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Angle, John

The Inequality Process Institute

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by

John Angle
angle@inequalityprocess.org
Inequality Process Institute
http://www.inequalityprocess.org
Post Office Box 215, Lafayette Hill, PA 19444



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# Socio-Economic Analogues of the Gas Laws (Boyle's and Charles')


## Abstract

Most social scientists would reject the possibility of socio-economic analogues of the gas laws (Boyle's and Charles') on verisimilitude grounds. The gas laws relate the variables temperature, pressure, and volume. The possibility of socio-economic analogues of the gas laws and their variables is suggested by the similarity of two mathematical models. One model is the Inequality Process (IP), a particle system model that explains a wide scope of socio-economic phenomena. The IP is isomorphic to the particle system of the Kinetic Theory of Gases (KTG) up to two differences. The KTG is the micro-level explanation of the gas laws. Given a map from the KTG into the IP, the IP implies empirically valid socio-economic analogues of Boyle's and Charles' Laws.



**Key Words:** Boyle's Law, Charles' Law, econophysics, income and wealth distribution, Inequality Process, Kinetic Theory of Gases

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John Angle
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## 1.0 Introduction

Most social scientists would likely dismiss the possibility of socio-economic analogues of Boyle's Law and Charles' Law on grounds of verismilitude: people are different from gas molecules. Boyle's and Charles' Laws relate the temperature, pressure, and volume of a gas. Despite appearances, there is a close similarity between two particle systems, one of which explains Boyle's and Charles' Laws at the level of molecular (particle) interactions. The other particle system has been shown to quantitatively explain a wide scope of phenomena related to income and wealth. The two particle systems are isomorphic up to two specific differences. Their similarity raises the theoretical possibility that there might be socio-economic analogues of Boyle's and Charles' Laws, indeed of a whole socio-economic analogue of thermodynamics.

Boyle's Law relates the volume of a gas to its pressure; Charles' Law relates the volume of a gas to its temperature. Boyle's and Charles' Laws are accounted for at the micro-level (particle interactions, gas molecules in collision, exchanging kinetic energy) by the particle system model of the Kinetic Theory of Gases (KTG). The KTG models gas molecules as perfectly elastic spheres colliding according to the laws of Newtonian mechanics. In the Kinetic Theory of Gases (KTG), temperature and pressure are different aspects of the kinetic energy of the molecules of a gas. Temperature is mean kinetic energy; pressure is the force exerted on the wall of the container and on other gas molecules from collisions with moving gas molecules, i.e. masses with kinetic energy. While introductory physics textbooks for secondary schools still mention Boyle's and Charles' Laws by name, they are found under the rubric 'ideal gas laws' in college level physics and thermodynamics texts (Feynman, Leighton, Sands, 1963; Gyftopoulos and Beretta, 2005; and Owen, 1984). They are the beginnings of thermodynamics.

The Inequality Process (IP) (Angle, 1983-2009) quantitatively explains a wide scope of socio-economic phenomena related to personal income and wealth. The IP is isomorphic to the KTG's particle system model up to: a) an IP parameter value set to a constant 1.0 (multiplicative identity) in the KTG, and b) a different stochastic driver of the exchange of the positive quantity between particles. This positive quantity is labeled 'kinetic energy' in the KTG's particle system, 'wealth' in the Inequality Process (IP). The IP's particles represent people with a very limited number of traits.

### 1.1 This Paper's Structure

To establish that there are socio-economic analogues of Boyle's and Charles' Laws this paper has to show:

> the interrelationships among the micro-level variables of the Kinetic Theory of Gases (KTG) that account for Boyle's and Charles' Laws, are approximately

preserved in a particular map of KTG variables into Inequality Process (IP) variables and parameters, such that the Inequality Process (IP) images of the KTG variables imply empirically valid socio-economic laws isomorphic to Boyle's and Charles' Laws.

Specifically in the case of the macro level variable temperature (of Charles' Law), this paper follows the map of temperature (T) into its KTG representation, mean kinetic energy of gas molecules, and from the map of mean kinetic energy into a variable or parameter of the Inequality Process (IP) and from there to the macro-level socio-economic variable implied by that IP variable or parameter. The paper follows the map trails of pressure (P) and volume (V) too. The map from KTG variables into IP variables and parameters seeks to preserve the characteristics and inter-relationships among KTG variables.

The map from the IP to macro-level socio-economic variables results from the aggregation of the micro-level entities. The micro-level socio-economic entities, people, are much more open to inspection and measurement than gas molecules. People are "taggable" (traceable as individuals over time). Gas molecules could not be imaged before the 20th century and still cannot be "tagged".

This paper proposes a triple map from macro-level gas law variables into micro-level KTG variables into micro-level IP variables and parameters and from IP variables and parameters into macro-level socio-economic variables, resulting in socio-economic laws isomorphic to Boyle's and Charles' Laws. See Table 1, the blanks of which are to be filled in by the present paper.

