Asymmetric Dominance, Deferral and Status Quo Bias in a Theory of Choice with Incomplete Preferences

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7 July 2012

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July 16, 2012

Abstract

This paper proposes a model of individual choice that does not assume completeness of the decision maker's preferences. The model helps explain in a natural way, and within a unified framework of choice in the presence of preference-incomparable options, three distinct behavioural phenomena: the *asymmetric dominance/attraction effect*, *choice deferral* and *status quo bias*. A decision maker who follows the decision rule featured in the model chooses an alternative from a menu if it is totally preference-undominated in that menu and at the same time is also partially preference-dominant. In situations where the decision maker's preferences are complete the model delivers strict utility maximisation.

*I thank Mark Dean, Robert Evans, Paola Manzini, Marco Mariotti, Yusufcan Masatlioglu, John Quah, Ran Spiegler, Jörg Stoye, Christopher J. Tyson and workshop/seminar audiences at Cambridge, Queen Mary-London and St Andrews for helpful comments. Partial financial assistance from SIRE is gratefully acknowledged. Preliminary versions of some of the material presented in this paper first appeared in the MPRA working paper 25481 titled "Rational Indecisive Choice" (October 2010).

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1 Introduction

This paper proposes a model of individual choice that does not assume completeness of the decision maker's preferences. The model helps explain in a natural way, and within a unified framework of choice in the presence of preference-incomparable options, three distinct behavioural phenomena: the *asymmetric dominance/attraction effect*, *choice deferral* and *status quo bias*.

The attraction effect concerns choice over multi-attribute alternatives and was originally reported by Huber, Payne, and Puto (1982). It comes about in two stages. In the first stage the decision maker is faced with the problem of choosing between two options, with each one dominating the other in some attribute. In the second stage the decision maker is presented with a menu that includes the original two options and a third one. This third option is dominated in all attributes by one of the original two alternatives (the "target" option) but neither dominates the other alternative nor is it dominated by it. The attraction effect is the observation that the choice probability of the asymmetrically dominant "target" option is significantly higher in the second, expanded menu where a "decoy" is present than in the smaller one where such a decoy is absent.

The introspectively familiar phenomenon of choice deferral (or choice avoidance) was described by Shafir, Simonson and Tversky (1993) as the occurrence of *"situations in which people (...) do not have a compelling reason for choosing among the alternatives and, as a result, defer the decision, perhaps indefinitely."* Although choice avoidance/deferral ultimately translates into status quo maintenance, one must be careful not to identify it with the distinct phenomenon of status quo *bias*. As defined in Anderson (2003), *"status quo bias is a decision maker's inflated preference for the current state of affairs."* This phenomenon was first documented by Thaler (1980), Knetsch and Sinden (1984) and Samuelson and Zeckhauser (1988). Unlike situations where deferral takes place and in which the status quo is irrelevant for the decision problem at hand, in situations where choice is biased towards the status quo the latter is directly relevant in the sense that it takes the form of an alternative similar to all others in the menu and which the decision maker actively compares with each of them. An example where deferral is the relevant phenomenon would be that of a consumer who avoids choice between two insurance policies and by so doing (s)he maintains the status quo of staying uninsured. By contrast, an example where status quo bias may be taking place would be one where the same person is facing the problem of choosing between the same two insurance policies but now holds one of them, and ultimately chooses to keep it.

The basic idea in the model that we propose in this paper to explain these phenomena is that a decision maker finds an alternative to be choosable in a menu if it is not dispreferred by any other feasible alternative *and* at the same time is preferred to at least one. It therefore builds on a special kind of preference maximisation that features the combination of the core rationality requirement of *total undomination* (TU) with the novel requirement of *partial dominance* (PD).

If there is a menu where the decision maker's preferences happen to be complete, then, since in this case a preference-undominated option is also preference-dominant, the TUPD rule prescribes utility-maximising behaviour at that menu. Rational choice is therefore included in the model as a special case. This property is demonstrated with novel axiomatic characterisations of rational choice. When preferences are incomplete, on the other hand, there is no guarantee that every menu will include an alternative that satisfies both requirements set out by the the TUPD rule. To this end, two variants of the model are studied, differing only in their predictions about what happens if such an alternative does not exist. The first variant builds on the typical assumption that the decision maker must always choose one of the feasible options, and predicts that in this case all feasible alternatives are choosable. By contrast, the second variant of the model is permissive of choice deferral and predicts that if the decision maker is unable to find a feasible option that satisfies both the TU and PD requirements, then she chooses nothing.

The typical findings on the attraction effect are explained in a simple way by the first variant of the model in the special case where preferences coincide with the usual coordinate-(attribute-)dominance partial ordering: The decision maker chooses either of the two alternatives in the first menu because she is unable to compare them, and chooses only the unique partially dominant alternative in the second menu. Moreover, the strengthening of the attraction effect as a result of choice deferral in pure-conflict menus that was reported in Dhar and Simonson (2003) are also both explained deterministically by the second variant of the model.

Finally, once the domain is expanded to include decision problems with a relevant status quo along the lines suggested by Masatlioglu and Ok (2005, 2010), then, in addition to the above two variants of the baseline model that apply to problems *without* such a status quo, a special application of the TUPD procedure can easily incorporate the well-known (Bewley, 1986/2002; Mandler, 2004) decision rule for problems *with* a relevant status quo, according to which the decision maker maintains the latter unless a feasible alternative that is preferred to it is found. In such an event, the agent abandons the status quo for one of the better options that is also no worse than anything else feasible. Evidently, in the case where this choice rule is coupled with the second variant of the baseline TUPD procedure that is permissive of deferral, the result is a unified model of choice that explains deferral, status quo bias and (the strengthening of) the attraction effect by attributing their occurrence to the presence of incomparable options.

The paper is organised as follows. Section 2 introduces the notation, discusses at length our proposed approach to model choice deferral (i.e. let the empty set be the optimal set in such cases) and reminds the reader of some key background results that are of relevance to this paper, thereby making the connection between the proposed theory and the existing ones more transparent. Section 3 introduces and discusses the basic axioms, lays out the two variants of the baseline TUPD model and analyses the behavioural implications associated with each of them. In section 4 the choice domain and the basic axioms are suitably augmented and the two variants of the TUPD choice rule are extended in the way outlined above to also account for status quo bias. Section 5 compares the paper's findings with those in existing related work. All proofs appear in the Appendix.

2 Preliminaries

We name our decision maker *Eve*. The grand set of all alternatives that Eve may be presented with is denoted X and is assumed finite. The collection of menus of alternatives drawn from X is denoted $\mathcal{M}$. For simplicity, we assume throughout that $\mathcal{M}$ includes all nonempty subsets of X (with suitable adjustments all results go through without this assumption).

Eve's preferences on X are captured by the binary relation $\succ$. When $x \succ y$ is the case it is understood, as usual, that x is preferred to y, but we will occasionally also write that y is *dispreferred* to x. The notation $x \not\succ y$ is used when it is understood that x is *not preferred* to y.

Eve's choice behaviour is described by a choice correspondence $C : \mathcal{M} \twoheadrightarrow X$, which is a mapping satisfying $C(A) \subseteq A$ for all $A \in \mathcal{M}$. Although the literature almost always takes nonempty-valuedness to be a defining characteristic of a choice correspondence, one notes that since the empty set is a subset of every set, the above definition does not a priori restrict C to be nonempty-valued. Moreover, nonempty-valuedness imposes the nontrivial behavioural restriction that Eve is able to find a choosable option in every menu. For this reason and the ones discussed just below, it will be treated as an explicit axiom in this paper:

Decisiveness (DEC)
If $A \in \mathcal{M}$, then $C(A) \neq \emptyset$.

Since its original conception by Samuelson (1938), the Weak Axiom of Revealed Preference (WARP) is generally accepted as the most fundamental postulate of choice consistency. According to an intuitive statement of WARP that is relevant in the present context, if some alternative x is chosen over another alternative y in an *arbitrary* menu, then y is never choosable in a menu where x is feasible. This statement can be formally written in the following familiar form:

Weak Axiom of Revealed Preference (WARP)
If $x \in C(A)$, $y \in A \setminus C(A)$ and $x \in B$, then $y \notin C(B)$.

