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Abstract

Conditional heteroskedasticity, skewness and leverage effects are well known features of financial returns. The literature on factor models has often made assumptions that preclude the three effects to occur simultaneously. In this paper I propose a conditionally heteroskedastic factor model that takes into account the presence of both the conditional skewness and leverage effects. This model is specified in terms of conditional moment restrictions and unconditional moment conditions are proposed allowing inference by the generalized method of moments (GMM). The model is also shown to be closed under temporal aggregation. An application to daily excess returns on sectorial indices from the U.K. stock market provides a strong evidence for dynamic conditional skewness and leverage with a sharp efficiency gain resulting from accounting for both effects. The estimated volatility persistence from the proposed model is lower than that estimated from models that rule out such effects. I also find that the longer the returns' horizon, the fewer conditionally heteroskedastic factors may be required for suitable modeling and the less strong is the evidence for dynamic leverage. Some of these results are in line with the main findings of Harvey and Siddique (1999) and Jondeau and Rockinger (2003), namely that accounting for conditional skewness impacts the persistence in the conditional variance of the return process.

Keywords: Factor models; conditional heteroskedasticity; conditional leverage; conditional skewness; temporal aggregation; generalized method of moments.

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1 Introduction

Conditional heteroskedasticity is a well-known feature of financial returns. In addition, returns are often characterized by the presence of skewness (i.e. returns have an asymmetric distribution) and leverage effects (i.e. the fact that a negative shock on returns has a larger impact on volatility than a positive shock of the same magnitude). One can refer to Nelson (1991), Glosten, Jagannathan and Runkle (1993) and Engle and Ng (1993) for studies documenting the presence of leverage effects in financial time series, and Ang and Chen (2002), Harvey and Siddique (1999, 2000) and Jondeau and Rockinger (2003) for the skewness effect.

The finance literature has recognized the importance of taking into account higher order moments in asset pricing models. An early example is Rubinstein (1973) (see also Kraus and Litzenberger (1976) for an empirical implementation of Rubinstein's (1973) model), who proposes an extension of the capital asset pricing model (CAPM) allowing for skewness in the unconditional distribution of returns. More recently, Harvey and Siddique (2000) have extended Kraus and Litzenberger's (1976) model to the dynamic context by allowing the third conditional moments to be time varying. In the univariate context, Harvey and Siddique (1999) and Jondeau and Rockinger (2003) confirm the presence of dynamic conditional third moments (leverage and skewness) in financial data. They also find that accounting for such skewness and leverage effects declines the volatility persistence estimates.

Although the finance literature in the univariate context has recognized the importance of modeling skewness and leverage effects, few attempts have been made to model both effects jointly in the multivariate framework. This is the case for the conditionally heteroskedastic factor model literature. The latent ARCH factor model of Diebold and Nerlove (1989) rules out both skewness and leverage effects through the conditional Gaussian return assumption. The QGARCH factor model of Fiorentini, Sentana and Shephard (2004) captures the conditional leverage but rules out possible conditional skewness through Gaussianity. Mencía and Sentana (2012) propose a conditionally heteroskedastic factor model in which the returns' innovations are assumed to follow a generalized hyperbolic conditional distribution that allows for dynamic conditional leverage and skewness in the returns. While Doz and Renault (2006) (henceforth DR (2006)) propose the stochastic volatility (SV)-factor models that are consistent with any dynamics in the conditional leverage and skewness of the returns. But the specific dynamics of conditional skewness and leverage are important for asset management. Conditional skewness and co-skewness are significant parts of stocks' risk premia (see Harvey and Siddique (2000)) while conditional leverage dynamics are useful guide for hedging strategies involving volatility-based products.

In this paper I propose some specification for the dynamics in the conditional skewness and leverage of the returns in the context of DR (2006) SV-factor models. Following DR (2006), and in opposition with Mencía and Sentana (2005), I consider a distribution-free approach in which conditional heteroskedasticity, leverage and skewness are all captured through conditional moment conditions. In this model, all the dynamics in the conditional moments (and cross sectional, or co-moments) of the returns are driven by common latent factors. The conditional heteroskedasticity in the common factors is captured by a square root stochastic autoregressive volatility model (SR-SARV) as proposed by Andersen (1994) (see Meddahi and Renault (2004) and DR (2006)). The leverage effect is modeled as an affine function of the conditional variance. This specification encompasses many of the existing models in the literature (e.g. the affine process of Dai and Singleton (2000)). By a temporal aggregation argument, the conditional skewness is also represented as an affine function of conditional variance. The resulting model is labeled the conditionally heteroskedastic factor model with asymmetries (ACHF[1]). Its closeness under temporal aggregation is established and unconditional moment conditions are proposed that allow for inference by generalized method of moments (GMM).

The ACHF model is then applied to a set of 23 daily stock index returns, including the FTSE 350 stock index return and 22 sectorial U.K. index returns.[2] I also estimate the DR (2006) version of this model in which the conditional skewness and leverage effects are not explicitly modeled. The first conclusion of the results is that there may be a substantial efficiency gain when both the conditional skewness and the leverage effects are suitably modeled. In this application, the GMM standard error estimates of the parameters shared by both models drop sharply in the ACHF model compared with the DR (2006) model. The results also suggest the presence of a significant leverage effect driven by common factors in daily UK sectorial returns, confirming the results in Sentana (1995) for monthly data. In addition, a more negatively skewed conditional distribution seems to be the typical response to an increase in volatility. I also notice that the volatility persistence estimated from the ACHF and the DR (2006) models are of the same magnitude and are significantly lower than the volatility persistence from the (Gaussian) dynamic conditional correlation (DCC-MV-GARCH) model of Engle (2002). This observation confirms the finding of Harvey and Siddique (1999) and Jondeau and Rockinger (2003) in a univariate framework that conditional variance is less persistent when the conditional skewness is not ruled out. Furthermore, this empirical application seems to show that conditional skewness and leverage become less pronounced as the asset returns' horizon becomes longer. Specifically, as one moves from daily data to weekly data, the volatility persistence of the factors drop substantially and a conditionally heteroskedastic single-factor model becomes a better representation than a 2-factor model narrowing the sources of variabilities in the conditional skewness and leverage in the assets to the volatility of that single factor. Thanks to the non-linear state-space representation of the SV-factor models, I propose an extended Kalman filter algorithm that provides filtered factors and

[1] I thank one anonymous referee for suggesting this acronym.

[2] A monthly version of these data has previously been modeled by Sentana (1995) and Fiorentini, Sentana and Shephard (2004) with a conditional heteroskedastic factor model. This empirical study differs from theirs in that I analyze daily data and I also specify explicitly the dynamics of conditional higher order moments beyond the first and the second moment.

volatility processes from past returns useful for reality checks for the models. This filter is conditional on the GMM estimate of the models and aims to compensate the failure of the GMM inference to deliver a proxy for the volatility process.

The remainder of the paper is organized as follows. In Section 2, I present the summary statistics for the data used in the empirical application. I also document the presence of dynamic leverage and skewness effects in the returns. Section 3 presents the ACHF model. Further examples are given to motivate the proposed specifications. A discussion on the implication for the conditional co-skewness and co-leverage is provided and the temporal aggregation properties of the model are also studied. The identification and estimation issues are discussed in Section 4 whereas Section 5 contains the empirical results and Section 6 concludes. The extended Kalman filter algorithm is presented in Appendix A. Appendix B contains the data description and all the tables related to the empirical application. The proofs of the propositions in Section 3 appear in the web supplement to this paper (Dovonon (2012)).

2 Empirical motivation

This section provides some empirical motivation for the need to account for asymmetric effects in both the conditional distribution (conditional skewness) and the conditional variances (leverage) of asset returns. The data set is downloaded from Datastream and consists of 23 daily UK stock market index returns, including the FTSE 350 and 22 other sectorial indices, all of which in the FTSE (see Appendix B for details). The sample runs from January 2, 1986 through July 7, 2009, for a total of 6037 daily observations. Only trading days are considered. For each index, the daily log excess return are computed, using the log return of the UK one month loan index (JPM UK CASH 1M) as risk free interest rate. Appendix B contains all the tables on the empirical results in the paper.

Table 1 in Appendix B is compiled based on the 3026 most recent observations in the sample ranging from October 27, 1997 through July 21, 2009. This table shows some summary statistics, the results of the tests for the impact of news on volatility as proposed by Engle and Ng (1993) and some additional results underlying the evidence for dynamic conditional skewness and leverage in the data.

Most of the sectorial indices have a quite large unconditional skewness ranging from -0.48 to 0.41 but are typically negative with a median among assets of -0.17 and an average of -0.11. The presence of skewness in the distributions of the daily excess returns in the U.K. sectorial indices analyzed here agrees with similar evidence for other financial return series found by Harvey and Siddique (1999), Ang and Chen (2002) and Jondeau and Rockinger (2003), among others.

The Ljung-Box statistics for autocorrelation up to lag 5 (QW(5)) reveal the presence of potential autocorrelation. This autocorrelation is filtered out in the empirical application in Section 5.

Table 1 also shows strong evidence of conditional heteroskedasticity as indicated by the results of the test for GARCH effect. The test for GARCH effect considered throughout is the GMM overidentification test (Hansen (1982)) of the moment condition $Var(Y_{i,t+1}|z_t) = cst$, where $Y_{i,t+1}$ is the excess return on asset i over period $(t, t+1)$ and the instrument $z_t = (1, y_t^2, \ldots, y_{t-k}^2)$; $k = 5$. Table 1 reports a strong rejection of the null hypothesis of conditional homoskedasticity for all series.

The results on the diagnostic tests for the impact of news on volatility proposed by Engle and Ng (1993) are also reported in Table 1. The sign bias, the negative size bias, and the positive size bias tests are considered. These tests are performed on the standardized index excess returns[3] (following Engle and Ng (1993), the GARCH(1,1) model is assumed under the null). The sign bias test is significant at 10% for 12 out of the 23 time series. The negative and the positive size bias tests are strongly significant for nearly all the indices and this translates into significant joint tests for all of the analyzed series. This brings to the conclusion of presence of leverage effects in the returns.

Next, I investigate whether the conditional skewness and leverage in the returns are time variant. They are respectively defined by $E(Y_{i,t+1}^3|J_t)$ and $Cov(Y_{i,t+1}, \Sigma_{ii,t+1}|J_t)$ where $\Sigma_{ii,t+1}$ denote the conditional variance of $Y_{i,t+2}$ at $t+1$ and J_t the set of information available at t. I regress $\varepsilon_{i,t+1} \times \hat{\Sigma}_{ii,t+1}$ and $\varepsilon_{i,t+1}^3$ on $\hat{\Sigma}_{ii,t}$ where $\varepsilon_{i,t+1}$ is the centered return and $\hat{\Sigma}_{ii,t}$ is the squared return at $t+1$ used as proxy for the conditional variance. The significance of the slopes (π_1 and h_1) of these regressions means that the leverage and/or skewness effects are dynamic.

From Table 1, π_1 and h_1 are strongly significant for all the sectorial indices except for 'Transport' and 'FTSE 350', π_1 and h_1 of which are not significant, respectively. I also use the (log) high-low range-based volatility estimator as a proxy for the conditional variance (see Brandt and Diebold (2004)) in regressions for 'FTSE 350' and both π_1 and h_1 appear strongly significantly negative.

I also investigate the empirical content of an asymmetric factor model for the returns. I argue that if the data have a factor representation, the FTSE 350 index excess return should be a good proxy for this factor. And, for an asymmetric factor model to hold, both the conditional leverage and the skewness effects in the returns should significantly be explained by the factor. Similar regressions to those previously described are performed using the proxy for 'FTSE 350' conditional variance throughout. From Table 1, the estimated co-leverage slopes $\pi_{f,1}$ are all strongly negatively significant while the co-skewness slopes $h_{f,1}$ are mostly significant and negative.

These features are in favor of a conditionally heteroskedastic factor structure with asymmetries. The model is introduced next.

3 Model specification

I build upon the stochastic volatility factor model proposed by DR (2006) that I specialize to explicitly take into account the conditional skewness and leverage effects.

[3]These tests have also been applied to the returns filtered for autocorrelation for similar results.

