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Abstract

We analyze a relation between interest rate controls and equilibrium determinacy using a two-country model featuring traded and non-traded goods. In addition, parameters of preference and production may differ between the two countries. We find that macroeconomic stability strongly depends on such heterogeneity including monetary policy, and that it is easier to generate determinate equilibrium under liberalization of the economy.

Keywords and Phrases: heterogeneity, Taylor rule, open economy, non-traded goods, equilibrium determinacy.

JEL Classification Numbers: E52, F41.

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1 Introduction

In this paper, we analyze equilibrium determinacy of a two-country model with traded and non-traded goods in which the monetary authority in each country may adopt different interest rate control rules, and the countries can have asymmetric production technologies and preferences.

There are a considerable number of studies concerning the stabilization effect of interest rate control rules in open economy settings that utilize small country models. For example, Chang, Chen, Lai, and Shaw (2008) examine an AK growth economy with a generalized Taylor rule in which the central bank controls nominal interest rate in response not only to inflation but also to the growth rate of income. [1] They show that the number of equilibrium paths is less than one, that is, equilibrium is determinate or source. [2] Carlstrom and Fuerst (1999), Kam (2004 and 2007), and Zanna (2003 and 2004) examine small-open economy models with Taylor-type monetary policy under sticky prices.

The role of interest rate controls in a world economy model with two-countries also has been extensively discussed in the literature. The New Keynesian models in Batini, Levine and Pearlman (2004); Benigno and Benigno (2006); Bullard and Schalling (2009); De Fiore and Liu (2005); and Airaudo and Zanna (2012a) are based on Clarida, Galí, and Gertler (2002). In these models with sticky price and monopolistic competition, preferences and production parameters are assumed to be identical in both countries, and the results are not analytically clear.

Moreover, non-traded goods are often ignored in open economy models. This is

[1] Such a monetary policy rule is also formalized in Fujisaki and Mino (2007).

[2] However, we should note that they assume sticky nominal interest rate.

because the law of one price is plausible only for traded goods and thus the non-arbitrage condition is described simply as the equivalence of real interest rates. For instance, Ono (2006) considers a two-country economy in which all goods are tradable, the utilities of consumption and money are additively separable, production is linear in labor and involuntary unemployment can emerge. He focuses on monetary policy such that the growth rate of real money balances equals to the deflation rate, that is, nominal money holdings are constant. Fujisaki (2012) revises his model by using an interest-rate control rule, and the utility of money need not be additively separable from consumption. The results mainly depend on the heterogeneity of interest rate controls and preferences, whereas productivity plays a limited role.

In order to check the robustness of the result and to obtain its implications for the openness of the economy, we construct a two-country version of the model in Airaudo and Zanna (2012b). They investigate small-country models with Taylor-type monetary policy [3] and they distinguish non-traded goods from tradable ones. If a continuous-time setting is used in their models, we only reconfirm the well-known results established in closed economy models: Taylor principle holds, which means that interest-rate control with an aggressive response to the rate of inflation generates equilibrium determinacy. They utilize discrete-time models for investigating the effect of timings of monetary dynamics [4] on equilibrium determinacy. In order to focus on heterogeneity between two countries, we assume a continuous-time model. That is, each country responds independently to its own inflation rate using

[3]A liquidity trap in which nominal interest rates cannot be negative is considered in Airaudo and Zanna (2004).

[4]For instance, monetary authority controls the current nominal interest rate in response to either the contemporaneous or forward-looking inflation rate.

interest rate controls, and parameters such as elasticities of labor used in production and intertemporal substitution may differ between the two countries. We suppose that production functions can be non-linear in labor by assuming a fixed productive factor as in Carlstrom and Fuerst (1999).

Using the Keynesian model with capital, McKnight (2011a) shows that real indeterminacy is considerably easier to obtain once trade liberalization is permitted. However, as his other papers McKnight (2011b) and McKnight and Mihailov (2007), they generally conclude that Taylor principle tends to hold regardless of the openness of the economy, and parameters about preferences and production in these models are assumed to be the same in both countries.

We show that heterogeneity has a significant effect on equilibrium determinacy and thus an appropriate combination of monetary policy is necessary to stabilize the world economic system. This does not necessarily mean that central banks in both countries should aggressively control nominal interest rates in response to inflation. Rather, passive monetary policy in one country may play a role for realizing the stable economy. Such results are similar to those in Fujisaki (2012), who assumes two kinds of tradable goods. In this paper, we consider the effect of non-traded goods that violates the law of one price. Then, the non-arbitrage condition may not imply the equivalence of real interest rates, and thus it becomes difficult to hold both the non-arbitrage condition and traded-goods equilibrium, which can be a source of indeterminacy. When non-traded goods do not exist, we can hardly assess macroeconomic stability only by preference and monetary policy, so that the heterogeneity of productivity becomes more significant. Liberalization might be effective for macroeconomic stability in that indeterminate equilibrium can be determinate by being all goods tradable.

2 The Model

2.1 Households in Country 1

We assume that there are two countries, Country 1 and Country 2, in the world economy. They produce and consume tradable and non-traded goods. Additionally, the structure of the economy in the two countries is similar, although they differ in the values of some parameters for preferences, production, and monetary policy. Our purpose is to investigate the effect of such heterogeneity on equilibrium determinacy.