Having reminded the reader of WARP's content, let us now go back to the discussion of the DEC axiom. An argument against letting C be possibly empty-valued in order to capture choice-avoidant behaviour is to introduce a special option, call it n, which will be feasible in every menu and which will be uniquely assigned by the choice correspondence whenever it is understood that Eve chooses none of the other, "real" feasible options. We claim, however, that when such behaviour has its roots in preference-incompleteness, following this approach can be misleading.

To illustrate with an example, imagine that Eve: a) prefers w to x and y; b) can't compare w to z; c) chooses something from a menu if and only if it is preferred to all others in that menu. In this case, w is chosen over n in $\{w, x, y, n\}$ and n is chosen over w in $\{w, z, n\}$. This is clearly a violation of WARP. Yet, Eve's behaviour is perfectly consistent with strict maximisation of her incomplete preferences. Why is it then that a WARP violation comes about? The reason is that once C is assumed nonempty-valued, n is treated by WARP just like any other alternative. This would be perfectly reasonable if deferral was due to the undesirability of some of the other alternatives, and if Eve also had no difficulty ordering all options (including n) according to her preferences. In such a case, n would be comparable with all alternatives and it would set a "choosability" threshold: if a feasible alternative better than n exists, Eve chooses something, otherwise she chooses nothing, i.e. n. In such a case one would indeed expect WARP to be operational in choices that involve n too. When deferral is caused by incompleteness, however, it doesn't seem to be the case that by choosing n from a menu Eve has actually compared it with all other options in that menu and has found it preferable to them. Utilising the empty set instead of the explicit no-choice alternative n for this purpose allows for the consistency restrictions to be applicable only when Eve does actually choose and therefore respects her possible cautiousness in choosing that follows from the incompleteness of her preferences.

Moving on to the behavioural implications of the above restrictions on a choice correspondence, it is well-known since Arrow (1959) that, in the current abstract choice framework, an individual's choice behaviour is a utility-maximising one if and only if it is consistent with both WARP and DEC.[1] For comparison purposes it will be useful to state this result in the following way (Kreps, 1988):

Proposition 1 (Arrow)
A choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC *and* WARP *if and only if there exists an asymmetric and negatively transitive relation $\succ$ on X such that, for all $A \in \mathcal{M}$*

$$C(A) = \{x \in A : y \nsucc x \text{ for all } y \in A\}. \tag{1}$$

Of course, if $\succ$ is asymmetric and negatively transitive and one defines the relation $\sim$ by $x \sim y$ if $x \nsucc y$ and $y \nsucc x$, then the relation that comes about by joining $\succ$ and $\sim$ is a weak order. Thus, when there are multiple choosable alternatives in a menu, the natural interpretation suggested by this model is that Eve is *indifferent* between them.

The choice rule set out in (1), however, obviously goes through even when $\succ$ is not the asymmetric part of a weak order and where indifference can therefore not be afforded. In particular, it may be that Eve's psychological primitive is a severely incomplete strict preference relation, which is nevertheless internally consistent in the sense that it contains no cycles. This more general model has also been studied in the literature and, as Schwartz (1976) showed, it is characterised by the following two well-known axioms, both of which are implied by WARP when DEC is assumed.

Independence of Irrelevant Alternatives (IIA)
If $x \in C(A)$, $B \subset A$ and $x \in B$, then $x \in C(B)$.

Expansion (EXP)
If $x \in C(A_1), \ldots, C(A_k)$, then $x \in C(\bigcup_{i=1}^{k} A_i)$.

As is evident, the former axiom imposes consistency restrictions across decision problems that involve moving from a large menu where one alternative is choosable to a smaller menu where the same alternative is still feasible. The latter axiom, on the other hand, imposes restrictions in the opposite direction. The behaviour characterised by these two axioms together with DEC is the one outlined above and formally stated below:

[1] Even though the role of DEC in this classic result is generally ignored, it should be emphasized that it is crucial. See Gerasimou (2012) for more details and for a generalisation of this result that comes about when DEC is not assumed.

Proposition 2 (Schwartz)
A choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC, EXP *and* IIA *if and only if there exists an acyclic relation $\succ$ on X such that, for all $A \in \mathcal{M}$*

$$C(A) = \{x \in A : y \nsucc x \text{ for all } y \in A\}. \tag{2}$$

One could think of Proposition 2 as the canonical model of choice with incomplete preferences.[2] The model has considerable normative appeal, taking into account the incompleteness (and, in fact, also the intransitivity) of the agent's preferences. However, the choice pattern that lies behind the attraction effect is outside this model's reach. Indeed, when faced with the problem of choosing between multi-attribute options x and y where each option dominates the other in some attribute, a decision maker whose preferences are captured by the usual partial ordering (defined on attribute space) would be unable to compare x and y. Modeled deterministically according to the Schwartz model, the choice behaviour of such a decision maker would dictate $C(\{x, y\}) = \{x, y\}$, which is a good proxy for the typical experimental findings of more or less equal choice probabilities for x and y in this problem. Yet, in the menu where the feasible alternatives are x, y and a third option y' (which is a "decoy" for y in the sense that it is worse than y in all attributes but both better and worse than x in some attribute), Schwartz's model would still predict $C(\{x, y, y'\}) = \{x, y\}$. This prediction, however, is at odds with the well-established empirical fact that the choice probability of y is significantly higher in this menu compared to the original two-element one where this probability was close to a half, and which would suggest that $C(\{x, y, y'\}) = y$ is a more accurate way of describing such behaviour. It will be shown below that a modification of Schwartz's basic model is able to generate this choice pattern in a simple and intuitive way.

3 Totally Undominated & Partially Dominant Choice

We set out the axioms that form the core of the models that follow. All axioms have a normative flavour in the sense that the restrictions imposed by them do not force Eve to behave in a way directly inconsistent with WARP. At the same time, they do not have the full normative power entailed by the combination of WARP and DEC which characterizes utility maximisation. As it will become clear below, the removal of some of the normative restrictions imposed by this combination of axioms is justified on descriptive grounds.

Reduced WARP (ReWARP)
If $C(\{x, y\}) = x$ and $x \in A$, then $y \notin C(A)$.

Although ReWARP places fewer restrictions than WARP, one could think of the two axioms as conveying the same normative message, namely that *"if x is revealed preferred*

[2] Eliaz and Ok (2006) axiomatise a similar but richer model where the agent's strict preferences are transitive and where indifference is also permissible and behaviourally distinguishable from incomparability. Salant and Rubinstein (2008) provide a reinterpretation of the model (also in the case where preferences are transitive) in a novel context of choice with frames.

to y, then y is not choosable in the presence of x". The difference lies in the fact that the former does so using a narrower definition of revealed preference than the latter. According to this line of reasoning one would think of WARP as taking x to be revealed preferred to y if there is *any* menu where x is chosen over y, and of ReWARP as doing the same only when x is chosen over y in the menu $\{x, y\}$. Choice-based inferences concerning preference between x and y are not affected by the presence of alternatives other than x and y in the latter situation, whereas they potentially are in the former. On these grounds one could therefore argue that a better indication of Eve's preference for x over y would come from knowing that $C(\{x, y\}) = x$ rather than knowing that there is some menu A where x is choosable and y is not. As far as the practical matters of relaxing WARP in this way are concerned, one observes that the restrictions imposed by ReWARP are compatible with the attraction effect, since, unlike WARP, they are permissive of the choice pattern $C(\{x, y, y'\}) = y$ and $C(\{x, y\}) = \{x, y\}$.[3]

Weak Partial Contraction (WPC)
If $x \in C(A)$ and $C(A) \subset A$, then $x = C(B)$ for some $B \subseteq A$ such that $|B| > 1$.

