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Enhancing balanced portfolios with CPPI methodologies – insights from a simulation exercise

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Summary

We investigate if using a CPPI-style methodology it is possible to “improve” the distribution of portfolio returns from the point of view of an investor holding a balanced portfolio with different allocations in Equities, and whose concern is to avoid significant negative returns and in general to maximize the skew of the returns distribution, with a yearly horizon. The starting point of the analysis is a traditional balanced portfolio investing in a constant mix of asset classes. The utility preference structure that underlies the analysis is that of an investor that is particularly adverse to large negative returns, and is willing to sacrifice (average) expected returns to reduce the severity of expected losses. This is very similar to a “safety first” approach. Hence, we will primarily be concerned with negative Skew, drawdown, volatility as negative properties of the analyzed portfolio strategies that we are seeking to minimize.

CPPI Portfolio technique

Financial strategies which are designed to limit downside risk and at the same time to profit from rising markets are summarized in the class of portfolio insurance strategies. The most prominent example of dynamic versions of such strategies is the “Constant Proportion Portfolio Insurance”, or CPPI: the investor sets a floor on the dollar value of his or her portfolio, then structures asset allocation around that decision. It was introduced by Perold (1986) (see also Perold and Sharpe (1988)) for fixed-income instruments and Black and Jones (1987) for equity instruments. In its simpler form, it uses two asset classes, a risky basket (usually equities or mutual funds), and a riskless asset of cash (equivalents) or Treasury bonds. The allocation to each depends on the “cushion” value, defined as (current portfolio value – floor value), and a multiplier coefficient, where a higher number denotes a more aggressive strategy. The beginning investment in the risky basket will be equal to:

(Multiplier) x (cushion value in dollars)

and the remainder will be invested in the riskless asset. As the portfolio value changes over time, the investor will rebalance accordingly. Ideally, the cushion value will grow over time, allowing greater allocation into the risky basket, while if the cushion drops the investor may need to sell a portion of it in order to safeguard the floor. If the portfolio falls to the floor value, the investor would move all assets to cash to preserve capital. The value of the multiplier is based on the investor's risk profile, the rebalancing frequency and the maximum one-period loss expected on the risky basket, typically being the inverse of that percentage. Multipliers between 3 and 6 are very common. Because of its simplicity and the possibility to customize it to the preferences of risk-conscious investors willing to benefit from rising markets, the CPPI has become very popular in recent years as the base for a wide range of financial products. Several forms of it have been introduced, including cash-based and option-based versions, in order to minimize some of the known drawbacks of the strategy, like for instance the negative impact of volatility that might cause

portfolios rebalanced according to a CPPI method to sell risky assets on the lows and buy them on the highs. For instance, Perold&Sharpe(1988) contrast static, constant mix and CPPI strategies, noting that no strategy will completely dominate the other, but that the rebalancing rule will dictate the shape of the payout function (linear, convex or concave), and that in non-directional, volatile markets constant mix should outperform CPPI, and vice versa. The focus of our article is the impact of CPPI methodologies on the distribution of returns, compared with a constant-mix asset allocation approach.

Dataset & Methodology

We think it is paramount to use a dataset that can span multiple periods of economic expansions and contractions, encompassing bull and bear markets for the different asset classes used. The longer and the more varied the asset class series, in terms of macroeconomic and financial environments they encompass, the more robust the conclusions we can draw, in terms of stability and significance of the analysis. To achieve this, we use US data on Equities, T-Bills and Aggregate Government Bonds, obtaining 371 monthly returns and 31 yearly returns since 1977, sourced from Bloomberg. Equities are proxied by the S&P500 while T-Bills and Government Bonds indexes are from Merrill Lynch.

We start from simplified balanced portfolios consisting of constant mixes of Equities and Government Bonds, rebalanced monthly. We define 12 portfolios with increasing equity content, grows by 5% each time from 10% to 40%, then by 10% up to 90%. The remaining portion of each portfolio invests in Government Bonds Index (“Govt All” henceforth). We compute monthly returns from 30/12/1977 to 31/10/2008. We then proceed to preliminary data analysis to uncover relationships that might be helpful in devising the strategies to be we are going to test later. Then, we build the strategies, compute their performance over the data

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sample and compare their results in terms of the main characteristics of the return distribution they generate. In particular, for each of the 12 balanced portfolios, we will compare a constant-mix investment in each portfolio with:

1. an investment in a CPPI strategy built using Equities, Cash and Bonds
2. an investment in a CPPI strategy built using Equities & Cash only

A preliminary data analysis, below, will highlight some properties of the series in our dataset, and help us choose other relevant parameters for our exercise.

