How Does Income Inequality Affect Market Outcomes in Vertically Differentiated Markets?

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Abstract

The distribution of consumer incomes is a key factor in determining the structure of a vertically differentiated industry when consumer's willingness to pay depends on his income. This paper computes the Shaked and Sutton (1982) model for a general specification of consumers' income distribution to investigate the effect of inequality on firms' entry, product quality, and pricing decisions. The main findings are that greater inequality in consumer incomes leads to the entry of more firms and results in more intense quality competition among the entrants. This is due to the elasticity of consumer demand for quality being higher in more inegalitarian economies. More intense quality competition among firms causes them to locate their products in higher ranges of the quality spectrum, closer to each other, decreasing the degree of product differentiation. Competition between more similar products tends to reduce their prices. However, when income inequality is very high, the top quality producer chooses to serve only the rich segment of the market, and the low price elasticity of demand of these consumers allows him to charge a higher price. The conclusion is that income inequality has important implications for the degree of product differentiation, price level, industry concentration, and consumer welfare.

Part I
Introduction

In this paper I study decisions of the firms operating in a vertically differentiated market. The products offered in such a market differ in quality. The consumers are perfectly informed of the products' characteristics and have the same ranking over the products, preferring higher quality products to inferior ones. Thus, if prices were the same, the consumers would all choose to buy the top quality good. In this type of market the demand for the products is directly affected by the properties of the consumers' income distribution. If the consumers have different incomes and thus, different willingness to pay for higher quality products, firms can profitably split the market by offering products differentiated in qualities at different prices. Therefore, in vertically differentiated markets, income inequality among consumers becomes a key factor in determining the product varieties offered by the firms.

The purpose of this paper is to study the effect of income inequality on market outcomes in vertically differentiated markets. The line of research linking income distribution of the consumers to the industry structure dates back to Gabszewicz and Thisse (1979), and has been cultivated by them (1980) as well as by Shaked and Sutton (1982, 1983, 1987). These authors demonstrate that the interplay of the industry cost structure and demand conditions, which are the outcome of the underlying income distribution, determine the degree of concentration and the maximum number of firms in vertically differentiated markets (Shaked and Sutton (1987)). They have almost nothing to say, though, about what kind of products these firms would be producing.

The paper most closely related to this one is Benassi, Chirco and Colombo (2006). These authors analyze the effect of income concentration on product differentiation and obtain solutions for quality and pricing decisions of duopolistic firms. To obtain analytical results they assume that consumer incomes are distributed with a trapezoid distribution, and that the market is not covered. In this paper I propose to further this research agenda by modifying the existing models to make them applicable for studying the effects of changes in the consumers' income distribution on the firms' entry decisions and the optimal choices of product attributes and prices for a more general specification of the income distribution function. I solve the model numerically to obtain the equilibrium number of firms in the market, the qualities they produce, and the prices they charge.

The most valuable insight from the present analysis is that income inequality among consumers affects the intensity of competition. The result that greater income inequality enables more firms to enter the industry with positive market shares dates back to Gabszewicz and Thisse (1979) and has been replicated in most of the works that followed. In this paper I am also able to demonstrate that income inequality impacts the degree of product differentiation in the market through its effect on the elasticity of consumer demand for product quality.

Greater inequality in consumer incomes results in more intense quality competition among firms. This is due to the elasticity of market demand for quality being higher in more inegalitarian economies. More intense quality competition among firms causes them to locate their products in higher ranges of the quality spectrum, closer to each other, decreasing the degree of product differentiation. Competition between more similar products tends to reduce their prices. However, when income inequality is very high, the top quality producer chooses to serve only the rich segment of the market, and the low price elasticity of demand of these consumers allows him to charge a higher price.

The theoretical tool used in this project is an extension of the Shaked and Sutton (1982) model. In their seminal paper the authors describe a model of monopolistic competition in a vertically differentiated industry. They use a three stage game to characterize industry equilibrium in which firms choose both the qualities of their products and their prices. The outputs of their model are the number of firms in the market, product qualities and prices, and the major input is the income distribution of the consumers. In order to be able to solve the model analytically, they make very specific assumptions about the distribution of consumer incomes (uniform on $[a, b]$, where $2a < b < 4a$). These assumptions enable the authors to obtain an analytical solution, but make the environment of the model unfit for studying the effects of changes in income distribution on market outcomes.

In this project I bypass the strict requirements for applicability of analytical tools by developing a computer code for solving the Shaked and Sutton (1982) model numerically for a more general and empirically relevant specification of income distribution. After describing the model in Part 2, I outline the solution method in Part 3. The discussion in this part also includes the issues of existence and uniqueness of equilibria. Part 4 of the paper gives the results of the model. Part 5 concludes with possible extensions and plans for future research.

Part II
The Model

The analysis here follows very closely that of Shaked and Sutton (1982). The economy is inhabited by two kinds of agents: consumers and firms. The firms produce distinct, substitute goods, that are differentiated by quality. Consumers are heterogeneous in income and have preferences over the goods produced by firms, with the ordering of preferences being identical for all consumers. They can choose to purchase only one good, basing their decision on the choice of qualities they face and prices, or make no purchase. These decisions generate demand functions for the firms, who face a more complicated oligopolistic competition problem.