We examine the structure of Country 1's economy. The consumer price index (CPI) p, the CPI-inflation rate π, and the price of traded-goods relative to non-traded goods' $\tilde{P}$ are

$$p \equiv \left(\frac{P^T}{\alpha}\right)^{\alpha}\left(\frac{P^N}{1-\alpha}\right)^{1-\alpha}, \pi = \alpha\pi^T + (1-\alpha)\pi^N, \tag{1}$$

$$\tilde{P} = \frac{P^T}{P^N}, \tag{2}$$

where $\pi^T \equiv \dfrac{\dot{P}^T}{P^T}$ $\left(\text{resp. } \pi^N \equiv \dfrac{\dot{P}^N}{P^N}\right)$ is the inflation rate of the price of traded goods P^T (resp. non-traded goods P^N) expressed in domestic currency, and $\alpha \in (0, 1]$ is the proportion of tradable goods among all commodities consumed in the country.

The production functions of traded and non-traded goods are respectively

$$y^T = (l^T)^{\theta^T}(\mathcal{L}^T)^{1-\theta^T}, \quad y^N = (l^N)^{\theta^N}(\mathcal{L}^N)^{1-\theta^N}, \quad 0 < \theta^N < 1, \quad 0 < \theta^T < 1,$$

where l^T and l^N are labor, and $\mathcal{L}^T$ and $\mathcal{L}^N$ are fixed factors. This formulation follows Airaudo and Zanna (2012b) and Carlstrom and Fuerst (1999). In the following, we suppose that the rent from the fixed factor is distributed to household and that $\mathcal{L}^T = \mathcal{L}^N = 1$. Then, income distribution is described as follows:

$$y^T = w^T l^T + h^T, \quad y^N = w^N l^N + h^N,$$

where $w^T = \dfrac{\theta^T y^T}{l^T}$ is a wage of labor and $h^T = (1-\theta^T)y^T$ is the rent from fixed factor for traded goods. (Notation of non-traded goods is similar.)

The budget constraint of representative household in nominal terms is

$$\dot{B} + \dot{M} = RB + P^T(y^T - c^T) + P^N(y^N - c^N),$$

where B denotes bonds, M nominal money holdings, R the nominal interest rate, c^T and c^N (resp. y^T and y^N) consumption (resp. output) of the tradable and non-traded goods. (We assume zero lump-sum taxes.) Employing notation $z \equiv \dfrac{Z}{p}$ that evaluates a nominal variable Z in real terms and $a \equiv b + m$ as real financial assets, we can describe

$$\frac{\dot{B} + \dot{M} - RB}{P^N}\frac{p}{p} = \frac{p}{P^N}(\dot{a} + \pi a - R(a - m)),$$

because

$$\frac{\dot{B} + \dot{M}}{p} = \frac{\dot{A}}{p} = \dot{a} + \pi a.$$

Using

$$\frac{p}{P^N} = \frac{\tilde{P}^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}}$$

from (1) and (2), we obtain the budget constraint in real terms as

$$\dot{a} = (R - \pi)a - Rm + \alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}[\tilde{P}(y^T - c^T) + (y^N - c^N)]. \tag{3}$$

The maximization problem of the representative household in Country 1 is

$$\max \quad \int_0^\infty u(c, m, l^T, l^N)e^{-\rho t}dt, \quad \rho > 0,$$

subject to (3), where ρ is the time discount rate and c is the consumption aggregator given by

$$c = (c^T)^\alpha (c^N)^{1-\alpha}, \quad 0 < \alpha \leq 1. \tag{4}$$

Additionally, the instantaneous utility is specified as

$$u(c, m, l^T, l^N) = \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{1-\sigma} + \psi(1 - l^T - l^N), \quad 0 < \gamma < 1, \quad \sigma > 0, \quad \psi > 0,$$

where σ indicates an inverse of the intertemporal elasticity of substitution (IES hereafter) [5].

Then, the Hamiltonian function for household's optimization is

$$\mathcal{H} = \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{1-\sigma} + \psi(1 - l^T - l^N) + \\ \lambda\{(R-\pi)a - Rm + \alpha^\alpha (1-\alpha)^{1-\alpha} \tilde{P}^{-\alpha}[\tilde{P}(y^T - c^T) + (y^N - c^N)]\},$$

where λ denotes the shadow value of assets. The first-order conditions are

$$\gamma\alpha \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{c^T} = \alpha^\alpha (1-\alpha)^{1-\alpha} \tilde{P}^{1-\alpha} \lambda, \tag{5}$$

$$\gamma(1-\alpha) \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{c^N} = \alpha^\alpha (1-\alpha)^{1-\alpha} \tilde{P}^{-\alpha} \lambda, \tag{6}$$

$$(1-\gamma) \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{m} = \lambda R, \tag{7}$$

$$\alpha^\alpha (1-\alpha)^{1-\alpha} \tilde{P}^{1-\alpha} \lambda \theta^T (l^T)^{-(1-\theta^T)} = \alpha^\alpha (1-\alpha)^{1-\alpha} \tilde{P}^{-\alpha} \lambda \theta^N (l^N)^{-(1-\theta^N)} = \psi, \tag{8}$$

$$\dot{\lambda} = [\rho + \pi - R]\lambda, \tag{9}$$

together with the transversality condition, $\lim_{t\to\infty} e^{-\rho t}\lambda_t a_t = 0$. We can rewrite these conditions in the following simpler manner:

$$\gamma \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{c} = \lambda, \tag{10}$$