Consider a menu A where an alternative x is choosable and not everything in the menu is. WPC states that there should be some smaller menu where x is the only choosable option. Natural candidates where one might expect this to be true are the binary menus consisting of x and each of the alternatives that are rejected in A. Yet, the axiom could also be satisfied even in non-binary menus that contain a single choosable alternative, without this alternative being the unique choosable one in *all* binary submenus. An example where this is true is the choice pattern that generates the attraction effect, where one has $C(\{x, y, y'\}) = y$, $C(\{y, y'\}) = y$ and $C(\{x, y\}) = \{x, y\}$.

Weak Expansion (WEXP)
If $C(A_1) = x$ and, for $i \geq 2$, either $x \in C(A_i)$ or $C(S_i) = \emptyset$ for all $S_i \subseteq A_i$ such that $x \in S_i$ and $|S_i| > 1$, then $x \in C(\bigcup_{i=1}^{k} A_i)$.

WEXP is similar to EXP but obviously weaker when DEC is also assumed. This is so due to the additional requirement that x must be the *unique* optimal choice in at least one of the menus $A_1, \ldots, A_k$, in addition to being choosable in all others, before it is considered choosable in the menu that comes about by merging all the A_is. The other novel component of the axiom becomes operational when DEC is not in force. In this case the original requirement of x being the unique choosable option in some menu remains, but now x can still be choosable in the expanded menu even if it is not choosable in any of the other A_is, provided that nothing is chosen in all these A_is and from all their submenus that contain x. Intuitively, even though x is not choosable in some A_is, it is true that nothing in these menus is chosen over x either. As such, these A_is are "neutral" when it comes to assessing the optimality of x in the expanded menu. The axiom states that the dominance of x in at least one of the components of the large menu provides sufficient reason for it to be choosable

[3]Under DEC, ReWARP is equivalent to Sen's (1977) *Property* $\alpha 2$, which states that if $x \in C(A)$ and $y \in A$, then $x \in C(\{x, y\})$. Sen introduced this axiom as a weakening of *Property* α, which is a synonym for IIA. However, ReWARP and $\alpha 2$ are logically distinct if DEC is not assumed. By contrast, ReWARP is weaker than WARP whether DEC is assumed or not.

there, even in the face of such neutral behaviour in the other components of the large menu.

Proposition 3

A choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC, ReWARP, WEXP *and* WPC *if and only if there exists an acyclic relation $\succ$ on X such that, for all $A \in \mathcal{M}$,*

$$C(A) = A \qquad \Longleftrightarrow \qquad x \not\succ y \text{ and } y \not\succ x \text{ for all } x, y \in A \tag{3a}$$

$$C(A) \subset A \qquad \Longleftrightarrow \qquad C(A) = \left\{ x \in M : \begin{array}{cc} z \not\succ x & \text{for all } z \in A \\ & \text{and} \\ x \succ y & \text{for some } y \in A \end{array} \right\} \tag{3b}$$

This model portrays Eve as a decision maker who operates procedurally and yet with a single incomplete and acyclic preference relation describing her tastes.[4] Her actions depend on whether the menu that she is presented with contains alternatives that are preference-ranked or not. If not, then all alternatives in the menu are mutually incomparable and, since the assumption that she always chooses something is built in the model by means of DEC, this fact leads to all feasible alternatives being choosable. This feature of the model is captured by (3a) and is shared with the two baseline models of rational choice, Propositions 1 and 2.

If, however, a preference comparison between some alternatives in the menu does exist, then Eve proceeds in two steps. First, she elicits those options in the menu that are *undominated* according to her preferences. In view of the acyclicity of $\succ$, this set is nonempty. Then, she searches within this smaller set of alternatives for ones that also *dominate* some other feasible options. Since it was assumed that there is at least one preference comparison in the menu, this set is also nonempty. Eve's choice is an element of this smaller set. This reasoning is captured by (3b).

We will refer to both a choice correspondence C that is characterized by the axioms of Proposition 3 and to the alternatives that are optimal according to such a correspondence as *totally undominated and partially dominant* (TUPD). Also, if C is a TUPD choice correspondence and $\succ$ is the preference relation that is associated with it in the sense of (3a) and (3b), then we will say that $\succ$ *induces* or *generates* C.

Similar to the model that builds on (2), the alternatives declared optimal by a TUPD choice correspondence C are *not* preferred in general to those that are rejected by C. Unlike (2), however, this is so even in those menus where C is single-valued. Furthermore, again similar to (2), a TUPD C includes the model of a rational choice single-valued function as a special case when the generating preference relation is complete. Indeed, if $\succ$ is complete, then the situation described in (3a) is ruled out from the outset (except at singleton menus), whereas for every menu A, an option $x \in A$ satisfies (3b) if and only if it is in fact preferred to all other options in A. The exact way in which the axiomatic system of Proposition 3 must be strengthened for it to deliver this model is studied below.

[4]The classic reference for two-stage choice procedures (based on two distinct, incomplete and possibly cyclic preference relations) in abstract choice theory is Manzini and Mariotti (2007).

A somewhat surprising feature of the TUPD procedure is that it predicts less consistency in Eve's cross-menu behaviour than the TU procedure of Proposition 2, even though she is actually portrayed as seeking *more reasons* than mere preference-undomination in the TUPD model before considering an alternative to be choosable in any given menu. More specifically, despite the fact that Eve will never choose a dominated option by following (3), her choices across menus may violate the IIA axiom. For example, suppose w, x, y, z are alternatives for which $w \succ x$, $y \succ z$ and no comparison is possible in all other pairs derived from this four-element set. Since w and z are undominated in the menu $\{w, x, y, z\}$ and both dominate something there, $C(\{w, x, y, z\}) = \{w, z\}$. Now consider the menu $\{w, y, z\}$. Since w is merely undominated here, whereas y is still both undominated and partially dominant, $C(\{w, y, z\}) = y$.

Despite its normative "shortcomings", it is straightforward that the TUPD choice correspondence explains the classic attraction effect in a simple way in the special case where the preference relation inducing it is the asymmetric part of the usual, coordinate-wise dominance partial ordering. Of course, the model is general enough to explain "idiosyncratic" attraction effects that are based on more subjective dominance relations. For instance, if Eve's incomplete preferences differ from those captured by the usual partial ordering, then there are alternatives x, y and z where x and y and also x and z are incomparable while x is preferred to z, where preference is not necessarily identified with attribute dominance. According to the model, y is the unique choosable option in the menu $\{x, y, z\}$.

A particularly desirable property of the TUPD choice procedure is its robustness with respect to the DEC axiom. In a decision environment where her preferences are possibly incomplete, the assumption that Eve is able to find a choosable option in every menu may be too strong. As mentioned above, a large body of experimental work in psychology and consumer research (Tversky and Shafir, 1992; Greenleaf and Lehmann, 1995; Luce, 1998) has demonstrated that people are very often unable to choose one of the available alternatives when decision conflict is present, because the tradeoffs involved may be severe enough that prevent them from finding an option whose choice can be justified (Shafir, Simonson, and Tversky, 1993). Moreover, for the particular case of the attraction effect the experimental findings in Dhar and Simonson (2003) suggest that the effect (as measured by the difference in the choice probabilities of the target option before and after the introduction of the decoy) becomes stronger when subjects are not forced to choose than when they are forced to do so. A candidate explanation for this is precisely the fact that many subjects choose nothing from the pure-conflict menu where the decoy is absent and at the same time most subjects choose the option that dominates the decoy when the latter is introduced.

To explain this behaviour we will modify the baseline model of Proposition 3 by removing DEC and by strengthening WPC in the following way:

Strict Partial Contraction (SPC)
If $x \in C(A)$, then $x = C(B)$ for some $B \subseteq A$ such that $|B| > 1$.

While WPC states that for x to be the only choosable option in a submenu of A it suffices that x be choosable in A *and* that something in A be rejected, according to SPC it is sufficient that x be merely choosable in A for the conclusion to follow. Although the axiom is now stronger, it is descriptively relevant in situations where choice is not forced and deferral is permissible. For instance, the behaviour reported in Dhar and Simonson (2003) would be best captured in this context by setting $C(\{x, y, y'\}) = y$ and $C(\{x, y\}) = \emptyset$. This choice pattern is consistent with SPC.

