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Abstract
Rising deficits and high debt ratios characterized currency crises in countries with low private savings rates and low population densities. But in emerging markets with large population transferring to more productive employment, sustainable debts and deficits may be higher. Debt ratios fall with growth rates. Higher private savings can compensate for government dissaving. An optimizing model of such an economy with dualistic labour markets and two types of consumers demonstrates these features but also shows debt ratios tend to rise in high growth phases. Policy conclusions for fiscal consolidation and coordination with monetary policy are derived in the Indian context.

JEL Codes: H63, E62, E52, D90
Key words: Deficits; Debt; Sustainability; Monetary-Fiscal Policy; Emerging Markets

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1. Introduction

Fund managers are trained to be doubtful about an emerging market with rising fiscal deficits and high debts ratios. But we argue risk assessments must be context sensitive. The Indian government’s fiscal deficit rose sharply in 2008-09 exceeding its targeted value by about 4 percent, although before that the government was on target to achieve the steady reduction in deficits as mandated by the 2003 Fiscal Responsibility and Budget Management (FRBM) Act. The lapse was partly driven by oil shock subsidies, unbudgeted pay commission awards and poor expenditure management. But it was required because of large external demand shocks, and a fear cum earlier domestic monetary tightening led decline in demand, which reduced output much below potential.

To associate high Indian deficit ratios with higher risk is to extrapolate unconditionally from past crises in Latin American countries where these features were found together. These countries had low savings rates and low population densities. In India higher private savings compensates for government dissaving. In this paper we show how high private saving reduces the current account deficit. Therefore twin deficits that provoke outflows become unlikely.

Fiscal stimuli contributed to making India one of the brighter spots in a dismal globe. There were fears the government may not have the capacity to reverse rising deficits, and with debt ratios already at 80 percent, an unsustainable rise in debt may result. But as growth revived, India became one of the first countries to resume fiscal consolidation in 2010. The second exercise in this paper shows how higher growth rates reduce debt ratios. In high population density emerging markets (EMs) in a catch-up phase, growth can be expected to rise. Examining India’s dynamic government budget constraint shows some decrease in debt in high growth periods. But in the post 2003 high growth phase this was less than feasible, despite the notification of the FRBM Act.

Third, introducing the dynamic debt equation in a DSGE open economy model used for monetary policy helps to explain these outcomes, and to derive optimal monetary policy
and its interaction with fiscal policy. This approach is relevant for the analysis of EMs because forward looking behaviour is combined with frictions that are important for EMs, DSGE models’ focus on labour productivity is appropriate especially during transition, and allowing policy to respond to outcomes rather than follow rules is useful in EMs where uncertainty is more. Backwardness in technology, infrastructure and education reduce potential output in EMs, but it rises in transition, and the correct fiscal interventions can support it. Our adaptation distinguishes between two types of consumers and suppliers of labour. It is therefore able to capture the effects of transition in populous EMs as the labour share in the more productive occupations rise, and to model the typical shocks.

In response to shocks to subsistence consumption the model throws up a large increase in debt. The rise is more if debt levels are already high, and if there is higher growth. This is the exact Indian fiscal response during the high growth period and during the financial crisis. Monetary accommodation reduces debt levels but can increase instability if there is a permanent rise in government expenditure. Higher transitional growth does reduce per capita debt ratios, but since the expenditure response can overwhelm it, some restrictions are required on expenditure.

Implications for Indian macroeconomic policy, explored in the final section of the paper, are revamping the Fiscal Responsibility and Budget Management (FRBM) legislation to emphasize expenditure caps, rather than deficits, thus improving compliance incentives, and shifting expenditure to components that enhance the supply response. With such fiscal consolidation, monetary policy can safely support growth.

The structure of the paper is as follows. Section 2 gives a contextual assessment of the twin deficit hypothesis, Section 3 analyses the factors affecting the evolution of government debt, Section 4 introduces fiscal policy in a model for optimal monetary policy: Section 4.1 presents the model, Section 4.2 adds government debt to it, Section 4.3 presents the simulation results, and Section 4.4 derives monetary and fiscal policy
combinations, Section 5 applies the analysis to Indian government deficits, before
Section 6 concludes. Details of the model are given in an appendix.

2. Twin Deficits and private savings
The balance of payments is the record of a country’s foreign transactions in an open
economy. Assume, for the time being, there is no income from abroad and no
government, then the basic macroeconomic income equal to expenditure identity is
\( Y = C + I + X - M \). Net exports are also the trade surplus, in this case equal to the current account
surplus of the balance of payments, \( CA \). Thus \( CA = X - M = NX \). Substituting savings, \( S \),
for \( Y - C \), in the basic identity, gives \( S - I = X - M \). Thus the \( CA \) is also equal to \( S - I \). The
popular conception of \( CA \) is identified with the net exports of goods and services. But
this derivation makes it clear that the \( CA \) depends on macro policies affecting \( S \) and \( I \), not
only on trade policies. If domestic savings exceed investment this must be reflected in
exports exceeding imports and vice versa. If investment exceeds savings there would be a
current account deficit: \( CAD = I - S = M - X = NM \).

On introducing government, the \( CAD \) can be derived as the excess of \( I \) over \( S \) plus the
excess of government expenditure over taxes or the government deficit (\( GD \)). From
\( Y - T - C + T - G - I = X - M \), we get \( S = S^P + T - G = I + NX \) or \( I = S^P + S^G + NM \). That is:

\[
CAD = I - S^P + GD
\]  

So as government expenditure exceeds taxes, unless \( I \) falls or \( S^P \) rises, the \( CAD \) will
widen. This is the twin deficits hypothesis, where one deficit is expected to lead to the
other, so that both can be expected to occur together. But a government deficit need not
necessarily imply positive \( CAD \) if private savings are high.

\[
S^P = I + CA + G - T
\]  

In an open economy, private savings can be used for domestic investment, acquiring
assets from foreigners and for buying government debt. In many Asian countries a rise in
income tends to raise savings more than consumption. In boom times investment may exceed savings but only marginally. For example, the current account deficit, which finances the difference between investment and domestic savings, remained around 1 percent of GDP in India. Capital flows much larger than the current account deficit were accumulated as reserves. With these cushions of domestic and foreign resources available, temporary government dissaving is not threatening.

3. Evolution of Government Debt

The other danger from a government deficit is that it adds to government debt. We turn, therefore, to examine the evolution of government debt. Assume a cashless economy in which all government debt consists of riskless one-period nominal debt. The maturity value of nominal government debt is $B_tP_t$. This changes over time as follows:

$$ B_tP_t = (1 + i_t)B_{t-1}P_{t-1} + (P_tG_t - T_t) \tag{3} $$

The maturity value of real public debt is $B_t$: Real government purchases are $G_t$ and nominal net tax collections are $T_t$ so that real tax collections are $\tau = T_t/P_t$. The real debt to output ratio is $b_t$. Dividing by $Y_t$, and making other manipulations, (3) can be written as:

$$ \frac{B_t}{Y_t} = (1 + i_t) \frac{B_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} \frac{P_{t-1}}{P_t} + \frac{G_t}{Y_t} \frac{P_t}{P_{t-1}} T_t \tag{4} $$

And further using $1 + g_t = Y_t/Y_{t-1}$, $1 + \pi_t = P_t/P_{t-1}$ and the approximation:

$$ (1 + i_t)/(1 + g_t)(1 + \pi_t) = 1 + i_t - g_t - \pi_t \tag{5} $$

We get:
\[ b_t - b_{t-1} = (i_t - \pi_t - g_t) b_{t-1} + \frac{G_t}{Y_t} - \frac{\tau_t}{Y_t} \]  \hspace{1cm} (6)

Equation (6) gives the evolution of the real debt ratio. Higher debt increases real debt as interest payments rise, as does the primary deficit ratio \((pd)\) or excess of real government expenditure over taxation as a ratio to output. Therefore high debt levels can imply exploding unsustainable debt. Falling real interest rates and rising growth rates effectively reduce government debt. Inflation and growth rates do not affect the nominal value of public debt, \(B_t P_t\), which increases in any year by nominal interest payments on debt plus the PD, \(P_t G_t - T_t\). The latter is the non-interest budget deficit, while the fiscal deficit includes interest payments and is the total government borrowing requirement. If the real interest rate equals the rate of growth, the PD ratio alone would add to the debt ratio.