[5]We can consider a more general form of disutility from labor in which the intertemporal elasticity of substitution in labor supply is not zero. Nevertheless, this is not essential so long as the marginal disutility is increasing in labor.

$$\frac{\psi}{\gamma\alpha(c^{\gamma}m^{1-\gamma})^{1-\sigma}/c^{T}} = \theta^{T}(l^{T})^{-(1-\theta^{T})}, \tag{11}$$

$$\frac{\psi}{\gamma(1-\alpha)(c^{\gamma}m^{1-\gamma})^{1-\sigma}/c^{N}} = \theta^{N}(l^{N})^{-(1-\theta^{N})}, \tag{12}$$

$$\frac{(1-\gamma)/m}{\gamma/c} = R, \tag{13}$$

$$\tilde{P}\frac{c^{T}}{c^{N}} = \frac{\alpha}{1-\alpha}: \quad c = \left(\frac{1-\alpha}{\alpha}\tilde{P}\right)^{1-\alpha} c^{T} = \left(\frac{1-\alpha}{\alpha}\tilde{P}\right)^{-\alpha} c^{N}, \tag{14}$$

$$\tilde{P} = \frac{\theta^{N}(l^{N})^{-(1-\theta^{N})}}{\theta^{T}(l^{T})^{-(1-\theta^{T})}}. \tag{15}$$

The market equilibrium condition for non-traded goods is

$$y^{N} = c^{N}, \tag{16}$$

while that for traded-goods is

$$y^{T} + y^{T*} = c^{T} + c^{T*}, \tag{17}$$

where y^{T*} and c^{T*} are production and consumption, respectively, of traded goods in Country 2 presented in the next subsection. From (10)-(16) and the fact that y^{T} can deviate from c^{T} since $(y^{T} - c^{T})$ is net export, important variables can be described as functions of R and λ;

$$c = \mathcal{C}(R^{(1-\gamma)(1-\sigma)}\lambda)^{-\frac{1}{\sigma}}, \tag{18}$$

$$m = \frac{1-\gamma}{\gamma}\mathcal{C}R^{-\frac{1-\gamma+\gamma\sigma}{\sigma}}\lambda^{-\frac{1}{\sigma}}, \tag{19}$$

$$y^{N} = c^{N} = (l^{N})^{\theta^{N}} = \mathcal{N}^{\mathcal{N}}(R^{1-\gamma}\lambda)^{-\frac{1-\sigma}{\sigma}\theta^{N}}, \tag{20}$$

$$y^{T} = (l^{T})^{\theta^{T}} = \mathcal{N}^{\mathcal{T}}(R^{\phi_1}\lambda^{\phi_2})^{\frac{\theta^{T}}{1-\theta^{T}}}, \tag{21}$$

$$c^{T} = \left(\frac{c}{(c^{N})^{1-\alpha}}\right)^{\frac{1}{\alpha}} = \mathcal{C}^{\mathcal{T}}R^{\chi_1}\lambda^{\chi_2}, \tag{22}$$

$$\tilde{P} = \mathcal{P}(R^{\nu_1}\lambda)^{\nu_2}, \tag{23}$$

where

$$
\begin{aligned}
\mathcal{C} &\equiv \gamma^{\frac{\sigma(1-\gamma)+\gamma}{\sigma}}(1-\gamma)^{\frac{(1-\gamma)(1-\sigma)}{\sigma}}, \\
\mathcal{N}^{\mathcal{N}} &\equiv \left\{(1-\alpha)\frac{\theta^N}{\psi}\gamma\left(\frac{1-\gamma}{\gamma}\right)^{(1-\gamma)(1-\sigma)}\mathcal{C}^{1-\sigma}\right\}^{\frac{1}{\theta^N}}, \\
\mathcal{N}^{\mathcal{T}} &\equiv \left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\theta^T(\mathcal{P})^{1-\alpha}}{\psi}\right]^{\frac{\theta^T}{1-\theta^T}}, \\
\mathcal{C}^{\mathcal{T}} &\equiv \left[\frac{\mathcal{C}}{\mathcal{N}^{\mathcal{N}}}\right]^{\frac{1}{\alpha}}, \\
\mathcal{P} &\equiv \left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\theta^N(\mathcal{N}^{\mathcal{N}})^{-\frac{1-\theta^N}{\theta^N}}}{\psi}\right]^{\frac{1}{\alpha}},
\end{aligned}
$$

are positive constants and

$$
\phi_1 \equiv \nu_1\nu_2(1-\alpha), \quad \phi_2 \equiv \frac{1-[\theta^N+\alpha(1-\theta^N)](1-\sigma)}{\sigma\alpha} > 0,
$$

$$
\chi_1 \equiv -\frac{(1-\gamma)(1-\sigma)[1-\theta^N(1-\alpha)]}{\sigma\alpha}, \quad \chi_2 \equiv -\frac{1-[\theta^N(1-\alpha)(1-\sigma)]}{\sigma\alpha} < 0,
$$

$$
\nu_1 \equiv \frac{(1-\gamma)(1-\theta^N)(1-\sigma)}{1-\theta^N(1-\sigma)}, \quad \nu_2 \equiv \frac{1-\theta^N(1-\sigma)}{\sigma\alpha} > 0.
$$

Properties of these variables are shown in Table 1. Any type of consumption increases with higher nominal interest rate when $\sigma > 1$, that is, consumption and real money balances are substitutes, since nominal interest rate represents an opportunity cost of holding money. This means higher relative marginal utility of leisure to that of traded goods' consumption so that marginal productivity of labor as the opportunity cost for enjoying leisure should rise. Therefore, the output decreases, in contrast to the production of non-traded goods equivalent to the consumption, and thus the tradable goods' relative price must be lower to satisfy the equation for the marginal values of product (15).