Preliminary analysis

We first look at the risk and correlation of the selected asset classes, unconditionally (using the full dataset) and then conditionally using some filters, to focusing on some important statistics like drawdown, or correlation.

Table 1- Unconditional Correlations

Unconditional Asset Class Correlation			
	Equity	T-Bill	Govt all
Equity	1		
T-Bill	0.05	1	
Govt 1-3	0.12	0.51	
Govt all	0.15	0.31	1

In the unconditional setting, the two government indexes are heavily correlated, while Equities are fairly uncorrelated with the rest of the asset classes. This is obviously a broad summary for the whole period and masks substantial variation in correlations. Looking at maximum drawdowns, Equities are clearly the main source of seriously negative returns at a portfolio level:

Table 2- Drawdowns

Drawdowns			
Period	Equity	T-Bill	Govt all
12m	-37.5%	0.11%	-4.48%
6m	-30.8%	0.11%	-7.90%
3m	-30.2%	0.11%	-6.41%

After drawdown, that looks at each asset class in isolation, the other filter we apply considers what happens to correlations when equities experience a large decline. In this conditional setting we capture only rolling periods when Equities lost cumulatively more than x%, with x taking on the following values: -5%, -10%, -15%, -20%, -25%, -27%. Equity returns are computed over rolling windows of different length: 3, 6 and 12 months. Conditional correlations reveal now a strong negative relation between Equity losses and corresponding returns on Bills and Bonds.

Table 3 - Conditional Correlations with Equities

Conditional Correlations* with Equities						
	12 Months		6 Months		3 Months	
Equity threshold	T-Bill	Govt	T-Bill	Govt	T-Bill	Govt
-5%	-0.67	-0.89	-0.68	-0.78	-0.71	-0.67
-10%	-0.69	-0.91	-0.73	-0.86	-0.80	-0.78
-15%	-0.68	-0.93	-0.84	-0.89	-0.91	-0.80
-20%	-0.86	-0.97	-0.73	-0.79	-0.94	-0.84
-25%	-0.90	-0.94	-1.00	-0.79	-1.00	-1.00
-27%	-0.89	-0.92	-1.00	-0.79	-1.00	-1.00

*=adjusted zero-mean correlations coefficients

This result is not new, and consistent with empirical evidence and financial theory, acknowledging that government instruments provide a hedge during times of uncertainty and risk aversion, as they benefit from investor seeking safe assets and moving out of risky assets. We could think of it as a mix of fear and macroeconomic rationale in times of bad economic news flow. In addition to conditional correlations, that capture the sign of the relationship well, we also compared conditional average returns, to assess the **scale** of the conditional returns and its interaction with the **time-horizon** of the analysis. To do this, we used an overlapping rolling window methodology, thereby increasing the number of observations obtaining a more robust representation of the dataset. The results clearly show that, keeping the equity negative return threshold constant, the **negative relationship between conditional equity returns and Government bond returns is stronger for windows of 3 months** then tends to fade using longer windows. In addition, keeping constant the window length, the **size of the conditional bonds returns seem to be little sensitive to the threshold loss on Equities¹**. We could spot a mildly diminishing tendency for T-Bills, but no clear pattern for Bonds. We will thus try and **incorporate the above results in a CPPI strategy to mitigate the portfolio impact of large equity drawdowns**. The main takeaways from the analysis of the conditionally filtered results are the following:

- a) We should probably **focus on short measurement windows** because with increasing time, other relations enter into play reducing the effectiveness of the hedge. We will then measure equity losses over a 3-months window to test our strategy
- b) Given that the size of bonds' positive returns is relatively insensitive to the threshold used to define what is a "significant" equity loss, we can **use a relatively "moderate" loss threshold of 10%**, that has the additional benefit of providing a larger number of observations when the threshold is actually exceeded (22)

¹ Charts of these results, omitted here for space constraints, are available on request to the Authors.