4. Introducing Fiscal Policy in the SOEME Model

So far we have considered the effects of fiscal policy on the government debt taking the interest and inflation rate as given. But the latter are determined by monetary policy. So we turn to examine (i) what are the restrictions on fiscal policy, which will allow optimal choice of interest rates in the context of a small open emerging market economy (SOEME), and (ii) if the structure of a SOEME gives greater degrees of freedom for policy.

4.1 Optimal monetary policy SOEME model

Optimal monetary policy has been derived in dynamic stochastic general equilibrium models with imperfect competition and nominal rigidities\(^1\), and is found to have substantial effects on real variables. Under monopolistic competition with product diversity individual producers have market power implying output is suboptimally low. This, together with some type of price stickiness, allows monetary policy to have real effects. This framework is promising for the analysis of EMs since the rigidities that dominate these markets can be introduced. Optimization over time by consumers,

workers and firms can be reduced to simple aggregate demand and supply curves with forward-looking variables. Being derived from basic technology, preferences and market structure, the coefficients of the equations are robust to policy changes, thus meeting the Lucas critique. The policy problem then simplifies to minimization of the deviation of output and inflation from steady-state values subject to these curves.

Goyal (2007, 2009, 2010) adapted the basic model\(^2\) to make it relevant to analyze monetary policy in EMs with a large share of less productive labour in the process of being absorbed into the modern sector. The steady-state full employment assumption of equilibrium models is far from adequate in these markets. At the very least, two types of consumers and workers need to be distinguished in the SOEME—those above subsistence (R), and those at subsistence (P). While the first are able to smooth consumption using international markets, those at subsistence cannot. Their intertemporal elasticity of consumption, productivity and wages are lower and their labor supply elasticity is higher, compared to the first group. All these follow from the key difference—high and low productivity. The product market structure, technology and preferences of R type consumers are the same across all economies. Productivity shocks differ since EMs are in transition stages of applying the new technologies becoming available. P type consumers are assumed to be at a fixed subsistence wage, financed in part by transfers from R types, mediated by the government.

The basic consumption Euler, household labor supply, risk sharing, aggregate equilibrium, and firms’ profit maximization is derived in the appendix following Galli and Monacelli (2005). Given first order conditions (FOCs), risk sharing only for the R type, exogenous subsistence level consumption of the P type, and the aggregate demand supply equality across countries, each of measure unity, the terms of trade, \( S_t \), are solved in terms of endogenous output, \( Y_t \), and exogenous variables, world output, \( Y_t^* \) and the consumption of the P type, \( C_{P,t} \). Substituting out the terms of trade, and taking deviations of output from the natural output, \( y_t - \bar{y}_t = x_t \), to write the FOCs as functions of log

\(^2\) The basic Gali and Monacelli (2005) (henceforth GM) SOE model is adapted to a SOEME by differentiating between the R and P types. Goyal (2007) offers a systematic comparison of results for a SOEME contrasted with a SOE.
output gap, \( x_t \), and domestic inflation, \( \pi_t \), then gives the final form of the two aggregate supply (AS) and aggregate demand (AD) equations. The level where marginal cost is at its desired steady-state level defines the natural output \( y^*_t \). Low productivity, poor infrastructure and other distortions keep the natural output in the SOEME below world levels and convergence to world levels is part of the process of development.

The intertemporal elasticity of consumption is \( (1/\sigma_i) \), labour supply elasticity \( (1/\varphi_i) \) with the subscript indicating the R or P type respectively; without the subscript it is the aggregate value. The population share of R is \( \eta \), \( 0<\eta<1 \). The share of foreign goods, \( \alpha \), \( 0<\alpha<1 \). The discount factor is \( \beta \), so that \( \rho \equiv \beta^{-1} - 1 \) is the time discount rate; \( i_t \) is the riskless nominal interest rate; \( \pi_t \equiv p_t - p_{t-1} \) is CPI (consumer price index) inflation (where \( p_t \equiv \log P_t \)); productivity is \( a_t \). It is easy to derive \( \pi_t = \pi_{H,t} + \alpha \Delta s_t \), where \( s_t = p_{F,t} - p_{H,t} \) is the log effective terms of trade or price of foreign goods in terms of domestic goods. These identities allow transformation of consumer to domestic price inflation and vice versa. Consumer prices, which enter the consumer’s maximand, have to be converted into producer or domestic prices. Lower case letters are logs of the respective variables.

The quadratic loss function (7) of the central bank (CB) is a weighted average of inflation, output and interest rate deviations from equilibrium values:

\[
L = q_Y x^2 + q_\pi \pi^2 + q_i i^2
\]  

(7)

The last is a smoothing term that prevents large changes in the policy rate. The CB minimizes this subject to the AD (8), AS (9) and the law of motion for real public debt (10). The last is derived in Section 4.2. Dynamic impulse responses to cost, natural rate, and government expenditure shocks, generated from the optimization, are presented in Section 4.3.

The dynamic AD equation for the SOEME is:
\[ x_t = E_t \{ x_{t+1} \} - \frac{1}{\sigma_D} \left( i_t - E_t \{ \pi_{H,t+1} \} - \bar{r}_t \right) \]  

Where \( \bar{r}_t = \rho - \sigma_D \left[ (1 - \rho_D) \sigma_D \right] \pi_t - \sigma_D \left( (1 - \eta + \Phi) E_t \{ \Delta c_{P,t+1} \} + \sigma_D \left( \Theta - \Psi \right) E_t \{ \Delta y_{t+1}^* \} \right) \)

and 

\[ \Theta = \alpha \left( \sigma - \eta \right), \quad d = \frac{1}{\sigma_D + \varphi}, \quad \Gamma = \frac{1 + \varphi}{\sigma_D + \varphi}, \quad \Psi = \eta \left( \sigma - \sigma_D \right) d, \]

\[ \Phi = d \left( (1 - \eta) \left( \sigma - \sigma_D \right) \right), \quad \sigma = \sigma_R + (1 - \alpha) \left( \sigma_R - 1 \right) \]

The dynamic AS and the change in public debt respectively are:

\[ \pi_{H,t} = \gamma_f \beta E_t \{ \pi_{H,t+1} \} + \kappa_D x_t + \gamma_b \pi_{H,t-1} \quad \gamma_f + \gamma_b = 1 \]

\[ \hat{b}_t = \beta^{-1} \left[ (1 - \tau_b) \bar{p}_{t-1} - \bar{b} \pi_t - \bar{b} g + (1 - \tau_g) \hat{g}_t \right] + \bar{b} \hat{i}_t \]

The natural interest rate, \( \rho \), is defined as the equilibrium real rate, consistent with a zero or target rate of inflation, when prices are fully flexible. Shocks that change the natural rate open an output gap and affect inflation. The shock or exogenous term \( \bar{r}_t \) that enters the AD is therefore the percentage deviation of the natural rate from its steady-state value. The deviation occurs due to real disturbances that change natural output; \( \bar{r}_t \) rises for any temporary demand shock and falls for any temporary supply shock. Optimal policy requires insulating the output gap from these shocks, so that the CB’s interest rate instrument should move in step with the natural rate. Thus the CB would accommodate positive supply shocks that raise the natural output by lowering interest rates. It would offset positive demand shocks that raise output about its potential by raising interest rates. In a SOEME a reduction in \( c_p \) is an additional large shock requiring reduction in the policy rate, since it increases the distance from the world consumption level (see Goyal 2009).