The central map of this triple map, the map from the KTG into the IP, is not one of algebraic transformation but rather one of similarity of concept. The '➡' in Table 1 indicates a map from a particular row in column 1 (variables of Boyle's and Charles' Laws) to the same row in column 4 (variables of macro level socio-economic laws isomorphic to Boyle's and Charles' Laws). The argument and evidence will link temperature, pressure, and volume to mean skill of wage earners, mean interpersonal competition for wealth, and mean wealth:

| | | |
|---|---|---|
| Temperature (T) | ➡ | Mean Skill of Labor Force (S) |
| Pressure (P) | ➡ | Mean Interpersonal Competition for Wealth (C) |
| Volume (V) | ➡ | Mean Wealth (W) |

While the map from the KTG's particle system into the IP is based on the subjective perception of similarity of function of each variable in the KTG to its analogue in the IP, this particular map results in valid macro-level socio-economic laws isomorphic to Boyle's and Charles' Laws.

Table 1 The Triple Map of Gas Law Variables into KTG Variables into IP Variables and Parameters into Macro-Level Socio-Economic Variables and Laws Isomorphic to Boyle's Law and Charles' Law

| Variables of Boyle's and Charles' Laws (macro level) | ➡ | Variables of the Kinetic Theory of Gases [KTG] (micro level) | ➡ | Variables of the Inequality Process [IP] (micro level) | ➡ | Variables of Socio-Economic Laws Isomorphic to Boyle's and Charles' Laws (macro level) |
|---|---|---|---|---|---|---|
| temperature | ➡ | mean molecular kinetic energy | ➡ | ? | ➡ | ? |
| pressure | ➡ | force exerted by kinetic energy of molecules on container and other molecules | ➡ | ? | ➡ | ? |
| volume | ➡ | 3-space, part of definition of kinetic energy, part of assumed image of model | ➡ | ? | ➡ | ? |

In Table 1 '➡' from column 1 to column 2 points from a macro-level gas law variable to the corresponding micro-level KTG variable or concept. '➡' from column 2 to column 3 points from a micro-level KTG variable or concept to the micro-level IP socio-economic variable, parameter, or concept that is most like the KTG variable. Finally, '➡' from column 3 to column 4 relates micro-level IP socio-economic variables to macro-level socio-economic variables that are related to each other in the same way that the macro-level gas law variables they are matched to, retracing the map path right to left across a row of Table1, are related to each other in Boyle's and Charles' Laws. The socio-economic images of Boyle's Law and Charles' Law under the map of Table 1 are called Boyle's Socio-Economic Law and Charles' Socio-Economic Law respectively.

## 2.0 The Gas Laws: Boyle's Law and Charles' Law

Boyle's Law relates the pressure of a gas to its volume, holding temperature constant. Charles' Law relates the temperature of a gas to its volume, holding pressure constant. These laws are called ideal gas laws because they are approximations to the thermodynamics of real gases in a familiar range of temperatures and pressures.

Boyle's Law asserts:

$$P_{t_1} V_{t_1} = P_{t_2} V_{t_2} \tag{1}$$

where,

$$P_{t_1} = \textit{the pressure in a container of gas at time} - \textit{step } 1$$
$$V_{t_2} = \textit{the volume of the container at time} - \textit{step } 2$$

given constant temperature (Fischer-Cripps, 2003). Visualize air in a metal piston with a good seal bathed in a fluid that evens out temperature differences.

Charles' Law asserts:

$$\frac{V_{t_1}}{T_{t_1}} = \frac{V_{t_2}}{T_{t_2}}$$

where, (2)

$$T_{t_1} = \textit{the temperature of the gas at time} - \textit{step } 1$$
$$V_{t_2} = \textit{the volume of the container at time} - \textit{step } 2$$

holding pressure constant (Fischer-Cripps, 2003). Visualize air being heated or cooled in a sealed rubber balloon able to exert an approximately equal pressure on the gas it contains over a range of degrees of inflation and temperatures.

## 3.0 Map of Column 1 into Column 2 of Table 1: Boyle's and Charles' Laws into the Kinetic Theory of Gases (KTG)

The Kinetic Theory of Gases (KTG) explains the thermodynamics of a sealed volume of gas in terms of elastic spheres (gas molecules) colliding according to the laws of Newtonian mechanics without a loss of kinetic energy in the collision. The KTG identifies heat energy as the kinetic energy of gas molecules in motion. Kinetic energy is defined in Newtonian mechanics as the product of mass and velocity squared, $m\ v^2$, m is mass and v velocity. Velocity is a vector variable, the gas molecule's speed in a direction in 3-space.

The KTG assumes a population of molecules that is completely isolated. No molecules enter or leave. The population of molecules is thermally isolated as well, unless otherwise stipulated, as for example, in Charles' Law.

The macro-level variable temperature (T) maps into the mean kinetic energy of gas molecules in the KTG. Pressure (P) is the force exerted by gas molecules colliding against an area of the wall of the container of the gas. Pressure (P) propagates through the volume of gas the same way. The macro level variables of the gas laws, temperature (T) and pressure (P), are thus just different aspects of molecular kinetic energy. The third macro level variable of Boyle's and Charles' Laws, volume (V) is the size of the interior space of a hollow object in 3-space. 3-space also appears at the micro level in the KTG in the variable 'velocity', speed in a direction in 3-space, a vector in 3-space. The particles of the KTG, gas molecules, move in 3-space. 3-space then is both a macro-level variable and part of a micro-level variable in the KTG.

