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Algorithm for construction of portfolio of stocks using Treynor's ratio

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Abstract

The aim of the paper is to implement the algorithm for selecting stocks from a pool of stocks listed in a single market index like S&P CNX 500(say) and finding the corresponding weights of the stocks in the optimized portfolio using Treynor's ratio, on the basis of historical data of Indian stock market when the short selling is not allowed. The effectiveness of this algorithm has been demonstrated with an example.

1. Introduction

Market offers several assets in various formats which are grounds for investing money and gaining returns after specific time periods. Investments are made in view of obtaining highest returns with lowest chance of losing money. The returns are however characterised by the nature of assets and the market factors that influence its pricing everyday. Since the returns cannot be foretold with certainty, the analysis of profitability in an asset becomes an objective of utmost priority in an investment procedure. A technique of judging the behaviour returns from an asset is historical data analysis of the asset with respect to market.

The classic portfolio theory of Markowitz[1] shows that a collective group of assets if formulated using mathematical modelled optimisation problem, can give higher returns with lower chance of losing money. In fact, Markowitz portfolio model gives the most basic and complete framework for investment decision. On the negative end, Markowitz theory exhausts the chances of selecting the asset that give unusual higher return on the cost of volatility.

This paper uses Treynor's ratio (i.e. excess return to beta) as the criteria for the selection of a stock in a portfolio as described in Elton, Gruber, Brown and Goetzmann[2]. The algorithm gives the percentage weights to be invested in each stock selected without short selling, for an optimum portfolio by evaluating the historical data available on stocks.

2. The decision making procedure of Treynor's ratio

Suppose we are having a pool of stocks of an index and we want to select stocks and calculate the weights of the selected stocks to be invested. The desirability of a stock is directly proportional to "excess return to beta" ratio i.e. Treynor's ratio. Excess return is the difference between the rate of return of the stock and the risk free rate of return as on Treasury bill (say). Beta specifies the non diversifiable risk (risk which cannot be eliminated by diversification).

$$\text{Treynor's ratio} = (E[R_i] - R_f) / \beta_i$$

where β_i = beta value of the stock (the relative change in excess return of stock with 1% change in market)

$$E[R_i] = \text{Expected return of } i^{\text{th}} \text{ stock}$$

$$R_f = \text{risk free rate}$$

Stocks with higher treynor's ratio means that the stock is more under valued and hence are ranked on its value (highest to lowest). A cut off rate C^* is calculated and

the stocks with the higher treynor's ratio than this C^* are selected for portfolio construction. The procedure of finding C^* , as given in Elton, Gruber, Brown and Goetzmann[2], involves finding C_i for each stock assuming i^{th} stock is present in our optimum portfolio.

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^i [(E[R_j] - R_f) \beta_j] / \sigma_{ej}^2}{1 + \sigma_m^2 \sum_{j=1}^i (\beta_j^2 / \sigma_{ej}^2)}$$

where σ_m = market volatility

σ_{ej}^2 = unsystematic risk of stock (risk not related to market but stock's nature)

β_j = beta value of the stock (index of systematic risk present in the j^{th} stock)

$E[R_j]$ = Expected return of j^{th} stock

R_f = risk free rate

Out of these C_i 's, there will be only one C_i for which treynor's ratio of all the stock preceding this i^{th} stock will be greater than this C_i and treynor's ratio of all the succeeding this i^{th} stock will be less than this C_i . This C_i will be our C^* and all the stocks preceding it will be selected in our optimum portfolio.

3. Calculation of weights of stock in our optimum portfolio

The weightage of capital invested in each selected stock (X_i) can be calculated as given in Elton, Gruber, Brown and Goetzmann[2].

$$X_i = \frac{Z_i}{\sum Z_i}$$

where $Z_i = (\beta_i / \sigma_{ei}^2) [(E[R_i] - R_f) / \beta_i] - C^*$

4. Algorithm

The algorithm requires an input of adjusted closing price of pool of stocks and of the market index. Following are the steps involved in our algorithm. The complete MATLAB program is given in the Appendix.