The dynamic AS derived in the appendix is:

\[ \pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa_D x_t \]

Since empirical estimations and the dominance of administered pricing in SOEMEs suggest that past inflation affects current inflation, the modification of the AS (9) was
used in the simulations, with $\gamma_b$ as the share of lagged and $\gamma_f$ the share of forward-looking inflation.

The slope of the AS for a SOEME is $\kappa_D = \lambda(\sigma_D + \varphi)$, compared to $\lambda (\sigma + \varphi)$ for a closed economy and $\lambda (\sigma_a + \varphi)$ for a SOE, where $\sigma_a = \frac{\sigma_R}{(1 - \alpha) + \sigma \alpha}$, $\sigma_D = \frac{\sigma_R}{\eta(1 - \alpha) + \sigma \alpha}$, $\sigma = \frac{\sigma_R}{\eta}$. Since R in the SOEME are identical to the representative SOE consumer, $\sigma_R$ is the numerator of $\sigma_a$ and $\sigma_D$. The slope is reduced in an open compared to a closed economy since $\sigma > \sigma_D > \sigma_a$, but the slope can be higher in the SOEME compared to a SOE, even though $\varphi$ is lower for the SOEME, since $\sigma_D > \sigma_a$. While $\sigma_a$=1 if $\sigma_R=1$, $\sigma_D$ always exceeds unity if $\alpha < 1$. The inequalities follow from the parameter values. In particular $\eta < 1$ reduces the denominator of $\sigma_D$ and $\sigma$. Similar results hold for the more general case of $\sigma_R \neq 1$. Since the gap between is large and varies with $\eta$ and $\alpha$, the slope for the SOEME remains larger than in the SOE.

Since $\sigma_D > \sigma_a$, the output gap, just like output, is less responsive to the excess of the policy rate over the natural interest rate in the SOEME compared to the SOE, while shocks to subsistence consumption, $c_P$, are a new source of shocks tending to reduce the natural rate below $\rho$ in a SOEME. From (8), if the policy rate exceeds the natural interest rate, AD would fall.

As $\eta$ approaches unity, or all the population reaches higher consumption levels, it implies the economy has developed, the $c_P$ term disappears and the equations collapse to those of the SOE.

4.2. Government debt

Next we derive the equation for the response of government debt to a shock that could include change in fiscal policy. Since we are interested in local equilibrium determination
it is sufficient to consider fiscal rules that are nearly consistent with a steady state. In a stable state with zero inflation and real disturbances both \( B_t \) and real tax collections \( \tau \equiv T_t / P_t \) are equal to values \( \bar{\tau}, \bar{B} > 0 \), and \( Y_t = \bar{Y} > 0, G_t = \bar{G} \geq 0 \), growing at a steady-state growth \( g \), and \( i_t = \bar{i} \equiv \beta^{-1} - 1 > 0 \). For consistency with the evolution of nominal debt (3), steady-state fiscal values must satisfy \( \bar{\tau} = \bar{G} + (1 - \beta)\bar{B} \).

To analyze the existence of equilibria near this steady state (3) can be linearized around the steady-state values, getting:

\[
\hat{b}_t = \beta^{-1} \left[ \hat{b}_{t-1} - \bar{b} \pi_t - \bar{b} g + \hat{G}_t - \hat{\pi}_t \right] + \bar{b} \hat{i}_t
\]

(12)

Where \( \hat{b}_t = (B_t - \bar{B})/\bar{Y} \), \( \hat{\pi}_t = (\pi_t - \bar{\pi})/\bar{Y} \), and \( \hat{G}_t = (G_t - \bar{G})/\bar{Y} \), so that \( \hat{i}_t = i_t - \bar{i} \) and \( \bar{b} \equiv \bar{B}/\bar{Y} \).

The term in steady-state growth \( g \) in equation (12) comes from following a process similar to the derivation of (6) from (4), by assuming a steady-state rate of growth of natural output \( \bar{y}_i \) so that \( \bar{y}_i = \bar{y}_{t-1} \). Such a growth is to be expected for an emerging market in the process of converging to world output levels.

Woodford (2003, pp.312) defines a fiscal or tax rule as locally Ricardian if on substituting into the flow budget constraint (6) or its local version (12) “it implies that \( \{b_t\} \) remains forever within a bounded neighborhood of \( \bar{B} \), for all paths of the endogenous variables \( \{\pi_t, Y_t, i_t\} \) that remain forever within some sufficiently small neighborhoods of the steady-state values \( (0, \bar{Y}, \bar{i}) \), and all small enough values of the exogenous disturbances (including \( \hat{G}_t \)).”

Under these circumstances the monetary policy rule and the outcomes of equilibrium inflation, output and interest rates do not depend on the paths of either of the purely fiscal variables \( \{B_t, \tau_t\} \) as they cancel out in the individual’s budget constraint. The fiscal policy rule can be neglected if it is Ricardian in this sense.

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3 The treatment in this section follows Woodford (2003), Chapter 4, Section 4.
Suppose a linear approximation to the tax rule is of the form:

$$\hat{\tau} = \tau_b \hat{b}_{t-1} + \tau_g \hat{G}_t$$

(13)

Where $\tau_b$ and $\tau_g$ are the respective response coefficients of taxes to deviations in debt ratio and in government expenditure. Substituting this into (12) gives the required law of motion for real government debt (10). The latter is stable or the tax rule (13) is locally Ricardian if and only if:

$$\left|\beta^{-1}(1 - \tau_b)\right| < 1$$

(14)

If $\tau_b \leq 1$, then fiscal policy or the tax rule is locally Ricardian if and only if $\tau_b > 1 - \beta$.

Defining equilibrium “to be (locally) determinate if and only if there are unique bounded equilibrium processes for all of the endogenous variables $\{b_t, \pi_t, Y_t, i_t\}$ for sufficiently tightly bounded processes for the exogenous disturbances (pp. 314)”, Woodford (2003) shows that if fiscal policy is locally Ricardian, equilibrium is determinate if and only if the response of monetary policy to inflation exceeds unity. The AD, AS system is stable with the latter condition. The three equation systems with debt require the additional stability condition (14). If fiscal policy is locally non-Ricardian, bounded paths for the endogenous variables will require monetary policy to violate the Taylor Principle and moderate its response to inflation. So unsustainable borrowing will require monetary accommodation.

Woodford, following earlier literature (see, for example, Davig and Leeper, 2009), then distinguishes between active and passive fiscal and monetary policies. If fiscal policy is Ricardian (passive) monetary policy can be active. If, however, fiscal policy is non-Ricardian (active) monetary policy must become passive (accommodating) to prevent instability. We explore some combinations of monetary and fiscal policy for an EM through simulations below.
4.3. Simulations

Calibration was loosely based on Indian stylized facts. The baseline coefficients are given in Table 1. The natural output \( \tilde{y} \) is optimally equivalent to a flexible price equilibrium, if a subsidy is set so as to correct for market power, openness and other distortions. This is calibrated for emerging markets in Goyal (2009) but is not required in the current simulations. The price setting parameters are such that prices adjust in an average of one year (\( \theta = 0.75 \)), giving \( \lambda = 0.24 \). The price response to output, \( \varphi \), is set at 0.25, which implies an average labour supply elasticity of 4. Because of less than perfectly flexible interest rates, lagged interest rate also enter the AD with a weight of 0.2.