Table 2 How Macro-level Variables of the Gas Laws Map into Micro-level Variables of the Kinetic Theory of Gases (KTG).

| Macro Level Gas Law | ➡ | Micro Level Kinetic Theory of Gases |
|---|---|---|
| Temperature (T) | ➡ | mean molecular kinetic energy |
| Pressure (P) | ➡ | force exerted by molecules colliding on an area of container |
| Volume (V) | ➡ | 3-space at micro level |

## 4.0 The Map from Column 2 into Column 3 of Table 1: The Particle System Model of the Kinetic Theory of Gases and the Inequality Process

This section examines the map from column 2 into column 3 of Table 1, the map from the micro-level variables of the Kinetic Theory of Gases (KTG) into the micro-level variables and parameters of the Inequality Process, a socio-economic particle system.

### 4.1 Similarity Between KTG and IP

Speculation about possible socio-economic analogues of Boyle's and Charles' Laws makes sense because of the similarity between the Kinetic Theory of Gases' (KTG) stochastic particle system model (presented in Whitney, 1990) and the Inequality Process (IP), a particle system with a wide empirical explanandum. Both the KTG and the IP are particle systems that scatter a positive quantity (kinetic energy in the KTG, wealth in the IP) among particles via encounters between randomly paired particles. In both particle systems, particles are immortal and collectively isolated. In both the aggregate sum of the positive quantity over all particles is constant. In both particle systems, an encounter between particles neither creates nor destroys the positive quantity exchanged. The KTG's stationary distribution of kinetic energy is a negative exponential distribution. The IP's stationary distribution is gammoidal (but not exactly a gamma pdf) and approximates the variety of distributions of personal wealth and income depending on the IP's parameter.

### 4.2 The Transition Equations of the KTG and IP

The equations for the exchange of kinetic energy, x, between a pair of colliding molecules in the KTG's stochastic particle system model is:

$$\begin{aligned} x_{it} &= \varepsilon_t \left( x_{i(t-1)} + x_{j(t-1)} \right) \\ x_{jt} &= (1-\varepsilon_t)\left( x_{i(t-1)} + x_{j(t-1)} \right) \end{aligned} \tag{3}$$

where

$$\begin{aligned} x_{i(t-1)} &= \textit{particle } i\textit{'s kinetic energy at time} - \textit{step } (t-1) \\ x_{jt} &= \textit{particle } j\textit{'s kinetic energy at time} - \textit{step } t \\ \varepsilon_t &= \textit{an i.i.d.}\,[0,1]\ \textit{uniform continuous r.v. at time} - \textit{step } t \end{aligned}$$

The equations for the exchange of wealth between two particles in the Two Parameter Inequality Process (TPIP), a modification of the particle system in Angle (1983,1986) is:

$$\begin{aligned} x_{it} &= x_{i(t-1)} + d_t \omega x_{j(t-1)} - (1-d_t)\omega x_{i(t-1)} \\ x_{jt} &= x_{j(t-1)} - d_t \omega x_{j(t-1)} + (1-d_t)\omega x_{i(t-1)} \end{aligned} \tag{4}$$

where,

$$x_{it} = \textit{particle i's wealth at time} - \textit{step } t$$
$$0 < x_{it}$$
$$d_t = \begin{pmatrix} 1 \textit{ with probability .5 in the absence of} \\ \quad \textit{organized competition, discrimination;} \\ 1 \textit{ with probability .5 when a member of coalition } \alpha \\ \quad \textit{encounters a member of coalition } \alpha\textit{, and when} \\ \quad \textit{a member of outgroup } \beta \textit{ encounters a member} \\ \quad \textit{of outgroup } \beta; \\ 1 \textit{ with probability } \delta_{\alpha\beta} \textit{ when a member of} \\ \quad \textit{coalition } \alpha \textit{ encounters a member of} \\ \quad \textit{outgroup } \beta, \textit{(minority group);} \\ 1 \textit{ with probability } (1-\delta_{\alpha\beta}) \textit{ when a member of} \\ \quad \textit{outgroup } \beta \textit{ encounters a member of coalition } \alpha; \\ 0 \quad \textit{otherwise} \end{pmatrix}$$

and,

$$0 < \omega < 1 \qquad \textit{the fraction of particle wealth lost in a loss}$$

where $w_\alpha$ is the fraction of the population in coalition $\alpha$, $w_\alpha + w_\beta = 1.0$ . $\delta_{\alpha\beta}$ is not determined endogenously.

Angle (1990) establishes that the IP's equations for the exchange of a positive quantity between two particles are isomorphic to those of the KTG's particle system model up to two differences. This near isomorphism is clearer if the IP is re-written as:

$$\begin{aligned} x_{it} &= (1-\omega)x_{i(t-1)} + d_t\,\omega(x_{j(t-1)} + x_{i(t-1)}) \\ x_{jt} &= (1-\omega)x_{j(t-1)} + (1-d_t)\omega(x_{j(t-1)} + x_{i(t-1)}) \qquad (5) \end{aligned}$$

The transformations, $d_t \rightarrow \varepsilon_t$ , and $\omega \rightarrow 1.0$, map the IP's transition equations into the KTG's.