3.1 The model

I first introduce the conditionally heteroskedastic factor model. Let Y_{t+1} be a $N \times 1$ vector of (excess) returns on N assets over the time period t to $t+1$, F_{t+1} a $K \times 1$ vector of K latent common factors, and U_{t+1} a $N \times 1$ vector of idiosyncratic shocks. The vector process Y_{t+1} is assumed to be described by the conditionally heteroskedastic factor representation:

$$Y_{t+1} = \mu(J_t) + \Lambda F_{t+1} + U_{t+1}, \tag{1}$$

with

$$\begin{array}{lcllcl} E\left(U_{t+1}|J_t\right) & = & 0, & Var\left(F_{t+1}|J_t\right) & = & D_t, \\ E\left(F_{t+1}|J_t\right) & = & 0, & Var\left(U_{t+1}|J_t\right) & = & \Omega, \\ E\left(U_{t+1}\, F'_{t+1}|J_t\right) & = & 0, & & & \end{array} \tag{2}$$

where J_t is a nondecreasing filtration defining the relevant conditioning information set containing the past values of $Y_\tau, \tau \leq t$ and $F_\tau, \tau \leq t$, $\mu(J_t)$ is a $N \times 1$ vector of J_t-adapted components representing the risk premia, Λ is the $N \times K$ $(N \geq K)$ full column rank matrix of factor loadings, D_t is a diagonal positive definite matrix of K factors' conditional variances; $\sigma^2_{k,t} : k = 1, \ldots, K$, and Ω is the conditional covariance matrix of the idiosyncratic shocks U_{t+1}.

The related literature has made several assumptions about Ω. The strict factor structures as in Diebold and Nerlove (1989), King, Sentana and Wadhwani (1994) and Fiorentini, Sentana and Shephard (2004) impose Ω to be diagonal while DR (2006) maintains an approximate factor structure that relaxes the diagonal condition of the idiosyncratic shocks' conditional variance matrix. An advantage of the approximate factor representation is that it is preserved by portfolio formation.

This conditionally heteroskedastic factor representation implies that the conditional variance of Y_{t+1} given the information available at time t is given by:

$$\Sigma_t \equiv Var\left(Y_{t+1}|J_t\right) = \Lambda\, D_t\, \Lambda' + \Omega. \tag{3}$$

Therefore, the heteroskedasticity in the returns is driven by the conditionally heteroskedastic factors. One of the nontrivial advantages of the factor models for volatility modeling is that the resulting conditional variance of the return process is guaranteed to be positive semi definite at each date under very mild conditions as shown by (3).

Following DR (2006), the factors have SR-SARV(1) dynamics so that the conditional variance of each factor k, $k = 1, \ldots, K$, has the dynamics:

$$E(\sigma^2_{k,t+1}|J_t) = \omega_k + \gamma_k \sigma^2_{k,t}; \quad \omega_k, \gamma_k \in \mathbb{R}_+. \tag{4}$$

I capture the conditional leverage effect by specifying for each factor, $F_{k,t}$ $(k = 1, \ldots, K)$, the conditional covariance $Cov(F_{k,t+1}, \sigma^2_{k,t+1}|J_t)$ as an affine function of its past conditional variance $\sigma^2_{k,t}$:

$$Cov(F_{k,t+1}, \sigma^2_{k,t+1}|J_t) = \pi_{0k} + \pi_{1k}\sigma^2_{k,t},\ \ \pi_{0k}, \pi_{1k} \in \mathbb{R};\ \ k = 1, \ldots, K. \tag{5}$$

This affine specification of the conditional leverage is quite common in the literature as illustrated in the next section. The leverage in the factors translates into leverage in the returns since

$$Cov(Y_{i,t+1}, \Sigma_{ii,t+1}|J_t) = \sum_{k=1}^{K} \Lambda_{ik}^3 Cov(F_{k,t+1}, \sigma_{k,t+1}^2|J_t). \tag{6}$$

I assume deriving this formula that:

Assumption 1 *(i) Conditionally on J_t, $F_{k,t+1}$ and $U_{i,t+1}$ are uncorrelated with $F_{k',t+2}^2$, for any $i = 1, \dots, N$ and $k, k' = 1, \dots, K$; $k \neq k'$. (ii) Conditionally on J_t, $U_{i,t+1}$, $U_{i,t+1}^2$, $F_{k,t+1}$ are respectively uncorrelated with $F_{k,t+1}^2$, $F_{k,t+1}$, $F_{k',t+2}^2$ for any $i = 1, \dots, N$ and $k, k' = 1, \dots, K$; $k \neq k'$.*

Assumption 1 is not particularly restrictive and is implied by the conditional independence of the factors and idiosyncratic shocks' processes. Assumption 1-*(i)* is useful to derive the conditional leverage while Assumption 1-*(ii)* serves for the conditional skewness. Under Assumption 1-*(ii)*,

$$E(Y_{i,t+1}^3|J_t) = \sum_{k=1}^{K} \Lambda_{ik}^3 E(F_{k,t+1}^3|J_t) + E(U_{i,t+1}^3|J_t). \tag{7}$$

Hence, the specification of the dynamics in the third conditional moments of the factors and the idiosyncratic shocks is enough to set-up a dynamic for the returns' conditional skewness. Since the idiosyncratic shocks are conditionally homoskedastic, it makes sense to consider their third conditional moments as time-invariant. It is also tempting to enforce a similar restriction on the factors.

However, a static conditional third moment for the factors would conflict with a possibly dynamic conditional leverage that they are supposed to support. Actually, as pointed out by Meddahi and Renault (2004), the conditional skewness and leverage are equivalent in continuous time since the time increment is virtually 0. This tight connection between conditional leverage and skewness in continuous time also appears in discrete time through temporal aggregation. The conditional third moment of returns at a lower frequency is time-variant if this returns exhibit a dynamic conditional leverage at some higher frequency. This means in particular that the conditional skewness and leverage can be considered to have the same sources of variability. I make the following assumption:

Assumption 2 *(i) $E(F_{k,t+1}^3|J_t) = h_{0k} + h_{1k}\sigma_{kt}^2$, h_{0k}, $h_{1k} \in \mathbb{R}$; $k = 1, \dots, K$. (ii) $E(U_{i,t+1}^3|J_t) = s_i^0$, $i = 1, \dots, N$ and $s_i^0 \in \mathbb{R}$, $\forall i$.*

The *conditionally heteroskedastic factor model with skewness and leverage effects (or asymmetries) (ACHF)* is defined by (1), (2), (4), (5) strengthened by Assumptions 1 and 2 and is summarized by:

$$\begin{aligned} & (1) \quad \& \quad (2) \\ E(\sigma_{k,t+1}^2|J_t) &= 1 - \gamma_k + \gamma_k \sigma_{kt}^2, \ \gamma_k \in (0,1), \ k = 1 \dots, K \\ Cov(F_{k,t+1}, \sigma_{k,t+1}^2|J_t) &= \pi_{0k} + \pi_{1k}\sigma_{kt}^2, \ k = 1, \dots, K \\ E\left(Y_{i,t+1}^3|J_t\right) &= \textstyle\sum_{k=1}^{K} \Lambda_{ik}^3 h_{1k}\sigma_{kt}^2 + s_i, \ i = 1, \dots, N. \end{aligned} \tag{8}$$

The restriction $\omega_k = 1 - \gamma_k$ in the factors' SR-SARV(1) dynamics is for identification purpose (See DR (2006)). This means in particular that $E(D_t) = Id_K$ and $Var(Y_t) = \Lambda\Lambda' + \Omega$ and helps prevents any transfer between the common component of the variance and its idiosyncratic component. (See also Fiorentini, Sentana and Shephard (2004).) The initial parameters h_{0k} and s_i^0 from Assumption 2 are identifiably represented by s_i in the specification of the ACHF model in Equation (8). The last equality in (8) implies that the unconditional third moment of the return on asset i is $E\left(Y_{i,t+1}^3\right) = \sum_{k=1}^K \Lambda_{ik}^3 h_{1k} + s_i$ and displays a common part and an idiosyncratic part controlled by s_i. This latter may set the third moment to 0 if no unconditional skewness is present in the data. It is worthwhile to mention that the ACHF model does not offer a specific treatment to the conditional mean $\mu(J_t)$ which is implicitly set to 0 and expected to be filtered out from the data prior to applications.

3.2 Supporting examples for the conditional leverage and skewness specifications

As it stands, even though backed by the regressions results from Section 2, the dynamics of conditional leverage given by (5) may still look arbitrary. But, as highlighted below, several examples of SR-SARV(1) volatility processes match that specification. (See Dovonon (2012) for the details.)

Example 1 *The $\mathbb{A}_1(3)$-affine family processes*[4] *(Dai and Singleton (2000), Singleton (2001)).*

Example 2 *The Quadratic GARCH (QGARCH(1,1)) of Sentana (1995).*

Example 3 *Heston-Nandi's (2000) GARCH process.*

Example 4 *The Inverse Gaussian GARCH(1,1) of Christoffersen, Heston and Jacobs (2006).*

The representation of the conditional skewness stems from the following observation. Assume that $E(f_{t+1}^3|J_t) = s$ is time invariant but the conditional leverage is given by (5). Assume now that the interest is in lower frequency returns over two successive time periods, $f_{2t}^{(2)}$. Since log-returns are considered, $f_{2t}^{(2)} = f_{2t} + f_{2t-1}$. Now, what is the most appropriate dynamics for $E(f_{2t}^{(2)^3}|J_{2(t-1)})$? Some straightforward expansions give: $E\left(f_{2t}^{(2)^3}|J_{2(t-1)}\right) = 2s + 3(\pi_0 + \pi_1\sigma^2_{2(t-1)})$. Hence, the dynamic leverage in the higher frequency process translates into dynamics in $E\left(f_{2t}^{(2)^3}|J_{2(t-1)}\right)$. This motivates the choice for the dynamics in Assumption 2-*(i)*. As shown in Section 3.4, these representations of conditional leverage and skewness are closed under temporal aggregation.

The representations in (5) and Assumption 2-*(i)* nest the standard GARCH(1,1) models (allowing for conditional skewness) when the standardized innovation in f_{t+1}, say η_{t+1}, has its third order conditional moment proportional to $1/\sigma_t$. This is the case for the standard Gaussian GARCH(1,1)

[4]Backus, Foresi and Telmer(2001) use the Euler scheme discrete time version of the Cox, Ingersoll and Ross' (1985) diffusion process to propose an affine model of currency. The affine process nests the square-root process of Heston (1993) and Cox, Ingersoll and Ross (1985).

model for which $E(\eta^3_{t+1}|J_t)$ is actually null. However it is worth mentioning that the standard GARCH(1,1) does not disentangle the conditional skewness from the leverage since these models imply that $Cov\left(f_{t+1},\, \sigma^2_{t+1}|J_t\right) = \alpha\, E\left(f^3_{t+1}|J_t\right)$, for some α. This is a drawback for this class of models as pointed out by Alami and Renault (2001). In contrast, the dynamics in (5) and Assumption 2-*(i)* allow for the two effects to be handled independently.

3.3 Handling of conditional correlation, co-leverage and co-skewness

The ACHF model described by (8) is quite suitable to handle joint dependencies. Dynamics in the conditional variance, leverage and skewness are driven by the common factors. The conditional covariance between the returns on assets i and j, $\Sigma_{ij,t}$, is given by $\Sigma_{ij,t} \equiv Cov(Y_{i,t+1}, Y_{j,t+1}|J_t) = \sum_{k=1}^{K} \Lambda_{ik}\Lambda_{jk}\sigma^2_{kt} + \Omega_{ij}$. And their conditional correlation, $\rho_{ij,t}$ is given by:

$$\rho_{ij,t} = \frac{\Sigma_{ij,t}}{\sqrt{\Sigma_{ii,t}}\sqrt{\Sigma_{jj,t}}}.$$

This *conditional correlation* is dynamic and would be time-invariant if the two asset return processes are conditionally uncorrelated. This feature makes the model bear some analogy with the Dynamic Conditional Correlations (DCC-MV-GARCH) model of Engle (2002).

Let $\ell_t = (\ell_{1t}, \ldots, \ell_{Kt})'$ with $\ell_{kt} = Cov(F_{k,t+1}, \sigma^2_{k,t+1}|J_t)$ the conditional leverage in the k-th factor. The conditional leverage in asset i, $\ell_{ii,t}$, is given by:

$$\ell_{ii,t} = \Lambda^{(3)}_{i\bullet}\ell_t; \quad \text{with} \quad \Lambda^{(p)}_{i\bullet} = \left(\Lambda^p_{i1}, \Lambda^p_{i2}, \ldots, \Lambda^p_{iK}\right), \text{ a } K\text{−row vector.}$$

This expression means that each asset has its conditional leverage positively correlated with the conditional leverage of factors it has a positive loadings for and negatively correlated with the other factors' conditional leverage.