Place Table 1 here

Since \( \sigma_R = 1 \) and \( 1/\sigma_P = 0 \), the implied average intertemporal elasticity of substitution is \( \eta(1-\alpha) + \alpha = 0.58; \beta = 0.99 \) implies a riskless annual steady-state return of 4 percent; and so the natural interest rate \( \rho = \beta^{-1} - 1 = 0.01 \). Consumption of the mature economy and of the rich is normalized at unity, five times that of the poor, so \( C_P = 0.2 \). Given \( \eta \), this gives consistent C values of 0.75, K (a measure of deviation from world output) of 1.1 so that \( c_P = -1.6 \) and \( \kappa = 0.1 \). Initial conditions are normalized at unity so the log value is zero. The \( \eta \) and \( \alpha \) parameters are calibrated to reflect the Indian unit benchmark and population share and export plus import share respectively.

A negative interest rate effect on consumption requires an intertemporal elasticity large enough so that the substitution effect is higher than the positive income effect of higher interest rates on net savers. Empirical studies have found real interest rates to have weak effects on consumption. Especially in low-income countries, subsistence considerations are stronger than intertemporal factors. This is particularly so when the share of food in

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4 The model was solved in state-space form by modifying Soderlind’s (2000) Matlab algorithms available at http://home.tiscalinet.ch/paulsoderlind. Modified versions used in SOEME simulations are available at igidr.ac.in/~ashima.
total expenditure is large. The elasticity Ogaki, Ostry and Reinhart (1996) estimate in a large cross-country study, varies from 0.05 for Uganda and Ethiopia to a high of 0.6 for Venezuela and Singapore. Our average elasticity of 0.58 compares well with these figures.

The weight on inflation in the CB’s loss function \( q_z \), exceeding unity satisfies the Taylor principle. The fiscal variables follow average Indian ratios: \( \tau_y \) is taken as 0.15 the average ratio of tax revenues to nominal debt in 2009; \( \tau_g \), is set at zero; \( \overline{b} \) is 0.8, and \( g \) is 0.006 for each simulation period since the annual rate of growth lie between 7 to 9 percent and the simulations are at the monthly frequency.

Cost shocks are frequent in SOEMEs; shocks to subsistence consumption imply a large shock to the natural rate, and populist pressures affect government expenditure, \( \hat{G} \).

Therefore the exogenous driving forces simulated are period one calibrated 0.2 standard deviation cost shock to domestic inflation, 0.01 negative shock to the natural rate, and 0.1 standard deviation shock to government expenditure. Each shock is of the same generic form. For example, persistence of the \( \hat{G} \) shock can be estimated from \( \hat{G}_t = \rho^{\hat{G}} \hat{G}_{t-1} + \varepsilon_t^{\hat{G}} \).

Table 2 reports some of the simulations. The benchmark simulations give consumer inflation, output gap, domestic inflation, debt (all as deviations from the steady-state) and the policy rate volatilities for each of the shocks. The initial values of \( \hat{b}_i \) and \( i_t \) are also reported. Figures 1 and 2 show the response of each of these variables to the cost and natural rate shock respectively over the 12 periods of the simulation. The table also reports sensitivity analysis with changes in the fiscal parameters and the weights of the CB’s loss function. Figures 3, and 4 show the response of \( \hat{b}_i \) to the variation in each of the parameters for the cost and natural rate shock respectively. Figure 5 gives the response of \( \hat{b}_i \) to \( \hat{G} \) shocks of varying persistence.
There are interesting insights from the simulations. The inflation following the cost shock leads to a fall in $\hat{b}$, but it returns to its steady state value in 4 periods. If natural rates fall $\hat{b}$ rises sharply, as the government borrows against the rise in potential output due to a positive supply shock, or spends to maintain demand due to a negative demand shock, or compensates for a fall in $c_p$. These are the factors that reduce natural rates. This fiscal response dominates the reduction in $\hat{b}$ due to the fall in interest rates, from equation (10). The net effect is from the working of the system as a whole. Since the policy rate falls less than the natural rate, the output gap rises, explaining some of the adjustments. Convergence back to the steady state is slow, not fully completed in the 12 periods.

Sensitivity analysis for both types of shocks is similar. A fall in $\tau_p$ reduces the deviation from steady state, since adjustment back would be more difficult. If $\bar{b}$ is lower, current borrowing requirement is reduced, and $\hat{b}$ rises less. A change in $g$ is the only one to have different effects under the 2 types of shocks. Under a cost shock, higher growth reduces the deviation of debt but for a natural rate shock it increases it. A shock to the consumption of the poor induces the government to borrow more if growth is high, but the reduction in the debt ratio under higher growth dominates under a cost shock.

In accordance with Woodford’s theoretical result, equilibrium does not exist if $q_\pi < 1$, but under both shocks, if $q_\pi = 1.1$ instead of 2, policy rates are lower. This monetary accommodation reduces the change in debt but increases deviations in the other macroeconomic variables. Table 2 reports a simulation where all the weights in the CB’s loss function are reduced to 0.5 and $\tau_p$ is put at zero. Equilibrium is determinate. The macroeconomic adjustment is of a similar order of magnitude as the case with $q_\pi = 1.1$ as the only change, but the fall in $\tau_p$ reduces the debt deviation as earlier. If the only change from benchmark shock is $\tau_p = 0$, equilibrium is determinate, the other macroeconomic
variables are the same as in the benchmark shock, only the deviation in debt is considerably reduced.

Considering a calibrated 0.01 shock to $\hat{\gamma}$ of persistence 0.25 the only effect is a temporary fall in debt. But equilibrium is indeterminate if persistence exceeds 0.9. With persistence of 0.85 equilibrium becomes determinate but unstable. Equilibrium is indeterminate if $q_{\pi} < 1$, determinate but even more unstable, for other macroeconomic variables, than the benchmark $\hat{\gamma}$ shock, if $q_{\pi} = 1.1$. The volatility of other macroeconomic variables is reduced if $\tau_b = 0$ since the initial rise in debt is considerably reduced, but $\hat{b}$ continues to be unstable remaining far from steady-state values at the end of 12 periods. As in simulations with earlier variants of SOEME models (Goyal 2007), a rise in the share of the rich and in openness reduces initial interest and therefore debt response, for both types of shocks, implying greater debt volatility is to be expected in a poorer less globally integrated country.

4.4. Monetary-Fiscal Policy Combination

Much more work needs to be done to explore the regions of determinacy and instability as a function of the parameters. The response to other types of shocks can also be explored. Results will change if debt or wealth enters the AD equation, and if the weights in the CB’s loss function are optimally derived accounting for public debt. Annicchiarico et. al. (2008) find the region of determinacy rises in an overlapping generation model with wealth effects, raising the scope for active monetary policy. Wealth effects through the balance of payments are an important contributor to persistent effects of monetary shocks in an open economy, but in a low per capita income economy taxes can be expected to neutralize wealth in order to smooth consumption of the P-type. Gali et. al. (2007) introduce rule-of-thumb consumers. Since fiscal expenditure raises rule-of-thumb consumption a stronger monetary response is required. But in our model, although the P-type are not forward looking, their consumption is fixed at subsistence, and subject to greater rigidities in an EME, so that the neutralization of a fiscal stimuli through expected

\footnote{Simulations not reported available on request.}
future taxes continues to be a valid first approximation. An EME typically has a large debt yet since it is in a transitional catch up phase, the borders between active and passive fiscal and monetary policy can differ, as we find. Net household exposure to stocks and bonds is low and can be expected to rise even as the denominator rises with growth. Davig and Leeper (2009) explore the consequences of the fiscal stimuli in a DSGE model with markov switching between different monetary and fiscal policies. More complex models will yield more insights but the key contribution of consumption shocks and growth in emerging markets will remain.

The lessons from the current simulations for the conduct and coordination of monetary and fiscal policy in an emerging market are: First, under an optimization respecting the law of motion of public debt, debt should optimally rise in response to a shock to subsistence consumption, and the rise will be higher the higher is debt, and the higher is growth, but should be moderated if the tax response to a rise in debt is low. Second, higher growth moderates debt volatility under cost shocks. Third, monetary accommodation is useful to moderate the debt response under shocks but is dangerous if the shock is a persistent rise in government expenditure.