Proposition 4
A choice correspondence $C : \mathcal{M} \twoheadrightarrow X$ satisfies ReWARP, SPC *and* WEXP *if and only if there exists an asymmetric relation $\succ$ on X such that, for all $A \in \mathcal{M}$ with $|A| > 1$,*

$$C(A) \quad = \quad \left\{ x \in M : \begin{array}{cc} z \not\succ x & \text{for all } z \in A \\ & \text{and} \\ x \succ y & \text{for some } y \in A. \end{array} \right\} \tag{4}$$

In Proposition 4 Eve is portrayed as a cautious decision maker who chooses nothing from a menu unless she can find a feasible option that meets the TUPD criterion. The fact that the relation $\succ$ here is merely asymmetric (and hence possibly cyclic) implies that, in addition to incomparability, inconsistency too is a potential source of deferral. Indeed, the presence of a preference cycle causes failure of meeting the TU requirement, while, as before, in the event of an absence of preference comparisons it is the PD condition that fails to be met. Whether one thinks of the possibly cyclic nature of Eve's preferences here as a virtue or an anomaly of the model, it is worth mentioning that acyclicity of these preferences is restored in the augmented model presented in the next section, without this affecting in any way Eve's ability to defer choice.

It is clear that the axiomatic system of Proposition 4 is neither stronger nor weaker than that of Proposition 3. As the following novel characterization of rational choice demonstrates, however, both results deliver strict utility maximisation (i.e. without indifference ties) as a special case.

Proposition 5
The following statements are equivalent:
(a) $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC, ReWARP *and* SPC*;*
(b) $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC, WARP *and* SPC*;*
(c) $C : \mathcal{M} \twoheadrightarrow X$ satisfies DEC, WARP *and single-valuedness;*
(d) There exists a strict linear order $\succ$ on X such that, for all $A \in \mathcal{M}$,

$$C(A) = \{x \in A : y \not\succ x \text{ for all } y \in A\}. \tag{5}$$

The equivalence between statements (c) and (d) in Proposition 5 is well-known. The novelty of the result lies in the fact that it also proves that these and the remaining statements are also equivalent. More specifically, the equivalence between (a) and (d) shows in a transparent way how the TUPD procedure reduces to rational choice. As a novel characterisation of rational choice, it is also of independent interest. The equivalence between (b) and (c) on the other hand identifies an axiom (i.e. SPC) which is satisfied in addition to DEC and WARP if and only if the benchmark model of a rational choice correspondence in Proposition 1 generates unique predictions.

4 Expanded Domain and Status Quo Bias

As set out in the previous section, the TUPD choice procedure was shown to be able to predict the occurrence of the attraction effect and of its strengthening when choice is not forced by also allowing for choice deferral in situations where the menu consists of mutually incomparable alternatives. In this section we show how the procedure can also be useful in explaining the phenomenon of status quo bias. To this end, we expand the choice domain in the way suggested by Masatlioglu and Ok (2005, 2010).

As before, $\mathcal{M}$ will denote the collection of all menus generated from a finite grand set X. A decision problem in the present context, however, is a pair (A, p), where $A \in \mathcal{M}$ is a menu and p is either an alternative contained in A or an object denoted by $\diamond$. The domain of all problems (A, p) is denoted Z. When $p \in A$, it is understood that the decision problem (A, p) features a status quo that is *relevant*, in the sense that it takes the form of a "real" feasible option, just like all others. When $p = \diamond$ on the other hand, we understand that the status quo is *irrelevant.*[5] This clearly differs from problem to problem. For instance, owning no car and choosing nothing when presented with the problem of choosing car A or car B corresponds to maintaining a different status quo than when one owns no fridge and chooses nothing when presented with fridges C and D. Despite this fact, since all decision problems of this kind share the feature of having a status quo that takes a different form than an alternative "just like all others", it is without any loss of conceptual relevance to use the notation $\diamond$ across all such problems.[6]

A choice correspondence is now a mapping $C : Z \twoheadrightarrow X$ that satisfies $C(A, p) \subseteq A$ for all $(A, p) \in Z$. For reasons of internal consistency, C is assumed to satisfy $C(A, s) \neq \emptyset$ for all $(A, s) \in Z$. The interpretation for such a restriction is obvious: Unlike the case of decision problems where doing nothing is associated with not choosing a feasible option, doing nothing in problems where the status quo takes the form of an alternative just like all others is obviously associated with keeping the status quo alternative. Thus, either the status quo or some other feasible option must be chosen in such decision problems, which makes the above requirement self-evident. Of course, this requirement does not automatically carry over to problems $(A, \diamond) \in Z$.

The next two axioms impose exactly the same restrictions than when they were originally introduced, but are now rewritten in a way that makes use of the current notation.

Decisiveness (DEC)
If $A \in \mathcal{M}$, then $C(A, \diamond) \neq \emptyset$.

Weak Partial Contraction (WPC)
If $x \in C(A, \diamond)$ and $C(A, \diamond) \subset A$, then $x = C(B, \diamond)$ for some $B \subseteq A$, $|B| > 1$.

Next, ReWARP is restated so as to make the axiom's restrictions applicable both in problems without a relevant status quo, as before, but also in ones where such a status quo

[5] Obviously, our terminology for relevant and irrelevant status quos is conceptually entirely unrelated to the axiom of *Status Quo Irrelevance* of Masatlioglu and Ok (2005, 2010).

[6] Masatlioglu and Ok (2005, 2010), who introduced this notation, interpret $\diamond$ as an element not included in the grand set X.

does exist. These restrictions are obviously applicable across problems with the *same* status quo.

ReWARP
If $C(\{x,y\},p)=x$, $x\neq p$ and $x\in A$, then $y\notin C(A,p)$.

A more substantial modification is due for the WEXP axiom. In its original version it was assumed that if an alternative x is uniquely chosen from a menu and, in addition, it is either choosable in a number of other menus or nothing is choosable in those menus and in all submenus that contain x, then x is also choosable in the large menu that is obtained by joining all individual ones. The axiom is now modified in two ways. First, the menu where x must be uniquely optimal is now required to be a binary one. Second, if all conditions are satisfied, then x is choosable not only in the status-quo-free problem that features the expanded menu, but also in the problem where the same expanded menu is involved and where the status quo is the option over which x was chosen in the binary menu.

Augmented Weak Expansion (AWEXP)
For $(A_1,\diamond),\ldots,(A_k,\diamond)\in Z$ with $A_1=\{x,s\}$ and $A=\bigcup_{i=1}^{k}A_i$, suppose that:

1. $C(A_1,\diamond)=x$;

2. For $i\geq 2$, either $x\in C(A_i,\diamond)$ or $C(S_i,\diamond)=\emptyset$ for all $S_i\subseteq A_i$ such that $x\in S_i$ and $|S_i|>1$.

Then $x\in C(A,\diamond)$ and $x\in C(A,s)$.[7]

The three axioms that are particular to decision problems with a relevant status quo are introduced next.

Consistent Strict Inferiority (CSI)
For all $x,s\in X$, $C(\{x,s\},\diamond)=x$ if and only if $C(\{x,s\},s)=x$.

The statement $C(\{x,y\},\diamond)=x$ in the expanded domain carries the same interpretation that the statement $C(\{x,y\})=x$ does in the standard domain. According to this interpretation, Eve strictly prefers x to y. The first part of the CSI axiom states that when Eve is faced with a binary problem where the status quo is strictly inferior to the other feasible option, then the fact that the inferior option also happens to be the status quo plays no role and Eve never chooses it even in that case. This restriction is compatible with the status quo bias phenomenon because the relevant experimental evidence seems to suggest that the inflated preference for the status quo comes about when neither of the two options is considered strictly superior to the other. One typically models a situation like this with $C(\{x,y\},\diamond)=\{x,y\}$. It should be emphasized that in such a case the CSI axiom *does not* require $C(\{x,y\},y)=\{x,y\}$, as the latter would indeed be at odds with status quo bias.