2.2 Households in Country 2

We represent variables and parameters in Country 2 by using asterisks, such as θ^{N*}, θ^{T*} and σ^*. We allow that $\theta^N \neq \theta^{N*}$, $\theta^T \neq \theta^{T*}$ and $\sigma \neq \sigma^*$. Since the structure of the economy is similar to that of Country 1 in that the forms of utility and production are identical, and since we allow the equivalence of parameters ρ, α and ψ between two countries, the Hamiltonian of utility maximization by Country 2's household is

$$\mathcal{H}^* = \frac{((c^*)^{\gamma^*}(m^*)^{1-\gamma^*})^{1-\sigma^*}}{1-\sigma^*} + \psi(1 - l^{T*} - l^{N*}) + \lambda^*\{(R^* - \pi^*)a^* - R^*m^* - \tau^*$$
$$+ \alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P^*}^{-\alpha}[\tilde{P^*}(y^{T*} - c^{T*}) + (y^{N*} - c^{N*})]\}.$$

The CPI expressed in Country 2's own currency is $p^* \equiv \left(\frac{P^{T*}}{\alpha}\right)^\alpha \left(\frac{P^{N*}}{1-\alpha}\right)^{1-\alpha}$ and $\tilde{P^*} = \frac{P^{T*}}{P^{N*}}$ denotes the relative price. Supposing that the quantities of fixed productive factors $\mathcal{L}^{T*} = \mathcal{L}^{N*} = 1$ and that the rent is again distributed to households, we can solve the above Hamiltonian as in the previous subsection. [6] Moreover, non-traded goods' market equilibrium in this country is

$$y^{N*} = c^{N*}$$

so that the reduced forms and properties of variables are the same as those displayed by (18)-(23) and Table 1, except that there are asterisks on variables and some parameters. For example, the relative price in Country 2 is

$$\tilde{P}^* = \mathcal{P}^*((R^*)^{\nu_1^*}\lambda^*)^{\nu_2^*}, \tag{24}$$

where

$$\mathcal{C}^* \equiv \gamma^{\frac{\sigma^*(1-\gamma)+\gamma}{\sigma^*}}(1-\gamma)^{\frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}},$$

[6] The transversality condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_t^* a_t^* = 0$.

$$\mathcal{N}^{\mathcal{N}*} \equiv \left\{(1-\alpha)\frac{\theta^{N*}}{\psi}\gamma\left(\frac{1-\gamma}{\gamma}\right)^{(1-\gamma)(1-\sigma^*)}\mathcal{C}^{*1-\sigma^*}\right\}^{\frac{1}{\theta^{N*}}},$$

$$\mathcal{P}^* \equiv \left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\theta^{N*}(\mathcal{N}^{\mathcal{N}*})^{-\frac{1-\theta^{N*}}{\theta^{N*}}}}{\psi}\right]^{\frac{1}{\alpha}},$$

$$\nu_1^* \equiv \frac{(1-\gamma)(1-\theta^{N*})(1-\sigma^*)}{1-\theta^{N*}(1-\sigma^*)}, \quad \nu_2^* \equiv \frac{1-\theta^{N*}(1-\sigma^*)}{\sigma^*\alpha} > 0.$$

2.3 Monetary Policy and Interest-Rate Condition

The central bank in each country adjusts nominal interest rate in response to the CPI inflation rate in its country: [7]

$$R = R(\pi) = \eta_\pi(\pi - \bar{\pi}) + \bar{R}, \quad \eta_\pi \geq 0,$$

$$R^* = R^*(\pi^*) = \eta_\pi^*(\pi^* - \bar{\pi}^*) + \bar{R}^*, \quad \eta_\pi^* \geq 0,$$

where $\bar{\pi}$ and $\bar{\pi}^*$ are the target rates of inflation in Country 1 and 2, respectively. [8] We rewrite these policy rules in the following manner:

$$\pi = \pi(R), \quad \pi'(R) = \frac{1}{\eta_\pi}, \tag{25}$$

$$\pi^* = \pi^*(R^*), \quad {\pi^*}'(R^*) = \frac{1}{\eta_\pi^*}. \tag{26}$$

Under this formulation, we define active (resp. passive) monetary policy as $\eta_\pi > 1$ or $\eta_\pi^* > 1$ (resp. $\eta_\pi < 1$ or $\eta_\pi^* < 1$), which implies that the real interest rate is higher (resp. lower) with inflation. We assume heterogeneity in the response of interest

[7] There are several types of Taylor rule that respond not only to inflation but also to an output gap as in Taylor's (1993) original idea or depreciation rate of currency as in Ball (1998). We do not introduce this expansion into a framework of this paper, as it cannot be beneficial except that it renders the model analytically difficult and thus greater variation in results may be expected.

[8] Around the steady state, $\rho = \bar{R} - \bar{\pi} = \bar{R}^* - \bar{\pi}^*$ hold from equation (9) and $\dot{\lambda^*} = [\rho + \pi^* - R^*]\lambda^*$.

rate controls to inflation as well as in production and preferences, that is, we allow $\eta_\pi \neq \eta_\pi^*$.