5. Indian Macroeconomic Policy Options
In this section we apply insights from the analysis to Indian policy dilemmas. Despite high growth over 2003-08 and the adoption of an FRBM Act, Indian debt levels did not come down much. And despite low fiscal space the government gave large fiscal boosts over 2008-10 after the global financial crisis. Does this imply an unsustainable fiscal path?

According to the analysis in the first section, high private savings, with a savings to GDP ratio exceeding 30, reduce the probability of a CAD and BOP crisis. Growth rates continued to be relatively high compared to most countries in the world, so capital inflows resumed quickly even in the period after the collapse of Lehman. Even as aggregate flows reduced, India got a higher share.
Next we turn to analysis of the evolution of Indian public debt, following the second section. Figure 6 shows the percentage change in $b$. It also shows the two components of the change in debt from Equation (6), over the Indian post-reform period. The nominal interest rate calculated is the implicit rate the central government pays on its debt. It is obtained by dividing actual interest payments in the budget documents $iPB$ by $PB$. The latter is also obtained from the budget documents\(^6\). Inflation calculated from the GDP deflator is subtracted from $i$, in order to get the real rate $r$. The growth rate used is the growth of real domestic product at market prices. Finally calculated $b_{t-1}(r_t - g_t)$ and actual reported $pd$ are graphed.

Figure 7 shows that $r$ was normally less than $g$ over this period and since the negative $b_{t-1}(r_t - g_t)$ value was larger in absolute value than a mostly positive $pd$, $b$ should have fallen much more than it did. The discrepancy is particularly acute in the high growth period after 2003, since the $pd$ was also falling over this period. Given the fall in $b$ that should have occurred due to $r$ and $g$, actual $b$ did not fall; implying deficits must have been higher. Reported deficits may have been doctored, to satisfy the FRBM Act\(^7\) targets applicable in this period. Government debt increased through off balance sheet items that were not included in deficits. Cash balances were also used to break the link between debt and deficits.

Figure 7 shows the movements in the relevant rates over the period. Although nominal interest rates were higher in the earlier years the consistent fall in inflation rates over the period meant the real interest rates on government debt rose from early strongly negative rates, to peak at 5 percent in 2000-01 before beginning to fall as nominal rates also fell.

\(^6\) Data for the period 1990-91 to 2008-09 is graphed. Calculations, available on request, were made from data available from ministry of finance and RBI websites. Debt and deficit figures are for the Central Government.

\(^7\) This was enacted by Parliament in 2003. The Rules accompanying the FRBM Act required the Centre to reduce the fiscal deficit to 3 per cent of GDP and, eliminate revenue deficit by March 31, 2008. There was also a ceiling on guarantees. But the ceilings may be exceeded during “national security or national calamity or such other exceptional grounds as the Central Government may specify”, so that the Government can legislate itself out of the commitments. In addition the budget has to each year place before Parliament the Medium Term Fiscal Policy, and Fiscal Policy Strategy and Macroeconomic Framework statements. Deficit financing or money creation is banned, but there are no restrictions on OMOs. Any deviations from the FRBM Act require the permission of Parliament.
Since real rates were low in both the high growth phases, \( r-g \) was strongly negative during these phases. The reported \( pd \) was also lower in high growth periods. In the second period revenues were high—the tax GDP ratio peaked at 11.5 percent. But the fall in \( b \) was larger in the briefer high growth period of the mid-nineties.

**Place Figures 6, 7 here**

For the first time \( pd \) turned into a surplus in 2004-05, but it had increased to 2.6 in 2008-09, the year of the global financial crisis, and exceeded 3 the next year. For the debt ratio to stay unchanged at around 0.8, at such a PD ratio, the growth rate must exceed the real interest rate by 3.75 basis points. With a \( PD \) ratio of 3, \( r = 4, g = 7 \), our formula implies that in the steady-state when \( b \) is not changing, \( b = PD/g-r = 100 \) percent. Unless growth revives and the \( pd \) is reduced India’s steady-state debt will rise. If the reverse happens debt can explode. Since even high growth together with the FRBM Act was insufficient to reduce India’s debt, a better conceived FRBM that improves incentives for compliance is required.

Growth did reduce debt levels, but large expenditures to increase the consumption of the poor, \( c_p \), given the government’s goal of inclusive growth, seem to have moderated deficit reduction in the second high growth period. The global shock also reduced employment and \( c_p \). In the SOEME model this constitutes a negative natural rate shock. The simulations suggest debt levels can be expected to rise, and they will be higher the higher is growth and the level of \( b \). Monetary accommodation can help reduce \( b \) to the extent soft commodity prices reduce inflation and the rise in \( G \) is temporary. It is very important that there is no permanent rise in \( G \) in excess of taxing capacity, or instability results. The excise cuts given as part of the post-crisis fiscal stimuli would be reversed and the 6th Pay Commission arrears and farm loan waivers were one-time payments. But new recurring expenditure commitments must be made only on a secure tax base.

A distinction should be made between structural and cyclical deficits. With private demand slowing, a cyclical deficit is needed. A structural deficit may also be defended in
a transitional high growth period, since growth reduces debt ratios, but only if it creates
supply-side capacity to enable growth. The level of debt and deficits should be reduced in
good times in order to create space for countercyclical fiscal policy. The inability to bring
down debt levels in the high growth period suggests that other measures are required to
ensure medium-term fiscal consolidation. We discuss these in order of importance.

The FRBM Act, brought down only reported deficits, which were on track to meet
announced targets before the oil shock hit. But the episode exposed the inadequate
attention paid to incentives and escape clauses in formulating the Act. Loopholes were
found to maintain the letter of the law even while violating its spirit. Off budget liabilities
such as oil bonds were used to subsidize some petroleum products. Targets were
mechanically achieved, compressing essential expenditure on infrastructure, health and
education, while maintaining populist subsidies. The Act should be reframed to improve
incentives for compliance. Expenditure caps that bite especially on transfers, while
protecting productive expenditure, will create automatic counter-cyclical stabilization as
tax revenue falls and deficits rise in a slowdown. They will also moderate the temptation
to raise expenditure when actual or potential revenues rise. In the Indian context, detailed
expenditure targets are required for individual ministries, and levels of government, as
part of improved accounting, including shifts from cash to accrual based accounts.

Productive expenditure is anything that improves human, social, and physical capital, and
therefore the supply response. Change in the composition of government expenditure
towards this will bring down debt ratios, by increasing the denominator of the ratio, and
raising revenues. Essential transfers must be better targeted to reduce waste, and the
effectiveness of government expenditure improved. Any permanent rise in G must be
linked to a specific tax source.

A more credible FRBM will allow better fiscal-monetary coordination. It will make
monetary accommodation during the crisis period safe. To use Woodford’s terminology,
a passive monetary policy can accompany an active fiscal policy during the crisis, as long
as they switch positions in the longer term. The more usual combination in post-reform
India, as the RBI gained greater independence, was for both to be active, which harmed growth, as monetary tightening sought to compensate for fiscal giveaways. When Indian interest rates fell after 2000, despite high government deficits, and aggressive sterilization, because international interest rates fell, growth was stimulated.

In the post-crisis circumstances, despite high government borrowing due to fiscal stimuli, lower inflows gave the RBI leeway to increase the share of government securities in the monetary base. This helped finance the deficit and limited crowding out as private borrowing revived. In addition to cuts in policy rates, quantitative easing with OMOs through the term structure was available to ease pressure on interest rates.