The second restriction imposed by CSI requires the converse to also be true, namely if Eve chooses x over s when faced with the pair $\{x,s\}$ and endowed with s, then she will also

[7] As far as the results that are presented in this section are concerned it is worth noting that if we impose the domain restriction that for all $A\in\mathcal{M}$ and all $s\in A$ it holds that $(A,s)\in Z$, then this axiom's requirement that $x\in C(A,\diamond)$ and $x\in C(A,s)$ can be reduced to $x\in C(A,s)$.

choose x over s when she is faced with the same problem and neither option is the status quo. This part of the axiom coincides with the second part of the *Weak Axiom of Status Quo Bias* suggested by Masatlioglu and Ok (2010).

Status-Quo-Independent Choice (SQIC)
If $x \in C(A, s)$ and $x \neq s$, then $x \in C(A, \diamond)$.

SQIC imposes the intuitive restriction that if there is a decision problem with a relevant status quo in which some feasible option other than the status quo is choosable, then that option must also be choosable in the same decision problem where there is no status quo. This axiom is normatively appealing. At the same time, it does not seem to run against existing experimental evidence or intuition.

Firm Status Quo Bias (FSQB)
If $x, s \in A$ and $x \neq C(\{x, s\}, s)$, then $x \notin C(A, s)$.

Consider a binary decision problem where the menu is $\{x, s\}$ and s is the status quo. If x is not chosen over s in this problem, then, reasoning in the way discussed above, one could interpret this as evidence that x is not considered superior to s. Consistent with the experimental evidence on status quo bias, FSQB requires that, in such a case, x be non-choosable in a decision problem where a larger menu A that includes both x and s is involved and where s is the status quo.

Proposition 6
A choice correspondence $C : Z \twoheadrightarrow X$ satisfies AWEXP, CSI, DEC, FSQB, ReWARP, SQIC *and* WPC *if and only if there exists an acyclic relation $\succ$ on X such that for all $A \in \mathcal{M}$*

$$C(A, \diamond) = A \quad \Longleftrightarrow \quad x \nsucc y \text{ and } y \nsucc x \text{ for all } x, y \in A \tag{6a}$$

$$C(A, \diamond) \subset A \quad \Longleftrightarrow \quad C(A, \diamond) = \left\{ x \in M : \begin{array}{cc} z \nsucc x & \text{for all } z \in A \\ & \text{and} \\ x \succ y & \text{for some } y \in A \end{array} \right\} \tag{6b}$$

$$C(A, s) \neq s \quad \Longleftrightarrow \quad C(A, s) = \left\{ x \in M : \begin{array}{cc} z \nsucc x & \text{for all } z \in A \\ & \text{and} \\ x \succ s & \end{array} \right\} \tag{6c}$$

$$C(A, s) = s \quad \Longleftrightarrow \quad z \nsucc s \text{ for all } z \in A. \tag{6d}$$

The first two elements of this model, (6a) and (6b), are identical to (3a) and (3b) in the baseline TUPD procedure and dictate Eve's behaviour in decision problems where the status quo is irrelevant. The latter two elements of the model, (6c) and (6d), formalize the incomplete-preference-based explanation of status quo bias that was referred to by Masatlioglu and Ok (2005) as the *Simon-Bewley* procedure (Simon, 1955; Bewley, 1986/2002). This procedure was described by Mandler (2004) in the following way: *"Agents with incomplete preferences maintain the status quo when they follow the simple rule of refusing to trade their endowment for unranked bundles and waiting until offered an alternative that is ranked superior"*.

Clearly, Proposition 6 provides a model that explains asymmetric dominance and status quo bias in a natural way within a unified framework of choice with incomplete preferences. The link between (6b) and (6c) is of particular interest. It shows that Eve's decision to abandon the status quo follows a special application of the TUPD procedure: In particular, for it to be choosable, a feasible alternative other than the status quo must be universally undominated and must also be partially dominant, not with respect to some arbitrary feasible alternative, but rather with respect to the status quo. Thus, the model features a reference-dependent, special application of the TUPD procedure when it comes to the decision of giving up the status quo. At the same time, status quo bias is captured by the fact that in all problems that feature a relevant status quo the latter is uniquely chosen if and only if it is merely undominated in the menu.

In order to make this expanded model permissive of choice deferral one need only follow the exact same steps that were taken previously, i.e. remove the DEC axiom and strengthen WPC with SPC:

Strict Partial Contraction (SPC)
If $x \in C(A, \diamond)$, then there exists $B \subseteq A$, $|B| > 1$, such that $x = C(B, \diamond)$.

Proposition 7
A choice correspondence $C : Z \twoheadrightarrow X$ satisfies AWEXP, CSI, FSQB, ReWARP, SPC *and* SQIC *if and only if there exists an acyclic relation $\succ$ on X such that, for all $A \in \mathcal{M}$ and $s \in X$,*

$$C(A, \diamond) \quad = \quad \left\{ x \in M : \begin{array}{cc} z \not\succ x & \text{for all } z \in A \\ & \text{and} \\ x \succ y & \text{for some } y \in A \end{array} \right\} \tag{7a}$$

$$C(A, s) \neq s \quad \Longleftrightarrow \quad C(A, s) = \left\{ x \in M : \begin{array}{cc} z \not\succ x & \text{for all } z \in A \\ & \text{and} \\ x \succ s & \end{array} \right\} \tag{7b}$$

$$C(A, s) = s \quad \Longleftrightarrow \quad z \not\succ s \text{ for all } z \in A \tag{7c}$$

Proposition 7 presents a further augemented model where asymmetric dominance, deferral and status quo bias can all be captured and are all attributed to the incompleteness of Eve's preferences. The parts that explain status quo bias, (7b) and (7c), remain exactly the same as in Proposition 6 and govern Eve's behaviour when she is faced with problems that have a relevant status quo. Her behaviour in decision problems without such a status quo on the other hand is guided by the variant of the TUPD procedure that allows for deferral and which was introduced in Proposition 4. Notably, Eve's preferences here are no longer merely asymmetric but have become acyclic. This is due to an interaction of the full domain assumption with ReWARP, CSI and the fact that $C(A,s) \neq \emptyset$ for all $(A,s) \in Z$ with $s \in A$. For more details the reader is referred to the proof in the Appendix.

Our last result identifies the way in which the model delivers rational choice as a special case when preferences are complete, irrespective of whether the decision problem is one where the status quo is relevant or not.

Proposition 8
The following statements are equivalent:
1. $C : Z \twoheadrightarrow X$ satisfies CSI, DEC, ReWARP, SPC and SQIC.
2. There exists a strict linear order $\succ$ on X such that, for all $(A,p) \in Z$

$$C(A,p) = \{x \in A : y \not\succ x \text{ for all } y \in A\}. \tag{8}$$

In addition to DEC, ReWARP and SPC that characterise strict utility maximisation in the standard domain (Proposition 5), Proposition 8 shows that CSI and SQIC are the key axioms relating behaviour in problems with and without a relevant status quo that are necessary and sufficient for Eve to be a strict utility maximiser in both types of problems. In such a case, of course, Eve is no longer susceptible to either the attraction effect or the phenomena of choice deferral and status quo bias.

5 Related Literature

We start by discussing the existing theoretical literature on the attraction effect. We note from the outset that despite the many models that can explain this effect, to our knowledge the only one that is also robust to the nonempty-valuedness assumption and can therefore explain the strengthening of the effect when choice is not forced is the TUPD model. Lombardi (2009) proposed a two-stage choice procedure which, like the one in this paper, builds on a *single* incomplete and acyclic preference relation and explains the attraction effect. The decision maker in his model chooses one of the feasible undominated options for which the set of worse alternatives in the given menu is not a proper subset of the same set corresponding to some other undominated option. For a given preference relation, an alternative that is choosable in Lombardi's model is also choosable in the TUPD model but the converse is not generally true.

Following a different approach, de Clippel and Eliaz (2012) explain the phenomenon with a model of intrapersonal bargaining between the agent's two selves. In their model, the individual assigns an ordered pair of "scores" to each alternative. The first (second) score is the number of feasible options that are worse than the given option according to the complete preferences of the first (second) self. the agent chooses one of the options with the largest minimum score. This model does not subsume rational choice, as one of its necessary conditions (*Existence of a Compromise*) predicts that, given any three alternatives with the property that both options are choosable in each of the three possible pairs, there is a unique choosable option in the menu consisting of all three alternatives (which violates WARP). The model, however, was also shown to be able to accommodate the compromise effect (Simonson, 1989).