I also define the *co-leverage* (or transversal leverage) between two assets i and j. This is given by $\ell_{ij,t} \equiv Cov\left(y_{i,t+1},\, \Sigma_{jj,t+1}|J_t\right)$. Note that the order of the arguments matters in this definition of the co-leverage. Specifically, $\ell_{ij,t}$ is typically different from $\ell_{ji,t}$. The co-leverage measures the impact of a shock on the return of asset i today on the volatility of asset j tomorrow. Under the maintained assumptions, it follows that

$$\ell_{ij,t} = \left(\Lambda_{i\bullet} \circ \Lambda^{(2)}_{j\bullet}\right)\ell_t,$$

where "$\circ$" denotes the Hadamard entry-wise product of vectors and $\Lambda_{i\bullet} \equiv \Lambda^{(1)}_{i\bullet}$. This co-leverage has a more transparent interpretation in the context of a *single factor model* where:

$$\ell_{ij,t} = \Lambda_i\, \Lambda^2_j \ell_t = \frac{\Lambda^2_j}{\Lambda^2_i}\left(\Lambda^3_i \ell_t\right) = \frac{\Lambda^2_j}{\Lambda^2_i}\, \ell_{ii,t}.$$

The last equality shows that the co-leverage of asset i on asset j has the same sign as the leverage effect for asset i. Thus, if asset i has a negative leverage effect, a positive shock on asset i' s return lowers its future volatility, which increases the confidence level in asset i's market, which ceteris paribus, propagates to the entire financial market. Thus, a positive shock on asset i's return reduces future volatility for all other assets, including asset j.

In a similar way, the *co-third conditional moments* (conditional co-skewness) between assets i and j is defined as $\mathsf{s}_{ij,t} = Cov(Y_{i,t+1}, Y^2_{j,t+1}|J_t)$. Note here also that $\mathsf{s}_{ij,t} \neq \mathsf{s}_{ji,t}$. Clearly,

$$\begin{aligned} \mathsf{s}_{ij,t} &= \textstyle\sum_{k=1}^{K} \Lambda_{ik}\Lambda^2_{jk} E(F^3_{k,t+1}|J_t) + E(u_i u_j^2|J_t) \\ &= \left(\Lambda_{i\bullet} \circ \Lambda^{(2)}_{j\bullet}\right) h_0 + \left(\Lambda_{i\bullet} \circ \Lambda^{(2)}_{j\bullet}\right)(h_1 \circ Diag(D_t)) + E(u_i u_j^2|J_t), \end{aligned}$$

where $h_0 = (h_{01}, \ldots, h_{0K})'$ and $h_1 = (h_{11}, \ldots, h_{1K})'$ and $Diag(D_t)$ is the vector of the diagonal entries of D_t. This expression shows that the co-skewness are also dynamic in ACHF. Their full specification requires the specification of the idiosyncratic co-skewness ($E(u_i u_j^2|J_t)$). Such attempt could easily lead to an inflation of parameters without any obvious added value. The main focus is on the marginal co-skewness which are suitably modeled in (8). Since this model does not restrict the idiosyncratic shocks' conditional co-skewness it is consistent with any dynamics that they may have.

3.4 Temporal aggregation properties of the model

Asset returns are available at many different frequencies. Because lower frequency returns are just a temporal aggregation of the higher frequency returns, an internally consistent model should be closed under temporal aggregation. Drost and Nijman (1993) show that the standard GARCH model is not closed under temporal aggregation and propose the weak GARCH model, which is closed under temporal aggregation. More recently, Meddahi and Renault (2004) propose the SR-SARV class of volatility processes and show that these processes are closed under temporal aggregation. See also Engle and Patton (2001) for a discussion of the merits of temporal aggregation.

This section discusses the properties of the ACHF model (8) related to temporal aggregation. While the specification of the conditional leverage agrees with several models in the literature, the specification of the conditional skewness is motivated by a temporal aggregation argument. Given the specified dynamics for conditional leverage, conditional skewness is specified so that the whole system is closed under temporal aggregation. This section gives a more detailed study of this property.

Suppose that one observes returns at $t = 1, 2, \ldots$. The relevant conditioning information set at time t is J_t, which contains the past observations up to time t. Suppose now that one observes returns at a lower frequency, in particular returns at tm intervals, where $t = 1, 2, \ldots$, and m is the time horizon. For example, if one moves from the daily to the weekly frequency, $m = 5$. In this case, the relevant conditioning information set depends on the observations dated at times tm. I denote this information set $J^{(m)}_{tm}$. In order to define $J^{(m)}_{tm}$, some additional notation are needed. Let $Y^{(m)}_{tm} \equiv \sum_{l=1}^{m} \alpha_l Y_{(t-1)m+l}$, $t \geq 1$, denote the process resulting from the temporal aggregation of Y_t

over the time horizon m. The coefficients α_l, $l = 1, \ldots, m$, are the aggregation coefficients. For a flow variable such as a log return, $\alpha_l = 1$, for all $l = 1, \ldots, m$, whereas for a stock variable, $\alpha_l = 1$ for $l = m$ and 0 otherwise. Similarly, let $F_{tm}^{(m)} \equiv \sum_{l=1}^{m} \alpha_l F_{(t-1)m+l}$ and $U_{tm}^{(m)} \equiv \sum_{l=1}^{m} \alpha_l U_{(t-1)m+l}$ be the temporal aggregation analogues of F_t and U_t. Following Meddahi and Renault (2004), I define $J_{tm}^{(m)} \equiv \sigma\left(Y_{\tau m}^{(m)}, F_{\tau m}^{(m)}, U_{\tau m}^{(m)}, D_{\tau m}; \tau \leq t\right)$, where, for any integer τ, $D_{\tau m} = Var\left(F_{\tau m+1} | J_{\tau m}\right)$, $J_{\tau m}$ is the same information set as J_t with $t = \tau m$ and $\sigma(X)$ denotes the σ-algebra generated by X. Meddahi and Renault (2004) show that the SR-SARV(1) model is closed under temporal aggregation with respect to the increasing filtration $J_{tm}^{(m)}$.

Proposition 3.1 *Let Y_t be defined by* (1) *and* (2)*. Assume that Y_t has a constant conditional mean μ. Then the temporally aggregated process $Y_{tm}^{(m)}$ of Y_t over the time horizon m has the following representation:*

$$Y_{tm}^{(m)} = \mu^{(m)} + \Lambda F_{tm}^{(m)} + U_{tm}^{(m)}, \tag{9}$$

such that, with $D_{tm}^{(m)}$ being a diagonal matrix and $\mu^{(m)}$ the time-invariant vector equal to $\left(\sum_{l=1}^{m} \alpha_l\right) \mu$,

$$\begin{aligned}
&E\left(U_{(t+1)m}^{(m)} | J_{tm}^{(m)}\right) = 0, \quad Var\left(F_{(t+1)m}^{(m)} | J_{tm}^{(m)}\right) = D_{tm}^{(m)}, \quad E\left(U_{(t+1)m}^{(m)} F_{(t+1)m}^{(m)\prime} | J_{tm}^{(m)}\right) = 0, \\
&E\left(F_{(t+1)m}^{(m)} | J_{tm}^{(m)}\right) = 0, \quad Var\left(U_{(t+1)m}^{(m)} | J_{tm}^{(m)}\right) = \left(\textstyle\sum_{l=1}^{m} \alpha_l^2\right) \Omega.
\end{aligned} \tag{10}$$

Proof: See Dovonon (2012).

Proposition 3.1 shows that $Y_{tm}^{(m)}$, the temporal aggregation of Y_t over the horizon m, has the same factor representation as Y_t, where the idiosyncratic shocks and the latent factors are the temporal aggregation analogues of the higher frequency idiosyncratic shocks and factors, respectively. Hence, if each factor is assumed to follow a SR-SARV(1) model, as in model (8), the results in Meddahi and Renault (2004) imply that each component of $F_{tm}^{(m)}$ inherits the SR-SARV(1) dynamics. We Therefore, under the maintained assumptions, the volatility specification assumed for the factor representation of Y_t in (8) is closed under temporal aggregation.

However, I would like to point out that even though the number of conditionally heteroskedastic factors is expected to remain the same by temporal aggregation, this may not be the case in empirical applications. In fact, from Meddahi and Renault (2004), the volatility persistence of the aggregated factors are γ_k^m which can be very small if γ_k is not large and m low. For example, if a factor has its volatility persistence $\gamma_k = 0.45$, observed every 5-period, the corresponding persistence would be $\gamma_k^5 = 0.018$. At such frequency, this factor is reasonably expected to drop out. The bottom line here is that higher frequency returns would require more factors than lower frequency data. This is further supported by the empirical findings in Section 5.

Next, I study the properties of temporal aggregation of the dynamics for the conditional leverage and skewness assumed in Model (8). For simplicity I consider a single factor model and let $\mu = 0$. The multi-factor setting presents only notational complication with no added value to the main conclusions.

The following proposition is auxiliary in proving the closeness of the skewness and leverage models for temporal aggregation. It provides some useful properties of SR-SARV(1) processes not yet established in the literature. In particular, this proposition gives the term structure of the conditional variance expressed as the expected value of the conditional variance given the information available at any earlier period. In addition, the conditional variance of an aggregated SR-SARV(1) process in terms of the conditional variance of the original process is also derived.

Proposition 3.2 *Let f_{t+1} follow a SR-SARV(1) model with volatility persistence and intercept γ and $1-\gamma$, respectively, and with conditional variance σ_t^2. Then, for all $l \geq 1$,*

$$E(\sigma^2_{tm+l-1}|J_{tm}) = 1-\gamma^{l-1}+\gamma^{l-1}\sigma^2_{tm}, \quad \textit{and}$$

$$\sigma^{(m)^2}_{tm} \equiv Var\left(f^{(m)}_{(t+1)m}|J^{(m)}_{tm}\right) = \sum_{l=1}^{m}\alpha_l^2\left(1-\gamma^{l-1}\right)+\sigma^2_{tm}\sum_{l=1}^{m}\alpha_l^2\gamma^{l-1} \equiv S_1^{(m)}+S_2^{(m)}\sigma^2_{tm}.$$

Proof: See Dovonon (2012).

The conditional leverage in the aggregated return $Y^{(m)}_{i,tm}$ of asset i is defined as $Cov\left(Y^{(m)}_{i,(t+1)m}, \sigma^{(m)^2}_{i,(t+1)m}|J^{(m)}_{tm}\right)$, with $\sigma^{(m)^2}_{i,tm} \equiv Var\left(Y^{(m)}_{i,(t+1)m}|J^{(m)}_{tm}\right)$. Given Assumption 1, it suffices to examine the leverage effect in the factor i.e. $Cov\left(f^{(m)}_{(t+1)m}, \sigma^{(m)^2}_{(t+1)m}|J^{(m)}_{tm}\right)$.

Proposition 3.3 *Let f_{t+1} follow a SR-SARV(1) model with volatility persistence and intercept γ and $1-\gamma$, respectively, and satisfying the conditional leverage dynamics in Equation* (5)*. Then:*

$$Cov\left(f^{(m)}_{(t+1)m}, \sigma^{(m)^2}_{(t+1)m}|J^{(m)}_{tm}\right) = \pi_0^{(m)}+\pi_1^{(m)}\sigma^{(m)^2}_{tm}; \qquad \pi_0^{(m)} \text{ and } \pi_1^{(m)} \in \mathbb{R}.$$

Proof: See Dovonon (2012).

Proposition 3.3 shows that the conditional leverage specification given by (5) is closed under temporal aggregation for the class of SR-SARV(1) processes. The next result establishes the closeness under temporal aggregation of the third conditional moment dynamics assumed in Assumption 2.