§ The data is read from the excel files and daily return of all the stocks as well as market is calculated as

$$r(i) = \frac{V(i+1) - V(i)}{V(i)}$$

§ Average daily return is calculated by using the geometric mean of the historical daily returns as given by

$$DR = ((1+r_1)(1+r_2)(1+r_3)(1+r_4)\dots(1+r_n))^{1/n} - 1$$

Standard deviation of the daily returns (Dstd) is calculated.

- § Average annual return is calculated from average daily returns assuming 252 trading days in a year.

$$AR = (1+DR)^{252}-1$$

Annual standard deviations can be calculated from the daily standard deviations as

$$Astd = \sqrt{252} \times Dstd$$

- § Capital Asset Pricing Model is used to calculate the raw beta for each stock which states the dependence of expected return of an asset on the market movement.

$$R_i(t) = R_f + \beta_i \{R_m(t) - R_f\} + \varepsilon_i$$

where $R_i - R_f =$ excess return of asset

$R_m - R_f =$ excess return of market

$\varepsilon_i =$ shock factor with mean =0

Linear regression is run for each stock on excess daily return of stock vs. excess daily return of market for finding the raw beta as per historical data. Then the adjusted beta ($A\beta$) is calculated from the raw beta ($R\beta$) assuming that the security's beta move towards the market average over time with a 67% confidence level as given by

$$A\beta = 0.67 * R\beta + 0.33 * 1$$

- § The stocks with β coming out to be -ve are removed.
- § Now the investment decision is made on the basis of Treynor's ratio as described above and the weights of the accepted stocks are calculated.

5. Application Example

We have considered index of S&P CNX 500 and the underlying stocks in the index for implementing the above algorithm using MATLAB, for the period June-2010 to June-2012.

Inputs :

- § The adjusted closing price of the stocks is taken from Prowess (CMIE database) and kept in the file "DATA.xlsx" as shown in the figure 1.