### 4.2.1 Consequences of the Differences Between the KTG and the IP

There are two differences between the particle system models of the KTG and the IP: a) a parameter in the IP that is set to 1.0 in the KTG, and b) different stochastic drivers of the exchange of a positive quantity between particles, a [0,1] continuous uniform random variable in the KTG, a 0,1 discrete random variable (Bernoulli variable) in the IP. In the simpler versions of the Inequality Process a 0 or 1 is equally likely. In the Two Parameter Inequality Process (TPIP), they are not necessarily. The stochastic drivers of the KTG and the IP have similar sounding names but distinct consequences. Among the consequences of the differences between the IP and KTG is the fact that the IP is time-asymmetric. The KTG is time-symmetric (impossible to identify the direction of time from a vector of consecutive observations on a particle's kinetic energy). The direction of time is readily determined from a vector of consecutive observations on the wealth of a particle in the IP (Angle, 2006). Another consequence of the difference between the particle systems of the KTG and IP is the stationary distribution of each. The KTG's particle system has a single stationary distribution, the negative exponential pdf, whereas the IP has a family of gammoidal stationary distributions depending on ω, the IP's particle parameter, the fraction of wealth a particle loses when it loses a competitive encounter with another particle.

4.2.2 Differences Between the KTG and the IP Due to Subject Matter

Because of the KTG's subject matter, it cannot be tested at the micro-level. It is not possible to tag and track the history of kinetic energies of individual gas molecules. The particles of the IP, on the other hand, are people. People can be identified and re-interviewed over time enabling the construction of a time-series of observations on them. Thus, the IP can be tested at the micro-level as well as the macro-level. An example of a macro-level test of the IP is the fitting of its stationary distribution to an empirical distribution of income and wealth. An example of a micro-level test of the IP's implications is the test of its implications for the pattern of autocorrelation of a particle's wealth over time against year to year data on people's income. And there is the difference that people can form coalitions and gas molecules do not.

## 5.0 The Map from the Kinetic Theory of Gases (KTG) into the Inequality Process (IP)

Temperature and pressure in the Gas Laws are closely related at the macro level in measurements on a contained volume of a gas because they are different aspects of the same variable at the micro level, the kinetic energy of gas molecules. Molecular kinetic energy is not a macro-level concept. 3-space appears in variables, on the other hand, at both the macro and the micro level. At the macro level it is volume, at the micro level it is the space that molecules move in and appears in the algebraic expression for kinetic energy, the product of mass by velocity squared. Velocity is speed in a direction in 3-space.

So the map of the KTG into the IP should require that the images of temperature and pressure in macro-level socio-economic variables (one row in Table 1) be implied by the IP images of mean molecular kinetic energy (temperature) and the force exerted by the kinetic energy of a molecule colliding with the container wall or another molecule (pressure) in the KTG. The macro level socio-economic analogue of volume should have the same meaning at the micro-level, since 3-space has the same meaning at the micro-level (KTG) as at the macro-level (Gas Laws).

5.1 The Analogue of Temperature in the IP and the Socio-Economic Analogue of Charles' Law

Temperature (mean kinetic energy at the micro level) drives the phase transitions of a gas at the macro-level. The states of matter of a gas with rising temperature go through the following phase transitions: solid -> liquid -> gas -> plasma (ionized gas). The image of mean kinetic energy in the IP should also drive phase changes at the micro and macro socio-economic levels. ω plays this role in the IP. ω is a micro-level variable that determines the shape of the IP's stationary distribution, a macro level concept.

(1-ω) in the IP's meta-theory represents a worker's skill level, operationalized as a worker's level of education in the fit of the IP's stationary distribution conditioned on ω to the distribution of labor income conditioned on education. The Inequality Process with Distributed Omega (IPDO) (Angle, 2002, 2006) is the IP's model of competition in an industrial labor market with workers with varying levels of skill as indicated by their terminal levels of education. The IPDO's transition equations for an encounter between particle i in the $\omega_\psi$ equivalence class and particle j in the $\omega_\theta$ equivalence class:

$$\begin{aligned} x_{i\psi t} &= x_{i\psi(t-1)} + d_t \omega_\theta \, x_{j\theta(t-1)} - (1-d_t)\omega_\psi x_{i\psi(t-1)} \\ x_{j\theta t} &= x_{j\theta(t-1)} - d_t \omega_\theta \, x_{j\theta(t-1)} + (1-d_t)\omega_\psi \, x_{i\psi(t-1)} \end{aligned} \tag{6}$$