Proposition 3.4 *Let f_{t+1} follow a SR-SARV(1) model with volatility persistence and intercept γ and $1-\gamma$, respectively, and satisfying the conditional leverage dynamics in Equation* (5) *along with Assumption 2-(i). Then:*

$$E\left(\left(f^{(m)}_{(t+1)m}\right)^3|J^{(m)}_{tm}\right) = h_1^{(m)}\sigma^{(m)^2}_{tm}+h_0^{(m)}.$$

If, in addition, the factor structure (1)-(2) *and Assumptions 1 and 2-(ii) are satisfied, then*

$$E\left(\left(Y^{(m)}_{i,(t+1)m}\right)^3|J^{(m)}_{tm}\right) = \lambda_i^3h_1^{(m)}\sigma^{(m)^2}_{tm}+s_i^{(m)},$$

for $i = 1, \dots, N$ and $t = 1, 2, \dots$, where, with $S_1^{(m)}$ and $S_2^{(m)}$ as defined in Proposition 3.2,

$$
\begin{aligned}
h_1^{(m)} &= \frac{1}{S_2^{(m)}}\left[h_1 \sum_{l=1}^{m} \alpha_l^3 \gamma^{l-1} + 3\pi_1 \times \sum_{l<l';\, l,l'=1}^{m} \alpha_l \alpha_{l'}^2 \gamma^{l'-2}\right],\\
h_0^{(m)} &= A^{(m)} - h_1^{(m)} S_1^{(m)},\\
A^{(m)} &= \sum_{l=1}^{m} \alpha_l^3 [h_0 + (1-\gamma^{l-1})h_1] + 3 \times \sum_{l<l';\, l,l'=1}^{m} \alpha_l \alpha_{l'}^2 \gamma^{l'-l-1}\left[\pi_0 + \pi_1(1-\gamma^{l-1})\right],\\
s_i^{(m)} &= \lambda_i^3 h_0^{(m)} + s_i^0 \left(\sum_{l=1}^{m} \alpha_l^3\right).
\end{aligned}
$$

Proof: See Dovonon (2012).

Proposition 3.4 shows that the conditional third order moment dynamics postulated in Assumption 2-*(i)* is closed under temporal aggregation in the set of SR-SARV(1) processes that have a conditional leverage dynamics fitting (5). In particular, the third conditional moments of excess aggregated returns follow an affine function of volatility. Moreover, if the conditional third moment of the underlying factor is time varying, it follows that the aggregated factor also has a dynamic conditional third moment, given that the aggregation coefficients α_ls are nonnegative.

The temporal aggregation properties of the temporally aggregated model summarizes as follows. First, the factor representation is preserved for the aggregated model, with the same factor loadings. Second, each aggregated factor has a conditional leverage and skewness, the specifications of which are affine functions of its volatility, just as assumed for the original factor itself. Third, the conditional skewness of the idiosyncratic shocks is constant if the same is true for the underlying non aggregated shocks, as assumed by Assumption 2-*(ii)*. These properties, together with the property of closeness under temporal aggregation of SR-SARV(1) models for conditional heteroskedasticity, prove that the ACHF model is closed under temporal aggregation. It is worth mentioning that the model is also closed under marginalization (portfolio formation) even though I do not give a formal treatment of this property here.

4 Identification and estimation of the model

The main goal of this section is to present some valid moment conditions for the ACHF model on which one can base a GMM inference. The GMM-based inference is distribution-free and is in several instances easier to perform than alternative methods often used in the conditionally heteroskedastic factor model literature, which also widely rely on distributional assumptions. I follow DR (2006) who are the first to propose a GMM-based inference method for conditionally heteroskedastic factor model.

As a specialization of DR's (2006) model, the overlap of the two models makes the use of the moment conditions that they propose useful for this model for the identification of the shared parameters such as Λ, γ and Ω. However, as they have pointed out, because the factors are not observable the proposed moment conditions cannot identify all of the parameters included. Actually, the model is only partially identified in the sense of Manski and Tamer (2002). Particularly, it identifies the whole set of parameters as a function of the loadings of a certain factor mimicking portfolio. The normalization approach sets these factor loadings to some specific values and allows the identification of the whole model given these values through some appropriate unconditional moment restrictions.

Since the matrix of factor loadings is of full column rank (with rank K equal to the number of factors), there is at least one set of K asset returns with factor loadings being a (K, K) nonsingular matrix. Such a set of assets is referred to as *factor mimicking portfolio.* I assume that the first K returns series are from a factor mimicking portfolio and set the corresponding loadings to a certain value $\bar{\Lambda}$ such that $0 < \bar{\Lambda}\bar{\Lambda}' < Var(\bar{Y}_t)$, with $\bar{Y}_t = (Y_{1t}, Y_{2t}, \ldots, Y_{Kt})'$ and $\bar{\bar{Y}}_t = (Y_{K+1,t}, Y_{K+2,t}, \ldots, Y_{Nt})'$.

Following DR (2006) and assuming that $E(Y_{t+1}|J_t) = 0$, from (8), I can write $\bar{Y}_{t+1} = \bar{\Lambda}F_{t+1} + \bar{U}_{t+1}$ thus $F_{t+1} = \bar{\Lambda}^{-1}(\bar{Y}_{t+1} - \bar{U}_{t+1})$. Letting $\bar{\bar{\Lambda}}$ denote the factor loadings associated to $\bar{\bar{Y}}_t$,

$$\bar{\bar{Y}}_{t+1} - \bar{\bar{\Lambda}}\bar{\Lambda}^{-1}\bar{Y}_{t+1} = \bar{\bar{U}}_{t+1} - \bar{\bar{\Lambda}}\bar{\Lambda}^{-1}\bar{U}_{t+1}. \tag{11}$$

Since the factors are conditionally orthogonal to the idiosyncratic shocks, (11) implies that:

$$E\left((\bar{\bar{Y}}_{t+1} - \bar{\bar{\Lambda}}\bar{\Lambda}^{-1}\bar{Y}_{t+1})\bar{Y}'_{t+1}|J_t\right) = \Omega_{K+1:N,1:K} - \bar{\bar{\Lambda}}\bar{\Lambda}^{-1}\Omega_{1:K,1:K}, \tag{12}$$

where $A_{a:b,c:d}$ denotes the submatrix of the matrix A at the intersection of the rows a through b and columns c through d. $A_{\bullet k}$ is the vector corresponding to the kth column of A.

The conditional moment conditions in (12) represent a set of $K(N - K)$ moment conditions that can identify $\bar{\bar{\Lambda}}$ and $\Omega_{K+1:N,1:K}$ if an appropriate choice of instruments is made. Note however that $\Omega_{1:K,1:K}$ is not identified by (12) since it cannot identify $\Omega_{1:K,1:K}$ and $\Omega_{K+1:N,1:K}$ at the same time. The conditional variance of the idiosyncratic shocks of the factor mimicking portfolio is identified through the SR-SARV(1) dynamics of the factors. I recall that:

$$E(\bar{Y}_{t+1}\bar{Y}'_{t+1}|J_t) = \bar{\Lambda}D_t\bar{\Lambda}' + \Omega_{1:K,1:K}. \tag{13}$$

Taking the diagonal of this matrix equality yields $E(\bar{Y}^{(2)}_{t+1}|J_t) = \bar{\Lambda}^{(2)}Diag(D_t) + Diag(\Omega_{1:K,1:K})$, where $A^{(p)}$ stands for the matrix A to the power p entry-wise. Hence,

$$Diag(D_t) = E(d_{t+1}|J_t); \quad d_{t+1} = \left(\bar{\Lambda}^{(2)}\right)^{-1}\left(\bar{Y}^{(2)}_{t+1} - Diag(\Omega_{1:K,1:K})\right).$$

The SR-SARV(1) dynamics for the factors translates into:

$$E\left(d_{t+2} - \gamma \circ d_{t+1} - (\mathbf{1} - \gamma)|J_t\right) = 0, \tag{14}$$

where $\gamma = (\gamma_1, \ldots, \gamma_K)'$ is the vector volatility persistence of the factors and $\mathbf{1}$ is a K-vector of ones. I recall that "$\circ$" denotes the Hadamard element-wise product of matrices. Up to a convenient choice of instruments, this conditional moment condition identifies γ as well as the diagonal of $\Omega_{1:K,1:K}$. The off-diagonal entries of $\Omega_{1:K,1:K}$ are identified by the covariances in (13):

$$E\left(Y_{i,t+1}Y_{j,t+1} - (\Lambda_{i\bullet} \circ \Lambda_{j\bullet})\, d_{t+1} - \Omega_{ij} \middle| J_t\right) = 0;\ 1 \leq i < j \leq K. \tag{15}$$

The block $\Omega_{K+1:N,K+1:N}$ of the idiosyncratic shocks' conditional variance is identified by:

$$EVech\left(\bar{\bar{Y}}_{t+1}\bar{\bar{Y}}'_{t+1} - \bar{\bar{\Lambda}}\tilde{D}_t\bar{\bar{\Lambda}}' - \Omega_{K+1:N,K+1:N} \middle| J_t\right) = 0, \tag{16}$$

$\tilde{D}_t$ is a diagonal matrix whose diagonal is equal to d_{t+1} and "$Vech$" is the usual half vectorization operator of square and symmetric matrices.

The moment conditions for the identification of the conditional leverage parameters can be obtained as a translation of (6) in terms of observable variables. This gives:

$$E\left(Y_{t+1} \circ Y_{t+2}^{(2)} - \sum_{k=1}^{K} \Lambda_{\bullet k}^{(3)} \pi_{1k} d_{t+1,k} - \sum_{k=1}^{K} \Lambda_{\bullet k}^{(3)} \pi_{0k} \middle| J_t\right) = 0 \tag{17}$$

and the conditional skewness parameters can be identified similarly from (7):

$$E\left(Y_{t+1}^{(3)} - \sum_{k=1}^{K} \Lambda_{\bullet k}^{(3)} h_{1k} d_{t+1,k} - s \middle| J_t\right) = 0, \tag{18}$$

$s = (s_1, \ldots, s_N)'$. The set of conditional moment conditions (12)-(14)-(15)-(16)-(17)-(18) identify the parameters $\bar{\bar{\Lambda}}$, $\gamma_1, \ldots, \gamma_K$, Ω, $\pi_{01}, \ldots, \pi_{0K}$, $\pi_{11}, \ldots, \pi_{1K}$, $h_{11}, \ldots, h_{1K}$ and s as long as an appropriate choice of instruments Z_t from J_t is made to translate the moment conditions into unconditional moment condition on which one can apply the GMM inference of Hansen (1982). Actually, one can consider using $Z_t = (1, z_t')'$ with z_t correlated with each component of d_{t+1}[5]. This ensures that the necessary first order local identification condition for the application of the asymptotic theory of Hansen (1982) is fulfilled. The resulting GMM estimator is asymptotically Gaussian and the GMM overidentification test statistic has asymptotically a chi-square distribution with degrees of freedom equal to the number of overidentifying moment conditions.

I conclude this section with some remarks. First, the number of parameters can grow very quickly with the number of assets in the factor structure depending on the restrictions on the variance matrix Ω of the idiosyncratic shocks. Typically, without any restriction on Ω, the number of parameters is of order $O(N^2)$ against $O(N)$ if this variance matrix is restricted to be diagonal. Thus, a free positive definite matrix Ω is tractable only in the case of a low number of assets (e.g. $N \leq 5$). For a larger number of assets, it would be more convenient to restrict Ω to be diagonal or at least block diagonal.

[5]The SR-SARV(1) dynamics for the factors implies an ARMA representation for the square of the factors (see Meddahi and Renault (2004)) so that past square returns are correlated with future square returns; past absolute returns also happen to be correlated with future square returns and any of them can suitably be used as instrument z_t.

5 Application to U.K. stock market excess returns

This empirical application considers daily log-returns on 23 sectorial indices from the U.K. stock market including FTSE 350, all of which are listed in the FTSE. (See Appendix B for details.) Three panels of data are considered. Panel A contains 5 series of daily excess returns on equally weighted portfolios of sectorial indices. Panel B contains (Wednesdays) weekly excess returns on the same assets as those in Panel A. And Panel C contains the daily excess returns on all the 23 sectorial indices. The data in Panel C cover the whole history of FTSE through 15/7/2009 while Panels A and B cover the most recent data within that time frame. I mainly apply two versions of the ACHF and the DR models. The first one considers the variance of the idiosyncratic shocks as free parameter while the second one restricts this variance to be diagonal. The unrestricted versions are applied to Panels A and B while the restricted versions are applied to Panel C.

The aim of applying the ACHF model to Panels A and B is to investigate its empirical performance through temporal aggregation while the application to Panel C illustrates possible application of the model to large portfolios under suitable restrictions on the parameters. The next section gives some details regarding the estimation and is followed by the results and the model validation.