Algorithm for construction of portfolio of stocks using Treynor's ratio

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	3I Infotech Ltd.	64.1	65.05	66.15	66.1	63.7	62.5	62.05	62.3	62.65	62.8	63.05	63	63.45	63.5	63.8	62.75	63.3
2	3M India Ltd.	2459.05	2493.9	2460.25	2467.95	2552.75	2510.9	2542.15	2550.1	2565.65	2583.85	2605.45	2638.4	2637.75	2670	2670	2665.85	2655.05
3	A B B Ltd.	854.3	854.45	859.35	858.55	858.75	852.9	855.1	858.3	858.85	861.1	862.8	861.25	865.55	861.35	869.85	871.1	872.8
4	A B G Shipyards Ltd.	244.55	249.45	253.65	250.35	250.55	246.95	246.95	250.2	248.6	251.75	253	253.55	255.45	257.05	258.5	258.2	256.7
5	A C C Ltd.	817.7	826.8	841.5	841.9	851.7	842.35	858.05	868.9	861.75	862.85	879.9	863.6	865.45	862.1	882.65	861.5	871.15
6	A I A Engineering Ltd.	405.05	411.75	403.85	401.5	394.65	393.6	398.5	398.4	404.35	400.6	399	394.05	397.55	395.35	396.45	401.1	400.05
7	Aarti Industries Ltd.	47.05	47.4	48.05	48	46.2	46	46.4	46.35	46.45	46.85	46.95	46.9	47.1	48	48.35	48	49.9
8	Aban Offshore Ltd.	691.15	694.9	695.45	699.95	674.45	658.5	647.9	645.6	657.9	662.85	678.95	708.65	730.8	731	744.85	749.05	741.25
9	Adani Enterprises Ltd.	536.5	540.85	550.2	551.05	538	535.1	534.85	545.35	542.25	542.3	544.5	544.65	545.5	543.1	539.85	537.1	537.15
10	Adani Ports & Special Econom	135.35	133.09	137.24	137.94	132.29	132.61	133.4	136.42	136.69	138.16	140.56	141.67	142.88	142.87	143.59	143.22	144.8
11	Aditya Birla Nuvo Ltd.	709.75	713.45	723.25	749.4	740.05	745.1	758.3	762.7	761.4	780.4	772.2	773.25	765.25	767.9	805.25	788.8	787.05
12	Advanta India Ltd.	424.8	423	427.25	428.35	420.25	413	413.6	416.3	418.85	419.35	419.35	420.95	418.95	412.85	416.45	414.9	416.65
13	Agro Tech Foods Ltd.	240.6	241.8	242.55	245.1	244.1	237.35	233.55	240.85	242.4	240.5	250.7	252	257.35	248.2	260	262.7	257
14	Akzo Nobel India Ltd.	594.4	599.1	600	611.5	619.35	629.4	657.75	644.05	662.2	697.55	724	737.25	721.7	736.25	742.7	734.5	751.35
15	Allahabad Bank	161.05	160.6	161.8	163.95	163.75	163.2	166	165.5	163.9	162.45	161.9	160.45	162	161.5	161.8	161.05	162.15
16	Allcargo Logistics Ltd.	163.8	159	164.65	163.35	164.45	162.8	160.4	165.65	168.2	167.55	167.65	172.1	171.3	170.6	173.75	170.9	169.95
17	Allok Industries Ltd.	19	19.1	19.1	18.6	18.35	17.85	18	18	18.1	19.45	19.7	19.25	19.3	19.1	19.45	20.4	20.95
18	Alstom India Ltd.	581.6	587.5	604.95	607.95	596.8	603.3	605.35	604.65	595.8	597.85	599.25	597.45	600.1	591.9	607.2	614.8	627.65
19	Alstom T & D India Ltd.	285.2	286.65	286.85	289.15	286.9	291.3	293.25	295.05	304.25	292.9	293.75	290.75	290.65	289.4	289.9	290.3	290.55
20	Amara Raja Batteries Ltd.	162.05	160.2	165.4	165.7	163.65	163.3	166.7	170.5	173.15	174.05	176.8	176.6	177.5	174.4	174.25	175.05	172.55
21	Ambuja Cements Ltd.	106.85	110.7	114.2	112.5	115.5	115.15	115.05	115.85	115.05	115.2	115.75	115.85	117	117.7	118.15	118.75	120.55
22	Antek Auto Ltd.	163.25	164.65	166.8	166.7	162.65	157.75	156.8	157.7	159.5	161.7	159.4	158.45	157.9	156.95	156.55	156.4	169.05
23	Antek India Ltd.	32.83	33.48	34.38	34.8	35.15	35.13	35.75	36.5	35.98	35.27	34.8	33.7	33.5	34.02	33.3	32.35	32.4
24	Anant Raj Inds. Ltd.	105.3	102.9	101.95	103.05	100.7	101	105.8	103.75	103.75	104.1	106.75	108.85	104.05	105.65	107.7	108.55	116.8
25	Andhra Bank	134.05	134	138.95	137.65	137.75	138.6	135.5	137.9	136	136.7	136.1	136.1	133.6	134.1	135.15	133.1	134.2

Figure 1 : Adjusted closing price of stock

§ The closing price of the market index is taken from NSE website and kept in the file “market.xlsx” as shown in the figure 2.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	4149.1	4189.75	4253.15	4263.45	4190	4152	4172	4228.95	4247.25	4299.05	4314.8	4319.95	4344.3	4328.65	4388.7	4368.7	4379.75	4372.35	4365.85	4429.25
2																				

Figure 2 : Closing price of market index (S&P CNX 500)

§ Risk free rate is taken as per rate of treasury bills given on RBI website.

Output of the above algorithm:

Figure 3 shows the stocks selected with respect to Security Market line. The stocks selected with their corresponding weights in are printed in “output.xlsx” as shown in the figure 4.