Each of the firms produces only one good. They compete in a three-stage non-cooperative game. In the first stage each of the firms chooses whether it would enter the market. In the second stage, upon observing the number of entrants, firms that have entered the industry choose the specifications of their product, that is, its quality. In the last stage firms observe both the number and quality choices of their rivals and set their prices.

The game is solved using Subgame Perfect Nash Equilibrium concept, beginning at the last stage of the game and moving up the game tree.

Stage 3. Choosing Optimal Prices.

Denote the number of firms that have entered the industry in stage 1 of the game by N. These firms produce distinct, substitute goods. Each firm $k = 1, ..., N$ produces a good of quality k. Denote the quality level of firm k's product by u_k. These u_k's have been chosen at stage 2 of the game and at the current stage are common knowledge. Assume these qualities are ordered $u_0 < u_1 < ... < u_N \leq \bar{u}$, where u_0 is the quality of the outside good, and $\bar{u}$ is an exogenous upper bound on quality. The price of the outside good is $p_0 = 0$. Let $c_k = c(u_k)$ be the marginal cost incurred in production of good k. Each firm k is choosing the price of its product p_k.

The economy is inhabited by a continuum (measure one) of consumers identical in tastes but heterogeneous in income. Each consumer has income t which is distributed with a cdf F with support on $[0, \infty)$.Consumers purchase only one good or make no purchase and consume an outside good $k = 0$. For every consumer good k is characterized by the level of utility he/she obtains from consuming good k, which is assumed to be equal to u_k, and price of this good p_k. The preferences of consumer with income t from consuming good (u_k, p_k) are described by utility function[1]

$$U(t, (u_k, p_k)) = u_k \ (t - p_k). \tag{1}$$

Define the income level t_k such that a consumer with income t_k is indifferent between purchasing good k at price p_k and good $k-1$ at price p_{k-1}. That is,

$$U(t_k, (u_k, p_k)) = U(t_k, (u_{k-1}, p_{k-1})).$$

Let

$$I_k \equiv \frac{u_k}{u_k - u_{k-1}},$$

where $k = 1, ..., N$. Note that $I_k > 1$ for all k.

[1] This utility function is the same as in Shaked and Sutton (1982). The important assumption on preferences is that consumer's willingness to pay for quality is increasing in income. Other types of utility functions can be used to describe preferences without significantly altering the results of the model, as long as they satisfy this assumption. This claim still needs to be tested, though, and the robustness tests are in the plans for the future versions of the paper.

Then $t_1 = p_1 I_1$ and

$$t_k = p_{k-1}(1 - I_k) + p_k I_k. \tag{2}$$

In this stage of the game the firms simultaneously choose their prices so as to maximize their profits taking as given the prices of their rivals.

The profit of firm k is[2]

$\Pi_k = (p_k - c_k)[prob(t_k < t \leq t_{k+1})] - C_k$, where $C_k = C(u_k)$ is the fixed cost of producing quality k product.

Thus,

$$\Pi_k = (p_k - c_k)\ [F(t_{k+1}) - F(t_k)] - C_k \tag{3}$$

for $k = 1, ..., N - 1$. The profit of firm $k = N$ is

$\Pi_N = (p_N - c_N)\ [1 - F(t_N)] - C_N$.

Each firm $k = 1, ..., N$ solves $\max_{p_k \geq 0} \Pi_k$. The solution is the best response function (possibly, a correspondence) of firm k

$$p_k^{BR} = p_k(p_1, ..., p_{k-1}, p_{k+1}, ..., p_N; u_1, ..., u_N).$$

The system of these best response functions for $k = 1, ..., N$ forms a vector valued best response function. The Nash equilibrium of this game is the set of price functions $\left\{p_k^{NE}(u_1, ..., u_N)\right\}_{k=1,...,N}$ that is a fixed point of this vector valued best response function.

Stage 2. Choosing Optimal Qualities.

In this stage of the game firms observe the number of entrants N and simultaneously choose the quality of their own product u_k, $k = 1, ..., N$.

Each firm solves:

$$\max_{u_0 < u_k \leq \bar{u}} \left\{(p_k^{NE} - c_k)\ [F(t_{k+1}^{NE}) - F(t_k^{NE})] - C_k\right\} \tag{4}$$

where $p_k^{NE} = p_k^{NE}(u_1, ..., u_N)$ and $t_k^{NE} = p_{k-1}^{NE}(1 - I_k) + p_k^{NE} I_k$. Recall also that c_k and C_k are functions of u_k, and I_k depends on both u_k and u_{k-1}. The equilibrium of this stage of the game is a vector of qualities $(u_1^*, ..., u_N^*)$, where u_k^* is firm k's best response to $u_{-k}^* = \left(u_1^*, ..., u_{k-1}^*, u_{k+1}^*, ...u_N^*\right)$ for all $k = 1, ..., N$. Denote by Π_k^* the maximized value of profits of firm k, $k = 1, ..., N$, at $(u_1^*, ..., u_N^*)$. The equilibrium qualities and profits depend on the number of entrants in stage 1 of the game N .

Stage 1. Entry.

