n} \equiv z^{piv}.
$$

Furthermore, if $z$ is indeed high enough to meet the condition (14), then by transver-
sality conditions of the dynamic optimization problem, convergence to the positive
steady state $(\ell^*_v, v^*)$ is guaranteed, and thus there will for sure be a positive amount
of bridging social capital in the long run if only $v(0) > 0$. Even more importantly,
from the comparative statics exercise we learn that the higher is the level of social
trust, the more social capital will eventually be accumulated.

If, however, the level of social trust $z$ fails to satisfy the above condition, then we
will observe a sustained decline in bridging social capital, leading to a zero value of
this variable in the long run, irrespective of $v(0)$. This leads to the announced result
of two equilibria (a corner and an interior one), indexed by the (exogenous) level of
social trust $z$.

For a numerical representation of the above statements, please consult Figure 1.

As far as the empirical content of the current multiple-equilibria result is con-
cerned, it should be noted that its foremost implication – that there should be discrete
gaps in cross-country distributions of social trust and bridging social capital stocks
- holds here only conditional on parameter values. These values might be country-
specific, however, and thus a reliable empirical verification of this prediction of the
current model would require using panel data, whereby the validity (or invalidity) of
the cross-country predictions of the current model could be identified thanks to the
temporal dimension of the data. Unfortunately, we are not aware of existence of any
Figure 1: The dependence of the steady state \((\ell^*_v, v^*)\) on social trust \(z\).

Source: own computations.
Notes: parameters used to produce this figure: \(\mu = 0.6, n = 1, \delta = 0.06, \xi = 0.5, \phi = 0.25, \theta = 0.3, \gamma = 0.4, \rho = 0.1\). The implied critical value of social trust \(z\) above which an interior steady state exists satisfies \(z_{piv} = 0.26639\). The relationships presented in this figure are in agreement with Proposition 3.

Panel datasets containing applicable measures of bridging social capital. In result, our theory must remain speculative in this respect for a while.

3.6 The inverse U-shaped relationship between bridging social capital, earnings, and well-being

When the economy finds itself in the interior equilibrium \((\ell^*_v, v^*)\), which is the case if \(z > z_{piv}\) as defined in (14), then the levels of earnings \(w^*\) and \(SWB^*\) are also uniquely determined. They are then simply a function of the underlying level of social trust and other parameters of the model. If these other parameters are fixed, then everything is pinned down by the underlying level of social trust \(z\). In particular, because \(\ell^*_v, v^*, w^*\) and \(SWB^*\) depend positively on trust, the steady-state relationship between bridging
social capital and earnings is unambiguously positive, too, and so is the steady-state relationship between bridging social capital and subjective well-being.

As it has been done in Growiec and Growiec (2010) in the case of a convex model of social capital formation, we may however trace the out-of-equilibrium pattern of dependence between the investment in bridging social capital, and earnings and well-being. It turns out not to be monotonic and follow an inverse U-shape. The following propositions hold.

**Proposition 5** For a given level of social trust $z \in (z^{piv}, 1]$, the relationship between bridging social capital and earnings in the vicinity of the interior steady state is inverse U-shaped: at low levels of bridging social capital, it increases earnings; at high levels, it decreases them. In the steady state, individuals invest less time in bridging social capital accumulation than is required to maximize instantaneous earnings, provided that

$$\varphi \left( \frac{\delta \theta \mu}{\rho \gamma + \delta \theta \mu} \right) < 0,$$

where the function $\varphi$ has been defined in (10). Conversely, individuals invest more time in bridging social capital accumulation than is required to maximize instantaneous earnings if the sign in inequality (15) is reversed.

**Proposition 6** For a given level of social trust $z \in (z^{piv}, 1]$, the relationship between bridging social capital and well-being in the vicinity of the interior steady state is inverse U-shaped: at low levels of bridging social capital, it increases well-being; at high levels, it decreases them. In the steady state, individuals invest less time in bridging social capital accumulation than is required to maximize instantaneous well-being.

The above finding, that the out-of-equilibrium relationship between bridging social capital and individuals’ earnings and well-being is inverse U-shaped, parallels the one put forward in Growiec and Growiec (2010). One must keep in mind two important differences though. Firstly, the relationship was treated as unconditional in the previous study, whereas here it is conditional on the level of social trust. As we have already argued, as the level of trust increases, bridging social capital, earnings and well-being go hand in hand instead of following and inverse U-shape.

Secondly, the above propositions are only valid in the vicinity of the interior steady state. If the economy finds itself in a corner equilibrium with $z \leq z^{piv}$, or if there
are multiple interior equilibria (discussed below), then the proposed inverse U-shaped relationship will break.

The current finding parallels also the inverse U-shaped relationship between genetic diversity and development, identified by Ashraf and Galor (2012). These authors have shown that diversity reduces trust, social capital and cooperation, but enhances innovation. Extremely high or low levels of genetic diversity are thus detrimental to economic development, whereas intermediate values of diversity provide most favorable conditions for development.

3.7 Dynamics

The dynamics of the model, summarized by equations (7)–(8), can be summarized in a sequence of phase diagrams. There are two cases which ought to be analyzed separately, namely that of \( z > z_{\text{piv}} \) where an interior steady state exists, and that of \( z \leq z_{\text{piv}} \) where social capital will gradually decay to zero.

In both cases, the \( \dot{v} = 0 \) locus in the \((v, \ell_v)\) space (\(\ell_v\) located on the vertical axis), is given as a graph of the function (see eq. (8)):

\[
\ell_v = \vartheta(v) = \left( \frac{\delta}{\xi(z_n - v)} \right)^{1/\mu}.
\]  

(16)

It is therefore increasing for all \( v \), begins at \( \vartheta(0) = \ell_{\text{piv}} \), and converges to a vertical asymptote as \( v \to z_n \).