The interest-parity condition is

$$R = \epsilon + R^*, \tag{27}$$

where $\epsilon \equiv \frac{\dot{\varepsilon}}{\varepsilon}$ is a depreciation rate of the nominal exchange rate ε. The law of one price holds for traded goods so that

$$\varepsilon = \frac{P^T}{P^{T*}}, \quad \pi^T = \epsilon + \pi^{T*}. \tag{28}$$

For example, the prices of traded goods are P^T yen in Japan and P^{T*} dollars in the United States if the exchange rate is \$1 =¥ε.

From (1), $\pi^* = \alpha\pi^{T*} + (1-\alpha)\pi^{N*}$, and (23)$-$(28), we obtain

$$r(R) = r(R^*) + (1-\alpha)\left[\nu_1\nu_2\frac{\dot{R}}{R} - \nu_1^*\nu_2^*\frac{\dot{R}^*}{R^*} + \nu_2\frac{\dot{\lambda}}{\lambda} - \nu_2^*\frac{\dot{\lambda}^*}{\lambda^*}\right], \tag{29}$$

where $r(R) = R - \pi(R)$ and $r^*(R^*) = R^* - \pi^*(R^*)$. We can interpret this equation as a non-arbitrage condition with non-traded goods. A higher real interest rate improves the capital account and thus trade balance as current account should worsen in order to balance total international payments. This is realized when the relative price of traded goods to non-traded as a benefit of trade declines. That is, equation (29) shows the equivalence of the effective rate of return from international payment. If all goods are tradable ($\alpha = 1$), we do not have to consider the difference between non-traded and tradable goods presented by

$$(1-\alpha)\left[\nu_1\nu_2\frac{\dot{R}}{R} - \nu_1^*\nu_2^*\frac{\dot{R}^*}{R^*} + \nu_2\frac{\dot{\lambda}}{\lambda} - \nu_2^*\frac{\dot{\lambda}^*}{\lambda^*}\right],$$

and thus

$$r(R) = r^*(R^*) \tag{30}$$

is a non-arbitrage condition.

3 Equilibrium Determinacy

Using equation (17), (21), (22), and their corresponding equations for Country 2, we obtain

$$\mathcal{Y}_R \frac{\dot{R}}{R} + \mathcal{Y}_\lambda \frac{\dot{\lambda}}{\lambda} = -\mathcal{Y}_R^* \frac{\dot{R}^*}{R^*} - \mathcal{Y}_\lambda^* \frac{\dot{\lambda}^*}{\lambda^*}, \tag{31}$$

where

$$\mathcal{Y}_R \equiv \frac{\theta^T}{1-\theta^T}\phi_1 y^T - \chi_1 c^T, \quad \mathcal{Y}_\lambda \equiv \frac{\theta^T}{1-\theta^T}\phi_2 y^T - \chi_2 c^T > 0,$$
$$\mathcal{Y}_R^* \equiv \frac{\theta^{T*}}{1-\theta^{T*}}\phi_1^* y^{T*} - \chi_1^* c^{T*}, \quad \mathcal{Y}_\lambda^* \equiv \frac{\theta^{T*}}{1-\theta^{T*}}\phi_2^* y^{T*} - \chi_2^* c^{T*} > 0,$$
$$\mathrm{sign}[\mathcal{Y}_R] = \mathrm{sign}[1-\sigma], \quad \mathrm{sign}[\mathcal{Y}_R^*] = \mathrm{sign}[1-\sigma^*].$$

Under the case where $\sigma = \sigma^* = 1$, the followings hold:

$$\nu_1 = \nu_1^* = \phi_1 = \phi_1^* = \chi_1 = \chi_1^* = 0,$$

$$\nu_2 = \nu_2^* = \phi_2 = \phi_2^* = -\chi_2 = -\chi_2^* = \frac{1}{\alpha} > 0.$$

Therefore, $r = r^*$ holds from equation (29) and $\frac{\lambda}{\lambda^*}$ is a constant. Additionally, the traded-goods equilibrium shows the relation between λ and λ^* whether all goods are tradable or not. Therefore, λ is uniquely determined regardless of policy stance and openness of the economy. In the following, we focus on the case except that $\sigma = \sigma^* = 1$. [9]

[9]We can also analyze the situation where either $\sigma = 1$ or $\sigma^* = 1$ holds by using discussion below.

3.1 Tradable Goods Only

The case where all goods are tradable ($\alpha = 1$) is a special situation in that the real interest rates are equivalent as in equation (30), since the benefit from the expensive non-traded goods becomes zero. Then, $\dfrac{\lambda}{\lambda^*}$ is a constant and the following holds:

$$R^* = R^*(R), \quad R^{*'}(R) = \frac{\eta_\pi^*(\eta_\pi - 1)}{\eta_\pi(\eta_\pi^* - 1)}, \quad \frac{\dot{R}^*}{R^*} = \frac{R^{*'}(R)R}{R^*(R)}\frac{\dot{R}}{R}, \tag{32}$$
$$\epsilon = \epsilon(R), \quad \epsilon^{'}(R) = 1 - R^{*'}(R) = \frac{\eta_\pi^* - \eta_\pi}{\eta_\pi(\eta_\pi^* - 1)}.$$

When $\dfrac{\eta_\pi - 1}{\eta_\pi^* - 1} < 0$, R and R^* are negatively correlated, and the currency in Country 1 appreciates to yield a lower nominal interest rate (i.e., inflation rate) in Country 1. If the central bank in Country 1 adopts passive monetary policy, real interest rates in both countries become higher from the non-arbitrage condition when the inflation in Country 1 decreases. Under active interest rate control in Country 2, this is accomplished by increasing the nominal interest rate in Country 2. From the interest-parity condition (27), it results in diminishing ϵ, indicating that currency in Country 1 is appreciated. This mechanism of currency fluctuation suggests that the change in nominal interest rates does not affect the depreciation of exchange rate ϵ if the strength of the response to inflation in each country is equivalent, $\eta_\pi = \eta_\pi^*$.