Denote by ε the entry cost for any firm k. If a firm chooses to enter this market it can expect to make $E_N\ [\Pi_k^*(N)]$. Thus, a firm will enter if

[2] There is a unit measure of consumers in this economy, thus, firm's per capita and total profits are the same.

$E_N\ [\Pi_k^*(N)] - \varepsilon \geq 0$. The number of firms in the market N^* is a Nash equilibrium if $\Pi_k^*(N^*+1) - \varepsilon < 0$ for some k. That is, the entry of an additional firm would lead to some firms making negative profits net of the entry cost.

In what follows the entry cost ε is assumed to be very small, so as to get the maximum possible number of entrants in the market. That is, N^* is considered to be an equilibrium number of firms if $\Pi_k^*(N^*+1) = 0$ for some k.

Part III
Solving the Model

In this section of the paper I discuss the computational algorithm and assumptions made in order to obtain the numerical solution of the model. In this paper the solution has been obtained for the specification of the model with no costs. Thus, for the rest of the paper I assume that $c_k = c(u_k) = 0$ and $C_k = C(u_k) = 0$ for all $k = 1, ..., N$.

3.1. Assumptions

3.1.1. Consumers' Income Distribution and Income Inequality

The consumers' income distribution is assumed to be lognormal[3] with cdf $F(\mu, \sigma)$. Since the purpose of the paper is to study the effect of income inequality on firms' decisions, parameters μ and σ are chosen so as to make the variance of the income distribution vary, while keeping the mean income constant. Denote the mean of the income distribution by A.

The standard measure of income inequality is the Gini coefficient. The Gini coefficient is a number between 0 and 1, with higher values corresponding to greater income inequality. According to the United Nations Development Programme's "Human Development Report 2006", it ranges from 0.19 in Azerbaijan to 0.74 in Namibia with an average of about 0.4 for the 126 countries in the report[4]. The Gini coefficient can be calculated for a given continuous cdf function as

[3] The lognormal distribution is often used to model the real world income distributions. The present computer code can be easily modified for another specification of the distribution function. It is important to keep in mind, though, that the choice of a different distribution function may affect the existence and uniqueness (or multiplicity) properties of the solution.

[4] According to the report, examples of countries with low income inequality include Denmark, Japan, and Sweden, all with Gini coefficients around 0.25 in 2006. In Europe Turkey has the highest measure of income inequality at 0.44. US has a Gini coefficient of 0.41, Canada - 0.33, and Mexico - 0.5.

$$G = 1 - \frac{1}{A}\int_0^{\infty} (1 - F(y))^2 \, dy,$$

For given μ and σ of the lognormal distribution the corresponding Gini coefficient can be computed using the formula above. The parameter σ is allowed to vary from 0.34 to 1.6. For each value of σ from this range, the value of the parameter μ is then chosen so as to keep the mean of the distribution constant at the chosen value for A. With these specifications the Gini coefficient varies from 0.19 to 0.74, which corresponds to the range observed in the data.

3.1.2. Parameters Choice

To compute the model numerically it remains to specify the values for the mean income A, the quality of the outside good u_0, and the upper bound on quality $\bar{u}$. Part 4 of the paper contains the results that have been obtained for $A = 15$, $u_0 = 1$, and $\bar{u} = 10$. The choice of these parameter values is arbitrary. Preliminary computations indicate that changes in the values of these parameters have quantitative implications, but do not affect the qualitative predictions of the model. More thorough tests are required, though, and are on the agenda.

3.2. Computational Algorithm

The issues of existence and uniqueness of equilibria for these types of models are typically not considered in the literature due to their extreme difficulty. Instead, the focus is on studying the characteristics which equilibria must have, if they exist. When looking for a numerical solution of the model, though, it is very important to know whether it exists and, if so, whether it is unique. The model here has multiple stages, and existence and uniqueness problems may arise at each of them. Unfortunately, due to the complexity of the problem the analytical proofs are not feasible for any part of the game. I turn to the numerical methods to verify existence and uniqueness or multiplicity of equilibria.

The computer code used to solve the model has been written with an explicit goal of making it possible to verify at any stage of the game that what is being found as a solution is in fact an equilibrium and, if so, whether there are other equilibria besides the one being computed. This requirement makes the computations more cumbersome and less efficient by necessitating that a different procedure be used for computing stages 2 and 3 of the model for each value of N . In the future I plan to rewrite the code relaxing this transparency requirement to make it more efficient, and testing the results against those obtained with the current version whenever in doubt.

The model is solved using Matlab software. The procedure is repeated for different values of μ and σ to study the effects of changes in income distribution function parameters on the model outcomes. For each value of N stages 2 and 3 of the model are written as functions. The stage 3 function takes as given the vector of qualities $(u_1, ..., u_N)$ and produces the vector of Nash equilibrium prices $\left(p_1^{NE}(u_1, ..., u_N), ..., p_N^{NE}(u_1, ..., u_N)\right)$. This function is called upon in the body of the stage 2 function, which, for a particular value of N, attempts to compute the Nash equilibrium qualities $(u_1^*, ..., u_N^*)$. If it finds that the equilibrium qualities converge to only one point, the one where all firms want to produce $\overline{u}$, it concludes that the Nash equilibrium with firms producing distinct qualities does not exist for a given number of entrants N [5]. The main body of the code then calls upon another stage 2 function, the one for smaller N, to see if there is an equilibrium with desired properties for a less crowded market. The procedure stops when it finds the maximum N for which there exists an equilibrium vector of qualities $(u_1^*, ..., u_N^*)$ with $u_i^* \neq u_j^*$ for all $i, j = 1, ..., N$, $i \neq j$ and u_i^*, $u_j^* \in \left(u_0, \overline{u}\right]$. This number of firms is the equilibrium N^*.