If the composition of fiscal expenditure changes, longer-term monetary policies can also be recast to support growth, and further boost the diversified sources that sustain Indian growth. These include domestic demand, agriculture, openness, technology, the demographic profile, the infrastructure cycle, and having crossed a critical threshold. As a net commodity importer India gains from lower global prices. Dependence on external demand is low compared to other Asian countries. So is the dependence on foreign capital. But although aggregate savings are high, about half of household savings are in physical form, making it difficult to finance high government and reviving private borrowing. Requirements of infrastructure finance may force development of the corporate bond market. RBI backing of credit to SMEs could better intermediate savings and raise India’s low credit/GDP ratio.

Lower tax response has a disciplining effect on debt expansion. But more efficient systems of tax collection will decrease debt if a more effective FRBM Act restrains the government. India does have improved technology-based tax systems, independent of government, that have delivered, and more improvements such as GST are on the way. There has been steady lowering of tax rates. Technology has been used to broaden coverage, and reduce loopholes. The experience with the destination based State level VAT since 2005 has been good. The proposed move to GST in 2011 should yield the large efficiency gains of one market. Continuing growth may protect some of the recent
buoyancy in tax revenue but revenue expansion due to improved compliance and broad basing can be expected to survive a slowdown. Indian fiscal policy made considerable progress in tax reform, but improvements in expenditure management are yet to come. Even so there was a steady increase in the quality of Indian institutions. The 13th Finance commission, whose report was put in the public domain in 2010, was asked to reset the path of fiscal consolidation. It tried to shift the composition of government expenditure more towards capital by imposing stricter reduction for the revenue deficit, and to make space for counter cyclical fiscal policy by imposing a long-term ceiling on the debt ratio. But for the central government at least incentives to improve compliance are still largely missing.

6. Conclusion
Rising fiscal deficits in the context of high debt ratios are a risk factor for any country especially with an open capital account. But two possible risk-mitigating factors are: high private savings, a large population transiting to higher productivity in a catch-up phase of higher growth. Higher private savings can compensate for government dissaving so that a fiscal deficit need not imply a balance of payment deficit. The second factor implies sustainable debts and deficits may be higher as growth rises. Analysis of twin deficits demonstrates the first and of the evolution of government debt shows how debt ratios fall with growth rates.

But an optimizing macroeconomic policy model of a small open emerging market economy (SOEME) with dualistic labour markets and two types of consumers, and the growth dividend built into the evolution of government debt, shows debt ratios tend to be higher in high growth phases. An application of the analysis to and assessment of post reform Indian debt and deficit ratios does show less than warranted reduction in debt ratios in the high growth phases. The implication is that stronger legislative restraints to ensure countercyclical deficits would allow better fiscal and monetary coordination and outcomes.
Even so, because of improvements in tax collection, and trends in growth and real interest rates, and the push for fiscal consolidation, Indian debt ratios are unlikely to become unsustainable, despite the fiscal stimulus after the global crisis. Just as Indian savings rates are rising to Chinese levels, Indian fiscal health can also approach Chinese levels if growth is sustained. Chinese government debt and deficits had peaked in the years of their big infrastructure push starting in the late nineties, but debt began to come down after 2005. China, as another populous emerging market, benefited from policies that harvested the growth dividend on deficits and debt. Improvements in fiscal institutions can, however, increase the dividend.

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Appendix: Deriving AD and AS from the SOEME model

Consumers and workers

A typical SOEME has two representative households: above subsistence (R) and at subsistence (P). The intertemporal elasticity of consumption (1/σ_{R}), productivity and wages (W_{R}) of R are higher, their labour supply elasticity (1/φ_{R}) is lower compared to the P, and they are able to fully diversify risk in international capital markets. Each type seeks to maximize the discounted present value of utility—a positive function of consumption and a negative function of labor supplied:

\[ E_o \sum_{t=0}^{\infty} \beta^t U\left(C_{i,t}, N_{i,t}\right) \]

\[ i = R, P \quad (A1) \]

\( N_{i,t} \) denotes hours of labor supplied by each type. Aggregate consumption \( C_t \) is a composite index of consumption of home (H) and foreign goods (F). Elasticity of substitution between H and F goods is assumed to be equal unity. In this case the CES aggregation simplifies to (A2) for consumption and (A3) for the price index. Each of \( C_{H,t} \), \( C_{F,t} \) are indices of a continuum of differentiated home and foreign goods respectively with elasticity of substitution between goods of different varieties, \( \varepsilon > 1 \), as is required for equilibrium under monopolistic competition. Simplifications are made to reduce the degree of disaggregation, and focus on disaggregation of consumption between the R and the P households.
The corresponding consumer price index can be derived from cost minimization of the consumption bundle as is standard in the literature. The share of foreign goods, \( \alpha \), \( 0 < \alpha < 1 \), defines the degree of openness. It is inversely related to the degree of home bias, and is assumed to be the same for R and P, since although P spend more on food, agricultural products are also traded goods.

\[
C_{i,t} = kC_{i,H,t}^{1-\alpha} C_{i,F,t}^\alpha \quad i = R, P
\]  

\( (A2) \)

Consumption of each type of good is a weighted average of consumption by the R and the P households, with \( \eta \) as the share of R.

\[
C_{H,t} = C_{P,H,t}^{1-\eta} C_{R,H,t}^\eta \\
C_{F,t} = C_{P,F,t}^{1-\eta} C_{R,F,t}^\eta
\]

Aggregate consumption \( C_t \) is distributed between R and P in the same proportion \( \eta \), where \( \eta \) is the share of above subsistence households in consumption. Substituting for \( C_{i,t} \) from (2) above and taking out the common constant \( k \) gives (3):

\[
C_t = k \left( C_{P,H,t}^{1-\alpha} C_{P,F,t}^\alpha \right)^{1-\eta} \left( C_{R,H,t}^{1-\alpha} C_{R,F,t}^\alpha \right)^\eta
\]

\( (A3) \)

The corresponding consumer price index can be derived from cost minimization of the consumption bundle as is standard in the literature. Given the constant \( k = \frac{1}{(1-\alpha)^\alpha} \), the price index can be written as:

\[
P_t = P_{H,t}^{1-\alpha} P_{F,t}^\alpha
\]

\( (A4) \)

Since the effective terms of trade, or price of foreign goods in terms of home goods, is \( S_t = P_{F,t}/P_{H,t} \), substituting in (A4) gives:

\[
P_t = P_{H,t}^{1-\alpha} S_t^\alpha
\]

\( (A5) \)

That is, consumer prices depend on domestic prices and the terms of trade.
A household’s period utility function is given the specific form:

\[ U(C_i, N_i) = \frac{C_i^{1-\sigma}}{1-\sigma_i} - \frac{N_i^{1+\varphi_i}}{1+\varphi_i} \quad i = R, P \]  

(A6)

The constant relative risk aversion (CRRA) utility function is defined for \( \sigma \) greater than zero and not equal to unity. At \( \sigma \) equal to unity it becomes \( \ln C \). Utility is maximized subject to a sequence of period budget constraints:

\[ P_i C_{i, t} + E_t \{ Q_{i, t+1} D_{i, t+1} \} \leq D_{i, t} + W_{i, t} N_{i, t} - T_{i, t} \]  

(A7)

Where \( W_{i, t} \) is the nominal wage paid to each type, \( Q_{i, t+1} \) is the stochastic discount factor corresponding to the random payoff \( D_{i, t+1} \) of the portfolio purchased at \( t \); \( I_t \) is the gross nominal yield on a riskless one period discount bond, paying one unit of domestic currency in \( t+1 \), so that \( I_t^{-1} = E_t \{ Q_{i, t+1} \} \) is the price of the discounted bond; \( T_{i, t} \) is lump sum taxes or transfers. Taxes \( T_{R, t} \) from \( R \) partly finance transfers \( T_{P, t} \) to \( P \); since the latter have zero savings \( D_P \) is zero. \( C_P \) must equal wages plus transfers. The government intermediates these transfers and runs a balanced budget so that \( \eta T_{R, t} + A_t = -(1-\eta) T_{P, t} \) where a negative tax is a transfer. \( A_t \) is government revenue from its international assets, net of any cost of accumulating foreign exchange reserves. The subsidy is calculated to give \( P \) subsistence consumption \( C^*_P \) if they work eight hours daily, but they are free to increase their consumption by working longer hours. On adding up across agents the fiscal variables drop out and do not affect individual decisions (Woodford, 2003). Lump sum transfers do not enter optimizing first order conditions. The economy is assumed to be cashless so monetary policy works by changing interest rates.