5.1 Estimation

I first filter out the possible autocorrelation in the returns (see the portmanteau tests in Table 1) by autoregression and estimate the models on filtered data. The 2-factor representation of the ACHF and DR models have been considered primarily. The factors mimicking portfolio for Panels A and B is ('FTSE 350', 'Oil Commod.') and for Panel C is ('FTSE 350', 'Leisure goods'). These two pairs of assets have been chosen because they have the lowest correlation. The factor loading of the mimicking portfolio is fixed to $\bar{\Lambda} = (\lambda_1|\lambda_2)$, with $\lambda_1 = (0.5, 0.3)'$ and $\lambda_2 = (0.3, 0.4)'$, up to which the full model is identified. (See Section 4 and DR (2006).) I also estimate a 1-factor structure of the ACHF model using Panel B. In this case, the factor mimicking asset is 'FTSE 350' and $\bar{\Lambda} = 0.5$.

The set of instruments used is $z_t = (1, \sum_{i=1}^n |Y_{i,t}|, \sum_{i=1}^n |Y_{i,t-1}|, \dots, \sum_{i=1}^n |Y_{i,t-k}|)'$ and is suggested by the Monte Carlo experiments reported in Dovonon (2012). I set $k = 5$ for Panel A resulting in 175 unconditional moment restrictions for the ACHF model with 34 parameters and 105 moment restrictions for the DR model with 23 parameters; $k = 2$ for Panel B yielding 100(100) moment restrictions and 34(28) parameters for the 2(1)-factor ACHF model. And, $k = 2$ for Panel C translating into 444(260) moment restrictions and 96(67) parameters for the 2-factor ACHF (DR) model. For comparison purpose, I also apply the DCC-MV-(E)GARCH model of Engle (2002) to Panel A. The computations are done with IMSL library routines for FORTRAN 1990.

5.2 Results and discussion

The estimation results are displayed by Tables 2 through 8. Table 2 shows the results for the ACHF model on Panel A. The factor loadings are all positive and strongly significant meaning that the common sources of heteroskedasticity tend to move the returns into the same direction. Also, while most of the conditional covariances of the idiosyncratic shocks are significant, some of them are not. This includes the pairs ('Oil Commod.', 'Fin. & Insur.') and ('Oil Commod.'; 'Serv. & other ind.'). The conditional third moments' intercepts (s_i) are all negatively significant except for 'Fin. & Insur.'. The two factors present some differences. Factor 1's volatility is less persistent ($\gamma_1 = 0.68$ against $\gamma_2 = 0.87$), the conditional variance influences positively the conditional leverage ($\pi_{11} = 0.42$) and negatively the conditional skewness ($h_{11} = -2.6$). This is in contrast with Factor 2 as $\pi_{12} = -3.09$ and $h_{12} = 9.69$. As a result, the effect of the factors' volatility on both the conditional skewness and leverage of the assets is rather conflicting. However, the magnitude of π_{12} means that Factor 2 may dominate Factor 1 leading to a negative relation between volatility and conditional leverage. The GMM overidentification test validates the model with a pvalue sufficiently large (0.999).

Table 3 shows the results of the DR model on Panel A. In comparison with Table 2, the parameters that the two models have in common have their estimates that are quite close. The GMM overidentification test also validates this model with a pvalue of 0.975. In spite of these similarities, it is important to highlight the sharp efficiency gain that the ACHF model offers for the shared parameters. Except for a few off-diagonal components of the idiosyncratic shocks variance matrix, all the other parameters have lower estimated standard errors in the ACHF model.

The estimates of the Gaussian DCC-MV-(E)GARCH model on Panel A are available in Table 4. The volatility persistence is the only parameter that can be compared across the models. The average persistence for individual assets (average of β_i) is 0.985 while the DCC persistence $\alpha + \beta$ is 0.99. This volatility persistence is large compared to the persistence that one gets from the ACHF and DR models. I interpret this as the consequence of setting actively the conditional skewness to 0 while it is not. This translates into a missing variable type of effect that inflates the volatility persistence.

The extended Kalman filter algorithm that I propose (see Appendix A) aims at filtering the future realizations of the factors and their volatilities from current and past returns. At best, the filtered factors and volatility processes can be considered as the best forecasts of the latent processes given the parameters' estimates while, for instance, the volatility processes from the DCC-MV-(E)GARCH are smoothed over the whole sample through the maximum likelihood fitting. Figure 1 shows the filtered volatility process for FTSE 350 from the ACHF, DR and DCC-MV-(E)GARCH models as well as the excess return on FTSE 350 over various part of the sample. As expected, the filtered volatilities from the ACHF and the DR models look noisier than the volatility process from DCC-MV-(E)GARCH. Focusing on the periods of noticeable crashes on the FTSE market that are contained within May

02-March 03 on the one hand and Sep 08-July 09 on the other, the 5-day moving average of the ACHF volatility shows clearly that the periods of high volatility in the market are suitably tracked by the filtered volatility from ACHF. The conditional leverage and skewness of FTSE 350 shown by this figure have similar pattern as they both increase in magnitude with volatility. But, as the conditional skewness can move into either direction, the conditional leverage is typically markedly negative in high volatility periods. This explains the interest of practitioners to invest in volatility products in periods of high volatility since they act as hedging products. Table 8 shows some relevant statistics indicating the relation between the returns processes and their filtered systematic parts.

5.3 Temporal aggregation

As an aggregation over trading weeks of Panel A, Panel B offers the possibility to evaluate empirically the temporal aggregation properties of the ACHF model. From the theory in Section 3.4, the factor structure is expected to hold but with volatility persistence for the factors of about $0.68^5 = 0.14$ and $0.87^5 = 0.49$ for the two factors, respectively. Table 5 shows the estimates of the ACHF model for weekly data. The estimated volatility persistence for Factor 1 is 0.03 whereas Factor 2 has a volatility persistence of 0.54. Also, no dynamics-related parameter (γ_1, π_{11} and h_{12}) for Factor 1 is significant while they all are significant for Factor 2. Coupling with the low volatility persistence of Factor 1, this suggests that two conditionally heteroskedastic factors are certainly not required to describe this data set. Table 6 shows the estimates of a 1-factor ACHF model. The significantly negative h_1 means that the conditional skewness is dynamic and that larger volatilities predict more negatively skewed returns. Besides, even though present and significantly negative through π_0, the leverage effect is not dynamic. The 1-factor ACHF also passes the global validation test but with a larger pvalue than the 2-factor version providing further evidence that this latter is less suitable for this weekly data set.

5.4 Large portfolios

The application of ACHF to Panel C of 23 sectorial indices of FTSE highlights the possibility of application to large portfolios of assets. The main source of limitation is the variance matrix of the idiosyncratic shocks. A portfolio of 23 assets requires 276 parameters as entries for this variance matrix. A free variance matrix in this application would inevitably lead to the curse of dimensionality. For practical purposes, I impose a diagonal variance matrix for the idiosyncratic shocks. As seen in the previous applications, some off-diagonal elements may be significant in some situations and this restriction may not be entirely realistic though common in the conditionally heteroskedastic factor literature (see e.g. Fiorentini, Sentana and Shephard (2004)).

The results are displayed by Table 7. Two factors have been considered for each model. First, observe that both models pass the GMM overidentification test for global validation meaning that there is no evidence against the specified models. Also, comparing the two models, similar conclusions to the application to Panel A can be drawn. The parameters that the two models share have their estimates of the same magnitude and the ACHF model estimates them more efficiently than does the DR model. This application also confirms the rather negative co-movement of conditional leverage and skewness with volatilities as π_{11}, h_{12}, π_{12} are all strongly significantly negative and h_{11} is also significantly negative, though at 10%. The intercept of the skewness equations (s_i) are also significantly negative throughout, except for the 'Forestry & Paper' sector.

5.5 Validation tests

The ACHF and the DR models are specified as overidentifying moment condition models and the GMM overidentification test of Hansen (1982) is useful for model validation. All the model estimation carried out in this paper actually pass that test. However, there is a need to be cautious in applying these tests here. Since there is a necessity to include several instruments in the estimation process, the degrees of freedom of the test statistic's distribution grows larger resulting in possible loss of power as the test becomes more and more conservative.

Even though the GMM tests implicitly validate the factor structure of the model, namely the number of conditionally heteroskedastic factors, it is of interest to be more specific in testing for this structure. A natural way to carry out such validations is to test whether there is any source of heteroskedasticity left in the co-feature portfolios, i.e. the portfolios that offset the conditionally heteroskedastic factors (the $N - K$ linear combinations of returns in the LHS of Equation (11)). The co-feature portfolios vector that I test for conditional homoskedasticity is $r_{t+1} \equiv \bar{\bar{Y}}_{t+1} - \hat{\bar{\bar{\Lambda}}}\bar{\Lambda}^{-1}\bar{Y}_{t+1}$. I basically carry out a GMM overidentification test of the restriction: $Vech\left(Var(r_{t+1}|J_t)\right) = cst$.

For Panels A and B, r_{t+1} has size 3(4) in 2(1)-factor models while r_{t+1} has size 21 for Panel C in 2-factor models. This size of r_{t+1} in Panel C means that one may easily face again some issue of power if enough lags are included as instruments. For this reason, I carry out the test for homoskedasticity pairwise for Panel C. The existence of one pair of returns that rejects the null of constant conditional variance gives evidence that the factor structure is misspecified.

Table 9 gives the results of the validation tests for the ACHF and DR models to Panels A and B. Conditional homoskedasticity cannot be rejected in any of the four cases considered as the pvalues are quite large. Non-significant unconditional skewness for the co-feature returns are also reported.

Table 10 reports the validation test results for Panel C. Out of the 210 pairs of co-feature returns, only 3 reject the null of constant conditional variance. These pairs are: ('Travel & leisure', 'Forestry & paper'), ('Industrials', 'Forestry & paper') and ('General retailers', 'Forestry & paper') for the DR model while only one pair, ('Travel & leisure', 'Forestry & paper'), rejects that null hypothesis for the ACHF model. Note that all these pairs include 'Forestry & paper' as component. This evidence against the 2-factor structures may, nevertheless, be considered marginal because of the small number of pairs and the fact that they consistently direct to the same asset. From a modeling perspective, the possible actions that one can take from this point may consist on removing 'Forestry & paper' from the panel and keep the factor structure or, alternatively, considering an additional factor with 'Forestry & paper' as part of the factor mimicking portfolio.

In the light of these empirical results, I can conclude that conditional skewness and leverage matter and have a rather negative relation with past volatility. A suitable modeling of these effects yields some efficiency gain on the volatility-related parameters. Also, these results suggest that volatility models that are not consistent with those effects may over estimate the volatility persistence. Moreover, the volatility persistence seems to decrease with returns of longer time horizon and shorter horizon returns may require more conditionally heteroskedastic factors in their representation. In addition, the evidence of dynamic conditional leverage seems weaker for longer horizon returns while a dynamic conditional skewness prevails. Some of these findings are in line with the literature.

Harvey and Siddique (1999), observe that taking into account the skewness impacts the persistence in the conditional variance. My findings also confirm, for daily data, the result of Sentana (1995) for monthly U.K. index excess returns, namely that there is significant leverage effect in sectorial returns through a common factor. See also Black (1976) and Nelson (1991). In addition, the dynamic conditional skewness supported by this application confirms the results of Harvey and Siddique (1999, 2000) and Jondeau and Rockinger (2003) for their respective data.

6 Conclusion

This paper builds upon the conditionally heteroskedastic factor model proposed by DR (2006) in which the common factors are the main source of conditional heteroskedasticity in the return processes and are assumed to follow a SR-SARV(1) dynamics. I exploit in this paper the fact that SR-SARV(1) processes are consistent with dynamic conditional skewness and leverage and propose some specification for those dynamics that lead to the conditional heteroskedastic factor model with asymmetries that I propose. This model, in addition to capturing the commonalities in volatility, conditional skewness and leverage, is shown to be closed under temporal aggregation. I derive some moment conditions implied by the model that allow for valid inference by GMM. The empirical application involves 23 index excess returns from the U.K. stock market and confirms some known features for asset returns in the literature and also uncover strong relations between conditional skewness and leverage with past volatility. The strength of these relations is also seen to depend on the horizon of the returns.

This work mainly addresses the specification and inference issues for the ACHF model and has the volatility forecast issue as a natural extension that I plan for future research. The volatility process filtered by the extended Kalman filter using the conditional heteroskedastic factor model estimates and the state-space representation of the model can rightly be considered as a one-step ahead forecast as only past returns are used as input for its determination. But, the quality of such forecasts along with the choice of tuning parameters for the Kalman filter need some thorough evaluation that is beyond the scope of this paper.