Here is a brief outline of the procedure:

I. Specify parameters μ and σ of the income distribution function.

II. Make a guess about the initial number of firms in the market N_0.

III. Call a stage 2 function for $N = N_0$ which seeks to find the Nash equilibrium qualities $(u_1^*, ..., u_N^*)$. This function uses the stage 3 function to compute Nash equilibrium prices for any distribution of firms' qualities.

IV. If this stage 2 function finds $(u_1^*, ..., u_N^*)$ with $u_i^* \neq u_j^*$, $i \neq j$ for all $i, j = 1, ..., N$, consider increasing N_0 to $N_0 = N_0 + 1$ to check that there does not exist a solution with desired properties for a greater number of firms in the market. That is, let $N_0 = N_0 + 1$ and go back to step **III**.

V. If the stage 2 function does not find $(u_1^*, ..., u_N^*)$ such that $u_i^* \neq u_j^*$, $i \neq j$ for all $i, j = 1, ..., N$, and instead concludes that the only solution is $u_k^* = \overline{u}$ for all $k = 1, ..., N$, then let $N_0 = N_0 - 1$.

VI. Call a stage 2 function for this smaller $N = N_0$.

VII. If the stage 2 function concludes that the only solution is $u_k^* = \overline{u}$ for all $k = 1, ..., N$, then let $N_0 = N_0 - 1$ and go back to step **VI**.

VIII. If the stage 2 function finds $(u_1^*, ..., u_N^*)$ with $u_i^* \neq u_j^*$, $i \neq j$ for all $i, j = 1, ..., N$, then call this number of firms the equilibrium N^* and the quality vector $(u_1^*, ..., u_N^*)$ the solution to the stage 2 of the game. Compute the Nash equilibrium prices for this vector of qualities using the stage 3 function.

[5] For $N > 2$ there is always an equilibrium with all firms producing the top quality $\overline{u}$. If two or more firms produce $\overline{u}$, the Bertrand competition at stage 3 ensures that all firms earn zero profits in equilibrium.

The optimal choice of initial N would be $N_0 = N^* + 1$, and depends on the parameters A, u_0, and $\bar{u}$. For their values specified above $N_0 = 4$ is sufficient.

Next I discuss the algorithms for computing stage 2 and 3 equilibria in greater detail, also addressing the issues of their existence and uniqueness.

3.2.1. Stage 3: Computing Optimal Prices

The input of the stage 3 function is a vector of firms' qualities $(u_1, ..., u_N)$. Each firm $k = 1, ..., N$ optimally chooses its price p_k so as to maximize its profit, taking the prices of other firms $p_{-k} = (p_1, ..., p_{k-1}, p_{k+1}, ..., p_N)$ as given. For a given vector p_{-k} this is a simple single-variable constrained optimization problem, and the best response function can be computed as $p_k^{BR} = p_k(p_{-k})$. Any point where the best response functions of all firms $k = 1, ...N$ intersect is a Nash equilibrium of this stage of the game. Visual tests conducted for different income distribution specifications, $N = 2, 3, 4$, and various combinations of qualities $(u_1, ..., u_N)$ lead to the conclusion that the point of intersection exists and is unique. I use the method of simple iterations on best response functions to find this unique Nash equilibrium. This method is the most simple and reliable. It can be slower than the alternative methods, but unreliability of other methods in this case prevents their meaningful use[6].