The \( \dot{\ell}_v = 0 \) locus is, on the other hand, identified as the zero contour of the bivariate function

\[
\Phi(\ell_v, v) = \rho + \left( \gamma\phi + \theta + \frac{v}{z_n - v} \right) \delta - \xi\ell_v(z_n - v)(\gamma\phi + \theta) \left( 1 + \frac{\mu}{\gamma} \left( 1 - \frac{\ell_v}{\ell_{\text{piv}}} \right) \right). 
\]

(17)

From the implicit function theorem, it is derived that the \( \dot{\ell}_v = 0 \) locus is unambiguously downward sloping in the \((v, \ell_v)\) space:

\[
\frac{\partial \ell_v}{\partial v} \Bigg|_{\Phi=0} = -\frac{\partial \Phi}{\partial v} \bigg|_{\Phi=0} < 0.
\]

(18)

Furthermore, it can be easily shown that for \( v \to z_n \), it must be the case that \( \ell_v \to 0 \) for the condition \( \Phi = 0 \) (eq. 17) to hold. Finally, we find that for \( v = 0 \), the
Figure 2: Phase diagram of the model with an interior steady state.

Source: own computations.

Notes: parameters used to produce this figure: \( \mu = 0.6, z = 1, n = 1, \delta = 0.06, \xi = 0.5, \phi = 0.25, \theta = 0.3, \gamma = 0.4, \rho = 0.1 \). Implied steady state satisfies: \( \ell_* = 0.1572, v* = 0.628 \). The condition (11) as well as the second order conditions (37) and (42) jointly hold.

The corresponding value \( \ell^0_v \) along the \( \dot{\ell}_v = 0 \) locus is larger than \( \ell^{piv} \) if and only if the inequality (14) holds (i.e., if social trust \( z \) exceeds the threshold value \( z^{piv} \)).\(^{14}\)

Knowing the behavior of the \( \dot{v} = 0 \) and \( \dot{\ell}_v = 0 \) loci, the dynamics of the two-dimensional system can be analyzed in phase diagrams, depicted in Figures 2–3. In Figure 2, we see that if an interior steady state exists, it is saddle-path stable and there exists a unique approach path, which will for sure be taken as it is the only one which satisfies the transversality condition. The saddle path is downward sloping, indicating that an individual who starts off with a low level of social capital (has a few acquaintances only) will initially invest more time in social capital formation that she will do in the long run, and vice versa.

\(^{14}\)Since \( \Phi \) is an increasing function of \( v \), the considered condition is equivalent to \( \Phi(\ell^{piv}, 0) < 0 \) which, after the necessary algebra, boils down to (14).
Figure 3: Phase diagram of the model without an interior steady state.

Source: own computations.
Notes: parameters used to produce this figure: $\mu = 0.6, z = 0.2, n = 1, \delta = 0.06, \xi = 0.5, \phi = 0.25, \theta = 0.3, \gamma = 0.4, \rho = 0.1$. Implied steady state satisfies: $\ell_v^* = 0, v^* = 0$. The condition (11) does not hold (indicating that there cannot exist an interior steady state), but the second order conditions (19) and (37) hold.

On the other hand, if $z \leq z^{piv}$ then the model implies a gradual decay of social ties (Figure 3), whose stock will tend to zero over the long run. In such case, the transversality condition imposes a further requirement on parameter values:

$$
\lim_{t \to \infty} \frac{\dot{\lambda}(t)}{\lambda(t)} = \delta(1 - \gamma\phi - \theta) - \rho < 0.
$$

(19)

3.8 Endogenizing social trust: threshold externalities and multiple interior equilibria

The model discussed until this point assumed an exogenous, constant level of social trust, captured by the parameter $z \in [0, 1]$. We would like to endogenize this parameter now, but in such case a few issues must be discussed first.
First of all, the amount of social trust cannot grow without bound: by construction, the highest possible level for this variable is $z = 1$ where each individual trusts everyone in the population and is ready to establish social ties with anyone else. Since steady-state earnings and subjective well-being uniformly increase with trust at all levels of this variable, it is concluded that earnings and well-being are maximized for $z = 1$. Hence, low trust might be a barrier to economic convergence in this model – a country with less trust will converge to a lower steady state than a country with more trust – and accumulating trust can help speed up convergence by shifting the country to approach paths of ever higher steady states, but trust cannot drive long-run growth here because it is bounded from above. This makes the current model applicable to the discussion on the catalysts and inhibitors of convergence, but not to the debate on sources of growth.15

Secondly, long-run levels of earnings and well-being are functions of the steady-state equilibrium values of social capital and social capital investment in the model, and social capital variables are in turn dependent on the magnitude of social trust $z$. Social trust should, however, be endogenized because it is argued in the literature that social ties with people outside of one’s family tend to increase general trust. Both in our model and in reality, a reverse causal link from trust to social capital formation might be also active, giving rise to a feedback loop of simultaneous co-dependence between these two variables.

Thirdly, social trust might change for reasons unrelated to the mechanisms dis-

\footnote{Social trust $z$ could drive long-run growth if it were not uniformly bounded from above, though. For instance, instead of taking the natural restriction $z \leq 1$ we could request that $zn \leq N$, where $n$ is the population “eligible” for establishing social ties and $N$ is total population (or some other arbitrary upper bound for $zn$). We could then fix $n$ but request $N$ to grow unboundedly with time and allow $z$ to follow it. A possible interpretation for the current case, in which $z$ grows over time given $n$ (so that $z$ and not $n$ drives long-run growth), would be that as total population grows larger, people can select the group of people “eligible” for establishing social connections more adequately, and thus they can trust such acquaintances more. Under some additional technical assumptions, according to equation (4), the number of social ties per person $v$ would then grow unboundedly over time. Consumption and output would follow, and thus we would observe endogenous growth. In reality, however, the number of social ties per person cannot be unboundedly expanding over time: people are born with no social ties outside family and all their social ties disappear when they die. Our aggregative model abstracts from finite lifetimes of individuals, though. Keeping this in mind, and having noted that there is no externality in equation (4), i.e. $v$ is the \textit{individual} number of social ties, we can apply the argument developed by Growiec (2010) here and demonstrate that accounting for finite lifetimes in our model would eliminate the currently discussed endogenous growth possibility. We can thus safely rule this case out from our present considerations.}
cussed in the current article: changes in the strength of institutions (e.g., enforcing contracts and property rights), changes in the ethnic, cultural, and income structure of the society, or shifts in attitudes, caused e.g. by technological change. All these issues must be properly accounted for in empirical studies.

