Convergence analysis: a new approach

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Economic growth and convergence is one of the most discussed fields in economics, as the long-run growth basically determines the welfare of countries. On the basis of neoclassical growth models, countries with lower GDP per capita will tend to grow faster than richer ones. However, convergence is not always confirmed. This means that economies are converging but the steady-state level is not always common, so countries may converge to different / own level of steady-states.

At the same time, the term 'convergence' can be interpreted by different ways. Therefore, multiple methods have to be applied to measure processes of convergence or divergence in a comprehensive way.

In this paper an indicator, called omega is presented in order to calculate convergence/divergence by a new approach. Omega is an adjusted weighted standard deviation of economic development (catching-up), which can be calculated on a single or multivariate basis.

The paper is organized as following. Section 1 briefly describes the definition and methodology of convergence. Section 2 outlines the model. In section 3 different types of convergence indicators are analysed and compared. Section 4 concludes.

Keywords: convergence, growth econometrics, growth theories

1. Introduction

Economic growth and convergence is one of the most discussed fields in economics, as the long-run growth basically determines the welfare of countries. Sometimes, it can be assumed that countries with lower GDP per capita tend to grow faster, than the richer ones. This process is called catching-up. However, the convergence process of lower income countries is not guaranteed. There are many factors leading to divergence: e.g. high level of net lending.

At the same time, the term 'convergence' can be interpreted in different ways, e.g.: catching-up to a reference value, decline of inequalities. Therefore, methods measuring convergence or divergence may also lead to different results. This also means that not only one but many indicators should be calculated in order to measure and analyze (e.g. determining time period for catching-up) convergence process in a comprehensive way.

2. Methodology

2.1. Definition of convergence

In economics the term convergence is used and defined in many approaches. On the basis of economic indicators several approaches exist: real convergence (e.g. GDP per capita), nominal convergence (e.g. interest rates) or structural convergence (e.g. agricultural employees). Maastricht criteria belong to the second group, while cohesion policy concentrates on the first one (Ferkelt and Gáspár, 2008).

In economics theory convergence can basically be interpreted in three ways:

- absolute convergence
- conditional convergence
- convergence clubs.

The term absolute or *unconditional convergence* (Barro and Sala-I-Martin, 2004) defines convergence process in a simple way. When economies are converging to the same level of steady state, unconditional convergence can be observed. It also means that disparities will diminish, as countries with lower income per capita are catching-up automatically.

However, there are many factors, which may lead to divergence. At the same time, countries with similar conditions, with same structural variables like savings, might converge to the same steady state. In this case *conditional convergence* (Sala-I-Martin, 1996a) can be confirmed. Countries with different structural variables will not automatically converge.

At the same times, initial conditions may also exist, which determine steady states. Economies, which have the same initial conditions, will only converge. In this case *convergence clubs* (Galor, 1996) exist.

The different types can also be mixed (Durlauf et al., 2004):

$$\lim_{t\to\infty} E\left(\log y_{i,t} - \log y_{j,t} \,|\rho_{i,0}, \theta_{i,0}, \rho_{j,0}, \theta_{j,0}\right) = 0, \text{ when } \theta_{i,t} = \theta_{j,t} (1)$$