3.2.2. Stage 2: Computing Optimal Qualities

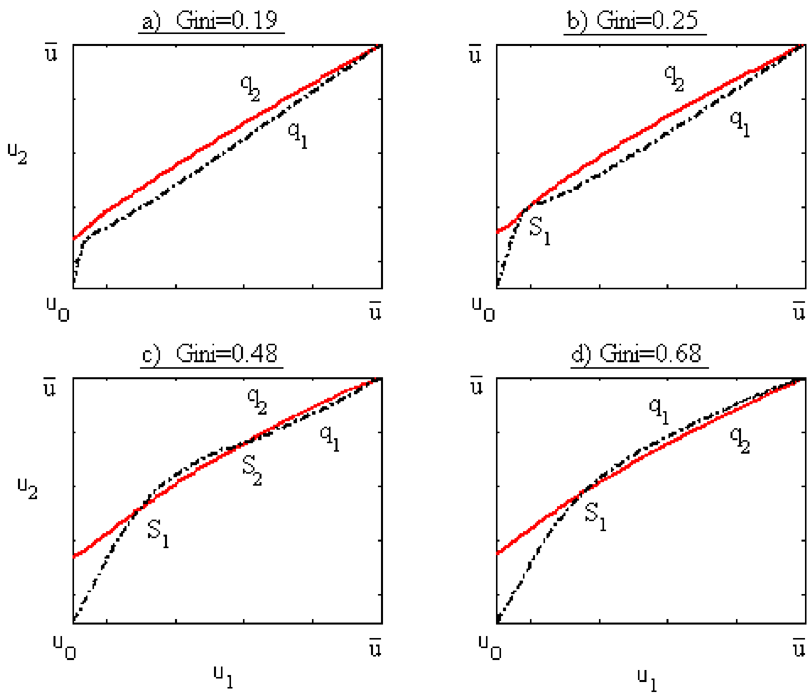
For a given N the stage 2 function searches for an equilibrium vector of qualities $(u_1^*, ..., u_N^*)$ such that no two elements of this vector are the same. Notice that each particular vector $(u_1^*, ..., u_N^*)$ corresponds to $N!$ equilibria in terms of the identities of the firms. To illustrate, suppose that two firms X and Y enter the market at stage 1. If there is an equilibrium with firm X producing u_1^* and firm Y producing u_2^*, then there is also an equilibrium with Y producing u_1^* and X producing u_2^*. For all purposes here these symmetric equilibria are considered to be identical and are treated as one equilibrium. Thus, when looking for equilibria with two firms producing distinct qualities I will assume that one of the firms is producing the lower quality good while the other one is making the higher quality one, and they both know their respective positions. For $N = 3$ the respective quality positions for each of the firms are "fixed" at low, middle, and high. There is a similar preassigned ordering for larger N.

In the model with no costs to producing higher quality the top quality firm's best response to any quality choices by its rivals is $u_N = \bar{u}$. For $N = 2$ the problem at this stage is a simple one of finding u_1 that maximizes firm 1's profit, taking as given $u_2 = \bar{u}$ and the price functions from stage 3 $\left(p_1^{NE}\left(u_1, \bar{u}\right),\ p_2^{NE}\left(u_1, \bar{u}\right)\right)$.

[6] An alternative solution method would involve solving the system of first-order conditions. The more efficient numerical methods for solving systems of nonlinear equations are based on replacing the problem with that of minimizing a functional. The surface of this functional turns out to have a very irregular shape due to the assumption of the lognormal probability density function. As a result, the solution obtained using these methods is very sensitive to the initial guess.

For $N = 3$ the procedure is looking for an intersection point of the quality best response functions for firms 1 and 2 when $u_3 = \bar{u}$ and the price functions from stage 3 are $\left(p_1^{NE}\left(u_1, u_2, \bar{u}\right),\ p_2^{NE}\left(u_1, u_2, \bar{u}\right),\ p_3^{NE}\left(u_1, u_2, \bar{u}\right)\right)$. Denote the quality best response function of firm 1 by $u_1 = q_1\left(u_2, \bar{u}\right)$ and that of firm 2 by $u_2 = q_2\left(u_1, \bar{u}\right)$. Below I plot four examples, each for an economy with a different value of the Gini coefficient, illustrating four possible situations for equilibria in this stage of the game.

Figure 1: Equilibria in stage 2 of the game, $N = 3$**.**



In Figure 1 a) the quality best response functions of firms 1 and 2 do not intersect in any point besides the one where they both produce $\bar{u}$. When this is the case, there does not exist an equilibrium with firms producing differentiated products. The best response functions in Figure 1 b) and d) coincide in one other point besides $\left(\bar{u}, \bar{u}\right)$, point S_1. The conclusion in these two cases is that the equilibrium with desired properties exists and is unique. In Figure 1 c) the best response functions intersect in two other points, S_1 and S_2, where $u_1, u_2 \neq \bar{u}$. Thus, potentially there are two solutions with firms producing distinct qualities. The equilibrium at S_2 cannot be computed by any procedure involving iterations, since it is non-stable. The code uses the simple iterations methods to compute the quality choices corresponding to S_1. The quality vector with thus chosen u_1 and u_2 is the solution for this stage of the game.

Similar graphs can be obtained for $N > 3$. Let $N = 4$ and denote the quality best response functions of the three lower quality firms by $u_1 = q_1 \left(u_2, u_3, \bar{u}\right)$, $u_2 = q_2 \left(u_1, u_3, \bar{u}\right)$, and $u_3 = q_3 \left(u_1, u_2, \bar{u}\right)$. For each $\left(u_3, \bar{u}\right)$ let u_1 be the solution to $u_1 = q_1 \left(q_2 \left(u_1, u_3, \bar{u}\right), u_3, \bar{u}\right)$ obtained by the method of simple iterations. Similarly, u_3 solves $u_3 = q_3 \left(u_1, q_2 \left(u_1, u_3, \bar{u}\right), \bar{u}\right)$ for every $\left(u_1, \bar{u}\right)$. Denote these solutions by $\hat{u}_1 = \hat{q}_1 \left(u_3, \bar{u}\right)$ and $\hat{u}_3 = \hat{q}_3 \left(u_1, \bar{u}\right)$. The problem becomes that of finding an intersection of functions $\hat{q}_1 \left(u_3, \bar{u}\right)$ and $\hat{q}_3 \left(u_1, \bar{u}\right)$, if it exists. This task is analogous to the one described for the case of $N = 3$ above. A similar procedure can be used for $N > 4$.