Capital markets are complete, but only the R-type of consumers can participate in them. Since \( P \) lack the ability to smooth consumption their intertemporal elasticity of consumption approaches zero. The aggregate intertemporal elasticity of substitution, \( 1/\sigma \), and the inverse of the labour supply elasticity, \( \varphi \), are weighted sums with population shares of \( R \) and \( P \) as weights,
\[
\frac{1}{\sigma} = \eta \frac{1}{\sigma_R} + (1 - \eta) \frac{1}{\sigma_p} \tag{A8}
\]
\[
\varphi = \eta \varphi_R + (1 - \eta) \varphi_p
\]
Since \( \sigma \) is the coefficient of relative risk aversion, an intertemporal elasticity of consumption approaching zero implies risk aversion approaching infinity for the poor.

The standard first order conditions for optimal allocation of consumption across home and foreign goods yield the demand functions:
\[
P_{H,t} C_{H,t} = (1 - \alpha) P_t C_t \tag{A9}
\]
\[
P_{F,t} C_{F,t} = \alpha P_t C_t \tag{A10}
\]
And from intertemporal optimization, we get the consumption Euler equation:
\[
\beta E_t \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\sigma_i} \left( \frac{P_t}{P_{t+1}} \right) = Q_{i,t+1} \tag{A11}
\]
Or:
\[
\beta I_t E_t \left[ \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{A12}
\]
The \( i \)th household’s labour supply is given by:
\[
C_{i,t}^{\sigma_i} N_{i,t}^{\varphi_i} = \frac{W_{i,t}}{P_t} \quad i = R, P \tag{A13}
\]
Since \( C_p \) is subsidized to prevent it falling below subsistence \( C^* \), we must have \( T_{P,t} = C_{P,t} - N_{P,t} \left( \frac{W_{P,t}}{P_t} \right) \), but we assume \( C_{P,t} > C^* \) since \( N_{P,t} > N^*_{P,t} \). That is, the poor work above 8 hours daily (\( N^* \)) to raise their consumption above subsistence. Equations (A12) and (A13) can be written in log linear form as:
\[
w_{i,t} - P_t = \sigma_i c_{i,t} + \varphi_i n_{i,t} \tag{A14}
\]
\[
c_{i,t} = E_t \left[ c_{i,t+1} \right] - \frac{1}{\sigma_i} \left( i_t - E_t \left( \pi_{i,t+1} \right) - \rho \right) \quad i = R \tag{A15}
\]
Although \( c_p \) is exogenously given it can rise with a rise in subsistence levels. Lower case letters are logs of the respective variables.

*Risk sharing*
Equating consumption Euler equations for the R consumer in the SOEME and consumers in a mature economy denoted by superscript \(i\), integrating over all \(i\) to get average world consumption \(C^*\), using it and the definition of the real exchange rate \(Z\), risk sharing gives:

\[ C_{R,t} = \nu C_i^* Z_i^{\sigma_s} \quad (A16) \]

Which can also be written as:

\[ c_{R,t} = c_i^* + \left( \frac{1 - \alpha}{\sigma_R} \right) s_t \quad (A17) \]

**Equilibrium and terms of trade**

The aggregate demand equal to supply equality implies for the EM:

\[ y_t = c_t + \frac{\alpha \sigma R}{\sigma_R} s_t + \kappa \quad (A18) \]

Where \(\sigma = \sigma_R + (1 - \alpha)(\sigma_R - 1)\), \(\kappa = \log K\), \(K = (1 - \alpha + \alpha K_R)\), and \(C_{R,t} = K_R C_t\) and for world output:

\[ y^* = c^* \quad (A19) \]

To solve for \(S_t\) in terms of endogenous \(Y_t\) and exogenous variables, first substitute \(C_{R,t}\) and \(C_{P,t}\) for \(C_t\) in the aggregate demand equal to supply equation and then substitute out \(C_{R,t}\) using risk smoothing. This gives:

\[ S_t = \left( \frac{Y_t}{Y_t^{\eta} C_{P,t}^{1-\eta}} \right) \sigma_D \quad (A20) \]

The terms of trade depreciate with a rise in \(Y_t\) and appreciate with a rise in \(Y_t^*\); but in a SOEME the former’s effect is magnified. \(C_{P,t}\) also affects \(S_t\), reducing the impact of \(Y_t^*\). The multiplier factor \(\sigma_D\), which affects only the SOEME, is large because the elasticity of substitution is lower for a SOEME. If \(\sigma_R = 1\), then \(\sigma = 1\), and if \(1/\sigma_P = 0\), then \(\sigma = \sigma_R/\eta\). It also follows that \(\sigma_D < \sigma\). Both rise as \(\eta\) falls or the proportion of \(P\) with low intertemporal elasticity of consumption \((1/\sigma_P = 0)\) rises. While \(\eta\) affects \(\sigma\), both \(\eta\) and \(\alpha\) affect \(\sigma_D\). As \(\alpha\) falls \(\sigma_D\) rises, and as \(\alpha\) approaches 0, or the economy becomes closed, \(\sigma_D\) equals \(\sigma\), which
is its upper bound. In a fully open economy $\alpha$ approaches unity, and $\sigma_D$ falls to its lower bound, which is unity.

**Firms**

**Technology:** A typical firm has a log-linear production technology, derived by aggregation over the individual firms producing the $j$ differentiated goods. It is written in log terms as:

$$y_t = a_t + n_t$$  \hspace{1cm} (A21)

Where $N_t = N_{P,t}^{1-\eta} N_{R,t}^\eta$ aggregates over the two types of labor in the economy and productivity $a_t \equiv \log A_t$ follows an AR (A1) process:

$$a_t = \rho^a a_{t-1} + \epsilon_t^a$$  \hspace{1cm} (A22)

**Price setting:** The real marginal cost in domestic prices, $mc_t$, is common across firms, as labor is mobile at the prevailing factor prices:

$$mc_t = -\nu - a_t + \eta(w_{R,t} - p_{H,t}) + (1-\eta)(w_{P,t} - p_{H,t})$$  \hspace{1cm} (A23)

Where $mc_t$ is the sum of real wages in terms of domestic prices paid to R and to P minus the aggregate productivity shock and $\nu = -\log(1-\tau)$ where $\tau$ can be understood as an employment subsidy paid to firms to counter market power, and other distortions due to terms of trade and the labor market, thus increasing their employment level to the optimal flexible price level. Adding and subtracting $p_t$ (A23) becomes:

$$mc_t = -\nu + \eta(w_{R,t} - p_t) + (1-\eta)(w_{P,t} - p_t) + (p_t - p_{H,t}) - a_t$$  \hspace{1cm} (A24)

Substituting from the consumers’ optimizing labour-leisure decision (A14), and from the log version of (A4) for the terms of trade:

$$mc_t = -\nu + \eta(\sigma_R c_{R,t} + \varphi_R n_{R,t}) + (1-\eta)(\sigma_P c_{P,t} + \varphi_P n_{P,t}) + \alpha s_t - a_t$$  \hspace{1cm} (A25)

Thus $s_t$ affects marginal cost since foreign prices affect domestic prices and costs. Using the identities (A8) and (A26) below,

$$\eta c_{R,t} + (1-\eta)c_{P,t} = c_t$$

$$\eta n_{R,t} + (1-\eta)n_{P,t} = n_t$$  \hspace{1cm} (A26)