Strategies definition

We compare the results of 3 different strategies.

1. The first is a constant-mix portfolio, chosen from our 12 Risk profiles detailed above, with fixed allocation weights rebalanced each month.
2. A CPPI strategy that allocates dynamically between Equities and T-Bills (our cash proxy) that provides protection over a yearly horizon
3. A CPPI strategy that allocates dynamically between Equities, Bonds and T-Bills, that provides the same protection over a yearly horizon and that in addition tries to capture the negative conditional correlation between Equities and Bonds shown above, with the aim to make the returns distribution the more skewed to the right as possible

Please note that we then have 12 different cases, one for each risk-profile, and for each one we compare the 3 strategies.

Both CPPI strategies aim to allocate dynamically between risky assets and T-Bills to avoid losing more than a specific percentage, called “*Max Loss*”. This percentage is computed, for each risk profile, as

$$\text{Max Loss} = \text{Equity Allocation} * -15\%$$

where the 15% number is arbitrarily chosen.

The *Max Loss* thus increases (from 1.5% to 13.5%) with growing initial equity allocation, consistently with the idea of an increasing willingness to bear absolute losses, while the relative loss in terms of the Equity component is constant and equals 15%.

Table 4 - Max Yearly Loss across Portfolios

Initial Equity Allocation	Max Loss Yearly CPPI
10%	1.5%
15%	2.3%
20%	3.0%
25%	3.8%
30%	4.5%
35%	5.3%
40%	6.0%
50%	7.5%
60%	9.0%
70%	10.5%
80%	12.0%
90%	13.5%

Both CPPI strategies also work with a yearly reset: at the onset of each year the risk budget is renewed.

Our two “CPPI” strategies are built using the same framework as the standard CPPI methodologies, i.e. the investment in risky assets varies proportionally to the surplus between the current value of the portfolio and the minimum value that must be preserved.

Without getting into details, in the simplest form of standard CPPI, the minimum value is the amount required to purchase a risk-free asset with maturity equal to the investment horizon, while the investment in risky assets is determined multiplying the portfolio surplus in excess of this value times a “multiplier”. The multiplier determines how aggressive the strategy is. Our CPPI strategies assume that the minimum value/maximum loss level is reached when the portfolio loses *Max Loss* = *Initial Cushion* = *Equity Allocation* * -15%. Thus, the maximum loss on the equity component is 15% across all portfolios, while in terms of the entire portfolio the absolute loss is equal to the product of 15% times the initial equity allocation, increasing with the risk profile. Our CPPI rebalancing rule works as follows:

- as the initial cushion is eroded, the initial equity allocation is eroded proportionally to the cushion reduction. This corresponds to using a constant multiplier of $1/0.15 = 6.6$
- when the *Max Loss* is reached, both strategies switch the entire portfolio allocation in T-Bills, that represent the risk-free asset. T-Bills never experience a monthly negative return within our data sample

More details on the functioning of each of the two CPPI strategies is presented below

1st CPPI – Equities/Bonds/TBILLS

This strategy tries to capture the negative conditional correlation uncovered above within a CPPI type of methodology, to try and create asymmetry in the pay-off. The idea is that losses are capped by the CPPI approach in any case, but in addition Bonds are kept in the portfolio together with equities so that they can partially offset Equities’ drawdowns. At the same time, as we have shown above that the negative correlation tends to fade with the time horizon, the strategy tries also to take this into account by selling out of Bonds after one of those “drawdowns” has occurred.

1. This strategy starts each year with an equity allocation equal to that of the constant-mix portfolio, which is also the maximum allowed equity allocation. Subsequently, and until the end of each calendar year, the equity allocation is diminished proportionally down to zero as and if the CPPI GAP (i.e. the distance from the protected floor) is eroded.
2. The Government bond allocation is also zero if the CPPI GAP is completely eroded. Until then, it is equal to the Equity allocation times the “Bond/Equity Ratio”
3. The “Bond/Equity Ratio” is equal to the ratio between the Govt Bond and the Equity components in the corresponding constant-mix portfolio. This is in order to ensure the results are comparable with that strategy, allowing to better isolate the impact of the

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CPPI strategy in determining the returns distribution².