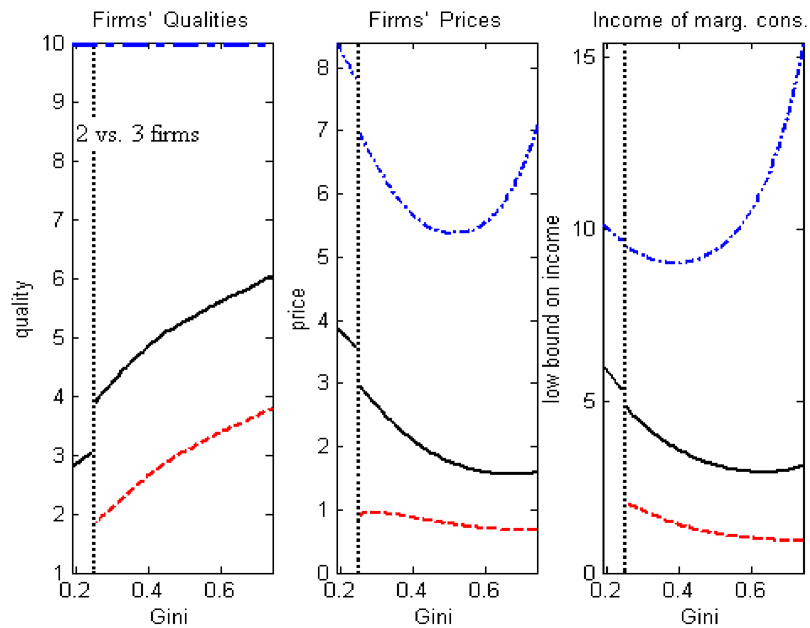
Part IV
Results: the Model with no Costs

The degree of income dispersion, measured by the Gini coefficient (G), determines the equilibrium number of firms in the market. In the economies with $G \leq 0.2492$ only two firms choose to enter the market, that is, $N^* = 2$. If another firm was to enter, the competition for the consumers with such small degree of heterogeneity would be so intense that it would drive the price of the top quality firm to zero, causing all firms to earn zero profits. Thus, in equilibrium only two firms in the market earn positive profits.

Economies with values of G above that threshold are inhabited by consumers whose incomes are distributed less equally. Greater degree of consumer heterogeneity gives the firms more "room" to compete. As a result, up to three firms can enter the market in these economies and earn positive profits and the equilibrium number of firms is $N^* = 3$. Thus, income inequality determines the number of firms that can coexist in a vertically differentiated industry with positive market shares, with more firms inhabiting the markets in less egalitarian economies.

In the model with no costs to producing higher quality the top quality firm always chooses to produce the highest possible quality $\bar{u} = 10$. The optimal quality choices of other firms depend on the degree of differentiation in consumers' incomes. Figures 2 a) and b) give, respectively, equilibrium quality and price decisions of firms as functions of the Gini coefficient G. The dotted vertical line is drawn at $G = 0.2492$ to separate the cases for $N^* = 2$ and $N^* = 3$. Figure 2 c) depicts the income levels of the marginal consumers, t_k's. Recall that a consumer with income t_k is indifferent between purchasing from firm k and $k - 1$. Thus, for example, in economies where three firms enter the market, the demand for the top quality firm is given by the fraction of population with incomes above t_3, the consumers with incomes between t_2 and t_3 buy from the second highest quality firm, those with incomes between t_1 and t_2 purchase the good of the lowest quality, and the rest choose not to buy and consume the outside good.

Figure 2: Results of the model with no costs.

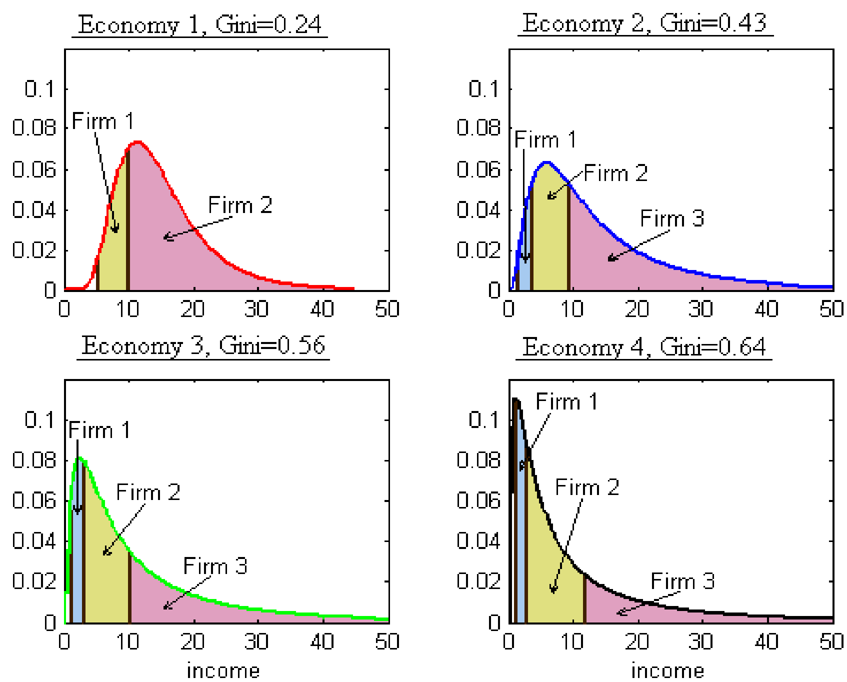


The optimal qualities are increasing functions of G, and equilibrium prices of firms producing the lowest and the second highest quality products are lower in the economies with higher levels of consumer income inequality. The price of the top quality product is decreasing at first, and then becomes an increasing function of G for the values of this parameter above some threshold value. To analyze these results of the model with no costs, consider four hypothetical economies, each characterized by a different value of the income inequality measure G.