Keeping these caveats in mind, we note that there exist multiple ways of conditioning social trust \( z \) on the stock of bridging social capital \( v \). For example, if one imposed a constant-elasticity spillover of \( v \) as in \( z = \zeta v^\beta \), this would lead to a unique interior steady state \((\ell^*_v, v^*)\) irrespective of \( v(0) \), and of the magnitude of \( z(0) \) – thereby removing the threshold effect present in the original model where trust needs to be sufficiently large for the interior steady state to be reached.

Consequently, in line with the primary hypothesis of the paper, we would like to link these two variables in a non-convex way, leading to multiple interior equilibria due to threshold externalities (Azariadis and Drazen, 1990). The simplest way to do so is to assume that people’s trust can be either high or low, depending whether their current stock of bridging social capital exceeds a threshold value \( \bar{v} \) or not:

\[
z = \begin{cases} 
    z_1, & v \leq \bar{v}, \\
    z_2, & v > \bar{v}, 
\end{cases}
\]  

where \( z_1 < z_2 \) and \( \bar{v} \) satisfies the inequality \( v^*_1 < \bar{v} < v^*_2 \), with \( v^*_i \) denoting the steady state value of bridging social capital \( v \) if social trust equals \( z_i, i = 1, 2 \). Furthermore, if \( z_1 < z^{piv} \), then the lower steady state has zero bridging social capital.

The interpretation of the current model is the following: one will generally trust others (high \( z \)) only provided that she is currently acquainted with sufficiently many people; if her stock of acquaintances falls short of the threshold, one would rather refuse to trust others (low \( z \)). The effect is non-linear here because our model assumes equality between individuals, and trust is a social phenomenon which builds on reciprocity (Simmel, 1950).

From the above analysis of model dynamics it follows that in the generalized model with a threshold externality of form (20), under the assumption \( z_1 > z^{piv} \), we observe the following regularities:

- If \( v(0) < v^*_1 \) then \( v \) will increase over time until it reaches the low equilibrium \( v^*_1 \). \( \ell_v \) will decrease over time until it reaches \( \ell^*_{v,1} \).
- If \( v^*_1 < v(0) < \bar{v} \) then \( v \) will decrease over time until it reaches the low equilibrium \( v^*_1 \). \( \ell_v \) will increase over time until it reaches \( \ell^*_{v,1} \).
Figure 4: Phase diagram of the model with a threshold externality on social trust.

Source: own computations.

Notes: parameters used to produce this figure: $\mu = 0.6, z_1 = 0.8, z_2 = 1, n = 1, \delta = 0.06, \xi = 0.5, \phi = 0.25, \theta = 0.3, \gamma = 0.4, \rho = 0.1, \bar{v} = 0.53$. The implied low steady state satisfies: $\ell_{v,1}^* = 0.1793, v_1^* = 0.4509$. The high steady state satisfies: $\ell_{v,2}^* = 0.1517, v_2^* = 0.628$. The condition (11) as well as the second order conditions (37) and (42) jointly hold in both steady states.

- If $\bar{v} < v(0) < v_2^*$ then $v$ will increase over time until it reaches the high equilibrium $v_2^*$. $\ell_v$ will decrease over time until it reaches $\ell_{v,2}^* < \ell_{v,1}^*$.
- If $v_1^* < v(0)$ then $v$ will decrease over time until it reaches the high equilibrium $v_2^*$. $\ell_v$ will increase over time until it reaches $\ell_{v,2}^* < \ell_{v,1}^*$.

The model therefore gives rise to multiple interior equilibria (see Figure 4). The choice of equilibrium converged upon depends on the initial stock of bridging social capital, $v(0)$.

Of course, the positive dependence of social trust on bridging social can be modeled in different, smoother ways, but as long as the dependence is sufficiently non-convex, the above result goes through. More importantly, it does not hinge on the assumption that the level of social trust is external, established after the agents have decided on
their dynamic paths of social capital formation. Although the level of social trust in
the society — i.e., how much trust we can expect to get from others — can indeed be
plausibly modeled as a threshold externality and treating it this way favors occurrence
of multiple equilibria, it does not need to be viewed as an externality for multiple
equilibria to appear. Indeed, even if the agents fully internalize the impact of their
decisions on the equilibrium level of social trust in the society, so that they directly
insert \( z = z(v) \) in their optimization problem, multiple equilibria can still potentially
occur provided that \( z(v) \) is sufficiently non-convex.\(^{16}\)

### 3.9 Introducing economic growth

As we already mentioned above, the current model can be used in analyses of the
impact of bridging social capital and social trust on the long-run level of economic
development. To make it applicable to economic questions of long-run growth and
convergence, one should however generalize it and incorporate some mechanism of
unbounded economic growth.

The simplest way to achieve this goal is to assume that “total factor productivity”
(TFP), i.e., the factor \( A \) in (2), grows exogenously at a rate of \( g > 0 \):
\[
A(t) = A_0 e^{gt}.
\]
Growth in \( A \) should then be incorporated in the individuals’ optimization problems,
partially counterbalancing the psychological discount rate \( \rho \). In this simplest case,
the total impact of TFP growth on the dynamic evolution of social capital formation
will actually consist in substituting \( \rho \) with \( \rho - g \) wherever the former appeared in
equations (7)–(8), as long as \( g < \rho \). Otherwise, an unwelcome possibility would appear
that aggregate discounted subjective well-being diverges to infinity, invalidating the
integrability condition.