Where

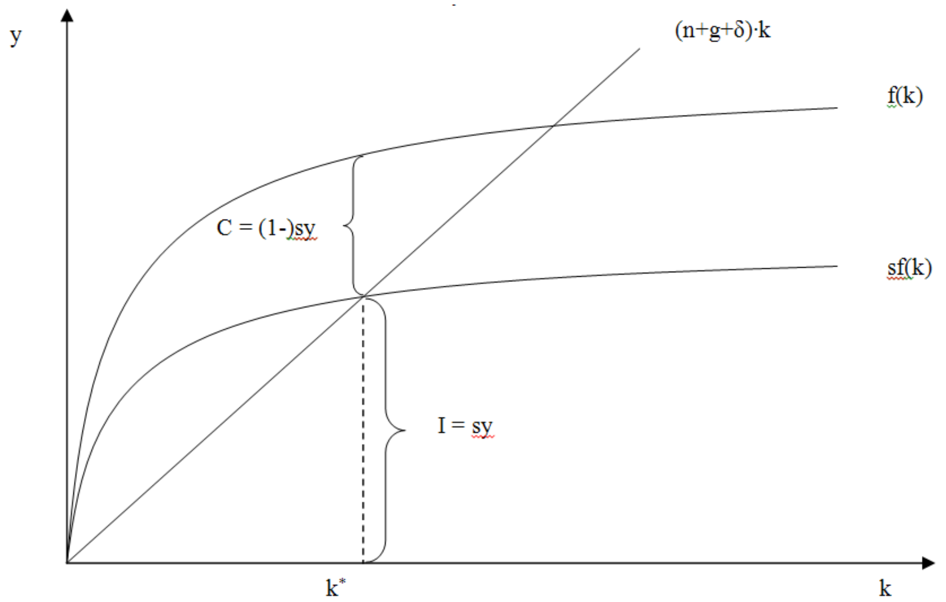
- i = observations
- t = time period
- y = measured variable (e.g. GDP per capita)
- ρ = initial conditions
- θ = structural variables

When equation (1) holds, convergence can be confirmed between observations when the initial conditions are also taken into account, however, the structural variables have to be identical.

2.2. Growth theories

Growth theories are connected to definitions of convergence, as well. On the basis of the Solow model (Solow, 1956) conditional convergence can be observed due to the diminishing return of capital:

Figure 1: Steady state in the Solow model



Source: Sorensen and Whitta-Jacobsen (2005)

Where:

- k = capital per effective labour
- y = income per effective labour (where y = kα)
- s = savings rate
- I = investment
- C = consumption
- n = growth of population
- g = growth of technical change
- δ = depreciation rate

This means that countries are converging to the steady state value of k* (and also for y*, as k basically defines y), where growth rate of k is zero: $k_{t+1} - k_t = 0$. The rate of convergence can be determined by linearization (Romer, 2006):

$$\dot{k} = \dot{k}(k) = sf(k) - (n + g + \delta) \cdot k$$

$$\dot{k}(k) \cong 0 + \left[\left. \frac{\partial \dot{k}(k)}{\partial k} \right|_{k=k^*} \right] \cdot \left(k_t - k^*\right)$$

Where:

$$-\lambda = \left[\left. \frac{\partial \dot{k}(k)}{\partial k} \right|_{k=k^*} \right]$$

And:

$$\dot{k} \cong -\lambda\left(k_t - k^*\right) \qquad (2)$$

And the speed of convergence:

$$-\lambda = \left.\frac{\partial \dot{k}(k)}{\partial k}\right|_{k=k^*} = -\left.\frac{\partial\left[sf(k)-(n+g+\delta)\cdot k\right]}{\partial k}\right|_{k=k^*} =$$

$$= -\left[sf'(k)-(n+g+\delta)\right] = \left[1-\alpha_K(k^*)\right]\cdot(n+g+\delta)$$

Where:

$$\frac{\alpha_K(k^*)}{1-\alpha_K(k^*)} = \frac{s}{y^*}\cdot\frac{\partial y^*}{\partial s} = \frac{k^* f'(k^*)/f(k^*)}{1-\left[k^* f'(k^*)/f(k^*)\right]}$$

This means that rate/speed of convergence (λ) is proportional to the difference from the steady state. Countries might converge fast, when they are far from steady state. This means that countries with lower GDP tend to grow faster, however, the convergence is conditional. Therefore, structural variables (savings rate, population growth, exogenous growth and depreciation) have to be identical, which is rarely the case, as developing countries usually have lower values of structural variables.