Figure 3 below gives the consumer income distributions for each of these four economies. The economies are ordered by the degree of inequality in the consumer incomes, with Economy 1 inhabited by consumers with the most equal distribution of incomes. The vertical lines mark the income levels of the marginal consumers, t_k's, and the shaded areas of the graphs represent the demands for each of the firms or, equivalently, their market shares.

As G increases, the income distribution becomes more skewed to the right. The most prevalent type of consumer (the income distribution peaks at his income level) becomes increasingly more poor from Economy 1 to Economy 4, while the fraction of consumers with incomes in the middle range is rapidly shrinking. The probability density functions of consumer incomes in the economies with greater income inequality are characterized by thicker tails, which means that these economies also have more consumers with incomes above the mean.

Figure 3. Probability density functions of consumer incomes in Economies 1 through 4 and market shares of firms.



Consumers with higher incomes constitute the more attractive market for the firms, since for each level of quality wealthier consumers have higher willingness to pay. In the economies with a more egalitarian distribution of incomes the most attractive market for the firms is composed of the middle income consumers, since they are the most prevalent type. Low variability of incomes in this group results in small differences in willingness to pay for higher quality, and thus lower elasticity of demand for the top quality product. This allows the top quality producer to capture most of the market by pricing low enough to keep its inferior quality competitor serving the relatively more poor part of the population.

Intuitively, greater homogeneity of consumer incomes leads to more intense price competition in the last stage of the game. Its effects can only be mitigated through greater degree of product differentiation. If more than two firms were to enter in the Economy 1, they would not be able to locate far enough from each other in the quality spectrum in stage 2 of the game to sufficiently lessen the intensity of price competition in stage 3.

Greater income inequality increases the variability of incomes of the consumers in the more attractive, higher income part of the market, making the demand for quality more elastic. The second highest quality firm can now benefit by increasing the quality of its product without causing a knockout price competition in the last stage of the game. Thus the quality of the second highest quality good increases and the prices of two higher quality firms decline until the middle income market becomes too small for both of the firms to share, and the highest quality good producer "gives up" these middle class consumers to serve exclusively the rich.

Figure 2 a) shows that the quality of the second highest quality good is increasing in G, that is, in the degree of income inequality. In Figure 2 b) the price of this good is decreasing in G, while the price of its higher quality competitor is "U" - shaped. The equilibrium price of the top quality product begins to increase in economies with very high levels of income inequality because the consumers purchasing it are so wealthy that their demand is inelastic for higher values of prices. Figure 2 c) also shows that the marginal consumer of the top quality firm (t_3) is becoming increasingly richer after some value of G.

Additional results are demonstrated in Figures 4 and 5. Figures 4 a) and b) give market shares and profits of firms. Increases in income inequality induce the low quality firm to produce better quality product and charge lower price. Combined with the increase in the proportion of the relatively poor consumers in the population, this leads to greater market share and higher profits for the low quality firm. The market share and profits of the top quality firm decrease in the level of income inequality of the consumers. Greater inequality of incomes results in more intense quality competition between the two top quality producers, enabling the second highest quality firm to steal some business from its top quality competitor. The shrinking middle class eventually leads to the decline in the second highest quality producer's market share as well. The market shares of all firms get closer to each other in size as the income distribution becomes more unequal, causing the concentration to fall with greater degree of income inequality (Figure 5 a).