$mc_t$ can be written as:
\[ mc_i = -\nu + \sigma c_i + \phi a_i + \alpha s_i - a_i \]  \hfill (A27)

Using risk sharing to eliminate \( c_{R,t} \), the production function (A21) to eliminate \( n \), and the approximation \( 1/\sigma = 0 \), so \( \sigma = \sigma_R/\eta \), the marginal cost can be written as a function of domestic output, \( c_{ov} \), and terms relating to the external sector.

\[ mc_i = -\nu + \sigma y_i^* + \phi y_i + \sigma(1-\eta)k_{p,t} - (1+\phi)a_t + s_t \]  \hfill (A28)

The equivalent relationship for a mature economy (GM (2005)) is:

\[ mc_i = -\nu + \sigma y_i^ + \phi y_i + s_t - (1+\phi)a_t \]  \hfill (A29)

Given marginal cost, prices are set according to the Calvo staggered pricing model, where each firm resets price with probability \( (1-\theta) \) each period implying that a measure \( (1-\theta) \) of randomly selected firms reset prices each period. Then the dynamics of domestic inflation are given by:

\[ \pi_{H,t} = \beta E_t \left[ \pi_{H,t+1} \right] + \lambda \hat{mc}_i \]  \hfill (A30)

Where \( \lambda = \frac{(1-\beta)(1-\theta)}{\theta} \), \( \hat{mc}_i \equiv mc_i - mc \) or the deviation of log marginal cost from its steady-state log value \( mc = -\mu \), determined by the elasticity of demand. The deviation of marginal cost from its steady-state value in terms of the output gap can be derived to be:

\[ \hat{mc}_i = (\sigma + \phi)x_i \]

Substituting in (A30) gives the AS equation (11) in the text.
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<th><strong>Baseline Calibrations</strong></th>
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<td>Price response to output</td>
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<tr>
<td>Steady state real interest rate or natural interest rate</td>
<td>$\rho$ or $i$</td>
<td>0.01</td>
</tr>
<tr>
<td>Variations in the natural interest rate due to temporary shocks</td>
<td>$\bar{rr}$</td>
<td>0.01 ±</td>
</tr>
<tr>
<td>Degree of openness</td>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Proportion of the R type</td>
<td>$\eta$</td>
<td>0.4</td>
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<tr>
<td>The intertemporal elasticity of substitution of the R type</td>
<td>$1/\sigma_R$</td>
<td>1</td>
</tr>
<tr>
<td>The intertemporal elasticity of substitution of the P type</td>
<td>$1/\sigma_P$</td>
<td>0</td>
</tr>
<tr>
<td>Consumption of the P type</td>
<td>$C_p$</td>
<td>0.2</td>
</tr>
<tr>
<td>Consumption of the R type</td>
<td>$C_R$</td>
<td>1</td>
</tr>
<tr>
<td>Share of backward looking inflation</td>
<td>$\gamma_b$</td>
<td>0.2</td>
</tr>
<tr>
<td>Share of forward looking inflation</td>
<td>$\gamma_f$</td>
<td>0.8</td>
</tr>
<tr>
<td>Response coefficient of taxes to the debt ratio</td>
<td>$\tau_b$</td>
<td>0.15</td>
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<tr>
<td>Response coefficient of taxes to G expenditure</td>
<td>$\tau_g$</td>
<td>0</td>
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<tr>
<td>Steady state public debt to output ratio</td>
<td>$\bar{b}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Monthly growth rate</td>
<td>$g$</td>
<td>0.006</td>
</tr>
<tr>
<td>Weight of output in the CB’s loss function</td>
<td>$q_y$</td>
<td>0.7</td>
</tr>
<tr>
<td>Weight of inflation in the CB’s loss function</td>
<td>$q_\pi$</td>
<td>2</td>
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<tr>
<td>Weight of the interest rate in the CB’s loss function</td>
<td>$q_i$</td>
<td>1</td>
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<table>
<thead>
<tr>
<th><strong>Implied parameters</strong></th>
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<tbody>
<tr>
<td>Weighted average elasticity of substitution</td>
<td>$1/\sigma$</td>
<td>0.58</td>
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<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
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<tr>
<td>Weighted average consumption level</td>
<td>$C$</td>
<td>0.75</td>
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<tr>
<td>Log deviation from world output</td>
<td>$\kappa$</td>
<td>0.1</td>
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<tr>
<td>Philips curve parameter</td>
<td>$\lambda$</td>
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<thead>
<tr>
<th><strong>Shocks</strong></th>
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<tbody>
<tr>
<td>Persistence of shock to G expenditure</td>
<td>$\rho^G$</td>
<td>0.25</td>
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<tr>
<td>Persistence of natural rate shock</td>
<td>$\rho^r$</td>
<td>0.75</td>
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<tr>
<td>Persistence of cost-push shock</td>
<td>$\rho^c$</td>
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<tr>
<td>Standard deviation of shock to G expenditure</td>
<td>$\sigma^G_\varepsilon$</td>
<td>0.1</td>
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<tr>
<td>Standard deviation of natural rate shock</td>
<td>$\sigma^r_\varepsilon$</td>
<td>0.01</td>
</tr>
<tr>
<td>Standard deviation of cost-push shock</td>
<td>$\sigma^c_\varepsilon$</td>
<td>0.2</td>
</tr>
<tr>
<td>Simulations</td>
<td>Parameters</td>
<td>Standard deviations of (in percentages):</td>
</tr>
<tr>
<td>-------------</td>
<td>------------</td>
<td>------------------------------------------</td>
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<tr>
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<td>Consumer inflation</td>
</tr>
<tr>
<td></td>
<td>$\tilde{b} = 0.8$, $\tau_b = 0.15$, $g = 0.006$</td>
<td>0.58</td>
</tr>
</tbody>
</table>

**Cost shock**
- Benchmark 0.58 0.36 1.08 0.45 (-0.0162) 0.70 (0.0256)
- $\tau_b=0.1$ 0.58 0.36 1.08 0.47 (-0.0169) 0.70 (0.0256)
- $g=0.008$ 0.58 0.36 1.08 0.42 (-0.0151) 0.70 (0.0256)
- $\tilde{b} =0.7$ 0.58 0.36 1.08 0.45 (-0.0162) 0.70 (0.0256)
- $\tilde{q}_n=1.1$ 0.71 0.18 1.21 0.16 (0.0056) 0.50 (0.0181)

**Natural rate shock**
- Benchmark 0.47 0.16 0.31 6.18 (0.2088) 0.39 (-0.0133)
- $\tau_b=0.1$ 0.47 0.16 0.31 3.90 (0.1319) 0.39 (-0.0133)
- $\tau_b=0$ 0.47 0.16 0.31 2.20 (0.0745) 0.39 (-0.0133)
- $g= 0.004$ 0.47 0.16 0.31 5.36 (0.1813) 0.39 (-0.0133)
- $\tilde{b} =0.9$ 0.47 0.16 0.31 7.27 (0.2456) 0.39 (-0.0133)
- $\tilde{q}_n=1.1$ 1.04 0.46 0.97 5.12 (0.1730) 0.83 (-0.0279)
- $\tilde{q}_n=0.5$, $\tau_b=0$ 1.01 0.42 0.91 1.81 (0.0614) 0.82 (-0.0276)

$\hat{G}$ shock
- Persistence= 0.25 0.00 0.01 0.00 1.14 (-0.0412) 0.00 (0.0000)
- Persistence =0.85, $\tilde{q}_n= 1.1$ 80.07 14.40 42.38 251.63 (-9.6890) 37.48 (-1.4167)
- Persistence = 0.85 87.47 08.67 28.19 935.66 (-3.5970) 37.65 (-0.1389)
- Persistence = 0.85, $\tau_b = 0$ 01.15 0.12 0.42 14.43 (-0.5551) 0.56 (-0.204)