There are many extensions of the Solow model, e.g. by endogenizing the savings rate or capital mobility (Barro and Sala-I-Martin, 2004). However, in all of these models countries are growing at an exogenous rate of technical change in the steady state. This means the ‘true growth’ is not explained actually.

In endogenous growth models the technical change is endogenously determined (e.g. by human capital or social transfers) but usually no steady state exists, as the marginal product of capital is not diminishing (Sorensen and Whitta-Jacobsen, 2005). This also means that these theories lead usually to divergence.

2.3. Measuring convergence

On the basis of growth theories it cannot be decided, whether convergence or divergence can be observed, as the prior conditions basically determine not only the speed but the existence of divergence of convergence.

In empirical studies the results are quite mixed, they depend on the countries/regions/country groups, on data sources, on models and time period analysed. However, the hypothesis of convergence cannot be accepted worldwide (Durlauf et al., 2004).

It is therefore important to measure convergence process in a comprehensive way, and in the second stage growth models can be applied to analyse converge or divergence in detail.

Different indicators can be applied to measure convergence process on the basis of the definitions of convergence:

- distributional approach
- beta convergence
- times series approach.

Among *distributional indicators*, sigma convergence is widely used. Sigma stands for the standard deviation of log GDP per capita values. In case of convergence, sigma shows a negative trend in a time period. This also means that inequalities were diminishing.

In case of *beta convergence*, countries with lower GDP per tend to grow faster than richer ones. Both cross sectional and panel models can be applied, however, panel models will often provide more comprehensive results.

Time series analyses are mostly based of stochastic approaches like cointegration. If the differences of times series (or other liner combination) contains a unit root than we face an extreme type of divergence.

These methods may lead to different results, as they measure different types of convergence. Some of them are rather statistical (especially the times series approaches), while others are rather theoretical. This means that they might assume one steady state, or multiply ones (or none at all).

It can be argued that multiple indicators should be calculated as they have different characteristics and restrictions. Therefore, it may occur that some countries converge only in specific conditions and it is important to reveal these features. In the following section, an indicator is presented, which measures convergence process but may also deliver some new characteristics.

3. Omega

3.1. The baseline model

Omega is a modified, weighted standard deviation of cluster differences (Gáspár, 2010b):

$$\Omega = \sum_{j=1}^{n} \omega_j = \frac{\sum_{j=1}^{n}\sum_{i=1}^{n_j}(K_{C\,ji} - K_{B\,ji}) \cdot DEV(x_{jiB}, x_{jiT} \mid \alpha_{jiB}, \alpha_{jiT})}{\left| \sum_{j=1}^{n}\sum_{i=1}^{n} DEV(x_{jiB}, x_{jiT} \mid \alpha_{jiB}, \alpha_{jiT}) \right|} \qquad (3)$$

Where:

- K: cluster
- C: current period
- B: base period
- DEV: development
- x: real GDP per capita
- i: observation (country)
- j: group (country group)
- T: time period
- α: other (possible) weights.

The intention behind this indicator is to measure convergence as a catching-up process, while it also measures standard deviation. This means supplementary calculations (like how many years are needed for country X to catch up with Y) can be directly made and the model can be directly connected to growth models/theories. At the same time, it might be easy to build a multivariate statistical model in order to extend omega (e.g. it might help to identify growth factors).

In this paper only the main features of the indicator are presented. In the first stage, clusters are created from the values of GDP per capita from different countries. Clusters are needed in order to take into account major rates of economic growth only. From these clusters differences (between a base and a current period) omega is measured.