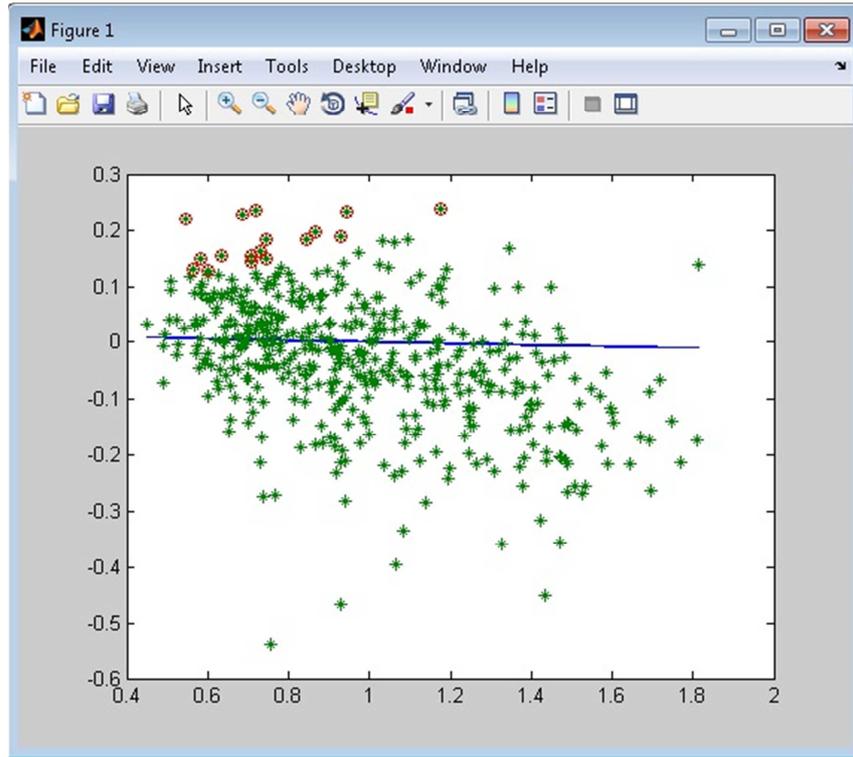


Figure 3 : Security Market Line(Blue Line)
Green points show the positions of stock and the selected stocks are circled with red colour

	A	B
1	Amtek India Ltd.	0.148446
2	Page Industries Ltd.	0.143936
3	V S T Industries Ltd.	0.15801
4	Bata India Ltd.	0.110855
5	F A G Bearings India Ltd.	0.07639
6	Supreme Industries Ltd.	0.082209
7	United Breweries Ltd.	0.025375
8	Hexaware Technologies Ltd.	0.04956
9	Astrazeneca Pharma India Ltd.	0.025055
10	Kajaria Ceramics Ltd.	0.053312
11	Eicher Motors Ltd.	0.033724
12	Gitanjali Gems Ltd.	0.018644
13	Blue Dart Express Ltd.	0.022886
14	Crisil Ltd.	0.040203
15	Wabco India Ltd.	0.005871
16	Federal-Mogul Goetze (India) Ltd.	0.002297
17	Hindustan Unilever Ltd.	0.003228

Figure 4 : The list of stocks selected for optimum portfolio with their corresponding weights

The performance of the portfolio constructed from the above selected 17 stocks is given in the table 1.

Annual Return	67.22%
Treynor's ratio	80.96%
Beta value	0.73
Volatility	16.44

Sharpe ratio	3.60
Jensen's alpha	65.28%

Table 1 : Performance of the obtained portfolio

6. Conclusion

We found that our optimum portfolio gives an annual return of 67.22% with respect to risk free return of 7.966%. This shows that the given method is very effective in investment decision making and even in today's economic scenario and investment environment in India, we can still achieve such good returns by selecting stocks using the described methodology and algorithm.

7. References

- [1] H.M. Markowitz, "Portfolio Selection", Journal of Finance,7(1952),77-91
- [2] E.J. Elton, M.J. Gruber, S.J. Brown, W.N. Goetzmann, Modern Portfolio Theory and Investment Analysis, (2009)

8. Appendix

```
%DEFINITION AND NAMES OF VARIABLES USED

%value=daily adjusted closing price values of all the stocks
%name=name of all the stocks
%n=number of stocks
%m=number of days of which data is available
%value=daily value of market index
%rfo=annual risk free rate
%rf=average daily risk free rate
%retrn=daily return on market
%mr=average daily return on market
%msig=daily volatility of market
%msigo=annual volatility of market
%retrn=daily return of stocks
%r=average daily return of stocks
%ro=average annual return of stocks
%beta=beta value of stocks
%er=expected average daily return of stock
%sig=daily non-diversifiable risk of stocks
%sigo=annual non-diversifiable risk of stocks
%s=treynor's ratio
%co=cut-off ratio
%xx=weights of the stocks selected in our optimum portfolio
%RP=average daily return of portfolio
%RPo=average annual return of portfolio
%betap=beta value of portfolio
%tr=treynor's ratio of portfolio
%cv=covariance matrix of the stocks selected
%sigp=daily volatility of portfolio
%sigpo=annual volatility of portfolio
%jensenalpha=Jensen's alpha of portfolio
```