4. The Government bond allocation goes also to zero if, in the current calendar year, Equities have a cumulated 3-month performance of at least $-X\%$, where X is a threshold level that we set to 10%. This point characterizes the strategy. We try to keep bonds as a hedge against equity drawdowns, that we measure over a 3-month period following our discussion above on the time-horizon. The 10% threshold also comes from our previous review where we noted that lower thresholds like of 5% and 10% provided a better return ratio, while also allowing for more occurrences to be tested.
5. The rest of the portfolio is invested in T-Bills
6. The portfolio is rebalanced monthly
7. At the end of each calendar year, the CPPI GAP is renewed.

2nd CPPI – Equities /TBILLS

This strategy uses only Equities and T-Bills. It starts each year with an equity allocation equal to that of the constant-mix portfolio, which is also the maximum allowed. Subsequently, and until the end of each calendar year, the equity allocation is diminished proportionally down to zero as and if the CPPI GAP (i.e. the distance from the protected floor) is eroded, and the rest of the portfolio is invested in T-Bills. Also this strategy is rebalanced monthly, and renewed with each calendar year.

Strategies comparison

Statistics collection

We compute statistics for the 3 strategies at a monthly as well as yearly frequency, sampled at the end of each calendar month/year. The yearly perspective is the most important, as our CPPI strategies have a yearly reset.

The statistics are the following:

- Skew
- Average Returns
- Volatility
- Minima
- Maxima
- Drawdown

Strategies comparison

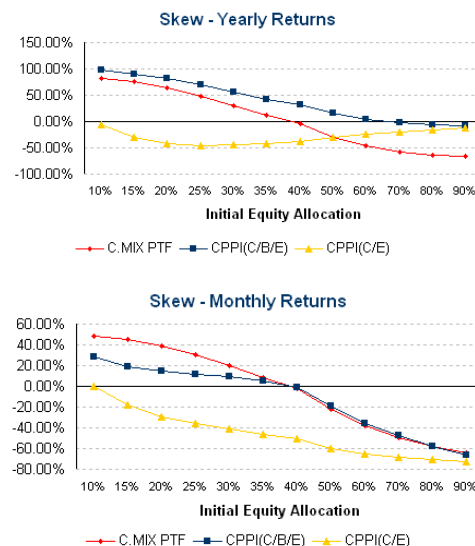
Here we compare the statistics – Average returns, Volatility, Skew, Extreme Values and Drawdown of the 3 trading strategies. As the analysis starts from the search for asymmetry, Skew will be the primary

focus, with Drawdown analysis providing a robustness check. We read and record the returns statistic with caution as with any evidence about past returns, despite the relatively long dataset. On the other hand, exactly this long dataset should make the rest of the statistics fairly robust as far as higher moments are concerned.

SKEW

- The introduction of Bonds versus a 100% equity allocation (in the constant-mix portfolio and in the B/C/E-CPPI) always reduces negative Skew
- The **B/C/E-CPPI always reduces Skew more than the C/E-CPPI, at both monthly and yearly intervals and for all levels of Equity allocations**
- At **yearly frequency, the B/C/E-CPPI provides the maximum skew among the three strategies**, having positive or nearly zero Skew across the whole spectrum of Equity allocations. We could interpret the difference between the blue and the red line as the increase in skewness resulting from the dynamic allocation methodology, while the difference between the blue and yellow line as the effect on the skewness brought by including bonds.
- At **monthly frequency, for low equity allocations, the constant-mix Portfolios provide greater positive Skew than the B/C/E-CPPI, but starting from Equity allocation of 40% upwards they behave more or less in line**

Table 5 - Skew of Strategies' Returns



RETURNS

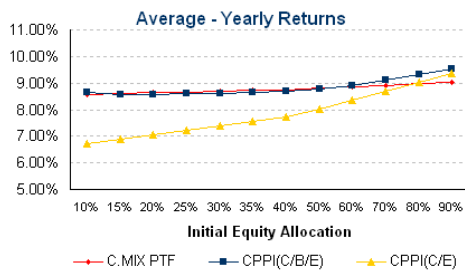
- In general, returns are very similar for the constant-mix portfolio and the B/C/E-CPPI, while the C/E-CPPI tends to lag, mainly because bonds on average offered a greater return than

²We recommend focusing on yearly statistics throughout the document as they capture more accurately our CPPI methodology.