Hence, the introduction of TFP growth lowers the effective discount rate. In
result, the following proposition holds (if an interior steady state exists):

**Proposition 7** The higher is TFP growth rate \( g \), or the lower is the discount rate
\( \rho \), the more time will be allocated to social capital accumulation in the interior steady
state \((l^*_v, v^*_v)\), and the more bridging social capital will be there in the long-run equilibrium
\((v^*_v)\).

\(^{16}\)As a numerical example, we have found multiple equilibria to occur in the case of a logistic
function \( z(v) = \frac{z_2}{1 + \left( \frac{z_1}{z_2 - z_1} \right)^{\frac{v}{v^*}}} \) which satisfies \( z(0) = z_1, \lim_{v \to \infty} z(v) = z_2 > z_1 \), and has its unique
inflection point at \( \bar{v} \). The results are available from the authors upon request.
In words, if individuals are more patient, or if there is faster growth in their productivity, they are also more willing to postpone consumption until later. Also the extra gains requested by them to do so are relatively smaller. The logic behind this finding is the following. Firstly, steady-state levels of subjective well-being and earnings are positively related to the amount of bridging social capital, so in the long run it pays to have more of it. Secondly, accumulating social capital requires time, and time invested in forming social ties must be subtracted from the amount if time spent on productive work which gives instant gratification. In result, the more patient are the individuals, the more time they spend on socializing with friends and acquaintances at the expense of earning for immediate consumption.

The important lesson here is that under exogenous TFP growth, levels of social capital will converge to the interior steady state, whereas consumption and earnings will grow at the balanced rate $\sigma g$.

Economic growth might also be linked to social trust in the considered model, however. Indeed, international evidence (e.g., Zak and Knack, 2001; Algan and Cahuc, 2010) suggests that wealthier societies are, on average, more willing to trust others – and vice versa. The social trust variable $z$ may thus be related to $A$ according to some increasing function. If we assume, for example,

$$z = 1 - \frac{\nu}{A}, \quad A(0) > \nu,$$

then it is implied that $z \to 1$ as $A \to \infty$ with time. Hence, the initial lack of social trust is only a temporary obstacle in economic convergence, one which gets less and less severe in the course of economic development and disappears in the long run. This means that in the current case, economic growth serves to alleviate the problem of low social trust. There is no multiplicity of equilibria: both the long-run equilibrium is uniquely given (for $z = 1$), and the approach path towards it.

It is important whether $z$ can take any value in the interval $(0, 1)$, as in the aforementioned example, or is bounded from above by some $\bar{z} < 1$. In the latter case, economic growth helps alleviate some of the negative effect of insufficient bridging social capital and trust, but not all of it, so a distrustful society still cannot fully converge in terms of economic development to a trusting one.

It must be therefore emphasized that for low social trust to constitute a long-term obstacle to economic convergence, it must not depend on the level of economic development in a monotonic and unbounded way. Only in such case it is possible for the
vicious circle of low bridging social capital and low social trust to work forever; otherwise, its workings will be counteracted, and eventually alleviated, by the increasing level of economic development.

3.10 Introducing heterogeneity: bridging social capital and employment

One further caveat with the model in the form developed in previous subsections is that it ignores the possibility that the population might be stratified according to some socio-economic dimension, generating differences in social trust, earnings potential, ability to form and maintain social ties with dissimilar others, etc. A prime example of such a dimension is employment status: some people are working, some are not.\footnote{Yet, there are also other dimensions of heterogeneity which may be relevant in this respect. Just to name a few: race and ethnicity (e.g., Alesina and la Ferrara, 2000), strength of family ties (e.g., Alesina and Giuliano, 2010), human capital and occupational category (e.g., Glaeser, Laibson and Sacerdote, 2002), sex, size of town of residence, etc.}

The optimization problems of the employed and the non-employed can easily be modelled differently from one another, though. First, the type of productive work which the non-employed might consider performing – home production – is typically less productive than wage work. Second, at the social margin, home production does not provide access to such social networks as employment does, and thus the accumulation of bridging social capital should be hampered in the case of the non-employed. Third, the spillover from bridging social capital to individuals’ earnings should be hampered as well: knowing people does not improve the productivity of home production but it might improve the earnings from paid work due to the benefits of cooperation, facilitated information flow, etc. (cf. Podolny and Baron, 1997; Durlauf and Fafchamps, 2005; Burt, 2005).

In consequence, analyzing the differences between the cases of the employed and the non-employed enables us to draw important conclusions on the interaction between the mechanism discussed throughout this paper and the employment rate in an economy. A corollary will be that, just like we argue in Growiec and Growiec (2011), one possible way to eradicate the low bridging social capital–low trust trap is
to provide strong enough increases in labor market participation. This could expose significantly more people to interactions with strangers and engage them in social learning with the ultimate lesson being that the non-kin could be trusted too, and that it is good to meet socially with people dissimilar to ourselves (Li, Pickles and Savage, 2005; Glanville and Paxton, 2007).

Our approach to capturing differences between the employed and the non-employed is the following. We shall assume that each employed person faces the optimization problem described in (5), and hence, provided that their level of trust \( z > z^{\text{piv}} \), their investment rate and stock of bridging social capital will converge to the unique interior steady state \((\ell^*_v, v^*)\). Non-employed persons, on the other hand, face a slightly modified optimization problem. Since they cannot consider colleagues from work as their potential new social ties, the pool of people with whom they can socialize is restricted. We assume that in their case, it is not \( zn \) but \( \chi zn \), with \( \chi \in (0,1) \). Analogously, we assume that their earnings from home production are positive but lower than those attainable in the market sector. We impose that their earnings are equal to:

\[
w_U = \kappa A^v (1 - \ell_v), \quad \kappa \in (0,1).
\] (22)

Hence, there are two differences between the earnings obtained from home production and from wage work: (i) the former ones are lowered by a fixed factor \( \kappa < 1 \), and (ii) in that case, there is no positive spillover from bridging social capital to earnings \( \phi \).