Figure 4: Additional results: market shares, profits, and market coverage

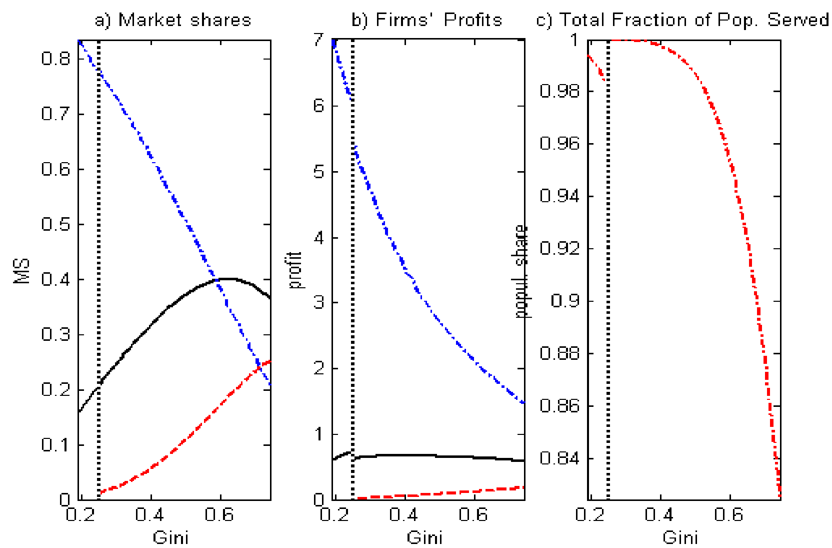


Figure 5: Additional results: concentration and consumer welfare

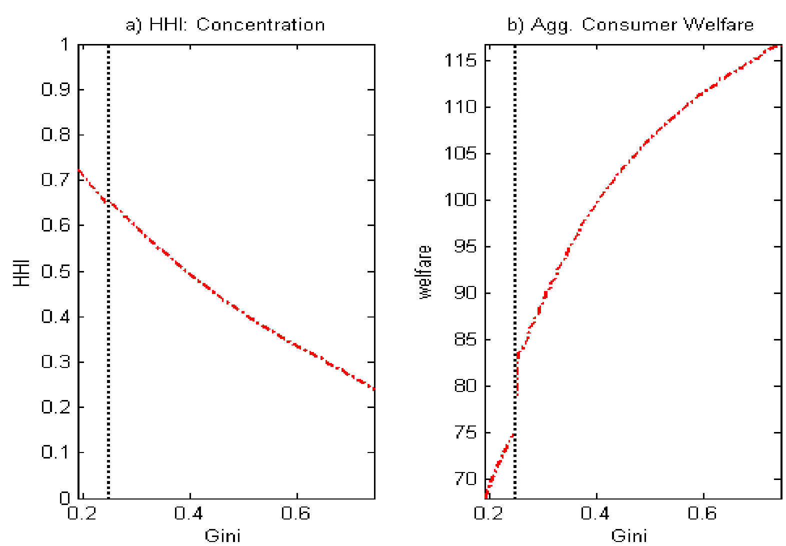


Figure 4 c) shows the total fraction of consumers in the market that choose to purchase from one of the firms as a function of the degree of inequality in incomes. Observe that at $G = 0.2492$, when an additional firm chooses to enter the market, the market is almost covered. Further increases in the income inequality measure are manifested in larger fraction of the consumers with low incomes, who cannot be induced to buy even the lowest quality good, notwithstanding its lower price and better quality. The consumers that do end up purchasing from one of the firms benefit from the more intense price and quality competition among the firms that accompany increases in income inequality. Thus, aggregate consumer welfare is higher in less egalitarian economies (Figure 5 b).

Part V

Conclusion and Further Plans

In this paper I study how income inequality among consumers affects the decisions of firms operating in vertically differentiated industries. The model used to address this question is due to Shaked and Sutton (1982). This model makes the following important assumptions: a) each consumer chooses at most one good out of a variety of products differentiated in quality; b) consumers have different incomes, and richer consumers are willing to pay more for better products; c) the products are supplied by firms that compete by choosing qualities and prices in a non-cooperative three-stage game, with each firm supplying only one type of quality; and d) there are no fixed or variable costs to producing higher quality products. In order to study the effects of changes in income inequality on model outcomes, I assume a lognormal distribution for consumer incomes and solve the model numerically, holding the mean of the distribution constant while changing the variance.

The results demonstrate that income inequality impacts the degree of product differentiation in the market through its effect on the elasticity of consumer demand for product quality. The industries in the economies with greater income inequality are characterized by a greater number of firms and more intense quality competition. This is due to the demand for quality being more elastic in the economies with less egalitarian distributions of incomes. In the model with no costs to producing higher quality the top quality product is always produced independently of the degree of consumer heterogeneity, but the qualities of other firms' products rise with the increases in income inequality. The more intense quality competition induces firms to locate their products in higher ranges of the quality spectrum and closer to each other. Thus, higher income inequality among consumers decreases the degree of product differentiation in the market. Lower heterogeneity of product qualities leads to more intense price competition, pushing down the prices of all firms in the market. However, in the economies where income inequality is very high, the top quality producer chooses to serve only the rich consumers; their demand is more price inelastic, which enables him to charge a higher price. Also, market shares and profits of all firms are distributed more equally in less egalitarian economies, and the consumers are better of in terms of aggregate welfare.

For the model described above, with all of the assumptions unaltered, it remains to properly investigate the robustness of results to changes in the values for the mean income A, the quality of the outside good u_0, and the upper bound on quality $\bar{u}$. Preliminary computations performed thus far indicate that the qualitative predictions of the model are not affected by different assumptions on these parameter values.

The robustness of the results to different specifications for the utility function will also be studied. Intuitively, the results should hold for any specification of the utility function as long as it has the property that willingness to pay for quality is increasing in consumer's income.

Another important assumption that needs to be relaxed is that of zero costs to producing higher quality. I intend to compute the model for nonzero cost functions $c(u)$ and $C(u)$. The results of the model with costs can then be tested empirically[7]. This is the subject of my next paper.

[7] Berry and Waldfogel (2003) empirically investigate how market size affects the level of top quality on offer and product concentration in vertically differentiated industries. One of the industries they study is the restaurants industry in the US. The data sources they use are rich enough to allow investigation of the effects of income inequality on the degree of product differentiation and price levels.

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