This work can also be extended to take into account the risk premium by modeling the conditional mean of the returns as a function of volatility as Fiorentini, Sentana and Shephard (2004) and DR (2006). As a main advantage, such an extension would make the proposed model consistent with the well-known volatility feedback feature that occurs in financial processes. This extension may also be particularly relevant for long horizon returns for which the risk premium is known to matter.

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Appendix A: An extended Kalman filter for the latent common factors and volatility processes

The aim of this extended Kalman filter algorithm is to deliver a path of the latent factors and their volatility processes given the data and the parameter estimates. These paths will be used for some model reality checks. In the application, I either use the GMM parameter estimates of the Doz and Renault (2006) model (DR) or the parameter estimates of the conditionally heteroskedastic factor model with asymmetries (ACHF).

The SR-SARV(1) conditionally heteroskedastic factor model of DR can alternatively be written as:

$$f_{j,t+1} = \sigma_{jt}\varepsilon_{j,t+1} \qquad \sigma^2_{j,t+1} = 1-\gamma_j + \gamma_j\sigma^2_{jt} + w_{j,t+1};\ \ j=1,2 \tag{19a}$$

$$Y_{t+1} = \mu + \Lambda F_{t+1} + U_{t+1}, \tag{19b}$$

with: $F_{t+1} \equiv (f_{1,t+1}, f_{2,t+1})'$, $E\left(w_{j,t+1}|J_t\right) = E\left(\varepsilon_{j,t+1}|J_t\right) = 0$, $E\left(U_{t+1}|J_t\right) = 0$, $E\left(\varepsilon^2_{j,t+1}|J_t\right) = 1$, $Var\left(U_{t+1}|J_t\right) = \Omega$ and $E(U_{t+1}\varepsilon_{j,t+1}|J_t) = E(U_{t+1}w_{j,t+1}|J_t) = 0$, $j = 1,2$ and $E\left(w_{1,t+1}w_{2,t+1}|J_t\right) = E\left(\varepsilon_{1,t+1}\varepsilon_{2,t+1}|J_t\right) = 0$.

In this factor representation, $Z_t \equiv (F_t, \sigma^2_{1,t}, \sigma^2_{2,t})'$ is unobservable; in fact, only the multivariate return process (Y_{t+1}) is observable. However, the latent process Z_t depends nonlinearly on its past value up to some random shocks. The extended Kalman filter's algorithm (see Sorenson, 1985) is proved to be of interest in this framework to filter Z_t from the observations provided that the parameters are known. The state equation is given by equations in (19a) while the measurement equation is (19b).

The specific problem that occurs in a such procedure is the positivity of σ^2_t. A naive application of that filter here could lead to non positive values for σ^2_t which, for obvious reasons, is not desirable. I instead propose to filter $z_t = (F_t, x_{1t}, x_{2t})'$ and then, get σ^2_{jt} by $\sigma^2_{jt} \equiv x^2_{jt}$. I rely on the following result:

If (x_{t+1}) is such that $x_{t+1} = \sqrt{\gamma}x_t + \sqrt{1-\gamma}v_{t+1}$; $E(v_{t+1}|J_t) = 0$, $E(v^2_{t+1}|J_t) = 1$; J_t an increasing filtration as the one introduced in the body of this paper, then (x^2_{t+1}) is an SR-SARV(1) process with persistence γ and intercept $1-\gamma$ with respect to J_t.

The state-space representation is:

$$f_{j,t+1} = \sqrt{x^2_{jt}}\varepsilon_{j,t+1} \qquad x_{j,t+1} = \sqrt{\gamma_j}x_{jt} + \sqrt{1-\gamma_j}v_{j,t+1};\ \ j=1,2 \tag{20a}$$

$$Y_{t+1} = \mu + \Lambda F_{t+1} + U_{t+1}, \tag{20b}$$

With: $E\left(v_{j,t+1}|J_t\right) = E\left(\varepsilon_{j,t+1}|J_t\right) = 0$, $E\left(U_{t+1}|J_t\right) = 0$, $E\left(\varepsilon^2_{j,t+1}|J_t\right) = 1$, $E\left(v^2_{j,t+1}|J_t\right) = 1$, $Var\left(U_{t+1}|J_t\right) = \Omega$ and $E(U_{t+1}\varepsilon_{j,t+1}|J_t) = E(U_{t+1}v_{j,t+1}|J_t) = 0$, $j=1,2$ and $E\left(v_{1,t+1}v_{2,t+1}|J_t\right) = E\left(\varepsilon_{1,t+1}\varepsilon_{2,t+1}|J_t\right) = 0$.

Let $A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\gamma_1} & 0 \\ 0 & 0 & 0 & \sqrt{\gamma_2} \end{pmatrix}$, $W_t = \begin{pmatrix} \sqrt{x^2_{1t}} & 0 & 0 & 0 \\ 0 & \sqrt{x^2_{2t}} & 0 & 0 \\ 0 & 0 & \sqrt{1-\gamma_1} & 0 \\ 0 & 0 & 0 & \sqrt{1-\gamma_2} \end{pmatrix}$, $H = (\Lambda|0)$, $Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$.

$Q_j : j = 1,2$ are the *tuning parameters* of the algorithm and need to be set to some reasonable values. They represent the conditional variance of $(\varepsilon_{j,t+1}, v_{j,t+1})$. I set $Q_j = \begin{pmatrix} 1 & \alpha_j \\ \alpha_j & 1 \end{pmatrix}$, with $-0.9 \leq \alpha_j \leq -0.1$ throughout the applications. The extended Kalman Filter algorithm is the following:

Initial value: $\hat{z}_0 = (0,0,1,1)'$, $P_0 = Id_4$.

Time Update ("Predict")	
1. *Project the state ahead:*	$z_t^- = A\hat{z}_{t-1}$
2. *Project the error covariance ahead:*	$P_t^- = AP_{t-1}A' + W_tQW_t'$
Measurement Update ("Correct")	
3. *Compute Kalman Gain:*	$K_t = P_t^- H'(HP_t^- H' + \Omega)^{-1}$
4. *Update estimate with measurement Y_t:*	$\hat{z}_t = z_t^- + K_t(Y_t - \mu - Hz_t^-)$
5. *Update the error covariance:*	$P_t = P_t^- - K_tHP_t^-$
6. $t = t + 1$, Go To 1.	

Appendix B: Data Appendix and Tables

The following table presents the indices used in this paper. The first index listed refers to the FTSE 350 index. All the 22 sectorial indices listed are in FTSE while 14 of them are also in the FTSE 350. The sectorial indices that are not listed in FTSE 350 are the following: (2)-Leisure goods, (14)-Transport, (15)-Gen. Financ., (16)-Personal goods, (17)-Gen. industrials, (18)-General retailers, (19)-Oil and Gas, (23)-Support Services.

The data are obtained from Datastream. With $p_{i,t}$ being the index i level at day t, the daily log-return series $r_{i,t}$ (in %) are obtained by: $r_{i,t} = 100 \times (\ln p_{i,t} - \ln p_{i,t-1})$. The log-return of the UK one month loan index JPM UK CASH 1M (r_t) as safe interest rate. The log-excess return of the index i is $Y_{i,t} = r_{i,t} - r_t$. The daily excess returns cover the period from January 2, 1986 through July 21, 2009 for 6037 trading days.

Number	Sectorial index	Number	Sectorial index
(1)	FTSE Actuaries 350		
(2)	Leisure goods	(13)	Travel & leisure
(3)	Banks	(14)	Transport
(4)	Beverages	(15)	Gen. Financ.
(5)	Cnstr. & bldg. mats.	(16)	Personal goods
(6)	Chemicals	(17)	Gen. industrials
(7)	Eng. & machinery	(18)	General retailers
(8)	Food & drug retailers	(19)	Oil and gas
(9)	Food producers	(20)	Forestry & paper
(10)	Non life insurance	(21)	Health
(11)	Life insurance	(22)	Pharm. & biotec.
(12)	Eqt invest. Inst.	(23)	Support services

I also consider 4 equally weighted portfolios of these sectorial indices. These portfolios and their constituents are detailed below:

Portfolio 1: Fin. & Insur.	*Portfolio 2:* Chem. & Food	*Portfolio 3:* Oil Commod.	*Portfolio 4:* Serv. & other ind.
Banks	Beverages	Oil and gas	Cnstr. & bldg. mats.
Non life insurance	Chemicals	Forestry & paper	Eng. & machinery
Life insurance	Food & drug retailers		Travel & leisure
Eqt invest. Inst.	Food producers		Transport
Gen. Financ.	Personal goods		Gen. industrials
	General retailers		Leisure goods
	Pharm. & biotec.		Health
			Support services

Panel A consists of daily excess returns on these 4 portfolios extended to the excess return on FTSE 350. The data in Panel A starts from October 27, 1997 through July 21, 2009 for 3026 observations.

Panel B consists of weekly excess returns on these 4 portfolios extended to the weekly excess return on FTSE 350. The returns are calculated over weeks ending on Wednesdays. I choose Wednesdays to avoid the well-known end/start of the week effects. The data in Panel B starts from January 5, 1994 through July 15, 2009 for 811 observations.

Panel C is made of the daily excess returns on each of the 23 sectorial indices including FTSE 350. This panel spans from January 2, 1986 through July 21, 2009 for 6037 observations.

Table 1: This table displays some summary statistics of daily excess returns on the sectorial indices considered (ranging from 27/10/1997 through 21/7/2009 for 3026 obs.) along with some diagnostic tests. "Mean:" average of log-returns in percent, "Std:" standard deviation, "Skew." and "Kurt." are the skewness and the kurtosis ($Skew. = \sum_{t=1}^{T} \tilde{Y}_{it}^3/T$, and $Kurt. = \sum_{t=1}^{T} \tilde{Y}_{it}^4/T$; $\tilde{Y}_{it} = (Y_{it} - \bar{Y}_i)/\sigma_i$; $\bar{Y}_i$ and σ_i the sample average and standard deviation of the returns on asset i). QW(5) is the test statistic of the usual Ljung-Box test for autocorrelation of the null that the first 5 autocorrelation coefficients are jointly equal to 0. The column "Test for Garch" gives the test statistic for GARCH effect in the returns. This statistic corresponds to the GMM overidentification test for $H_0: \; Vech\,(Var(Y_{it+1}|J_t)) = cst$, where the instruments $z_t = (1, Y_{it}^2, \ldots, Y_{it-k}^2)'$; $k = 5$ is used throughout. The results of the impact of news on volatility as proposed by Engle and Ng (1993) are reported. These tests aim to investigate whether positive shocks on returns have a different marginal impact on future volatility. I consider the sign bias test, the positive/negative size bias test and the joint test. I assume a standard Gaussian GARCH(1,1) volatility under the null. Dynamic conditional leverage and skewness are investigated through the regressions $\varepsilon_{it}\sigma_{it}^2 = \pi_0 + \pi_1\sigma_{it-1}^2 + v_t$ and $\varepsilon_{it}^3 = h_0 + h_1\sigma_{it-1}^2 + w_t$, respectively ($\varepsilon_{it} = Y_{it} - \bar{Y}_i$). The estimated slopes are reported. The commonality in this dynamics across assets is evaluated through the regression of $\varepsilon_{it}\sigma_{it}^2$ and ε_{it}^3 on the conditional variance of FTSE 350. The estimates of the slopes ($\pi_{f,1}$ and $h_{f,1}$) of these regressions are reported. The square returns is used as proxy for unobserved conditional variance.