Under the conditions \( \delta - \xi \chi zn < \rho \) and

\[
p^{\text{piv}}_{v,U} = \left( \frac{\delta}{\xi \chi zn} \right)^{1/\mu} < \frac{\delta \mu \theta}{\rho + \delta \mu \theta},
\] (23)

guaranteeing that a unique interior steady state exists, the results are as follows.

**Proposition 8** Employed persons have an unambiguously higher steady-state level of bridging social capital than non-employed ones: \( v^* > v^*_U \). The amounts of time spent on social capital accumulation by the employed and non-employed cannot be unambiguously ordered. They are higher for the employed \((\ell^*_v > \ell^*_{v,U})\) if and only if the spillover from bridging social capital and wages of the employed is strong enough:

\[
\phi > \frac{\xi zn (\ell^*_v)^{\mu+1}}{\delta \mu (1 - \ell^*_v)} (1 - \chi).
\] (24)

As a corollary from this proposition, we note that if \( \chi = 1 \), i.e., the pool of potential acquaintances is equally large for both considered groups, then the non-employed
Figure 5: The dependence of the steady state \((\ell^*_v, v^*)\) on social trust \(z\): comparison of the situation of the employed and the non-employed.

Source: own computations.
Notes: parameters used to produce this figure: \(\mu = 0.6, n = 1, \delta = 0.06, \xi = 0.5, \phi = 0.25, \theta = 0.3, \gamma = 0.4, \rho = 0.1, \chi = 0.8\). The implied critical value of social trust \(z\) above which an interior steady state exists satisfies \(z^{\text{piv}} = 0.26639\). For the non-employed, it is \(z^{\text{piv}}_U = 0.3798\). The relationships presented in this figure are in agreement with Proposition 3. In the long-run equilibrium, the non-employed spend less hours socializing than the employed do, irrespective of the level of social trust \(z\) in the society. They also possess less bridging social capital. Their earnings in equilibrium are less than those of the employed only if inequality (26) holds, which implies that \(\kappa < k^{\text{piv}}(z)\). In the numerical example, \(k^{\text{piv}}(z)\) is an increasing function.

devote unambiguously less time to social capital formation than the employed \((\ell^*_v > \ell^*_v,U)\); and if \(\phi = 0\), i.e., there is no positive impact of a larger stock of bridging social capital on individuals’ earnings, then the relationship is reversed \((\ell^*_v < \ell^*_v,U)\). This is because in each of those two cases, one of the two counteracting channels – lowered ability to form social ties, or the impossibility to improve earnings via social ties – is shut down. If both these mechanisms are at work, then the net result depends on their relative strength, captured by \(\chi\) and \(\phi\), respectively. A numerical example (see
Figure 5) shows that the non-employed will generally spend less time socializing with acquaintances in equilibrium even if $\chi$ is reasonably large.

It can also be easily shown that equilibrium earnings of the non-employed, $w^*_U$ and their subjective well-being depend positively on social trust $z$. The proof is analogous the ones presented for the case of the employed.

Let us now pass to the question of determination of the level of total output in the society and the aggregate level of well-being. We will now condition these two aggregate statistics on (i) the share of the employed in the society, and (ii) the societal level of social trust.

Under the assumption that goods produced in the market sector and within home production are perfect substitutes, output per worker can be computed as

$$y = A^\sigma \left[ \varepsilon (1 - \ell^*_v) (v^*)^\phi + (1 - \varepsilon) \kappa (1 - \ell^*_v, U) \right]$$  \hspace{1cm} (25)

where $\varepsilon \in [0, 1]$ captures the share of the employed in total population.

As both $w^*$ and $w^*_U$ increase with social trust $z$, we conclude that $y$ increases with $z$ as well: more social trust implies a higher level of output in the economy, irrespective of the employment rate. The same result carries forward directly to aggregate well-being, under the (empirically falsified) assumption that $H$ is constant across individuals.

Let us now proceed to the analysis of the impact of the labor participation rate on total output and well-being in the economy. We have:

$$\frac{\partial y}{\partial \varepsilon} = A^\sigma \left( (1 - \ell^*_v) (v^*)^\phi - \kappa (1 - \ell^*_v, U) \right) > 0$$  \hspace{1cm} (26)

$$\Leftrightarrow \kappa < \kappa^{\text{pivot}}(z) \equiv \left( \frac{1 - \ell^*_v(z)}{1 - \ell^*_v, U(z)} \right) (v^*(z))^\phi.$$

and hence, total output increases with employment rate if:

- the parameter $\kappa$ is small enough, i.e., the relative productivity of the non-employed is low enough,

- the steady-state stock of bridging social capital of the employed $v^*$ is high enough,

- the spillover $\phi$ from bridging social capital to earnings of the employed is weak enough (if $\phi$ satisfies (24), then $\ell^*_v, U > \ell^*_v$ so the left-hand side formula is greater than one, validating the inequality).
It must be noted that the inequality condition (26) is very weak, especially if the
considered population \( n \) is sufficiently large (and thus the steady-state bridging social
capital level \( v^* \) is sufficiently high), so we expect it to be satisfied under all “reasonable”
parametrizations of the model. It is already satisfied in our benchmark numerical
example presented in Figure 5.

Naturally, the proposed dimension of heterogeneity – employed vs. non-employed –
is not the only one which could be incorporated in the above model. This example was
given only because of its important interpretation and empirical relevance. Adding
further dimensions of heterogeneity into the model is a very promising avenue for
further research, for two reasons. Firstly, the more empirical regularities related to
social capital, trust, and economic performance of individuals are accounted for in the
model, the better our understanding of the discussed mechanisms, especially that it is
a well-established empirical fact that these three variables are correlated with, among
others, individuals’ human capital, occupational category, sex, race and ethnicity.