(6) is isomorphic to (4) except that particle i is in the $\psi^{th}$ ω equivalence class (all particles whose parameter is $\omega_\psi$), while particle j is in the $\theta^{th}$ ω equivalence class and there are no coalitions. Particles i and j are distinct although they may be drawn from the same equivalence class, i.e. it is possible that $\omega_\psi = \omega_\theta$. The stationary distribution of wealth in each IPDO $\omega_\psi$ equivalence class is not in general equal to that of (4) with equal $\omega_\psi$ unless the $\omega_\psi$ equivalence class includes the entire particle population and there are no coalitions. Another difference between the particle system of (4) and that of (6) is that in (4) mean wealth, μ is exactly 1.0 [by assumption for numerical reasons, without loss of generality], whereas in (6), only the unconditional mean of wealth, μ, is exactly 1.0 [by assumption for numerical reasons, without loss of generality]. In (6), mean wealth in the $\omega_\psi$ equivalence class, $\mu_\psi$, is not constrained except that the $\mu_\psi$'s, weighted by the fraction of particles in each $\omega_\psi$ equivalence class, must sum to the unconditional mean, μ.

With data based estimates of the $\mu_\psi$'s and thus μ, Angle (2002, 2006, 2007) shows that the $\mu_\psi$'s and μ are approximately related to the $\omega_\psi$'s as:

$$\mu_\psi \approx \frac{\tilde{\omega}\mu}{\omega_\psi} \tag{7}$$

where

$$\tilde{\omega} = \textit{the harmonic mean of the } \omega_\psi\textit{'s}$$

The analogue of volume in the Gas Laws in the Inequality Process with Distributed Omega (IPDO) is the unconditional mean of wealth, μ. It has the same meaning at both the micro and macro levels. The IPDO expression for estimating the unconditional mean of wealth, μ, from the distribution of the $\omega_\psi$'s, their harmonic mean $\tilde{\omega}$, and mean wealth, $\mu_\psi$, in each $\omega_\psi$ equivalence class is:

$$\mu \approx \frac{\omega_\psi \mu_\psi}{\tilde{\omega}} \tag{8}$$

The numerator of (8) is an approximate constant across $\omega_\psi$ equivalence classes as can be seen in the graph of change in particle wealth in a particular IPDO from time step t-1 to t against particle wealth at time t-1, Figure 1. Notice the y = $-\omega_\psi\mu_\psi$ line; it is the reflection in the x-axis of $y = \tilde{\omega}\mu$, each approximately in absolute value equal across all $\omega_\psi$ equivalence classes. (8) is isomorphic to Charles' Law given the substitution of $1/\tilde{\omega}$ for temperature and the unconditional mean of wealth, μ, for volume.

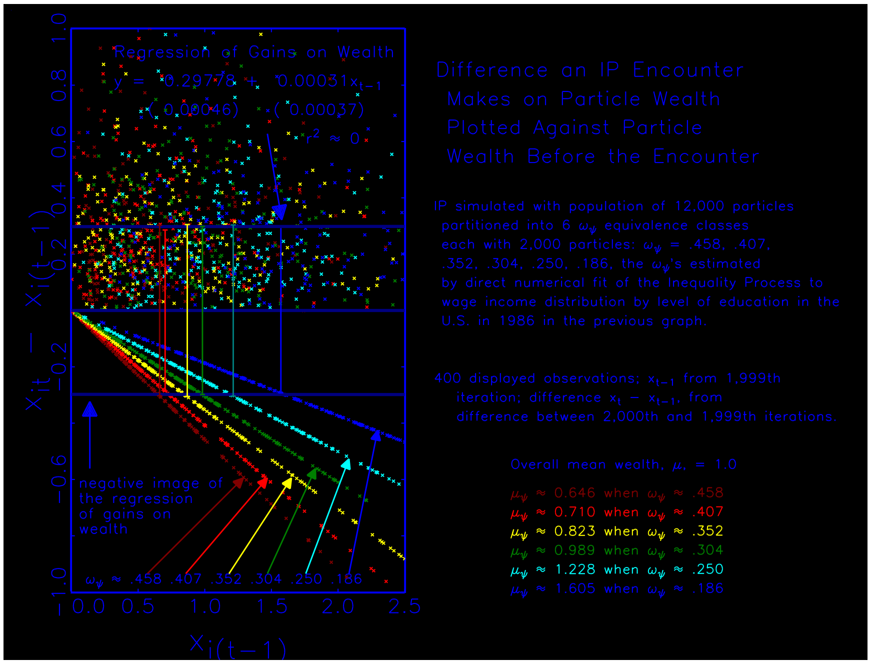

**Figure 1**

5.1.1 Why Not Map Temperature into Mean Wealth in the IP?

Dragulescu and Yakovenko (2000) proposed the particle system of the KTG as a model of wealth distribution without modification other than a relabeling of 'molecule' as 'person' and 'kinetic energy', the x variable in (3), as 'wealth'. Their hypothesis commits them to the hypothesis that the KTG's stationary distribution of income, a negative exponential pdf, is a general model of income and wealth distribution. It is not, as is readily demonstrated (Angle, 1983, 1986, 1996, 2002, 2006, 2007, and forthcoming). The Dragulescu and Yakovenko (2000) model implies that the image of temperature under their relabeling of the KTG's particle system is mean wealth. But as noted, this hypothesis does not make sense for the Inequality Process whose properties are determined not by mean wealth but by ω and δ. In the IP's meta-theory, it is worker skill, and by extension the technological level of the society, operationalized by ω in the OPIP or $\tilde{\omega}$ in the IPDO that determines μ, the unconditional mean of wealth in the population.