Algorithm for construction of portfolio of stocks using Treynor's ratio

```
%MAIN PROGRAM

close all;
clear all;

[value,name]=xlsread('DATA.xlsx');
[n,m]=size(value);
[mvalue]=xlsread('market.xlsx');
rfo=7.966;
rf=100*((1+rfo/100)^(1/m))-1;

for i=1:(m-1)
    mretrn(i)=100*(mvalue(i+1)-mvalue(i))/mvalue(i);
end
mr=100*(geomean(1+mretrn/100)-1);
msig=std(mretrn);
msigo=sqrt(252)*msig;
for i=1:(m-1)
    retrn(:,i)=100*(value(:,i+1)-value(:,i))./value(:,i);
end
for i=1:n
    r(i)=100*(geomean(1+retrn(i,:)/100)-1);
    ro(i)=100*((1+(r(i)/100))^252)-1;
end
for i=1:n
    y=retrn(i,:)'-rf;
    x=(mretrn-rf)';
    temp1=ones(m-1,1);
    x=[temp1 x];
    temp2=inv(x'*x)*x'*y;
    beta(i)=temp2(2);
    beta(i)=0.67*beta(i)+0.33;
end

flagg=0;
while(flagg==0)
for i=1:n
if(beta(i)<0 || beta(i)==NaN)
        beta(i)=[];
        value(i,:)=[];
        retrn(i,:)=[];
        r(i)=[];
        ro(i)=[];
        n=n-1;
break;
        flagg=0;
else
        flagg=1;
end
end
end

er=rf+beta*(mr-rf);
plot(beta,er)
hold on;
for i=1:n
    sig(i)=sqrt((std(retrn(i,:)-rf)^2)-(beta(i)^2)*(std(mretrn-rf)^2));
    sigo(i)=sqrt(252)*sig(i);
    s(i)=(ro(i)-rfo)/beta(i);
end

[s,order]=sort(s,'descend');
beta=beta(order);
name=name(order);
sig=sig(order);
sigo=sigo(order);
```

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```
r=r(order);
ro=ro(order);
retrn=retrn(order,:);

sum1=0;
sum2=0;
flag=0;
for i=1:n
    sum1=sum1+((ro(i)-rfo)*beta(i))/(sigo(i)^2/100);
    sum2=sum2+(beta(i)^2)/(sigo(i)^2/100);
    c(i)=(msigo^2/100)*(sum1)/(1+(msigo^2/100)*(sum2));
if(c(i)<s(1:i))
if(c(i)>s((i+1):n))
    co=c(i);
    io=i;
    flag=1;
end
end
end

scatter(beta,r,'*')
hold on;

if(flag==1)
for i=1:io
    z(i)=(beta(i)/(sigo(i)^2/100))*((ro(i)-rfo)/beta(i))-co);
end
for i=1:io
    xx(i)=z(i)/sum(z);
end
    xlswrite('output.xlsx',name(1:io),'A1:A10000');
    xlswrite('output.xlsx',xx(1:io),'B1:B10000');
    scatter(beta(1:io),r(1:io),'red')
    RP=sum(xx(1:io).*r(1:io));
    RPo=100*((1+RP/100)^252)-1
    betap=sum(xx(1:io).*beta(1:io))
    tr=((RPo-rfo)/betap)
    cv=cov(retrn(1:io,:));
    sigp=sqrt(xx*cv*xx');
    sigpo=sqrt(252)*sigp
    sr=(RPo-rfo)/sigpo
    temp=RP-(rf+betap*(mr-rf));
    jensenalpha=100*((1+temp/100)^252)-1
else
'None of the stock is good for investing.'
end
```