Secondly and even more importantly, heterogeneity in itself is a powerful source for
generating multiple equilibria as well. In the current model, their existence is a con-
sequence of the non-convexity in social capital accumulation as well as the threshold
mechanism between social capital and trust; numerous scholars have shown, however,
that it can also be generated by endogenous stratification mechanisms within het-
erogenous societies (cf. Galor, 1996; Acemoglu, 2009, Ch. 21). One potential idea
would be to allow for some fixed costs, e.g., analogously to Galor and Zeira (1993),
but this time, either in the accumulation of social capital or in the production of
individual trust. Coupled with sufficient individual heterogeneity (for instance in the
initial income distribution), they may also give rise to poverty traps. Another possi-
bility would be to introduce convex saving functions, associated to some imperfections
(Moav, 2002).\(^{19}\)

4 Conclusion

Let us now wrap all above findings together. Most generally, the contribution of
the current paper to the literature has been to put forward a model formalizing a
novel mechanism where bridging social capital and social trust feed back on each
other, creating either a “vicious” or a “virtuous” circle depending on initial conditions,
capable of generating poverty traps and multiple equilibria in economic development

\(^{19}\)We are grateful to the anonymous Referee for pointing out these interesting possibilities.
thanks to a non-convexity in the process of social capital formation.

More specifically, we have argued that:

- whether there will be multiple equilibria or not, depends on the underlying initial level of social trust;

- low enough initial levels of social trust push the economy towards a poverty trap;

- steady-state levels of bridging social capital, trust, earnings, and subjective well-being are positively related, but the steady-state amount of time devoted to social capital accumulation decreases with social trust;

- the out-of-equilibrium relationship between bridging social capital and earnings (or subjective well-being) is inverse U-shaped;

- if the initial level of social trust is high enough, individuals who start off with low levels of bridging social capital will initially invest more time in social capital creation that they will do in the long run, and vice versa;

- if the initial level of social trust is low enough, then the model implies a gradual decay of social ties, whose stock will tend to zero over the long run;

- there might also be multiple interior equilibria if one incorporates a bi-directional feedback effect between social capital and social trust in the model;

- in a growing economy, low social trust constitutes a persistent obstacle to economic convergence only if it does not depend on the level of economic development; otherwise, its workings will be counteracted, and eventually alleviated, by the increasing level of economic development;

- employed persons should have unambiguously higher steady-state levels of bridging social capital than the non-employed individuals, with all its impacts on social trust, earnings, and subjective well-being.

What remains to be done is to carry out quantitative empirical tests of the numerous predictions of the model. Admittedly, its workings are in agreement with a wide array of contributions in sociology and social psychology, but their “goodness of fit” has been evaluated qualitatively rather than quantitatively. One step in this direction has been taken in our companion paper, Growiec and Growiec (2011), where
we find corroborating evidence for the hypothesized mechanism in individual-level (World Values Survey) cross-sectional data for Central and Eastern European countries. The literature is still in need for a wider coverage of countries and the use of panel datasets, though.

References


A Appendix

A.1 Proofs of Propositions

Proof of Proposition 1. The function $\varphi$ is continuous for all $\ell_v \in [0, 1]$. Furthermore, we have:

$$\varphi(0) = -\frac{\delta \mu}{\gamma} (\gamma \phi + \theta) < 0$$
$$\varphi(1) = \xi z n + \rho - \delta > 0$$

where the last inequality holds by assumption. From the Darboux property of continuous functions, there must exist at least one solution $\ell_v^* \in (0, 1)$ such that $\varphi(\ell_v^*) = 0$. The second derivative of $\varphi$ is

$$\varphi''(\ell_v) = \xi z n (1 + \mu) \mu \ell_v^{\mu-1} > 0,$$

indicating that $\varphi$ is strictly convex and thus the solution $\ell_v^*$ is unique. □

Proof of Proposition 2. From (10), it can be easily seen that the condition $\ell_v^* > \ell_{pv}$ is equivalent to the request that $\varphi(\ell_{pv}) < 0$. Inserting the appropriate formula into (10) and rearranging leads to (11). □

Proof of Proposition 3. To compute $\left. \frac{\partial \ell_v}{\partial z} \right|_{\ell_v=\ell_v^*}$, we shall apply the implicit function theorem to equation (10), treating $\varphi$ as a function of both $\ell_v$ and $z$. We obtain:

$$\left. \frac{\partial \ell_v}{\partial z} \right|_{\ell_v=\ell_v^*} = -\left. \frac{\partial \varphi}{\partial z} \right|_{\ell_v=\ell_v^*} = \frac{\xi n \ell_v^{\mu+1}}{\xi z n \mu \ell_v^\mu + \frac{\delta \mu}{\gamma \ell_v} (\gamma \phi + \theta)} < 0.$$ (27)

Furthermore, applying the chain rule to the steady state relationship $v^* = zn - \frac{\delta}{\xi (\ell_v^*)^\mu}$, we obtain:

$$\left. \frac{\partial v}{\partial z} \right|_{v=v^*} = n + \frac{\delta \mu}{\xi (\ell_v^*)^\mu+1} \left. \frac{\partial \ell_v}{\partial z} \right|_{\ell_v=\ell_v^*} = n \left( \frac{(\xi n \ell_v^\mu - \delta) + \frac{\delta (\gamma \phi + \theta)}{\gamma \ell_v}}{\xi z n \ell_v^\mu + \frac{\delta (\gamma \phi + \theta)}{\gamma \ell_v}} \right) > 0.$$ (28)