5.2 The Analogue of Pressure in the IP and the Socio-Economic Analogue of Boyle's Law

Pressure is a macro-level variable in the gas laws: the force exerted by a gas on an area of its container. The micro-level KTG manifestation of pressure is the force exerted by the collisions of molecules on an area of the container (as well as each other). This force is produced by molecular kinetic energy. So, temperature and pressure at the macro level are different aspects of molecular kinetic energy at the micro-level.

The socio-economic analogue of pressure is competition for scarce resources. Population biologists analogize competition, called ‘crowding’ and in the case of population size curves ‘density dependence’ (Thieme, 2003), to pressure exerted by niche constraints on a growing population, similar to the way decreasing the volume in a piston increases pressure and, at the molecular level, the density and kinetic energy of molecule collisions. The increased pressure in the case of population biology comes either from population growth or contraction of the carrying capacity of the species niche. Competition spaces out individual members of a species throughout its niche, as gas pressure spaces out gas molecules, filling in any relative vacuum. Competition forces organisms into the margins of the species niche, where survival and reproduction are more difficult. Those that survive on the niche margin – or beyond – enlarge the niche, as gas pressure inflates a balloon. There is no gas analogue of “fitter” molecules occupying subsets of the volume that are “better”. The ideal gas metaphor for population size and species niche carrying capacity is imperfect.

Unlike gas molecules, competitive pressure leads people to form coalitions. A coalition able to exert competitive pressure on others takes wealth away from them. People form coalitions to benefit themselves at the expense of others. The impulse to do so is particularly strong when wealth decreases (increased pressure from niche carrying capacity contraction). The behavior occurs in small groups and whole societies. Coalition formation occurs over the entire arc of techno-cultural evolution from hunter/gather to industrial society. An argument can be made that people are physically predisposed to act this way because they are descendents of such coalitions formed in hunter/gatherer groups, i.e., most of the history of the species, which formed during famines. Sociologists call the coalition that benefits the ‘majority group’ and the victimized residual, the ‘minority group’.

5.2.1 The Two Parameter Inequality Process (TPIP)

The Inequality Process (IP) of (4), the Two Parameter Inequality Process (TPIP), models the pressure that the majority coalition puts on the minority outgroup by the majority’s power to increase the probability of a member’s winning a competitive encounter with a nonmember. In (4) the probability of a member of the majority group, call it coalition $\alpha$, winning wealth from a member of its own coalition is 1/2. The probability of a member of coalition $\alpha$, the majority group, winning an encounter with a nonmember, a member of the minority group $\beta$, is $\delta_{\alpha\beta}$. (4) asserts that every member of the population is either a member of coalition $\alpha$ or coalition $\beta$. The intersection of the two groups is empty. So the intensity of the victimization of the minority, called ‘discrimination’ by sociologists in the case of milder, indirect victimization, is given by the expression ($\delta_{\alpha\beta}$ - .5), or disregarding whether $\alpha$ or $\beta$ is the majority group, $|\delta_{\alpha\beta} - .5|$ since $|\delta_{\alpha\beta} - .5| = |.5 - \delta_{\beta\alpha}| = |\delta_{\beta\alpha} - .5|$ because $\delta_{\alpha\beta} = 1 - \delta_{\beta\alpha}$. Note that in the TPIP, (4), $\delta_{\alpha\beta}$ is set exogenously. It is posited here that coalition $\alpha$ has more than 50% of the population so that it is literally the majority group and has the power to increase $\delta_{\alpha\beta}$ above .5.

A particle in coalition $\alpha$ has an unconditional (i.e., not conditioned on the coalition membership of the particles it encounters) of winning an encounter with another particle of :

$$\delta_\alpha = w_\alpha (1/2) + (1 - w_\alpha)(\delta_{\alpha\beta})$$

(9)

where $w_\alpha$ is the fraction coalition α is of the population of particles and, by assumption, $w_\alpha > .5$. $w_\beta = 1 - w_\alpha$. A particle in minority group β has an unconditional chance of winning of:

$$\delta_\beta = (1 - w_\alpha)(1/2) + w_\alpha (1 - \delta_{\alpha\beta})$$

(10)

Given the negative binomial pf approximation to the run-like solution of the transition equations of the Inequality Process and its stationary distribution (Angle, 2002, 2006), the expected wealth of a member of coalition α is, approximately, for $|\delta_{\alpha\beta} - .5|$ not far from zero (modeling mild antagonism by the majority):

$$\mu_\alpha \approx w_\alpha \mu + (1 - w_\alpha)\left(\frac{\delta_\alpha}{(1 - \delta_\alpha)}\right)\mu$$

(11)