Another model that explains the attraction effect is due to Ok, Ortoleva and Riela (2010). This model features reference-dependent choice and the main idea behind it is that a feasible alternative may act as a reference point that favors those alternatives that dominate it. Once this set of dominating options is determined, the decision maker chooses among the elements of this set that maximise a real-valued function. In another theory of bounded-rational behaviour, Masatlioglu, Nakajima, and Ozbay (2012) studied a decision maker who does not generally consider all feasible alternatives due to attention constraints. Letting y' be a decoy for alternative y, their model accommodates choice behaviour where $C(\{x, y\}) = x$ and $C(\{x, y, y'\}) = y$, which the authors interpret as another instance of the attraction effect.[8] Natenzon (2011) explains asymmetric dominance with a similarity-based model of learning and stochastic choice.

As far as deferral is concerned, the idea that it can be modeled with an empty optimal set goes back to at least Hurwicz (1986). Clark (1995) argued that WARP rationality is not incompatible with choice deferral modeled in this way, without, however, characterizing a choice procedure that features such behaviour. This task is carried out in Gerasimou (2012) and the result provided there includes Arrow's theorem (Proposition 1 above) as a special case when preferences are complete. Gaertner and Xu (2004) also model an individual who occasionally chooses nothing by letting the empty set describe behaviour in such cases, but the sources of choice avoidance in their theory are procedural aspects behind the feasibility of some alternatives and not preference incompleteness. Buturak and Evren (2010) investigate preference for flexibility as an explanation of choice deferral.

Concerning status quo bias and incomplete preferences, it was demonstrated in Mandler (2004) that a decision maker who follows the choice procedure that was formally embedded in Propositions 6 and 7 above cannot be led to exploitations of the money-pump kind.[9] Masatlioglu and Ok (2005, 2010) provided the first axiomatic treatment of the phenomenon within the framework of multi-(weak)-utility representations of the agent's incomplete preferences, demonstrating also its compatibility with rational behaviour. Our model (both Propositions 6 and 7) is logically distinct from theirs. In terms of differences in behavioural predictions, their model (in particular, Theorem 3 in Masatlioglu and Ok, 2010) portrays an agent who never defers choice and is generally immune to the attraction effect in problems without a status quo. In problems *with* a status quo, the two models coincide in the prediction that unless something better than the status quo is feasible, the agent keeps the latter. In those cases where at least one better option exists, however, the two models' predictions are generally distinct.[10]

Dean (2008) investigated the role of complexity (as captured by the size of a menu) in status quo bias. In contrast to Masatlioglu and Ok (2005, 2010) who build on WARP, Dean

[8]The model by Ok, Ortoleva, and Riella (2010) also captures this IIA-inconsistent choice pattern.

[9]Further discussion on the possible exploitation of an agent with incomplete preferences is given in Mandler (2009) and Danan (2010).

[10]In such cases the model of Masatlioglu and Ok (2010) predicts that the agent chooses one of the options for which the sum of all weak utilities is maximised. Thus, an option may exist that is incomparable to these alternatives and which may not be choosable even though it is strictly better than the status quo and worse than no other alternative.

(2008), among other things, relaxes WARP by requiring it to hold only in problems without a status quo. Apesteguia and Ballester (2009) studied status quo bias and reference-dependent choice in a model where behaviour is rationalized by a collection of binary relations, one for each possible reference point. An alternative approach based on maximisation of a strict linear ordering is followed in Apesteguia and Ballester (2012). Finally, Ortoleva (2010) studied status quo bias in the context of choice under uncertainty, showing that the presence of a status quo can make the decision maker become more ambiguity averse compared to cases where a status quo is absent.

6 Concluding Remarks

In this paper we showed that the phenomena of asymmetric dominance, choice deferral and status quo bias can be explained in a natural way within a unified framework of choice with incomplete preferences. The novel element in the choice procedure that we proposed for this purpose is the combination of the criteria of total preference undomination and partial preference dominance. Although the proposed model is one of bounded-rational choice, the sole source of bounded rationality is the incompleteness of the decision maker's preferences. As such, in decision problems where the agent's preferences over the feasible alternatives happen to be complete, the model prescribes and predicts utility-maximising behaviour. The theoretical framework developed in this paper is therefore able to explain the above three phenomena with minimal deviations from fully rational behaviour.

Appendix

Proof of Proposition 3:

Suppose there exists an acyclic relation $\succ$ on X such that (3) holds. Let $C(A) = \emptyset$ for some $A \in \mathcal{M}$. Clearly, $C(A) \subset A$. It follows then from (3b) that there is no $x \in A$ such that (i) $z \not\succ x$ for all $z \in A$, and (ii) $x \succ y$ for some $y \in A$. Since $\succ$ is acyclic, there exists $x \in A$ that satisfies (i). It follows then that there is no $x \in A$ satisfying (ii). These two facts in turn imply that $x \not\succ y$ and $y \not\succ x$ for all $x, y \in A$. But in this case (3a) ensures that $C(A) = A$. Since $A \neq \emptyset$ for all $A \in \mathcal{M}$, this is a contradiction. Hence, DEC is satisfied.

Assume that ReWARP is violated. There exist $x, y \in X$ and $A \in \mathcal{M}$ such that $C(\{x, y\}) = x$, $x \in A$ and $y \in C(A)$. From $C(\{x, y\}) = x$ and (3b) it follows that $x \succ y$. Since $y \in C(A)$ it also follows from (3a) or (3b) (if $C(A) = A$ or $C(A) \subset A$, respectively) that $x \not\succ y$. This is a contradiction.

Now let $A_1, \ldots, A_k \in \mathcal{M}$ be such that $x = C(A_1)$, $|A_1| > 1$ and $x \in C(A_i)$ for all $i \geq 2$. Define $A := \bigcup_{i=1}^{k}$. It is implied by (3b) and $C(A_1) = x$ that $z \not\succ x$ for all $z \in A_1$ and $x \succ y$ for some $y \in A_1$. Moreover, $x \in C(A_i)$ for $i \geq 2$ and (3a) or (3b) (depending on whether $C(A_i) = A_i$ or $C(A_i) \subset A_i$, respectively) imply $z \not\succ x$ for all $z \in A_i$, for all $i = 2, \ldots, k$. Thus, $z \not\succ x$ for all $z \in A$ and $x \succ y$ for some $y \in A$. From (3b), this implies $x \in C(A)$. Hence, WEXP is satisfied.

Finally, let $A \in \mathcal{M}$ be such that $x \in C(A)$ and $C(A) \subset A$. From (3b) it follows that $x \succ y$ for some $y \in A$. Since $\{x, y\} \in \mathcal{M}$, $x \succ y$ and (3b) together imply $C(\{x, y\}) = x$.

This proves that WPC is also satisfied.

Conversely, assume that DEC, ReWARP, WEXP and WPC hold. Define the relation $\succ$ on X by $x \succ y$ if $C(\{x,y\}) = x$. By definition, $\succ$ is asymmetric. Suppose $\succ$ is not acyclic. There exist $x_1, x_2, \ldots, x_k \in X$ such that $x_1 \succ x_2 \succ \ldots \succ x_k \succ x_1$. By assumption, $B := \{x_1, x_2, \ldots, x_k\} \in \mathcal{M}$, while DEC implies $C(B) \neq \emptyset$. Thus, $x_i \in C(B)$ for some $i \leq k$. From the $\succ$-cycle above it follows that there is $x_j \in B$ such that $x_j \succ x_i$, i.e. $C(\{x_i, x_j\}) = x_j$. This contradicts ReWARP.