Proof of Proposition 4. Using the results presented in the proof of proposition (3), it suffices to show that

$$\left. \frac{\partial \varphi}{\partial \phi} \right|_{\ell_v=\ell_v^*} = \delta \mu (\ell_v^* - 1) < 0$$

to imply that

$$\left. \frac{\partial \ell_v}{\partial \phi} \right|_{\ell_v=\ell_v^*} > 0$$
and
\[ \frac{\partial v}{\partial \phi} \Big|_{v=v^*} = \frac{\delta \mu}{\xi (\ell^* v)^{\mu+1}} \frac{\partial \ell}{\partial \phi} \Big|_{\ell=\ell^*} > 0. \]

**Proof of Proposition 5.** Recall that \( w = n A^\sigma (1 - \ell_v) v^\phi \). Using the steady-state relationship \( v^* = zn - \frac{\delta}{\xi (\ell_v)^{\mu}} \), in the vicinity of the steady state it holds that
\[ w \approx \omega(\ell_v) \equiv n A^\sigma (1 - \ell_v) \left( zn - \frac{\delta}{\xi (\ell_v)^{\mu}} \right)^\phi. \]

To prove that, for a fixed value of \( z \), the function \( \omega(\ell_v) \) is inverse U-shaped, note that
\[ \frac{\partial \omega}{\partial \ell_v} = n A^\sigma (1 - \ell_v) \left( zn - \frac{\delta}{\xi (\ell_v)^{\mu}} \right)^\phi \left( \frac{1}{1 - \ell_v} + \frac{\phi \mu \delta}{\xi (\ell_v)^{\mu+1}} \right). \]
From simple algebra it follows that the term inside the last brackets, and hence the whole derivative, is positive if and only if
\[ \varphi_w(\ell_v) = \xi zn \ell_v^{\mu+1} + (\phi \mu - 1) \delta \ell_v - \phi \mu \delta < 0, \]
and negative if the sign is reversed.

We note that \( \varphi_w(0) = -\phi \mu \delta < 0 \), and \( \varphi_w(1) = \xi zn - \delta > 0 \). The last inequality follows from the condition \( \ell^*_v > \ell^*_{\ell v} \), guaranteeing existence of an interior equilibrium. Furthermore,
\[ \varphi''_w(\ell_v) = (\mu + 1) \mu \xi zn \ell_v^{\mu-1} > 0, \quad \forall (\ell_v \in (0, 1]). \]
Hence, the continuous function \( \varphi_w \) intersects zero exactly once in the interval \((0, 1]\), from below, at a point which we denote \( \ell^*_v \). It follows \( \omega(\ell_v) \) is increasing for all \( \ell_v < \ell^*_v \), and decreasing for all \( \ell_v > \ell^*_v \). Hence, \( \omega \) is inverse U-shaped.

Let us now compare \( \ell^*_v \) with \( \ell^*\). Using (10), we have
\[ \ell^*_v < \ell^*_v \iff \varphi_w(\ell^*_v) < 0 \iff \frac{\theta}{\gamma} \mu \delta (1 - \ell^*_v) - \rho \ell^*_v < 0, \]
and hence, if \( \ell^*_v > \frac{\delta \mu \theta}{\rho \gamma + \delta \mu \theta} \), or equivalently if \( \varphi \left( \frac{\delta \mu \theta}{\rho \gamma + \delta \mu \theta} \right) < 0 \), with \( \varphi \) defined as in (10).

**Proof of Proposition 6.** Recall that \( SWB = HA^{\gamma \sigma} (1 - \ell_v)^{\gamma \phi + \theta} \). Using the steady-state relationship \( v^* = zn - \frac{\delta}{\xi (\ell_v)^{\mu}} \), in the vicinity of the steady state it holds that
\[ SWB \approx \Omega(\ell_v) \equiv HA^{\gamma \sigma} (1 - \ell_v)^{\gamma \left( zn - \frac{\delta}{\xi (\ell_v)^{\mu}} \right)^{\gamma \phi + \theta}}. \]
To prove that, for a fixed value of \(z\), the function \(\Omega(\ell_v)\) is inverse U-shaped, note that

\[
\frac{\partial \Omega}{\partial \ell_v} = HA^{\sigma}(1 - \ell_v)^\gamma \left( zn - \frac{\delta}{\xi \ell_v^m} \right)^{\gamma\phi+\theta} \left( -\frac{\gamma}{1 - \ell_v} + \frac{(\gamma\phi+\theta)\mu\delta}{zn - \frac{\delta}{\xi \ell_v^m}} \right).
\]

From simple algebra it follows that the term inside the last brackets, and hence the whole derivative, is positive if and only if

\[
\varphi_s(\ell_v) = \xi zn \ell_v^m + \left( \frac{\gamma\phi+\theta}{\gamma} - 1 \right) \delta \ell_v - \frac{\gamma\phi+\theta}{\gamma} \mu \delta < 0,
\]

and negative if the sign is reversed.

We note that \(\varphi_s(0) = -\frac{\gamma\phi+\theta}{\gamma} \mu \delta < 0\), and \(\varphi_s(1) = \xi zn - \delta > 0\). The last inequality follows from the condition \(\ell_v^* > \ell_v^\text{max}\), guaranteeing existence of an interior equilibrium. Furthermore,

\[
\varphi_s''(\ell_v) = (\mu + 1)\mu \xi zn \ell_v^{m-1} > 0, \quad \forall (\ell_v \in (0, 1]).
\]

Hence, the continuous function \(\varphi_s\) intersects zero exactly once in the interval \((0, 1]\), from below, at a point which we denote \(\ell_v^\text{max}\). It follows that \(\Omega(\ell_v)\) is increasing for all \(\ell_v < \ell_v^\text{max}\), and decreasing for all \(\ell_v > \ell_v^\text{max}\). Hence, \(\Omega\) is inverse U-shaped.