The difference $\mu_\alpha - \mu$ is the expected gain from membership in coalition α for one of its members where μ is the unconditional expectation of wealth (i.e., not conditioned on membership in coalition α) and $\mu_\alpha$ is the expectation of wealth in the coalition α. Note that $\mu_\alpha$ is a scale transformation (multiplication by a positive constant) of μ, the unconditional mean (i.e., not conditioned on coalition membership) of particle wealth:

$$\mu_\alpha \approx \left[ w_\alpha + (1 - w_\alpha)\left(\frac{\delta_\alpha}{(1 - \delta_\alpha)}\right)\right]\mu$$

(12)

The expected wealth of a member of minority group β, the minority group, is:

$$\mu_\beta \approx (1 - w_\alpha)\mu + w_\alpha \left(\frac{\delta_\beta}{(1 - \delta_\beta)}\right)\mu$$

\

(13)

The difference $\mu - \mu_\beta$ is the expected loss of a member of minority group β, the minority group. Note that $\mu_\beta$ is a scale transformation of μ, and different from that of coalition α.

$$\mu_\beta \approx \left[ (1 - w_\alpha) + w_\alpha \left(\frac{\delta_\beta}{(1 - \delta_\beta)}\right)\right]\mu$$

(14)

The constants in brackets on the RHS of (12) and (14) are the scale transformations that map the unconditional distribution of wealth of the TPIP into those of the majority and minority groups respectively. Their inverses map the majority and minority wealth distributions back into the unconditional distribution. The inverses can be thought of as “discrimination reversing” scale transformations.

While it is a feature of the Inequality Process that all transfers of wealth between particles are zero-sum, and coalition $\alpha$'s gain is minority group $\beta$'s loss, it does not follow that each member of coalition $\alpha$ gains as much as each member of minority group $\beta$ loses. Coalition $\alpha$'s gain is diluted by division among a larger population than that of minority group $\beta$.

(10) through (14) depend on the validity of the negative binomial pf approximation to the solution of the Inequality Process' transition equations (Angle, 2002, 2006). That approximation deteriorates as $|\delta_{\alpha\beta} - .5|$ becomes large, because as $|\delta_{\alpha\beta} - .5|$ becomes large, the wealth of members of minority group $\beta$ becomes small enough that fewer losses are required to nearly zero it out. That number of losses plus 1 determines the shape parameter of the approximating gamma pdf. So the effect of discrimination on the shape of the distribution becomes confounded with that of scale as $|\delta_{\alpha\beta} - .5|$ becomes large. To the extent the Two Parameter Inequality Process (TPIP) is a useful model of the effect of discrimination on the distributions of wealth of a majority coalition and its victimized minority group, then the inverse of the scale transformation of $\mu$ into $\mu_\alpha$ of (12) maps $\mu_\alpha$ into $\mu$. Similarly for minority group $\beta$, the inverse of (14) mapping $\mu$ into $\mu_\beta$ maps $\mu_\beta$ into $\mu$.

Scale transformations operate not only on means but on every wealth percentile equally. The effect can be reproduced by a scale transformation of every observation on wealth. Every percentile before the transformation is mapped into the identical percentile after the transformation. It is difficult to estimate the mean of a wealth or income distribution because small and large amounts of wealth or income are seriously affected by non-sampling error or not available, i.e., top-coded or bottom-coded. Usually, the median is in the best measured part of an income or wealth distribution, the distribution of incomes least affected by the measurement issues associated with small and large incomes. The inverse scale transformations to those of (12) and (14) reverse the effect of discrimination on the wealth distribution of majority and minority respectively. The scale transformation that reverses the effect of discrimination on the minority group when multiplied by minority group wealth is:

$$\frac{x_{\{50\}}}{x_{\{50\}\beta}}$$

the ratio of the unconditional median of wealth to that of minority group $\beta$.

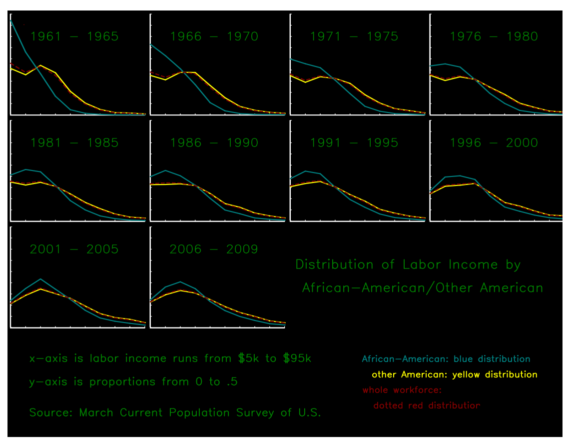

Figure 2

Historically for U.S. sociology, the classic minority group has been African-Americans. Figure 2 graphs the distribution of annual labor income of African-Americans averaged into half decade periods over the last half century. Notice that the blue piecewise linear curve (the African-American distribution) is above the red dashed curve (the unconditional distribution) over small incomes, below the red dashed curve over large incomes, especially in the earlier half-decades. If the TPIP is valid, multiplication of African-American labor incomes should result in a relative frequency distribution that approximately overlaps the unconditional distribution. It does. See Figure 3.