If $\alpha = 1$, equations (31) and (32) hold, and thus the system equation is summarized as a function of R,

$$\dot{R} = \frac{\dfrac{c(R)}{\sigma} + \dfrac{c^*(R)}{\sigma^*} + \dfrac{\theta^T}{1-\theta^T}y^T(R) + \dfrac{\theta^{T*}}{1-\theta^{T*}}y^{T*}(R)}{c(R)\dfrac{(1-\gamma)(1-\sigma)}{\sigma} + c^*(R)\dfrac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\dfrac{R^{*'}(R)R}{R^*(R)}}[r(R) - \rho]R, \tag{33}$$

since

$$\phi_1 = \phi_1^* = 0, \quad \phi_2 = \phi_2^* = 1,$$

$$\chi_1 = -\frac{(1-\gamma)(1-\sigma)}{\sigma}, \quad \chi_1^* = -\frac{(1-\gamma)(1-\sigma^*)}{\sigma^*},$$

$$\chi_2 = -\frac{1}{\sigma} < 0, \quad \chi_2^* = -\frac{1}{\sigma^*} < 0.$$

We evaluate this equation around the steady state in order to examine local determinacy;

$$\dot{R}_R|_{ss} = \frac{\dfrac{c(\bar{R})}{\sigma} + \dfrac{c^*(\bar{R})}{\sigma^*} + \dfrac{\theta^T}{1-\theta^T} y^T(\bar{R}) + \dfrac{\theta^{T*}}{1-\theta^{T*}} y^{T*}(\bar{R})}{c(\bar{R})\dfrac{(1-\gamma)(1-\sigma)}{\sigma} + c^*(\bar{R})\dfrac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\dfrac{R^{*'}(\bar{R})\bar{R}}{\bar{R}^*}}[1-\pi'(R)]\bar{R}. \quad (34)$$

As shown in the following propositions, the result is essentially the same in the case investigated in Fujisaki (2012) where two kinds of goods are tradable:

Proposition 1 *If the value of IES in a country is 1, another country's central bank can make equilibrium determinate by using a policy rule such that* $(\eta_\pi - 1)(1-\sigma) > 0$ *(* *or* $(\eta_\pi^* - 1)(1-\sigma^*) > 0$*).*

Proposition 2 *When the signs of both* $(\eta_\pi - 1)(1-\sigma)$ *and* $(\eta_\pi^* - 1)(1-\sigma^*)$ *are positive (resp. negative), equilibrium is determinate (resp. indeterminate). If one of these signs is positive and the other negative, either determinacy or indeterminacy may emerge.*

3.2 Including Non-Traded Goods

Next, we examine the generalized case where both traded and non-traded goods exist ($0 < \alpha < 1$) to compare with the fully-open economy ($\alpha = 1$) in the previous subsection. In this case, we can summarize the system equations consisted by two jump variables, R and R^*:

$$\dot{R} = -\frac{R}{\mathcal{Y}_R + \mathcal{Y}_R^* \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}} \left\{ \frac{\mathcal{Y}_R^*}{\nu_1^*\nu_2^*} \left(-\frac{r(R) - r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*)) \right) \right.$$
$$\left. + \mathcal{Y}_\lambda(\rho - r(R)) + \mathcal{Y}_\lambda^*(\rho - r^*(R^*)) \right\}, \quad (35)$$

$$\dot{R}^* = \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\frac{R^*}{R}\dot{R} + \frac{R^*}{\nu_1^*\nu_2^*}\left(-\frac{r(R) - r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*)) \right). \quad (36)$$

Detailed derivation is in Appendix. We linearize these equations around the steady state and equilibrium is locally determinate if both det$J$ and trace$J$ are positive:

$$\dot{x}_t = J\hat{x}_t,$$

where $J = \begin{bmatrix} \dot{R}_R & \dot{R}_{R^*} \\ \dot{R^*}_R & \dot{R^*}_{R^*} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial \dot{R}_t}{\partial R_t}\right|_{ss} & \left.\frac{\partial \dot{R}_t}{\partial R_t^*}\right|_{ss} \\ \left.\frac{\partial \dot{R}_t^*}{\partial R_t}\right|_{ss} & \left.\frac{\partial \dot{R}_t^*}{\partial R_t^*}\right|_{ss} \end{bmatrix}$ and $\hat{x}_t = \begin{bmatrix} R_t - \bar{R} \\ R_t^* - \bar{R}^* \end{bmatrix}$. The characteristic equation is