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cash, which is not made-up by the methodology.

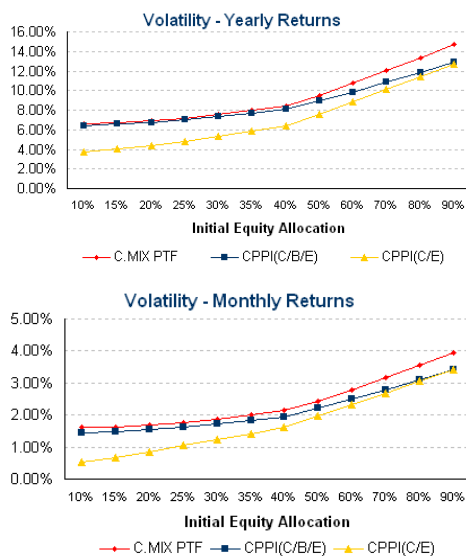
Table 6 - Average Strategies' Returns



VOLATILITY

- Volatility is decreased by CPPI strategies. However, the B/C/E-CPPI mitigates volatility in a meaningful way only for high-equity profiles, while the C/E-CPPI decreases volatility significantly across all allocations. This is due to the fact that Govt Bonds also contribute to portfolio volatility.

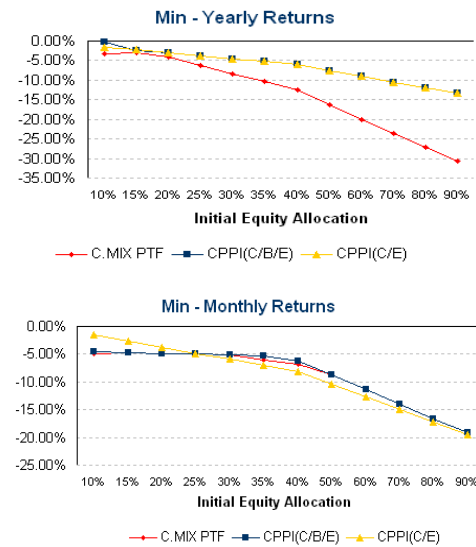
Table 7 - Strategies Volatility Returns



EXTREMES – MINIMA

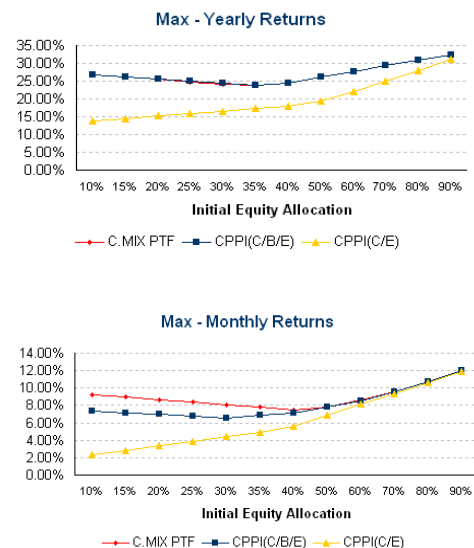
- At yearly frequency, CPPI strategies obviously significantly reduce extreme portfolio losses across all equity allocations (the methodology is built to provide a floor at yearly frequency). The effect is more pronounced for higher equity allocations, and the two CPPIs offer broadly the same results in terms of maximum realized loss

Table 8 – Strategies' Minimum Returns Across Different Portfolio Profiles



EXTREMES – MAXIMA

Table 9 – Strategies' Maximum Returns



In line with the above considerations on returns, the C/E-CPPI lags the other two.

DRAWDOWN FUNCTION ANALYSIS

The statistics collected so far can be complemented by an analysis of the profile of the Drawdown function for the various strategies. The drawdown function shows how much the portfolio value falls below the all-time high ever reached, over the entire history of the strategy. It thus answers the question “how much did the strategy lose versus its most recent peak?”, at any point in time. As the all-time high is never reset, while the CPPI methodology we use is reset annually, we are aware this introduces an inconsistency in the results³. We take this into account focusing again on yearly frequency data.