Suppose $A \in \mathcal{M}$ is such that $x \not\succ y$ and $y \not\succ x$ for all $x, y \in A$. From DEC, $C(A) \neq \emptyset$. Let $x \in C(A)$ and $C(A) \subset A$. Repeated application of WPC shows that $x \succ y$ for some $y \in A$. This is a contradiction. It holds, therefore, that $C(A) = A$. In the other direction, let $C(A) = A$ and suppose $x \succ y$ for some $x, y \in A$. Since $y \in C(A)$ by assumption, this is a violation of ReWARP. Hence, (3a) is established.

To also establish (3b), let $C(A) \subset A$ for some $A \in \mathcal{M}$. Suppose first that $x \in C(A)$. If $z \succ x$ for some $z \in A$, then ReWARP is violated. Let $x \not\succ y$ for all $y \in M$. WPC implies that $x = C(B)$ for some $B \subseteq A$ with $|B| > 1$. Applying WPC repeatedly yields $x = C(\{x, w\})$ for some $w \in A$, which is equivalent to $x \succ w$. This is a contradiction. Thus, $x \in C(A)$ and $C(A) \subset A$ implies $z \not\succ x$ for all $z \in A$ and $x \succ y$ for some $y \in A$. In the other direction, suppose there is $x \in A$ such that $z \not\succ x$ for all $z \in A$ and $x \succ y$ for some $y \in A$. This implies $C(\{x,y\}) = x$ for some $y \in A$ and, in view of DEC, $x \in C(\{x,z\})$ for all $z \in A$. Let $F := \{x, y\}$ and label all other elements in A by $z_1, \ldots, z_k$. Also, let $G_i := \{x, z_i\}$. Since $x = C(F)$, $x \in \bigcap_{i=1}^k C(G_i)$ and $\bigcup_{i=1}^k G_i \cup F = A$, it follows from WEXP that $x \in C(A)$. Suppose, finally, that $C(A) = A$. Then $y \in C(A)$, $x \in A$ and $C(\{x,y\}) = x$ contradicts ReWARP. Hence, $C(A) \subset A$. ■

Proof of Proposition 4:

Suppose there is an asymmetric relation $\succ$ on X such that (4) holds. Let $x, y \in X$ be such that $C(\{x,y\}) = x$ and let $x \in A$ and $y \in C(A)$. It follows from $C(\{x,y\}) = x$ and (4) that $x \succ y$. From $y \in C(A)$ and (4) it also holds that $x \not\succ y$, a contradiction. Hence, ReWARP is satisfied. Next, let $w \in C(B)$ for some $B \in \mathcal{M}$. It is implied by (4) that $w \succ y$ for some $y \in B$, and therefore, again from (4), $C(\{w,y\}) = w$. Thus, SPC is satisfied too.

Now let A_1 be such that $|A_1| > 1$ and $x = C(A_1)$, and consider a collection $A_2, \ldots, A_k \in \mathcal{M}$. Suppose that, for all $i = 2, \ldots, k$, either $x \in C(A_i)$ or $C(S_i) = \emptyset$ for all $S_i \subseteq A_i$ such that $|S_i| > 1$ and $x \in S_i$. In view of (4), it holds in both cases that $z \not\succ x$ for all $z \in A_i$ and all $i = 2, \ldots, k$. Moreover, (4) also implies $z \not\succ x$ for all $z \in A_1$ and $x \succ y$ for some $y \in A_1$. Let $A := \bigcup_{i=1}^k A_i$. The above implies $z \not\succ x$ for all $z \in A$ and $x \succ y$ for some $y \in A$. It follows then from 4) that $x \in C(A)$. This shows that WEXP is satisfied too.

Conversely, suppose C obeys WEXP, SPC and ReWARP. Again, define the asymmetric relation $\succ$ on X by $x \succ y$ if $C(\{x,y\}) = x$. It follows from ReWARP that $x \in C(A)$ implies $z \not\succ x$ for all $y \in A$. It also follows from repeated application of SPC that $x \in C(A)$ implies $x \succ y$ for some $y \in A$. In the other direction, suppose that $x \in A$ is such that $z \not\succ x$ for all $z \in A$ and $x \succ y$ for some $y \in A$. It holds that $C(\{x,y\}) = x$ for some $y \in A$ and, in view of SPC, either $C(\{x,z\}) = x$ or $C(\{x,z\}) = \emptyset$ for all other $z \in A$. Label all such $z \in A$ as $z_1, \ldots, z_k$ and let $F := \{x, y\}$ and $G_i := \{x, z_i\}$, $i \leq k$. Since $C(F) = x$, $x = C(G_i)$ or $C(G_i) = \emptyset$ for all $i \leq k$, and $A := \bigcup_{i=1}^k G_i \cup F$, it is implied by WEXP that $x \in C(A)$. This establishes (4). ■

Proof of Proposition 5:

(a) $\implies$ (b): Suppose DEC, SPC and ReWARP are satisfied and WARP is not. Then, $x \in C(A)$, $y \in A \setminus C(A)$, $y \in C(B)$ and $x \in B$ for some $x, y \in X$ and $A, B \in \mathcal{M}$. Consider $\{x, y\} \in \mathcal{M}$. DEC and SPC imply $C(\{x, y\}) = x$ or $C(\{x, y\}) = y$. In the former case the postulate $y \in C(B)$ and $x \in B$ is contradicted, whereas in the latter case the postulate $x \in C(A)$ and $y \in A$ is contradicted.

(b) $\implies$ (c): Suppose DEC, SPC and WARP are satisfied and that C is not single-valued. There exists $A \in \mathcal{M}$ such that $x, y \in C(A)$ for some distinct $x, y \in A$. As before, from DEC and SPC we have $C(\{x, y\}) = x$ or $C(\{x, y\}) = y$. In view of ReWARP, either case contradicts the postulate $x, y \in C(A)$.

(c) $\implies$ (d): The argument here is well-known.

(d) $\implies$ (a): Suppose C satisfies (5) and let $\succ$ be the strict linear order with which it does so. Let C violate DEC. There exists $A \in \mathcal{M}$ such that $C(A) = \emptyset$. Since $\succ$ is a strict linear order, there is a unique $x \in A$ such that $y \not\succ x$ for all $y \in A$. It follows from (5) that $C(A) = x$, a contradiction. Suppose now that C violates ReWARP. There exist $x, y \in X$ and $A \in \mathcal{M}$ such that $C(\{x, y\}) = x$, $x \in A$ and $y \in C(A)$. Since $\succ$ is a strict linear order and (5) holds, $C(\{x, y\}) = x$ implies $x \succ y$, while $x \in A$ and $y \in C(A)$ implies $x \not\succ y$, a contradiction. Finally, let $x \in C(A)$ for some $A \in \mathcal{M}$. Since $\succ$ is a strict linear order and (5) holds, we have $x \succ y$ for all $y \in A$, and therefore $C(\{x, y\}) = x$ for all such y. This shows that SPC is also obeyed by C.

Proof of Proposition 6:

Suppose there is an acyclic relation $\succ$ on X such that (6a)–(6d) hold. The relevant argument in the proof of Proposition 3 shows that (6a), (6b) and acyclicity of $\succ$ together imply DEC. Similarly, the relevant arguments in that proof also show that (6a) and (6b) imply WPC and ReWARP for $p = \diamond$.

Now consider the case of ReWARP when $p \neq \diamond$. If the axiom is violated, there exist $x, s \in X$ and $A \in \mathcal{M}$ such that $C(\{x, s\}, s) = x$ and $s \in C(A, s)$. It follows from (6c) and $C(\{x, s\}, s) = x$ that $x \succ s$. It also follows from (6d) and $s \in C(A, s)$ that $x \not\succ s$. This is a contradiction.

Next, let $A_1, A_2, \ldots, A_k \in Z$ be such that $A_1 = \{x, s\}$, $x = C(A_1, \diamond)$ and $x \in C(A_i, \diamond)$ for all $i = 2, \ldots, k$. The former postulate and (6b) together imply $x \succ s$. The latter postulate and (6b) together imply $z \not\succ x$ for all $z \in A_i$ and all $i = 2, \ldots, k$. Thus, $z \not\succ x$ for all $z \in \bigcup_{i=1}^{k} A_i := A$ and $x \succ s$. In view of (6b) and (6c) this implies $x \in C(A, \diamond)$ and $x \in C(A, s)$ respectively, which shows that AWEXP is also satisfied.