$$p(\mu) = \mu^2 - A_1\mu + A_0,$$

where $A_1 = trace(J) = \mu_1 + \mu_2 = \dot{R}_R + \dot{R^*}_{R^*}$, and $A_0 = det(J) = \mu_1\mu_2 = \dot{R}_R\dot{R^*}_{R^*} - \dot{R}_{R^*}\dot{R^*}_R$. The followings are the components in the matrix J:

$$\dot{R}_R = \frac{\bar{R}}{\mathcal{Y}_R + \mathcal{Y}_R^* \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}} \left\{ \frac{\mathcal{Y}_R^*}{\nu_1^*\nu_2^*}\left(\frac{1}{1-\alpha} + \nu_2 \right) + \mathcal{Y}_\lambda \right\} r'(\bar{R}),$$

$$\dot{R}_{R^*} = -\frac{\bar{R}}{\mathcal{Y}_R + \mathcal{Y}_R^* \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}} \left\{ \frac{\mathcal{Y}_R^*}{\nu_1^*\nu_2^*}\left(\frac{1}{1-\alpha} + \nu_2^* \right) - \mathcal{Y}_\lambda^* \right\} r^{*\prime}(\bar{R}^*),$$

$$\dot{R^*}_R = \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\frac{\bar{R}^*}{\bar{R}}\dot{R}_R - \frac{\bar{R}^*}{\nu_1^*\nu_2^*}\left(\frac{1}{1-\alpha} + \nu_2 \right) r'(\bar{R}),$$

$$\dot{R^*}_{R^*} = \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\frac{\bar{R}^*}{\bar{R}}\dot{R}_R^* + \frac{\bar{R}^*}{\nu_1^*\nu_2^*}\left(\frac{1}{1-\alpha} + \nu_2^* \right) r^{*\prime}(\bar{R}^*).$$

Therefore, the signs of trace and determinant of the Jacobian matrix J are

$$\text{sign}[A_1] = \text{sign}[trace(J)] = \text{sign}\left[\frac{r'(\bar{R})}{1-\sigma} + \frac{r^{*\prime}(\bar{R}^*)}{1-\sigma^*} \right], \quad (37)$$

$$\text{sign}[A_0] = \text{sign}[det(J)] = \text{sign}\left[\frac{r'(\bar{R})}{1-\sigma}\cdot\frac{r^{*'}(\bar{R}^*)}{1-\sigma^*}\right]. \tag{38}$$

Propositions below show the results in this case with non-traded goods:

Proposition 3 *If either* $(\eta_\pi - 1)(1-\sigma)$ *or* $(\eta_\pi^* - 1)(1-\sigma^*)$ *is negative, equilibrium is indeterminate. Otherwise, it is determinate.*

3.3 Intuitive Mechanism of Determinacy

As summarized in Table 2, we find that coexistence of non-traded and tradable goods can easily make equilibrium indeterminate.

Now, assume that interest-rate control rules in both countries are passive, that $\sigma < 1$, and that inflation in Country 1 becomes lower. Then, the nominal rate of interest as an opportunity cost for holding money falls so that consumption in Country 1 which is complements of money increases. On the other hand, the real interest rate rises and thus production in Country 1 is smaller, since the growth rate of shadow value of real assets diminishes.

In a fully-opened economy, the real rate of interest in Country 2 is also higher from the non-arbitrage condition, and the nominal rate falls. If consumption and money are separable (i. e., $\sigma^* = 1$), the net export in Country 2 becomes lower, which contradicts the traded-goods equilibrium so that indeterminacy holds. (This discussion is similar to the case with non-traded goods.) In contrast, if $\sigma^* > 1$, consumption decreases and then the traded-goods equilibrium (and thus determinacy) can be satisfied. This equilibrium holds more easily if $\eta_\pi^* < 1$ is enough high, because a decrease of consumption becomes larger.

However, when the non-traded goods coexist, the relative inflation between traded and non-traded goods in Country 1 is lower so that the non-arbitrage condition is violated if the real interest rates are equivalent between the two countries. Therefore, indeterminate equilibrium tends to emerge under an imperfect open economy, since it is hard to satisfy both the complicated non-arbitrage condition with non-traded goods and equilibrium condition of the two types of goods.

4 Conclusion

We consider equilibrium determinacy in the economy which consists of two countries with tradable and non-traded goods. Additionally, the countries are heterogeneous in that parameters about preference, production and monetary policy of interest-rate control type.

Liberalization of economy which means that a fraction of traded goods is close to one can make equilibrium determinate under the case where equilibrium is indeterminate in the not-fully open economy. However, since full liberalization is not pragmatic, central banks had better take an appropriate stance of monetary policy according to the heterogeneity of economic structure such as preference and production for realizing stable economy. We should note that active policy is not always good and thus the international combination is required.

Introducing capital stock and fiscal policy for a means of stabilizer as well as monetary policy may be future research. Additionally, it may also be beneficial to investigate the relation between social-status preference in open economy as in Farmer and Lahiri (2005) and Valente (2006, 2009) and Taylor-type monetary policy.