³ For example, for a CPPI with a 20% maximum annual loss, if we had a yearly return of -8% for 3 consecutive years, the yearly CPPI floor would certainly be preserved, but the drawdown function would show a final value of roughly $(1-8\%)^3 = -22\%$. The drawdown function we use does not reset the maximum level at the start of the year.

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Below, we compare the constant-mix strategy and the C/B/E CPPI. We plot the difference between the annual drawdown functions. A positive (negative) difference means that the drawdown of the CPPI strategy is less (more), thus the CPPI strategy has protected better (worse) the all-time maximum. A zero value, lying on the x-axis, means the two strategies ended the year at the same distance from the all-time maximum.

We group portfolio by buckets of 4 portfolios each, according to the level of the Equity component:

- Group 1 = Initial Equity allocation between 10% and 25%
- Group 2 = Initial Equity allocation between 30% and 50%
- Group 3 = Initial Equity allocation between 60% and 90%

Table 10 - Yearly drawdown function differences - constant-mix Portfolio and C/B/E CPPI

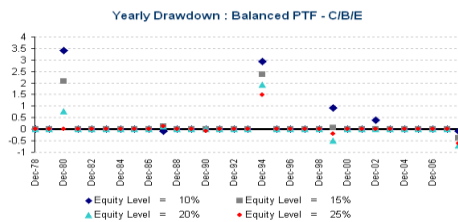
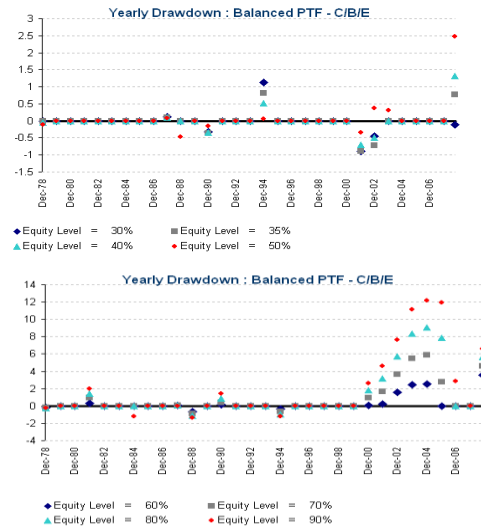


Table 11 - (continued) Yearly drawdown function differences - constant-mix Portfolio and C/B/E CPPI



In general, the CPPI strategy provides a better protection against drawdown. However, we can spot some instances where the constant-mix portfolio provides better results; we attribute this effect to periods of sustained bond rallies in the context of volatile equity market. In this environment, the CPPI strategy would have reduced the allocation to both risky assets, thereby underperforming.

Conclusions

We document a negative, consistent relationship between conditional equity returns and Government bond returns, whose effect is stronger for short time windows (3 months) then tends to fade, while it is little sensitive to the threshold used to determine the conditional environment. To try to capture this effect to mitigate the portfolio impact of equity drawdowns, we focus on dynamic CPPI methodologies using Equities, T-Bills and Government Bonds as eligible asset classes. A 3-month measurement window is used, with a 10% conditional threshold to gather a larger number of conditional observations. We compare a constant-mix portfolio with 2 CPPI strategies, a standard Equity/TBills one and one including also a dynamic allocation to Government bonds. Our results are the following:

- **We show that negative skew, induced by the equity allocation, can be reduced through an allocation to government bonds and through the CPPI methodology. At yearly frequency, the B/C/E-CPPI provides the maximum skew among the three strategies.**
- CPPI strategies also offer, by construction, the best protection against the most severe losses on a yearly basis. The effect is more pronounced for higher equity allocations.
- **Overall, it seems that using B/C/E-CPPI strategy could improve the returns distribution properties of a constant-mix portfolio investing in bonds and equities. The benefits are expected across the full range of Equity allocations, although they are more pronounced for higher risk profiles. Further analysis is needed to quantify more robustly the size of the improvement and also to test it over different datasets and scenarios.**

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