The less developed the countries are and the higher the rate of economic growth is, the more significant the convergence will be. However, cluster differences should be weighted, as significant economic growth of developed countries should be interpreted as divergence, while for developing countries this case should be measured as convergence. The weights (DEV) can be calculated by different ways, for example:

$$\mathrm{DEV}=\sum_{j=1}^{n}\sum_{i=1}^{n}\left[\left(\frac{f_{jiB}\cdot x_{jiB}+f_{jiC}\cdot x_{jiC}}{f_{jiB}+f_{jiC}}\right)-\left(\frac{f_B\cdot \overline{x}_B+f_C\cdot \overline{x}_C}{f_B+f_C}\right)\right] \quad (4)$$

where: f = number of population

$$DEV=\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\frac{f_{jiB}\cdot\left[x_{jiB}-me(x_B)\right]+f_{jiC}\cdot\left[x_{jiC}-me(x_C)\right]}{f_{jiB}+f_{jiC}}\right) \quad (5)$$

or:

$$\mathrm{DEV}=\alpha_{ji}\cdot\frac{1}{2}\cdot\sum_{j=1}^{n}\sum_{i=1}^{n}\left[x_{jiB}-me(x_B)\right]+\left[x_{jiC}-me(x_C)\right] \quad (6)$$

where: $\alpha_{ji}=\frac{f_{jiB}+f_{jiC}}{2}$

There are significant differences among these weights presented in (4) to (6). In (4) development is measured as a weighted mean of the base and current period of population and income (GDP per capita). However, the distribution of world income is asymmetric, so calculating the median (instead of the mean) in (5) and (6) might be a much better choice. In (6) the values are weighted directly, which might the best choice, as the number of population is taken into account directly.

3.2. Restrictions and assumptions

There are some restrictions, which should be taken into account:

- *Significant differences*:

If: $(K_{C ji}-K_{B ji})$ = 0, for all ij (7)

Than: ω = 0

- *Optimal number of clusters (e.g. dendogram should be taken into account)*
- *Assumption of computability*:

$$\mathrm{DEV}(x_{jiB}|\alpha_{jiB})\neq\mathrm{DEV}(\tilde{x}_B|\alpha_{jiB}) \text{ and } \mathrm{DEV}(x_{jiT}|\alpha_{jiT})\neq\mathrm{DEV}(\tilde{x}_T|\alpha_{jiT}) \quad (8)$$

where: for all $x_{jiB},\alpha_{jiB},\tilde{x}_B,\alpha_{jiT},x_{jiT},\tilde{x}_T\geq 0$; $\tilde{x}$ = level of development (e.g. median).

- Aggregation:

$$\Omega = \sum_{j=1}^{n} \omega_j \tag{9}$$

(7) means that omega equals to zero for a certain country, when the economy is clustered into the same group in the base and in the current period. This also means that the country's growth was not significant (Gáspár, 2010a).

Omega depends on the number of clusters, therefore, these values should be optimal.

For all countries it should be decided, whether they are developing or developed ones (8). If that is not the case, omega cannot be calculated for that specific country (which might happen very rarely). However, if this is the case, more or less data (countries) should be used.

Omega might also be calculated for country groups and the main indicator can be created (9) by weighting.

4. Data analysis

4.1. Empirical analysis

In my previous studies (Gáspár, 2010a, b) significant differences was found among omega and other convergence indicators. In least developed and rest of world countries (mostly non-developed) convergence, while in OECD countries divergence was found. The speed of convergence is very slow; more than 100 years are needed for significant catching up. However, the rate of divergence of OECD members is much stronger than the rate of convergence in least developed countries. This means that on the basis of omega only partial convergence can usually be measured.

The sign of the convergence process differs often among indicators. This is due to fact that they might be biased, e.g. cross sectional regression might be biased because of endogenity. It is also important to note again that the term convergence also differs substantially. In many countries I found divergence on the basis of sigma, while on the basis of omega convergence was estimated at the same time. This is not surprising. Several developing countries had high growth, while others had only slow ones, which also means that the country group as a whole showed divergence as inequalities increased. However, on the basis of omega this may be interpreted as convergence, as some of the countries caught up, which had high weights (low income in the base/current period and/or high number of population), while others had only slow growth rates but had low weights (high income in the base period/current and/or low number of population). This might be an important feature of omega, as it is a standard deviation-type indicator while it also measures catching-up. Beta convergence might lead to the same conclusions but regressions especially cross sectional ones are often biased in growth equations. For that reason distributional approaches might provide better results but they cannot be conditioned easily. But as omega measures catching up it can be extended in a simple way in order to measure conditional convergence.