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Appendix: Derivation of System Equations in Section 3.2 (The Case Including Non-Traded Goods)

Combining equilibrium condition for bond market

$$b + b^* = 0,$$

and the budget constraints of households in both countries, and goods-market equilibrium, we acquire the equilibrium condition for money:

$$\dot{m} + \dot{m^*} = -\pi m - \pi^* m^* + \alpha^\alpha (1-\alpha)^{1-\alpha} (y^T - c^T)[\tilde{P}^{1-\alpha} - (\tilde{P}^*)^{1-\alpha}]. \tag{39}$$

From (19) and the correspondence in Country 2, the equilibrium condition (39) can be rewritten in the following:

$$\frac{1-\gamma+\gamma\sigma}{\sigma} m \frac{\dot{R}}{R} + \frac{1-\gamma+\gamma\sigma^*}{\sigma^*} m^* \frac{\dot{R^*}}{R^*} + \mathcal{M} = 0, \tag{40}$$

where

$$\mathcal{M} \equiv \frac{m}{\sigma}(\rho - r(R)) + \frac{m^*}{\sigma^*}(\rho - r^*(R^*)) - \pi(R)m - \pi^*(R^*)m^* + \alpha^\alpha (1-\alpha)^{1-\alpha}(y^T - c^T)[\tilde{P}^{1-\alpha} - (\tilde{P}^*)^{1-\alpha}].$$

Substituting (29) into (40), we obtain

$$\frac{\dot{R}}{R} = -\left(\frac{1-\gamma+\gamma\sigma}{\sigma} m + \frac{1-\gamma+\gamma\sigma^*}{\sigma^*} m^* \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right)^{-1} \cdot$$
$$\left[\frac{1-\gamma+\gamma\sigma^*}{\sigma^*} m^* \frac{1}{\nu_1^*\nu_2^*}\left(-\frac{r(R) - r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*))\right) + \mathcal{M}\right]. \tag{41}$$

On the other hand, from (31) and

$$\frac{\dot{R^*}}{R^*} = \left(\frac{1-\gamma+\gamma\sigma}{\sigma} m + \frac{1-\gamma+\gamma\sigma^*}{\sigma^*} m^* \frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right)^{-1} \cdot$$
$$\left[\frac{1-\gamma+\gamma\sigma}{\sigma} m \frac{1}{\nu_1^*\nu_2^*}\left(-\frac{r(R) - r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*))\right) - \mathcal{M}\frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right], \tag{42}$$

the dynamic equation is

$$\frac{\dot{R}}{R} = -\Bigg[\mathcal{Y}_R^*\left(\frac{1-\gamma+\gamma\sigma}{\sigma}m + \frac{1-\gamma+\gamma\sigma^*}{\sigma^*}m^*\frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right)^{-1} \cdot$$
$$\left\{\frac{1-\gamma+\gamma\sigma}{\sigma}m\frac{1}{\nu_1^*\nu_2^*}\left(-\frac{r(R)-r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*))\right) - \mathcal{M}\frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right\}$$
$$+ \mathcal{Y}_\lambda(\rho - r(R)) + \mathcal{Y}_\lambda^*(\rho - r^*(R^*))\Bigg]\frac{1}{\mathcal{Y}_R}. \quad (43)$$

Comparing (41) and (43), we find that

$$\mathcal{M}\left(\mathcal{Y}_R + \mathcal{Y}_R^*\frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right) =$$
$$[\mathcal{Y}_\lambda(\rho - r(R)) + \mathcal{Y}_\lambda^*(\rho - r^*(R^*))]\left(\frac{1-\gamma+\gamma\sigma}{\sigma}m + \frac{1-\gamma+\gamma\sigma^*}{\sigma^*}m^*\frac{\nu_1\nu_2}{\nu_1^*\nu_2^*}\right) +$$
$$\left(-\mathcal{Y}_R\frac{1-\gamma+\gamma\sigma^*}{\sigma^*}\frac{m^*}{\nu_1^*\nu_2^*} + \mathcal{Y}_R^*\frac{1-\gamma+\gamma\sigma}{\sigma}\frac{m}{\nu_1^*\nu_2^*}\right) \cdot$$
$$\left(-\frac{r(R)-r^*(R^*)}{1-\alpha} + \nu_2(\rho - r(R)) - \nu_2^*(\rho - r^*(R^*))\right), \quad (44)$$

This equation suggests that λ is a function of R and R^*, since λ^* is the one of R, R^*, and λ from the goods-market equilibrium.

Table 1: Properties of Variables

	$\tilde{P}$	$y^N = c^N$	y^T	c^T	c	m
R	$(1-\sigma)$	$-(1-\sigma)$	$(1-\sigma)$	$-(1-\sigma)$	$-(1-\sigma)$	$-$
λ	$+$	$-(1-\sigma)$	$+$	$-$	$-$	$-$

ex) sign$[\tilde{P}_R]$ =sign$\left[\dfrac{\partial \tilde{P}}{\partial R}\right]$ = sign$(1-\sigma)$, $\tilde{P}_\lambda > 0$, and $c^T_\lambda < 0$.

Table 2: Equilibrium Determinacy

	$(\eta^*_\pi - 1)(1-\sigma^*) > 0$	$\sigma^* = 1$	$(\eta^*_\pi - 1)(1-\sigma^*) < 0$
$(\eta_\pi - 1)(1-\sigma) > 0$	(iii)	(iii)	(ii)
$\sigma = 1$	(iii)	(iii)	(i)
$(\eta_\pi - 1)(1-\sigma) < 0$	(ii)	(i)	(i)

(i) Indeterminate for any $\alpha \in (0,1]$;

(ii) Determinate or Indeterminate if $\alpha = 1$, Indeterminate if $0 < \alpha < 1$;

(iii) Determinate for any $\alpha \in (0,1]$.