Let us now compare \(\ell_v^\text{max}\) with \(\ell_v^*\). Using (10), we have

\[
\ell_v^* < \ell_v^\text{max} \iff \varphi_s(\ell_v^*) < 0 \iff -\rho \ell_v^* < 0,
\]

which is trivially satisfied. Hence, it is always the case that \(\ell_v^* < \ell_v^\text{max}\). \(\blacksquare\)

**Proof of Proposition 7.** From (10) it is derived that \(\frac{\partial \varphi}{\partial \rho}\bigg|_{\ell_v=\ell_v^*} = \ell_v^* > 0\) and hence, because we also know that \(\frac{\partial \varphi}{\partial \ell_v}\bigg|_{\ell_v=\ell_v^*} > 0\), it follows that \(\frac{\partial \ell_v}{\partial \rho}\bigg|_{\ell_v=\ell_v^*} < 0\).

As \(\rho\) has no direct impact on \(v^*\) apart from the one through \(\ell_v^*\), from the chain rule it follows that

\[
\frac{\partial v}{\partial \rho}\bigg|_{v=v^*} = \frac{\partial v}{\partial \ell_v}\bigg|_{v=v^*} \frac{\partial \ell_v}{\partial \rho}\bigg|_{\ell_v=\ell_v^*} = \frac{\delta \mu}{\xi \ell_v^{m+1}} \frac{\partial \ell_v}{\partial \rho}\bigg|_{\ell_v=\ell_v^*} < 0. \quad (29)
\]

**Proof of Proposition 8.** For the non-employed, the steady state is computed as an implicit solution to the following system of two equations:

\[
\varphi_U(\ell_v) = \xi \chi zn \ell_v^m + \left( \frac{\delta \mu}{\gamma} + \rho - \delta \right) \ell_v - \frac{\delta \mu}{\gamma} \theta = 0, \quad (30)
\]

\[
v = \chi zn - \frac{\delta}{\xi \ell_v^m}. \quad (31)
\]
Because we know that \( v^* \) depends positively both on \( z \) and \( \phi \), and both these parameters are lower for the non-employed than for the employed (\( z \) is replaced with \( \chi z \), and \( \phi \) is replaced with 0), it follows that \( v^*_U \) is unambiguously lower than \( v^* \).

Turning to the issue of \( \ell^*_v, v, U \), we note that it must satisfy the condition \( \varphi(v, U) = 0 \) whereas \( \ell^*_v \) satisfies \( \varphi(v) = 0 \). Both these functions are increasing at their respective zeros, and hence it suffices to analyze the sign of \( \varphi(v) - \varphi_U(v) \). We find:

\[
\varphi(v) - \varphi_U(v) = \xi z n \ell^{\mu+1}_v (1 - \chi) - \phi \delta \mu (1 - \ell_v). \tag{32}
\]

To find the conditions under which \( \ell^*_v > \ell^*_v, U \), we ought then to find the conditions for which the above difference is positive at \( \ell^*_v \) (steady-state value for the employed), i.e., the conditions for which \( \varphi_U(v) < 0 \). Simple algebra completes the proof.

### A.2 Second order conditions

The current appendix justifies that the Mangasarian second order condition holds for our model (cf. Chiang, 1992) and thus it correctly describes a maximum of the Hamiltonian 5, if only a certain parametric condition is met.

Differentiating the maximand function \( F \)

\[
F = H A^{\gamma} (1 - \ell) v^{\gamma \phi + \theta} e^{-\rho t}
\]

(33)
twice with respect to \( \ell_v \) and \( v \), we obtain:

\[
\frac{\partial^2 F}{\partial \ell_v^2} = \gamma H A^{\gamma} v^{\gamma \phi + \theta} e^{-\rho t} (\gamma - 1)(1 - \ell_v)^{\gamma - 2},
\]

(34)

\[
\frac{\partial^2 F}{\partial \ell_v \partial v} = -\gamma (\gamma \phi + \theta) H A^{\gamma} v^{\gamma \phi + \theta - 1} e^{-\rho t} (1 - \ell_v)^{\gamma - 1},
\]

(35)

\[
\frac{\partial^2 F}{\partial v^2} = (\gamma \phi + \theta)(\gamma \phi + \theta - 1) H A^{\gamma} v^{\gamma \phi + \theta - 2} e^{-\rho t} (1 - \ell_v)^{\gamma}.
\]

(36)

It is automatically verified that \( \frac{\partial^2 F}{\partial \ell^2} < 0 \) and \( \frac{\partial^2 F}{\partial v^2} < 0 \). Some more algebra is necessary to ensure that the determinant of the Hessian is positive, and thus the matrix is negative definite, if

\[
\gamma + \gamma \phi + \theta < 1.
\]

(37)

If the second order condition (37) holds, then \( F \) is concave with respect to both variables jointly.

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\(^{20}\)The impact of \( \kappa \) on \( v^*_U \) is nil.
Differentiating the constraint function $G$ given as

$$G = \xi \ell_v^n v(zn - v) - \delta v$$  \hspace{1cm} (38)

twice with respect to $\ell_v$ and $v$, we obtain:

$$\frac{\partial^2 G}{\partial \ell_v^2} = \mu (\mu - 1) \xi \ell_v^{\mu - 2} v(zn - v),$$  \hspace{1cm} (39)

$$\frac{\partial^2 G}{\partial \ell_v \partial v} = \xi \mu \ell_v^{\mu - 1} (zn - 2v),$$  \hspace{1cm} (40)

$$\frac{\partial^2 G}{\partial v^2} = -2 \xi \ell_v^\mu.$$  \hspace{1cm} (41)

It is automatically verified that $\frac{\partial^2 G}{\partial \ell_v^2} < 0$ and $\frac{\partial^2 G}{\partial v^2} < 0$. Some more algebra is necessary to ensure that the determinant of the Hessian is positive, and thus the matrix is negative definite, if

$$\frac{\mu}{2} (zn)^2 < (1 + \mu) v^*(zn - v^*).$$  \hspace{1cm} (42)

If both (37) and (42) hold simultaneously, then by Mangasarian’s theorem, the described time path of $(\ell_v, v)$ describes a maximum of the Hamiltonian.