	Descriptive statistics						Engle and Ng diagnostic tests for the impact of news on volatility				Evidence for dynamic conditional skewness and leverage			
	Mean	Std	Skew.	Kurt.	QW(5)	Test for Garch	Sign bias	Negative size bias	Positive size bias	Joint test	$\hat{\pi}_1$	$\hat{h}_1$	$\hat{\pi}_{f,1}$	$\hat{h}_{f,1}$
FTSE 350	-0.008	1.26	-0.17	8.8	63.6^a	24.08^a	3.12^a	-5.49^a	-6.24^a	45.2^a	-0.95^a	-0.01	-0.95^a	-0.01
Leisure goods	-0.035	1.48	0.29	14.3	9.7^c	28.23^a	1.68^c	0.38	0.41	8.8^b	0.47^a	1.01^a	-0.29^a	-0.15
Banks	-0.020	2.00	0.01	13.3	36.5^a	46.85^a	2.66^a	-3.83^a	-4.39^a	24.3^a	-0.47^a	0.51^b	-1.20^a	2.29^b
Beverages	0.014	1.51	0.07	7.1	44.4^a	31.41^a	1.86^c	-5.45^a	-5.22^a	40.4^a	-0.76^a	0.69^a	-0.70^a	0.18
Cnstr. & bldg. mats.	0.016	1.37	0.03	8.2	64.3^a	26.85^a	0.71	-1.68^c	-2.39^b	10.1^a	-0.51^a	0.67^a	-0.23^a	-0.89^a
Chemicals	0.005	1.56	-0.22	8.1	9.3^c	48.91^a	1.83^c	-2.31^b	-2.84^a	10.3^a	-0.61^a	-2.14^a	-1.02^a	-2.07^a
Eng. & machinery	-0.011	1.60	-0.04	7.9	58.6^a	36.20^a	0.71	-1.33	-2.45^b	11.0^a	-0.39^a	1.19^a	-0.91^a	-1.15^a
Food & drug retailers	0.009	1.45	-0.05	6.3	52.6^a	21.94^a	2.98^a	-5.76^a	-6.11^a	43.6^a	-0.89^a	-0.36^a	-0.92^a	-1.10^a
Food producers	-0.001	1.34	-0.21	6.8	23.0^a	16.35^a	0.88	-2.19^b	-1.76^c	8.5^b	-0.29^a	-0.95^a	-0.68^a	-0.13
Non life insurance	-0.024	1.83	-0.37	9.1	23.5^a	45.47^a	1.56	-2.85^a	-2.59^a	10.7^a	-0.83^a	-5.67^a	-0.72^a	-2.60^a
Life insurance	-0.022	2.30	-0.25	15.1	67.1^a	43.96^a	1.02	-1.16	-0.79	4.9	-0.58^a	-7.69^a	-1.37^a	5.81^a
Eqt invest. Inst.	-0.004	1.22	-0.32	7.8	18.7^a	28.96^a	3.77^a	-4.73^a	-6.70^a	47.5^a	-0.62^a	-0.63^a	-0.83^a	-0.28^a
Travel & leisure	-0.008	1.44	-0.24	6.6	37.4^a	50.44^a	2.62^a	-2.64^a	-4.01^a	18.1^a	-0.69^a	-1.81^a	-0.82^a	-1.15^a
Transport	-0.015	1.21	-0.09	8.4	34.2^a	35.30^a	0.61	0.93	0.18	10.5^a	0.05	0.48^a	-0.63^a	-0.85^a
Gen. Financ.	-0.002	1.71	-0.33	9.5	10.2^c	26.54^a	2.86^a	-3.91^a	-5.59^a	37.9^a	-0.77^a	-2.73^a	-1.04^a	-2.14^a
Personal goods	0.020	1.81	-0.24	10.5	9.9^c	34.18^a	0.97	0.24	1.39	13.5^a	0.54^a	-4.65^a	-0.38^a	-0.36
Gen. industrials	-0.009	1.30	-0.46	6.8	46.7^a	33.15^a	2.44^b	-4.53^a	-5.40^a	34.6^a	-0.78^a	-2.55^a	-0.77^a	-0.53^a
General retailers	-0.021	1.46	-0.20	7.8	27.2^a	72.31^a	1.50	-1.25	-1.96^b	8.4^b	0.12^b	-1.93^a	-0.62^a	0.49^a
Oil and gas	0.001	1.73	0.07	6.8	50.3^a	18.80^a	1.49	-2.96^a	-3.53^a	14.7^a	-0.38^a	1.60^a	-0.95^a	0.87^a
Forestry & paper	-0.023	2.26	0.41	13.3	11.4^b	32.03^a	1.69^c	-4.15^a	-3.23^a	28.3^a	-0.60^a	9.64^a	-0.76^a	2.31^c
Health	-0.010	1.62	0.18	11.9	21.8^a	39.13^a	-1.11	2.32^b	2.24^b	18.1^a	1.98^a	2.34^a	-0.84^a	-0.55
Pharm. & biotec.	-0.006	1.64	0.07	7.3	26.6^a	32.27^a	1.15	-2.59^a	-4.15^a	27.4^a	-0.74^a	1.92^a	-0.77^a	0.04
Support services	-0.021	1.23	-0.48	5.9	24.9^a	46.48^a	2.86^a	-4.01^a	-5.34^a	30.9^a	-0.69^a	-2.55^a	-0.66^a	-0.67^a

The upper script "a", "b" and "c" denote significance at 1%; 5% and 10%, respectively.

Table 2: Parameter estimates of the 2-factor conditionally heteroskedastic factor model with asymmetries (**ACHF**) using the data in Panel A. The vector of instruments used is $z_t = (1, \sum_{i=1}^{n} |Y_{it}|, \ldots, \sum_{i=1}^{n} |Y_{it-k}|)$, with $k = 5$; $n = 5$ is the number of assets in the model.

	Factor loadings				Idiosyncratic shocks variance matrix										s_i	
	Factor 1		Factor 2													
	est.	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*
FTSE 350	0.50	-	0.30	-	1.67	(1.27)									-4.59	(6.07)
Oil Commod	0.30	-	0.40	-	1.41	(1.93)	2.81	(2.81)							-5.85	(12.2)
Fin & Insur	0.48	(0.04)	0.36	(0.02)	1.89	(2.39)	1.57	(12.3)	2.62	(1.96)					5.52	(9.36)
Chem & Food	0.31	(0.03)	0.23	(0.01)	1.22	(1.25)	1.03	(4.09)	1.38	(1.24)	1.26	(0.79)			-2.51	(2.55)
Serv. & other ind.	0.28	(0.03)	0.25	(0.01)	1.16	(1.92)	1.06	(6.38)	1.40	(1.43)	0.90	(0.90)	1.16	(1.04)	-0.13	(2.93)

Factor 1			Factor 2		
γ_1	0.68	(0.55)	γ_2	0.87	(0.26)
π_{01}	3.91	(3.42)	π_{02}	2.08	(61.3)
π_{11}	0.42	(2.00)	π_{12}	-3.09	(1.42)
h_{11}	-2.60	(4.86)	h_{12}	9.69	(4.17)

$T = 3026$ $\quad J = 70.75$ $[pValue = 0.999]$

Table 3: Parameter estimates of the 2-factor conditionally heteroskedastic factor model of Doz and Renault (2006) (**DR**) using the data in Panel A. Same instruments as in Table 2.

	Factor loadings				Idiosyncratic shocks variance matrix									
	Factor 1		Factor 2											
	est.	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*
FTSE350	0.50	-	0.30	-	1.76	(1.68)								
Oil Commod	0.30	-	0.40	-	1.47	(2.30)	2.87	(3.33)						
Fin & Insur	0.48	(0.06)	0.36	(0.03)	1.98	(2.02)	1.62	(2.81)	2.73	(2.34)				
Chem & Food	0.31	(0.05)	0.23	(0.03)	1.28	(1.28)	1.07	(1.78)	1.44	(1.50)	1.32	(0.95)		
Serv. & other ind.	0.28	(0.04)	0.25	(0.02)	1.20	(1.39)	1.08	(1.97)	1.45	(1.72)	0.95	(1.09)	1.19	(1.23)

Factor 1			Factor 2		
γ_1	0.69	(0.82)	γ_2	0.86	(0.33)

$T = 3026$ $\quad J = 58.46$ $[pValue = 0.975]$

Notes: est.: Estimate; s.e.: Estimated GMM standard error × 10 (in parenthesis); s_i: Conditional 3rd moment model's intercept

Table 4: Parameter estimates of the DCC(1,1)-MV-(E)GARCH model of Engle (2002) using the data in Panel A. Each asset i's excess return is represented by a Gaussian EGARCH$(1,1)$ with conditional variance σ_{it}^2: $\ln \sigma_{it}^2 = \omega_i + \gamma_i \frac{Y_{it}}{\sigma_{it-1}} + \alpha_i \frac{|Y_{it}|}{\sigma_{it-1}} + \beta_i \ln \sigma_{it-1}^2$.

	Asset specific EGARCH$(1,1)$ parameter estimates							
	ω_i		γ_i		α_i		β_i	
	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.
FTSE350	-0.078	(0.158)	-0.106	(0.138)	0.096	(0.256)	0.987	(0.006)
Oil Commod	-0.093	(0.272)	-0.058	(0.155)	0.128	(0.537)	0.993	(0.009)
Fin & Insur	-0.089	(0.210)	-0.105	(0.150)	0.120	(0.369)	0.985	(0.008)
Chem & Food	-0.085	(0.291)	-0.098	(0.143)	0.104	(0.477)	0.986	(0.010)
Serv. & other ind.	-0.143	(0.630)	-0.095	(0.143)	0.172	(0.952)	0.975	(0.037)
	DCC parameters estimates							
	α		β		Log-Likelihood			
	0.036	(0.012)	0.954	(0.025)	-15386.0			

Notes: est.: Estimate; s.e.: Estimated standard error × 1000 (in parenthesis)

Figure 1: Excess return on FTSE 350, filtered-volatilities for FTSE 350 (from the 2-factor ACHF and DR models and the DCC-MV-(E)GARCH model), and filtered conditional leverage and skewness of FTSE 350 (from the 2-factor ACHF model) using Panel A.

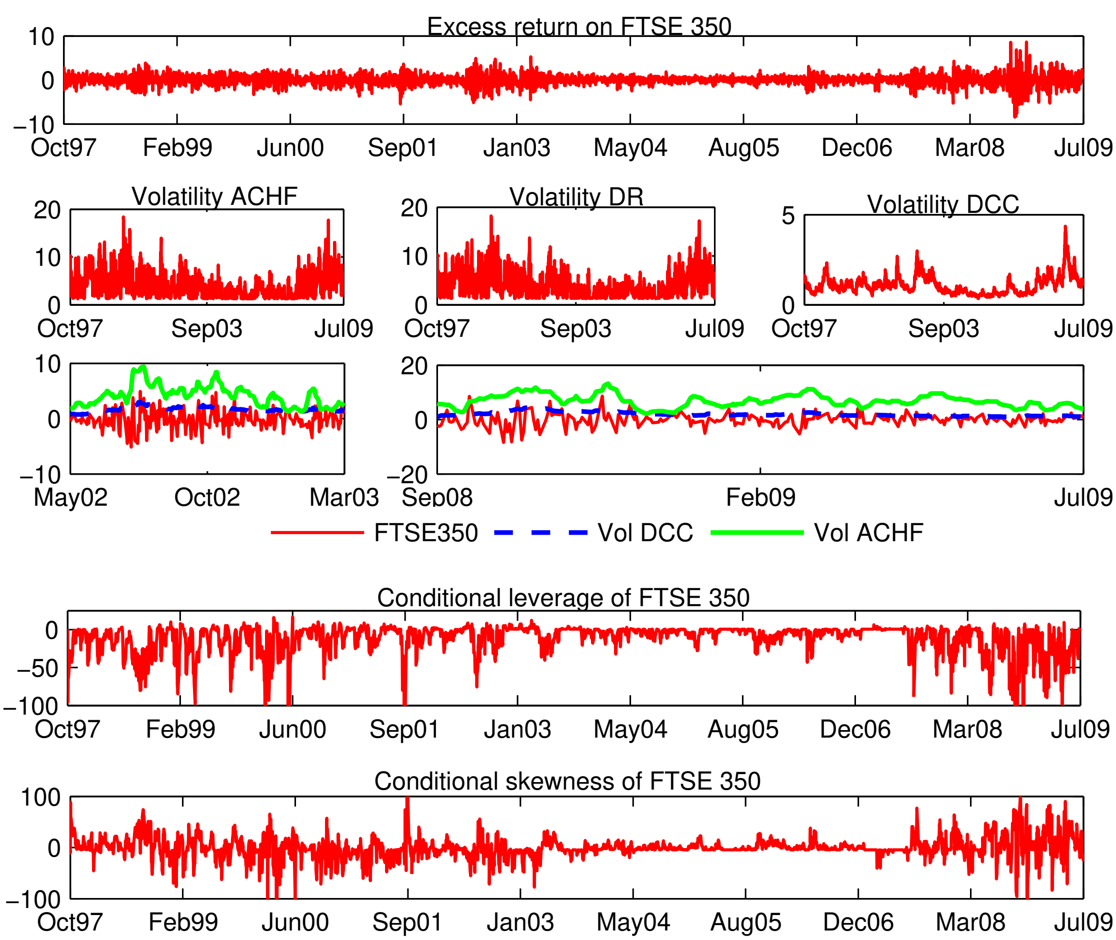

Table 5: Parameter estimates of the 2-factor conditionally heteroskedastic factor model with asymmetries (**ACHF**) using the data on weekly excess returns in **Panel B**. The vector of instruments used is $z_t = (1, \sum_{i=1}^{n} |Y_{it}|, \ldots, \sum_{i=1}^{n} |Y_{it-k}|)$, with $k = 2$; $n = 5$ is the number of assets in the model.