4.2. Monte Carlo analysis

In order to compare omega and other convergence indicators a more comprehensive analysis should be made. Statistical indicators can be compared in different ways. In inferential statistics estimators have to fulfil different conditions: functions of unbiasedness, robustness, consistency and efficiency. However, the term convergence can be interpreted in different ways, therefore, the variable which should be estimated from the sample is not a fixed one.

For that reason Monte Carlo simulation is applied. Two samples are generated: a base and a current period sample on the basis of lognormal distribution (as incomes likely follow lognormal distribution). The sampling size is 200 for both periods, as approximately 200 economies exist in the world. The only difference between the two samples are the variance and the expected value, which are estimated from the sample mean and variance of real GDP per capita values from the Penn World Table for 1970 and 2007. The number of simulations amounts to 10000.

It is assumed that the samples are independent, which of course is a strong assumption, as the macroeconomic variables are usually autocorrelated. However, the aim of this simulation is to 'purely'/mathematically compare convergence indicators. This means that indicators might be better compared when autocorrelation is not taken into account. For the same reason, weights are not used and 10 clusters are applied for all simulations.

Table 1 illustrates in how many cases the indicators showed same results (convergence or divergence):

Table 1: Monte Carlo Simulation (%)

Indicator	Sigma	Beta	Omega
Sigma	100 (0)		
Beta	68 (1,416)	100 (0)	
Omega	50 (1,427)	50 (1,428)	100 (0)

Source: Author's calculations based on data of Penn World Table v6.3 (2010).

The table is – of course – symmetric. Out of 10000 simulations, in 6800 cases had sigma and beta the same results. Both indicators measure convergence but sigma measures inequalities, while beta calculates catching-up. It may happen that developing countries converge so dynamic that they become the developed ones, while inequalities remain the same. In this case these two indicators lead to completely different results (Sala-I-Martin, 1996b).

At the same time, only 50% of omega-convergence values had the same sign as sigma and beta. This means that omega may include important characteristic of the convergence process. Of course, a very low rate would also raise lot of questions, too.

However, only by the sign of convergence or divergence the indicators cannot be compared in a comprehensive way. It may occur that indicators have the same sign but the speed of convergence or divergence is completely different. For that reason, the values are normalized and the standard deviations are calculated from the data (which are shown in brackets).

Omega differs from beta and sigma in a stronger way that beta differs from sigma. But the differences of the standard deviations are not substantial.

5. Conclusions

The term convergence can be defined in lot of ways and for that reason convergence indicators may also lead to different results. In this paper a new indicator of convergence analysis is briefly presented and analysed. Omega is a modified, weighted standard deviation of cluster differences generated from GDP per capita values.

On the basis of a Monte Carlo simulation it was found that the indicator differs substantially from other types of convergence indicators. This could mean that some characteristics may exist in data which might include important information for the convergence process.

At the same time, differences of the standard deviations of the convergence indicators were not substantial. This can be interpreted as a feature similar to consistency. Different indicators may lead to different results. However, it is expected that in specific cases (e.g. substantial economic growth) they have similar results (if no special circumstances exist, like substantial country weights), which means that they should produce similar standard deviations. In this sense the indicator should be interpreted as 'consistent'.

However, all of the indicators contain important information. Therefore, for convergence analysis not only one but many indicators have to be calculated and compared. In the next stage growth or multivariate statistical models can also be built in order to detect and analyse growth determinants or catching-up rates in detail. Omega can be extended easily, as it measures catching-up, which is a common basis of comparison for growth models.

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