Now consider $(A, s) \in Z$ and suppose $x \in C(A, s)$ for some $x \neq s$. Since it follows from (6c) that $x \succ s$, it also follows from (6c) that $C(\{x, s\}, s) = x$. This shows that FSQI (specifically, its contrapositive) is satisfied. Moreover, in view of (6c), $x \in C(A, s)$ and $x \neq s$ also implies $z \not\succ x$ for all $z \in A$. It follows then from (6b) that $x \in C(A, \diamond)$ too, which proves that SQIC is also satisfied.

Finally, to show that C obeys CSI let $C(\{x, s\}, s) = x$. From (6c), this implies $x \succ s$, which, in view of (6b) also implies $C(\{x,s\},\diamond) = x$. Conversely, suppose $C(\{x,s\},\diamond) = x$. From (6b), $x \succ s$. It is implied by (6c) that $C(\{x,s\},s) = x$ too.

Suppose now that C is consistent with the axioms and define the asymmetric relation $\succ$ on X by $x \succ y$ if $C(\{x,y\},\diamond) = x$. The relevant argument in the proof of Proposition 3 that establishes acyclicity of $\succ$ from DEC, ReWARP and the full domain assumption is valid here too. Hence, $\succ$ is acyclic.

Next, let $x \in C(A,\diamond)$. Repeated application of WPC eventually gives $C(\{x,y\},\diamond) = x$ and therefore $x \succ y$ for some $y \in A$. Moreover, if $z \succ x$ for some $z \in A$, then ReWARP is violated. Hence, $z \not\succ x$ for all $z \in A$. Conversely, suppose $(A,\diamond) \in Z$ is such that $x \succ y$ for some $y \in A$ and $z \not\succ x$ for all $z \in A$. Let $A_1 := \{x,y\}$ and label all elements of A other than x and y by $z_2,\dots,z_k$. For all $i \geq 2$, define $A_i := \{x, z_i\}$. We have $C(A_1,\diamond) = x$. Moreover, in view of DEC and $z \not\succ x$ for all $z \in A$, we also have $x \in C(A_i,\diamond)$ for all $i = 2,\dots,k$. Since $A = \bigcup_{i=1}^{k} A_i$, it follows from AWEXP that $x \in C(A,\diamond)$. This establishes (6b).

Now suppose $x \in C(A,s)$ and $x \neq s$. If $C(\{x,s\},s) \neq x$, then FSQB is violated. Thus, $C(\{x,s\},s) = x$. CSI implies $C(\{x,s\},\diamond) = x$ too, and therefore $x \succ s$. Now let $z \succ x$ for some $z \in A$. Since $x \in C(A,s)$ and $x \neq s$, SQIC implies $x \in C(A,\diamond)$. But since $z \succ x$ and $z \in A$, this contradicts ReWARP. Conversely, suppose $(A,s) \in Z$ is such that $x \succ s$ and $z \not\succ x$ for all $z \in A$. By defining $A_1 := \{x,s\}$ and then repeating the AWEXP-based argument in the previous paragraph one obtains $x \in C(A,s)$ and hence establishes (6c).

Finally, let (A,s) be such that $z \not\succ s$ for all $z \in A$. Suppose $x \in C(A,s)$ for some $x \neq s$. It is implied by FSQB that $C(\{x,s\},s) = x$, which, in view of CSI, contradicts $z \not\succ s$ for all $z \in A$. Then, since it holds by construction that $C(A,s) \neq \emptyset$ for all $(A,s) \in Z$, it follows that $C(A,s) = s$. Conversely, suppose $C(A,s) = s$ and assume that $z \succ s$ for some $z \in A$. CSI implies $C(\{z,s\},s) = z$, which, together with ReWARP, implies $s \notin C(A,s)$. This contradiction establishes (6d). ■

Proof of Proposition 7

It is known from Propositions 4 and 6 that the axioms are necessary and sufficient for (7a)–(7c) to hold with respect to an asymmetric (in the case of (7a)) and an acyclic (in the case of (7b)–(7c)) relation, respectively. It remains to be shown that under the stated axioms the relation $\succ$ defined by $x \succ y$ if $C(\{x,y\},\diamond) = x$ is acyclic.

Suppose not. There exist $x_1, x_2, \dots, x_k \in X$ such that $x_1 \succ x_2 \succ \dots \succ x_k \succ x_1$. Let $A := \{x_1,\dots,x_k\}$. Suppose $x_i \in C(A,\diamond)$ for some $x_i \in A$. Since $x_{i-1} \succ x_i$ (or, if $i = 1$, $x_k \succ x_1$), this contradicts ReWARP. Hence, $C(A,\diamond) = \emptyset$. Next, consider $(A, x_i) \in Z$ for some $x_i \in A$. By assumption, $C(A,x_i) \neq \emptyset$. Suppose $x_j \in C(A,x_i)$ for some $x_j \neq x_i$. Since $x_j \notin C(A,\diamond) = \emptyset$, this contradicts SQIC. Now suppose $x_i \in C(A,x_i)$. Since $x_{i-1} \succ x_i$, we have $C(\{x_{i-1},x_i\},\diamond) = x_{i-1}$ and, from CSI, $C(\{x_{i-1},x_i\},x_i) = x_{i-1}$. Since $x_{i-1} \in A$, this contradicts ReWARP. ■

Proof of Proposition 8:

Suppose $C : Z \twoheadrightarrow X$ satisfies CSI, DEC, ReWARP, SPC and SQIC. Define the relation $\succ$ as before, i.e. $x \succ y$ if $C(\{x,y\},\diamond) = x$. We know from Proposition 5 that (8) holds for all $(A,\diamond) \in Z$. By way of contradiction, let there be $(A,s) \in Z$ for which (8) does not hold. Then, $C(A,s) \neq \{x \in A : y \not\succ x \text{ for all } y \in A\}$.

By construction, $C(A,s) \neq \emptyset$. Moreover, it is known from Proposition 5 that DEC, ReWARP and SPC ensure that $\succ$ is a strict linear order. Thus, there exists $x \in A$ such that $x \succ y$ for all $y \in A$, $x \neq y$. Suppose $s \in C(A,s)$. It is true that $y \succ s$ for some $y \in A$. In view of CSI, this implies $C(\{y,s\},s) = y$. Since $s \in C(A,s)$ and $y \in A$, this contradicts ReWARP. Now suppose $w \in C(A,s)$ for some $w \neq s$. As before, $y \succ w$ holds for some $y \in A$. It is implied by SQIC that $w \in C(A,\diamond)$. Since $y \succ w$, ReWARP is contradicted. This establishes that (8) holds for all $(A,s) \in Z$ as well.

Conversely, let there be a strict linear order $\succ$ such that (8) holds. For all problems $(A,\diamond) \in Z$ with $A \in \mathcal{M}$ we know from Proposition 5 that DEC, SPC and ReWARP are satisfied. Suppose ReWARP is violated for a problem (A,s) with $s \in A$. Then $C(\{x,s\},s) = x$ for some $x \in A$ and $C(A,s) = s$. It follows from (8) and $C(\{x,s\},s) = x$ that $x \succ s$. It also follows from (8) and $C(A,s) = s$ that $x \not\succ s$, which is a contradiction. Thus, ReWARP is satisfied for both types of problems. To show that C also obeys CSI consider $x, y \in X$ and let $C(\{x,y\},\diamond) = x$ and $C(\{x,y\},y) \neq x$ or $C(\{x,y\},y) = x$ and $C(\{x,y\},\diamond) \neq x$. Since (8) holds, the first statement in both cases implies $x \succ y$ and the second implies $x \not\succ y$, a contradiction. Finally, suppose $C(A,s) = x$ for some $x \neq s$. It follows from (8) and the fact that $\succ$ is a strict linear order that $x \succ y$ for all $y \in A$, $y \neq x$. In view of (8) again, $C(A,\diamond) = x$ too. Thus, C is also consistent with SQIC. ■

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