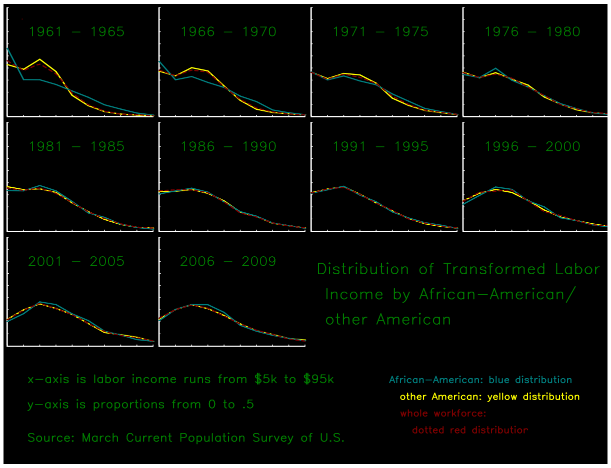

Figure 3

5.3 Volume

The macro-level gas law variable volume is unchanged at the micro level. It is the concept of 3-space at both the micro and macro levels. There is only one variable left in the IP that the concept of 3-space can be mapped into in the IP. It is wealth. Wealth is the space in which particles move in the IP. Wealth in the IP at the micro-level has the same meaning at the macro socio-economic level, although it is more readily measured in its flow form at the macro level.

Table 3 How Micro-level Variables of the Kinetic Theory of Gases (KTG) Map into the Micro-Level Variables of the Inequality Process (IP).

| Micro Level Kinetic Theory of Gases (KTG) | ➡ | Micro Level Inequality Process (IP) |
|---|---|---|
| mean molecular kinetic energy | ➡ | $1/\tilde{\omega}$ where omega tilde is the harmonic mean of the $\omega$'s, the fraction of wealth lost when a particle loses an encounter in (6), and in the IP's meta-theory, the indicator of worker skill level. Smaller $\omega$ -> greater skill. |
| force exerted by molecular collisions on unit area of container | ➡ | the power of a coalition $\alpha$, the majority group, to increase its chance above .5 of its members winning wealth from those of outgroup $\beta$, $\delta_{\alpha\beta}$ - .5 . |
| 3-space at micro level | ➡ | mean wealth, the space in which particles move, $\mu$ |

## 6.0 The Map from Column 3 into Column 4 of Table 1: The Inequality Process into Socio-Economic Laws Isomorphic to Boyle's and Charles' Laws

6.1 The Socio-Economic Image of Boyle's Law

The map from column 1 of Table 1 to column 4, that is, across each row of Table 1 is:

Temperature (T) ➡ Mean Skill of Labor Force (S)

Pressure (P) ➡ Mean Competition (C) for Wealth via Coalitions

Volume (V) ➡ Mean Wealth (W)

since micro-level variables in the Inequality Process scale up to macro-level variables. They do so because the micro-level IP particles are "taggable" and because human society is scaleable, i.e., work skill, competition, and wealth have the same meaning at the societal level as in small groups.

The Charles' Socio-Economic Law asserts that mean worker skill (S) covaries with aggregate wealth (W), holding constant mean competition (C) organized into coalitions. Techno-cultural evolution achieves greater aggregate wealth with a more skilled labor force. Charles' Socio-Economic Law is an empirical truism. This law is:

$$\frac{V_{t_1}}{T_{t_1}} = \frac{V_{t_2}}{T_{t_2}} \quad \Rightarrow \quad \frac{\mu_{t1}}{(1/\tilde{\omega}_{t1})} \approx \frac{\mu_{t2}}{(1/\tilde{\omega}_{t2})} \quad \Rightarrow \quad \frac{W_{t_1}}{S_{t_1}} \approx \frac{W_{t_2}}{S_{t_2}}$$

pressure (P) constant competition (C) constant

The approximate equation holds because of (8), as illustrated in Figure 1.

The socio-economic image of Boyle's Law, Boyle's Socio-Economic Law, is:

$$P_{t_1} V_{t_1} = P_{t_2} V_{t_2} \quad \Rightarrow \quad C_{t_1} W_{t_1} = C_{t_2} W_{t_2}$$

T assumed constant S assumed constant

The Boyle's Socio-Economic Law says that keeping mean worker skill level (S) constant, intensity of competition (C) for wealth organized into coalitions moves inversely to mean wealth (W). It has been demonstrated that such is the case for a minority group. More generally, if as Angle (2006a,2007b) argues, the Inequality Process (without coalitions biasing the probability of winning up or down) is an algorithm that maximizes wealth by transferring wealth to workers most productive of wealth, then the effect of coalitions on the distribution of wealth depresses aggregate economic product.. This conclusion is also that of conventional economic theory: discrimination reduces economic product by interfering with the operation of the labor market: members of a minority group are under-rewarded for their efforts and investments in skill acquisition, while members of the majority group are over-rewarded per capita but not to the extent that minority group members are under-rewarded per capita. From the point of view of gross economic product, the effect of discrimination against a minority group is to waste human resources. Boyle's Socio-Economic Law is obvious in cases of intra-societal victimization more intense than ordinary labor market discrimination since conflict disrupts the economy.

## 7.0 References


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