	Factor loadings				Idiosyncratic shocks variance matrix										s_i	
	Factor 1		Factor 2													
	est.	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*
FTSE350	0.50	-	0.30	-	2.00	(2.31)									-1.02	(6.93)
Oil Commod	0.30	-	0.40	-	1.40	(2.94)	4.13	(4.65)							1.67	(28.4)
Fin & Insur	0.49	(0.06)	0.36	(0.03)	2.38	(2.93)	1.37	(3.59)	3.73	(3.52)					-2.43	(18.2)
Chem & Food	0.31	(0.06)	0.22	(0.03)	1.48	(1.87)	0.99	(2.42)	1.67	(2.39)	1.98	(1.37)			-0.22	(4.08)
Serv. & other ind.	0.28	(0.06)	0.25	(0.03)	1.58	(2.11)	1.25	(2.68)	1.96	(2.86)	1.25	(1.87)	2.06	(2.16)	-0.73	(5.65)

Factor 1			Factor 2		
γ_1	0.03	(0.71)	γ_2	0.54	(0.25)
π_{01}	-1.99	(33.6)	π_{02}	10.10	(159.)
π_{11}	0.04	(5.45)	π_{12}	-0.65	(2.52)
h_{11}	-0.64	(5.64)	h_{12}	-1.14	(5.73)

$T = 811$ $\quad J = 24.71$ $[pValue = 0.999]$

Table 6: Parameter estimates of the 1-factor conditionally heteroskedastic factor model with asymmetries (**ACHF**) using the data on weekly excess returns in **Panel B**. The vector of instruments used is the same as in Table 5.

	Factor loadings		Idiosyncratic shocks variance matrix										s_i	
	est.	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*	*est.*	*s.e.*
FTSE350	0.50	-	1.93	(2.14)									-1.07	(6.14)
Oil Commod	0.48	(0.06)	1.31	(2.11)	4.08	(3.76)							0.85	(21.9)
Fin & Insur	0.48	(0.04)	2.34	(2.33)	1.38	(2.45)	3.74	(1.98)					-2.56	(15.7)
Chem & Food	0.30	(0.05)	1.43	(1.55)	0.99	(1.82)	1.67	(1.41)	1.92	(1.07)			-0.11	(3.50)
Serv. & other ind.	0.27	(0.04)	1.58	(1.52)	1.30	(1.58)	1.99	(1.31)	1.25	(1.06)	2.09	(1.25)	-0.71	(4.72)

γ_1	0.53	(0.21)	π_1	-0.77	(27.9)
π_0	0.95	(1.35)	h_1	-0.64	(2.57)

$T = 811$ $\quad J = 25.45$ $[pValue = 0.999]$

Notes: est.: Estimate; s.e.: Estimated GMM standard error × 10 (in parenthesis); s_i: Conditional 3rd moment model's intercept

Table 7: Parameter estimates of the 2-factor conditionally heteroskedastic factor model with asymmetries (**ACHF**) and the **DR** model using the data on daily excess returns in **Panel C**. The vector of instruments used is $z_t = (1, \sum_{i=1}^{n} |Y_{it}|, \ldots, \sum_{i=1}^{n} |Y_{it-k}|)$, with $k = 2$; $n = 23$ is the number of assets in the model.

	Factor loadings								Idiosyncratiques variances				s_i	
	ACHF				DR				ACHF		DR			
	Factor 1		Factor 2		Factor 1		Factor 2							
	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.	est.	s.e.
FTSE 350	0.50	-	0.30	-	0.50	-	0.30	-	0.21	(0.26)	0.22	(0.49)	-0.47	(3.62)
Leisure goods	0.30	-	0.40	-	0.30	-	0.40	-	0.15	(2.84)	0.12	(6.22)	-0.35	(16.5)
Banks	0.68	(0.22)	0.38	(0.15)	0.68	(0.91)	0.38	(0.65)	1.73	(3.15)	1.81	(6.73)	-0.44	(10.6)
Beverages	0.47	(0.17)	0.28	(0.10)	0.47	(0.64)	0.28	(0.41)	0.50	(1.99)	0.50	(4.18)	-0.42	(11.8)
Cnstr. & bldg. mats.	0.46	(0.19)	0.31	(0.12)	0.46	(0.71)	0.31	(0.47)	1.06	(1.52)	1.10	(2.77)	-0.44	(6.70)
Chemicals	0.50	(0.18)	0.30	(0.12)	0.50	(0.66)	0.30	(0.45)	1.15	(1.74)	1.19	(3.33)	-0.45	(10.5)
Eng. & machinery	0.50	(0.18)	0.32	(0.13)	0.50	(0.75)	0.32	(0.49)	1.22	(1.41)	1.27	(2.76)	-0.47	(6.31)
Food & drug retailers	0.41	(0.18)	0.24	(0.12)	0.41	(0.75)	0.24	(0.45)	0.79	(2.00)	0.81	(3.17)	-0.46	(6.34)
Food producers	0.41	(0.15)	0.25	(0.10)	0.41	(0.59)	0.25	(0.35)	0.94	(1.21)	0.97	(2.41)	-0.51	(5.49)
Non life insurance	0.51	(0.22)	0.30	(0.14)	0.51	(0.95)	0.30	(0.57)	2.11	(2.62)	2.19	(4.50)	-0.44	(13.4)
Life insurance	0.61	(0.22)	0.32	(0.18)	0.61	(0.86)	0.32	(0.66)	3.49	(4.11)	3.67	(6.27)	-0.49	(13.1)
Eqt invest. Inst.	0.47	(0.11)	0.29	(0.08)	0.47	(0.43)	0.29	(0.28)	0.86	(1.57)	0.93	(2.49)	-0.47	(3.73)
Travel & leisure	0.50	(0.16)	0.31	(0.10)	0.51	(0.62)	0.31	(0.39)	1.12	(1.31)	1.16	(2.14)	-0.44	(5.13)
Transport	0.40	(0.13)	0.26	(0.09)	0.40	(0.52)	0.26	(0.35)	0.26	(1.04)	0.26	(2.69)	-0.45	(4.52)
Gen. Financ.	0.57	(0.21)	0.34	(0.14)	0.57	(0.72)	0.34	(0.46)	1.71	(1.65)	1.78	(3.07)	-0.45	(8.11)
Personal goods	0.38	(0.22)	0.24	(0.14)	0.38	(0.79)	0.24	(0.52)	1.19	(3.04)	1.19	(6.39)	-0.36	(19.4)
Gen. industrials	0.49	(0.12)	0.30	(0.07)	0.49	(0.49)	0.30	(0.30)	0.94	(1.10)	0.97	(2.28)	-0.49	(5.77)
General retailers	0.48	(0.17)	0.30	(0.12)	0.48	(0.65)	0.30	(0.42)	0.92	(1.18)	0.95	(2.23)	-0.46	(5.38)
Oil and gas	0.54	(0.18)	0.30	(0.12)	0.54	(0.70)	0.30	(0.44)	1.67	(1.81)	1.71	(3.40)	-0.43	(8.41)
Forestry & paper	0.39	(0.34)	0.26	(0.25)	0.39	(1.21)	0.26	(0.81)	3.48	(5.70)	3.55	(10.5)	-0.31	(38.9)
Health	0.46	(0.17)	0.29	(0.13)	0.46	(0.69)	0.29	(0.53)	1.75	(3.08)	1.80	(6.90)	-0.50	(17.2)
Pharm. & biotec.	0.53	(0.21)	0.31	(0.14)	0.53	(0.75)	0.31	(0.46)	1.90	(2.76)	1.93	(4.75)	-0.49	(16.9)
Support services	0.46	(0.13)	0.29	(0.09)	0.46	(0.47)	0.29	(0.30)	0.32	(1.01)	0.33	(1.80)	-0.48	(5.19)

ACHF						DR		
γ_1	0.83	(1.60)	γ_2	0.84	(0.72)	γ_1	0.84	(6.37)
π_{01}	-0.51	(11.5)	π_{02}	0.09	(58.6)	γ_2	0.83	(2.77)
π_{11}	-0.43	(10.7)	π_{12}	-0.52	(6.92)			
h_{11}	-0.43	(25.9)	h_{12}	-0.47	(19.9)			

$T = 6025$ $\quad J = 142.22$ $[pValue = 0.999]$ $\qquad J = 134.70$ $[pValue = 0.999]$

Notes: est.: Estimate; s.e.: Estimated GMM standard error × 100 (in parenthesis); s_i: Conditional 3rd moment model's intercept

Table 8: Summary statistics of the filtered factors and correlations of the asset returns with their filtered systematic components.

	Model	2F-ACHF	2F-DR	2F-ACHF	2F-DR
	Dataset	Panel A	Panel A	Panel C	Panel C
Mean:	Factor 1	0.018	0.006	-0.010	-0.009
	Factor 2	-0.022	-0.019	0.009	0.008
Standard deviation:	Factor 1	1.943	1.904	1.323	1.831
	Factor 2	3.291	3.270	1.942	2.258
Skewness:	Factor 1	0.374	0.332	-0.252	-0.717
	Factor 2	0.093	-0.061	-1.522	0.894
Systematic component	Std.	1.058	1.056	0.830	0.859
of FTSE 350:	Skewness	-0.135	-0.075	-0.729	-0.884
	Corr. with FTSE 350	0.916	0.910	0.892	0.893
Systematic components	Aver. std.	0.965	0.965	0.811	0.837
of other assets:	Aver. skewness	-0.168	-0.128	-0.755	-0.832
	Aver. Corr. with assets	0.794	0.796	0.661	0.668

Table 9: (Validation) This table displays the largest absolute skewness of the estimated co-feature portfolios (portfolios that offset the conditional heteroskedasticity feature) along with the pvalues of the related significance tests. This test is obtained by applying the Δ-method to the relevant sample means. This table also displays the results of the test for joint conditional homoskedasticity in the co-feature portfolios. This is a GMM overidentification test for the moment condition $Vech\,(Var(r_{t+1}|J_t)) = cst$, where r_{t+1} is the vector of estimated co-feature portfolios. $z_t = (1, \sum_{i=1}^{n} r_{it}^2, \ldots, \sum_{i=1}^{n} r_{it-k}^2)$ is used as instruments with $k = 5$ for Panel A and $k = 4$ for Panel B. 'nF' and 'DoF' stand for 'n-factor' and 'degrees of freedom', respectively.

		Skewness		Multivariate volatility test		
Model	Dataset	Max. Abs. Skewness	Min pvalue	J-test stat.	DoF	Pvalue
2F-ACHF	Panel A	0.146	0.454	45.809	36	0.127
2F-DR	Panel A	0.146	0.456	45.829	36	0.126
2F-ACHF	Panel B	0.759	0.055	35.560	30	0.223
1F-ACHF	Panel B	0.764	0.128	29.211	30	0.507

Table 10: (Validation for large portfolios: Panel C) This table displays the 90th percentile of the absolute skewness of the 21 estimated co-feature portfolios as well as the 10th percentile of the pvalues of their significance tests. This latter is at about 0.10 for both models meaning that the evidence for unconditional skewness is rather weak in these portfolios. The 21 co-feature portfolios induce 210 pairs for which the moment conditions: $Vech\,(Var(r_{kl,t+1}|J_t)) = cst$; $k, l = 1, \ldots, 21$ where $r_{kl,t+1} = (r_{k,t+1}, r_{l,t+1})'$ is a pair of co-feature portfolios returns, have been each tested through the GMM overidentification test. Some percentiles of the distribution of the resulting 210 pvalues are displayed and the pairs of assets that reject the null of conditional homoskedasticity are listed.

		Skewness		Distribution of pvalues for bi-variate tests for homoskedasticity				
Model	Dataset	90-perc. abs. Skewness	10-perc pvalue	Percentiles 90th	10th	min	Rejection at 5% Pairs	Pvalue
2F-ACHF	Panel C	0.467	0.109	0.504	0.171	0.010	(13)-(20)	0.010
2F-DR	Panel C	0.599	0.090	0.361	0.101	4.5e-5	(13)-(20)	4.5e-5
							(17)-(20)	0.006
							(18)-(20)	0.020

(13): 'Travel & leisure'; (17): 'Gen. industrials'; (18): 'General retailers'; (20): 'Forestry & paper'